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Multi-Hole Waveguide Directional Couplers

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Additional information is available at the end of the chapter

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1. Introduction

The directional couplers are inherently assumed as four-port devices, which consisted of two transmission lines that are electromagnetically coupled to each other. The first port is named as input, and the second one as output or transmitted, the third one as sampling or coupled and the fourth one as isolated or terminated. By using a special design the input power is divided between output and coupled port in a certain ratio named coupling factor. The required value for coupling factor P_1/P_3 defines the range of applications for directional couplers. Based on the application, coupling factor could be any value like 3, 6, 10, 20 dB and even more. The performance of the directional coupler is usually evaluated by its directivity between port 3 and 4. The directivity is a calculated parameter from isolation and coupling factor and shows how the two components of wave cancel each other at port 4. Though we prefer to have high value for directivity as much as possible, but in real situation this could be happened only around center frequency of designing band. The waveguide directional couplers have a good directivity compared to microstrip or stripline couplers and in spite of their bulky size, give us a low loss, high power handling, good characteristics and low cost due to use of just a simple waveguide.

Nowadays the numerical methods are widely used for simulation and optimization of high frequency structures. Some of them such as HFSS and FEKO, are well commercialized and used widely by researchers and engineers.. But for designing procedure and for starting point we need an initialization value to input into simulator and then optimize the parameters by its internal routines.

In this chapter we focus on the waveguide directional couplers and we try to give a good reference as well as finalized designing formulas in closed form and tables to be used individually or as initial values for numerical software. The full generalized field theory and its

equations for designing based on multi-hole coupling structure will be introduced, though more detailed basic information could be found in given references.

Moreover, by solving the equations, the recursive procedure is employed in a computer program to adjust the required directivity, coupling and frequency or waveguide type to specify the number of coupling holes, individual diameters and locations of holes in waveguide's coordinate. Besides of those parameters, the length of coupler, matched load and other sizes of structure for fabricating, will be defined too. By using the different methods like binomial, Chebyshev, and super imposed to calculate the coupling of each hole, the wide bandwidth response is achieved. At the end, a number of books and papers are given as good references for further study.
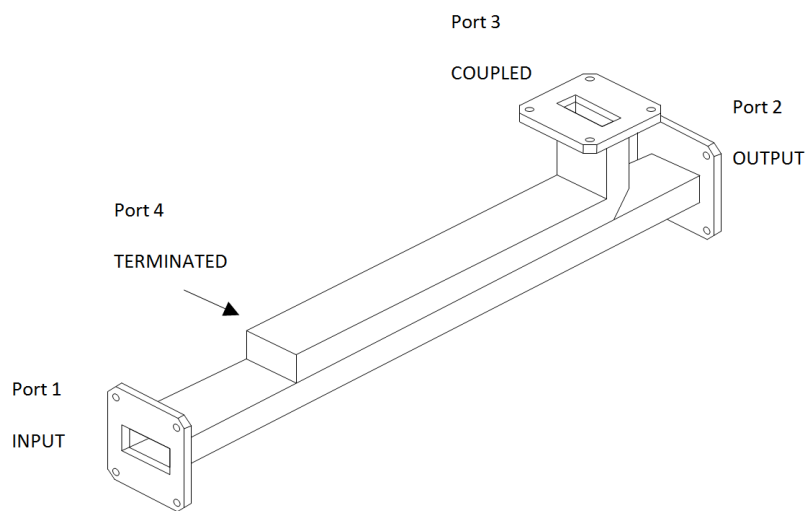

Figure 1. An Ordinary Broad-Wall Waveguide Directional Coupler and its ports

1.1. Definitions

As mentioned, couplers are considered as 4-port passive devices in which, a part of input wave reaches to output port 2 and the remained would be coupled to the coupled port 3. Port 4 usually internally is matched to damp the residual internally reflected waves from port 2 and port 3. Ideally there is no wave reach port 4. Port 4 usually is terminated by a full band load as shown in Figure1 and Figure2.

In waveguide couplers, the coupling method is done by putting a waveguide on top of another one and by making some aperture holes in their common wall a determined portion of wave would be leaked into the other waveguide. Though the waveguides axis and coupling apertures can be chosen arbitrary [2, 3], but for adequate specific usage and for easy derivation of design equations, we consider that two waveguide's are lay exactly on each other. Here the broad-wall coupling configuration is more interested and concentrated. Though, for side-wall the derivation of the design equations are so simple but the bandwidth is limited in spite of higher power handling.
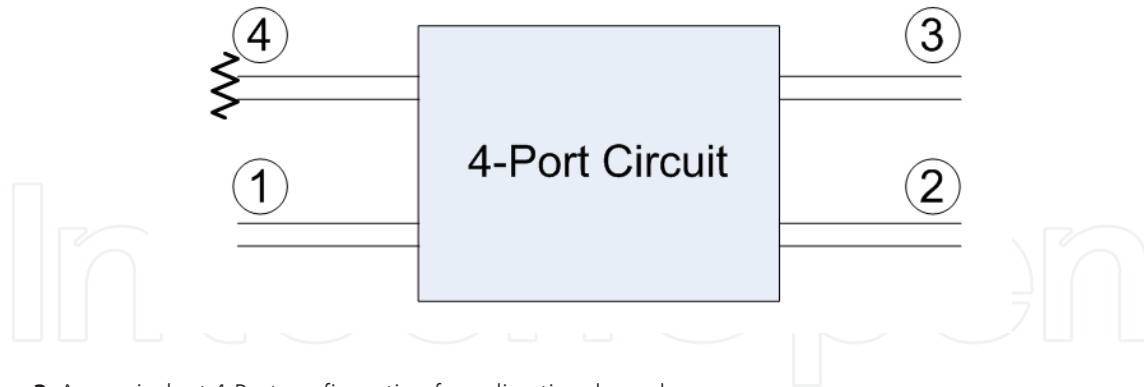
Figure 2. An equivalent 4-Port configuration for a directional coupler

Since 1945, extensive studies have been conducted on the issue and many researchers have tried to optimize the designing equations to make the result more accurate. For instance, different slot shapes had been introduced to increase the bandwidth and modifying the specifications. [12, 25, 27]

In order to start the calculations, there are three major parameters that we need to define for each coupler:

- Coupling factor "C" in dB, that represents the power received by port 3 as:

$$C = 10\log \frac{P_i}{P_f} \quad \text{or} \quad C = -10\log \frac{P_f}{P_i} \tag{1}$$

  The coupling factor shows the ratio of power at port 3 to input power at port 1. Typically we prefer to have one of the 3, 6, 10, 20 or 30 dB as standard values but for specific application it also can be defined freely.

- Directivity "D", is the ratio of output power at port 3 to received power at port 4. Since we prefer to eliminate the power at port 4, therefore the high values for "D" is more interested. The "D" in terms of dB is defined as:

$$D = 10\log \frac{P_f}{P_b} \quad \text{or} \quad D = -10\log \frac{P_b}{P_f} \tag{2}$$

- Bandwidth "BW", which depends on directivity. By increasing the number of coupling apertures, the order of coupler increases (similar to the order of filter) hence the directivity is increased. Meanwhile, higher bandwidth is also achieved. Therefore, by choosing the required minimum directivity, D_{min}, the available bandwidth is calculated.

