19

Survey on Design of Truss Structures by Using Fuzzy Optimization Methods

Aykut Kentli
Marmara University Engineering Faculty,
Mechanical Engineering Department,
Turkey

1. Introduction

This study aims to reveal the studies on design optimization of trusses using fuzzy logic. In literature there are many surveys on truss optimization or on fuzzy logic, but, none of them is focused on fuzzy design optimization of truss. We believe that this study will help the researcher willing to study on this area by drawing a framework of the studies and by showing the lack of the area.

Firstly, a brief information fuzzy logic and optimization will be given. Then, studies will be classified under the topics related with the type of optimization problem and used method. In each topic, application area of fuzzy logic and main difference of the study will be explained. Classifications will also be shown as tables to show the overall picture. Lack of the area will be given in conclusion.

2. Fuzzy logic

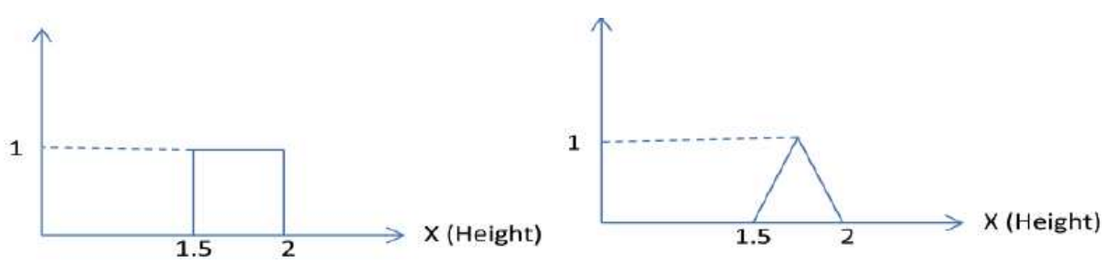
Fuzzy sets are generalized sets introduced by Professor Zadeh as a mathematical way to represent and deal with vagueness in everyday life (Zadeh, 1965). Indeed, Zadeh informally states what he calls the principle of incompatibility: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics".

Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth – the truth values between "completely true" and "completely false". A type of logic that recognizes more than simple true and false values. With fuzzy logic, propositions can be represented with degrees of the truthfulness and falsehood. For example, the statement, today is sunny, might be 100% true if there are no clouds, 80% true if there are a few clouds, 50% true if it's hazy and 0% true if it rains all day.

Even though fuzzy sets were introduced in their modern form by Zadeh (1965), the idea of a multi-valued logic in order to deal with vagueness has been around from the beginning of the century. Peirce was one of the first thinkers to seriously consider vagueness; he did not believe in the separation between true and false and believed everything in life is a

continuum. In 1905 he stated: "I have worked out the logic vagueness with something like completeness" (Peirce, 1935). Other famous scientists and philosophers probed this topic further. Russell (1923) claimed, "All language is vague" and went further saying; "vagueness is a matter of degree" (e.g., a blurred photo is vaguer than a crisp one, etc.). Einstein said that "as far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality" (Black, 1937). Lukasiewicz took the first step towards a formal model of vagueness, introducing in 1920 a three-valued logic based on true, false, and possible (Lukasiewicz, 1970). In doing this he realized that the laws of the classical two-valued logic might be misleading because they address only a fragment of the domain. A year later Post outlined his own three-valued logic, and soon after many other multi-valued logics proliferated (Godel, von Neumann, Kleene etc.) (McNeill & Freiberger, 1993). A few years later, Black (1937) outlined his precursor of fuzzy sets. He agreed with Peirce in terms of the continuum of vagueness and with Russell in terms of the degrees of vagueness. Therefore, he outlined a logic based on degrees of usage, based on probability that a certain object will be considered belonging to a certain class. Finally, Zadeh (1965) elaborated a multi-valued logic where degrees of truth (rather than usage) are possible.

Fuzzy set theory generalizes classical set theory in that the membership degree of an object to a set is not restricted to the integers 0 and 1, but may take on any value in [0,1]. By elaborating on the notion of fuzzy sets and fuzzy relations we can define fuzzy logic systems (FLS). FLSs are rule-based systems in which an input is first fuzzified (i.e. converted from a crisp number to a fuzzy set) and subsequently processed by an influence engine that retrieves knowledge in the form of fuzzy rules contained in a rule-base. The fuzzy sets computed by the fuzzy inference as the output of each rule are then composed and defuzzified (i.e., converted from a fuzzy set to a crisp number). A fuzzy logic system is a nonlinear mapping from the input to the output space.



(a) (b)

Fig. 1. Representation of set of heights from 1.5 to 2 meter for crisp (a) and fuzzy (b)

As Figure 1 shows, crisp set is defined by membership of element X of set A . Fuzzy set contain objects that satisfy imprecise properties of membership .

3. Optimization problem types

An optimization or a mathematical programming problem can be stated as follows:

$$\text{Find which minimizes } f(X) \tag{1}$$

Subject to the constraints:

$$g_j(X) \leq 0,\ j=1,2,\ldots,m \quad \text{and} \quad l_j(X)=0,\ j=1,2,\ldots,p \tag{2}$$

where X is an n-dimensional vector called the design vector, f(X) is called the objective function and gj(X) and lj(X) are, respectively, the inequality and the equality constraints. The number of variables n and the number of constraints m and/or p need not be related in any way.

Design vector. Any engineering system or component is described by a set of quantities some of which are viewed as variables during the design process. In general certain quantities are usually fixed at the outset and these are called preassigned parameters. All the other quantities are treated as variables in the design process and are called design or decision variables xi, i=1,2,…,n.

Design constraints. In many practical problems, the design variables cannot be chosen arbitrarily; rather they have to satisfy certain specified functional and other requirements. The restrictions that must be satisfied in order to produce an acceptable design are collectively called design constraints.

Objective functions. The conventional design procedure aim at finding an acceptable or adequate design, which merely satisfies the functional and other requirements of the problem. In general, there will be more than one acceptable designs and the purpose of optimization is to choose the best one out of the many acceptable design available. Thus a criterion has to be chosen for comparing the different alternate acceptable designs and for selecting the best one. The criterion with respect to which design is optimized, when expressed as a function of the design variables is known as criterion or merit or objective function.

Optimization problems can be classified in several ways as described below. This classification is extremely useful from the computational point of view since there are many methods developed solely for the efficient solution of a particular class of problems. This will, in many cases, dictate the types of solution procedures to be adopted in solving the problem.

3.1 Classification based on the existence of constraints

As indicated earlier, any optimization problem can be classified as a constrained or an unconstrained one depending upon whether the constraints exist or not in the problem. Previously defined problem is called a constrained optimization problem. Used methods will differ according to problem type and at each following topic, appropriate methods will be given. Some optimization problems do not involve any constraints and can be stated as:

$$\text{Find which minimizes } f(X) \tag{3}$$

Such problems are called unconstrained optimization problems. Mostly known methods are Hooke-Jeeves Pattern Search Method and Powell's Conjugate Gradient Method. Some methods (Penalty Function etc.) transform the constrained problem into unconstrained problem and then use mentioned methods (Rao, 1984).

3.2 Classification based on the nature of equations involved

Another important classification of optimization problems is based on the nature of expressions for the objective function and the constraints. According to this classification,

optimization problems can be classified as linear, nonlinear and dynamic programming problems.

Nonlinear Programming Problem: If any functions among objective and constraints functions are nonlinear, the problem is called a nonlinear programming (NLP) problem. This is the most general programming problem and all other problems can be considered as special cases of the NLP problem. There are several type methods. Complex method is using only the function value to find optimum. On the other hand, feasible direction algorithm uses the derivative of the objective and constraints.

