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Electromagnetic Wave Packets in the Theory of Bremsstrahlung and Transition Radiation by High-Energy Electrons

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1. Introduction

A lot of high-energy physical processes develop within large domains of space along the direction of particle motion (see, for example monographs [1-3] and references in them). In the case of electromagnetic processes the size of these domains can substantially exceed sometimes not only interatomic distances of substance but the size of experimental facility (detectors) as well [1,2,4-13]. Essential in this case is the fact that interaction of particles with atoms and experimental facility situated within such domains and outside them can substantially differ. Such situation arises, for example, when considering long-wave radiation in processes of bremsstrahlung and transition radiation by ultra relativistic electrons. Therefore, it is necessary to know what happens within such regions and what the peculiarities of evolution of such processes in space and time are. The present chapter is dedicated to the consideration of different aspects of this problem, which concern the behavior of high-energy wave packets, which take place in processes of bremsstrahlung and transition radiation.

We begin with consideration of the behavior of localized high-energy wave packets of a scalar massive particle in wave mechanics [2,14], of Gaussian packets of free electromagnetic waves and of packets, which take place in the equivalent photon method [15]. It is shown that in all cases high energies make the stabilizing effect upon the packet motion. Some peculiarities of dispersion of such packets and their reconstruction into the packets of diverging waves are considered as well.

Further we show that the discussed wave packets naturally arise in processes of bremsstrahlung and transition radiation by high-energy electrons. For this purpose, firstly, the process of bremsstrahlung at an ultra relativistic electron instantaneous scattering to a large angle is considered [16,17]. The consideration is made on the basis of classical electrodynamics. In this case the moving electron is considered as a charge with its own coulomb field moving together with it. At the instantaneous scattering the perturbation of this field occurs. This perturbation is treated here as appearance of a packet of free plane electromagnetic waves, which reconstructs then into a packet of diverging waves. For ultra relativistic particles, however, this does not happen at once. The length within which this process develops has a name of the coherence length of the radiation process [1,2]. It is $2\gamma^2$ times larger than the length λ of the considered radiated waves (γ is here the electron's Lorentz-factor). We show that within this length the field around the electron substantially differs from the coulomb one. This leads, in particular, to the fact that the bremsstrahlung characteristics in this case substantially depend on both the detector's size and its position relative to the scattering point.

It is shown further that analogous effects take place in the process of transition radiation by an ultra relativistic electron during its traverse of thin ideally conducting plate as well [16-18]. The picture of evolution in space and time of the electromagnetic field, which arises before and after the electron traverse of thin metallic plate is considered here. The main attention is paid to effects in the process of backward transition radiation. In this case the wave packets of the field reflected from the plate are the packets of free waves, which reconstruct into the field of transition radiation. We show that the structure of these packets is in many respects analogous to the structure of the packets, which take place at instantaneous scattering of the particle to a large angle. This fact explains the presence of analogous effects in transition radiation and bremsstrahlung in the considered cases.

The special attention is drawn to the transition radiation by a scattered electron, which own field is not totally reconstructed after the scattering [17-19]. During a long period of time in this case the electron is in 'half-bare' state, which is the state in which some Fourier harmonics in the field around the electron are suppressed compared to the equilibrium coulomb field. The large values of distances, which the electron covers in this state allow us to place the plate within these distances and to consider the transition radiation by such electron on this plate. We show that in this case characteristics of backward transition radiation substantially differ from transition radiation characteristics in the case when the target is situated on large distances from the scattering point. The effect of transition radiation suppression and the effect of oscillatory dependence of transition radiation characteristics on the distance between the plate and the scattering point take place in this case. The causes of such effects are discussed.

2. High energy wave packets

2.1. Dispersion of relativistic wave packets

The general solution of the wave equation can be presented in the form of a wave packet, which spatially disperses in course of time. In semiclassical approximation such packet does not disperse. It moves according to the laws of classical mechanics (see, for example [2, 20]). It is going beyond the semiclassical approximation that leads to the packet dispersion. The high-energy wave packets are of special interest because the speed of their dispersion decreases with the increase of their energy. Let us pay attention to some peculiarities of dispersion of such packets. Significant here is the fact that characteristic features of this dispersion are similar for all fields. Therefore it is sufficient to consider just scalar field.

The general solution of the wave equation

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 - m^2\right)\phi(\vec{r}, t) = 0 \quad (1)$$

for a scalar particle with the mass m can be written in the following form of the expansion of the field $\phi(\vec{r}, t)$ over plane waves:

$$\phi(\vec{r}, t) = \int \frac{d^3\kappa}{(2\pi)^3} e^{i(\vec{\kappa}\vec{r} - \omega t)} C_{\vec{\kappa}}^- \quad (2)$$

where $\omega = \sqrt{\vec{\kappa}^2 + m^2}$ and $C_{\vec{\kappa}}^-$ are the expansion coefficients. Here and further we will use the system of units in which the speed of light c and the Plank constant \hbar equal unit.

Let us consider the dispersion of the wave packet, which at the initial moment of time coincides with the Gaussian packet modulated by the plane wave with large value of the momentum \vec{p} [2,14]. Moreover we will assume that the initial widths of the packet $a_{||}$ and a_{\perp} parallel and perpendicular to the particle momentum \vec{p} are different. For such packet at the initial moment of time the field $\phi(\vec{r}, t)$ has the following form:

$$\phi(\vec{r}, t) = e^{i\vec{p}\vec{r} - \frac{z^2}{2a_{||}^2} - \frac{\vec{\rho}^2}{2a_{\perp}^2}} \quad (3)$$

where z and $\vec{\rho}$ are the coordinates parallel and orthogonal to \vec{p} . At the moment of time t this packet will be defined by the relation (2) with

$$C_{\vec{\kappa}}^- = (2\pi)^{3/2} a_{||} a_{\perp}^2 e^{-\frac{(p-\kappa_z)^2 a_{||}^2}{2} - \frac{\kappa_{\perp}^2 a_{\perp}^2}{2}} \quad (4)$$

We can write the obtained expression for the field $\phi(\vec{r}, t)$ in the form

$$\phi(\vec{r}, t) = Ae^{i(\vec{p}\vec{r} - \varepsilon t)} I(\vec{r}, t) \quad (5)$$

in which $A = a_{||} a_{\perp}^2$, $\varepsilon = \sqrt{\vec{p}^2 + m^2}$ and

$$I(\vec{r}, t) = (2\pi)^{-3/2} \int d^3\kappa e^{i(\vec{\kappa} - \vec{p})\vec{r} - \frac{(p - \kappa_z)^2 a_{||}^2}{2} - \frac{\kappa_{\perp}^2 a_{\perp}^2}{2} - i(\omega_{\kappa} - \varepsilon)t} \quad (6)$$

Having made in this expression the variable substitution $\vec{\kappa} = \vec{p} + \vec{q}$ we find that

$$I(\vec{r}, t) = (2\pi)^{-3/2} \int d^3q e^{i\vec{q}\vec{r} - \frac{q_z^2 a_{||}^2}{2} - \frac{q_{\perp}^2 a_{\perp}^2}{2} - i(\omega_{\vec{p}+\vec{q}} - \varepsilon)t} \quad (7)$$

In the case of large energies it is possible to make the expansion over $|\vec{q}|/p$ in the quantity $(\omega_{\vec{p}+\vec{q}} - \varepsilon)$ in (7). Having preserved the quadratic terms of expansion we obtain

$$\omega_{\vec{p}+\vec{q}} - \varepsilon \approx vq_z + \frac{q_z^2}{2\varepsilon\gamma^2} + \frac{q_{\perp}^2}{2\varepsilon} \quad (8)$$

where $v = p/\varepsilon$ and $\gamma = (1 - v^2)^{-1/2}$. Substituting this expression into (6) after simple calculations we obtain

$$I(\vec{r}, t) = \frac{1}{\sqrt{a_{||}^2 + i\frac{t}{\varepsilon\gamma^2}}} \frac{1}{a_{\perp}^2 + i\frac{t}{\varepsilon}} \exp\left\{-\frac{(z - vt)^2}{2\left(a_{||}^2 + i\frac{t}{\varepsilon\gamma^2}\right)} - \frac{\rho^2}{2\left(a_{\perp}^2 + i\frac{t}{\varepsilon}\right)}\right\} \quad (9)$$

The formula (9) can be written in the following form as well:

$$I(\vec{r}, t) = A(t) \exp\left\{i\alpha(\vec{r}, t) - \frac{(z - vt)^2}{2\Delta_{||}^2(t)} - \frac{\rho^2}{2\Delta_{\perp}^2(t)}\right\} \quad (10)$$

in which $A(t)$ is a slowly changing quantity

$$A(t) = \frac{1}{\left(a_{\perp}^2 + i\frac{t}{\varepsilon}\right) \sqrt{a_{||}^2 + i\frac{t}{\varepsilon\gamma^2}}} \quad (11)$$

$\alpha(\vec{r}, t)$ is the real phase

$$\alpha(\vec{r}, t) = \frac{(z-vt)^2}{2} \frac{t/\varepsilon\gamma^2}{a_{||}^4 + (t/\varepsilon\gamma^2)^2} + \frac{\rho^2}{2} \frac{t/\varepsilon}{a_{\perp}^4 + (t/\varepsilon)^2} \quad (12)$$

$\Delta_{||}(t)$ and $\Delta_{\perp}(t)$ are the longitudinal and transverse widths of the packet at the moment of time t

$$\Delta_{||}^2(t) = a_{||}^2 + \left(\frac{t}{a_{||}\varepsilon\gamma^2} \right)^2, \quad \Delta_{\perp}^2(t) = a_{\perp}^2 + \left(\frac{t}{a_{\perp}\varepsilon} \right)^2 \quad (13)$$

In the case of $a_{||} = a_{\perp}$ the obtained above formulae coincide with the corresponding result of the paper [14].

The formulae (13) show that in longitudinal and transverse directions the squares of the widths of the packet $\Delta_{||}^2(t)$ and $\Delta_{\perp}^2(t)$ grow with time proportionally to $t^2 m^4 / \varepsilon^6$ and t^2 / ε^2 . In nonrelativistic case these quantities do not depend on the particle energy ($t^2 / \varepsilon^2 = t^2 / m^2$). In relativistic case the quantities $t^2 m^4 / \varepsilon^6$ and t^2 / ε^2 are substantially smaller than the corresponding values for nonrelativistic particles. Let us note that the additional factor m^4 / ε^4 exists for longitudinal direction in $\Delta_{||}^2(t)$. It leads to the substantial decrease of the speed of the packet dispersion in this direction compared to the speed of the packet dispersion in transverse direction. Thus the relativistic effects do the stabilizing influence upon the wave packets.

In conclusion let us note that while deriving the formula (9) we neglected the terms proportional to tq^4 / ε^3 in the exponential factor in (7). Therefore the formula (9) is valid during the interval of time, which satisfies the condition

$$tq_{eff}^4 / \varepsilon^3 \ll 1 \quad (14)$$

where q_{eff} are the characteristic values of the variable \vec{q} in (7), which make contribution to this integral. For $a_{||} \approx a_{\perp}$ the inequality (14) can be written in the form

$$\frac{t}{a_{\perp}^2 \varepsilon} \frac{1}{(a_{\perp} \varepsilon)^2} \ll 1 \quad (15)$$

Thus the formula (9) is valid for the time interval t , which satisfies the inequality (15). For ultra relativistic particles this interval rapidly grows with the increase of the particle's energy.

2.2. Dispersion of a high-energy packet of electromagnetic waves

Now let us consider high energy packets of free electromagnetic waves. Scalar and vector potentials of such packets are the solutions of the wave equation (1) with $m=0$. Therefore in order to analyze the peculiarities of dispersion of such packets we can use the formulae of

the previous section assuming that all the terms in them containing the Lorentz-factor γ equal zero. In this case for scalar potential we find that

$$\phi(\vec{r}, t) = Ae^{i(\vec{k}\vec{r} - \omega t)} I(\vec{r}, t) \quad (16)$$

where \vec{k} and ω are the wave vector and the frequency of the electromagnetic wave and

$$I(\vec{r}, t) = A(t) \exp \left\{ i\alpha_k(\vec{r}, t) - \frac{(z-t)^2}{2\Delta_{\parallel}^2(t)} - \frac{\rho^2}{2\Delta_{\perp}^2(t)} \right\} \quad (17)$$

Here

$$A(t) = \frac{1}{a_{\parallel}(a_{\perp}^2 + it/\omega)}, \quad \alpha(\vec{r}, t) = \frac{\rho^2}{2} \frac{t/\omega}{a_{\perp}^4 + (t/\omega)^2} \quad (18)$$

and

$$\Delta_{\parallel}^2(t) = a_{\parallel}^2, \quad \Delta_{\perp}^2(t) = a_{\perp}^2 + (t/a_{\perp}\omega)^2$$

The obtained formulae show that the initially Gaussian packet does not disperse in the direction parallel to the \vec{k} vector. In transverse direction the square of the packet widths grows proportionally to $(t/\omega)^2$. Thus the speed of the packet dispersion decreases with the increase of the wave frequency ω .