For a 10 dB coupling or having a 0.1 of input power to port 3, we would have:

$$10\log \frac{P_f}{P_b} = 10 \quad \rightarrow \quad P_f = \frac{1}{10} P_i \tag{3}$$

In the same way, for a 3dB coupling, half of input power will receive to port 3:

$$10\log \frac{P_i}{P_f} = 3 \quad \rightarrow \quad P_f = \frac{1}{2}P_i \tag{4}$$

And if we consider $D = 40$ dB for directivity:

$$10\log \frac{P_f}{P_b} = 40 \quad \rightarrow \quad P_f = 10000 P_b \tag{5}$$

It is the adequate value for designing a good directional coupler.

A number of references, which have studied the couplers and have given the relationship between number of aperture holes "n" and directivity "D" are listed in references [12, 25, 23]. In addition to number of aperture holes"n", in the designing procedure for directional couplers, certain parameters should be well defined as:
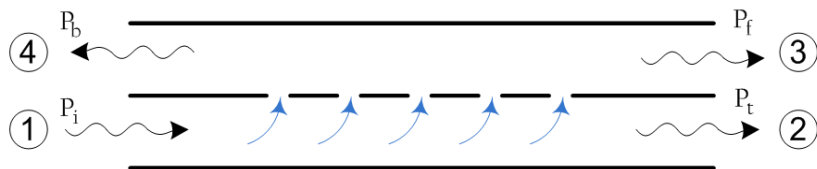
• Distances between the holes

• Distances between holes to side-wall (holes center offset from waveguide axis)

• The holes dimensions (diameter of holes for circular holes).

It has been shown that to have an optimum coupling around a certain frequency, the criteria (6) should be kept in which "x" is the distance between the holes centers to the side-wall and "a" is the broad wall size of waveguide: [24]

$$\frac{x}{a} \leq 0.25 \tag{6}$$

Furthermore, by precise study, the best design value for ratio of (6) is given as [23]

$$\frac{x}{a} = 0.203 \tag{7}$$

Figure 3. Cross section of the directional coupler and coupling holes

The distances between holes should be about $\lambda_g/4$ however a question remains, what is the proper value of λ_g when the bandwidth is limited to the λ_{g_1} to λ_{g_2} interval? To answer the question, there are three definitions used for λ_g:

1. λ_g is the average of wavelength of lower band λ_{g1} and upper band λ_{g2} so:

$$\lambda_g = \frac{\lambda_{g1} + \lambda_{g2}}{2} \quad \rightarrow \quad \frac{\lambda_g}{4} = \frac{1}{8}\left(\lambda_{g1} + \lambda_{g2}\right) \tag{8}$$

2. λ_g can be considered as geometric mean between λ_{g1} and λ_{g2}:

$$\lambda_g = \sqrt{\lambda_{g1}.\lambda_{g2}} \rightarrow \frac{\lambda_g}{4} = \frac{1}{4}\sqrt{\lambda_{g1}.\lambda_{g2}} \tag{9}$$

3. λ_g can be considered as mean value between λ_{g1} and λ_{g2}:

$$\frac{2}{\lambda_g} = \frac{1}{\lambda_{g1}} + \frac{1}{\lambda_{g2}} \quad \rightarrow \lambda_g = \frac{2\lambda_{g1}\lambda_{g2}}{\lambda_{g1} + \lambda_{g2}} \quad \rightarrow \quad \frac{\lambda_g}{4} = \frac{\lambda_{g1}\lambda_{g2}}{2(\lambda_{g1} + \lambda_{g2})} \tag{10}$$

The best choice for defining the centers of two holes is the 3rd definition since it has been practically approved too [23]. So the wavelength would be derived from (10). Therefore, in order to define the dimensions of each hole (or diameter in case of circular hole type), the each hole's coupling should be calculated first, and the hole's diameter would be derived consequently.

1.2. Fields Equations

In order to calculate the coupling of each hole and by using the required D_{min} that we need here, the number of holes "n" will be derived in two different ways:

i- The coupling coefficient mapped to coefficients of nth order of Chebyshev polynomial.

ii- The coupling coefficient mapped to coefficients of nth order of Binomial polynomial.

By assuming the same order for polynomials "*i*" and "*ii*" and by noticing that the directivity slope in case of "*i*" is higher, we expect to have higher bandwidth in comparizon to "*ii*" and limited ripple in pass-band. In case of "*ii*" though there is no ripple in pass-band but the slope of directivity is lower than "*i*" with same order of polynomial, therefore the band-width is lower than "*i*". For years many of manufacturers chose the "*i*" method and considering the number of holes n = 20. Here, the "*i*" method is chosen, however the number of holes "n" would be defined from D_{min} and it will be not fixed anymore.

In fabricating the couplers, any arbitrary shape for holes can be used but the circular; elliptic and rounded-edge rectangle has been widely studied, simulated and used in research reports. [30] Here, the circular holes have been adopted. The circular holes can be aligned in one, two or three parallel rows, but in our case, 2-rows are used. In order to calculate the coupling coefficients and related field equations the "Bethe's small-hole coupling theory" is used as the main computational method. Further, by using a correcting function, the theory is expanded to use big-size holes as well [12, 26, 27]. In that way, Levi's work would be followed to find the effect of wall-thickness "t" and also the relationship between variations of directivity "D" and coupling error "ΔC". [27]. Levi showed that if "D" increases, "ΔC" will decrease.

In special case, if we require high directivity "D", like "D = 50 dB" for small bandwidth like 8.9 to 9 GHz, only 2 holes are needed to synthesis the coupler.

For calculating the distance of holes' centers to side-wall "x", the equation $\sin\frac{\pi x}{a} = \frac{\lambda_0}{\sqrt{6a}}$ is used in which "a" is broad-side of waveguide and λ_0 is the wavelength in the middle of the band. [2]

The coupled wave equations for incident wave A_1 and reflected B_1 by assuming the same amplitude for waves are as follows:

$$A_1 = j\frac{2\pi}{ab\lambda_g}[M_x H_x^{(1)} H_x^{(2)} + M_z H_z^{(1)} H_z^{(2)} - P E_y^{(1)} E_y^{(2)}] \tag{11}$$

$$B_1 = j\frac{2\pi}{ab\lambda_g}[-M_x H_x^{(1)} H_x^{(2)} + M_z H_z^{(1)} H_z^{(2)} - P E_y^{(1)} E_y^{(2)}] \tag{12}$$

In which, "a" and "b" are the waveguide dimensions, M_x and M_z are the magnetic polarization components in "x" and "z" axis and P is electrical polarization. The $H_x^{(1)}$ is the amplitude's wave component in the first waveguide and $H_x^{(2)}$ is for second and so on. If two waveguides are identical then $H_x^{(1)} = H_x^{(2)}$ and $H_z^{(1)} = H_z^{(2)}$. Then the fields' components are expressed as:

$$H_x = -\operatorname{Sin}\frac{\pi x}{a} e^{-j\gamma z} \tag{13}$$

$$H_z = j\frac{\lambda_g}{2a}\operatorname{Cos}\frac{\pi x}{a} e^{-j\gamma z} \tag{14}$$

$$E_y = \frac{\lambda_g}{\lambda}\operatorname{Sin}\frac{\pi x}{a} e^{-j\gamma z} \tag{15}$$

