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Power Control for Cognitive Radios: A Mixed-Strategy Game-Theoretic Framework

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1. Introduction

Motivated by the spectrum drought and explosive growth of increasing quality of service requirements, cognitive radio as a promising technique has attracted a significant attention in wireless community. In this chapter, we investigate and summarize the following contents:

- After a simple introduction of cognitive radio development in recent years, we focus on the issue of how to implement interference mitigation by power control techniques amongst multiple cognitive radios. An overview of concurrent power control schemes is provided first, then we point out the existing problems and new challenges of power control in cognitive radio networks, which leads us to concentrate on a novel mathematical model-game theory.

- Game theory, which captures the dynamic decision-making behavior of selfish and rational players have attracted a wide attention from cognitive radio community, specifically for the game theory-based power control in cognitive radio networks. Several specific game models which suit cognitive radios well are explored and introduced, and these models are with typical good properties, for instance, they can well guarantee the existence and uniqueness of the celebrated Nash equilibrium solution.

- There are many impractical assumptions in these existing literature, for example, complete information and rationality and so on. In cognitive radio networks/systems, the complexities of mobility and traffic models, coupled with the dynamic topology and the unpredictability of link quality make these convectional game models meet with limited success. So that we employ mixed-based power control game (MPCG) to deal with the discrete power control issue. MPCG provides a novel point of view to investigate other resource management problems in the uncertain environment including cognitive radio context.

- We also discuss several related open problems, such as the lack of proper models for dynamic and incomplete information games. We use the application prospect of game theory to conclude this chapter.

We will relax the full nature of the information requirements, and investigate the effective power control from a very creative perspective termed as mix-strategy based matrix power control game (MPCG) model. The typical max-min fairness criteria is chosen as the fair and

optimal criteria for the mixed-strategy based power control algorithm, so that, the scheme proposed in this chapter can greatly improve the fairness of the multi-user located at different distance and with diversities of channel state information. The contributions in this chapter are summarized as:

- An efficient and fair (max-min fairness) discrete power control scheme is proposed in this chapter. Our scheme is based on the mixed-strategy of the matrix power control game model. The convergence and uniqueness properties are well guaranteed as long as available strategy space, e.g., the available power level is in a finite countable number.

- Additionally, mixed-strategy provides much larger strategy space for each player, so that the opportunity for each player to achieve the Nash Equilibrium Solution can well satisfy the Pareto optimality (effectiveness criteria). Because the pure strategy of the traditional game-theoretic model is a special case of the mixed-strategy with the determinate distribution.

- Last but not the least, the power control algorithm is greatly simplified by employing an amazing transformation from the mathematical point of view. With the conventional simplex method, the reformulated system model can be efficiently solved.

2. Background

The wireless industry is witnessing an explosive growth due to the increase in the number of the mobile users, paralleled by the widespread deployment of heterogeneous wireless networks. The requirements of the high transmit rate is becoming serious, and the high wide-band data service urgently requires more spectrums. Unfortunately, the available spectrum has been allocated completely. Meanwhile, recent measurement studies suggest that radio spectrum is gradually becoming an under-utilized resource that should be better explored. According to FCC, 15% to 85% assigned spectrum is used with large temporal and geographical variations (1). By now, it has been recognized that the scarcity of radio spectrum is mainly due to inefficiency of traditional static spectrum-allocation policies (1; 2). Motivated by the promising cognitive radio (CR) technology, both academic and industrial communities have shifted attention to dynamic spectrum access to alleviate alleviate spectrum scarcity and improve spectrum efficiency. Dynamic spectrum access represented as cognitive radio technology attracts wide attention to improve the spectrum-hunger situation. An introduction to CR basics, different spectrum sharing models, and challenges and issues in designing dynamic spectrum access networks can be found in (1–4).

While the cognitive radio community has had significant success popularizing the concept of cognitive radio and developing prototypes, applications, and critical components, the community has had a surprisingly difficult time agreeing upon exactly what is and is not a cognitive radio beyond. Some commonalities have developed different definitions of cognitive radios. However, as the original cognition cycle shown in Figure 1, the basic characteristics can be summarized as follows: First, all of these definitions assume that cognition will be implemented as a control process, presumably as part of a software defined radio. Second, all of the definitions at least imply some capability of autonomous operation. In detail, Observation: whether directly or indirectly, the radio is capable of acquiring information about its operating environment. Adaptability: the radio is capable of changing
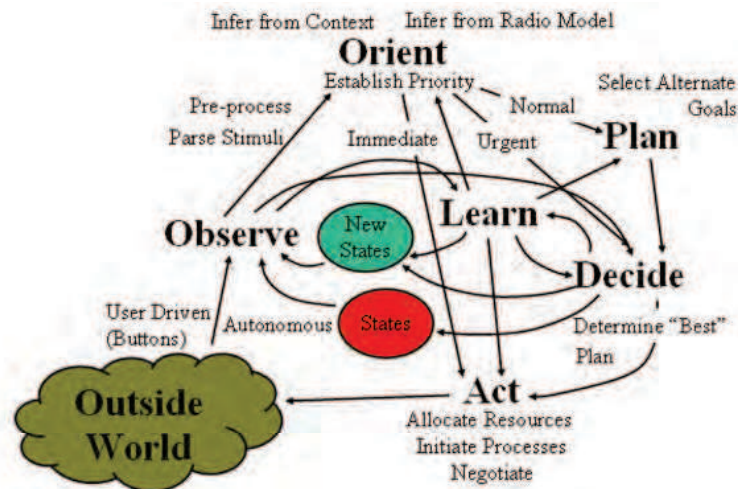
Fig. 1. Cognitive radio and cognition cycle (1; 2).

its waveform. Intelligence: the radio is capable of applying information towards a purposeful goal (6). The interference avoidance problem between the primary user and the secondary user is a critical issue for the cognitive radio networks.

