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The Role of the Gearbox in an Automatic Machine

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1. Introduction

A *machine* is a system realized by many parts with different functions, linked each other to reach a defined task. Depending on the task, a classification of machines equipped with moving parts can be done. In particular, one can distinguish:

- Drive machines (*motors*): these machines deliver mechanical power from other forms of energy. If their purpose is simply to make placements or generate forces/torques, they are called *actuators*.

- Working machines (*users*): these machines absorb mechanical power to accomplish a specific task (machine tools, transportation, agricultural machinery, textile machinery, machine packaging, etc.).

- Mechanical transmissions: these machines transmit mechanical power by appropriately changing values of torques and speed. Mechanical transmissions are generally made up of *mechanisms* that have been studied (mainly from the point of kinematic view) to connect motors and users.

The combination of a motor, a transmission and a mechanical user is the simplest form of machine.

In servo-actuated machines, the choice of the electric motor required to handle a dynamic load, is closely related to the choice of the transmission Giberti et al. (2011).

The choice of the transmission plays an important role in ensuring the performance of the machine. It must be carried out to meet the limitations imposed by the motor working range and it is subjected to a large number of constraints depending on the motor, through its rotor inertia J_M or its mechanical speed and on the speed reducer, through its transmission ratio τ, its mechanical efficiency η and its moment of inertia J_T.

This chapter critically analyzes the role of the transmission on the performance of an automatic machine and clarifies the strategies to choose this component. In particular, it is treated the general case of coupled dynamic addressing the problem of inertia matching and presenting a methodology based on a graphical approach to the choice of the transmission.

The identification of a suitable coupling between motor and transmission for a given load has been addressed by several authors proposing different methods of selection. The most common used procedure are described in Pasch et al. (1984), Van de Straete et al. (1998), Van de Straete et al. (1999), Roos et al. (2006). In these procedures, the transmission is
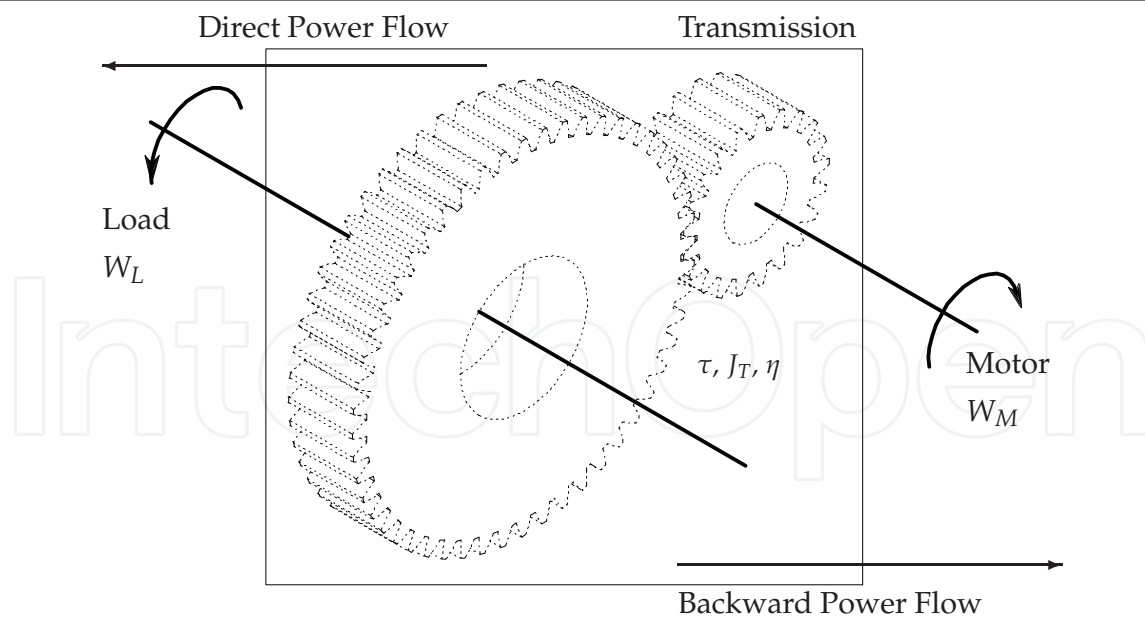
Fig. 1. A scheme of a simple transmission

approximated to an ideal system in which power losses are neglected, as the effects of the transmission inertia. Only after the motor and the reducer are selected, the transmission mechanical efficiency and inertia are considered to check the validity of the choice. Naturally, if the check gives a negative result, a new motor and a new transmission should be selected and the entire procedure has to be performed again. Differently, in Giberti et al. (2010a) the effects of transmission efficiency and inertia are considered since the beginning.

This chapter is structured as follows. Paragraph 2 gives an overview of main features of a mechanical transmission, while Par.3 describes the functioning of a generic machine when a given load is applied. Paragraph 4 describes the conditions, in terms of useful transmission ratios, for which a motor-load combination is feasible. Paragraph 5 gives the guidelines for the selection of both the gearbox and the electric motor, neglecting the effects of the transmission mechanical efficiency and inertia. Theoretical aspects are supported by a practical industrial case. Paragraphs 6 and 7 extend results previously reached considering the effects of the transmission mechanical efficiency and inertia. Finally conclusions are drawn in paragraph 8.

All the symbols used through the chapter are defined in Par.9.

2. The transmission

To evaluate the effect of the transmission in a motor-load coupling, the transmission can be considered as a black box in which the mechanical power flows (Figures 1, 2, 4). The mechanical power is the product of a torque (extensive factor) for an angular speed (intensive factor).

A mechanical transmission is a mechanism whose aim is:

1. to transmit power
2. to adapt the speed required by the load
3. to adapt the torque.

and it is characterized by a transmission ratio τ and a mechanical efficiency η.

Conventional mechanical transmissions with a constant ratio involve the use of *friction wheels*, *gear*, *belt* or *chains*. The choice of the most suitable transmission for a given application depends on many factors such as dimensions, power, speed, gear ratio, motor and load characteristics, cost, maintenance requirements. Table 1 gives an overview of the most common applications of mechanical transmissions.

	Power max (kW)	τ minimum	Dimensions	Cost	Efficiency	Load on bearings
Friction wheels	1/6	20	low	medium	0.90	high
Spur gear	750	1/6	low	high	0.96	low
Helical gears	50000	1/10	low	high	0.98	low
Worm gears	300	1/100	low	high	0.80	medium
Belt	200	1/6	high	low	0.95	high
Trapezoidal belt	350	1/6	medium	low	0.95	high
Toothed belt	100	1/6	medium	low	0.90	low
Linkages	200	1/6	medium	medium	0.90	low

Table 1. Typical characteristics of mechanical transmissions.

2.1 The transmission ratio

The *transmission ratio* τ is defined as:

$$\tau = \frac{\omega_{out}}{\omega_{in}} \tag{1}$$

where ω_{in} and ω_{out} are the angular velocity of the input and the output shafts respectively. This value characterizes the transmissions. If $\tau < 1$ the gearbox is a *speed reducer*, while if $\tau > 1$ it is a *speed multiplier*.

The mechanical transmission on the market can be subdivided into three main categories: transmissions with constant ratio τ, transmissions with variable ratio τ and transmissions that change the kind of movement (for example from linear to rotational).

Since it is generally easier to produce mechanical power with small torques at high speeds, the transmission performs the task of changing the distribution of power between its extensive and intensive factors to match the characteristics of the load.

It is possible to define the term μ as the *multiplication factor of force (or torque)*:

$$\mu = \frac{T_{out}}{T_{in}} \tag{2}$$

where T_{in}, and T_{out} are respectively the torque upstream and downstream the transmission.

2.2 The mechanical efficiency

If the power losses in the transmission can be considered as negligible, it results:

$$\mu = \frac{1}{\tau}. \tag{3}$$

However, a more realistic model of the transmission has to take into account the inevitable loss of power to evaluate how it affects the correct sizing of the motor-reducer coupling and the resulting performance of the machine.

In general, transmissions are very complex, as the factors responsible for the losses of power. In this chapter they are taken into account just considering the transmission mechanical efficiency η defined as the ratio between the power outgoing from the transmission (W_{out}) and the incoming one (W_{in}), or through the extensive factors (T_{in}, T_{out}) and the intensive ones (ω_{in}, ω_{out}) of the power itself:

$$\eta = \frac{W_{out}}{W_{in}} = \frac{T_{out}}{T_{in}} \frac{\omega_{out}}{\omega_{in}} = \mu\tau \leq 1 \tag{4}$$

The loss of power within the transmission leads to a reduction of available torque downstream of the transmission. Indeed, if the coefficient of multiplication of forces for an ideal transmission is $\mu = \tau^{-1}$, when $\eta \neq 1$ it becomes $\mu' < \mu$, thus leading to a consequent reduction of the transmitted torque ($T'_{out} < T_{out}$).

Let's define W_M and W_L respectively as the power upstream and downstream of the transmission. When the power flows from the motor to the load the machine is said to work with *direct power flow*, otherwise, the functioning is said to be *backward*. Depending on the machine functioning mode, the transmission power losses are described by two different mechanical efficiency values η_d and η_r, where:

$$\eta_d = \frac{W_L}{W_M} \quad \text{(direct power flow)} \qquad \eta_r = \frac{W_M}{W_L} \quad \text{(backward power flow)}. \tag{5}$$

3. A single d.o.f. machine

An automatic machine is a system, usually complex, able to fulfill a particular task. Regardless of the type of machine, it may be divided into simpler subsystems, each able to operate only one degree of freedom and summarized by the three key elements: motor, transmission and load (Fig. 2).