The field equations are given separately for "Narrow wall" and "Broad wall" cases. Here, we briefly introduce them and give the relations for our interested one (i.e., Broad wall):
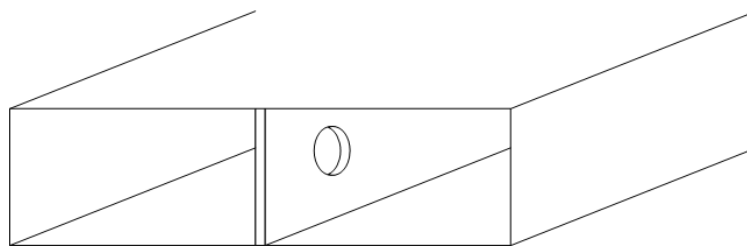
1.2.1. Narrow wall

By referring to Figure 5, since $x = 0$, the equations could be simplified as:

$$A_1 = A_2 = -\frac{j\lambda_g M_z}{2a^3 b} \tag{16}$$

In which, M_z is independent from frequency. In other words, "Narrow wall" coupling has significant difference comparing to "Broad wall" coupling.
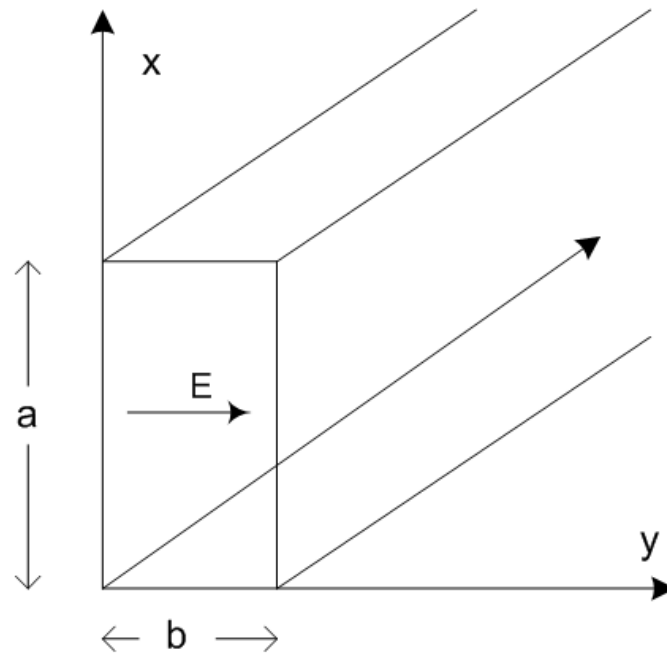
Figure 4. The geometry of field equations and waveguide
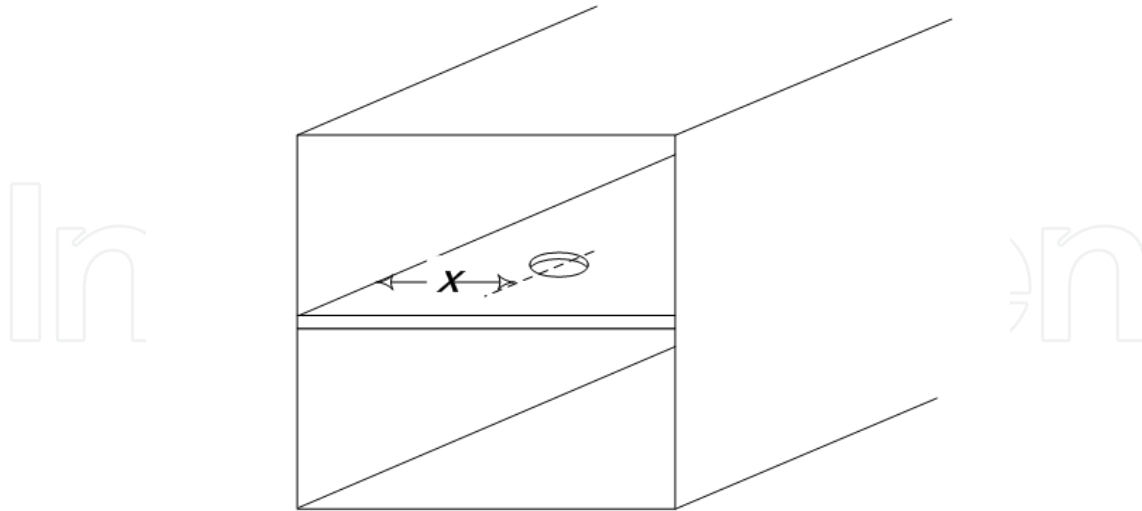

Figure 5. Narrow wall coupling

1.2.2. Broad wall

This case is shown in Fig.6 and equations are expressed as following:

$$A_1 = \frac{j2\pi}{ab\lambda_g}\left[\left\{M_x - \left(\frac{\lambda_g}{\lambda}\right)^2.P\right\}\sin^2\frac{\pi x}{a} + M_z\left(\frac{\lambda_g}{2a}\right)^2\cos^2\frac{\pi x}{a}\right] \tag{17}$$

$$B_1 = \frac{j2\pi}{ab\lambda_g}\left[\left\{M_x + \left(\frac{\lambda_g}{\lambda}\right)^2.P\right\}\sin^2\frac{\pi x}{a} - M_z\left(\frac{\lambda_g}{2a}\right)^2\cos^2\frac{\pi x}{a}\right] \tag{18}$$

Figure 6. Broad-wall coupling

If the holes are circular with a diameter "d" the parameters "P, M_x and M_z" are cal-
culated as:

$$M_x = M_z = \frac{d^3}{6} \tag{19}$$

$$P = \frac{d^3}{12} \tag{20}$$

By putting (19 & 20) in (17 &18) we will have simplified form:

$$A_1 = \frac{j\pi d^3}{12a^2b}\left[\frac{2a}{\lambda_g} + \frac{\lambda_g}{2a} - \frac{a\lambda_g}{\lambda^2}\right] \tag{21}$$

$$B_1 = \frac{j\pi d^3}{12a^2b}\left[\frac{2a}{\lambda_g} - \frac{\lambda_g}{2a} + \frac{a\lambda_g}{\lambda^2}\right] \tag{22}$$

And if we put $\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} - \frac{1}{(2a)^2}$ we will have:

$$A_1 = \frac{j\pi d^3}{24a^2b}\left[\frac{2a}{\lambda_g} + \frac{\lambda_g}{2a}\right] \tag{23}$$

$$B_1 = \frac{j\pi d^3}{24a^2b}\left[\frac{6a}{\lambda_g} - \frac{\lambda_g}{2a}\right] \tag{24}$$

By putting $\lambda_g = 2a$ then minimum value for A_1 is calculated as:

$$A_{10} = \frac{j\pi d^3}{12a^2b}$$ (25)