2.1 Background of game theory for wireless communications

In recent years, game theory has found an increasingly important role, especially for the issues of radio resource management (17; 18). There are many prior game theory literature which investigate various issues in wireless communications, especially, in the context of Cognitive Radios (CRs). Game theory is a powerful tool to analyze the interactions among decision-makers with conflicting interests and finds a rich extent of application in communication systems including network routing, load balancing, resource allocation, flow control and power control. There is an extensive power control strategies based on game theoretic and utility theory (2). Meanwhile, they did achieve certain progress and better results, especially for resource management issue for cognitive radios. But most of them are based on the Nash game (5), which is essentially a non-cooperative game model.

Based on game theory, there are extensive research on the radio resource management (RRM) issues, we cite some here, including the power control (4), spectrum sharing (2), spectrum access (13), channel selection (12) and congestion control (19). However, the concurrent research on the basis of the game theory almost all focuses on characterization description and identification of the feasible equilibrium operating point, e.g., the typical Nash equilibrium (6–9) and Nash bargaining solution (5), also including some other extensive equilibrium solutions , e.g., Stackelberg equilibrium solution and correlated equilibrium solution(23). Some others concentrate the existence and uniqueness of the equilibrium solution. For example, the investigations in the potential game (18) and the super-modular game (6), which are all game models with some nice properties guaranteeing the existence and uniqueness. Actually, this is guaranteed by the specific utility function design in the game model (6–9; 18).

A great number of resource allocation and management problems in communication networks can be formulated as game models, which are summarized in (17). There are also lots of works on dealing with a diversity of new issues in current wireless networks. (1; 2) investigate

the non-cooperative selfish behavior and spectrum sharing games of multiple WiFi access point in the open/unlicensed spectrum, considers the interference management problem in the ad hoc networks using the super-modular games (3). For example, (8; 9) investigate the pricing function design for improving the Pareto optimality of the Nash equilibrium solution in the power control games in CDMA systems, and others study the bandwidth allocation in broadband networks, channel allocation in OFDMA networks, and the resource management in the multi-media transmission networks, respectively from the cooperative game-theoretical perspective, for example, the Nash bargaining game, and coalition formulation games. On the other hand, some drawbacks and disadvantages have been found and encountered of the traditional mathematical tools, which are unprecedentedly faced before. For example, the convex optimization can not well formulate the dynamic decision making problem of the multiple CRs. In addition, the decision making process is interactive, coupled among each other, inter-dependently. Meanwhile, the dynamic topology and changing radio spectrum holes, the opportunistic spectrum access and various service characteristics cause people to find new mathematical molding tool in CR context. How to devise an adaptive QoS measurement for the cognitive radios is really full of absolute challenge in CRNs. In addition, from the concurrent research, we have seen that the game theory is really suitable for analysis of cognitive radios, which is shown in Figure 2.
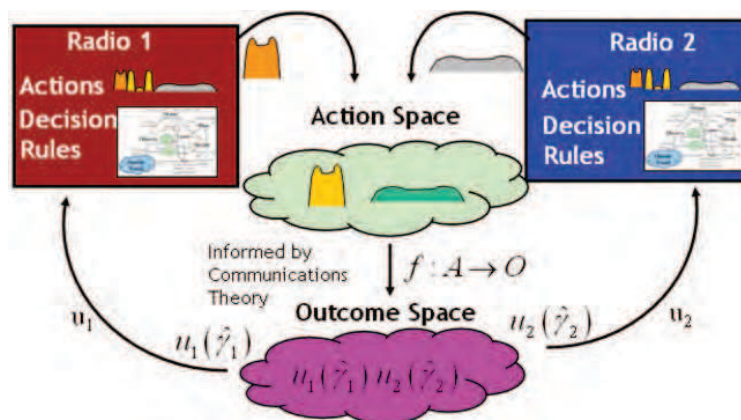

Fig. 2. Mapping of cognitive radio to game model (6).

2.2 Mixed-strategy considerations

John von Neumann's (1928) theoretical formulation and analysis of such strategic situations is generally regarded as the birth of game theory. von Neumann introduced the concept of a mixed strategy: a mixed strategy is a probability distribution one uses to randomly choose among available actions in order to avoid being predictable. In a mixed strategy equilibrium each player in a game is using a mixed strategy, one that is best for him against the strategies the other players are using. John Nash (1950) introduced the powerful notion of equilibrium in games (including non-zero-sum games and games with an arbitrary number of players): an equilibrium is a combination of strategies (one for each player) in which each player's strategy is a best strategy for him against the strategies all the other players are using. An equilibrium is thus a sustainable combination of strategies, in the sense that no player has an incentive to change unilaterally to a different strategy. A mixed-strategy equilibrium (MSE) is one in which each player is using a mixed strategy; if a game's only equilibria are mixed,

we say it is an MSE game. In two-person zero-sum games there is an equivalence between minimax and equilibrium: it is an equilibrium for each player to use a minimax strategy, and an equilibrium can consist only of minimax strategies. Non-zero-sum games and games with more than two players often have mixed strategy equilibria as well. Important examples are decisions whether to enter a competition (such as an industry, a tournament, or an auction), 'wars of attrition' (decisions about whether and when to exit a competition), and models of price dispersion (which explain how the same good may sell at different prices), as well as many others. Every finite n-person strategic game has a mixed Nash Equilibrium.

To the best of our knowledge, first, there is always a impractical complete information assumption, which is players know choices of strategies and corresponding payoffs of other players [1] (but not their actions [2]). Second, the previous mentioned work mostly focused on research of continuous power control scheme, since under the continuous assumption, it is easy to deal with from the mathematical perspective. Traditional discrete power control is based on the continuous power space, which is adaptive to the practical scenario and the traditional method "discretizing "the continuous value that will not always guarantee the convergence and uniqueness of continuous power control.