3.1 The load and the servo-motor

The power supplied by the motor depends on the external load applied T_L and on the inertia acting on the system $J_L\dot{\omega}_L$. Since different patterns of speed ω_L and acceleration $\dot{\omega}_L$ generate different loads, the choice of a proper law of motion is the first project parameter that should be taken into account when sizing the motor-reducer unit. For this purpose, specific texts are recommended (Ruggieri et al. (1986), Melchiorri (2000)).

Once the law of motion has been defined, all the characteristics of the load are known.

Electric motor, and among these brushless motors, are the most widespread electrical actuators in the automation field, and their working range can be approximately subdivided into a *continuous* working zone (delimited by the motor rated torque) and a *dynamic* zone (delimited by the maximum motor torque $T_{M,max}$). A typical shape of the working zones is displayed in Fig.3. Usually the motor rated torque decreases slowly with the motor speed ω_M from T_{M,N_c} to $T_{M,N}$. To simplify the rated torque trend and to take a cautious approach, it's usually considered constant and equal to $T_{M,N}$ up to the maximum allowed motor speed
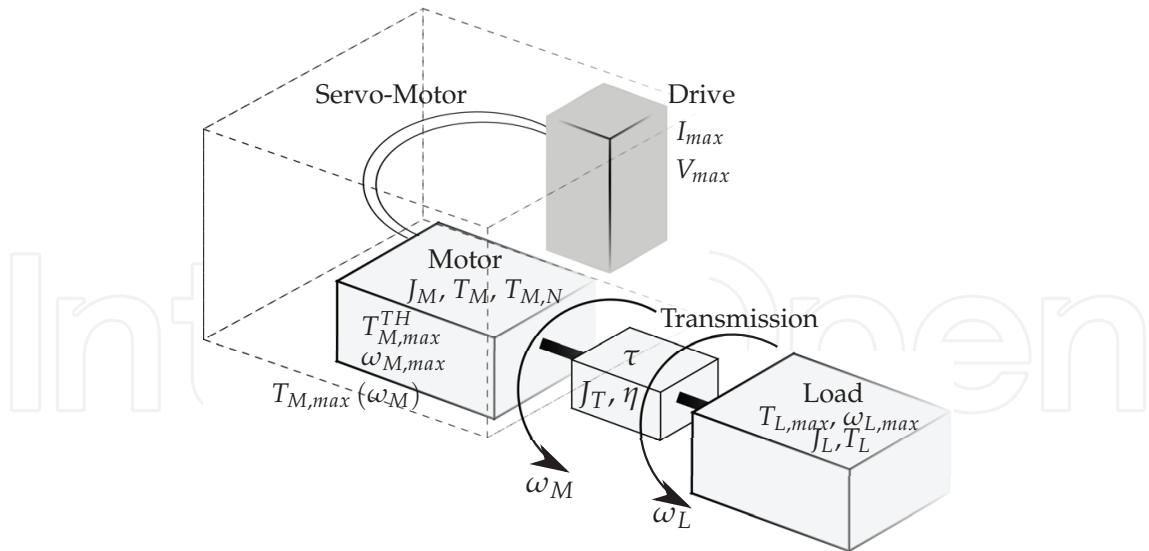
Fig. 2. Scheme of a generic machine actuated by an electric motor
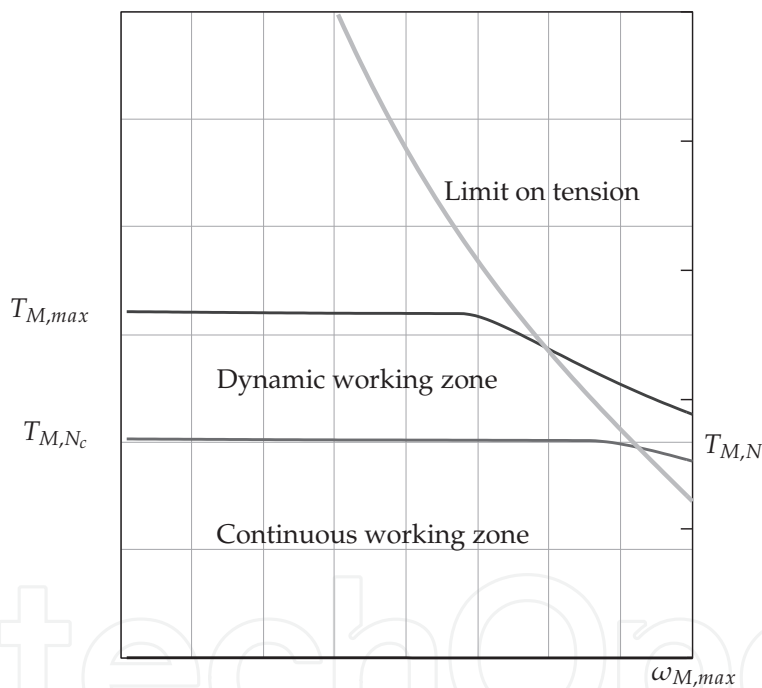

Fig. 3. An example of a speed/torque curve of a common brushless motor

$\omega_{M,max}$, whereas $T_{M,max}$ decreases from a certain value of ω_M. The nominal motor torque $T_{M,N}$, which is specified by the manufacturer in the catalogs, is defined as the torque that can be supplied by the motor for an infinite time, without overheating. Conversely, the trend of the maximum torque $T_{M,max}$ is very complex and depends on many factors. For this reason it is difficult to express it with an equation.

3.2 Conditions to the right coupling between motor and load

Frequently in industrial applications, the machine task is cyclical with period t_a much smaller than the motor thermal time constant. Once the task has been defined, a motor-task
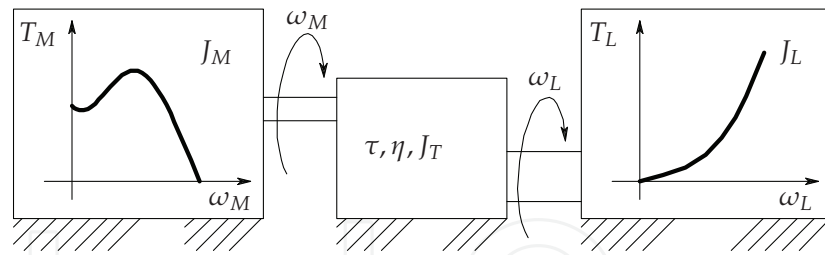
Fig. 4. Scheme of a generic single degree of freedom machine

combination is feasible if there exists a transmission ratio such that:

1. the maximum speed required from the motor is smaller than the maximum speed achievable ($\omega_{M,max}$);

$$\omega_M \leq \omega_{M,max} \tag{6}$$

2. a certain p-norm [1] of the motor torque $\|T_M(t)\|_p$ is smaller than a corresponding motor specific limit T_p. The norms that have most physical meaning for motor torques are Van de Straete et al. (1998):

$$\|T_M(t)\|_2 = T_{rms} = \left(\frac{1}{t_a} \int_0^{t_a} T_M^2(t)dt \right)^{\frac{1}{2}} \tag{7}$$

$$\|T_M(t,\omega)\|_\infty = T_{Max}(\omega) = \max |T_M(t,\omega)| \quad 0 \leq t \leq t_a \tag{8}$$

Since the motor torque is proportional to the current, 2-norm is a measure of mean square current through the windings and it should be limited by the nominal torque $T_{M,N}$ to avoid overheat. The ∞-norm is a measure of peak current and its limit is the maximum torque that can be exerted $T_{M,max}$. These limits translate into:

$$T_{rms} \leq T_{M,N} \tag{9}$$

$$T_{Max}(\omega) \leq T_{M,max}(\omega) \qquad \forall \omega \tag{10}$$

where $T_{M,max}$ depends on ω as shown in Fig. 3.

While the terms on the right of inequalities (6), (9), (10) are characteristics of each motor, ω_M, T_{rms} and T_{Max} depend on the load and on the mechanical features of the motor and the speed reducer.

3.3 Mathematical model of a machine

The machine functioning can be described as an instantaneous balance of power. In the case of direct power flow it can be expressed as:

$$\eta_d \left(\frac{T_M}{\tau} - \frac{J_M \dot{\omega}_L}{\tau^2} \right) = T_L + (J_L + J_T) \dot{\omega}_L \tag{11}$$

---

[1] In general, the p-norm $\|\cdot\|_p$ of a time function $f(t)$ over the period t_a is defined as:

$$\|f(t)\|_p = \left(\frac{1}{t_a} \int_0^{t_a} f^p(t)dt \right)^{\frac{1}{p}}$$

while in backward power flow it is:

$$\left(\frac{T_M}{\tau} - \frac{J_M \dot{\omega}_L}{\tau^2} \right) = \eta_r \left[T_L + (J_L + J_T) \dot{\omega}_L \right] \tag{12}$$

Equations (11), (12) can be written as:

$$T_{M,d} = \frac{\tau T_L^*}{\eta_d} + J_M \frac{\dot{\omega}_L}{\tau} \tag{13}$$

$$T_{M,r} = \tau T_L^* \eta_r + J_M \frac{\dot{\omega}_L}{\tau} \tag{14}$$

where:

$$T_L^* = T_L + (J_L + J_T) \dot{\omega}_L \tag{15}$$

is the total torque applied to the outgoing transmission shaft.

To unify these different operating conditions, a general mechanical efficiency function is introduced by Legnani et al. (2002). It is defined as:

$$\eta = \begin{cases} \eta_d & \text{(direct power flow)} \\ 1/\eta_r & \text{(backward power flow)} \end{cases} \tag{16}$$

where η, η_d and η_r are constants.

In the case of backward power flow it results $\eta > 1$. Note that this does not correspond to a power gain, but it is simply an expedient for unifying the equations of the two working conditions (direct/backward power flow). In fact, in this case, the effective efficiency is $\eta_d = 1/\eta < 1$.