Linear Programming Problem: If the objective function and all the constraints are linear functions of the design variables, the mathematical programming problem is called a linear programming (LP) problem. A linear programming problem is often stated in the following standard form:

Find which minimizes (4)

subject to the constraints , j=1,2,….,m (5)

and xi≥0, i=1,2,…,n where c,ajk and bj are constants.

Although allocating resources to activities is the most common type of application, linear programming has numerous other important applications as well. Furthermore, a remarkably efficient solution procedure called the Simplex method, is available for solving linear programming problems of even enormous size.

The Simplex method is a general procedure for solving linear programming problems and developed by George Dantzig in 1947 (Dantzig, 1963). It has proved to be a remarkably efficient method that is used routinely to solve huge problems on today's computers.

Dynamic Programming: In most practical problems, decisions have to be made sequentially at different points in time, at different points in space, and at different levels, say, for a component, for a subsystem, and/or for a system. The problems in which the decisions are to be made sequentially are called sequential decision problems. Since these decisions are to be made at a number of stages, they are also referred to as multistage decision problems. Dynamic programming is a mathematical technique well suited for the optimization of multistage decision problems. This technique was developed by Richard Bellman in the early 1950s.

3.3 Classification based on the permissible values of the design variables

Depending on the values permitted for the design variables, optimization problem is called a real-valued programming problem.

Integer Programming Problem: If some or all of the design variables x1,x2,…xn of an optimization problem are restricted to take on only integer (or discrete) values, the problem is called an integer programming problem. Branch-and-Bound methods are widely used in this area. Local Search methods (GA etc.) are also used for this problem type. Moreover, there are hybrid applications like GA+ANN. The genetic algorithm (GA) is an optimization and search technique based on the principles of genetics and natural selection. The method

was developed by John Holland over the course of the 1960s and 1970s and finally popularized by one of his students, David Goldberg, who was able to solve a difficult problem involving the control of gas-pipeline transmission for his dissertation. Holland's original work was summarized in his book (Holland, 1995).

3.4 Classification based on the number of objective functions

Depending on the number of objective functions to be minimized, optimization problem can be classified as single and multi-objective programming problem.

Multiobjective Programming Problem: Multiobjective optimization in last two decades has been acknowledged as an advanced design technique in structural optimization (Eschenawer et.al., 1990). The reason is that most of the real-world problems are multidisciplinary and complex, as there is always more than one important objective in each problem. To accommodate many conflicting design goals, one needs to formulate the optimization problem with multiple objectives. One important reason for the success of the multiobjective optimization approach is its natural property of allowing the designer to participate in the design selection process even after the formulation of the mathematical optimization model. The main task in structural optimization is determining the choice of the design variables, objectives, and constraints. Sometimes only one dominating criterion may be a sufficient objective for minimization, especially if the other requirements can be presented by equality and inequality constraints. But generally the choice of the constraint limits may be a difficult task in a practical design problem. These allowable values can be rather fuzzy, even for common quantities such as displacements, stresses, and natural frequencies. If the limit cannot be determined, it seems reasonable to treat that quantity as an objective. In addition, usually several competing objectives appear in a real-life application, and thus the designer is faced with a decision-making problem in which the task is to find the best compromise solution between the conflicting requirements.

A multiobjective optimization problem can be formulated as follow:

$$\text{Min } [f1(x), f2(x),..., fn(x)] \tag{6}$$

subject to

$$gj(x) \geq 0j = 1,2,\ldots,m \tag{7}$$

$$hj(x) = 0j = 1,2,...,p< n \tag{8}$$

where x is n-dimensional design variable vector, fi(x) is objective function.

A variety of techniques and applications of multiobjective optimization have been developed over the past few years. The progress in the field of multicriteria optimization was summarized by Hwang and Masud (1979) and later by Stadler (1984). Stadler inferred from his survey that if one has decided that an optimal design is to be based on the consideration of several criteria, then the multicriteria theory (Pareto theory) provides the necessary framework. In addition, if the minimization or maximization is the objective for each criterion, then an optimal solution should be a member of the corresponding Pareto set. Only then does any further improvement in one criterion require a clear tradeoff with at

least one other criterion. Radfors, et al (1985) in their study has explored the role of Pareto optimization in computer-aided design. They used the weighting method, noninferior set estimation (NISE) method, and constraint method for generating the Pareto optimal. The authors discussed the control and derivation of meaning from the Pareto sets.

Pareto optimality serves as the basic multicriteria optimization concept in virtually all of the previous literature (Grandhi & Bharatram, 1993). A general multiobjective optimization problem is to find the vector of design variables $X = (x_1, x_2, \ldots, x_n)T$ that minimize a vector objective function F(X) over the feasible design space X. It is the determination of a set of nondominated solutions (Pareto optimum solutions or noninferior solutions) that achieves a compromise among several different, usually conflicting, objective functions. The Pareto optimal is stated in simple words as follows: A vector X* is Pareto optimal if there exists no feasible vector X which would increase some objective function without causing a simultaneous decrease in at least one objective function. This definition can be explained graphically. An arbitrary collection of feasible solutions for a two-objective maximization problem is shown in Figure 2. The area inside of the shape and its boundaries are feasible. The axes of this graph are the objectives F1 and F2. It can be seen from the graph that the noninferior solutions are found in the portion of the boundary between points A and B. Thus, here arises the decision-making problem from which a partial or complete ordering of the set of nondominated objectives is accomplished by considering the preferences of the decision maker. Most of the multiobjective optimization techniques are based on how to elicit the preferences and determine the best compromise solution.

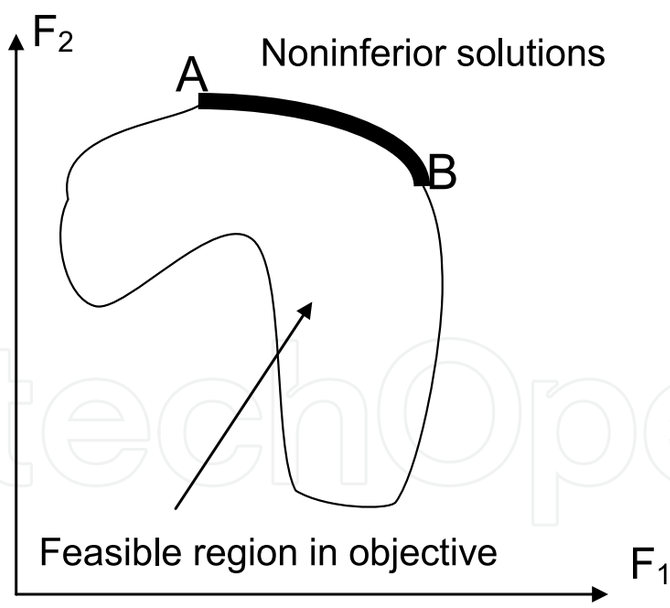


Fig. 2. Graphical Interpretation of Pareto Optimum

Nearly all of the solution schemes used in multiobjective optimization involve some sort of scalarization of the vector optimization problem. The vector problem is replaced by some equivalent scalar minimization problem. Because the Pareto set is generally infinite, an additional use of scalarization is the selection of a unique member of the Pareto set as the optimum for the vector optimization problem. Usually, a problem is scalarized either by

defining an additional supercriterion function or by considering the criteria sequentially. There are various techniques for generating noninferior solutions (Stadler, 1984;Radford et.al., 1985; Grandhi & Bharatram, 1993).

Weighting Method: This technique is based on the preference techniques of the weights' prior assessment for each objective function. It transforms the multicriteria function to a single criterion function through a parameterization of the relative weighting of the criteria. With the variation of the weights, the entire Pareto set can be generated. Because the results of solving an optimization problem can vary significantly as the weighting coefficients change, and very little is usually known about how to choose these coefficients, a necessary approach is to solve the same problem for many different values of weighting factors. However, because the shape and distribution characteristics of the Pareto set are unknown, it is difficult to determine beforehand the nature of the variations required in the weights so as to produce a new solution at each pass. The second important disadvantage of the method is that it will not identify the Pareto solutions in a nonconvex part of the set.