We assume that there exists only one time step, which means that the players have only one move as a strategy. In game-theoretic terms, this is called a single stage or static game. Please note that the definition of a static game means that the players have only one move as a strategy, but this does not necessarily correspond to the time slot of an underlying networking protocol. In many strategic situations a player's success depends upon his actions being unpredictable. Competitive sports are replete with examples. One of the simplest occurs repeatedly in soccer (football): if a kicker knows which side of the goal the goaltender has chosen to defend, he will kick to the opposite side; and if the goaltender knows to which side the kicker will direct his kick, he will choose that side to defend. In the language of game theory, this is a simple 2x2 game which has no pure strategy equilibrium. So that, mixed strategy-based game theoretical formulation with nice existence of equilibrium solutions has received a great attention.

2.3 Power control in cognitive radio systems

In a cognitive radio network, proper power control is of importance to ensure efficient operation of both primary and secondary users. Even without the presence of primary users, power control is still an issue among secondary users since the signal of one user may cause interference to the transmissions of others. Thus, how to develop an efficient power allocation scheme that is able to jointly optimize the performance of multiple cognitive radios in the presence of mutual interference is of interest to such a system.

2.3.1 Power control

Power control mitigates unnecessary interference, and it can save the battery life of the mobile devices, hence, increasing the network capacity and prolong battery's life. Centralized power control requires extensive information interaction between the base station and the mobile

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[1] In this chapter, we use player, secondary user and CR interchangeably.
[2] We also use strategy, action and power level interchangeably throughout this chapter.

terminal, which is applied inefficiently in practice. The distributive versions only depend on local information, e.g., the received interference power or the Signal-to-interference and noise ratio (SINR) to adaptively adjust the power level until each user obtains the objective SINR threshold.

Recently, the max-min fairness criterion is widely accepted as the fairness criteria. The max-min fairness is regarded as the one that the player can not increase the utility without decreasing the utility of his components. It is a standardized fairness concept in the ATM networks, and now it is widely accepted as the fairness criterion of the resource allocation technique in the wireless communication networks. E.g. (9) addresses the joint transmit power control and beam-forming technology if the multi-antenna systems with the aid of two different objective function design. The same as (10), the authors of the (11) investigate the max-min fairness for the MISO downlink systems. Based on the max-min fairness framework, a distributed power control algorithm is proposed for the Ad-hoc networks.

2.3.2 Game theoretical consideration for power control

Autonomously dynamic behavior and performance analysis is of great importance in dynamic spectrum sharing scenario, especially, when context information perceived by multiple secondary users (SUs) of different levels of cognition is asymmetric, which is definitely necessary in cognitive radio networks (CRNs) (1; 2). Game theory, which captures the dynamic decision-making behavior of selfish and rational players have attracted a wide attention from wireless community (6–8). Meanwhile, its excellent predictability of next action employed by the player, along with well established equilibrium solution concepts, lends itself well to the design and analysis of CRNs. A survey of game theory for wireless engineers is provided in (5), and its increasing use for spectrum management is exemplified in CRNs (6–10).

2.4 Special game models

Only when the game has certain special structure, the gaming iteration algorithm can be converged and lead to equilibrium solution, especially, in the distributed decision making context, for instance, the cognitive radio networks, since there is only local information support. There are several special cases of utility function design besides the above mentioned several design criteria.

2.4.1 Potential games

A potential game is a special type of game where $\mathcal{U}$ are such that the change in value seen by a unilaterally deviating player is reflected in the potential function $\mathcal{V}$. A game $\mathcal{G} = \{\mathcal{N}, \mathcal{S}, \mathcal{U}\}$ is a potential game if there is a potential function $\mathcal{V} : S \to \mathbb{R}$ such that one of the following conditions holds.

- $U_i(s_i, s_{-i}) - U_i(s'_i, s_{-i}) = \mathcal{V}_i(s_i, s_{-i}) - \mathcal{V}_i(s'_i, s_{-i})$, where for any $i \in N$, $s \in S$, and $s'_i \in S_i$;
- $sgn\{U_i(s_i, s_{-i}) - U_i(s'_i, s_{-i})\} = sgn\{\mathcal{V}_i(s_i, s_{-i}) - \mathcal{V}_i(s'_i, s_{-i})\}$, where sgn is the signal function.

It is an exact potential game, if and only if the first condition above denoted is satisfied. In addition, the necessary and sufficient condition for a game to be an exact potential game is

$$\frac{\partial^2 u_i(s_i, s_{-i})}{\partial s_i \partial s_j} = \frac{\partial^2 u_i(s_i, s_{-i})}{\partial s_j \partial s_i}, \forall j \neq i \in N. \tag{1}$$

There are also some other special potential game, which requests different properties owned by the utility function. For example, the coordination-dummy games, self-motivated games, and bilateral symmetric interaction games. Detailed about this can be found in (1).

2.4.2 S-modular games

An S-modular game restricts $\{u_i\}$ such that for $i \in N$, either the following two equations (2) or (3) is satisfied.

$$\frac{\partial^2 u_i(s_i, s_{-i})}{\partial s_i \partial s_j} \geq 0, \forall j \neq i \in N \tag{2}$$

$$\frac{\partial^2 u_i(s_i, s_{-i})}{\partial s_i \partial s_j} \leq 0, \forall j \neq i \in N \tag{3}$$

When (2) is satisfied, the game is said to be super-modular; when (3) is satisfied, the game is said to be sub-modular. Myopic games whose stages are S-modular games and potential games with a unique Nash equilibrium solution(NES) and follow a best response dynamic converge to the NES when the NES is unique.

3. System model and problem formulation

In this chapter, a distributed scenario is considered as Figure 3, multiple secondary users (SUs) opportunistically access in the spectrum holes of the GSM system by sensing technology who works as the primary users (PUs), we don't care about how the SUs access and how to obtain such access opportunities in this chapter, but focus on how the multiple SUs choose the optimal power control strategy to mostly improve performance of the secondary system and maximize the payoff function of the individual SU.

