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Performance Robustness Criterion of PID Controllers

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1. Introduction

PID is one of the earliest and most popular controllers. The improved PID and classical PID have been applied in various kinds of industry control fields, as its tuning methods are developing. After the PID controller was first proposed by Norm Minorsky in 1922, the various PID tuning methods were developing and the advanced and intelligent controls were proposed. In the past few decades, Z-N method which is for first-order-plus-time-delay model was proposed by Ziegler and Nichols (Ziegler & Nivhols, 1943), CHR method about generalized passive systems was proposed by Chien, Hrones and Reswick (Chien et al., 1952), and so many tuning methods were developed such as pole assignment and zero-pole elimination method by Wittenmark and Astrom, internal model control (IMC) by Chien (Chien & Fruehauf, 1990). The gain and phase margin (GPM) method was proposed by Åström and Hägglund (Åström & Hägglund, 1984), the tuning formulae were simplified by W K Ho (Ho et al., 1995).

In classical feedback control system design, the PID controller was designed according to precise model. But the actual industrial models has some features as follows:

1. The system is time variant and uncertain because of the complex dynamic of industrial equipment.
2. The process is inevitably affected by environment and the uncertainty is introduced.
3. The dynamic will drift during operation.
4. The error exists with the dynamic parameter measurement and identification.

So there are two inevitable problems in control system designing. One is how to design robust PID controller to make the closed-loop system stable when the parameters are uncertain in a certain range. The other is the performance robustness which must be considered seriously when designing PID controllers. The performance robustness is that when the parameters of model change in a certain interval, the dynamic performances of system are still in desired range.

This chapter discusses the new idea mentioned previous – Performance Robustness. Based on the famous Monte-Carlo method, the performance robustness criterion is proposed. The performance robustness criterion could give us a new view to study the important issue that how the PID controller performs while the parameters of model are uncertain. Not only the stability, but also the time-domain specifications such as overshoot and adjusting time, and the frequency-domain specifications such as gain margin and phase margin can be obviously clear on the specification figures.

The structure of this chapter is as follows. A brief history of Monte-Carlo method is given in section 2. The origin, development and latest research of Monte-Carlo method are introduced. The performance robustness criterion is discussed in detail. This section also contains several formulas to explain the proposed criterion. In section 3, the performance robustness criterion is applied on typical PID control systems comparison, the detailed comparisons between DDE method and IMC method, and between DDE method and GPM method. Finally, section 4 gives out a conclusion.

2. Monte-Carlo method in performance robustness criterion

2.1 A brief history of Monte-Carlo method

Monte-Carlo method is also called random sampling technology or statistical testing method. In 1946, a physicist named Von Neumann simulated neutron chain reaction on computer by random sampling method called Monte-Carlo method. This method is based on the probability statistics theory and the random sampling technology. With the further development of computer, the vast random sampling test became viable. So it was consciously, widely and systematic used in mathematical and physical problems. The Monte-Carlo method is also a new important branch of computational mathematics.

In the late 20th century, Monte-Carlo method is closely linked the computational physics, computational statistical probability, interface science of computer science and statistics, and other boundary discipline. In addition, the Monte-Carlo method also plays a role for the development of computer science. In order to show the new performance evaluation method of mainframe which has multi-program, variable word length, random access and time-shared system, the performance of developed computer was simulated and analysed on the other computer. The relationship could be clear via the study on different target.

Large numbers of practical problems on nuclear science, vacuum technology, geological science, medical statistics, stochastic service system, system simulation and reliability were solved by Monte-Carlo method, and the theory and application results have gained. It was used in simulation of continuous media heat transfer and flow (Cui et al., 2000), fluid theory and petroleum exploration and development (Lu & Li, 1999). Monte-Carlo method was combined with heat network method to solve the temperature field of spacecraft, and the steady-state temperature field of satellite platform thermal design was calculated and analysed (Sun et al., 2001). In chemical industry, Yuan calculated the stability of heat exchanger with Monte-Carlo method, and it was used in selection and design (Yuan, 1999).