4. Selecting the transmission

Conditions (6), (9), (10) can be expressed, for each motor, as constraints on acceptable transmission ratios. However, each transmission is characterized not only by its reduction ratio, but also by its mechanical efficiency and its moment of inertia. This chapter analyzes how these factors affect the conditions (6), (9), (10) and how they reduce the range of suitable transmission ratios for a given motor.

4.1 Limit on the maximum achievable speed

Since each motor has a maximum achievable speed $\omega_{M,max}$, a *limit transmission ratio* τ_{kin} can be defined as the minimum transmission ratio below which the system cannot reach the requested speed:

$$\tau_{kin} = \frac{\omega_{L,max}}{\omega_{M,max}} \tag{17}$$

The condition of maximum speed imposed by the system can be rewritten in terms of minimum gear ratio τ_{kin} to guarantee the required performance. Eq.(6) becomes:

$$\tau \geq \tau_{kin} \tag{18}$$

4.2 Limit on the root mean square torque

When the direction of power flow during a working cycle is mainly direct or mainly backward, using the notation introduced in eq. (16), the root mean square torque can be expressed as:

$$T_{rms}^2 = \int_0^{t_a} \frac{T_M^2}{t_a} dt = \int_0^{t_a} \frac{1}{t_a} \left(\frac{\tau T_L^*}{\eta} + J_M \frac{\dot{\omega}_L}{\tau} \right)^2 dt \tag{19}$$

Developing the term in brackets and using the properties of the sum of integrals, it is possible to split the previous equation into the following terms:

$$\int_0^{t_a} \frac{1}{t_a} \left(\frac{\tau T_L^*}{\eta} \right)^2 dt = \frac{T_{L,rms}^{*2} \tau^2}{\eta^2}$$

$$\int_0^{t_a} \frac{1}{t_a} \left(J_M \frac{\dot{\omega}_L}{\tau} \right)^2 dt = \frac{J_M^2}{\tau^2} \dot{\omega}_{L,rms}^2 \tag{20}$$

$$\int_0^{t_a} \frac{1}{t_a} \left(\frac{2 T_L^* J_M \dot{\omega}_L}{\eta} \right)^2 dt = 2 \frac{J_M}{\eta} \left(T_L^* \dot{\omega}_L \right)_{mean}$$

In cases in which the power flow changes during the cycle it is not possible to choose the proper value of η and eq.(19) is no longer valid. In this circumstance, any individual working cycle must be analyzed to check if one of the two conditions (eq.(11) or eq.(12)) can be reasonably adopted. For example, in the case of purely inertial load ($T_L = 0$) it is[2]:

$$\left| T_{M,d} \right| \geq \left| T_{M,r} \right| \tag{21}$$

and the equation for direct power flow can be prudentially adopted. Same considerations can be done when the the load is mainly resistant (limited inertia). For all these cases, the condition of eq.(9) becomes:

$$\frac{T_{M,N}^2}{J_M} \geq \frac{\tau^2}{J_M} \frac{T_{L,rms}^{*2}}{\eta^2} + J_M \frac{\dot{\omega}_{L,rms}^2}{\tau^2} + 2 \frac{\left(T_L^* \dot{\omega}_L \right)_{mean}}{\eta} \tag{22}$$

4.2.1 The accelerating factor and the load factor

Considering eq.(22) two terms can be introduced: the *accelerating factor* Legnani et al. (2002)

$$\alpha = \frac{T_{M,N}^2}{J_M} \tag{23}$$

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[2] For purely inertial load eq.(15) can be written as:

$$T_L^* = (J_L + J_T) \dot{\omega}_L$$

Then eq.(21) becomes:

$$\left(\frac{\tau (J_L + J_T)}{\eta_d} + \frac{J_M}{\tau} \right) |\dot{\omega}_L| \geq \left(\tau (J_L + J_T) \eta_r + \frac{J_M}{\tau} \right) |\dot{\omega}_L|$$

and thus:

$$\frac{1}{\eta_d} \geq \eta_r$$

which is always true since both η_d and η_r are defined as positive and smaller than 1.

that characterizes the performance of a motor Giberti et al. (2010b), and the *load factor*:

$$\beta = 2\left[\dot{\omega}_{L,rms}T^*_{L,rms} + (\dot{\omega}_L T^*_L)_{mean}\right] \tag{24}$$

defining the performance required by the task. The unit of measurement of both factors is *W/s*.

Coefficient α is exclusively defined by parameters related to the motor and therefore it does not depend on the machine task. It can be calculated for each motor using the information collected in the manufacturers' catalogs. On the other hand, coefficient β depends only on the working conditions (applied load and law of motion) and it is a term that defines the power rate required by the system.

4.2.2 Range of suitable transmission ratios

Introducing α and β, equation (22) becomes:

$$\alpha \geq \frac{\beta}{\eta} + \left[\frac{T^*_{L,rms}}{\eta}\left(\frac{\tau}{\sqrt{J_M}}\right) - \dot{\omega}_{L,rms}\left(\frac{\sqrt{J_M}}{\tau}\right)\right]^2 \tag{25}$$

Since the term in brackets is always positive, or null, the load factor β represents the minimum value of the right hand side of equation (25). It means that the motor accelerating factor α must be sufficiently greater than the load factor β/η, so that inequality (22) is verified. This check is a first criterion for discarding some motors. If $\alpha > \beta/\eta$, a range of useful transmission ratio exists and can be obtained by solving the biquadratic inequality:

$$\left(\frac{T^*_{L,rms}}{\eta^2 J_M}\right)\tau^4 + \left(\frac{\beta}{\eta} - \alpha - 2\frac{T^*_{L,rms}}{\eta}\dot{\omega}_{L,rms}\right)\tau^2 + J_M\dot{\omega}^2_{L,rms} \leq 0 \tag{26}$$

Inequality (26) has 4 different real solutions for τ. As the direction of the rotation is not of interest, only the positive values of τ are considered. A range of suitable transmission ratios is included between a minimum τ_{min} and a maximum gear ratio τ_{max} for which the condition in equation (9) is verified:

$$\tau_{min}, \tau_{max} = \eta\sqrt{J_M}\frac{\sqrt{\alpha - \frac{\beta}{\eta} + \frac{4\dot{\omega}_{L,rms}T^*_{L,rms}}{\eta}} \pm \sqrt{\alpha - \frac{\beta}{\eta}}}{2T^*_{L,rms}} \tag{27}$$

$$\tau_{min} \leq \tau \leq \tau_{max} \tag{28}$$

>From equation 27 it is evident that a solution exists only if $\alpha \geq \frac{\beta}{\eta}$.

4.2.3 The optimum transmission ratio

The constraint imposed by equation (25) becomes less onerous when a suitable transmission is selected, with a transmission ratio τ that annuls the terms in brackets. This value of τ is called *optimum transmission ratio*. Considering an ideal transmission ($\eta = 1$, $J_T = 0\,kgm^2$) one gets:

$$\tau = \tau_{opt} = \sqrt{\frac{J_M\dot{\omega}_{L,rms}}{T^*_{L,rms}}} \tag{29}$$

that, for a purely inertial load ($T_L = 0$), coincides with the value introduced in Pasch et al. (1984):

$$\tau' = \sqrt{\frac{J_M}{J_L}} \qquad (30)$$

The choice of the optimum transmission ratio allows system acceleration to be maximized (supplying the same motor torque) or to minimize the torque supplied by the motor (at the same acceleration).

For real transmissions, eq.(29) takes the general form:

$$\tau = \tau_{opt,\eta} = \sqrt{\frac{J_M \dot{\omega}_{L,rms}}{T^*_{L,rms}} \eta} = \tau_{opt} \sqrt{\eta} \qquad (31)$$

showing the dependence of the optimum transmission ratios on the mechanical efficiency η. Eq. (31) shows how the optimization of the performance of the motor-reducer unit through the concept of inertia matching is considerably affected by the mechanical efficiency. In the following (par. 6.3) this effect will be graphically shown.

4.3 Limit on the motor maximum torque

As shown in Fig.3, each motor can supply a maximum torque $T_{M,max}(\omega_M)$ that depends on the speed ω_M. However this relationship cannot be easily described by a simple equation. As a result, it is difficult to express condition (10) as a range of suitable transmission ratios.

Moreover the maximum torque that can be exerted depends on the maximum current supplied by the drive system. For this reason, these conditions will be checked only once the motor and the transmission have been chosen. It has to be:

$$T_{M,max}(\omega_M) \geq \max \left| \frac{\tau T_L}{\eta} + \left(\frac{J_M + J_T}{\tau} + \frac{J_L \tau}{\eta} \right) \dot{\omega}_L \right| \qquad \forall \omega. \qquad (32)$$

This test can be easily performed by superimposing the motor torque $T_M(\omega_M)$ on the motor torque/speed curve.

4.4 Checks

Once the transmission has been chosen, it is important to check the operating conditions imposed by the machine task satisfy the limits imposed by the manufacturers. Main limits concern:

- the maximum achievable speed;
- the maximum torque applicable on the outcoming shaft;
- the nominal torque applicable on the outcoming shaft;
- the maximum permissible acceleration torque during cyclic operation (over 1000/h) using the load factor.

There is no a standard procedure for checking the transmission. Each manufacturer, according to his experience and to the type of transmission produced, generally proposes an empirical procedure to check the right functioning of his products.

These verifications can be carried out by collecting information from catalogs.

5. Ideal transmission

If both the inertia of the transmission and the power losses are negligible, the gearbox can be considered as ideal ($J_T = 0$ kgm^2, $\eta = 1$). In this cases the problem is easier and the equations describing the dynamical behavior of the machine and the corresponding operating conditions can be simplified.