In Figure 3, the rectangles represent the transmitters of the cognitive radio, and the circles represent the respective receiver, the communication link is tagged as the solid lines with the arrow. The lines depict the interference links of the CR-transmitter to the base station (BS) of the primary system, e.g. the GSM system; and also including the mutual interference between the multiple CR-transmitter and the specific CR-receiver. Here, we assume that each CR-transmitter can well obtain the necessary information, e.g. the channel state information and the interference situation of the considering scenario with the help of the BS. Consider the heterogeneous networks, and the GSM coexists with the secondary network composed by the multiple CRs who will employ the same available power levels as the GSM users.

PU   SU

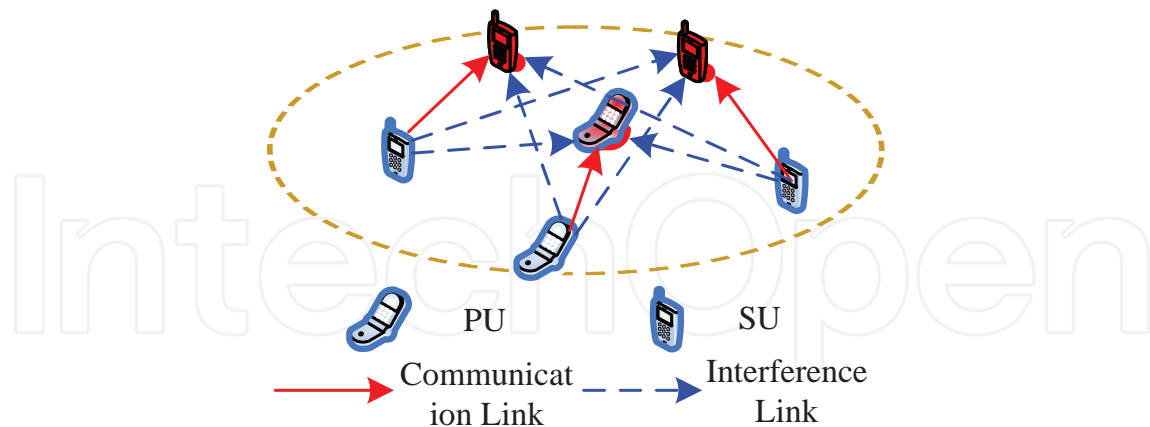Communicat _ _ _ _ Interference
ion Link   Link

Fig. 3. Secondary system model: where GSM as primary user who is the provider of the spectrum holes; and multiple secondary form a ad hoc cognitive scenario to implement an opportunistically spectrum accessing fashion.

A typical strategic power control game model is a three-tuple defined as $\mathcal{G} = \{\mathcal{N}, \mathcal{A}, \mathcal{U}\}$, where $\mathcal{N}$ is the player set, $\mathcal{A} = \prod_i \mathcal{A}_i$ is the action set, and $\mathcal{U}$ denotes the utility function which depicts the preference relationship of the various players in the game model. Here, we summarize a general joint rate and power control game model, which means that the action set is (R_i, p_i). There exists a tradeoff relationship among large SINR, low power consumption and high transmit rate, which are shown in Figure 4. As Figure 4 shown, we have some intrinsic characterizes summarized as follows.

- when the SINR λ_i and the transmit rate R_i are fixed, the utility function U_i won't increase with the increasing power level. This is partially due to the more power introduced into the game process, the more mutual interference power to the other players in the same gaming situation. That means when a player achieve the available SINR threshold, then increasing more power will not do good to the performance improvement, but damage it.

- Meanwhile, if one player has obtained the required QoS, that means more power consumption will shorten battery life of the equipment. So that the power control is necessary.

- If the consuming power is fixed and one player is transmit in the fixed transmit rate, the utility perceived by the player will increase with the SINR, which is illustrated in Figure 4. This tells us that when the higher SINR is guaranteed, the spectrum efficiency will be higher too.

- In addition, we capture the case of the fixed power level, when the SINR is also maintained on some fixed level, we can see that the utility leads a proportional relationship with respect to the transmit rate as Figure 4 shown.

The utility functions denoted in the (6–9; 17; 18) are all satisfied above mentioned these observations. From the typical power control game, we have some conclusions on the concept of utility function. The design or selection of a suitable utility function form in the extension of game theory for communications networks is always the bottleneck factor.
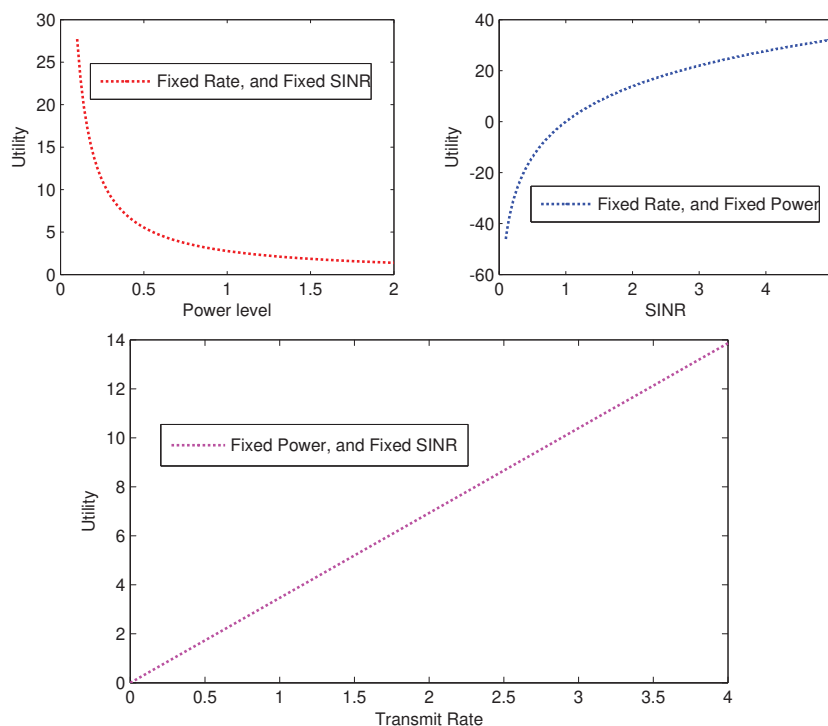
Fig. 4. Characteristics description of the utility function in the power control games, with the power level p_i, transmission rate R_i and the obtained SINR λ_i into consideration. The typical properties of the utility function (here, we choose the utility function as $u_i(p_i, R_i, \lambda_i) = \frac{R_i log(\lambda_i)}{p_i}$ in the similar form of (7; 8)) are reflected among these impacting parameters.