In power system, Monte-Carlo method was applied in reliability assessment of generation and transmission system, the software was design and the application was successful (Ding & Zhang, 2000).

2.2 Performance robustness criterion based on Monte-Carlo method

Consider the SISO system as follows:

$$G(s) = \frac{N(s)}{D(s)} e^{-Ls} \tag{1}$$

In this system, N(s) and D(s) are coprime polynomials, and D(s)'s order is greater than or equal N(s)'s order, L is rational number greater than or equal to zero. The controlled model is some uncertain, and the parameters of N(s) and D(s) are variable in bounded region. So, the model is a group of transfer function denoted by {G(s)}. The control system is shown in figure 1.

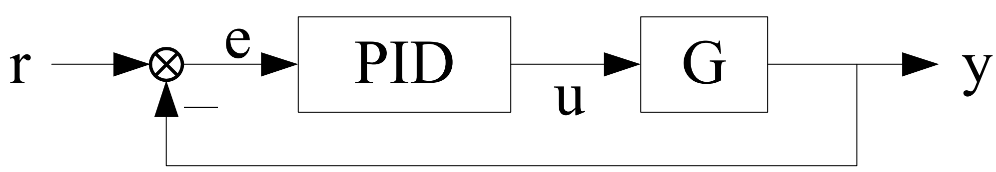


Fig. 1. Control system structure

The controller is PID controller:

$$u(s) = K_p(1 + \frac{1}{T_i s} + T_d s)e(s) \tag{2}$$

or

$$u(s) = (K_p + \frac{K_i}{s} + K_d s)e(s)$$

The parameters K_p, K_i, K_d are positive number, and all of the PID controllers compose a controller group denoted by {PID}.

The PID tuning methods are used on the nominal controlled models, and the closed-loop systems are obtained. The overshoot $\sigma\%$ and adjustment time T_s are considered as dynamic performance index. Because the controlled models are a group of transfer function, the dynamic performance index is a collection, denoted by:

$$\{\sigma\%, T_S\} \tag{3}$$

Obviously, it is a collection of two-dimension vector an area in plane plot. The distance between this area and origin reflects the quality of control system, and the size of this area shows the dispersion of performance index, that is the performance robustness of control system.

The comparison study on PID tuning methods should follow the steps below:

1. Confirm the controlled model transfer function and parameter variety interval, and the transfer function group is obtained.
2. Confirm the compared PID tuning methods, and choose the appropriate experiment times N to ensure the dispersion of performance index invariable when the N is larger.
3. Tuning PID controller for the nominal model.
4. In every experiment, a specific model is selected from the transfer function group by a rule (random in this paper). With the PID controller obtained in step three, the step response of closed-loop PID control system is tested, and the overshoot and adjustment time could be measured.
5. Repeat the step 4 N times, and plot the performance index on coordinate diagram. So, the N points compose an area on the coordinate diagram.
6. Repeat the step 3-5 by different tuning methods.
7. Compare the performance index of different tuning methods.

In next section, performance robustness is applied on PID control system comparison.

3. Performance robustness comparisons

3.1 Performance robustness comparison of typical PID control systems

In this section, we consider four typical models as follows:

1. First-order-plus-time-delay model (FOPTD)

$$G(s)=\frac{k}{1+sT}e^{-sL} \quad \text{k, T, L>0.} \tag{4}$$

2. Second-order-plus-time-delay model (SOPTD)

$$G(s)=\frac{k}{(1+sT_1)(1+sT_2)}e^{-sL} \quad \text{k, } T_1, T_2, L>0 \tag{5}$$

or

$$G(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}e^{-sL} \quad \omega_n>0\text{ , }1>\xi>0\text{ ,L>0.}$$

3. High-order model

$$G(s)=\frac{k}{(1+sT)^n} \quad \text{k, T>0, } n\geq 3 \text{ and } n\in N\text{ .} \tag{6}$$

4. Non-minimum model

$$G(s)=\frac{k(-s+a)}{(1+sT_1)(1+sT_2)} \quad \text{k ,} T_1, T_2, \text{a>0.} \tag{7}$$

The classical PID tuning methods are showed in table 1.