When considering a process of radiation by relativistic electrons it is often necessary to deal with packets, which are constructed of plane waves with wave vectors, which directions are close to the direction of a given vector \vec{k} . Such wave packets differ somehow from the ones considered above. Let us consider some peculiarities of dispersion of such packets assuming for simplicity that at the initial moment of time $t=0$ the distribution of the waves over the wave vectors is Gaussian relative to the given vector \vec{k} [15]. For such distribution in the initial moment of time the scalar potential $\phi_k(\vec{r}, 0)$ has the following form:

$$\phi_k(\vec{r}, 0) = \frac{1}{\pi\bar{\Delta}_{\vartheta}^2} \int d^2\vartheta e^{-\vartheta^2/\bar{\Delta}_{\vartheta}^2} e^{i\vec{k}\vec{r}} \quad (19)$$

where ϑ is the angle between the packet wave vector and the wave vector \vec{k} , $\bar{\Delta}_{\vartheta}^2$ is the average value of the square of the angle ϑ , $\bar{\Delta}_{\vartheta}^2 \ll 1$.

The coefficients C_q of the Fourier expansion (2) for such initial packet have the following form

$$C_{\vec{q}} = (2\pi)^3 \int \frac{d^2\vartheta}{\pi \bar{\Delta}_{\vartheta}^2} e^{-\vartheta^2 / \bar{\Delta}_{\vartheta}^2} \delta(\vec{k} - \vec{q}) \quad (20)$$

in which $\delta(\vec{k} - \vec{q})$ is the delta-function. As a result we come to the following expression for the scalar potential

$$\phi_k(\vec{r}, t) = \frac{1}{1 + ikz \bar{\Delta}_{\vartheta}^2 / 2} e^{ik(z-t) - \frac{(k\rho/2)^2 \bar{\Delta}_{\vartheta}^2}{1 + ikz \bar{\Delta}_{\vartheta}^2 / 2}} \quad (21)$$

where z and $\vec{\rho}$ are the coordinates parallel and orthogonal to \vec{k} .

Let us note that the considered case corresponds to the wave packet, which consists of the plane waves the directions of the wave vectors of which have some small scatter around the z axis. If the initial packet has some small variation in frequency ω , the formula (21) should be averaged over this variation of the waves in frequency. Assuming for simplicity that the distribution of the waves over frequency has the Gaussian form with the average value of the square of the frequency variation $\bar{\Delta}_{\omega}^2$ we find that

$$\langle \phi_k(\vec{r}, t) \rangle = \frac{1}{1 + i\omega z \bar{\Delta}_{\vartheta}^2 / 2} e^{i\omega(z-t) - \frac{(z-t)^2 \bar{\Delta}_{\omega}^2}{4} - \frac{(\omega\rho/2)^2 \bar{\Delta}_{\vartheta}^2}{1 + i\omega z \bar{\Delta}_{\vartheta}^2 / 2}} \quad (22)$$

The given expression for the wave packet has the same structure as the corresponding expression for the packet (16). If the substitutions $(t/a_{\perp}^2 \varepsilon) \rightarrow (\omega z \bar{\Delta}_{\vartheta}^2 / 2)$ and $a_{\parallel}^2 \rightarrow 4 \bar{\Delta}_{\omega}^2$ are made in the latter expression the both formulae for the wave packet will become identical.

The formula (22) shows that for $\omega z \bar{\Delta}_{\vartheta}^2 / 2 \ll 1$

$$\langle \phi_k(\vec{r}, t) \rangle \approx \exp \left\{ i\omega(z-t) - \frac{(z-t)^2 \bar{\Delta}_{\omega}^2}{4} - \left(\frac{\omega\rho}{2} \right)^2 \bar{\Delta}_{\vartheta}^2 \right\} \quad (23)$$

and for $\omega z \bar{\Delta}_{\vartheta}^2 / 2 \gg 1$

$$\langle \phi_k(\vec{r}, t) \rangle \approx -\frac{2i}{\omega z \bar{\Delta}_{\vartheta}^2} \exp \left\{ i\omega(z-t) - \frac{(z-t)^2 \bar{\Delta}_{\omega}^2}{4} + i\omega \frac{\rho^2}{2z} - \frac{\rho^2}{z^2 \bar{\Delta}_{\vartheta}^2} \right\} \quad (24)$$

For $z \gg \rho$ the latter formula can be written in the form of a diverging wave

$$\langle \phi_k(\vec{r}, t) \rangle \approx -\frac{2i}{\omega r \bar{\Delta}_{\vartheta}^2} \exp \left\{ i\omega(r-t) - \frac{(z-t)^2 \bar{\Delta}_{\omega}^2}{4} - \frac{\rho^2}{z^2 \bar{\Delta}_{\vartheta}^2} \right\} \quad (25)$$

where $r = \sqrt{z^2 + \rho^2} \approx z + \rho^2/2z$. Thus on distances z from the center of the initial packet, which satisfy the condition

$$\omega z \bar{\Delta}_y^2 / 2 \ll 1 \quad (26)$$

the form of the packet (22) coincides with the form of the packet at $t=0$. Only on the distances, which satisfy the condition

$$\omega z \bar{\Delta}_y^2 / 2 \gg 1 \quad (27)$$

the transformation of the packet (22) to the packet of spherical diverging waves occurs.

Let us note that in the theory of radiation of electromagnetic waves by a moving electron the spatial region in which the formation of spherical diverging waves occurs has a name of the wave zone (see for example [21]). In particular, for nonrelativistic charged particles the wave zone begins on distances from the radiation region, which exceed the length of the radiated wave λ . However, the condition (27) shows that for $\bar{\Delta}_y^2 \ll 1$ the wave zone formation occurs not on distances $z \gg \lambda$ as in the case of a nonrelativistic particle but on distances

$$z \gg 2\lambda / \bar{\Delta}_y^2 \quad (28)$$

which are much larger than the wave length $\lambda = 1/\omega$. For sufficiently small values of $\bar{\Delta}_y^2$ the length $z = 2\lambda / \bar{\Delta}_y^2$ can reach macroscopic size.

2.3. Wave packets in the equivalent photon method

The problem of dispersion of wave packets naturally arises in the equivalent photon method (or the method of virtual photons) in which at the certain moment of time ($t=0$) the electron's coulomb field is substituted by a packet of free electromagnetic waves (see [22] and references there). Let us consider some peculiarities of wave packets behavior in this method [15].

For this purpose we write the scalar potential of the coulomb field of the electron moving along the z axis with the velocity \vec{v} in the form of the following Fourier-expansion:

$$\phi_C(\vec{r}, t) = \text{Re} \int \frac{d^3k}{(2\pi)^3} e^{i(\vec{k}\vec{r} - \vec{k}\vec{v}t)} C_k^C \quad (29)$$

in which

$$C_k^C = \frac{8\pi e \theta(k_z)}{k_{\perp}^2 + k_z^2 / v^2 \gamma^2} \quad (30)$$

Here e is the electron's charge, γ is the Lorentz-factor, k_z and k_{\perp} are the components of vector \vec{k} parallel and orthogonal to z axis, $\theta(k_z)$ is the Heaviside step function.

In the equivalent photon method it is assumed that for $t=0$ the packet (2), which consists of free electromagnetic waves coincides with the packet (29). It corresponds to the Fourier expansion (2) with the coefficients $C_{\vec{k}}^C = C_k^C$. Let us consider the behavior of such packet in ultra relativistic case in which $\gamma \gg 1$.

For $\gamma \gg 1$ the main contribution to (2) are made by the values of \vec{k} with directions close to the direction of the electron's velocity \vec{v} . Taking this fact into account we can present the packet (2) in the following form:

$$\phi(\vec{r}, t) = \text{Re} \int_0^{\infty} dk \phi_k(\vec{r}, t) \quad (31)$$

where

$$\phi_k(\vec{r}, t) = \frac{2}{\pi} e^{ik(z-t)} \int_0^{\infty} \frac{\vartheta d\vartheta}{\vartheta^2 + \gamma^{-2}} J_0(k\rho\vartheta) e^{-ikz\vartheta^2/2} \quad (32)$$

Here ϑ is the angle between \vec{k} and \vec{v} ($\vartheta \ll 1$) and $J_0(x)$ is the Bessel function.

The function $\phi_k(\vec{r}, t)$ has the same structure as the function (21) corresponding to the Gaussian distribution of vectors \vec{k} over the angles ϑ . Namely, if $kz\vartheta^2/2 \ll 1$, the main contribution to the integral (32) is made by the values $\vartheta \approx \gamma^{-1}$ and

$$\phi_k(\vec{r}, t) \approx \frac{2}{\pi} K_0(k\rho/\gamma) e^{ik(z-t)} \quad (33)$$

where $K_0(x)$ is the modified Hankel function. In this case after integration over \vec{k} in (31) we find that

$$\phi(\vec{r}, t) = \frac{e}{\sqrt{\rho^2 \gamma^{-2} + (z-t)^2}} \quad (34)$$

The main contribution to (31) is made by the values $k \approx \gamma / \rho$, hence expression (34) is valid in the range of coordinates ρ and z that satisfy the condition $z < \gamma \rho$. In this range of coordinates the packet under consideration moves with the velocity of light in the z axis direction.

So on distances $z < 2\gamma^2 \lambda$ the considered wave packet practically coincides with the initial one (at $t=0$). Substantial transformation of the packet would happen only on distances

$$z > 2\gamma^2 \lambda \quad (35)$$

In this case for the evaluation of the integral in (32) over ϑ one could apply the method of stationary phase. As a result of using of this method we find that

$$\phi_k(\vec{r}, t) = -\frac{2i}{\pi} \frac{1}{\theta_0^2 + \gamma^{-2}} \frac{1}{kr} e^{ik(r-t)} \quad (36)$$

where $r \approx z + \rho^2 / 2z$ and $\vartheta_0 = \rho / z$ is the point of stationary phase of the integral (32). We see that the components (36) of our packet have in the case under consideration the form of diverging spherical waves. Under this condition the angle ϑ_0 corresponds to the direction of radiation, and the function before the diverging wave describes the angular distribution of the radiation. So, the condition (35) draws out the wave zone in application to given problem.

The value $2\gamma^2 \lambda$ presenting in the condition (35) is known in the theory of radiation by ultra-relativistic particles as the formation length or the coherence length [1, 2].

3. The bremsstrahlung at an electron instantaneous scattering

3.1. The electromagnetic field structure at an electron instantaneous scattering. The 'half-bare' electron

The electromagnetic wave packets similar to the ones considered above arise, for example, in the processes of bremsstrahlung by relativistic electron at its instantaneous scattering to a large angle and in the process of transition radiation during an electron traverse of thin metallic plate in vacuum. The present section is dedicated to the analysis of evolution of electromagnetic wave packets and peculiarities of formation of radiation by relativistic electron in the wave and the pre wave zones in the first process, while the next one – to the analogous questions concerning the second one.

Let a relativistic electron move along the z axis with the velocity \vec{v} and at the moment of time $t=0$ in the point $z=0$ change it abruptly from \vec{v} to \vec{v}' (Figure 1).