The limit on the maximum achievable speed remains the same (6), since it arises from kinematic relationships, while limits on the root mean square torque (7) and maximum torque (8) are simplified. Inequality (22) can be written as:

$$\frac{T_{M,N}^2}{J_M} \geq \tau^2 \frac{T_{L,rms}^{*2}}{J_M} + J_M \frac{\dot{\omega}_{L,rms}^2}{\tau^2} + 2(T_L^* \dot{\omega}_L)_{mean}. \tag{33}$$

whose solutions are between

$$\tau_{min}, \tau_{max} = \frac{\sqrt{J_M}}{2T_{L,rms}^*} \left[\sqrt{\alpha - \beta + 4\dot{\omega}_{L,rms} T_{L,rms}^*} \pm \sqrt{\alpha - \beta} \right]. \tag{34}$$

Accordingly, the condition on the maximum torque can be expressed as:

$$T_{M,max}(\omega_M) \geq \max \left| \tau T_L + \left(\frac{J_M}{\tau} + J_L \tau \right) \dot{\omega}_L \right| \qquad \forall \omega. \tag{35}$$

5.1 Selection of gearbox and motor

The main steps to select the gear-motor are:

STEP 1: Creation of a database containing all the commercially available motors and reducers useful for the application. For each motor the accelerating factor (α_i) must be calculated. Once the database has been completed it can be re-used and updated each time a new motor-reducer unit selection is needed.

STEP 2: Calculation of the load factor β, on the basis of the features of the load (T_L^*).

STEP 3: Preliminary choice of useful motors: all the available motors can be shown on a graph where the accelerating factors of available motors are compared with the load factor. All the motors for which $\alpha < \beta$ can be immediately rejected because they cannot supply sufficient torque, while the others are admitted to the next selection phase. Figure 7 is related to the industrial example discussed in par.5.2 and displays with circles the acceleration factors α_i of the analyzed motors, while the horizontal line represents the load factor β.

STEP 4: Identification of the ranges of useful transmission ratios for each motor preliminarily selected in step 3. For these motors a new graph can be produced displaying for each motor the value of the transmission ratios τ_{max}, τ_{min}, τ_{opt} and $\tau_{M,lim}$. The graph is generally drawn using a logarithmic scale for the y-axis, so τ_{opt} is always the midpoint of the adoptable transmission ratios range. In fact:

$$\tau_{opt}^2 = \tau_{min}\tau_{max} \quad \Leftrightarrow \quad \log \tau_{opt} = \frac{\log \tau_{min} + \log \tau_{max}}{2} \tag{36}$$

A motor is acceptable if there is at least a transmission ratio τ for which equations (18, 28) are verified. These motors are highlighted by a vertical line on the graph. Figure 8 is related to the same industrial example and shows the useful transmission ratios for each motor preliminarily selected.
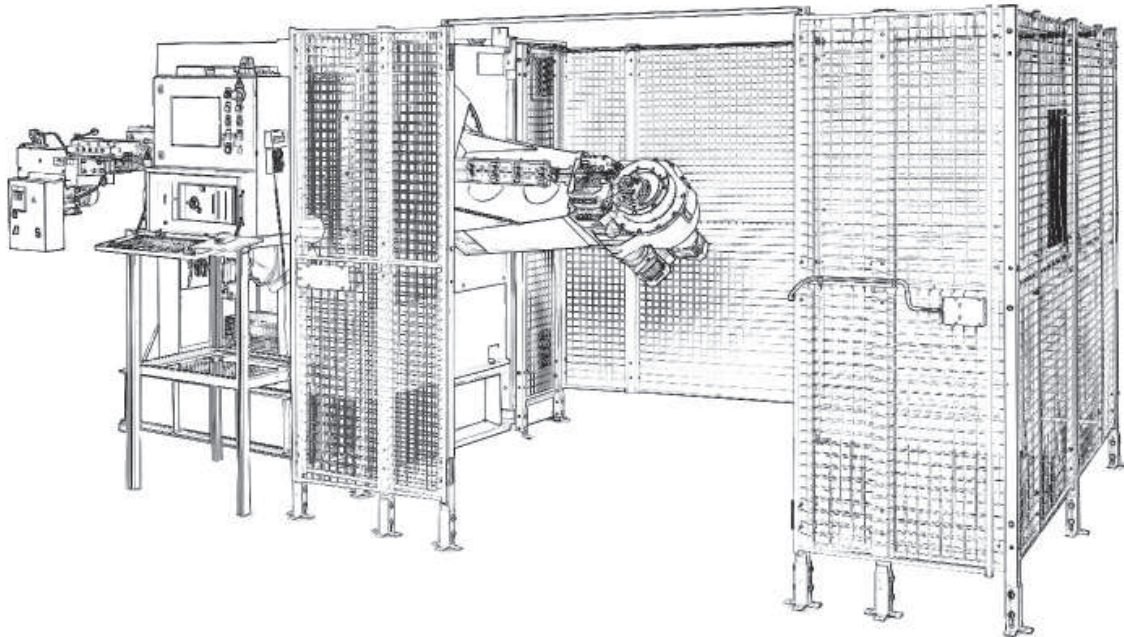
Fig. 5. CNC wire bending machine

STEP 5: Identification of the useful commercial speed reducers: the speed reducers available are represented by horizontal lines. If one of them intersects the vertical line of a motor, this indicates that the motor can supply the required torque if that specific speed reducer is selected. Table 2 sums up the acceptable combinations of motors and speed reducers for the case shown in figure 8. These motors and reducers are admitted to the final selection phase.

STEP 6: Optimization of the selected alternatives: the selection can be completed using different criteria such as economy, overall dimensions, space availability or any other depending on the specific needs.

STEP 7: Checks (see Sec.4.4).

5.2 Example

Fig. 5 shows a CNC wire bending machine. The system automatically performs the task of bending in the plane, or three-dimensionally, a wire (or tape) giving it the desired geometry. The machine operation is simple: semi-finished material is stored in a hank and is gradually unrolled by the unwinding unit. The straightened wire is guided along a conduit to the machine's bending unit, which consists of a rotating arm on which one or more bending heads are mounted. Each head is positioned in space by a rotation of the arm around the axis along which the wire is guided in order to shape it in all directions 6.

The production capacity of the machine is related to the functionality of the heads, while bending productivity depends strongly on arm speed, which allows the heads to reach the position required for bending. The design of the system actuating the rotating arm (selection of motor and speed reducer) is therefore one of the keys to obtaining high performance.
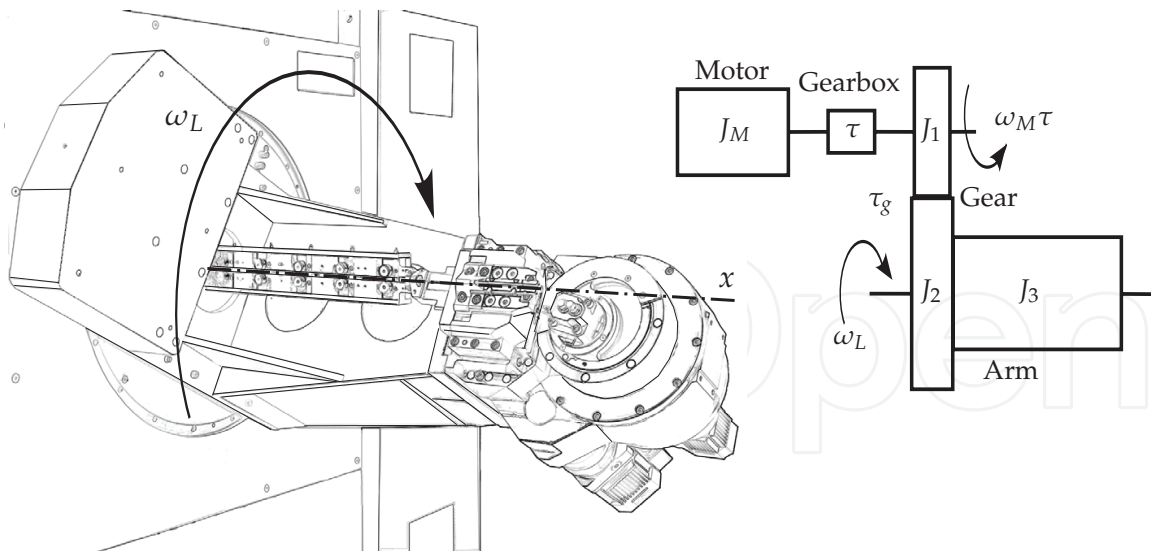
Fig. 6. The moving arm and mechanical system layout

Consider now only the bending unit: the motor, with its moment of inertia J_M, is connected through a planetary reducer with transmission ratio τ to a pair of gear wheels that transmits the rotation to the arm.

The pair of gear wheels has a ratio $\tau_2 = 1/5$ which is dictated by the overall dimension of the gearbox and cannot be modified. Putting $J_1 = 0.0076$ [kgm^2], $J_2 = 1.9700$ [kgm^2] and $J_3 = 26.5$ [kgm^2] respectively as the moment of inertia referred to the axes of rotation of the two wheels and of the arm, the comprehensive moment of inertia referred to the output shaft of the planetary gear is:

$$J_L = J_1 + (J_2 + J_3)\tau_2^2 = 1.1464 \text{ [kgm}^2]$$

Since the load is purely inertial[3], the load factor can easily be calculated as:

$$\beta = 4J_L\dot{\omega}_{L,rms}^2 \qquad (37)$$

where $\dot{\omega}_{L,rms}$ is a function of the law of motion used. The choice of the law of motion depends on the kind of operation requested and, in the most extreme case, it consists of a rotation of $h = 180°$ in $t_a = 0.6$ [s]. After this, a stop of $t_s = 0.2$ [s] before the next rotation is normally scheduled.