Lemma 1. *Utility function development for the investigated resource management issue, for example, power control, must satisfy two basic criteria: 1) utility function can be with physical meaning of the formulated problem as described in Figure 4. 2) utility function should well capture the characteristics of the preference of the users/players in the resource management game, for example, the relationship of resource consumption and the QoS satisfaction perceived by users.*

Definition 1. *Utility Function: Without loss of generality, in this chapter, we employ the Shannon channel capacity as the utility function $U_i(p_i, \lambda_i)$, which is shown as*

$$U_i(p_i, \lambda_i) = \log(1 + \lambda_i), \tag{4}$$

The terms p_i, g_i and N represent the transmit power, the channel gain of CR_i, and the CR transceiver pairs number, where the signal-to-interference and noise ratio λ_i is defined as

$$\lambda_i = \frac{p_i g_i}{\sum\limits_{j=1, j \neq i}^{N} p_j g_j + \sigma^2}, \tag{5}$$

where σ^2 is the power density of background noise, and $\sum_{j=1, j \neq i}^{N} p_j g_j$ represents the total interference power perceived by the SU i, which is introduced by the other players who are sharing the same spectrum hole.

In this chapter, we apply the max-min fairness as the fair and optimal power control criterion, which is shown to be Pareto optimal and fair. As the primary user in the scenario considered is the listened user, the CR opportunistic access can not damage the performance of the PU, and the interference temperature constraints of the PU firstly satisfied, the discrete power control problem can be formulated as

$$\max_{p_i \in P_i} \min_i \; U_i(p_i, \lambda_i) = \log(1 + \lambda_i)$$

$$\text{subject to } \sum_{i=1}^{N} p_i h_{i,k} \leq T_k, k \in K, \tag{6a}$$

$$p_i \in [p_{i,\min} : p_{i,step} : p_{i,\max}], \tag{6b}$$

where $\sum_{i=1}^{N} p_i h_{i,k} \leq T_k, k = 1, ..., K$ is the interference temperature constraint of the GSM BS. Each user selfishly chooses the optimal power level to max-minimize the utility function. Basically speaking, the problem is still a non-cooperative game model, for the cognitive radio the interference temperature is introduced to constraint the power control of the secondary spectrum utilization in order that the data transportation of the primary user is guaranteed. In this chapter, we assume that the available discrete power level for each secondary user is well satisfied the ITL constraints of the primary users.

Notice: In this chapter, we assume that the various SUs can observe the possible policy (actions, e.g., the available power level) and further to derive the possible utility function, which forms the payoff matrix, but they cannot determine what is the exact strategy during the gaming process. But to exploit the mixed-strategy to guess. And the biggest probability of some specific power level for each SU deserves to most exact strategy in this decision-making step.

3.1 MPCG: Matrix discrete power control game

The decision-making flow chart of the matrix-game model-based discrete power control approach can be shown as Figure 5.

Each CR-transmitter can achieve the context information including the channel state information (CSI) and the interference temperature constraint thresholds of the primary user for some specific spectrum by using sensing techniques. For instance, the signal sensing in Gaussian noise environment can be carried out using higher order statistics (HOS). Then CR-transmitters determine suitable discrete power level according to the context information sensed/reasoned and the available power policy set. For instance, each CR-transmitter has 13 power level, which can be used accordingly in set $P = [5dBm : 2dBm : 33dBm]$.

The most important thing in the proposed MPCG model is to compute the utility matrix due to based on which, we can determine the useful power in line with mixed policy design. Therefore, the utility value achieved by each CR when it selects the available power level can be predicted according to the power strategy interactions and combinations among multiple CR-transmitters. After each CR gets the final utility value, then the utility matrix is made up. Here, it deserves to pay attention to that not all the power level in the strategy space can

be utilized because the interference power level of the primary user must be satisfied first. By now, we know it is of great interest to find the threshold of the available power level, which is not the main objective in this chapter. However, in this chapter we assume that every power strategy in the strategy space can be employed during the next analysis. In the following step, the adaptively mixed-strategy selecting algorithm that will be implemented by each CR-transmitter.
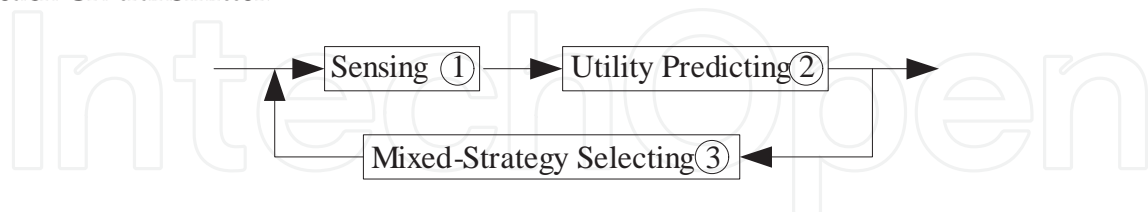

Fig. 5. Stream architecture

On the strategy space design, for instance, we choose the available power levels for each CR from $5dBm$ to $33dBm$, which is in accordance with the GSM system with the updating step $2dBm$. The channel state information of each CR is $G = [g_{=}0.5632, g_2 = 1.2321]$. Therefore, the power level of CR1 and CR2 can select is defined as $P = [5dBm : 2dBm : 33dBm]$.