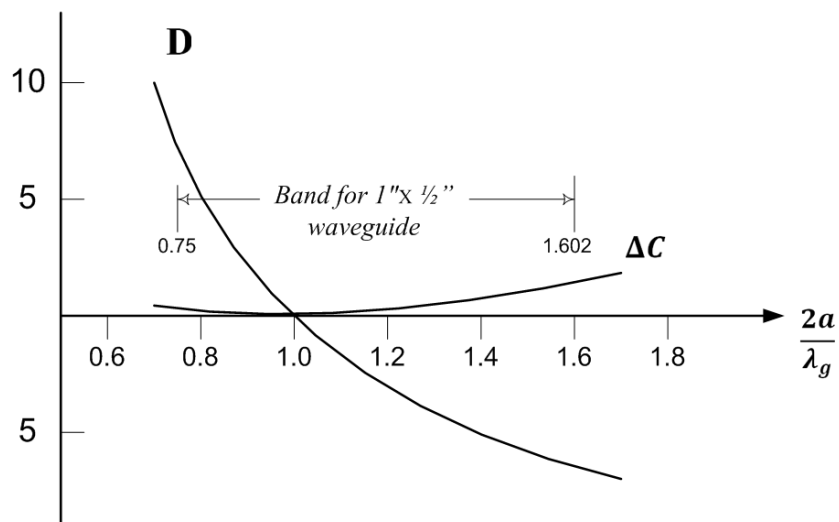And coupling variation to frequency changing is:

$$\Delta C = 20\log\left[\frac{\frac{2a}{\lambda_g} + \frac{\lambda_g}{2a}}{2}\right]$$ (26)

On the other hand, variation of directivity to frequency is given by:

$$D = 20\log\frac{A_1}{B_1} = 20\log\left[\frac{\frac{2a}{\lambda_g} + \frac{\lambda_g}{2a}}{\frac{6a}{\lambda_g} - \frac{\lambda_g}{2a}}\right]$$ (27)

The solved equations for R100 waveguide is given in Fig.7

Figure 7. Theoretical coupling variation and directivity for transverse arrangements of 1, 2 or 3 holes in common broad-wall of R100 waveguide.[14]

When a number of single holes are aligned in a row, the array's directivity will be added to basic value of directivity.

As it has been shown earlier, the Broad-wall couplers have better characteristics in wide frequency spans comparing to Narrow-wall couplers while the power handling characteristic of Narrow-wall couplers are better.

1.3. Coupling by big holes

The equations given in section 1.3 were valid for small holes. By considering the wall-thickness "*t*" and big hole's by surface size of "*A*", the equations should be corrected. This has already done and the results will be used now. [15]

1.3.1. Corrections for wall thickness "t"

When the coupling holes have thickness "t", it equals to a short-length circular waveguide, which is working under its cut-off frequency (when frequency is lower than resonance frequency). Therefore the amplitude of the coupled wave attenuates by following term:

$$exp\left\{\frac{-2\pi t A}{\lambda_c}\left[1-\left(\frac{\lambda_c}{\lambda}\right)^2\right]^{1/2}\right\} \tag{28}$$

In which, the λ_c is the cut-off wavelength of hole and λ is the wavelength in operating frequency, "*t*" is the wall-thickness and "*A*" is the correction factor which includes the interactions of adjacent fields and is defined practically. [15] For elliptic holes (Narrow slot) the "A" factor is in 3rd order but for circular holes it is close to first order. If the fields have multi components, correction factor (28) should be applied on all of the Bethe's equations. It means that in equations (17) and (18) the magnetic field correction factor for coupling have a parameter λ_c that is due to excitation by TE_{11} mode so for this mode $\lambda_c \rightarrow \lambda_H = 1.705d$ and correction factor for electric field coupling uses a λ_c, which relates to TM_{01} mode and equals to $\lambda_c \rightarrow \lambda_E = 1.305d$.

1.3.2. Corrections for big size holes

A good study for such big size holes have been done by an equivalent circuit. [15] In this way, the hole's effect will be simulated by a two port network in parallel or in series between two waveguides which have mutual coupling. This network is a lossless and should be defined by Foster's reactance theorem. Here, the impedance will be defined by locations of its zeroes and poles in addition to the multiplications coefficients.

In simple expression of Bethe's small sized coupling hole theory, this impedance has a simple form. For example the reactance of a small hole in a thin diaphragm at the cross section of a rectangular waveguide, working in TE$_{10}$ is:

$$X = \frac{4\pi M Z_0}{ab\lambda_g} \tag{29}$$

In which, the "a" and "b" are the waveguide dimensions that have been shown in Fig.4

The term λ_g is the guided wavelength, "M" magnetic polarization, Z_0 characteristic impedance of the waveguide. The Z_0 has a direct relation to term $\frac{\lambda_g}{\lambda}$, which shows that lumped reactance "X" has a direct relation to frequency ($X \propto f$) too.

Therefore the small hole coupling theory assumes that the "X" would be a constant reactance but it is not true. Because, there are a few numbers of unwanted resonances occurred in the frequency band. For this reason the (29) would be a good definition when the operating frequency is somehow lower than the first resonance. For considering the resonance effect in equation (29), the corrected "M" would be expressed by introducing a new term that considers the effect of cut-off wavelength $\dfrac{M}{1 - \left(\frac{f^2}{f_c^2}\right)}$ and the result is as follows:

$$\frac{X}{Z_0} = \frac{4\pi M}{ab\lambda_g\left(1 - \left(\frac{f^2}{f_c^2}\right)\right)} \tag{30}$$

From measurements, it has been shown that the above correction factor gives a good approximation.

The attenuation definition (28) can be combined to (30) to give us a general correction factor for big size holes:

$$\frac{exp\left\{\frac{-2\pi t A}{\lambda_c}\left[1 - \left(\frac{f^2}{f_c^2}\right)\right]^{1/2}\right\}}{1 - \left(\frac{f^2}{f_c^2}\right)} \tag{31}$$

1.4. Multi holes coupling

A longitudinal coupling consists of a series of holes by center distance of $\frac{\lambda_g}{4}$ that has a great coupling in forward and weak coupling in backward direction.
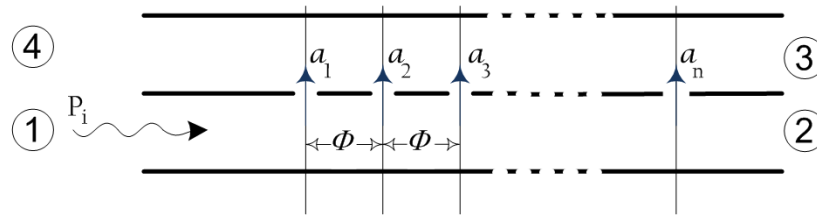
The slight coupling for a single hole has been studied and the directivity introduced by:

$$Directivity\ (dB) = 20\log \frac{A_1}{B_1} \tag{32}$$

In which: [15]

$$\frac{A_1}{B_1} = \frac{\int_{-L/2}^{L/2} \phi_{(x)}dx}{\int_{-L/2}^{L/2} \phi_{(x)}\exp\left(-2j\beta x\right)dx} \tag{33}$$

In the Fig.8 a series of "n" holes in one row is shown. The coupling voltage of the series is named $a_1, a_2,...,a_n$.