The idea of this technique consists in adding all the objective functions together using different coefficients for each. It means that we change our multicriteria optimization problem to a scalar optimization problem by creating one function of the form

$$f(x) = \sum_{i=1}^{k} w_i f_i(x) \tag{9}$$

where wi ≥ 0 are the weighting coefficients representing the relative importance of the criteria. It is usually assumed that

$$\sum_{i=1}^{k} w_i = 1 \tag{10}$$

Since the results of solving an optimization model using Eq. (9) can vary significantly as the weighting coefficients change and since very little is usually known about how to choose these coefficients, a necessary approach is to solve the same problem for many different values of w_i.

Note that the weighting coefficients do not reflect proportionally the relative importance of the objectives but are only factors which when varied locate points in the domain. For the numerical methods of seeking the minimum of Eq. (10) this location depends not only on values of w_i but also on units in which the functions are expressed.

The best results are usually obtained if objective functions are normalized. In this case the vector function is normalized to the following form

$$\tilde{f}(x) = [\tilde{f}_1(x), \tilde{f}_2(x), \ldots\ldots, \tilde{f}_k(x)]^T \tag{11}$$

where $\tilde{f}_i(x) = \dfrac{f_i(x)}{f_i^o}$

Here, fio is generally the maximum value of ith objective function. A condition fio≠0 is assumed and if it is not satisfied which rarely happens; another value of normalizing function must be chosen by the decision maker.

Game Theory: Game theory deals with decision situations in which two intelligent opponents with conflicting objectives are trying to outdo one another. It is a mathematical theory that deals with the general features of competitive situations like these in a formal, abstract way. It places particular emphasis on the decision-making processes of the adversaries. Typical examples include launching advertising campaigns for competing products and planning strategies for warring armies.

In a game conflict, two opponents, known as players, will each have a (finite or infinite) number of alternatives or strategies. Associated with each pair of strategies is a payoff that one player receives from the other. Such games are known as two-person zero-sum games because a gain by one player signifies an equal loss to the other. It suffices, then, to summarize the game in terms of the payoff to one player.

Because games are rooted in conflict of interest, the optimal solution selects one or more strategies for each player such that any change in the chosen strategies does not improve the payoff to either player. These solutions can be in the form of a single pure strategy or several strategies mixed according to specific probabilities (Frederick & Gerald, 2001).

Goal Programming: Goal programming was proposed by Charnes & Cooper (1961) for a linear model. It has been further developed by others (Ijiri, 1965; Charnes & Cooper, 1977). This method requires the decision maker (DM) to set goals for each objective that he wishes to attain. A preferred solution is then defined as the one, which minimizes the deviations from the set goals. Thus a simple GP formulation of the multiobjective optimization problem is given by

$$\text{Min}\left\{\sum_{j=1}^{k}(d_j^- + d_j^+)^p\right\}^{1/p}, \quad p \geq 1 \tag{12}$$

Subject to:

$$\begin{aligned} &G_i(x) \leq 0, \; i = 1, 2, \ldots, m \\ &F_j(x) + d_j^- + d_j^+ = b_j \quad , j = 1, 2, \ldots, k \\ &d_j^-, d_j^+ \geq 0 \text{ and } d_j^- \cdot d_j^+ = 0 \quad \text{for all } j \end{aligned} \tag{13}$$

where bj's are the goals set by the DM for the objectives, and are respectively the under-achievement and over-achievement of the jth goal. The value of p is based on the utility function of the DM. Other than p = 1 results in a nonlinear goal programming problem.

The most common form of GP requires that the DM, in addition to setting the goals for objectives, also be able to give an ordinal ranking of the objectives. This may result in a nonlinear goal-programming problem if objectives or constraints are nonlinear.

Goal Attainment Method, Global Criterion Method and Utility Function Method are also used to solve multiobjective optimization problems.

4. Fuzzy optimization

The available general model of a programming with fuzzy resources can be formulated as:

$$\min f(X) \tag{14}$$

$$\text{subject to , } i=1,2,\ldots,n \tag{15}$$

$$XL \leq X \leq XU \tag{16}$$

where the objective function and the ith in-equality constrained function are indicated as f (X) and gi(X) , respectively. The fuzzy number ,∀i are in the fuzzy region of [b_i , b_i + p_i] with given fuzzy tolerance pi. Assume the fuzzy tolerance pi for each fuzzy constraint is known, then, , will be equivalent to (b_i + θ p_i) ,∀i where θ is in [0 , 1]. Several methods are described in the following section. All methods, except the first one (R.E. Bellman and L.A. Zadeh's approach), are derivatives of the level cuts method and generally using ordinary crisp optimization methods by converting problem into crisp optimization problem.

4.1 R.E. Bellman and L.A. Zadeh's approach

In Bellman and Zadeh (1970) approach, the problem in fuzzy environment can be stated as,

$$\text{Find X which minimizes f(X)} \tag{17}$$

$$\text{subject to } gj(X) \in Gj \quad j=1,2,\ldots,n \tag{18}$$

where ordinary subset Gj denotes the allowable interval for the constraint function gj, Gj =[gj(l) , gj(u)] the bold face symbols indicate that the operations or variables contain fuzzy information. The constraint gj(X) ∈ Gj means that gj is a member of a fuzzy subset Gj in the sense of μGj(gj) > 0. The fuzzy feasible region is defined by considering all the constraints as

$$S = \bigcap_{j=1}^{m} G_j \tag{19}$$

And the membership degree of any design vector X to fuzzy feasible region S is given by

$$\mu_S(X) = \min_{j=1,2,\ldots,m} \left\{ \mu_{Gj}[g_j(X)] \right\} \tag{20}$$

i.e., the minimum degree of satisfaction of the design vector X to all of the constraints.

A design of vector X is considered feasible provided μS(X) > 0 and the differences in the membership degrees of two design vectors X1 and X2 imply nothing but variations in the minimum degrees of satisfaction of X1 and X2 to the constraints. Thus the optimum solution will be a fuzzy domain D in S with f(X). The fuzzy domain D is defined by

$$D \quad {}_f(X) \quad {}_{j\ 1,2,\ldots,m} \quad {}_{Gj}(g_j(X)) \tag{21}$$

that is

$$\mu_D(X) = \min\left\{\mu_f(X), \min_{j=1,2,\ldots,m} \mu_{Gj}[g_j X]\right\} \tag{22}$$

If the membership function of D is unimodal and has a unique maximum, then the maximum solution X* is one for which the membership function is maximum:

$$\mu_D(X^*) = \max \mu_D(X), X \in D \tag{23}$$

4.2 Verdegay's approach: α-cuts method

Verdegay (1982) considered that if the membership function of the fuzzy constraints has the following form:

$$\mu_{g_i}(X)\begin{cases} 1 & \text{if } g_i(X) < b_i \\ 1 - \dfrac{g_i(X) - b_i}{p_i} & \text{if } b_i + p_i \le g_i(X) \le b_i \\ 0 & \text{if } g_i(X) > b_i + p_i \end{cases} \tag{24}$$

Simultaneously, the membership functions of μgi(X), ∀I, are continuous and monotonic functions, and trade-off between those fuzzy constraints are allowed; then problem is equivalent to the following formulation:

$$\text{Min } f(X) \tag{25}$$

$$\text{Subject to } X \text{ ε } X\alpha \tag{26}$$

where Xα= {x ¡ $\mu g_i(X) \ge \alpha$, ∀i X ≥ 0}, for each αε [0, 1]. This is the fundamental concepts of α-level cuts method of fuzzy mathematical programming. The membership function indicates that if $g_i(X)$ ε(b_i,b_i + p_i); then the memberships functions are monotonically decreasing. That also can means, the more resource consumed, the less satisfaction the decision maker thinks. One can then obtain the following formulation:

$$\text{Min } f(X) \tag{27}$$

$$\text{subject to } g_i(X) \le b_i + (1-\alpha)\, p_i, \forall i \tag{28}$$

where XL ≤X ≤XU and αε[0, 1]. Thus, the problem is equivalent to a crisp parametric programming formulation while α= 1-θ. For each α, one will have an optimal solution; therefore, the solution with αgrade of membership function is fuzzy. This model was applied by Wang & Wang (1985) and Rao (1987a) in structural design problems.