According to definition of the utility function, we can obtain the following utility matrix when different CRs choose multi-power levels. For CR1, the transmit power level is denoted as $P = [5dBm : 2dBm : 33dBm]$, and the channel gain between the CR1 transmitter to the receiver is g_1 and the channel gain between the CR2 transmitter to the CR1 receiver is termed as g_{21}, so the interference power of CR2 introducing to the CR1 is $p_2 g_{21}$, and the utility function of CR1 takes the form of

$$U_1(p_1, p_2) = \log(1 + p_1 g_1 \big/ (p_2 g_{21} + \sigma^2)). \tag{7}$$

Similarly, the utility function of CR2 takes the form of

$$U_2(p_1, p_2) = \log(1 + p_2 g_2 \big/ (p_2 g_{12} + \sigma^2)). \tag{8}$$

From above utility function design, we can see that the optimal utility value achieved by each CR depends not only its own power selection but also the others in the wireless environment. And the strategy space of CR2 is also $P_2 = [5dBm : 2dBm : 33dBm]$.

3.2 Basics of matrix game definition

Consider the complex interference radio environment, we assume that each CR will adjust the power level entirely, e.g. when CR1 chooses the power level of 5dBm, the utility of the CR1 maybe of 12 different cases, since CR2 has 12 different power level to choose in the set of $P_2 = [5dBm : 2dBm : 33dBm]$. So that, we get a utility matrix A_{12*12} for a two-player MPCG model, where $A = \{a_{ij}, i = 1, ..., M, j = 1, ..., M\}$ represents utility achieved when the CR1 selects any power strategy p_i in set $P_1 = [5dBm : 2dBm : 33dBm]$, and the CR2 selects any power strategy p_j in set $P_2 = [5dBm : 2dBm : 33dBm]$.

Definition 2. *Mixed-strategy definition: A Mixed-strategy for CR1 $S_1 = \{x_i, i = 1, .., M\}$ is denoted as the distribution function of the pure strategy $P_1 = [5dBm : 2dBm : 33dBm]$. That is to say, the*

CR1 chooses the pure power level p_i at a probability of x_i. These vectors form a novel strategy space which is referred as the mixed-strategy.

Therefore, we can conclude that the pure strategy can be considered as a special case of that the CR1 choose the power level p_i with a probability of "1". The vector x_i satisfies $0 \leq x_i \leq 1$, and $\sum_{i=1}^{M} x_i = 1$. In fact, the mixed-strategy provides more opportunity for each user, in other words, the strategy space for each user is becoming larger so they all achieve more available strategy to choose, the optimal strategy can more easily achieved. For CR2, we can also similarly denote the mixed-strategy is $S_2 = \{y_i, i = 1, ..., M\}$.

Throughout this chapter, we assume that the available power level for each user is with the same dimension, and the Matrix Power Control Game (MPCG) model can be defined as

Definition 3. *MPCG: a three-element $G = \langle S_1, S_2, A \rangle$ is called as matrix game, where $S_1 = \{x_i, i = 1, ..., M\}$ and $S_2 = \{y_i, i = 1, ..., M\}$ is the mixed-strategy of CR1 and CR2, respectively. The term $A_{M \times M}$ is the utility matrix as the above section described which is also where the concept comes out.*

3.3 How to solve matrix power control game

First, the expected utility function must be clearly described for the matrix power control game. As the utility matrix definition, the term represents the utility obtained when the CR1 selects the power strategy x_i and the CR2 selects the power strategy y_j. For the CR1, the matrix power control game model can be formulated as

$$U_{CR_1} = \max_{X \in S_1} \min_{1 \leq j \leq M} \sum_{i=1}^{M} a_{i,j} x_i$$

subject to $0 \leq x_i \leq 1,$ (9a)

$$\sum_{i=1}^{M} x_i = 1,$$ (9b)

where X is the mixed power vector selected by CR1. For simplicity, we utilize the term $L(X)$ represent $\min_{1 \leq j \leq m} \sum_{i=1}^{M} a_{i,j} x_i$, which means $L(X) = \min_{1 \leq j \leq m} \sum_{i=1}^{M} a_{i,j} x_i$. Then the matrix power control game model with the mixed strategy design can be reformulated as

$$U_{CR_1} = \max_{X \in S_1} L(X)$$

subject to $L(X) \leq \sum_{i=1}^{M} a_{i,j} x_i, j = 1, 2, ..., M,$ (10a)

$$0 \leq x_i \leq 1,$$ (10b)

$$\sum_{i=1}^{M} x_i = 1.$$ (10c)

Let $x_i = x_i/L(X)$, and the model of (10) can be further represented as

$$U_{CR_1} = \min_{X \in S_1} \sum_{i=1}^{M} x_i'$$

$$\text{subject to } \sum_{i=1}^{M} a_{i,j} x_i' \geq 1, j = 1, 2, ..., M, \tag{11a}$$

$$0 \leq x_i' \leq 1. \tag{11b}$$

Theorem 1. *(10) is equivalent to (11).*

Proof. The optimal utility of $\tilde{G} = \langle S_1, S_2, \tilde{A} \rangle$ is termed as $\tilde{U}_1^*$ and U_1^* for $G = \langle S_1, S_2, A \rangle$. Using $x_i' = x_i/L(X)$, that is $x_i = x_i'L(X)$ to take place of the x_i in the Eq. (10), the objective function is temporally unchanged, and next we focus on the constraint conditions. We assume that the term $L(X) > 0$ always holds, if it initially can not guaranteed, luckily, we further prove during the sequel section provides us a powerful technique to satisfy the assumption. First, we see the constraint condition (3) in the Eq. (10) $\sum_{i=1}^{M} x_i'L(X) = 1$, that is $\sum_{i=1}^{M} x_i' = 1/L(X)$. So observe the objective function again, we can conclude that the objective function takes the form of $U_{CR_1} = \max_{X \in S_1} L(X)$, which can represent as $U_{CR_1} = \min_{X \in S_1} 1/L(X)$, and it finally appears as $U_{CR_1} = \min_{X \in S_1} 1/L(X) = \min_{X \in S_1} \sum_{i=1}^{M} x_i'$, as the Eq. (11) shown. Because we assume that $L(X) > 0$ always holds, and the constraint condition $x_i \geq 0, i = 1, 2, ..., M$, when $x_i = x_i'L(X)$, and we can conclude that $x_i'L(X) = x_i \geq 0$, that is $x_i' \geq 0, i = 1, 2, ..., M$. Further, we only need to represent the term x_i as x_i', $L(X) \leq \sum_{i=1}^{M} a_{i,j} x_i = \sum_{i=1}^{M} a_{i,j} x_i'L(X), j = 1, 2, ..., M$, and the $L(X) > 0$ always holds, and it can be missed, so the constraint condition transforms into $\sum_{i=1}^{M} a_{i,j} x_i' \geq 1, j = 1, 2, ..., M$. We can conclude that Eq. (10) is equivalent to Eq. (11). $\qquad\square$