Tuning methods	K_p	T_i	T_d
Z-N	1.2T/kL	2L	L/2
CHR	0.6T/kL	T	L/2
Cohen-Coon	$\frac{1.35T}{kL}\left(1+\frac{0.18L}{T}\right)$	$\frac{0.5L+2.5T}{0.61L+T}L$	$\frac{0.37T}{0.19L+T}L$
IMC	$\frac{0.5L+T}{k(L+T_f)}$	T+L/2	$\frac{LT}{L+2T}$
IST^2E	$\frac{0.968}{k}\left(\frac{L}{T}\right)^{-0.904}$	$\frac{T}{0.977-0.253(L/T)}$	$0.316T\left(\frac{L}{T}\right)^{0.892}$
GPM	$\frac{W_p T}{A_m k}$	$\left(2W_p-\frac{4W_p{}^2 L}{\pi}+\frac{1}{T}\right)^{-1}$	
	$W_p=\frac{A_m\Phi_m+0.5\pi A_m(A_m-1)}{(A_m{}^2-1)L}$		

Table 1. Formulas of classical PID tuning method

If the tuning object is zero overshoot, the selection of IMC method free parameter T_f will only correlate to delay-time L. We fit the approximate relation between L and T_f.

$$\begin{cases} T_f = p_1 L^3 + p_2 L^2 + p_3 L + p_4 & L \le 100 \\ T_f = L/2 & L > 100 \end{cases} \tag{8}$$

where

$p_1=-1.7385\times10^{-5}$，$p_2=3.0807\times10^{-3}$，$p_3=0.3376$，$p_4=5.6400$.

The different transfer function models can be simplified and transferred to FOPTD model(Xue, 2000).

Suppose the FOPTD (4).

Calculate the first and second derivative and then we obtain

$$\frac{G_1'(s)}{G_1(s)} = -L - \frac{T}{1+Ts} \tag{9}$$

and

$$\frac{G_1''(s)}{G_1(s)} - \left(\frac{G_1'(s)}{G_1(s)}\right)^2 = \frac{T^2}{(1+Ts)^2}\,. \tag{10}$$

when s=0,

$$T_{ar} = -\frac{G_1'(0)}{G_1(0)} = L + T \tag{11}$$

and

$$T^2 = \frac{G_1''(0)}{G_1(0)} - T_{ar}^2 \tag{12}$$

We can get L and T from equation above, and the system gain can be obtained directly by k=G(0).

So, in actual application, if we have the transfer functions, the more accurate FOPTD equivalent models will be get.

For example, the transfer function is

$$G(s) = \frac{1}{(20s+1)^3} \ . \tag{13}$$

The approximate FOPTD model is

$$G_1(s) = \frac{1}{34.64s+1} e^{-25.36s} \ . \tag{14}$$

The step response is shown in figure 2.

For FOPTD model (4), the L/T is very important. So, there are three cases to be discussed L<T, L≈T and L>T. The parameters and simulation results are shown in table 2, 3, figure 3, 4 and 5.