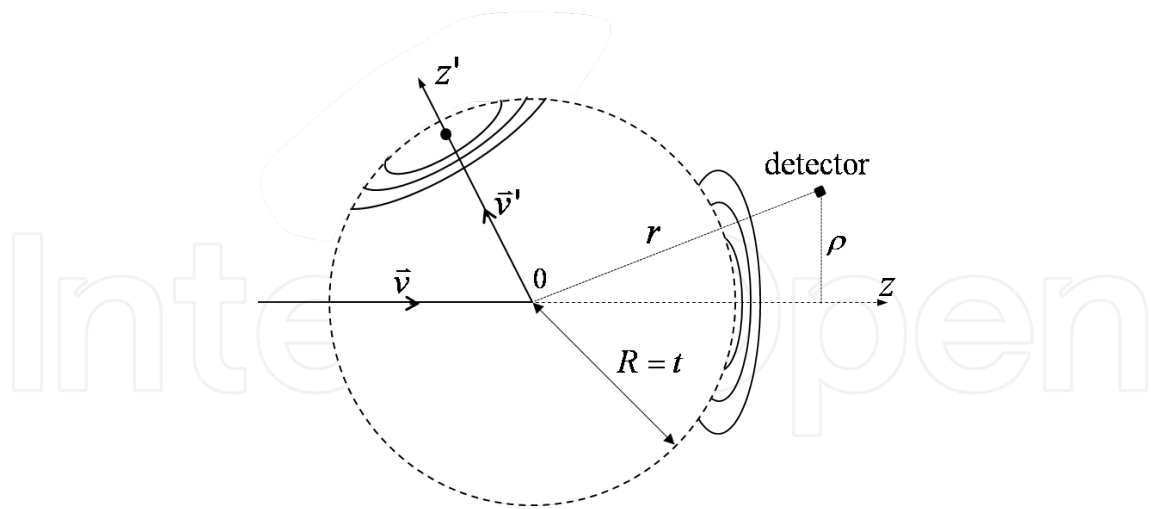


Figure 1. The picture of total field after instantaneous scattering of an electron to a large angle.

Scalar and vector potentials of the total electromagnetic field, which takes place in such process can be defined from inhomogeneous Maxwell equations

$$\begin{aligned} \Delta\phi - \frac{\partial^2\phi}{\partial t^2} &= -4\pi\rho(\vec{r}, t), \\ \Delta\vec{A} - \frac{\partial^2\vec{A}}{\partial t^2} &= -4\pi\vec{j}(\vec{r}, t) \end{aligned} \tag{37}$$

in which ρ and \vec{j} are respectively the charge and the current density of the particle, which in the case of a single point particle motion should be taken as

$$\begin{aligned} \rho(\vec{r}, t) &= e\delta(\vec{r} - \vec{r}(t)) \\ \vec{j}(\vec{r}, t) &= e\vec{v}(t)\delta(\vec{r} - \vec{r}(t)) \end{aligned} \tag{38}$$

where $\delta(\vec{r})$ is the delta-function and $\vec{r}(t)$ and $\vec{v}(t)$ are respectively the particle's trajectory and its velocity. The solution of the set of equations (37) has the physical sense if it has the form of the retarded potentials. In our case of a point particle the required form of the solution of (37) and (38) is the following [21]:

$$\{\vec{A}, \phi\} = e \int_{-\infty}^{+\infty} \frac{dt'}{|\vec{r} - \vec{r}(t')|} \{\vec{v}(t'), 1\} \delta(t' - t + |\vec{r} - \vec{r}(t')|) \tag{39}$$

In the case of a uniform particle motion with the velocity \vec{v} along the z axis (39) leads to the following explicit expressions for the potentials:

$$\vec{A}_v(\vec{r}, t) = \frac{e\vec{v}}{\sqrt{\rho^2\gamma^{-2} + (z-vt)^2}}, \quad \phi_v(\vec{r}, t) = \frac{e}{\sqrt{\rho^2\gamma^{-2} + (z-vt)^2}} \quad (40)$$

which are the coulomb potentials of the moving particle. Here $\gamma = E/m$ is the particle's Lorentz-factor and ρ is the absolute value of the coordinate $\vec{\rho}$ of the observation point in the plane orthogonal to the z axis. The formula (40) for $\phi(\vec{r}, t)$ shows that the equipotential surfaces of the field of the particle, which moves uniformly straightforward are the ellipsoids flattened in the direction of particle motion.

In order to obtain the solution of (37) for the considered case of the particle instantaneous scattering it is convenient to express the potentials in the form of Fourier-integrals. Let us consider, for example, the vector potential:

$$\vec{A}(\vec{r}, t) = \frac{1}{(2\pi)^3} \int \vec{A}_k(t) e^{i\vec{k}\vec{r}} d^3k \quad (41)$$

In order to obtain the expansion in the form of the retarded potential we should calculate the Fourier-component $\vec{A}_k(t)$ with the use of the inverse Fourier-transform of the expression (39) for the vector potential:

$$\begin{aligned} \vec{A}_k(t) &= \int \vec{A}(\vec{r}, t) e^{-i\vec{k}\vec{r}} d^3r = \\ &= e \int_{-\infty}^{+\infty} dt' \int d^3r e^{-i\vec{k}\vec{r}} \frac{\vec{v}(t')}{|\vec{r}-\vec{r}(t')|} \delta(t'-t + |\vec{r}-\vec{r}(t')|) \end{aligned}$$

Making here the substitution $\vec{R} = \vec{r} - \vec{r}(t')$ and integrating the expression over d^3R it is possible finally to present the Fourier-expansion (41) of the vector potential in the following form:

$$\vec{A}(\vec{r}, t) = -\frac{e}{2\pi^2} \text{Im} \int \frac{d^3k}{k} e^{-i(\vec{k}\vec{r} - \vec{k}\vec{v}t)} \int_{-\infty}^t dt' \vec{v}(t') e^{i(\vec{k}\vec{r} - \vec{k}\vec{v}t')} \quad (42)$$

Substituting into (42) $\vec{v}(t') = \vec{v}\theta(-t') + \vec{v}'\theta(t')$, where $\theta(x)$ is the Heaviside theta function, we have for $t < 0$ (which is before the scattering moment):

$$\vec{A}(\vec{r}, t) = \frac{e}{2\pi^2} \text{Re} \int \frac{d^3k}{k} \frac{\vec{v}}{ck - \vec{k}\vec{v}} e^{i(\vec{k}\vec{r} - \vec{k}\vec{v}t)} \quad (43)$$

As integration over \vec{k} may show, (43) is nothing else than the Fourier-expansion of the own coulomb field (40) of a uniformly moving particle.

The different and rather interesting situation takes place after the scattering moment (for $t > 0$). In this case from (42) we obtain:

$$\vec{A}(\vec{r}, t) = \frac{e}{2\pi^2} \text{Re} \int \frac{d^3k}{k} \left\{ \frac{\vec{v}'}{k - \vec{k}\vec{v}'} [1 - e^{-i(k - \vec{k}\vec{v}')t}] e^{-i\vec{k}\vec{v}'t} + \frac{\vec{v}}{k - \vec{k}\vec{v}} e^{-ikt} \right\} e^{i\vec{k}\vec{r}} \quad (44)$$

The integration over \vec{k} gives in this case:

$$\vec{A}(\vec{r}, t) = \vec{A}_{\vec{v}'}(\vec{r}, t)\theta(t - r) + \vec{A}_{\vec{v}}(\vec{r}, t)\theta(r - t) \quad (45)$$

where $\vec{A}_{\vec{v}'}$ and $\vec{A}_{\vec{v}}$ are respectively the coulomb potentials of the particles, which uniformly move in the directions of z and z' axes. The analogous expressions for the scalar potential can be easily obtained by the same method.

The expression (43) shows that before the scattering moment the total field around the electron coincides with its own coulomb field, which moves with the velocity \vec{v} together with the electron. After the scattering, according to (44) and (45), the total field around the electron breaks into two parts represented by two items in the considered expressions.

The first item in braces in (44) corresponds to the nonequilibrium field, which the scattered electron has already managed to rebuild around itself by the moment of time t . This field consists of the own coulomb field of the electron, which moves along the z' axis with the velocity \vec{v}' (the first item in square brackets in (44) corresponds to it) and the packet of free waves, which moves in the same direction (it is described by the second item in square brackets). There is substantial interference between these fields due to which (as the first item in (45) shows) the total field vanishes in the region $r > t$, which the signal about the electron scattering at the moment of time $t = 0$ has not yet reached. Inside the sphere of radius $r = t$ the total field coincides with the equilibrium coulomb field of the electron.

The second item in braces in (44) describes the field, which as though 'tears away' from the electron at the scattering moment. It is a packet of free electromagnetic waves, which moves in the direction of the initial electron's velocity \vec{v} and gradually transforms into bremsstrahlung. This field is different from zero outside the sphere $r = t$ and vanishes inside it.

The equipotential surfaces of the scalar potential of the field around the electron after its scattering to a large angle are presented on Figure 1.

The behavior of certain Fourier-components of the nonequilibrium field of the electron after its scattering and of the field 'torn away' from the electron at its scattering is of special interest. According to (44), the values of k , which make contribution to each item of the nonequilibrium field of the electron has directions close to the direction of the electron's final velocity \vec{v}' . During the period of time $t < (k - \vec{k}\vec{v}')^{-1}$, however, both of these items substantially cancel each other due to the interference. Therefore the coulomb field, which corresponds to the scattered electron, does not appear instantly. Namely, during the period of time $t \leq (k - \vec{k}\vec{v}')^{-1}$ the Fourier components with wave vector k are nearly absent in the field around the scat-

tered electron. As the main contribution to the coulomb field is made by \vec{k} with directions close to the direction of the velocity \vec{v} this period of time is defined by the relation $t \leq 2\gamma^2/k$. Figuratively speaking we can say that after the scattering during such period of time the electron is in 'half-bare' state, which means without considerable part of its field. Such state of electron with nonequilibrium field manifests itself during further scatterings of the electron, which causes different effects of bremsstrahlung suppression during an electron's motion in substance, such as Landau-Pomeranchuk-Migdal effect [23-25], the effect of radiation suppression in thin layer of substance (TSF-effect [26-28]), etc. Recently the series of detailed experimental investigations of these effects at ultra high energies was performed on SLAC [29,30] and CERN[31,32] accelerators, which confirmed the main theoretical predictions.

The notion of a 'half-bare' electron was introduced in the papers of E.L. Feinberg [33,34] who studied the time evolution of the state vector of the system 'electron + photon' after the scattering of a fast electron to a large angle on atom. The classical theory of this effect was given in [2, 27, 35].

Let us note that during the period of time $t = 2\gamma^2/\omega$ the electron covers the distance

$$l_C = 2\gamma^2 v / \omega \quad (46)$$

In the theory of radiation by relativistic electrons the length l_C has a name of the coherence length of the radiation process [1,2]. Within this length the interference effects in radiation are significant. For ultra relativistic electrons in the region of small frequencies of radiated waves the length l_C can have macroscopic size, which exceeds not only interatomic distance of the substance but the size of the experimental facility as well. Indeed, in the millimeter range of the waves radiated by electrons with the energy of 50MeV the coherence length is $l_C = 20\text{m}$. It means that within such length after scattering the electron is in 'half-bare' state, which means that the Fourier-components of the corresponding wavelengths are absent in the field around the electron.

The field, which 'tears away' from the electron at its scattering (the second item in braces in (44)) has the structure similar to the one, which has the packet of free waves considered above in the equivalent photons method. Therefore the main peculiarities of the reconstruction of the 'torn away' field to the field of radiation will be the same as the considered above peculiarities of the wave packets evolution. Let us consider this process in detail.

3.2. The problem of measurement of bremsstrahlung characteristics

The results presented above show that for ultra relativistic electrons the radiation formation process develops on large distances along the initial and final directions of the electron motion, which can be of macroscopic size. In this case a detector, which registers the radiation characteristics can be situated both in the wave zone (which means on large distances $r \rightarrow \infty$ from the scattering point) and in the pre wave zone (which means on within the coherence length l_C of the radiation process). Let us show that the results of measurements can sub-

stantially differ in these cases. For this purpose let us consider the peculiarities of the formation of radiation from the 'torn-away' field on different distances from the scattering point [17]. The consideration will be made for a point detector situated in the point with coordinates $\vec{r}=(\vec{\rho}, z)$. Here the z axis is parallel to the vector of the electron's initial velocity and $\vec{\rho}$ are the coordinates in the orthogonal plane. In this case by the point detector we mean the detector, which measures the characteristics of electromagnetic waves (wave packet), which fall on a small element of surface situated in the point with the coordinates $\vec{r}=(\vec{\rho}, z)$ and seen at solid angle $d\Omega$ from the scattering point.

Making in the second item in (44) the variable substitution $k_z \rightarrow k$ by $k_z = \sqrt{k^2 - q^2}$ (k_z and q are respectively the components of the wave vector \vec{k} along the z axis and orthogonal to it) and denoting $k = \omega$, it is possible to present the Fourier-expansion of the 'torn-away' field scalar potential in the following form:

$$\phi(\vec{r}, t) = \frac{e}{\pi v^2} \int_{-\infty}^{+\infty} d\omega e^{-i\omega t} \int_0^{|\omega|} dq \frac{q J_0(q\rho)}{q^2 + \omega^2/v^2\gamma^2} Q(z) \tag{47}$$

where

$$Q(z) = \frac{\omega \cos(\sqrt{\omega^2 - q^2} z)}{\sqrt{\omega^2 - q^2}} + i v \sin(\sqrt{\omega^2 - q^2} z) \tag{48}$$

and $\rho = |\vec{\rho}|$.