3.4 Simplex algorithm for linear programming problem

Finally, the problem is transformed into the simple linear programming as the Eq. (11) shown, which can be easily solved using the simplex method. Meanwhile, for the power level of each CR is limited, the matrix power control game model is absolutely guaranteed to have at least one mixed-strategy. But in the context of a more practical scenario when CR2 employs a so large power level that introduces more interference power to CR1, the utility achieved by CR1 maybe zero even negative. We find the matrix game will be hard to be solved from the mathematical perspective. Fortunately, we find a powerful tool to deal with this situation.

If we directly to employ the simplex method to search the mixed-strategy optimal power level, the computation complexity will be very high, now to simply the problem an equivalent mathematical model is introduced. If the original matrix power control game model takes the form of $G = \langle S_1, S_2, A \rangle$, and $A = \{a_{ij}, i = 1, ..., M, j = 1, ..., M\}$. Meanwhile, we define the novel matrix game $\tilde{G} = \langle S_1, S_2, \tilde{A} \rangle$, where $\tilde{A} = \{\tilde{a}_{ij} = a_{ij} + d, i = 1, ..., M, j = 1, ..., M\}$.

Lemma 2. *The optimal mixed-strategy of original model is the same as the newly-designed matrix power control model. The optimal utility $\tilde{G} = \langle S_1, S_2, \tilde{A} \rangle$ of is termed as $\tilde{U}_1^*$ and U_1^* for $G = \langle S_1, S_2, A \rangle$. Meanwhile, we can conclude that $\tilde{U}_1^* = U_1^* + d$.*

Proof. For $G = \langle S_1, S_2, A \rangle$, we assume that the optimal mixed-strategy for CR1 and CR2 is X_1^* and Y_2^* respectively. So that, we know

$$U_1^* = E(X_1^*, Y_2^*) = \max_{X \in S_1} \min E(X, Y_2^*). \tag{12}$$

Adding a suitable parameter d to the both sides of the Eq. (12), and we get

$$U_1^* + d = E(X_1^*, Y_2^*) + d = \max_{X \in S_1} \min E(X, Y_2^*) + d, \tag{13}$$

further, mathematically we get

$$\begin{aligned}
\tilde{U}_1^* &= E(\tilde{X}_1^*, \tilde{Y}_2^*) \\
&= \max_{X \in S_1, 1 \leq j \leq M} \min \sum_{i=1}^{M} \tilde{a}_{i,j} x_i \\
&= \max_{X \in S_1, 1 \leq j \leq M} \min \sum_{i=1}^{M} (a_{i,j} + d) x_i \\
&= \max_{X \in S_1, 1 \leq j \leq M} \min \sum_{i=1}^{M} a_{i,j} x_i + d \\
&= E(\tilde{X}_1^*, \tilde{Y}_2^*) + d \\
&= U_1^* + d.
\end{aligned} \tag{14}$$

That is to say, for any X, and any optimal mixed-strategy of X_1^* and Y_2^*, the Eq. (14) always holds. So the conclusion always holds. □

Lemma 2 tells us that the objective function $L(X) > 0$, if we choose the suitable constant parameter of d. So that, we conclude thta the proposed simplex method based power control approach always works well.

3.5 Proposed algorithm

In this subsection, the proposed discrete power control algorithm is depicted as the pseudo-code:

- Each CR collects the channel information (g_i) and the accumulated interference temperature (T) of the primary user from the sensing block of the CR system;

- In line with the necessary information achieved in the last step, each CR determines minimum available discrete power level (p_i) from the strategy space;

- The utility matrix (U_i) is predicted in this step, meanwhile, according to the method introduced in section IV the matrix is simplified to easily deal with;

- Using the simplex method to find the optimal discrete power level (P_i^*) and obtain the optimal utility (U_i^*) of each CR.

4. Numerical results

We consider a time-varying channel model which obeys the Raleigh distribution, and the channel characteristics change with the time, each CR must dynamically and adaptively the power level according to the wireless environment, e.g., the channel state information and the

interference temperature of the requirement of the PU which are all considered in this chapter. The background noise is in accordance with the Gauss distribution, and the power density is 1e-5 w.