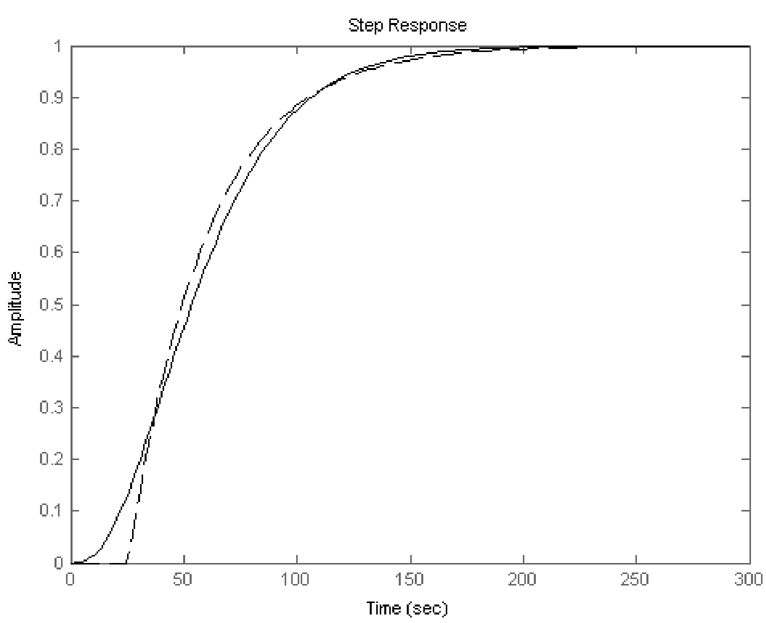


Fig. 2. Step response comparison (the solid line is original system and the dotted line is approximate system)

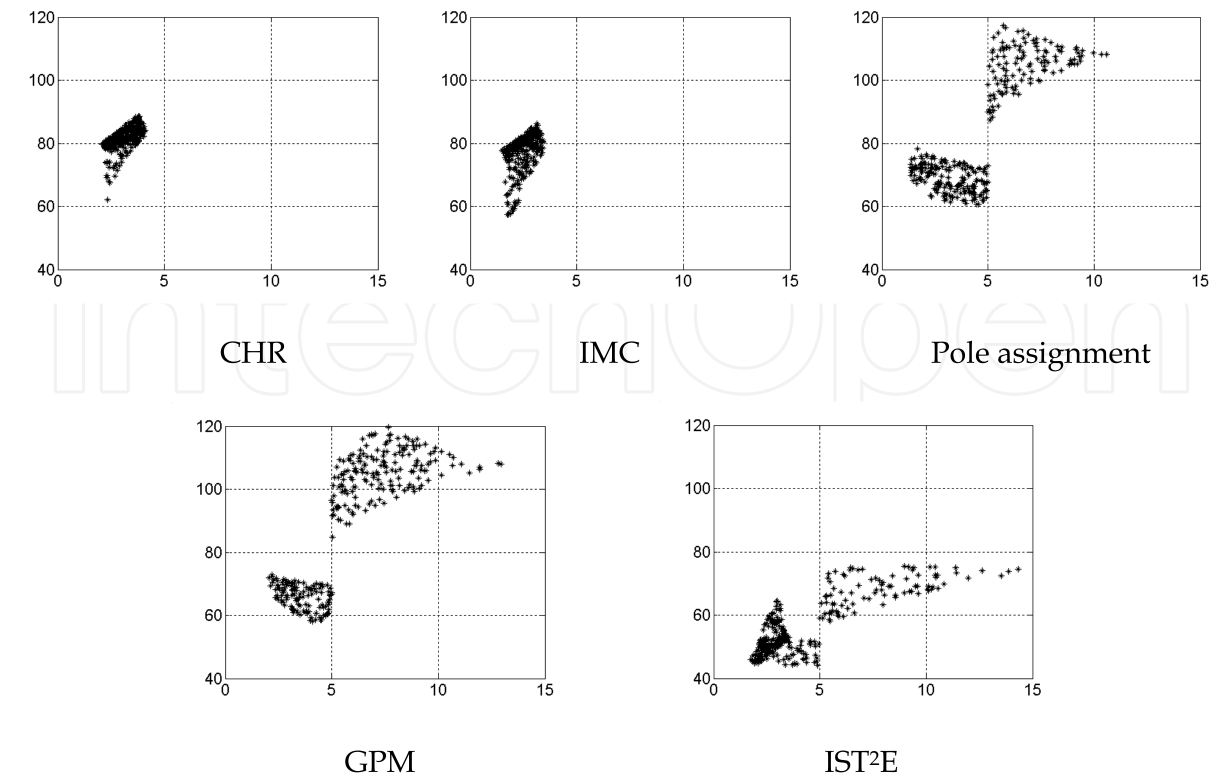


Fig. 3. Simulation results of FOPTD model when L<T (the abscissa represents overshoot and the ordinate represents adjustment time)