The value of $\dot{\omega}_{L,rms}$ can be expressed through the mean square acceleration coefficient ($c_{a,rms}$) using the equation:

$$\dot{\omega}_{L,rms} = c_{a,rms}\frac{h}{t_a^2}\frac{1}{\tau_2}\sqrt{\frac{t_a}{t_a + t_s}}$$

As is known (Van de Straete et al., 1999), the minimum mean square acceleration law of motion is the cubic equation whose coefficient is $c_{rms} = 2\sqrt{3}$. Moreover, this law of motion has the advantage of higher accelerations, and therefore high inertial torques, corresponding to low velocities. Substituting numerical values in eq.(37) one gets: $\beta = 7.8573 \cdot 10^4$ [W/s].

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[3] In the selecting phase frictions are not considered.

Considering the same law of motion, maximum acceleration and maximum speed can easily be obtained by:

$$\dot{\omega}_{L,max} = c_a \frac{h}{t_a^2} \frac{1}{\tau_2} \simeq 261.8 \ [\text{rad/s}^2]; \quad \omega_{L,max} = c_v \frac{h}{t_a} \frac{1}{\tau_2} \simeq 39.3 \ [\text{rad/s}] \tag{38}$$

where $c_a = 6$ and $c_v = 1.5$.

Knowing the load factor β and after selecting the motors and transmissions available from catalogs, the graph shown in Fig.7 can be plotted. Available motors for this application are synchronous sinusoidal brushless motors[4]. Manufacturer's catalogs give information on motor inertia, maximum and nominal torque. A first selection of suitable motors can be performed. Motors whose accelerating factor α_i is lower than the load factor β can be discarded.

For all the accepted motors a new graph can be produced. It displays, for each motor, the corresponding minimum and maximum transmission ratios and the optimum and the minimum kinematic transmission ratios.

They can be obtained using the simplified expression for the purely inertial load case:

$$\tau_{min}, \tau_{max} = \frac{\sqrt{J_M}}{2T_{L,rms}^*} \left[\sqrt{\alpha} \pm \sqrt{\alpha - \beta} \right]. \tag{39}$$

and eq.(29),(17). Commercial transmissions considered for the selection are planetary reducers[5].

The graph in Fig.8 shows all the available couplings between the motors and transmissions considered. Three of the eleven motors (M1, M2 e M7) are immediately discarded, since their accelerating factors α are too small compared with the load factor β. Motors M3, M4, M5, and M6 are eliminated because their maximum speed is too low. Suitable motors are M8, M9, M10 and M11. The selection can be completed evaluating the corresponding available commercial speed reducers whose ratio is within the acceptable range. Motor M8 is discarded since no transmission can be coupled to it. Suitable pairings are shown in Tab.2.

Motor	Speed reducer
M9	$\tau = 1/10, \tau = 1/7$
M10	$\tau = 1/5, \tau = 1/4$
M11	$\tau = 1/5, \tau = 1/4, \tau = 1/3$

Table 2. Combination of suitable motors and speed reducers for the industrial example discussed in par.5.2

The final selection can be performed using the criterion of cost: the cheapest solution is motor M9 and a reducer with a transmission ratio $\tau = 1/10$. The main features of the selected motor[6] and transmission[7] are shown in table 3.

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[4] Produced by "Mavilor", *http://www.mavilor.es/*.

[5] produced by "Wittenstein", *http://www.wittenstein.it/*.

[6] Model Mavilor BLS 144.

[7] Model Alpha SP+140.
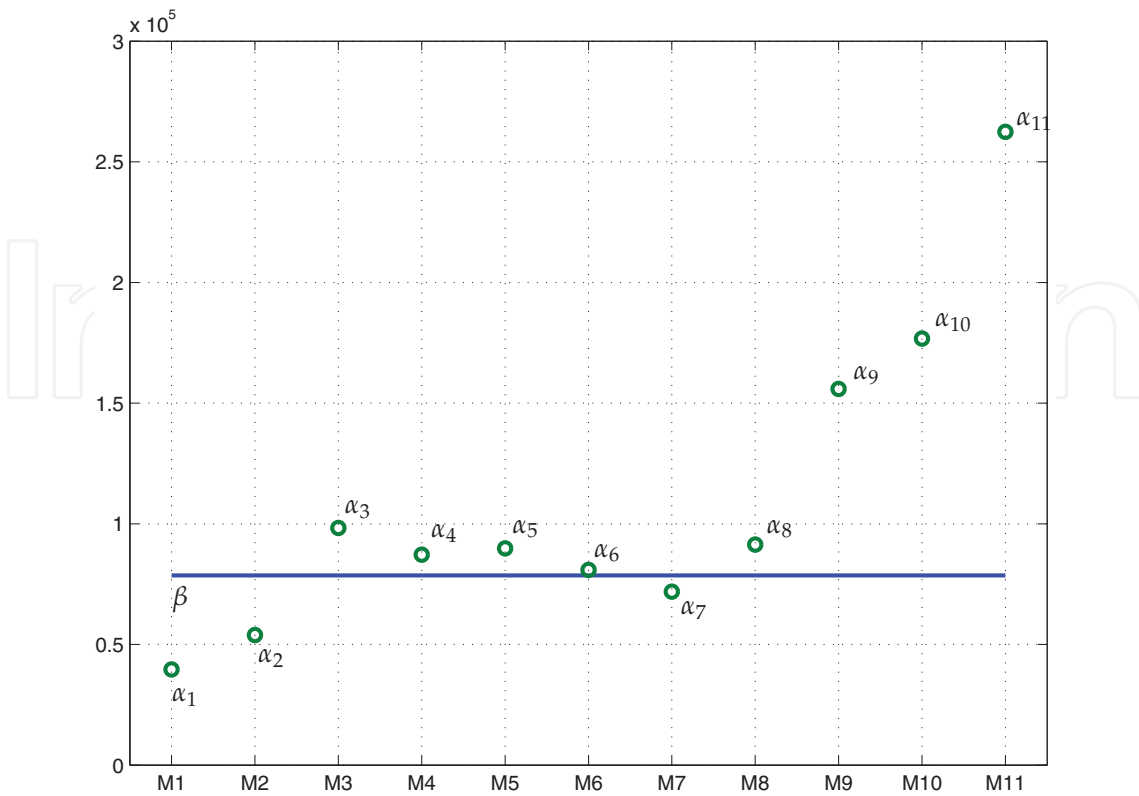
Fig. 7. A first selection of suitable motors

Motor M9	
moment of inertia	$J_M = 0.0046$ [kgm^2]
nominal torque	$T_{M,N} = 26.7$ [Nm]
maximum Torque	$T_{M,max}^{TH} = 132$ [Nm]
maximum achievable speed	$\omega_{M,max} = 5000$ [rpm]
Speed reducer $\tau = 1/10$	
moment of inertia	$J_T = 5.8 \cdot 10^{-4}$ [kgm^2]
nominal torque	$T_{T,N} = 220$ [Nm]
maximum Torque	$T_{T,max} = 480$ [Nm]
maximum endurable speed	$\omega_{T,max} = 4000$ [rpm]
nominal speed	$\omega_{T,N} = 2600$ [rpm]
mechanical efficiency	$\eta = 0.97$