4.3 Werner's approach: Max-α method

Werner's (1987) proposed the objective function should be fuzzy due to the fuzziness existing in fuzzy inequality constraints. For solving equations, one needs to define f_{max} and f_{min} as follows:

$$f_{max} = \text{Min } f(X), \text{ s.t. } g_i(X) \le b_i \ \forall i \text{ , and } XL \le X \le XU \tag{29}$$

$$f_{min} = \text{Min } f(X), \text{ s.t } g_i(X) \le b_i + p_i, \forall i, \text{ and } XL \le X \le XU \tag{30}$$

The membership function mf(X) of the objective function is stated as:

$$\mu_f(X) \begin{cases} 1 & \text{if } f(X) < f_{min} \\ 1 - \dfrac{f(X) - f_{min}}{f_{max} - f_{min}} & \text{if } f_{min} \le f(X) \le f_{max} \\ 0 & \text{if } f(X) > f_{max} \end{cases} \tag{31}$$

One can consequently apply the max-min operator to obtain the optimal decision. Then, equations can be solved by the strategy of max-α, where

$\alpha = \min[\mu f(X), \mu g_1(X), \mu g_2(X), \ldots, \mu g_m(X)]$. That is:

$$\begin{gathered} \text{Max } \alpha \\ \text{Subject to } \alpha \le \mu f(X) \\ \alpha \le \mu g_i(X), \forall i \end{gathered} \tag{32}$$

where α ε 0, 1] and XL ≤X ≤XU. This model is similar to the model proposed by Zimmermann (1978) and applied in structural design by Rao (1987b) and Rao et.al. (1992).

4.4 Xu's approach: Bound search method

Suppose there are a fuzzy goal function f and a fuzzy constraint C in a decision space X, which are characterized by their membership functions μf(X) and μC(X), respectively. The combined effect of those two can be represented by the intersection of the membership functions and the following formulation.

$$\begin{aligned} \mu_D(X) &= \mu_{f \cap C}(X) = \mu_f(X) \wedge \mu_C(X) \\ &= \min\{\mu_f(X), \mu_C(X)\} \end{aligned} \tag{33}$$

Then Bellman & Zadeh (1970) proposed that a maximum decision could be defined as:

$$\mu_D(X^M) = \max \mu_D(X) \tag{34}$$

If μD(X) has a unique maximum at XM, then the maximizing decision is a uniquely defined crisp decision. From equations and following the procedure given, one can obtain the particular optimum level α* corresponding to the optimum point XM such that:

$$\mu_D(X) = \max_{X \in C_{\alpha^*}} \mu_f(X) \tag{35}$$

where Cα* is the fuzzy constraint set C of α*-level cut.

Xu (1989) used a goal membership function of f(X) as following:

$$\mu_f(X) = \frac{f_{min}}{f(X)} \tag{36}$$

where fmin has been defined as before. It is apparent that the upper and lower bound of this goal membership function is between 1 and fmin /fmax. As a result, the optimum α* can be achieved through an iteration computation. This method has been called the 2nd phase of α-cuts method in his paper(Xu, 1989).

4.5 Single level cuts method

It is observed in Xu's approach where maximizing μf(X) is similar to maximizing α in Werner's approach; therefore, it is predicted the final result of those two approaches have the similar tendency, even though the form of their membership function is not the same, in which Werner's approach uses the linear function and Xu's approach uses the nonlinear function.

For obtaining the unique solution of the original α-level cuts approach in nonlinear programming problem with fuzzy resources, another alternative single level-cut approach called the single level-cut approach of the second kind is proposed (Shih et.al., 2003). This approach contains both linear membership function and nonlinear membership function of objective function.

$$\mu_f(X) = \frac{f_{max} - f(X)}{f_{max} - f_{min}} = 1 - \frac{f(X) - f_{min}}{f_{max} - f_{min}} \tag{37}$$

The mathematical formulation of the fuzzy problem with unique α–cut level can be written in the following:

$$\begin{gathered}
\text{Find } [X, \alpha]T \\
\min f(X) \\
\text{subject to } f(X)-[f_{max}-\alpha(f_{max}-f_{min})]=0 \text{ (for linear } \alpha f(X)) \\
f(X)-(f_{min}/\alpha)=0 \text{ (for nonlinear } \alpha f(X)) \\
g_i(X) \le b_i+(1-\alpha p_i),\ \forall i \\
\alpha \in [0,1] \text{ (for linear } \alpha f(X)) \\
\alpha \in [f_{min}/f_{max}] \text{ (for linear } \alpha f(X))
\end{gathered} \tag{38}$$

where XL ≤ X ≤ XU and f(X) can be nonlinear or linear membership functions.

There are also new approaches in literature, based on fuzzy set theory like Evidence Theory. Evidence theory is based on the Belief (Bel) and Plausibility (Pl) fuzzy measures. Fuzzy measures provide the foundation of fuzzy set theory.

5. Fuzzy design optimization applications

This section will classify the applications using previously mentioned methods. As mentioned earlier, optimization can be classified according to how many objectives problem have. Investigated literature studies are shown in Table 1 from objective perspective.

OBJECTIVES	
Single	Multi
(Jensen, 2001; Yeh & Hsu, 1990; Mohandas et.al., 1990; Maglaraset.al., 1997; Fang et.al., 1998; Hsu et.al., 1995; Tonona & Bernardini, 1998; Liu, 2006; Shih, 1997; Arakawa et.al., 1999; Sarma & Adeli, 2000b; Yang & Soh, 2000; Joghataie & Ghasemi, 2001; Sarma, 2001; Xiong, 2002; Shih et.al., 2003; Marler et.al., 2004; Shih & Lee, 2004; Xiong & Rao, 2005; Shih & Lee, 2006; Khorsand & Akbarzadeh, 2007)	(Rao, 1987b; Chen & Wang, 1989; Rao et.al., 1992; Yu & Xu, 1994; Shih & Lai, 1994; Forouraghi et.al., 1994; Shih & Wangsawidjaja, 1995; Shih & Chang, 1995; Shih & Wangsawidjaja, 1996; Cheng & Li, 1997; Shih, 1997; Shih et.al., 1997; Yoo, 2000; Sarma & Adeli, 2000a; Yoo & Hajela, 2001; Kiyotaet.al., 2001; Sarma, 2001; Xiong, 2002; Kiyotaet.al., 2003; Wang et.al., 2005; Kelesoglu & Ulker, 2005a; Rao & Xiong, 2005a; Rao & Xiong, 2005b; Kelesoglu & Ülker, 2005b; Kelesoglu, 2007)

Table 1. Investigated Literature

Shih et.al. (2003) developed and proposed three alternative α-level-cuts approaches: single-cut, double-cuts, and multiple-cuts, for solving nonlinear programming design problems of structuring engineering with fuzzy resources. The approaches have performed better than that of conventional α-level-cuts method.

Hsu et.al. (1995) considered the optimization process as a closed-loop control system. Traditional "controllers", the numerical optimization algorithms, are usually "crisply" designed for well defined mathematical models. However, when applied to engineering design optimization problems in which function evaluations can be expensive and imprecise, very often the crisp algorithms will become impractical or will not converge. They presented how the heuristics of this human supervision can be modeled into the optimization algorithms using fuzzy control concept.