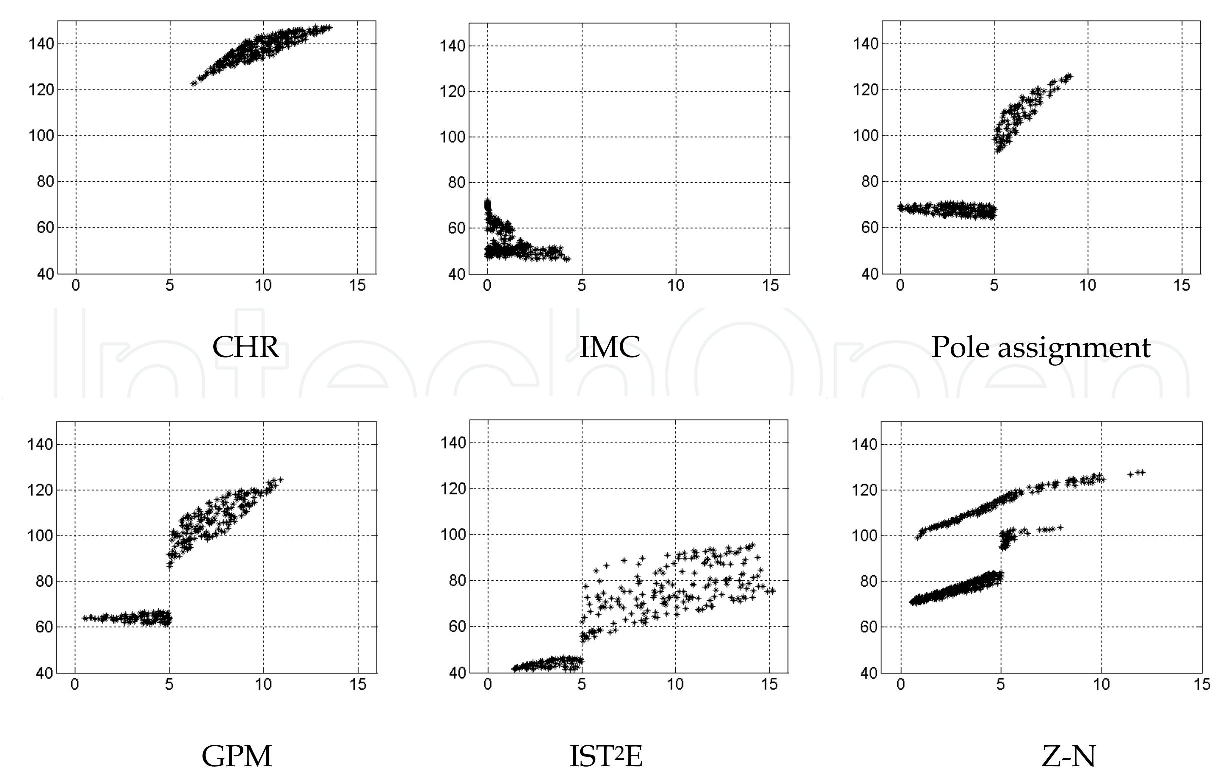


Fig. 4. Simulation results of FOPTD model when L≈T (the abscissa represents overshoot and the ordinate represents adjustment time)