Figure 8. The cross section of n-Hole array and coupling coefficients

All the hole's center distances and electrical length are the same and are considered in the middle of the band. If the input wave to port 1 has constant amplitude and matched to other 3 ports, the reflected wave can be expressed by:

$$B_1 = a_1 + a_2 Exp(-2\,j\phi) + a_3 Exp(-4\,j\phi) + \ldots + a_n Exp(-2(n-1)\,j\phi) \tag{34}$$

The interesting and useful case is when the coefficients of the series being symmetrical from center. Therefore:

$$a_1 = a_n, \qquad a_2 = a_{n-1}, \qquad a_K = a_{n-K+1} \tag{35}$$

So by putting the values (35) into (34):

$$B_1 = \begin{cases} \left[2a_1 \cos(n-1)\phi + 2a_2 \cos(n-2)\phi + \ldots + 2a_{n/2}\cos\phi \right] e^{\,j(n-1)\phi} & n \ even \\ \left[2a_1 \cos(n-1)\phi + 2a_2 \cos(n-2)\phi + \ldots + \frac{a_{n+1}}{2} \right] e^{\,j(n-1)\phi} & n \ odd \end{cases} \tag{36}$$

The direct coupled wave at port 3 will be:

$$A_1 = \sum_{r=1}^{n} a_r\, e^{\,j(n-1)\phi} \tag{37}$$

And the directivity "D" is calculated by normalizing B_1 to A_1 in (36) by dividing the sum of each coupling voltages. In special case if there are "n" identical holes, therefore:

$$\frac{B_1}{A_1} = \frac{\sin n\phi}{n \sin \phi} \tag{38}$$

2. Design methods based on arrays

2.1. Chebyshev Array

If the minimum voltage over the full bandwidth to reach a good directivity "D" is needed, the Chebyshev polynomial can be used for distribution function of each hole's voltage. Such coefficients are derived by putting the B_1 in (36) by considering the following equal ripple's directivity function as following:

$$B_1 = a_m T_{n-1}\left(\frac{\cos \phi}{\cos \phi_0}\right) \tag{39}$$

In which the a_m is the maximum of B_1 over the coupling bandwidth that given by following:

$$\phi_0 \leq \phi \leq \pi - \phi_0 \tag{40}$$

The a_m is calculated by putting the $\phi = 0$ in (39):

$$a_m = \frac{\sum\limits_{r=1}^{n} a_r}{T_{n-1}\left(\frac{1}{\cos \phi_0}\right)} = \frac{|A_1|}{T_{n-1}\left(\frac{1}{\cos \phi_0}\right)} \tag{41}$$

In (36) if we put $\phi = 0$:

$$B_1 = \sum_{r=1}^{n} a_r \tag{42}$$

Therefore the minimum directivity over the bandwidth would be:

$$D_{min} = 20\log\left[T_{n-1}\left(\frac{1}{\cos \phi_0}\right)\right] \tag{43}$$

Comparing this method to method of Binomial polynomial is very informative that has been done by Levi. In this case we should have: [16]

$$B_1 = a_m \cdot \left(\frac{\cos \phi}{\cos \phi_0}\right)^{n-1} \tag{44}$$

In which:

$$a_m' = \frac{\sum\limits_{r=1}^{n} a_r}{\left(\frac{1}{\cos \phi_0}\right)^{n-1}} = \frac{|A_1|}{\left(\frac{1}{\cos \phi_0}\right)^{n-1}} \tag{45}$$

The minimum directivity at the edge of the band for this case is:

$$D_{min} = 20\log\left[\left(\frac{1}{\cos\phi_0}\right)^{n-1}\right] \tag{46}$$

Obviously the (46) always is significantly lower than the value for Chebyshev case (43).

The coupling equation for Chebyshev case is derived by putting the identical coefficients of $\cos\phi$ in (36). Young gave such coefficients for $3 \le n \le 8$ [3]. But here, the generalized case is obtained by a computer program for $1 \le n \le 25$.

For coupling $C = 0$ these coefficients are changed into Pascal's triangle that for $C = 0$ the infinite directivity over a zero bandwidth obtained.

The hole's size is derived by coupling of each hole in dB. That relation for r th hole is as follows:

$$C_r = 20\log\left(\frac{\sum\limits_{r=1}^{n} a_r}{a_r}\right) \tag{47}$$

Since $\sum\limits_{r=1}^{n} C_r = 1$, all the theoretically given hole couplings, transferred all power by assuming the 0dB in the formula. Therefore in order to design a "C dB" coupler the "C" is added to C_r in (47). The entire hole sizes by this way and by given theory for small size holes (or if we need by using the correction coefficient curves given by references) can be computed. [17, 18]

In addition to both mentioned series for calculating the coefficients (Chebyshev and Binomial), there is another method that actually derived from them. It is named "Super Imposed Arrays".

2.2. Super Imposed Array

When the strong coupling is needed, i.e. 3dB or 6dB, it is not possible to use the one row of holes (single array), since diameter of holes will be increased. Therefore it is more convenient to have approximately same diameter for all to get good coupling quality. For this case the super imposed array is used. As first step, we need the coefficient series in which the holes get bigger. It would be happened when $n > 4$. For starting we can use Chebyshev or binomial coefficient series in one line. Then the same series should be written in second line but in shifted position. It means, first coefficient of line 2 in under the 4th coefficient of line 1 and so on. By adding the two lines we would have a new series that its coefficients (or holes' diameters) alternately are the same. For example by a 6-element binomial series, we can make a 9-element super imposed series:

As it has shown, the elements in new series are alternately identical. This can be done by any other number of elements or polynomials. If we wanted to add more number of holesTable 1, the same way is chosen:

1	5	10	10	5	1			
		1	5	10	10	5	1	
1	5	10	11	10	11	10	5	1

Table 1. A 9-element super imposed array based on 6-element binomial array

1	5	10	11	10	11	10	5	1		
				1	5	10	10	5	1	
1	5	10	11	10	11	11	10	11	10	5	1

Table 2. Added more number of elements to table 1.

For special characteristics of Chebyshev series, hereinafter, it will be used as basic polyno-
mial for further design. In super imposed method the number of elements can be any value
and we would have longer coupler. If we chose other two methods the the holes diameter
get bigger and bigger and it may exceeds the broad-wall size of waveguide. By considering
the $\sum_{r=1}^{n} C_r = 1$ it is cleared that each C ,must be less than 1 and in super imposed this would be
happened.

For strong couplings and the holes with the same shape the following relation is proposed
by Cohn: [14]

$$A_m = \sin\left(\frac{m\pi}{4m_{3dB}}\right) \tag{48}$$

In (48) the m_{3dB} is the number of series for obtaining the 3 dB coupling. For example, if a 6-
element series is needed for 3dB coupling, therefore the coupling for one series will be:

$$A_1 = \sin\left(\frac{\pi}{24}\right) = 0.1305 \tag{49}$$

That equals to 17.68 dB. The directivity for Chebyshev-based super imposed array is greater
than the single array. The reason can be explained by this fact that returned waves are add-
ed in phase and amplitude, so the maximum amplitude for returned wave will not exceed
from single array in any case and it will damped soon.