In (47) and (48) the square root $\sqrt{\omega^2 - q^2}$ is considered to be a single-valued branch of the analytical function, which is equal to $|\sqrt{\omega^2 - q^2}|$ for $\omega > q$ and $-|\sqrt{\omega^2 - q^2}|$ for $\omega < -q$.

In ultra relativistic case ($\gamma \gg 1$) the range of q , which make the main contribution to the integral (47) is $q \leq \omega/\gamma \ll \omega$ and it is possible to expand the square roots $\sqrt{\omega^2 - q^2}$ in (48) in the small factor q/ω . Let us leave the items proportional to the second power of q/ω in the arguments of sine and cosine, while in the other parts of the expression (48) neglect them. Moreover the integration over q can be extended to the region $0 < q < \infty$. This leads to the following expression for the 'torn-away' field potential in ultra relativistic case:

$$\phi(\vec{r}, t) = \frac{e}{\pi} \int_{-\infty}^{+\infty} d\omega \int_0^{+\infty} dq \frac{q J_0(q\rho)}{q^2 + \omega^2/v^2\gamma^2} e^{i\omega(z-t) - iq^2 z/2\omega} \tag{49}$$

The equations (37) are presented in Lorentz gauge

$$\operatorname{div} \vec{A} + \frac{\partial \phi}{\partial t} = 0 \quad (50)$$

If we know $\phi(\vec{r}, t)$, we can derive the vector potential component A_z from this equation ($\vec{A}_\perp = 0$ in this case). In the considered case the vector potential equals to the scalar one up to the neglected items of the order of $(q/\omega)^2$.

In order to determine the total energy radiated in the direction of a small area $d\vec{s} = \vec{n} r^2 d\omega$ situated in the point $\vec{r} = (\vec{\rho}, z)$ we can calculate the flux of the Poynting vector through this area during the whole time of the particle motion

$$d\varepsilon = \frac{r^2 d\omega}{4\pi} \int dt (\vec{E} \times \vec{H}) \cdot \vec{n}$$

where $d\omega$ is the element of solid angle in the direction of radiation $\vec{n} = \vec{r}/r$. The fields \vec{E} and \vec{H} in this expression are the fields of the packet of free electromagnetic waves (the field 'torn-away' from the electron at its scattering). They are related to scalar ϕ and vector \vec{A} potentials of the radiation field by

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \phi, \quad \vec{H} = \nabla \times \vec{A} \quad (51)$$

Proceeding to the Fourier-expansions of the fields ϕ and \vec{A} over frequency ω we obtain the following expression for the radiation spectral-angular density:

$$\frac{d\varepsilon}{d\omega d\omega} = \frac{r^2}{4\pi^2} [\vec{E}_\omega(\vec{r}) \times \vec{H}_{-\omega}(\vec{r})] \cdot \vec{n} \quad (52)$$

in which $\omega \geq 0$.

With the use of the Maxwell equation $\vec{H}_\omega = (-i/\omega) \nabla \times \vec{E}_\omega$, the radiation spectral-angular density can be expressed in the terms of Fourier-component of the electric field alone:

$$\frac{d\varepsilon}{d\omega d\omega} = \frac{ir^2}{\omega} [\vec{E}_\omega \times (\nabla \times \vec{E}_{-\omega}(\vec{r}))] \cdot \vec{n} \quad (53)$$

Let us note that the formulae (52) and (53) are valid for arbitrary distances from the scattering point. Therefore they can be used for radiation consideration both in the wave and the pre wave zones. In the wave zone (which means in the region $r \rightarrow \infty$) they can be considerably simplified. Therefore, firstly, let us dwell on the consideration of this case.

On large distances from the scattering point ($r \rightarrow \infty$) a Fourier-component of the field of the radiation waves $\vec{E}_\omega(\vec{r})$ reconstructs into a packet of diverging waves, which amplitude is

proportional to the factor $r^{-1} \exp(i\omega r)$. The action of the Hamilton operator ∇ in (53) upon $E_{-\omega}$ in this case can be applied only to the factor $\exp(i\omega r)$:

$$\nabla \times \vec{E}_{-\omega} = -i\omega \vec{n} \times \vec{E}_{-\omega} \tag{54}$$

As a result we obtain the following expression for the radiation spectral-angular density for $r \rightarrow \infty$:

$$\frac{d\varepsilon}{d\omega d\Omega} = \frac{r^2}{4\pi^2} |\vec{E}_{\omega}(\vec{r})|^2 \tag{55}$$

Let us note that the formula (55) is valid on large distances from the scattering point ($r \rightarrow \infty$). While proceeding from (53) to (55) we took into account that $\vec{n} \vec{E}_{\omega} = 0$. This relation directly comes from the expression of Fourier-components of electric and magnetic fields in terms of potentials $\phi_{\omega}(\vec{r})$ and $\vec{A}_{\omega}(\vec{r})$

$$\vec{E}_{\omega} = -\nabla \phi_{\omega}(\vec{r}) + i\omega \vec{A}_{\omega}(\vec{r}), \quad \vec{H}_{\omega} = \nabla \times \vec{A}_{\omega}(\vec{r}) \tag{56}$$

where $\phi_{\omega}(\vec{r})$ and $\vec{A}_{\omega}(\vec{r})$ are defined from the expression (49).

In the case of ultrarelativistic particle in the region of characteristic for this process small radiation angles $\vartheta \ll 1$ the 'torn-away' electric field can be considered as transverse having only E_{\perp} component orthogonal to z axis. In this case the formula (55) for the radiation spectral-angular density has the following form:

$$\frac{d\varepsilon}{d\omega d\Omega} = \frac{r^2}{4\pi^2} |E_{\omega\perp}(\vec{r})|^2 \tag{57}$$

Using (49) we can derive the electric field Fourier-component orthogonal to z axis. Substituting it into (57) for the spectral-angular distribution of bremsstrahlung we achieve:

$$\frac{d\varepsilon}{d\omega d\Omega} = \left(\frac{ez}{\pi}\right)^2 \left| \int_0^{\infty} dq \frac{q^2 J_1(q\rho)}{q^2 + \omega^2/v^2\gamma^2} e^{-i\frac{q^2 z}{2\omega}} \right|^2 \tag{58}$$

For large distances from the scattering point, namely in the wave zone of the radiation process ($z \gg 2\gamma^2/\omega$) the integral in (58) can be calculated with the use of stationary phase method [36]. It leads to the well known expression for radiation distribution from the bremsstrahlung theory [1,4,37]:

$$\frac{d\mathcal{E}}{d\omega d\vartheta} = \frac{e^2}{\pi^2} \frac{\vartheta^2}{(\vartheta^2 + \gamma^{-2})^2} \tag{59}$$

where $\vartheta = \rho / z$ is the angle between the direction of radiation and the z axis. As we can see from (59) in the wave zone the radiation is mainly concentrated within characteristic angles $\vartheta \approx 1/\gamma$. The schematic form of the bremsstrahlung angular distribution in the wave zone is presented on the Figure 2 by the solid curve.

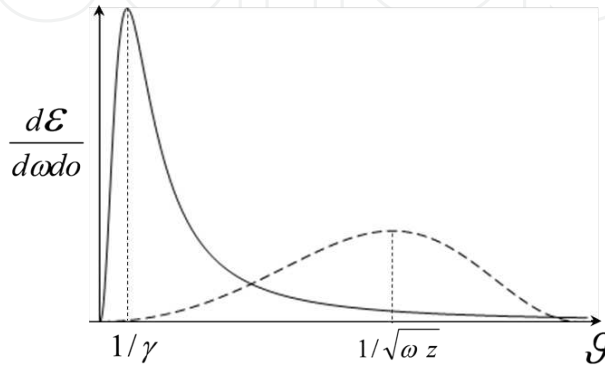


Figure 2. Bremsstrahlung angular distribution given by a point detector in the wave zone (solid curve) and in the pre-wave zone (dashed curve) for a certain frequency ω

For ultra high energies of the radiating particle in the region of characteristic small angles of radiation the expression (57) (and hence the expression (58)) is valid for the description of radiation spectral-angular density on small distances from the scattering point as well (in particular, in the pre-wave zone ($z \ll 2\gamma^2/\omega$) of the radiation process). Indeed, as the electric field can be considered transverse in this case than $\vec{E}_\omega = -\nabla\phi_\omega = -\partial\phi_\omega/\partial\vec{\rho}$. The equality of $\phi_\omega(\vec{r})$ and $A_\omega(\vec{r})$ leads to the following expression for the magnetic field:

$$\vec{H}_\omega = \nabla \times \vec{A}_\omega = \nabla\phi_\omega \times \vec{e}_z$$

where \vec{e}_z is a unit vector in the direction of z axis. Taking into account the orthogonality of $\nabla\phi_\omega$ and \vec{e}_z for the absolute value of the magnetic field we obtain $|\vec{H}_\omega| = |\nabla\phi_\omega|$, which is equal to the absolute value of the electric field. Moreover, these fields are orthogonal ($\vec{E}_\omega \vec{H}_\omega = \nabla\phi_\omega (\nabla\phi_\omega \times \vec{e}_z) = 0$). Taking into account the fact that the magnetic field $\vec{H}(\vec{r}, t)$ is the real function, which leads to the relation $\vec{H}_{-\omega} = \vec{H}_\omega^*$, we can present the general expression (52) in the following form:

$$\frac{d\mathcal{E}}{d\omega d\vartheta} = \frac{r^2}{4\pi^2} [E_{\omega\perp} E_{\omega\perp}^*(\vec{r})] \vec{e}_z \vec{n}$$

which coincides with (57) at small angles between \vec{e}_z and \vec{n} . Now let us consider radiation in the pre-wave zone on the basis of this formula.

In the pre-wave zone ($z \ll 2\gamma^2/\omega$) of the radiation process it is not possible to use the stationary phase method for the analysis of radiation characteristics. Here, making the substitutions $q = \omega x / \gamma$ and $\rho = z\vartheta$, we can present the integral (58) in the form:

$$\frac{d\varepsilon}{d\omega d\vartheta} = \left(\frac{e\omega z}{\pi\gamma}\right)^2 |I_1 - I_2|^2 \quad (60)$$

where

$$I_1 = \int_0^{+\infty} dx J_1(\omega z \gamma^{-1} x \vartheta) e^{-i \frac{\omega z}{2\gamma^2} x^2}$$

$$I_2 = \int_0^{+\infty} dx \frac{J_1(\omega z \gamma^{-1} x \vartheta)}{x^2 + 1} e^{-i \frac{\omega z}{2\gamma^2} x^2}$$

In the case $\gamma \gg 1$ the absolute value of the integral I_2 is negligibly small comparing to the corresponding value of I_1 and for spectral-angular density of bremsstrahlung in the pre-wave zone we obtain:

$$\frac{d\varepsilon}{d\omega d\vartheta} = \left(\frac{e\omega z}{\pi}\right)^2 |I_1|^2 = \frac{4e^2}{\pi^2} \frac{1}{\vartheta^2} \sin^2\left(\frac{\omega z \vartheta^2}{4}\right) \quad (61)$$

From (61) we can conclude that in the pre-wave zone the radiation is mainly concentrated within angles $\vartheta \approx 2/\sqrt{\omega z}$, which exceed the characteristic angles $\vartheta \approx 1/\gamma$ of the wave zone. Therefore in the pre-wave zone ($z \ll 2\gamma^2/\omega$) the point detector gives broader angular distribution of radiation (dashed curve on Figure 2) than in the wave zone ($z \gg 2\gamma^2/\omega$). Moreover this distribution depends on the frequency ω of the radiated waves.

By the point detector we mean here the detector of the smaller size $\delta\rho$ than the transversal radiation length of the process $l_T \approx \gamma/\omega$, which is the characteristic transversal distance on which at the moment of time $t=0$ the Fourier harmonics of frequency ω are concentrated in the wave packet (49). Such detector registers the radiation of frequency ω , which falls on a small domain of space, where the detector is situated.