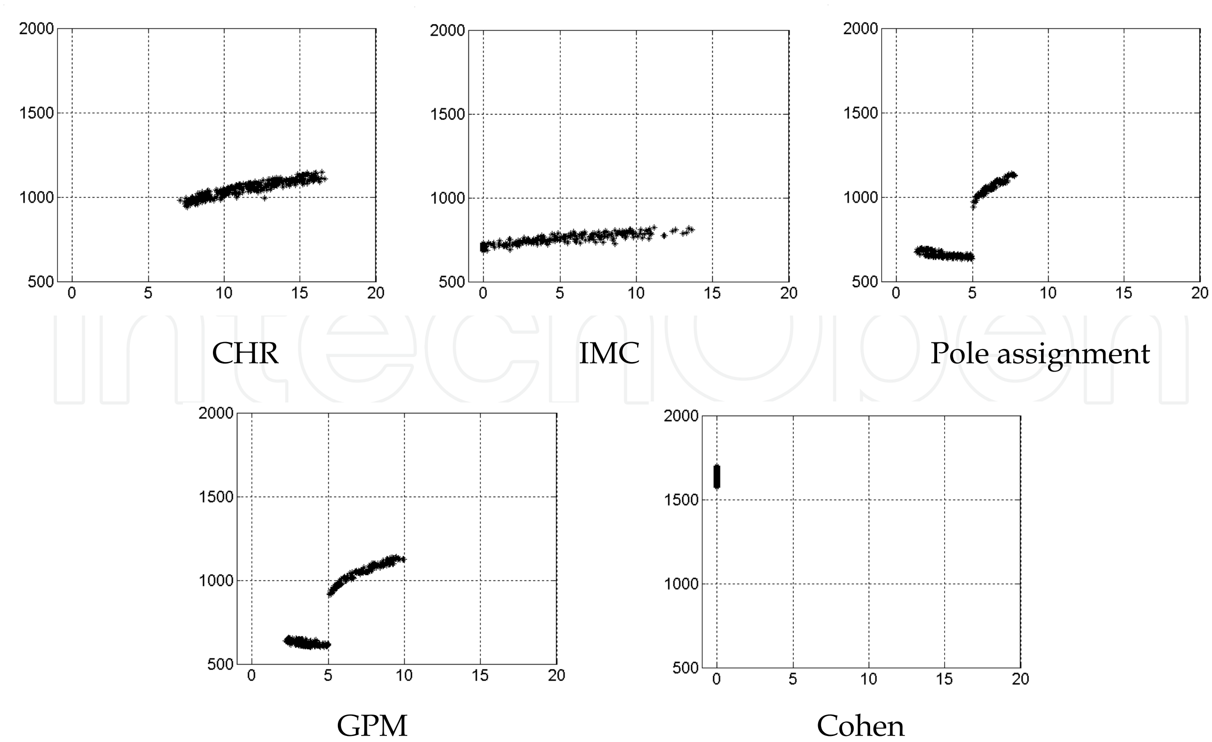


Fig. 5. Simulation results of FOPTD model when L>T (the abscissa represents overshoot and the ordinate represents adjustment time)

In order to compare different method visualized, the figures which have too long adjustment time or too large overshoot are not included in figure 3, 4, 5, 7 and 8.