2.3. Transverse groups of holes

It maybe concluded that by using two or three rows of holes in broad wall, the stronger cou-
pling would be obtained. It is true if there are two rows and the distance of center of holes to
side walls being equal to $x = 0.25a$ but for three rows the result is not good. For two rows the
coupling and directivity are derived from (23) and (27). See Fig. 9
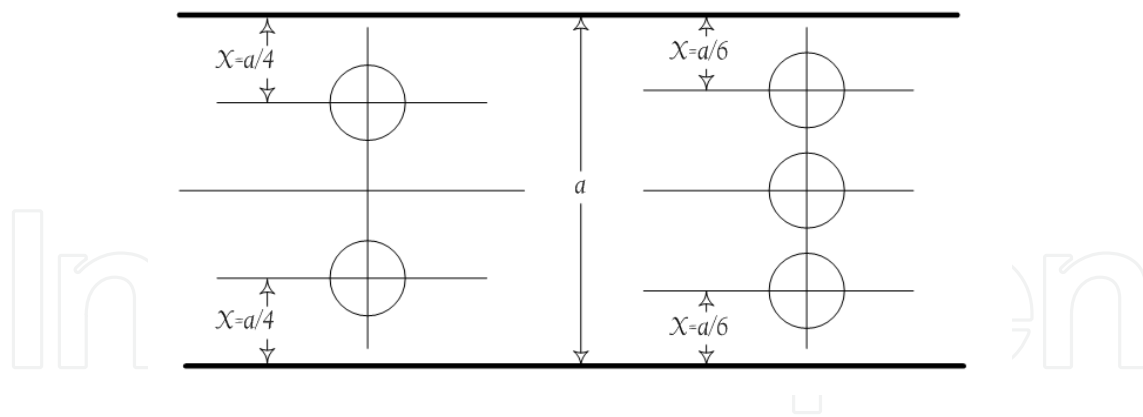
Figure 9. Groups of two or three rows holes.

The test results of multi-hole couplers are as following:

A 6dB coupler by $1" \times 1/2"$ waveguide in 8.2~12.4GHz band is designed by 21-elements superimposed array based on 6-element Chebyshev in two rows (totally 42 holes). The distance from side-wall for circular-holes is $x = 0.25a = a/4$ and distance between hole centers is $\lambda_g/4$. The obtained directivity "D" is more than 40dB and coupling deviation ΔC is about $\pm 0.5dB$. [19]

Shelton has tested multi-rows couplers and has given the coupling curve in terms of holes diameters for X and Ku bands waveguides. His efforts by using 3 rows were not successful. [20] For $1" \times 1/2"$ waveguide, Cohn used rounded rectangular holes in two rows. His research approved that 2-rows is better than 3-rows and for shortening the length it is not possible to use 3-rows holes.

It was also declared that in case of 3-rows, the resonance in upper band will happen. The reason is, where the electric vector is in parallel to broad wall, the even mode is excited and coupler acts as side wall coupler. Such case is not happened for 2-rows couplers. By reducing the height of "b" it is possible to put the resonance frequency of even modes out of operating frequency band. Only a slight reduction in "b" is needed since the resonance is occurred when the coupling region has length of $\lambda_g/2$ and it is near cut-off for even modes. Indeed, as an example, the "b" should be reduced from 0.4" to 1/16". This reduction in "b" increases the coupling of each hole. See (17). So a few number of holes needed to make 3dB coupling. As an example, a 2nd order Chebyshev transformer has a theoretical VSWR of 1.01 over its bandwidth. For better matching the waveguide height should be reduced at the two ends. Each series of holes at each side of coupler should be located inside of transformer in the way that it does not change the length. The final length would be 3.9 inches consisting of 2-rows of 10 holes that gives a coupling of $3 \pm 0.5dB$ and Isolation of more than 30dB for X-band 8.2~12.4GHz.

3. Practical designing

3.1. A real sample

After reviewing the basics of directional coupler, we start to design a coupler practically. First of all it is better to introduce the abbreviations that we use. They are listed in following table:

C	= Coupling in dB
λ_{g_1}	= Guided wavelength at the lower end of the required bandwidth (mm)
λ_{g_2}	= Guided wavelength at the upper end of the required bandwidth (mm)
$\lambda_{g_{mid}}$	= Mean guided wavelength
N	= number of coupling elements in basic array
D_{min}	= minimum directivity (dB)
	$= 180 \left/ 1 + \dfrac{\lambda_{g_1}}{\lambda_{g_2}} \right.$ (deg)
λ_g	= Guided wavelength (at the center frequency of the wave-guide bandwidth) (mm)
X	= Axis across broad dimensional of waveguide
A	= Broad dimension of a waveguide wall (mm)
B	= Narrow dimension of waveguide wall (mm)
d	= Diameter of hole in millimeter (mm)
A'	$= 1 - \left(\dfrac{1.71d}{\lambda_0}\right)^2$ Term giving correction of resonance phenomena
λ_0	= free space wavelength (mm)
T	= wall thickness (mm)
A''	$= 32\left(\dfrac{t}{d}\right)\left[1 - \left(\dfrac{1.71d}{\lambda_0}\right)\right]^{1/2}$ term giving correction to the attenuation effect on a finite wall thickness
X_0	$= 1 / \cos$

Table 3. The terms and abbreviations that used in design procedure.

As it is mentioned before in (10):

$$\frac{\lambda_{g_{mid}}}{4} = \frac{\lambda_{g1}\lambda_{g2}}{2(\lambda_{g1} + \lambda_{g2})} \tag{50}$$

The number of holes can be defined by minimum directivity D_{min} as: [23]

$$n = 1 + \cosh^{-1}\left[\frac{10^{\frac{D_{min}}{20}}}{\cosh^{-1}\left(\frac{1}{\cos\phi}\right)}\right] \tag{51}$$

The starting coefficient in Chebyshev argument is calculated as:

$$X_0 = \frac{1}{\cos\phi} = \frac{1}{\cos\left(\frac{180}{1 + \frac{\lambda_{g_1}}{\lambda_{g_2}}}\right)} \tag{52}$$

For X-band that we have $\lambda_{g_1} = 6.089\ Cm$ and $\lambda_{g_2} = 2.489\ Cm$ the $X_0 = 1.853$ is obtained. Next step is to find the Chebyshev polynomial coefficients by computer program that gives:

{40.507, 172.277, 355.449, 445.373} (Notice that, only a half of the coefficients are enough due to symmetric specification of Chebyshev polynomial).

Then the coefficients are normalized to least element that gives following table:

A	B	C	D
1.0	4.253	8.775	10.995

Table 4. Normalizing the Chebyshev coefficients.

Therefore the whole structure of the holes will be as follows:

A	B	C	D	C	B	A

Table 5. Sequence of holes and its related Chebyshev coefficients for coupler synthesis.