The measurements, however, can be made by the extended detector of the larger size than the characteristic transversal length of the radiation process, so that $\delta\rho \gg l_T$. Such detector registers not only the waves of frequency ω , which fall on the small element of surface with coordinates $\vec{\rho}$ and z , as the point detector does, but all the electromagnetic waves of frequency ω , which propagate in the direction of wave vector k ($|k| = \omega$). In order to calculate the bremsstrahlung spectral-angular distribution, which is registered by an extensive detector, which is a plate of large size, we need to integrate the expression (58) over the entire considered plate and express the obtained result in the form of an integral over the directions of

wave vectors of radiated waves. The integrand in this case will be nothing else than the required distribution. In our case after performing the procedures described above we can present the expression (58) in the following form:

$$\frac{d\varepsilon}{d\omega d\vartheta_\gamma} = \frac{e^2}{\pi^2} \frac{\vartheta_\gamma^2}{(\vartheta_\gamma^2 + \gamma^{-2})^2} \quad (62)$$

where $\vartheta_\gamma = q/\omega$ is the angle between the direction of the wave vector \vec{k} and the z axis. Hence the bremsstrahlung spectral-angular distribution obtained by the extended detector coincides with the one (59) obtained by the point detector in the wave zone. But unlike the case with point detector this distribution does not depend on the distance from the scattering point and is the same both in the wave and the pre-wave zones.

4. Transition radiation by relativistic electron on thin metallic plate

4.1. Scalar and vector potentials of transition radiation field

The electromagnetic wave packets of the structure analogous to the one considered in the process of an electron instantaneous scattering take place also in the process of relativistic electron traverse of thin conducting plate. Let us consider a problem about transition radiation that arises during normal traverse of thin ideally conducting plate, situated in the plane $z=0$, by an electron, which moves along the z axis from $z=-\infty$ to $z=+\infty$ (Figure 3). Let us investigate the structure of electromagnetic fields that take place before and after the electron's traverse of the plate in vacuum [16-18].

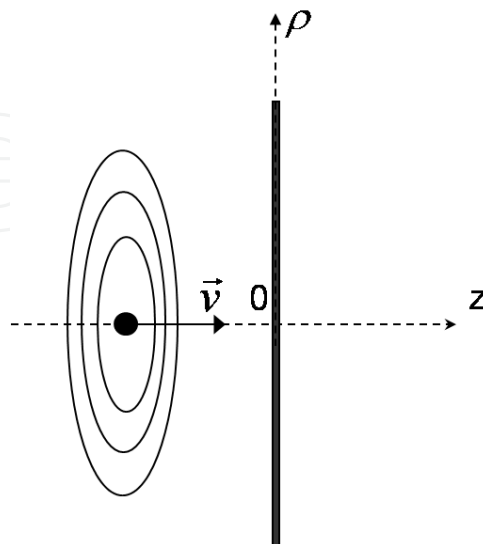


Figure 3. The electron normal traverse of thin metallic plate.

Scalar and vector potentials of the electromagnetic field, which is generated by an electron moving in vacuum are the solutions of inhomogeneous Maxwell equations (37). The equations set (37) in the considered problem should be supplemented by a boundary condition, which corresponds to the fact that on the plate's surface the tangential component of the total electric field equals zero. The general solutions of the equations (37) for the electron, which moves uniformly with the velocity \vec{v} can be represented in the form of the following Fourier-expansions of the potentials $\phi(\vec{r}, t)$ and $\vec{A}(\vec{r}, t)$:

$$\begin{aligned} \phi(\vec{r}, t) &= \int \frac{d^3k d\omega}{(2\pi)^4} e^{i(\vec{k}\vec{r} - \omega t)} [\phi_{k,\omega}^C \delta(\omega - \vec{k}\vec{v}) + \phi_{k,\omega}^f \delta(k^2 - \omega^2)] \\ \vec{A}(\vec{r}, t) &= \int \frac{d^3k d\omega}{(2\pi)^4} e^{i(\vec{k}\vec{r} - \omega t)} [\vec{A}_{k,\omega}^C \delta(\omega - \vec{k}\vec{v}) + \vec{A}_{k,\omega}^f \delta(k^2 - \omega^2)] \end{aligned} \quad (63)$$

The first items in (63) are the Fourier-expansion of the electron's Coulomb field, for which

$$\phi_{k,\omega}^C = -\frac{8\pi^2 e}{\omega^2 - k^2}, \quad \vec{A}_{k,\omega}^C = \vec{v} \phi_{k,\omega}^C \quad (64)$$

The second items in (63) are the Fourier-expansion of the field of induced surface currents on the plate (we will name it the free field), for which in vacuum $|\vec{k}| = |\omega|$. On large distances from the region in which the transformation of the surrounding electron field takes place this items form the transition radiation field. It can be derived from the boundary condition for the total electric field $\vec{E} = \vec{E}^C + \vec{E}^f$ on the surface of the plate:

$$\vec{E}_{\perp}^C(\vec{\rho}, z=0, t) + \vec{E}_{\perp}^f(\vec{\rho}, z=0, t) = 0 \quad (65)$$

Here \vec{E}^C is the electron's Coulomb field and \vec{E}^f is the field of induced surface currents on the plate. Moreover it is required that the free field produced by the plate propagates on the left and on the right of the plate respectively in the negative and positive directions of z axis. The scalar and the vector potentials define electric and magnetic fields by the relation (51). Due to the symmetry of the problem the vector potential is directed along the particle velocity \vec{v} :

$$\vec{A}(\vec{r}, t) = \vec{v} A(\vec{r}, t) \quad (66)$$

and the relation

$$A^C(\vec{r}, t) = \phi^C(\vec{r}, t) \quad (67)$$

for the particle's field in vacuum is valid. In general case such relation between potentials is not valid.

Using Fourier expansion (63) it is possible to derive the potential $\phi(\vec{r}, t)$ from the condition (65). For this let us perform the integration over the component k_z in (63). Taking also into account the relation between potentials and fields (51) we obtain the following expression for transversal component of the electric field:

$$\vec{E}_\perp(\vec{r}, t) = -\frac{\partial}{\partial \rho} \int \frac{d^2 k_\perp d\omega}{(2\pi)^4} e^{i(k_\perp \vec{\rho} + k_z z - \omega t)} \left[\phi_{k, \omega}^C \Big|_{k_z = \frac{\omega}{v}} + \frac{1}{2 \sqrt{\omega^2 - k_\perp^2}} \left(\phi_{k, \omega}^f \Big|_{k_z = \sqrt{\omega^2 - k_\perp^2}} + \phi_{k, \omega}^f \Big|_{k_z = -\sqrt{\omega^2 - k_\perp^2}} \right) \right] \quad (68)$$

For $z=0$, according to (65) this component should equal zero. From this we find that

$$-\phi_{k, \omega}^C \Big|_{k_z = \omega/v} = \frac{1}{2 \sqrt{\omega^2 - k_\perp^2}} \left(\phi_{k, \omega}^f \Big|_{k_z = \sqrt{\omega^2 - k_\perp^2}} + \phi_{k, \omega}^f \Big|_{k_z = -\sqrt{\omega^2 - k_\perp^2}} \right) \quad (69)$$

The values $k_z = \pm \sqrt{\omega^2 - k_\perp^2}$ satisfy the dispersion relation $\omega^2 = k_z^2 + k_\perp^2$, which is defined by the respective δ -function in (63). The sign before the square root $\sqrt{\omega^2 - k_\perp^2}$ determines the direction of propagation of plane waves (Fourier components) with given values of ω and $|k_\perp|$ [1, 4]. Indeed, the equation of a plane wave constant phase along the z -axis is $isk_z z - \omega t = \text{const}$. The plane waves, which the free field produced by the plate consists of, should propagate away from the plate. Hence for $\omega > 0$ on the right of the plate (which means $z > 0$) it is necessary to take into account only Fourier-components with positive sign before the root $\sqrt{\omega^2 - k_\perp^2}$ in (62), while for $\omega < 0$ and $z > 0$ only Fourier-components with negative sign before this root in (68). In the region $z < 0$ for $\omega > 0$ and $\omega < 0$ in (68) we should take into account the items with opposite signs before the root $\sqrt{\omega^2 - k_\perp^2}$ relatively to the case for $z > 0$. The value of the square root itself is considered either positive or to belong to the upper complex half plane.

Thus, taking into account all requirements mentioned above we can write the scalar potential of the free field in the following form:

$$\phi^f(\vec{r}, t) = -\frac{e}{2\pi^2 v} \int d^2 k_\perp \int_{-\infty}^{\infty} d\omega \frac{1}{k_\perp^2 + \omega^2 / p^2} e^{i(z\omega \sqrt{1 - k_\perp^2 / \omega^2} - \omega t + k_\perp \vec{\rho})} \quad (70)$$

where $p = v\gamma$ (γ – electron's Lorentz-factor). It is a packet of free electromagnetic waves, which gradually turns into the field of transition radiation in such way that each harmonic with frequency ω reconstructs into diverging spherical wave on distance $z > l_C \approx 2\gamma^2 / \omega$, which is the formation length of the radiation process.

Let us note that the value of $|k_{\perp}|$ in (63) is arbitrary. Therefore it is necessary to perform the integration in (63) not only over travelling waves $k_{\perp}^2 < \omega^2$, but over surface ones $k_{\perp}^2 > \omega^2$ as well.

Using (70) and the Lorentz gauge (50) in which the equations (37) are presented we can derive the vector potential:

$$A^f(\vec{r}, t) = -\frac{e}{2\pi^2} \int d^2k_{\perp} \int_{-\infty}^{\infty} d\omega \frac{1}{k_{\perp}^2 + \omega^2/p^2} \frac{1}{\sqrt{1 - k_{\perp}^2/\omega^2}} e^{i(z\omega\sqrt{1 - k_{\perp}^2/\omega^2} - \omega t + k_{\perp}\rho)} \quad (71)$$

Making in (70) the substitution $|k_{\perp}| = |\omega|x$ and separating the contributions to the potential by the free field of travelling and surface waves, we can write the potential of this field in the following form:

$$\phi^f(\vec{r}, t) = \Phi_1(\vec{r}, t) + \Phi_2(\vec{r}, t) \quad (72)$$

where

$$\Phi_1(\vec{r}, t) = -\frac{2e}{\pi v} \int_0^1 \frac{xdx}{x^2 + p^{-2}} \int_0^{\infty} d\omega J_0(\omega x \rho) \cos[\omega(|z|\sqrt{1-x^2} - t)] \quad (73)$$

$$\Phi_2(\vec{r}, t) = -\frac{2e}{\pi v} \int_1^{\infty} \frac{xdx}{x^2 + p^{-2}} \int_0^{\infty} d\omega J_0(\omega x \rho) \cos(\omega t) e^{-|z|\omega\sqrt{x^2-1}} \quad (74)$$

Deriving (72), we performed in (70) integration over azimuth angle between k_{\perp} and ρ and proceeded from integration over ω along the interval $-\infty < \omega < +\infty$ to integration over only positive values of this variable.

The corresponding expressions for vector potential $A^f = A_1 + A_2$, according to (71), differ from (73) and (74) only by additional factor $1/\sqrt{1-x^2}$ in the integrands.

4.2. The structure of transition radiation field

Let us discuss the structure of the fields that arise during the electron traverse of thin ideally conducting plate. Firstly, let us consider the structure of this field along the z axis for $\rho=0$ [16]. This case is interesting for the fact that the calculation of all the integrals in (73) and (74) essentially simplifies for $\rho=0$. Nevertheless, in this case all the main peculiarities of free waves formation in the considered process remain intact.

As a result of rather simple calculations (see Appendix) we obtain the following expression for the field produced by the plate on the z axis for $\rho=0$:

$$\phi^f(z, t) = -\frac{e}{|z| - vt} \theta(|z| - t) - \frac{e}{|z| + vt} \theta(t - |z|) \quad (75)$$

The total field produced by the electron and the plate can be obtained by addition of the expression (75) and the electron's own coulomb field on the z axis:

$$\phi^c(z, t) = e / |z - vt|$$

The obtained results show that for $t < 0$ the total field produced by the electron and the plate in the region $z < 0$ is the electron's own coulomb field, which moves towards the plate and the field, which coincides with the field of electron's image inside the plate. By the field of electron's image we assume the field, which is created by an imaginary particle with a charge of the opposite sign, which is situated on the opposite side of the plate and moves symmetrically to the electron relatively to the plate. In the region $z > 0$ the total field equals zero for $t < 0$.

For $t > 0$ the picture of the total field distribution is the following. In the region $z < 0$ for $|z| > t$ the total field is defined by the electron's field in this coordinate region (the electron in this case is situated on the right of the plate) and the field of its 'image', which moves in the direction opposite to the z axis. In the region $z < 0$ for $|z| < t$ the total field equals zero. In the region of positive z values for $z > t$ the total field equals zero while for $z < t$ this field is the sum of the field of the electron, which is situated in this coordinate region for $t > 0$ and the field of its 'image' situated on the left of the plate.