	L	T	k
L<T	[18,22]	[180,220]	1
L≈T	[18,22]	[18,22]	1
L>T	[180,220]	[18,22]	1

Table 2. Parameters of FOPTD model

		CHR	IMC	Pole assignment	GPM	IST²E	Cohen	Z-N
L<T	Overshoot (%)	2.08~4.08 (3.10)	1.49~3.41 (2.48)	1.37~10.6 (4.54)	2.04~12.9 (5.86)	1.75~14.3 (4.42)	64.4~122 (91.2)	49.6~102 (74.3)
	Adjustment time	62.2~88.6 (81.6)	57.4~86.1 (77.2)	60.7~117 (83.2)	58.1~120 (89.7)	44.1~75.5 (56.3)	113~477 (181)	105~214 (140)
L≈T	Overshoot (%)	6.22~13.5 (9.83)	0~4.27 (1.13)	0~9.03 (4.20)	0.50~10.9 (5.93)	1.40~15.2 (7.48)	21.6~52.5 (36.8)	0.57~12.0 (3.58)
	Adjustment time	122~147 (138)	46.3~72.3 (54.8)	64.3~126 (83.4)	61.2~125 (91.6)	41.3~95.4 (64.3)	74.8~225 (136)	70.5~128 (90.1)
L>T	Overshoot (%)	7.11~16.6 (11.6)	0~13.6 (4.01)	1.36~7.79 (4.34)	2.20~9.94 (5.65)	Not stable	0	0
	Adjustment time	940~1147 (1051)	681~821 (743)	635~1137 (815)	602~1140 (855)	Not stable	1571~1701 (1642)	>6000

Table 3. Performance index of FOPTD model

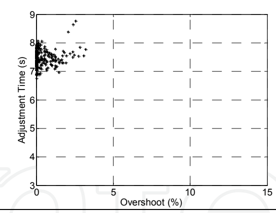
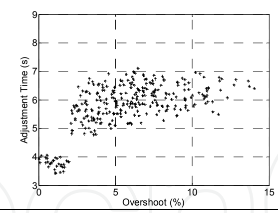
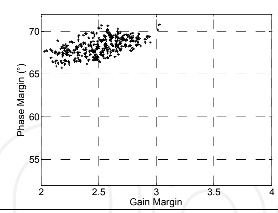
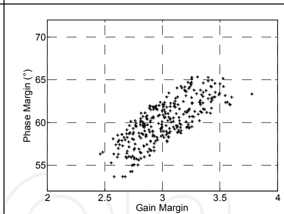
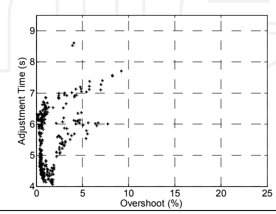
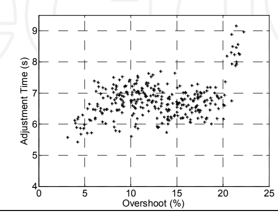
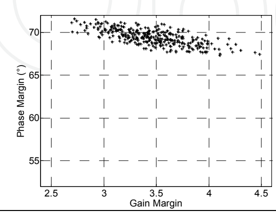
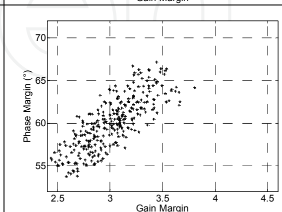
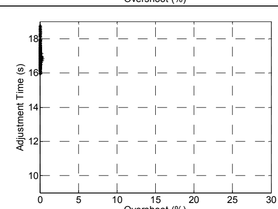
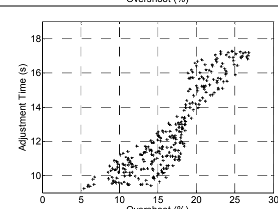
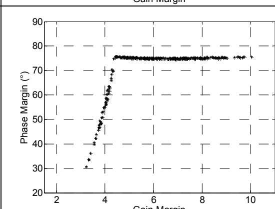
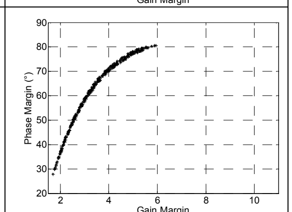
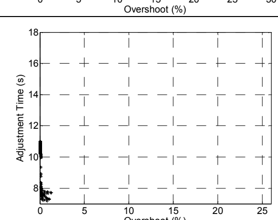
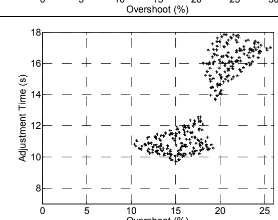
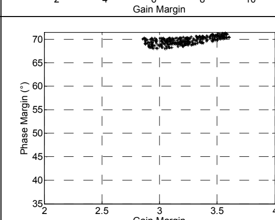
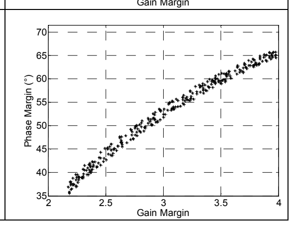
Types of models	Time-domain performance robustness		Frequency-domain performance robustness	
	DDE method	GPM method	DDE method	GPM method
G_{P1}				
G_{P2}				
G_{P3}				
G_{P4}				