Now we add them all together:

$$2(A + B + C) + D = 39.051 \tag{53}$$

The coupling for each hole will be defined in dB as follows:

Coupling for Holes $A = 20\log\frac{39.051}{1}\ dB = 31.832\ dB$

Coupling for Holes $B = 20\log\frac{39.051}{4.253}\ dB = 19.259\ dB$

Coupling for Holes $C = 20\log\frac{39.051}{8.775}\ dB = 13.968\ dB$

Coupling for Holes $D = 20\log\frac{39.051}{10.051}\ dB = 11.776\ dB$

Now, consider that we want to design a 10dB coupler, so we add a 10dB to each coefficient:

C_A=41.832 dB, Holes C_B=29.259 dB, Holes C_C=23.968 dB, Holes C_D=21.776 dB

Finally the achieved numbers should be inserted into Bethe's formula for small size holes: [12]

$$C = 20\log\left\{\frac{12a^2b}{\pi d^3}\left[1 - \left(\frac{1.71d}{\lambda_0}\right)^2\right]\right\} + 20\log\left\{32\left(\frac{t}{d}\right)\left[1 - \left(\frac{1.71d}{\lambda_0}\right)\right]^{1/2}\right\} \tag{54}$$

Now we solve the above equation (for each hole) by iteration method and the diameter of each hole would be determined. By considering the distance of circle centers to side wall as $x=0.203a$ (7) following values for diameters would be obtained:

A=0.234 inch, B=0.343 inch, C=0.397 inch, D=0.421 inch

Note that the solved example is for single array. If we wanted to have the double rows we should put the (C+6) dB instead of C dB (that we considered 10 dB in above example).

Notice: an approximation way to define the number of holes "n" is using the D_{min} in equal to maximum coupling between holes plus 3 ~ 5 dB. For instance in the solved example, the maximum coupling is belonged to "A" that was C_A=41.832 dB. So:

$$D_{min} = C_A + 5 = 47\ dB \tag{55}$$

And the number of holes would be:

$$n = 1 + \cosh^{-1}\left[\frac{10^{\frac{47}{20}}}{\cosh^{-1} X_0}\right] \approx 6.9 \ \rightarrow n = 7 \tag{56}$$

Therefore if we wanted to have a good directivity, a directivity higher than 47dB then we should have 7 holes in the coupler.

In practice, for eliminating the effect of wall thickness "t", it is possible to remove one broad wall of a waveguide and mill- the next wall to have half thickness between to waveguides. [23]

The real designed 20 dB coupler by R70 waveguide and 14 holes in two rows (each row has 7 holes) is fabricated and tested. The results are given in Fig.10

In Fig. 11 and 12, another directional coupler for $C = 10$ dB is sketched. The diameter of circles for its five categories of holes, are:

Holes number 4 =4.16 mm

Holes number 3 =6.45 mm

Holes number 2 =8.00 mm

Holes number 1 =8.66 mm

Holes number 0 =8.94 mm

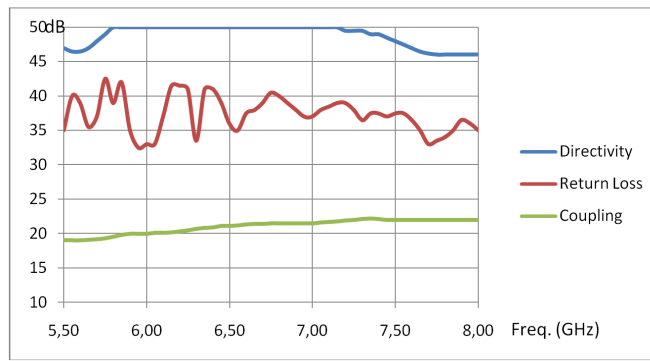For further information see Fig. 11 and 12.

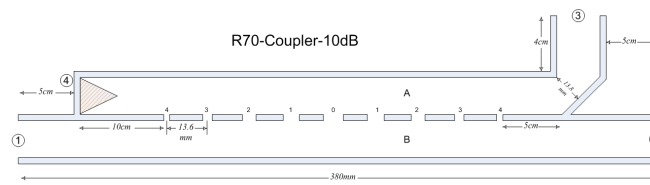**Figure 10.** The test result for a multi-hole waveguide R70, directional coupler C=20dB

Figure 11. The R70 directional coupler, C = 10 dB by two rows of 9-elements
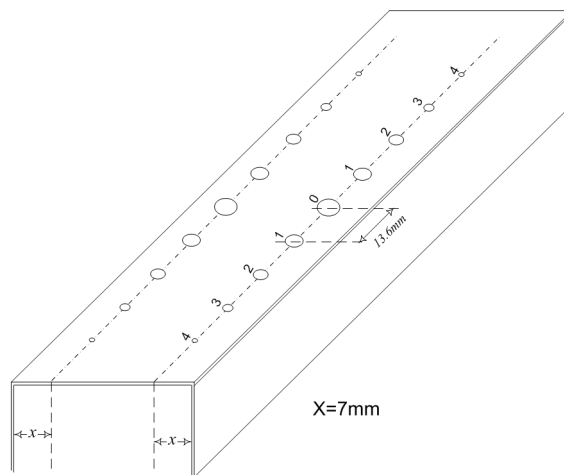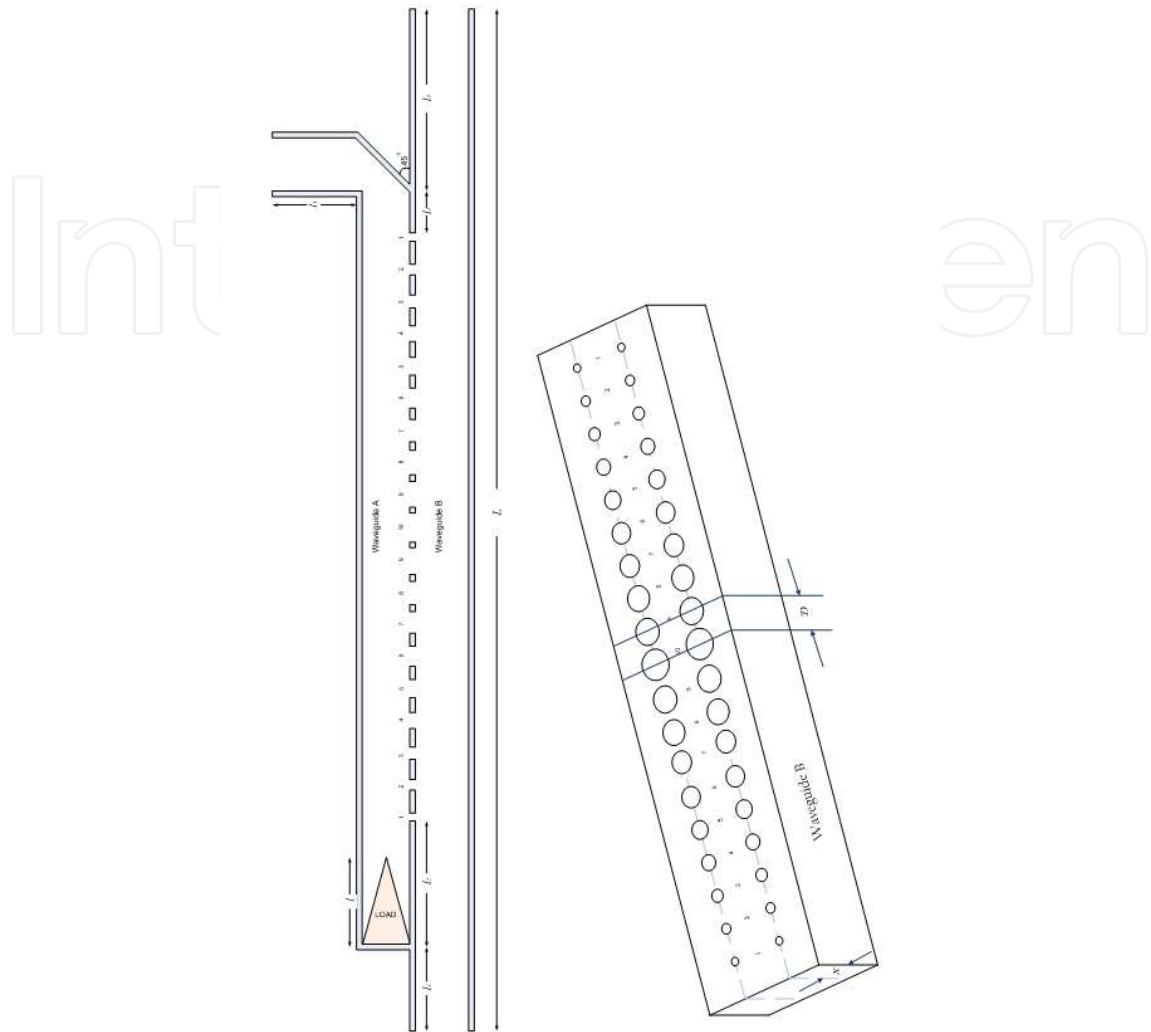