Shih (1997) employed three fuzzy models to combine with an improved imposed-on penalty approach for attacking a nonlinear multiobjective in the mixed-discrete optimization problem. He presented a penalty method, including the forms of penalty function and the values of each parameter. The presented strategy is suggested as appropriate for solving a generalized mixed-discrete optimization problem.

Arakawa et.al. (1999) showed the effectiveness of the use of fuzzy members as design variables, by comparing with the other robust design methods. They proposed a way to raise certainties in estimating robustness by using approximation concepts in operation of fuzzy function.

Fang et.al. (1998) considered an approach to the optimum design of structures, in which uncertainties with a fuzzy nature in the magnitude of the loads. The optimization process under fuzzy loads is transformed into a fuzzy optimization problem based on the notion of Werners' maximizing set by defining membership functions of the objective function and constraints. An example of a ten-bar truss is used to illustrate the present optimization process. The results are compared with those yielded by other optimization methods.

Mohandas et.al. (1990) has combined Zadeh's approach in Eq. (20) with goal programming. They implemented this approach to single objective optimization problems. As example problems, optimization of four bar and ten bar truss are selected. No comparison is made in this work.

Yang and Soh (2000) proposed a fuzzy logic integrated genetic programming (GP) based methodology to increase the performance of the GP based approach for structural optimization and design. Fuzzy set theory is employed to deal with the imprecise and vague information, especially the design constraints, during the structural design process.

Joghataie and Ghasemi (2001) implemented fuzzy membership functions in the multistage optimization technique to improve its performance for the minimum weight design of truss structures of fixed topology. It has been found that this technique has significantly improved the convergence speed at the expense of increasing the minimum weight by a negligible amount.

Shih et.al. (2004) presented new method (Two single level cut approach). Also, new method is implemented on three bar, ten bar and 25 bar truss optimization problems and objective function values are compared with Verdegay's approach in section IV.2, Werner's approach in section IV.3 and Xu's approach in section IV.4.

Shih and Lee (2006) presented the modified double-cuts approach for large-scale fuzzy optimization, typically in 25-bar and 72-bar truss design problems. The proposed approach is better than the single-cut approach and easy programming for use to instead of multiple-cuts approach.

Maglaras et.al. (1997) compared probabilistic and fuzzy set based approaches in designing a damped truss structure.

Sarma (2001) developed a fuzzy discrete multicriteria cost optimization model by considering three criteria 1) minimum cost 2) minimum weight and 3) minimum number of section types. In the design, the uncertainty of fuzziness of the AISC code based design constraints is considered.

Sarma and Adeli (2000a, 2000b) presented a fuzzy augmented Lagrangian GA for optimization of steel structures subjected to the constraints of the AISC allowable stress design specifications taking into account the fuzziness in the constraints. The algorithm is applied to two space axial-load structures including a large 37-story structure with 1310 members.

Rao and Xiong (2005) presented a new method in which the fuzzy lambda-formulation and game theory techniques are combined with a mixed-discrete hybrid genetic algorithm for solving mixed-discrete fuzzy multiobjective programming problems. They dealt with three example problems: the optimal designs of a two-bar truss, a conical convective spine and a twenty-five bar truss.

Wang et.al. (2005) studied the principle of solving multiobjective optimization problems with fuzzy sets theory. Membership functions based on functional-link net have been used in multiobjective optimization.

Yoo and Hajela (2001) have dealt with a genetic algorithm based optimization procedure for solving multicriterion design problems where the objective or constraint functions may not be crisply defined.

Forouraghi et.al. (1994) introduced a new methodology in which multiobjective optimization is formulated as unsupervised learning through induction of multivariate regression trees. In particular, they showed that learning of Pareto-optimal solutions can be eficiently accomplished by using a number of fuzzy tree-partitioning criteria. The widely used problem of design of a three-bar truss is presented.

Shih et.al. (1997) introduced a design method using fuzzy logic to find the best stochastic design by maximizing Hasofer-Lind's (H-L's) reliability and simultaneously optimizing design goals. The objective weighting strategy in multiobjective fuzzy formulation is adopted to represent the importance among the design goals.

Rao (1987b) has used Werner's approach in section IV.3. This approach is presented to solve multiobjective optimization problems. Sample problems are three bar and 25 bar truss optimization problems. No comparison is made in this work.

Shih & Chang (1995) has combined Werner's approach in section IV.3 with Global Criterion method and implemented on multiobjective optimization problems. As sample cases, three bar truss and 11 bar truss are solved and results (objective function values) are compared.

Chen and Wang (1989) proposed a general fuzzy programming with wide generality in order to consider the overall fuzzy factors and fuzzy information in optimum design of engineering structures.

Shih & Lai (1994) has used two weighting strategies to get Pareto optimum values: objective weighting and membership weighting strategies. Three bar truss optimization problem is selected as sample multiobjective optimization problem. Objective function values are presented as comparison criteria.

Rao et.al. (1992) have used two methods: Verdegay's approach in section IV.2 and Werner's approach in section IV.3 for multiobjective optimization problems. As sample cases, optimization of three bar and 25 bar truss systems are selected. Objective function values are used to compare methods.

Kiyota et.al. (2001, 2003) described a fuzzy satisficing method for multiobjective optimization problems using Genetic Algorithm (GA). A multiobjective design problem with constraints is expressed as a satisficing problem of constraints by introducing an aspiration level for each objective.

Kelesoglu & Ulker (2005a) optimized space truss systems by using fuzzy sets. The algorithm of multi-objective fuzzy optimization was formed using the macros of Ms-Excel.

Cheng and Li (1997) presented a constrained multiobjective optimization methodology by integrating Pareto Genetic algorithm with fuzzy penalty function method. A 72-bar space truss with two criteria and a 4-bar truss with three criteria were investigated.

Kelesoglu & Ulker (2005b) presented a general algorithm for nonlinear space truss system optimization with fuzzy constraints and fuzzy parameters. The analysis of the space truss system is performed with the ANSYS program.

Kelesoglu (2007) proposed a genetic algorithm to solve fuzzy multiobjective optimization of space truss. This method enables a flexible method for optimal system design by applying fuzzy objectives and fuzzy constraints. An algorithm was developed by using MATLAB programming. The algorithm is illustrated on 56-bar space truss system design problem.

At following pages, these studies will be classified according to used methods and application area.

5.1 Single objective applications

Table 2 shows the objectives in literature. It is seen that minimizing weight is the most common objective for single objective optimization studies. Minimizing failure possibility and natural frequency are also used even though found rarely.

SINGLE OBJECTIVE	
Minimize Weight	(Yeh & Hsu, 1990; Mohandas et.al., 1990; Hsu et.al., 1995; Shih, 1997; Fang et.al., 1998; Tonona & Bernardini, 1998; Arakawa et.al., 1999; Sarma & Adeli, 2000b; Yang & Soh, 2000; Joghataie & Ghasemi, 2001; Jensen, 2001; Sarma, 2001; Xiong, 2002; Shih et.al., 2003; Marler et.al., 2004; Shih & Lee, 2004; Xiong & Rao, 2005; Shih & Lee, 2006; Liu, 2006)
Minimize the system failure possibility	(Maglaraset.al., 1997)
Maximize the fundamental natural frequency	(Khorsand & Akbarzadeh, 2007)

Table 2. Objectives in single objective problems

Used methods differ at each study. But, generally there are two different applications: Direct methods and Hybrid Methods. Also, hybrid methods differ according to at where fuzzy logic is applied. Sometimes, fuzzy logic assists to another optimization method and sometimes vises versa. Table 3 shows the studies in literature according to used method type. Table 4 shows the hybrid methods.