Table 20. Monte-Carlo simulations

Types of models	Tuning method	Overshoot (%)			Adjustment time (s)			Gain margin			Phase margin (°)		
		Scope	Mean	Variance	Scope	Mean	Variance	Scope	Mean	Variance	Scope	Mean	Variance
G_{P1}	DDE method	0.00-3.20	0.35	0.0000	6.76-8.77	7.47	0.08	2.03-3.03	2.50	0.05	65.72-70.76	68.23	1.08
	GPM method	0.00-14.04	5.75	0.0009	3.43-7.11	5.75	0.73	2.46-3.78	3.02	0.06	53.68-65.33	60.11	6.52
G_{P2}	DDE method	0.05-9.19	1.61	0.0003	4.08-8.61	5.53	0.89	2.69-4.48	3.46	0.12	67.33-71.59	69.46	0.85
	GPM method	3.14-22.26	12.79	0.0021	5.43-9.16	6.77	0.33	2.44-3.81	3.02	0.08	53.78-67.15	60.07	9.24
G_{P3}	DDE method	0.00-0.35	0.01	0.0000	15.94-18.79	17.13	0.41	3.23-10.05	6.04	2.65	30.71-75.76	71.34	84.1
	GPM method	5.38-26.78	16.63	0.0023	9.25-17.28	12.66	5.83	1.70-5.94	3.52	1.19	27.95-80.70	61.67	196
G_{P4}	DDE method	0.00-1.28	0.09	0.0000	7.19-11.02	9.78	1.54	2.86-3.60	3.23	0.04	68.07-71.38	69.80	0.58
	GPM method	10.35-25.67	18.10	0.0013	9.73-17.88	13.45	7.60	2.17-3.98	3.06	0.30	35.61-65.80	52.75	76.8

Table 21. Comparison of performance index

The detailed comparison is shown in table 22. Obviously, DDE method has better performance than GPM method. Especially in time-domain, DDE method has nearly zero overshoot and equivalent adjustment time compared with GPM method. In most industry field, the unknown model is inevitable, the simple tuning method, small overshoot and good performance robustness are needed. So the 2-DOF DDE method is available for industry field to meet the high performance requirement.

		DDE Method	GPM Method
Controller Structure		2-DOF	1-DOF
Approximation of Model		No	Yes
Demand of Model		Relative Order	Precise
Complicacy of Tuning Method		Simple	Simple
Design Basis		Time-domain	Frequency-domain
Overshoot		Small	Large
Performance Robustness	Time-domain	Good	Bad
	Frequency-domain	Mostly Good	Mostly Bad

Table 22. Comparison of DDE method and GPM method

4. Conclusions

Combined the Monte-Carlo method, this chapter gives a new method to test the performance robustness of PID control system. This method do not need complex mathematical reasoning, but the simple simulations and visible results are easy to be accepted by engineers. The large numbers of simulations have been done to study the performance robustness of different PID tuning method with the proposed criterion. We can see that the IMC method and GPM method are superior to other classical method. Then the DDE method which does not base on precise model is compared with IMC method and GPM method. The simulation results show that the DDE method perform better than the other two methods in general, especially on the models which the IMC method and GPM method have to design controllers based on approximate model. So, the proposed performance robustness criterion is effective to test PID type controller.

Although PID control is the most popular control method in the industry field, the advanced control theory is developing all the time. We are making effort to apply proposed performance robustness criterion on other type controller.

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