Table 3. Main features of selected motor and transmission

Figure 9 shows the required motor torque as a function of speed during the working cycle. It has been calculated considering the inertia of both the motor and the gearbox and the mechanical efficiency of the transmission. Since the mechanical efficiency of the speed reducer in backward power flow mode is not available, it is assumed to be equal to that in direct power flow. To verify the condition on the maximum torque reported in eq.(32), the curve has to be contained within the dynamic working field. Note how the maximum torque achieved by the motor is limited by the drive associated with it. From Fig.9 it is possible to check that the condition on the maximum torque is verified.
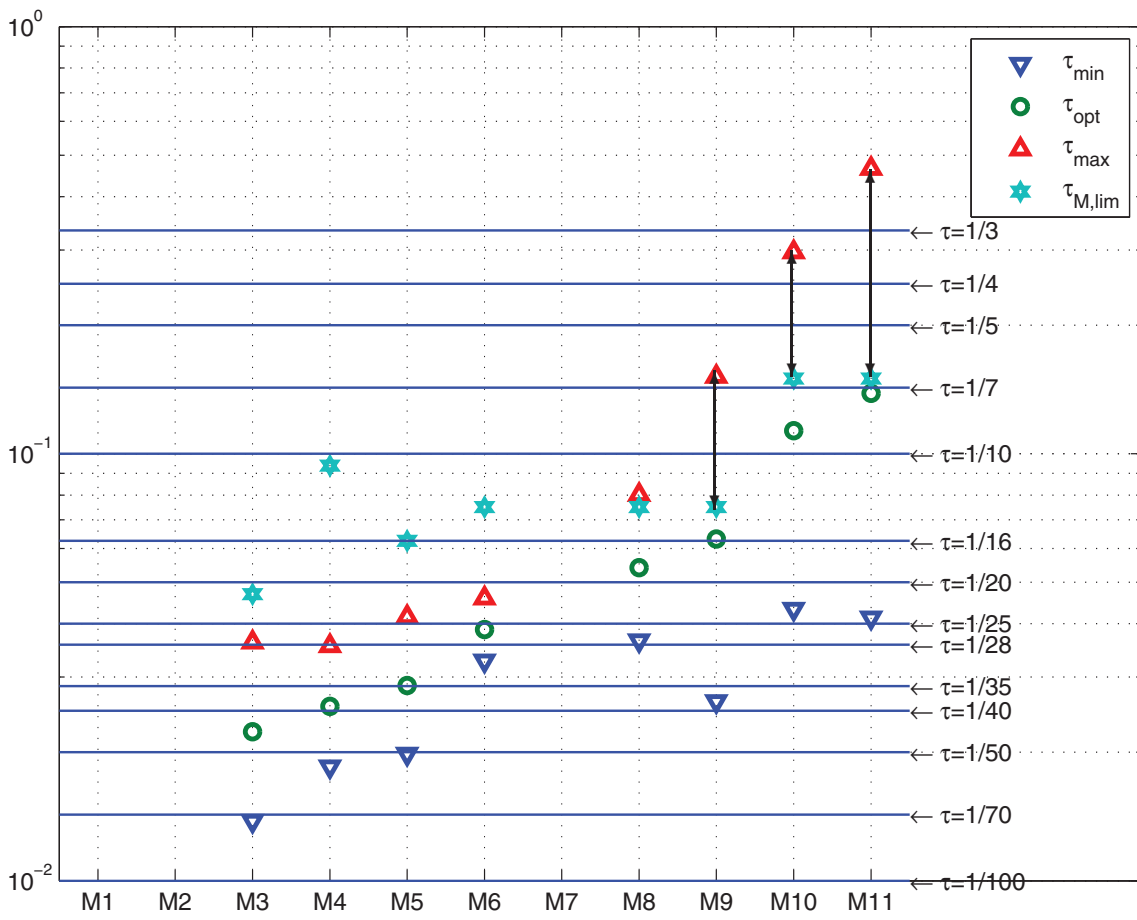
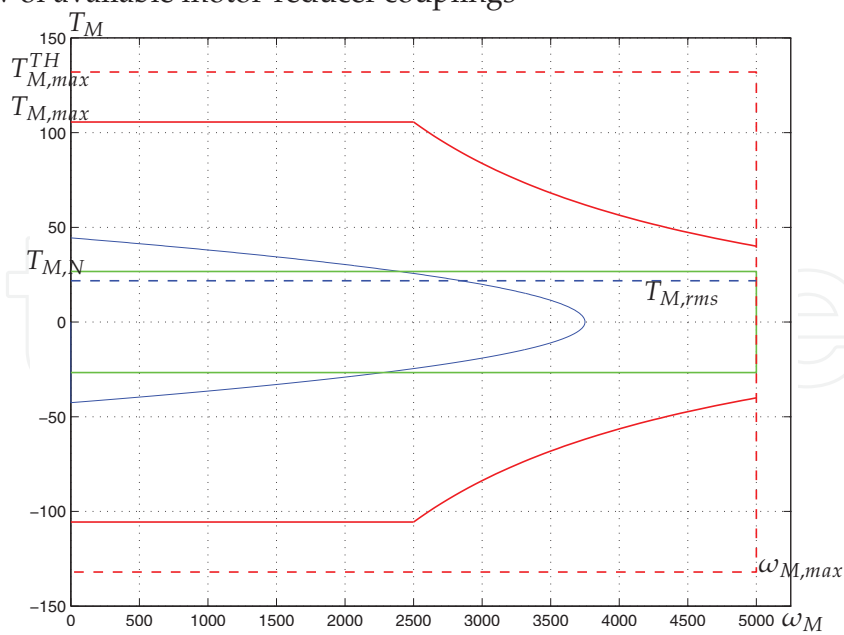Fig. 8. Overview of available motor-reducer couplings

Fig. 9. Checkouts on maximum and nominal torque.

The motor root mean square torque can now be updated, considering the inertia of the transmission and its mechanical efficiency.

Finally, checks should be carried out on the reducers following the manufacturer's guidelines. In this case they mainly consist of verifying that both the maximum and the nominal torque applied to the transmission incoming shaft are lower than the corresponding limits shown in the catalog ($T_{T,max}$, $T_{T,N}$).

$$T_{max} \simeq 300[Nm] < T_{T,max} = 480[Nm]$$

$$T_n \simeq 150[Nm] < T_{T,N} = 220[Nm]$$

In addition, the maximum and the mean angular speed of the incoming shaft have to be lower than the corresponding limits on velocity ($\omega_{T,max}$, $\omega_{T,N}$).

$$n_{max,rid} \simeq 3750[rpm] < \omega_{T,max} = 4000[rpm]$$

$$n_{mean,rid} \simeq 1873[rpm] < \omega_{T,N} = 2600[rpm]$$

The selected motor-transmission pairing satisfies all the checks and provides margins for both the motor ($\approx 20\%$ on the nominal torque) and the reducer.

6. Real transmission

For machines working with direct power flow, a decrease of the performance of the transmission that corresponds to an increase in the power dissipation may result in a motor overhead which makes it non longer adequate.

6.1 The mechanical efficiency limit

A motor which is able to perform the task planned in ideal conditions ($\eta = 1$), when coupled with a transmission characterized by poor efficiency, could be discarded. Referring to equation (27), once β is known, a minimum transmission mechanical efficiency exists, for each motor, below which τ_{min} and τ_{max} are undefined. The limit value is called the transmission *mechanical efficiency limit* and it is defined as the ratio between the load factor and the accelerating factor:

$$\eta \geq \eta_{lim} = \frac{\beta}{\alpha} \tag{40}$$

This parameter gives to the designer a fundamental indication: if the task required by the machine is known (and thus the load factor can be calculated), for each motor there is a minimum value of the transmission mechanical efficiency below which the system can not work. This limit is not present in the case of backward power flow functioning.

6.2 Restriction of the range of useful transmission ratios

For each selectable motor it is possible to graphically represent the trend of both the minimum and maximum transmission ratios. Combining equations (27) and (40), the two functions τ_{min} and τ_{max} are respectively defined as:

$$\tau_{min}, \tau_{max} = \begin{cases} \eta\sqrt{J_M}\dfrac{\sqrt{\alpha - \frac{\beta}{\eta} + \frac{4\dot{\omega}_{L,rms}T^*_{L,rms}}{\eta}} \pm \sqrt{\alpha - \frac{\beta}{\eta}}}{2T^*_{L,rms}} & \text{if } \eta \geq \eta_{lim} \\ \text{undefined} & \text{if } \eta < \eta_{lim} \end{cases} \tag{41}$$
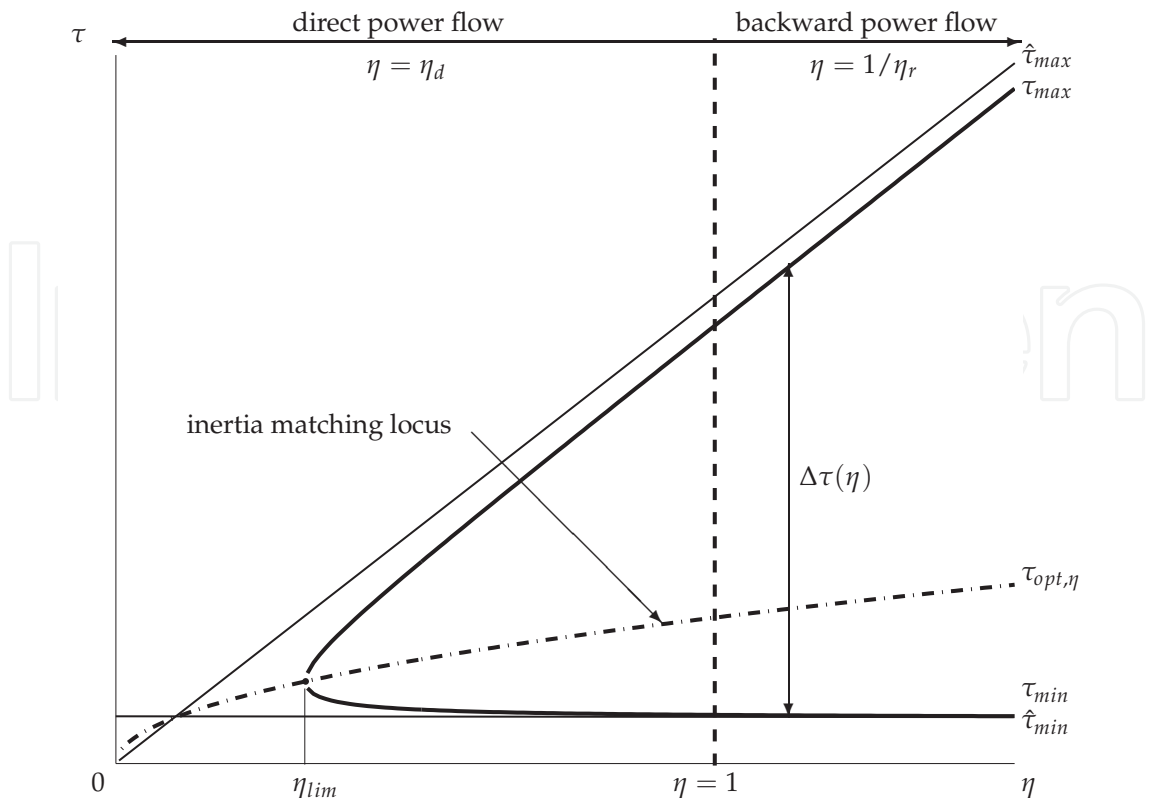
Fig. 10. Trends of τ_{min} and τ_{max} as functions of mechanical efficiency (using the notation introduced in eq. (16)), for a certain motor.

For each motor it is possible to plot τ_{min} and τ_{max} as functions of the mechanical efficiency, highlighting a region in the plane $\eta\tau$ satisfying the condition on the root mean square torque (Fig.10). Depending on the machine functioning mode (direct or backward power flow) the left side or the right side of the graph should be taken into account. On the same graph the trends of the optimum transmission ratio $\tau_{opt,\eta}$ and the breadth $\Delta\tau$ of the range of useful transmission ratios is shown. Note that the range grows monotonically with the difference between the accelerating and load factors. In particular, the range breadth is:

$$\Delta\tau(\eta) = \frac{\sqrt{J_M}}{T^*_{L,rms}}\eta\sqrt{\alpha - \frac{\beta}{\eta}} \tag{42}$$

and decreases appreciably with the transmission mechanical efficiency. However, while the limit on the maximum transmission ratio τ_{max} varies considerably, the minimum transmission ratio τ_{min} remains almost constant. This behavior is clearly visible in the $\eta\tau$ graph which plots the asymptotes of the two functions (Fig. 10) described respectively by:

$$\hat{\tau}_{max} = \frac{T_{M,N}}{T^*_{L,rms}}\eta \qquad \hat{\tau}_{min} = \frac{J_M}{T_{M,N}}\dot{\omega}_{L,rms} \tag{43}$$

It is interesting to observe that, while $\hat{\tau}_{max}$ depends on the reducer, $\hat{\tau}_{min}$ depends only on the chosen motor and on the law of motion defined by the task. This is because the transmission ratio τ is so small that the effect of the load is negligible compared to the inertia of the motor.