Direct	Hybrid
(Maglaraset.al., 1997; Fang et.al., 1998; Arakawa et.al., 1999; Jensen, 2001; Shih et.al., 2003; Shih & Lee, 2004; Marler et.al., 2004; Shih & Lee, 2006)	(Yeh & Hsu, 1990; Mohandas et.al., 1990; Hsu et.al., 1995; Shih, 1997; Tonona & Bernardini, 1998; Sarma & Adeli, 2000b; Yang & Soh, 2000; Joghataie & Ghasemi, 2001; Sarma, 2001; Xiong, 2002; Xiong & Rao, 2005; Liu, 2006; Khorsand & Akbarzadeh, 2007)

Table 3. Used methods in single objective problems

HYBRID TECHNIQUES	
Unconstrained	(Yeh & Hsu, 1990)
Penalty Function	(Joghataie & Ghasemi, 2001; Shih, 1997)
PD Controller	(Liu, 2006)
Goal Programming	(Mohandas et.al., 1990)
Evidence Theory	(Tonona & Bernardini, 1998)
Linear Programming	(Hsu et.al., 1995)
GA	(Sarma & Adeli, 2000b; Yang & Soh, 2000; Sarma, 2001; Xiong, 2002)
ANN+GA	(Khorsand & Akbarzadeh, 2007)
Dynamic Programming	(Xiong & Rao, 2005)

Table 4. Hybrid methods in single objective problems

Mentioned methods are applied to different truss structures. These structures are listed in Table 5.

	PLANAR (2D)	SPACE(3D)
2 bar:	(Khorsand & Akbarzadeh, 2007)	
3 bar:	(Yeh & Hsu, 1990; Hsu et.al., 1995; Shih, 1997; Arakawa et.al., 1999; Shih et.al., 2003; Shih & Lee, 2004; Liu, 2006)	
4 bar:	(Mohandas et.al., 1990; Fang et.al., 1998; Xiong, 2002; Xiong & Rao, 2005)	
10 bar:	(Mohandas et.al., 1990; Tonona & Bernardini, 1998; Yang & Soh, 2000; Joghataie & Ghasemi, 2001; Shih & Lee, 2004)	
25 bar:		(Jensen, 2001; Shih & Lee, 2004; Marler et.al., 2004; Shih & Lee, 2006)
30 bar:		(Maglaraset.al., 1997)
46 bar:	(Joghataie & Ghasemi, 2001)	
72 bar:		(Sarma & Adeli, 2000b; Sarma, 2001; Shih & Lee, 2006)
135 bar:	(Joghataie & Ghasemi, 2001)	
1310 bar:		(Sarma & Adeli, 2000b; Sarma, 2001)

Table 5. Truss structures in single objective problems

5.2 Multi objectives applications

Table 6 shows the objectives in literature. It is seen that minimizing weight is still the most common objective for single objective optimization studies. Minimizing deflection and natural frequency are also used.

MULTIOBJECTIVE	
Minimize weight	(Rao, 1987b; Chen & Wang, 1989; Rao et.al., 1992; Yu & Xu, 1994; Shih & Lai, 1994; Forouraghi et.al., 1994; Shih & Chang, 1995; Shih & Wangsawidjaja, 1995; Shih & Wangsawidjaja, 1996; Cheng & Li, 1997; Shih et.al., 1997; Shih, 1997; Yoo, 2000; Sarma & Adeli, 2000a; Sarma, 2001; Kiyotaet.al., 2001; Yoo & Hajela, 2001; Xiong, 2002; Kiyotaet.al., 2003; Wang et.al., 2005; Kelesoglu & Ulker, 2005a; Rao & Xiong, 2005a; Rao & Xiong, 2005b; Kelesoglu & Ülker, 2005b; Kelesoglu, 2007)
Minimize deflection	(Rao, 1987b; Chen & Wang, 1989; Rao et.al., 1992; Forouraghi et.al., 1994; Yu & Xu, 1994; Shih & Lai, 1994; Shih & Chang, 1995; Shih & Wangsawidjaja, 1995; Shih & Wangsawidjaja, 1996; Shih, 1997; Yoo, 2000; Kiyotaet.al., 2001; Yoo & Hajela, 2001; Xiong, 2002; Kiyotaet.al., 2003; Kelesoglu & Ulker, 2005a; Wang et.al., 2005; Rao & Xiong, 2005a; Rao & Xiong, 2005b; Kelesoglu & Ülker, 2005b; Kelesoglu, 2007)
Minimize number of cross-sections	(Sarma & Adeli, 2000a; Sarma, 2001)
Minimize cost	(Sarma & Adeli, 2000a; Sarma, 2001)
Minimize Hasofer's and Lind's reliability	(Shih et.al., 1997)
Minimize strain energy	(Cheng & Li, 1997)
Minimize control effort	(Cheng & Li, 1997)
Maximize the fundamental natural frequency	(Rao, 1987b; Rao et.al., 1992; Xiong, 2002; Rao & Xiong, 2005a; Rao & Xiong, 2005b)

Table 6. Objectives in multi objective problems

Table 7 shows the studies in literature according to used method type. Table 8 shows the hybrid methods.

Direct	Hybrid
(Rao, 1987b; Chen & Wang, 1989; Rao et.al., 1992; Yu & Xu, 1994; Forouraghi et.al., 1994; Shih & Chang, 1995; Shih & Wangsawidjaja, 1995; Shih & Wangsawidjaja, 1996; Shih et.al., 1997; Yoo, 2000; Kelesoglu & Ulker, 2005a; Kelesoglu & Ülker, 2005b)	(Shih & Lai, 1994; Cheng & Li, 1997; Sarma & Adeli, 2000a; Sarma, 2001; Kiyotaet.al., 2001; Yoo & Hajela, 2001; Xiong, 2002; Kiyotaet.al., 2003; Wang et.al., 2005; Rao & Xiong, 2005a; Rao & Xiong, 2005b; Kelesoglu, 2007)

Table 7. Used methods in multiobjective problems

HYBRID TECHNIQUES	
ANN	(Wang et.al., 2005)
Weighting	(Shih & Lai, 1994)
Game Theory	(Rao & Xiong, 2005a; Rao & Xiong, 2005b)
GA	(Cheng & Li, 1997; Sarma & Adeli, 2000a; Sarma, 2001; Kiyotaet.al., 2001; Yoo & Hajela, 2001; Xiong, 2002; Kiyotaet.al., 2003; Kelesoglu, 2007)

Table 8. Hybrid methods in multi objective problems

Mentioned methods are applied to different truss structures. These structures are shown in Table 9.

	PLANAR (2D)	SPACE (3D)
2 bar:	(Xiong, 2002; Rao & Xiong, 2005a; Rao & Xiong, 2005b)	
3 bar:	(Rao, 1987b; Chen & Wang, 1989; Rao et.al., 1992; Yu & Xu, 1994; Shih & Lai, 1994; Forouraghi et.al., 1994; Shih & Chang, 1995; Shih & Wangsawidjaja, 1995; Shih & Wangsawidjaja, 1996; Shih et.al., 1997; Yoo, 2000; Yoo & Hajela, 2001; Wang et.al., 2005)	
4 bar:	(Kiyota et.al., 2001; Kiyota et.al., 2003)	(Cheng & Li, 1997; Kelesoglu & Ülker, 2005b)
9 bar:		(Kelesoglu & Ulker, 2005a)
10 bar:	(Shih, 1997)	
11 bar:	(Yoo & Hajela, 2001)	
25 bar:		(Rao, 1987b; Rao et.al., 1992; Xiong, 2002; Rao & Xiong, 2005a; Rao & Xiong, 2005b; Kelesoglu & Ülker, 2005b)
56 bar:		(Kelesoglu, 2007)
72 bar:		(Cheng & Li, 1997)
120 bar:		(Kelesoglu & Ulker, 2005a)
244 bar:		(Kelesoglu & Ulker, 2005a)
1310 bar:		(Sarma & Adeli, 2000a; Sarma, 2001)

Table 9. Truss structures in multi objective problems

6. Conclusions

Design of structural systems has always been one of the most important topics to study. But, over the years, optimization of structural system has gained popularity. Today, there are a few conferences and journals concerning only the optimization of structural systems. This study aimed to summarize the studies on using fuzzy logic in optimization of structural systems. Following results has been found as remarkable to notice:

*Fuzzy logic is applied to different variety of structural design problems (single and multiobjective problems, simple and complex problems etc.)