The integrals in (73) and (74) can be analytically calculated as well. After rather long calculations we finally obtain the following expression for $\phi^f(\vec{r}, t)$:

$$\phi^f(\vec{r}, t) = -\frac{e}{\sqrt{\rho^2 \gamma^{-2} + (|z| - vt)^2}} \theta(r - t) - \frac{e}{\sqrt{\rho^2 \gamma^{-2} + (|z| + vt)^2}} \theta(t - r) \quad (76)$$

The electron's own coulomb field has the following form:

$$\phi^c(\vec{r}, t) = \frac{e}{\sqrt{\rho^2 \gamma^{-2} + (z - vt)^2}} \quad (77)$$

The structure of the expressions for scalar potential is the same as the structure of these expressions for $\rho=0$. If $t < 0$ then, according to (76) and (77), the total field in the left half-space is equal to the sum of the electron's coulomb field in this region and the field of its image:

$$\phi(\vec{r}, t) = \frac{e}{\sqrt{\rho^2 \gamma^{-2} + (z - vt)^2}} - \frac{e}{\sqrt{\rho^2 \gamma^{-2} + (|z| - vt)^2}}$$

In the right half-space the total field equals zero for $t < 0$.

After electron's traverse of the plate, which means for $t > 0$, according to (76) and (77), the total field in the left half-space is defined by the formula

$$\phi(\vec{r}, t) = \left[\frac{e}{\sqrt{\rho^2 \gamma^{-2} + (z - vt)^2}} - \frac{e}{\sqrt{\rho^2 \gamma^{-2} + (|z| - vt)^2}} \right] \theta(r - t) \quad (78)$$

In the right half-space, where the electron is situated after the traverse of the plate, the total field has the following form:

$$\phi(\vec{r}, t) = \left[\frac{e}{\sqrt{\rho^2 \gamma^{-2} + (z - vt)^2}} - \frac{e}{\sqrt{\rho^2 \gamma^{-2} + (z + vt)^2}} \right] \theta(t - r) \quad (79)$$

Thus for $t > 0$ the picture of the total field, which is created by the electron-plate system is as following. In the left half-space in the coordinate region $r > t$ it is a sum of the electron's coulomb field of the opposite sign reflected from the plate and the own field of electron, which is situated on the right of the plate. The reflected field in this case moves with velocity $-\vec{v}$ in the direction opposite to the direction of electron's motion. In the coordinate region $r < t$, which the signal about the electron's traverse of the plate at $t = 0$ has already reached, the total field equals zero. In the right half-space for $t > 0$ and $r < t$ the total field equals the sum of the fields of the electron and its 'image' on the left of the plate. For $r > t$ the total field equals zero.

The analogous expressions can be obtained for vector potential as well. Namely, for $t > 0$ the total field vector potential (the sum of particle's coulomb field and radiation potentials) has the following form:

$$\vec{A}(\vec{r}, t) = \vec{v} \left[\frac{e}{\sqrt{\rho^2 \gamma^{-2} + (z - vt)^2}} + \frac{e}{\sqrt{\rho^2 \gamma^{-2} + (|z| - vt)^2}} \right] \theta(r - t) \quad (80)$$

The expression in square brackets in (80) differs from the same expression for scalar potential (78) only by the sign of second item. The reason of this can be understood from the following reasoning. For $z < 0$ in the region $r > t$ the field (78) is the difference between two coulomb fields, the sources of which are the electron and its image. The vector potential of the coulomb field is related to its scalar potential by $\vec{A} = \vec{v}\phi$, where \vec{v} is the velocity of the field source. As electron moves with velocity \vec{v} and its image - with velocity $-\vec{v}$, their vector potentials respectively equal $\vec{A}_e = \vec{v}\phi_e$ and $\vec{A}_i = -\vec{v}\phi_i$. It is the presence of the 'minus' sign in the expression for \vec{A}_i that causes the discussed difference in the signs of the items in square brackets in (78) and (80).

In the region $z > 0$ for $t > 0$ the vector potential has the following form:

$$\vec{A}(\vec{r}, t) = \vec{v} \left[\frac{e}{\sqrt{\rho^2 \gamma^{-2} + (z - vt)^2}} + \frac{e}{\sqrt{\rho^2 \gamma^{-2} + (z + vt)^2}} \right] \theta(t - r) \quad (81)$$

The obtained results are valid for arbitrary electron velocities. The case of an ultra relativistic particle is of special interest because for such particles the reconstruction of the total field, created by the plate and the electron after its traverse of the plate, into the field of radiation occurs on large distances. The results obtained in this case are illustrated by Figure 4. Here the equipotential surfaces of the scalar potential of the field reflected to the left half-space and the field around the electron on the right of the plate are presented for $t > 0$.

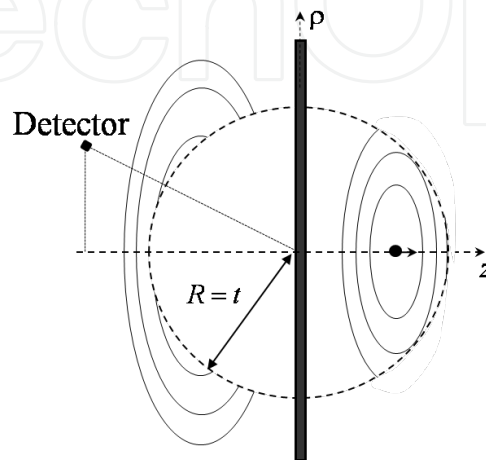


Figure 4. The total field in ultra relativistic case for $t > 0$.

In order to understand what occurs with the electric field on the surface of the sphere of radius $r = t$ with the center in the point of the electron traverse of the plate (θ - sphere) let us consider the structure of the force lines of the total field for $t > 0$ and, for example, $z < 0$.

While building the force lines it is necessary to take into account that they should originate or end either on charges or in the infinity and not to cross each other. In the case of an infinite plate all the field lines originate and end either on the surface charges of the plate or on the electron, which traverses it.

Thus each force line of the total field (78), which originates on a surface charge of the plate somewhere in the area $\rho > t$ and stretches through the space region $r > t$ to the θ - sphere, should be refracted and stretch further along the surface of the sphere, ending on another surface charge of the plate at $\rho = t$ (Figure 5). It is the force lines, which overlap each other on the θ - sphere are the force lines of the transition radiation field. Indeed, the field on the θ - sphere propagates in the radial direction with the speed of light and is perpendicular to this direction. Moreover, as will be shown in further discussion, this field decreases with the distance as $1/r$.

It is necessary to note that the given picture of the force lines indicates the necessity of the existence of the field (78) outside the θ - sphere along with the radiation field on it. It is only in this case that the force lines of the radiation field, which originate (end) on the surface charges of the plate and stretch along the θ - sphere can proceed to the region $r > t$ forming the lines of the field (78) and not intersect or break on the θ - sphere.

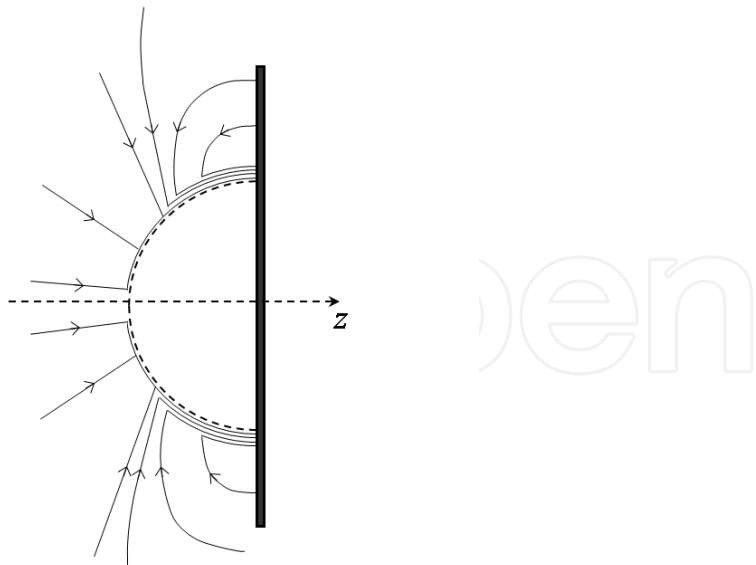


Figure 5. The picture of the total field force lines for $z > 0, z < 0$.

4.3. The transition radiation field and its intensity

In ultra relativistic case ($\gamma \gg 1$) the range of k_{\perp} , which make the main contribution to the integral (70), which represents the transition radiation field is $k_{\perp} \leq \omega / \gamma \ll \omega$ and it is possible to make in (70) the same expansion of the square root $\sqrt{\omega^2 - k_{\perp}^2}$ in the small factor k_{\perp} / ω as we did in (47) and (48) for the bremsstrahlung field. Such expansion represents the fact that transition radiation is considered at small angles around the particle's trajectory $\vartheta = k_{\perp} / \omega \approx 1 / \gamma$ within which the most part of the radiation is concentrated in ultra relativistic case. In the result of the expansion of (70) we obtain the expression for the transition radiation field, which coincides with the analogous expression (49) for the bremsstrahlung field. This means that the packets of free electromagnetic waves, which arise in the processes of electron instantaneous scattering and the particle traverse of thin metallic plate have similar (but not totally identical) structure and the radiation effects, which take place in these processes should be analogous.

Let us consider the radiation, which arises during an electron normal traverse of thin ideally conducting plate [16-18]. In this case by radiation we mean the part of the electromagnetic energy, which belongs to the frequency interval $(\omega, \omega + d\omega)$ and falls into the small detector, which is placed on different distances r from the point of electron traverse of the plate. The detector's position relative to this point is defined by the transverse $\vec{\rho}$ and longitudinal z coordinates (Figure 4). The radiation direction ϑ is then defined by the ratio of these coordinates:

$$\vartheta = \text{arctg} \frac{\rho}{|z|} \quad (82)$$

In further discussion we will mainly concentrate our attention on the peculiarities of the radiation formation process in the left half-space, which is the region of negative z . Naturally, the general expression (55) for the radiation spectral-angular density on large distances from the target is valid in this case as well. The discussed structural similarity of the fields in the processes of electron instantaneous scattering and electron traverse of thin metallic plate also gives us the possibility to use the expression (57) for the description of the transition radiation field in the range of small angles $\vartheta \approx 1/\gamma$ characteristic to this process on arbitrary distances z from the plate. Firstly, let us consider the transition radiation in the wave zone.

Let us show that in the region of large distances from the target ($r \rightarrow \infty$) the formula (55) gives the well known result from the theory of transition radiation by electron on metallic plate [1, 4, 37]. Indeed, in differentiating of (78) and (80) the items proportional to derivatives of the square brackets will give the total field outside the θ - sphere, which decreases with the distance r faster than r^{-1} . The item proportional to the derivative of the θ - function is on large distances the diverging wave, which amplitude is proportional to r^{-1} . This item defines all the transition radiation characteristics.

The scalar ϕ^f and vector \vec{A}^f potentials on the θ - sphere can be written as

$$\phi = -2v \frac{e}{r} \frac{\cos \vartheta}{1 - v^2 \cos^2 \vartheta} \theta(r-t), \quad \vec{A} = -\vec{v} \frac{\phi}{\cos \vartheta} \quad (83)$$

where ϑ is the angle between r and $-\vec{v}$. So the transition radiation electric field is totally defined by the spatial derivative of the scalar potential:

$$\vec{E}^f(\vec{r}, \vartheta, t) = 2v \frac{e}{r} \frac{\delta(r-t)}{1 - v^2 \cos^2 \vartheta} \left(\frac{\vec{v}}{v} + \vec{n} \cos \vartheta \right) \quad (84)$$

The Fourier-component of this expression has the following form:

$$\vec{E}_\omega^f(\vec{r}, \vartheta) = 2v \frac{e}{r} \frac{1}{1 - v^2 \cos^2 \vartheta} \left(\frac{\vec{v}}{v} + \vec{n} \cos \vartheta \right) e^{i\omega r} \quad (85)$$

Let us note that the relation $\vec{n} \vec{E}_\omega = 0$ is valid for (81) as $\vec{n} \vec{v} = -v \cos \vartheta$.