Figure 12. The circle centers and distance to side walls are the same.

Figure 13. Cross section for a 38-holes (double rows of 19-elements) directional coupler. The lossy load maching using ferro-based materials (courtesy H. Mottaghi: hossain_mtg@yahoo.com)

The port-4 in the couplers is matched by a conical or pyramidal load. To obtain the lowest reflection from this port, the tapered structure is used to make a slight change in characteristic impedance of the waveguide along its length. Since the power loss at this port, naturally is not significant, the medium or low power absorber or load is enough.

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References

[1] Gandhi, O. P. (1981). *Microwave Engineering and Applications*, Pergamon Press.

[2] Collin, R. E. (1966). *Foundations for Microwave Engineering*, Mc. Graw Hill.

[3] Young, Leo. (1966). *Advances in Microwave, volume I*, Academic Press.

[4] Miller, S. E. (1954). Coupled Wave Theory and Waveguide Applications. *Bell system Tech. J.*, 33, 661.

[5] Riblet, H. J. (1954). The short- slot hybrid Junction. *Proc, IRE*, 40, 180.

[6] Tomiyasu, K., & Cohn, S. B. (1953). The Transvar Directional Coupler. *Proc. IRE*, 41, 922.

[7] Cook, J. S. (1955). Tapered velocity couplers. *Bell system Tech. J.*, 34, 807-822.

[8] Fox, A. G. (1955). Wave coupling by warped normal modes. *Bell System Tech. J.*, 34, 823.

[9] Louisell, W. H. (1955). Analysis of a single tapered mode. *Bell System Tech. J.*, 34, 853.

[10] Miller, S. E., & Mumford, W. W. (1952). Multi-element directional couplers. *Proc. IRE*, 40, 1070.

[11] Bethe, H. A. (1943). Theory of side windows wave guides. *Mass. Inst. Technology Radiation Lab Report* [43].

[12] Bethe, H. A. (1944). Theory of diffraction by small holes. Phy. Rev. , 66, 163.

[13] Surdin, M. (1946). Directive couplers in wave guides. J. Inst. Elec. Engrs (London) , 93(IIIA), 725.

[14] Cohn, S. B., Knootz, R. H. S., & Wehn, S. L. (1962). Microwave hybrid coupler study program. *Rantec- Corporation Report* [61], 361-363.

[15] Cohn, S. B. (1952). Microwave coupling by large aperture. *Proc IRE*, 40, 697.

[16] Levy, R. H. (1959). Guide to the practical application of Chebyshev functions to the design of microwave components. *Proc. Inst. Elec. Engrs. (London) C*, 106, 193.

[17] Harrison, J. R. (1945). Design considerations for directional couplers. *MIT Radiation Lab. Rept.* [724].

[18] Saad, T. (1964). The Microwave Engineers handbook. *109 Boston, Massachusetts*.

[19] Hensperger, E. S. (1959). The design of multi-hole coupling arrays. *Microwave J.*, 2, 38.

[20] Shelton, W. (1961). Compact multi-hole waveguide directional couplers. *Microwave J.*, 4, 89.

[21] Geranpaye. (1985). Waveguide standards. *ITRC, Tehran, Iran*.

[22] Alison, W. B. W. A. (1987). A handbook for the mechanical tolerance of waveguide components. *Artech house*.

[23] Hancock, K. E. (1967). The design and manufacture of waveguide Tchebyshev directional-coupler. Electron. Eng , 39, 292.

[24] Liang, C. H., & Cheng, D. K. (1982). On performance limitations of aperture coupling between rectangular waveguide. *MTT-30* [5].

[25] Levy, R. (1968). Analysis and synthesis of waveguide multi-aperture directional couplers. *MTT-16* [12].

[26] Mc Donald, N. A. (1972). Electric and Magnetic coupling through small apertures in shield wall of any thickness. *MTT-20* [10].

[27] Levy, R. (1980). Improved single and multi- aperture waveguide coupling theory including explanation of mutual interactions. *MTT-28* [4].

[28] Riblet, H. J. (1947). A Mathematical theory of differential coupler. *Proc. IRE*, 35, 1307.

[29] Crompton, J. W. (1957). A contribution to the design of multi-element directional couplers. *Proc. IEE*, 104C(6TD), 398-402.

[30] Matthaei, G. L. (1964). *Microwave Filters impedance matching networks and coupling structures*, Mc Graw Hill.

[31] Wheeler, H. A. (1964). Coupling holes between resonant cavities or waveguides evaluation terms of volume ratios. *MTT*, 231.

[32] Sangster, A. J. (1973). Slot Coupling between uniform rectangular waveguides. *MTT-27* [7], 705.

[33] Pandhuri, Pande. V. H., & Das, B. W. (1979). comments on coupling of waveguides through large aperture. *MTT-27* [7], 702.

[34] Pandhuri, Pande. V. H., & Das, B. W. (1978). coupling of waveguide through large aperture. *MTT-26* [3], 209.

[35] Schmiedel, H., & Arndt, F. (1986). Field Theory Design of Rectangular Waveguide Multiple-Slot Narrow-Wall couplers. *MTT-34* [7], 791.

[36] Oliner, A. H. (1960). Equivalent circuits for small symmetrical longitudinal aperture and obstacles, . *IRE MTT*, 72.

[37] Hakkak, M., & Barakat, F. (1984). Waveguide bend characteristics. *ITRC publicationsTehran, Iran*.

[38] Mottaghi, H. (2010). Measuring the Load absorption parameters in waveguide. *IT&mmwaves SDN BHD, Kuala Lumpur*.