*Most important objectives in designing optimal structures are minimizing weight and deflection.

*Both direct and hybrid methods are used. Especially using GA together with fuzzy logic has given better performance. It is recommended to the researchers to use also other evolutionary algorithms (Simulated Annealing, Particle Swarm Optimization etc.)

*Mostly used case examples are 3 bar and 25 bar truss systems.

7. References

Arakawa, M.; Yamakawa, H.; Ishikawa, H. (1999). Robust Design Using Fuzzy Numbers with Intermediate Variables. *Proceedings of 3rd ISSMO/UBCAD/UB/AIAA World Congress on Structural and Multidisciplinary Optimization*, NewYork, USA.

Bellman, R.E.; Zadeh, L.A. (1970). Decision-making in a Fuzzy Environment. *Management Science*, Vol. 17, pp. B141-B164.

Black, M. (1937). Vagueness: An Exercise in Logical Analysis. *Philosophy of Science*, Vol. 4, pp. 427-455.

Borkowski, L. (1970). *Selected works of J. Lukasiewicz*. North Holland, London.

Charnes, A.; Cooper, W.W. (1961). *Management Models and Industrial Applications of Linear Programming*, John Wiley & Sons, New York.

Charnes, A.; Cooper, W.W. (1977) Goal programming and multiple objective optimizations. *European Journal of Operational Research*, Vol. 1, pp. 39-54.

Chen, S.-X.; Wang, G.-Y. (1989). Theory and solution of general fuzzy optimization for structures. *Acta Mechanica Solida Sinica*, Vol. 2, No 3, pp. 271-283.

Cheng, F.Y.; Li, D. (1997). Multiobjective optimization design with pareto genetic algorithm. *Journal of Structural Engineering*, Vol.123, No 9, pp. 1252-1261.

Dantzig, G.B. (1963). *Linear programming and extensions*, Princeton University Press.

Holland, J.H. (1995). *Hidden order: how adaptation builds complexity*. Addison-Wesley, Reading, Mass.

Eschenawer, H.; Koski, J.; Osyczka, A. (1990). *Multicriteria Design Optimization-Procedures and Applications*, Springer, Berlin.

Fang, J.; Smith, S.M.; Elishakoff, I. (1998). Combination of Anti-Optimization and Fuzzy-Set-Based Analyses for Structural Optimization under Uncertainty. *Mathematical Problems in Engineering*, Vol. 4, pp. 187-200.

Forouraghi, B.; Schmerr, L.W.; Prabhu, G.M. (1994). Fuzzy multiobjective optimization with multivariate regression trees. *Proceedings of IEEE World Congress on Computational Intelligence*, Orlando, USA.

Grandhi, R.V.; Bharatram, G. (1993). Multiobjective Optimization of Large-Scale Structures. *AIAA Journal*, Vol. 31, pp. 1329-1337.

Hartshorne, C.; Weiss,P. (1935). *Collected Papers of Charles Sanders Peirce*. Harvard University Press, Cambridge, MA.

Hillier, F.S.; Lieberman, G.J. (2001). *Introduction to Operations Research* (7th Ed), McGraw-Hill.

Hsu, Y.L.; Lin, Y.F.; Sun, T.L. (1995). Engineering Design Optimization as a Fuzzy control process. *Proceedings of International Joint Conference of the Fourth IEEE International Conference on Fuzzy Systems and The Second International Fuzzy Engineering Symposium,*Yokohoma, Japan.

Hwang, C.L.; Masud, A.S.M. (1979). *Multiple Objective Decision Making Methods and Applications*, Springer, Berlin.

Ijiri, Y. (1965). *Management Goals and Accounting for Control*, North-Holland.

Jensen, H.A. (2001). Structural Optimal Design of Systems with Imprecise Properties: a Possibilistic Approach. *Advances in Engineering Software*, Vol. 32, pp. 937-948.

Joghataie, A.; Ghasemi, M. (2001). Fuzzy Multistage Optimization of Large-Scale Trusses. *Journal of Structural Engineering*, Vol. 127, pp. 1338–1347.

Kelesoglu, O.; Ulker, M. (2005a). Fuzzy optimization of geometrical nonlinear space truss design. *Turkish Journal of Engineering & Environmental Sciences*, Vol. 29, pp. 321-329.

Kelesoglu, O.; Ulker, M. (2005b). Multi-objective fuzzy optimization of space trusses by MS-Excel. *Advances in Engineering Software*, Vol. 36, pp. 549-553.

Kelesoglu, O. (2007). Fuzzy multiobjective optimization of truss-structures using genetic algorithm. *Advances in Engineering Software*, Vol. 38, pp. 717-721.

Khorsand, A.R.; Akbarzadeh, M.R. (2007). Multi-objective meta level soft computing-based evolutionary structural design. *Journal of the Franklin Institute*, Vol. 9, No 5, pp. 595-612.

Kiyota, T.; Tsuji, Y. Y.; Kondo, E. (2001). An interactive fuzzy satisficing approach using genetic algorithm for multi objective problems. *Proceedings of Joint 9th IFSA World Congress and 20th NAFIPS International Conference*, Vancouver, Canada.

Kiyota, T.; Tsuji, Y.; Kondo, E. (2003). Unsatisfying Functions and Multiobjective Fuzzy Satisficing Design Using Genetic Algorithms. *IEEE Transactions on Systems, Man and Cybernetics – PART B: Cybernetics*, Vol. 33, No 6, pp. 889-897.

Liu, T.C. (2006). *Developing a fuzzy proportional-derivative controller optimization engine for engineering design optimization problems*. PhD Thesis, Yuan-Ze University.

Maglaras, G.G.; Nikolaidis, E.; Haftka, R.T.; Cudney, H.H. (1997). Analytical-experimental comparison of probabilistic methods and fuzzy set based methods for designing under uncertainity. *Structural Optimization*, Vol. 13, pp. 69-80.

Marler, R.T.; Yang, J.; Rao, S.S. (2004). A Fuzzy Approach for Determining a Feasible Point in a Constrained Problem. *Proceedings of ASME/JSME Pressure Vessels and Piping Division Conference*, San Diego, USA.

McNeill, D.; Freiberger, P. (1993). *Fuzzy Logic/the Discovery of a Revolutionary Computer Technology and How It Is Changing Our World*, Simon & Schuster, New York.

Mohandas, S.U.; Phelps, T.A.; Ragsdell, K.M. (1990) Structural optimization using a fuzzy goal programming approach. *Computers & Structures*, Vol. 37, pp. 1-8.

Radford, A.D.; Gero, J.S.; Roseman, M.A.; Balachandran, M. (1985). Pareto Optimization as a Computer Aided Design Tool. In: *Optimization in Computer-Aided Design,* J.S. Gero, Springer, Berlin.

Rao, S.S. (1984). *Optimization, Theory and Applications*, John Wiley & Sons.

Rao, S.S. (1987a). Description and Optimum Design of Fuzzy Mechanical Systems. *ASME Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 109, pp. 126-132.

Rao, S.S. (1987b). Multiobjective Optimization of Fuzzy Structural Systems. *International Journal for Numerical Methods in Engineering*, Vol. 24, pp. 1157-1171.

Rao, S.S.; Sundararaju, K.; Prakash, B.G.; Balakrishna, C. (1992). Multiobjective Fuzzy Optimization Techniques for Engineering Design. *Computers & Structures*, Vol. 42, pp. 37-44.