4.1 Analysis of proposed algorithm

Figure 6 is the utility obtained of CR1 and CR2 after a limited iterations, because the interaction of the strategy (e.g. the power level) and the interference between them, the strategy choosing process is harshly hard. Fortunately based on the basics of the non-cooperative game theory, the mixed-strategy optimal power level for each CR is always existed. From the figures, we can see that in the context of the max-min fairness criterion we finally achieved the optimal power level for CR1 and CR2, respectively, which are shown as the peak value of the two three-dimensions pictures. The max-min utilities obtained of the CR1 and CR2 are 7.3584 and 7.5888, respectively. From the numerical results, we can conclude that our algorithm is fair, for the two CRs obtain the similar utility. Simultaneously, the existence of the proposed algorithm is investigated as well.
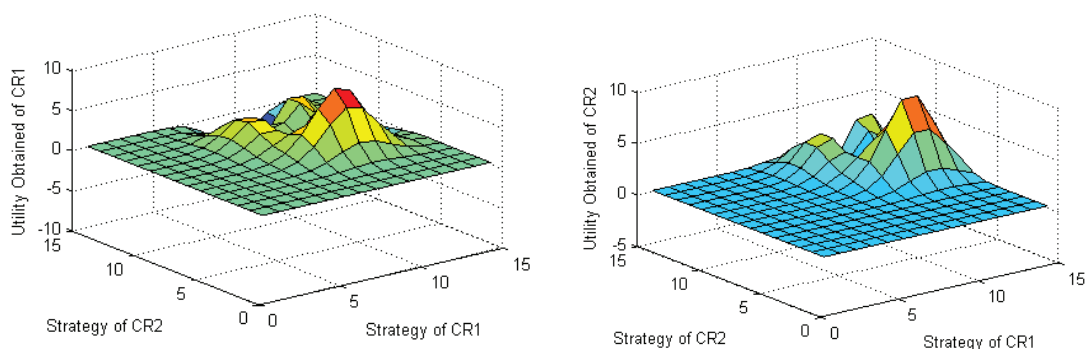
Fig. 6. Utility of CR1 and CR2 (max-min) (27).
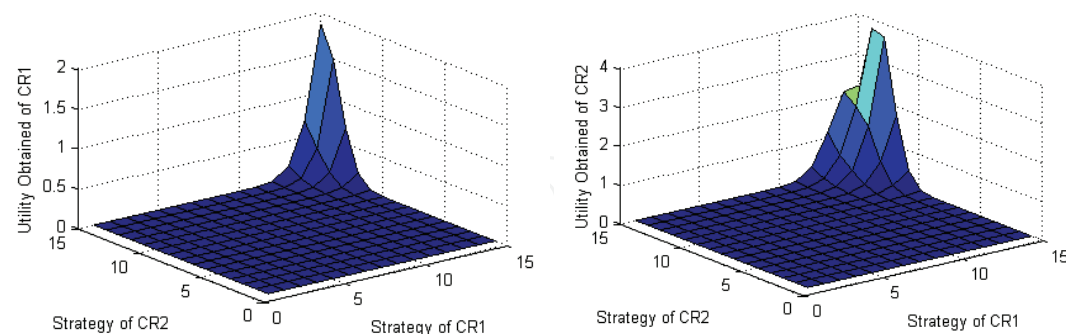
Fig. 7. Utility of CR1 and CR2 (max-max) (27).

4.2 Performance comparison

To show the effectiveness of our algorithm, here we compare with the greedy scheme which is based on the max-max principle. When other factors are all the same as the proposed algorithm, we obtain Figure 7. Though the fairness criterion changed into the max-max principle, the optimal power level can also achieved, which tells that the proposed algorithm

is robust. The most important issue reflected in the pictures that when the user become greedily to pursue the maximum utility, first the fairness between them is damaged entirely, and the entire performance also declined absolutely as a result of the selfish behavior. The utilities obtained of the CR1 and CR2 using the max-max based proposed algorithm are 1.8512 and 3.7225, respectively. Observing from the perspectives of the two numerical results, we can conclude that our algorithm can guarantee the existence, the fairness and effectiveness.

5. Conclusion and prospect of game theory for wireless communications

Due to the typical dynamic behavior of PU, various categories requirement for service, limited resource constraints and complicated interactive context of the cognitive radio context, the traditional mathematical tools encounter the unprecedented drawback in the multiple users who interact with each other including the context/user information. Meanwhile, the behavior of a given wireless device may affect the communication capabilities of a neighboring device, notably because the radio communication channel is usually shared in wireless networks. Game theory is a discipline aimed at modeling situations in which decision makers have to make specific actions that have mutual, possibly conflicting, consequences. It has been used primarily in economics, in order to model competition between companies: for example, should a given company enter a new market, considering that its competitors could make similar (or different) moves? Game theory has also been applied to other areas, including politics and biology. So that, from the engineering point of view, researchers capture the similar characteristics between the issues from communication networks and the game theory. Therefore, game-theoretic framework have been widely developed in this cognitive radio scenarios.

The proposed discrete power control approach highlights a practical approach for the cognitive system designer. The proposed mixed-strategy based scheme can entirely avoid the convergence issue of the "discretizing scheme". The max-min fairness improves a lot of the fairness and spectrum efficiency. Meanwhile, the Matrix game based scheme can easily extend to multiple secondary users cognitive context but with huge computation complexity. This is our next focus research topic as how implementation our proposed scheme in a multiple user case, but is still represents a potential point in this chapter.

Game theory is a fascinating field of study. Due to the development of game theory, there are always novel game model suits a great number of issue the wireless communication networks, for instance, the conventional radio resource management, access control and the media access control policy design. For example, potential game, super-modular game and Markovian game have found a lot of use in wireless communication networks.

6. Acknowledgement

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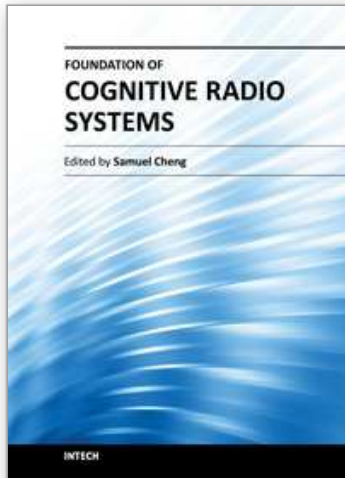
Foundation of Cognitive Radio Systems

Edited by Prof. Samuel Cheng

The fast user growth in wireless communications has created significant demands for new wireless services in both the licensed and unlicensed frequency spectra. Since many spectra are not fully utilized most of the time, cognitive radio, as a form of spectrum reuse, can be an effective means to significantly boost communications resources. Since its introduction in late last century, cognitive radio has attracted wide attention from academics to industry. Despite the efforts from the research community, there are still many issues of applying it in practice. This books is an attempt to cover some of the open issues across the area and introduce some insight to many of the problems. It contains thirteen chapters written by experts across the globe covering topics including spectrum sensing fundamental, cooperative sensing, spectrum management, and interaction among users.

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