Substituting (85) into (55) we obtain the well known expression for spectral-angular density of backward transition radiation [1, 4, 5]:

$$\frac{d\varepsilon}{d\omega d\Omega} = \frac{e^2 v^2}{\pi^2} \frac{\sin^2 \vartheta}{(1 - v^2 \cos^2 \vartheta)^2} \quad (86)$$

The presented derivation of the formula (86) for the transition radiation spectral-angular density is based on the analysis of the electron's field reflected from the plate on large distances $r \rightarrow \infty$ from the plate. In this case by large distances we mean the distances from the target, which substantially exceed the coherence length of the radiation process [1, 2]

$$l_C(\vartheta) = \frac{\omega^{-1}}{1 - v \cos \vartheta} \quad (87)$$

For ultrarelativistic electrons the transition radiation is mainly concentrated inside a narrow cone with the opening angle $\vartheta \approx 1/\gamma$. In this region of angles ϑ the coherence length (87) quickly increases with the increase of the electron's Lorentz-factor:

$$l_C(\vartheta) = \frac{2\gamma^2/\omega}{1 + \gamma^2\vartheta^2} \quad (88)$$

For large γ and small ω this length can have ultra large values, which allow, for example, a detector to be situated within this length. The analysis of the transition radiation process in this case requires the development of the methods, which allow to consider the radiation process not only on large distances from the target comparing to the coherence length, l_C , but on the small ones as well. One of such methods is based on the application in the considered problem the Fourier-expansions of scalar, ϕ^f , and vector, \vec{A}^f , potentials of the free field, which is generated by the plate during its traverse by the electron. Primarily, it is necessary to note that these fields are the wave packets, which consist of plane electromagnetic waves. At large values of the time interval after the interaction of the electron with the target these wave packets reconstruct into spherical diverging waves of radiation. On small distances from the target the spatial structure of these packets is close to the structure of the packet formed by the electron's own field, which is reflected from the plate.

Firstly, let us consider on the base of this method the transition radiation on large distances from the plate $|z| \gg 2\gamma^2/\omega$. In this case the integrals over x in (72) can be calculated with the use of the stationary phase method. Using for this purpose the asymptotic expression for the Bessel function $J_0(\omega x \rho)$ for large values of its argument we can write $\phi_\omega^f(\vec{r})$ in the next way:

$$\phi_\omega^f(\vec{r}) = -\frac{2e}{v} \operatorname{Re} \int_0^1 \frac{x dx}{x^2 + p^{-2}} \sqrt{\frac{2}{\pi \omega \rho x}} \left\{ e^{i(\omega \rho x - \frac{\pi}{4} + |z| \omega \sqrt{1-x^2})} + e^{i(\omega \rho x - \frac{\pi}{4} - |z| \omega \sqrt{1-x^2})} \right\} \quad (89)$$

The first item in the braces gives the stationary phase point

$$x_0 = \frac{\rho / |z|}{\sqrt{\rho^2 / z^2 + 1}} = \sin \vartheta \quad (90)$$

where the ratio $\rho / |z|$ is defined by the relation (82). As stationary phase point is absent in the second item in (89) in the considered region of the variable x , we can neglect the contribution of this item to the integral over x . For the same reason we neglect the contribution to $\phi_\omega^f(\vec{r})$ of the item in (72), which contains the integration over the values $x > 1$. In the result for $\phi_\omega^f(\vec{r})$ we obtain:

$$\phi_\omega^f(\vec{r}) = -\frac{2e}{\omega vr} \frac{\cos^2 \vartheta}{\sin^2 \vartheta + p^{-2}} e^{i(\omega r - \pi/2)} \quad (91)$$

We took into account the fact that in spherical coordinate system $\rho = r \sin \vartheta$ and $|z| = r \cos \vartheta$.

With the use of (56) the expression $(\vec{E}_\omega \times \vec{H}_{-\omega}) \vec{n}$ in (52) can be written in the form:

$$(\vec{E}_\omega \times \vec{H}_{-\omega}) \vec{n} = (-\partial_z \phi_\omega^f + i\omega \vec{A}_\omega^f) \vec{n}_\perp \cdot \nabla_\perp A_{-\omega}^f - \cos \vartheta \nabla_\perp \phi_\omega^f \cdot \nabla_\perp A_{-\omega}^f \quad (92)$$

In the point of stationary phase for $r \rightarrow \infty$ $A_\omega^f = -\phi_\omega^f / \cos \vartheta$. Taking into account $\frac{\partial}{\partial \rho} \phi_\omega = i\omega \sin \vartheta \phi_\omega$ and $\frac{\partial}{\partial z} \phi_\omega = i\omega \cos \vartheta \phi_\omega$ we find that

$$(\vec{E}_\omega \times \vec{H}_{-\omega}) \vec{n} = \omega^2 t g^2 \vartheta \phi_\omega^f \phi_{-\omega}^f \quad (93)$$

Substituting the asymptotic (91) for the potential ϕ_ω into this expression we obtain the formula (86) for the radiation spectral-angular density.

For ultra relativistic particles characteristic values of the radiation angles $\vartheta \approx \gamma^{-1}$ are much less than unit. The radiation spectral-angular density (86) in this case has the following form:

$$\frac{d\varepsilon}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{\vartheta^2}{(\gamma^{-2} + \vartheta^2)^2} \quad (94)$$

In this region of radiation angles the characteristic values of the variable x , which make contribution to the integral over x in (72) are small $x_{eff} \approx \gamma^{-1}$. The stationary phase method is valid for calculation of this integral if

$$\frac{|z|\omega}{2\gamma^2} \gg 1$$

In other words, it is required that the distance between the target and the detector should substantially exceed the coherence length of the radiation process, which means the radiation should be considered in the wave zone.

As it was pointed out, the length l_C can have macroscopic values and the detector can be situated in the pre-wave zone, which is on smaller distances than the coherence length of the radiation process. The stationary phase method is not applicable for calculation of the integral over x in (72) in this case. For this purpose the expansion of the phase over small values of x in the exponential factor in (72) can be made. Neglecting the items proportional to higher powers than x^2 we find that

$$\phi_{\omega}^f = -\frac{2e}{\pi v} \operatorname{Re} \int_0^{\infty} \frac{x dx}{x^2 + p^{-2}} J_0(\omega x \rho) e^{i(\omega |z| - \omega |z| x^2/2)} \quad (95)$$

In the region of small radiation angles for which $\vartheta^2 |z| \omega \leq 1$, the convergence of the integral (95) is determined by the values $x \approx 1/\sqrt{|z| \omega}$, therefore the items proportional to x^2 should be preserved in the exponent phase. With the same accuracy in the region of small angles the vector potential A_{ω}^f is related ϕ_{ω}^f by

$$A_{\omega}^f = -\phi_{\omega}^f$$

In this case $(\vec{E}_{\omega} \times \vec{H}_{-\omega}) \vec{n} = \nabla_{\perp} \phi_{\omega}^f \cdot \nabla_{\perp} \phi_{-\omega}^f$ and the radiation spectral-angular density has the following form:

$$\frac{d\varepsilon}{d\omega d\vartheta} = \left(\frac{2e^2}{\pi v} \right)^2 |B(z)|^2 \quad (96)$$

where

$$B(z) = \omega |z| \int_0^{\infty} x^2 dx \frac{J_1(x\omega |z| \vartheta)}{x^2 + \gamma^{-2}} e^{-i\omega |z| x^2/2} \quad (97)$$

Let us note that during the derivation of the expression (97) we only took into account the fact that the consideration of the radiation process was made in the region of small angles of radiation. Therefore the formula (97) is valid both for large ($z \gg l_C$) and small ($z \ll l_C$) distances between the detector and the target. If $|z| \gg l_C$, the calculation of the integral $B(z)$ can be made with the use of the stationary phase method. As a result formula (97) gives the corresponding result of the transition radiation theory (see formula (94)). In the region $|z| < l_C$ the formula (96) leads to the broader radiation angular distribution than the one defined by the expression (94). The possibility of existence of such effect was considered in the paper [10]. Its theory was elaborated in the paper [8]. According to it, in particular, in the region of small angles on distances $|z| < l_C$ the radiation spectral-angular density is defined by the formula:

$$\frac{d\varepsilon}{d\omega do} = \frac{4e^2}{\pi^2} \frac{1}{\vartheta^2} \sin^2\left(\frac{\omega |z| \vartheta^2}{4}\right) \quad (98)$$

Such modification of the radiation angular distribution for $|z| < l_C$ comparing to the case of $|z| \gg l_C$ is caused by the fact that in the region $|z| < l_C$ only part of the waves reflected from the plate fall into the point detector (see [8]). It is necessary to note that for $|z| < l_C$ the radiation angular distribution depends on the frequency of the radiated electromagnetic wave. If the measurements are performed by the detector of the larger size than the characteristic transversal length of the radiation process the radiation spectral-angular density is defined by the expression (94) on arbitrary distances from the plate and the discussed above pre-wave zone effects disappear.

The obtained results show that the effects analogous to the ones which take place in the process of bremsstrahlung at the instantaneous scattering of the electron to a large angle (the broadening of the radiation angular distribution and its dependence on the frequency of the registered photon in the measurements performed by a point detector on small distances from the scattering point) take place also for backward transition radiation in the process of an electron traverse of metallic plate. For ultra relativistic particles, according to (94) and (98), the radiation is mainly concentrated in the region of small angles $\vartheta \ll 1$: $\vartheta_{eff} \approx 1/\gamma$ in the wave zone, in the pre-wave zone $\vartheta_{eff} \approx 2/\sqrt{\omega |z|}$.

5. Transition radiation by 'half-bare' electron

The wave packets, which arise at ultra relativistic electron instantaneous scattering to a large angle, reconstruct into radiation field on distances along the initial and final directions of the electron's velocity, which are of the order of the coherence length of the radiation process. For large energies of the electron and low frequencies of the radiated waves, as was stated above, this length can be of macroscopic size. In this case the possibility of investigating of the evolution of such wave packets in space and time by macroscopic devices appears. In [15] one of such possibilities, which concerns the reflection of wave packets from an ideally conducting plate situated on different distances from the scattering point, was discussed. In this case if the plate is situated in the direction of motion of the scattered electron perpendicular to its velocity the reflected field is the backward transition radiation. However, the peculiarity of this process lies in the fact that unlike the ordinary backward transition radiation the considered one is the radiation by the particle with non-equilibrium field. The plate in this case can be considered as an element of the radiation detector. Let us obtain the formulae, which describe the given process and discuss some of its peculiarities on their basis [17].

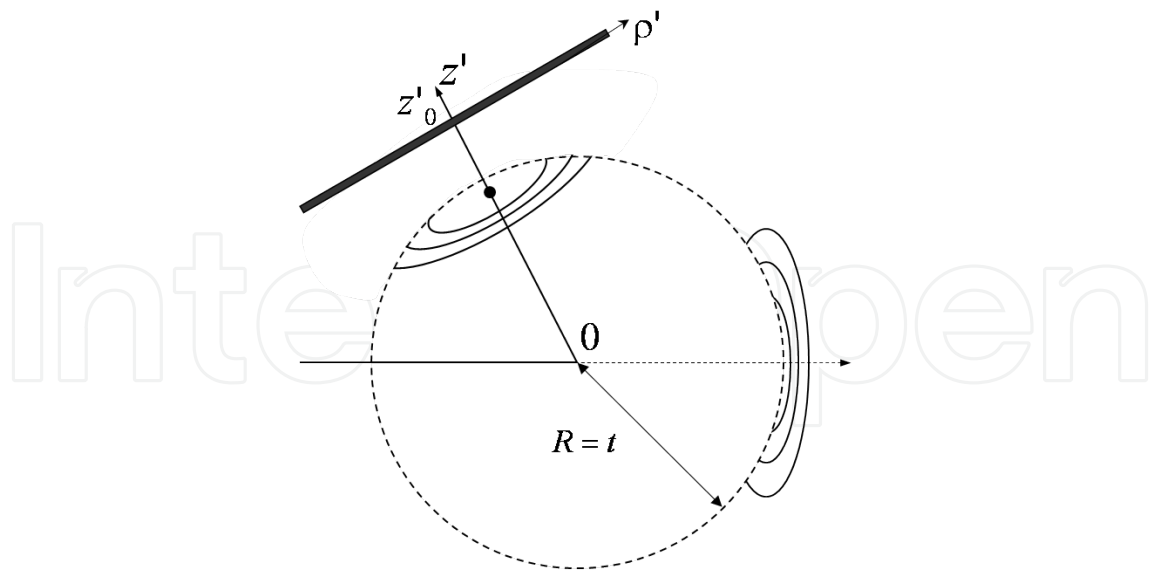


Figure 6. Normal incidence of the scattered 'half-bare' electron on thin metallic plate.