The power supplied, therefore, is used just to accelerate the motor itself. Note that, for values of $\eta > 1$, that is backward power flow functioning, the range of suitable transmission ratios is wider than in the case of direct power flow functioning, which is the most restrictive working mode. For this reason, in all cases where the direction of power flow is not mainly either direct or backward, the first functioning mode can be considered as a precautionary hypothesis on the root mean square torque and the left side of the $\eta\tau$ graph ($\eta < 1$) can be used.

6.3 The extra-power rate factor

Inequality (25) can be written as:

$$\alpha \geq \frac{\beta}{\eta} + \gamma(\tau, \eta, J_M) \qquad (44)$$

where:

$$\gamma(\tau, \eta, J_M) = \left[\frac{T^*_{L,rms}}{\eta} \left(\frac{\tau}{\sqrt{J_M}} \right) - \dot{\omega}_{L,rms} \left(\frac{\sqrt{J_M}}{\tau} \right) \right]^2 \qquad (45)$$

The term γ is called the *extra-power rate factor* and represents the additional power rate that the system requires if the transmission ratio is different from the optimum ($\tau \neq \tau_{opt}$).

Figure 11 shows the trends of the terms of the γ function when the transmission efficiency changes. Note that, when the transmission ratio is equal to the optimum ($\tau = \tau_{opt}$), the curve γ reaches a minimum. For this value the convexity of the function is small and, even for large variations of the transmission ratio, the extra-power rate factor appears to be contained in eq.(44).

With the mechanical efficiency decreasing, two effects take place: first the optimum transmission ratio decreases, moving on the left of the graph, secondly the convexity of the curve γ is more pronounced and the system is more sensitive to changes in τ with respect to the optimum. For transmissions characterized by poor mechanical efficiency, in the case of the direct power flow mode, the choice of a gear ratio different from the optimum significantly affects the choice of the motor.

6.4 Effect of the transmission inertia

The inclusion of the transmission inevitably changes the moment of inertia of the system. With J_T as the moment of inertia of the speed reducer, referred to its outgoing shaft, the resistive torque is generally given by eq.(15). Entering this new value in eq.(24), the load factor can be updated. This change makes the system different from that previously studied with the direct consequence that the limit on the mean square torque can no longer be satisfied.

In particular, for the ith transmission, characterized by a moment of inertia $J_{T,i}$, the limits on the transmission ratio to satisfy the root mean square torque condition can be expressed as $\tau_{min,i}$ and $\tau_{max,i}$. Let's consider, as example, a machine task characterized by a constant resistant load ($T_L = cost$). Figure 12 shows how the range of suitable transmission ratios is reduced when the inertia of the transmission increases. The same reduction can be observed in the $\eta\tau$ graph (Fig. 13) for a purely inertial load. Note that, even for the moment of inertia, there is a limit value beyond which, for a given motor, there is no suitable transmission ratio.
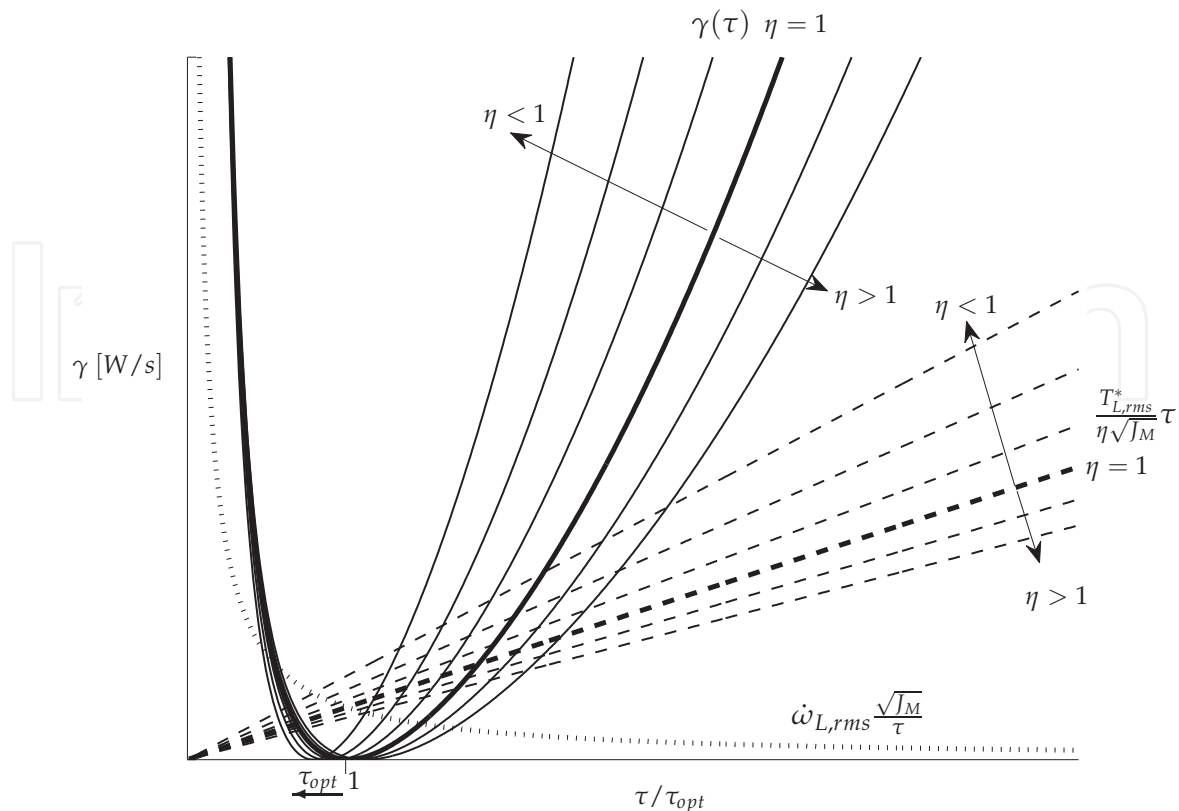
Fig. 11. Variation of the extra-power rate factor as a function of the transmission ratio and mechanical efficiency.

7. Guidelines for the motor-reducer selection

The theoretical steps presented can be summarized by a series of graphs for evaluating the effect of the transmission on the choice of the motor that make the selection process easy to use.

Firstly, to ensure the condition on the root mean square torque it should be verified that the accelerating factor α of each available motor is greater than the limit β/η. However, since the transmission has not yet been selected and thus its efficiency and inertia are still unknown, it is possible to perform only a first selection of acceptable motors, eliminating those for which $\alpha < \beta$.

For each selectable motor a $\eta\tau$ graph can be plotted (Fig. 14), with the limits on the transmission ratio defined by equations (18), (28). Since these functions depend on the transmission moment of inertia, such limits are plotted for each available transmission.

Available transmissions can be inserted in the $\eta\tau$ graph (there is an example in Fig. 14) using their coordinates (η_{di}, τ_i) and (η_{ri}, τ_i) which can easily be found in manufacturers' catalogs. Each speed reducer appears twice: to the left of the dashed line for direct power flow, to the right for backward power flow.

Remember that for tasks characterized by mainly backward power flow, only the transmissions on the right half plane should be considered ($\eta > 1$). For all other cases only the transmissions on the left half plane ($\eta < 1$) should be taken into account.

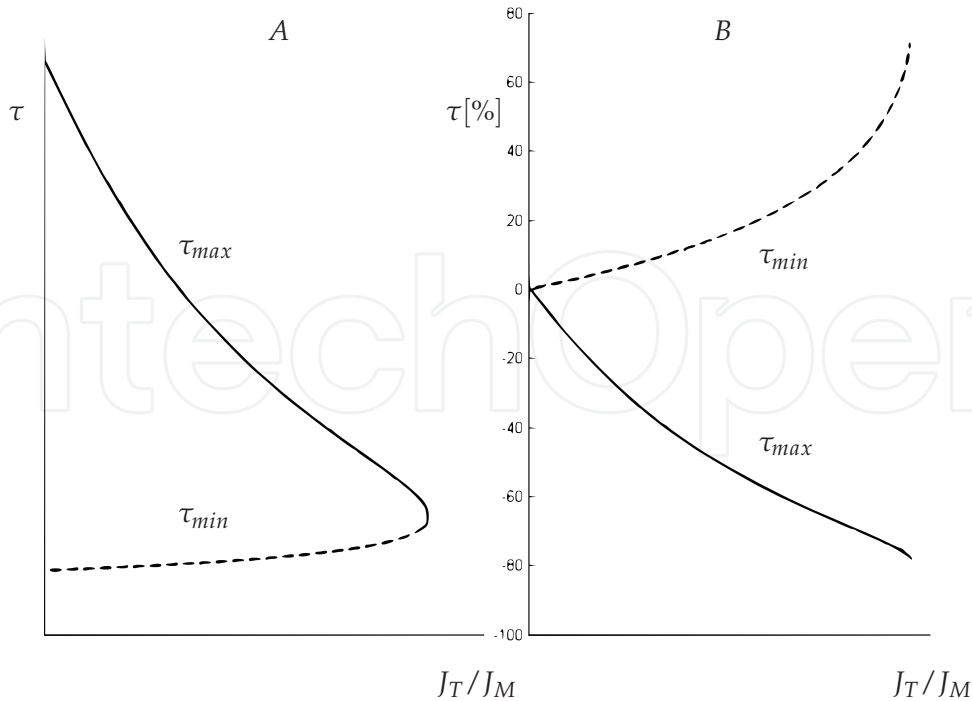Fig. 12. A - reduction of range of useful transmission ratio as function of J_T/J_M (for $T_L = $ cost); B - percentage change of maximum and minimum transmission ratios as function of J_T/J_M (for $T_L = $ cost).
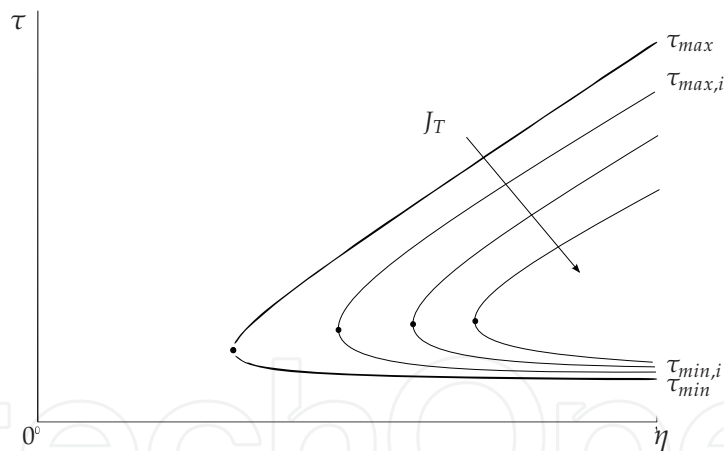