Rao, S.S.; Xiong, Y. (2005a). Mixed-discrete fuzzy multiobjective programming for engineering optimization using hybrid genetic algorithm. *AIAA Journal,* Vol. 43, No 7, pp. 1580-1590.

Rao, S.S.; Xiong, Y. (2005b). Mixed-discrete fuzzy multiobjective optimization of structures using a hybrid genetic algorithm. *Proceedings of 6th World Congress on Structural and Multidisciplinary Optimization*, Rio de Janeiro, Brazil.

Russell, B. (1923). Vagueness. *Australian Journal of Psychology and Philosophy*, Vol. 1, pp. 84-92.

Sarma, K.C.; Adeli, H. (2000a). Fuzzy Discrete Multicriteria Cost Optimization of Steel Structures. *Journal of Structural Engineering*, Vol.126, pp. 1339–1347.

Sarma, K.C.; Adeli, H. (2000b). Fuzzy Genetic Algorithm for Optimization of Steel Structures. *Journal of Structural Engineering*, Vol. 126, pp. 596–604.

Sarma, K.C. (2001). *Fuzzy discrete multicrteria cost optimization of steel structures using genetic algorithm*. PhD Thesis, Ohio State Univeristy.

Shih, C.J.; Lai T.K. (1994). Fuzzy weighting optimization with several objective functions in structural design. *Computers & Structures*, Vol. 52, pp. 917-924

Shih, C.J.; Wangsawidjaja, R.A.S. (1995). Multiobjective fuzzy optimization with random variables in a mix of fuzzy and probabilistic environment. *Proceedings of International Joint Conference of the Fourth IEEE International Conference on Fuzzy Systems and The Second International Fuzzy Engineering Symposium*, Yokohoma, Japan.

Shih, C.J.; Chang, C.J. (1995). Pareto Optimization of Alternative Global Criterion Method for Fuzzy Structural Design. *Computers & Structures*, Vol. 54, pp. 455–460.

Shih, C.J.; Wangsawidjaja, R.A.S. (1996). Mixed fuzzy-probabilistic programming approach for multiobjective engineering optimization with random variables. *Computers & Structures*, Vol. 59, pp. 283-290.

Shih, C. J. (1997). Fuzzy and Improved Penalty Approaches for Multiobjective Mixed-Discrete Optimization in Structural Systems. *Computers & Structures*, Vol. 63, pp. 559-565.

Shih, C.J.; Wang, C.S.; Lin, Y.Y. (1997). Hasofer-Lind's Reliability Based Optimization for Multiobjective Fuzzy and Stochastic Design Problem. 6nd IEEE International Conference on Fuzzy Systems, (1997) 1191- 1196.

Shih, C.J.; Chi, C.C.; Hsiao, J.H. (2003). Alternative α-level-cuts Methods for Optimum Structural Design with Fuzzy Resources. *Computers & Structures*, Vol. 81, pp. 2579–2587.

Shih, C.J.; Lee, H. W. (2004). Level-Cut Approaches of First and Second Kind for Unique Solution Design in Fuzzy Engineering Optimization Problems. *Tamkang Journal of Science and Engineering - An International Journal*, Vol. 7, pp. 189-198.

Shih, C.J.; Lee, H.W. (2006). Modified double-cuts approach in 25-bar and 72-bar fuzzy truss optimization. *Computers and Structures*, Vol. 84, pp. 2100-2104.

Stadler, W. (1984). Multicriteria Optimization in Mechanics (A Survey). *Applied Mechanics Reviews*, Vol. 37, pp. 277-286.

Tonona, F.; Bernardini, A. (1998). A Random Set Approach to the Optimization of Uncertain Structures. *Computers & Structures*, Vol. 68, pp. 583-600.

Verdegay, J. L. (1982). Fuzzy Mathematical Programming. In: *Approximate reasoning in decision analysis,* M.M. Gupta, E. Sanchez, pp. (231-236), North-Holland Pub. Co.

Wang, G.Y.; Wang W.Q. (1985). Fuzzy Optimum Design of Structures. *Engineering Optimization*, Vol. 8, pp. 291-300.

Wang, P.; Huang, H.; Zuo, M.J.; Wu, W.; Liu, C. (2005). Functional-link net based multiobjective fuzzy optimization. In: *Advances in Neural Networks – Lecture Notes in Computer Science*, J. Wang et.al., pp. (800-804), Springer, Berlin .

Werner, B. (1987). Interactive Fuzzy Programming Systems. *Fuzzy Sets and Systems*, Vol. 23, pp. 131-147.

Xiong, Y. (2002). *Mixed Discrete Fuzzy Nonlinear Programming for Engineering Design Optimization*. Ph.D. thesis, University of Miami.

Xiong, Y.; Rao, S.S. (2005). A fuzzy dynamic approach for the mixed-discrete design optimization of mechanical systems. *ASME Journal of Mechanical Design*, Vol. 127, pp. 1088-1099.

Xu, C. (1989). Fuzzy Optimization of Structures by the Two-phase Method. *Computers & Structures,* Vol. 31, pp. 575-580.

Yang, Y.; Soh, C.K. (2000). Fuzzy logic integrated genetic programming for optimization and design. *Journal of Computing in Civil Engineering*, Vol. 14, pp. 249-254.

Yeh, Y.C.; Hsu, D.S. (1990). Structural optimization with fuzzy parameters. *Computers & Structures*, Vol. 37, pp. 917-924.

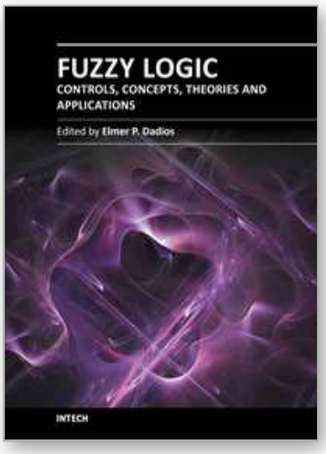
Yoo, J.S. (2000). *Adaptation of soft computing methods in multidisciplinary and structural optimization*. PhD Thesis, Rensselaer Polytechnic Institute.

Yoo, J.; Hajela, P. (2001). Fuzzy Multicriterion design using immune network simulation. *Structural and Multidisciplinary Optimization*, Vol. 22, pp. 188-197.

Yu, M.; Xu, C. (1994) Multi-objective Fuzzy Optimization of Structures Based on Generalized Fuzzy Decision-Making. *Computer & Structures*, Vol. 53, pp. 411-417.

Zadeh, L.A. (1965). Fuzzy Sets. *Information and Control*, Vol. 8, pp. 338-352.

Zimmermann, H.J. (1978). Fuzzy Programming and Linear Programming with Several Objective Functions. *Fuzzy Sets and Systems*, Vol. 1, pp. 45-55.



Fuzzy Logic - Controls, Concepts, Theories and Applications
Edited by Prof. Elmer Dadios

ISBN 978-953-51-0396-7
Hard cover, 428 pages
Publisher InTech
Published online 28, March, 2012
Published in print edition March, 2012

This book introduces new concepts and theories of Fuzzy Logic Control for the application and development of robotics and intelligent machines. The book consists of nineteen chapters categorized into 1) Robotics and Electrical Machines 2) Intelligent Control Systems with various applications, and 3) New Fuzzy Logic Concepts and Theories. The intended readers of this book are engineers, researchers, and graduate students interested in fuzzy logic control systems.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

Aykut Kentli (2012). Survey on Design of Truss Structures by Using Fuzzy Optimization Methods, Fuzzy Logic - Controls, Concepts, Theories and Applications, Prof. Elmer Dadios (Ed.), ISBN: 978-953-51-0396-7, InTech, Available from: http://www.intechopen.com/books/fuzzy-logic-controls-concepts-theories-and-applications/design-of-truss-structures-by-using-fuzzy-optimization-methods

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821