Let the ideally conducting plate be situated in the plane $z' = z'_0$ (Figure 6). The Fourier-expansion of the field around the scattered electron

$$\phi(\vec{r}, t) = \frac{e}{2\pi^2} \text{Re} \int d^3k \left[\frac{e^{i\vec{k}(\vec{r}-\vec{v}'t)}}{k - k\vec{v}'} - \frac{e^{i\vec{k}\vec{r}-ikt}}{ck - k\vec{v}'} \right] \quad (99)$$

consists of two parts, the first of which describes the equilibrium coulomb field of the electron, which moves with the velocity \vec{v}' along the direction of scattering, while the second part is the nonequilibrium field, which is structurally equal to the 'torn away' field (it is equal to equilibrium coulomb field outside the θ - sphere and vanishes inside it). Hence, the second part of the field (99) can be presented in the form (47) with a mere substitution $\vec{v} \rightarrow \vec{v}'$. The first part of the field (99) can be presented in the analogous form by making the substitution $k_z \rightarrow k$ from $k = \sqrt{k_z^2 + q^2}$ and denoting $k_z v' = \omega$. From the expression for scalar potential obtained by the considered transformations we can derive the expression for the Fourier-component of the electric field perpendicular to z axis, which in ultra relativistic case is:

$$E_{\perp}(\vec{r}, \omega) = 2e \int_{-\infty}^{+\infty} d\omega e^{i\frac{\omega z'}{v'}} \int_0^{\infty} dq \frac{q^2 J_1(q\rho)}{q^2 + \omega^2/v'^2 \gamma^2} \left[1 - e^{-i\frac{\omega z'}{2v'^2}(\gamma^{-2} + q^2 v'^2/\omega^2)} \right] \quad (100)$$

From (100) it follows that the rebuilding of the field around the electron occurs in such way that each Fourier-harmonic of frequency ω_0 totally reconstructs and becomes the harmonic of equilibrium coulomb field on the distance from the scattering point, which coincides with radiation formation length ($|z'| \approx 2\gamma^2/\omega_0$) for this ω_0 . It is possible to place the plate quite

close to the scattering point so that at the moment of electron's traverse of the plate the Fourier-harmonics of certain frequencies $\omega < \omega_0$ will have not yet reconstructed. In other words it is possible to place the plate in the pre-wave zone for these frequencies. In this case the incident electron will be 'half-bare' and its transition radiation should differ from such radiation by electron with equilibrium field.

The total field of the electron-plate system consists of the field of 'half-bare' electron E_{\perp} and the field E_{\perp}^f of currents induced on the surface of the plate. Applying the boundary condition for electric field on the surface of the plate

$$E_{\perp}|_{z'=z'_0} + E_{\perp}^f|_{z'=z'_0} = 0$$

we can find the expression for the Fourier-harmonic of the field of induced surface currents:

$$E_{\perp}^f(\vec{r}, \omega) = 2e \frac{e^{i\omega R}}{R} \frac{\vartheta}{\vartheta^2 + \gamma^{-2}} [F_{\omega}(\vec{r}) - 1] \quad (101)$$

$$\text{where } F_{\omega}(\vec{r}) = \frac{1}{v} \frac{\rho'^2 + \gamma^{-2}(z' - 2z'_0)^2}{\rho'^2 + \gamma^{-2}(z' - z'_0)^2} \exp\left\{ \frac{i\omega z'_0}{2} \left[\frac{1}{v^2 \gamma^2} + \frac{\rho'^2}{(z' - z'_0)(z' - 2z'_0)} \right] \right\},$$

R is the distance between the point of the electron's traverse of the plate and the point where the field is considered, $R \approx z'_0 - z' + \rho'^2 / 2(z' - z'_0)$ and ϑ is counted from the direction of $-\vec{v}'$. This field gradually transforms into backward transition radiation.

The expression (101) can be simplified for $-z' \gg 2\gamma^2 / \omega$. In this case:

$$E_{\perp}^f(\vec{r}, \omega) = 2e \frac{e^{i\omega R}}{R} \frac{\vartheta}{\vartheta^2 + \gamma^{-2}} \left[\frac{1}{v} e^{i \frac{z'_0 \omega}{2\gamma^2} (1 + \gamma^2 \vartheta^2)} - 1 \right] \quad (102)$$

and using (57) for spectral-angular density of transition radiation by 'half-bare' electron we obtain:

$$\frac{d\varepsilon}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{\vartheta^2}{(\vartheta^2 + \gamma^{-2})^2} 2 \left\{ 1 - \cos \left[\frac{\omega z'_0}{2} (\gamma^{-2} + \vartheta^2) \right] \right\} \quad (103)$$

The expression (103) differs from the corresponding expression for transition radiation by electron with equilibrium field by the interference factor inside the braces and the coefficient two in front of them. As we can see from (103), when the distance z'_0 between the scattering point and the plate is much less than the radiation formation length ($l_c \approx 2\gamma^2 / \omega$) the radiation is highly suppressed. For larger values of z'_0 the dependence of the radiation intensity on z'_0 has the oscillation type with the period of the order of the formation length:

$$\Lambda = \frac{4\pi}{\omega(\vartheta^2 + \gamma^{-2})} \quad (104)$$

Due to the nonzero frequency resolution $\Delta\omega$ of the detector it is possible to observe such oscillations only in the area limited by the condition

$$z'_0 < \frac{2\pi}{\Delta\omega(\vartheta^2 + \gamma^{-2})} \quad (105)$$

Also due to the nonzero size and, therefore, angular resolution of the detector the oscillations can be observed only inside the region

$$z'_0 < \frac{\pi}{\omega\vartheta\Delta\vartheta} \quad (106)$$

For large distances $z'_0 \gg \Lambda$ the considered oscillations disappear and the detector registers an incoherent sum of contributions to transition radiation by electron's own field reflected from the plate and by the field of bremsstrahlung in this direction.

6. Conclusion

The behavior of localized high-energy electromagnetic wave packets, which take place in processes of transition radiation and bremsstrahlung by relativistic electrons has been considered. It was shown that with the increase of the energy the stabilization of characteristics of motion of such packets takes place, which consists in substantial decrease of the speed of their dispersion. Essential here is the fact that at high energies the lengths, on which the reconstruction of the form of such packets into packets of diverging waves takes place, can reach macroscopic size, which can exceed the size of experimental facility. In this case both the size of the used detector and its position relative to the region of the wave packet formation become essential for measurements.

Such situation takes place, for example, after the sharp scattering of an electron to a large angle. It was shown that as a result of such scattering the electron's own coulomb field tears away from it and turns into a localized packet of free electromagnetic waves, which transforms into a packet of diverging waves on large distance from the scattering point. For ultra relativistic electrons such transformation of certain Fourier-harmonics of the packet field takes place within the coherence length of the radiation process, which substantially exceeds the length of the considered wave of radiation. In the case of low-frequency radiation this coherence length can be macroscopic. It gives birth to the problem of bremsstrahlung characteristics measurement by different detectors, which consists in the dependence of the results of measurement on the detector's size and its position relative to the scattering point.

In the final direction of the electron motion (after the scattering) certain Fourier-components of the field around it do not appear at once. The regeneration of these Fourier-components occurs within the coherence length of the radiation process $l_C \approx 2\gamma^2 / \omega$. In this case during a long period of time the ultra relativistic electron is in 'half-bare' state without a certain part of Fourier-components of its field. Such state of electron manifests itself, for example, during further collisions of the electron with atoms of a substance, which causes different effects of bremsstrahlung suppression (Landau-Pomeranchuk-Migdal effect, the effect of radiation suppression in thin layer of substance – TSF-effect, etc.). We have shown that the 'half-bare' state of the scattered electron should manifest itself in the process of further transition radiation by such electron as well. This manifestation consists in the fact that characteristics of transition radiation by such electron substantially depend on the position of the metallic plate relative to the scattering point. In this case both the transition radiation suppression and the oscillatory dependence of its characteristics on the distance between the plate and the scattering point take place.

The transition radiation formation process also develops within distances of the order of the coherence length of the radiation process, l_C . Such situation takes place both in the direction of the electron motion and in the opposite one. In the case of transition radiation, as in the case of bremsstrahlung, these distances can be macroscopic. The analysis of development of this process in space and time has shown that the packet of the waves reflected from the plate has the same structure as the wave packet torn away from the electron at its instantaneous scattering to a large angle. The structure of the field around the electron after its traverse of thin metallic plate is similar to the structure of the field, which forms around the electron after its scattering to a large angle. In other words, in this case the electron can be in the 'half-bare' state during a long period of time: a certain part of the Fourier components in the field around it is suppressed. The mentioned analogies cause the existence of the similar effects in transition radiation and bremsstrahlung. Thus the possibility of long existence of electron in the 'half-bare' state and of different manifestations of such state of electron can be investigated on the basis of the process of transition radiation by such electron.

Appendix

According to (72) the potential $\phi^f(\vec{r}, t)$ of the free field of electromagnetic waves is the sum of the contributions to it by the propagating $\Phi_1(\vec{r}, t)$ and the surface $\Phi_2(\vec{r}, t)$ waves. While deriving these functions we will pay special attention to the reasons of the step functions appearance in (75).

In order to calculate $\Phi_1(\vec{r}, t)$ we will present this function in the form:

$$\Phi_1(\vec{r}, t) = -\frac{2e}{\pi} \lim_{\eta \rightarrow 0} \int_0^1 \frac{xdx}{x^2 + p^{-2}} \int_0^\infty d\omega \cos\omega(|z| + \sqrt{1-x^2} - t) e^{-\eta\omega} \quad (107)$$

where $\eta > 0$. Then after the variable substitution $y = \sqrt{1-x^2}$ and integration over ω it is easy to obtain that

$$\Phi_1(\vec{r}, t) = -\frac{2e}{\pi v} \lim_{\eta \rightarrow 0} \int_0^1 \frac{y dy}{v^2 - y^{-2}} \frac{\eta}{\eta^2 + (y|z| - t)^2} \quad (108)$$

Integration over y in this expression leads to

$$\begin{aligned} \Phi_1(\vec{r}, t) = & -\frac{2e}{\pi v} \lim_{\eta \rightarrow 0} \frac{\eta}{1 - v^2(\eta^2 + t^2 + z^2/v^2)^2 / (4z^2 t^2)} \left\{ -\frac{v^2(\eta^2 + t^2 + z^2/v^2)}{4z^2 t^2} \ln \gamma \right. \\ & - \frac{v}{2zt} \ln \gamma (1+v) - \frac{v^2(\eta^2 + t^2 + z^2/v^2)}{4t^2} \left[\frac{1}{2z^2} \ln \frac{\eta^2 + (|z| - t)^2}{\eta^2 + t^2} \right. \\ & \left. \left. + \frac{t}{z^2 \eta} \left(\arctg \frac{|z| - t}{\eta} + \arctg \frac{t}{\eta} \right) \right] + \frac{v^2(\eta^2 + t^2)}{2zt} \frac{1}{z\eta} \left(\arctg \frac{|z| - t}{\eta} + \arctg \frac{t}{\eta} \right) \right\} \end{aligned} \quad (109)$$

Taking into account that for $\eta \rightarrow 0$ only items proportional to $(\arctg[(|z| - t)/\eta] + \arctg[t/\eta])$ remain in (109) and that for these items

$$\arctg \frac{|z| - t}{\eta} + \arctg \frac{t}{\eta} = \frac{\pi}{2} [\text{sign}(|z| - t) + \text{sign}(t)] = \frac{\pi}{2} \theta(|z| - t) \theta(t)$$

we obtain the following expression for $\Phi_1(\vec{r}, t)$:

$$\Phi_1(\vec{r}, t) = \frac{e}{2} \left(\frac{1}{|z| + vt} - \frac{1}{|z| - vt} \right) \theta(|z| - t) \theta(t) \quad (110)$$

Thus the appearance of the step function $\theta(|z| - t)$ in $\Phi_1(\vec{r}, t)$ is connected with the limiting procedure $\eta \rightarrow 0$ in (109).

The calculation of $\Phi_2(\vec{r}, t)$ does not represent any difficulties as the integral over ω in it converges. As a result of the variable substitution $u = \sqrt{1-x^2}$ and elementary integration we find that:

$$\begin{aligned} \Phi_2(\vec{r}, t) = & \frac{e}{2} \left(\frac{1}{|z| - vt} - \frac{1}{|z| + vt} \right) \text{sign}(t) - \frac{e}{2} \left(\frac{1}{|z| - vt} + \frac{1}{|z| + vt} \right) = \\ = & -\frac{1}{|z| - vt} \theta(-t) - \frac{1}{|z| + vt} \theta(t). \end{aligned} \quad (111)$$

Substituting the obtained expressions (110) and (111) for Φ_1 and Φ_2 into (72) we obtain the expression (75) for the potential of the free waves.

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