Fig. 13. Effect of the transmission moment of inertia on the $\eta\tau$ graph (for $T_L = $ cost).

The ith speed reducer is acceptable if it lies inside the area limited by the limit transmission ratio (τ_{kin}) and the corresponding maximum and minimum ratios ($\tau_{max,i}$, $\tau_{min,i}$). These reducers are highlighted on the graph with the symbol $\odot$, unacceptable ones with $\otimes$

Table 4 resumes all the alternatives both for the direct and backward power flow modes.

Note that, for the direct power flow mode, transmissions which would be acceptable with normal selection procedures, are now discarded (e.g. transmission T_2 because of insufficient efficiency, reducer T_4 because of its excessively high moment of inertia).

Moreover it is evident that, for the motor considered, there are no acceptable transmissions with a ratio equal to the optimum. More generally it could happen that a motor, while meeting
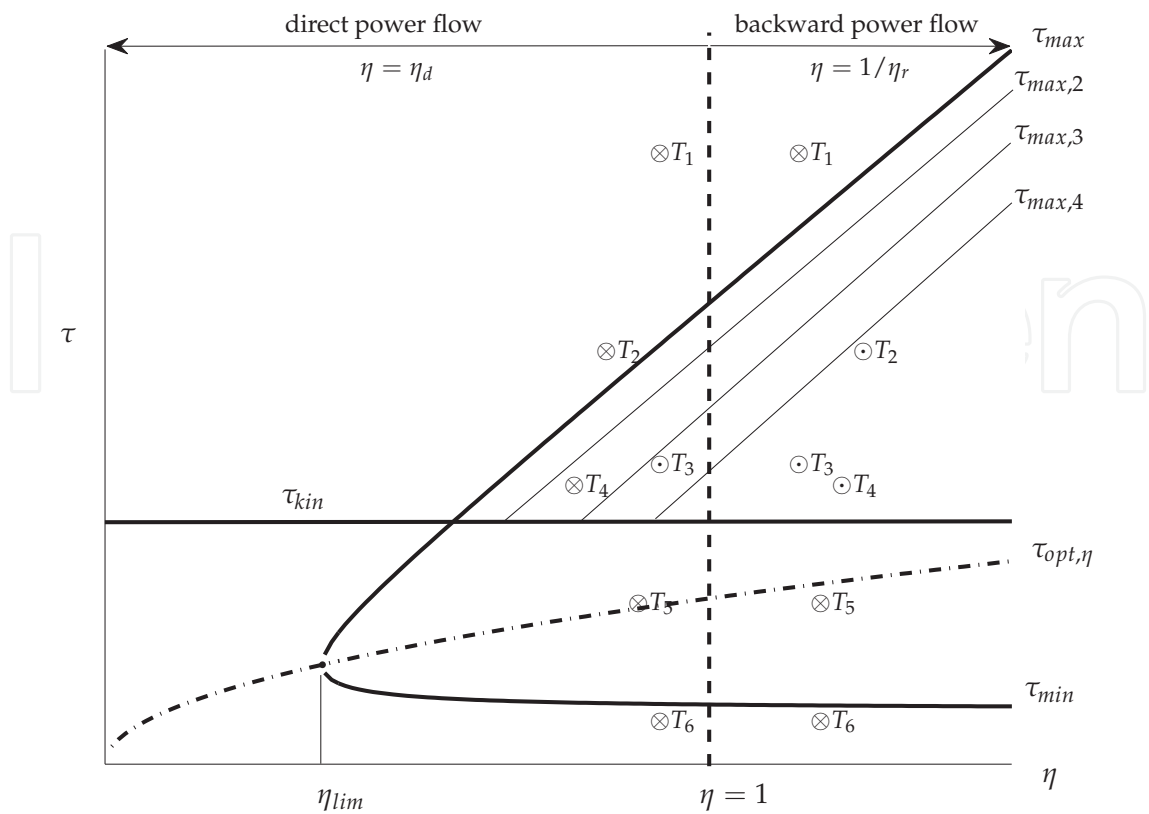
Fig. 14. Graph $\eta\tau$ for the selection of the speed reducer for a specific motor: ($\odot$ acceptable transmission, $\otimes$ unacceptable transmission)

Transmission	Direct P.F.	Cause	Backward P.F.	Cause
T_1	unaccept.	$\tau_1 > \tau_{max}$	unaccept.	$\tau_1 > \tau_{max}$
T_2	unaccept.	$\tau_2 > \tau_{max}$	accept.	—
T_3	accept.	—	accept.	—
T_4	unaccept.	$\tau_4 > \tau_{max,4}$	accept.	—
T_5	unaccept.	$\tau_5 < \tau_{kin}$	unaccept.	$\tau_5 < \tau_{kin}$
T_6	unaccept.	$\tau_6 < \tau_{kin} \wedge \tau_6 < \tau_{min}$	unaccept.	$\tau_6 < \tau_{kin} \wedge \tau_6 < \tau_{min}$

Table 4. Overview of acceptable and unacceptable transmissions for the example in Fig. 14

all the constraints mentioned, may have a range of acceptable gear ratios within which there is no reducer commercially available. For this reason it cannot be chosen.

The choice of the motor-reducer unit is made easy by comparing the $\eta\tau$ graphs for each selectable motor.

The resulting graphs give an overview of all the possible pairings of motors and transmissions that satisfy the original conditions. They allow the best solution to be selected from the available alternatives, in terms of cost, weight and dimensions, or other criteria considered important according to the application.

Once the motor and the transmission have been selected and all their mechanical properties are known, the final checks can be performed.

8. Conclusions

The correct choice of the motor-reducer unit is a key factor in automation applications. Such a selection has to be made taking into account the mechanical constraints of the components, in particular the operating ranges of the drive system and the mechanical features of the transmission. The paper investigates the effects of these constraints on the correct choice of the motor-reducer unit at the theoretical level and illustrated a method for its selection that allows the best available combination to be chosen using a practical approach to the problem.

It identifies the influence of the transmission's mechanical efficiency and inertia on the coupling between motor and reducer itself, showing how they affect the optimum solution. The procedure, based on the production of a chart containing all the information needed for the correct sizing of the system, sums up all the possible solutions and allows them to be quickly compared to find the best one.

9. Nomenclature

Symbol	Description
T_M	motor torque
J_M	motor moment of inertia
$T_{M,N}$	motor nominal torque
$T_{M,max}$	motor maximum torque
$\omega_M, \dot{\omega}_M$	motor angular speed and acceleration
T_L	load torque
J_L	load moment of inertia
T_L^*	generalized load torque
$T_{L,rms}^*$	generalized load root mean square torque
$T_{L,max}$	load maximum torque
$\omega_L, \dot{\omega}_L$	load angular speed and acceleration
$\dot{\omega}_{L,rms}$	load root mean square acceleration
t_a	cycle time
$\tau = \omega_L/\omega_M$	transmission ratio
τ_{opt}	optimal transmission ratio
η	transmission mechanical efficiency
η_d	transmission mechanical efficiency (direct power flow)
η_r	transmission mechanical efficiency (backward power flow)
J_T	transmission moment of inertia
α	accelerating factor
β	load factor
γ	extra-power rate factor
τ_{min}, τ_{max}	minimum and maximum acceptable transmission ratio
τ_{kin}	minimum kinematic transmission ratio
$\omega_{M,max}$	maximum speed achievable by the motor
$\omega_{L,max}$	maximum speed achieved by the load
W_M	motor side power
W_L	load side power

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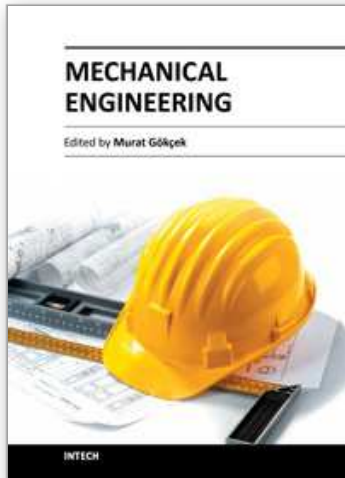
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