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Using the Smith Chart in an E-Learning Approach

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1. Introduction

Prior to the advent of digital computers and calculators, engineers developed all sorts of aids (tables, charts, graphs) to facilitate their calculations for design and analysis in different areas in particular for line transmission problems. To reduce the tedious manipulations involved in calculating the characteristics of transmission lines, graphical tools have been developed. The Smith chart is the most commonly used of these graphical techniques. It is basically a graphical indication of the impedance change along a transmission line as one moves along it. It becomes easy to use after a small amount of experience. We will first explain how the Smith chart is constructed and then how to use it to calculate transmission line characteristics such as: the reflection coefficient (ρ), the Voltage Standing Wave Ratio (VSWR), the impedance along the line ($Z(d)$), the maximum and minimum voltage localization and impedance matching. For the majority of these Smith chart applications lossless lines will be assumed, although this is not absolutely required.

Since the main topic of this book is concerned with e-learning, the aim of this chapter is to help the reader understand and learn how to use the Smith chart, following step by step procedure based on *MATLAB* scripts that will be available for download and should be used when reading this chapter. This approach should teach the students how to solve several kinds of transmission line problems by themselves, in a paper chart using a pencil, a ruler and a compass.

MATLAB scripts are a good tool to help students better understand the Smith chart and explain, step by step, several procedures related to transmission line problems, (Mak & Sundaram, 2008), (Pereira & Pinho, 2010).

The goals of the chapter are to explain the reasons why using and understanding the Smith chart is still important nowadays, despite the present generalization of personal computers and powerful calculators. It is easy to plug a few numbers into a program and have it spit out solutions. When the solutions are complex and multifaceted, having a computer to do the grunt work is especially handy. However, knowing the underlying theory and principles that have been ported to computer platforms, and where they came from, makes the engineer or designer a more well-rounded and confident professional, and makes the results more reliable. Moreover it is interesting to note that these kinds of graphical tools are still useful nowadays. For example some types of modern laboratory equipment, such as

network analyzers still have displays that imitate the Smith chart. Another example is the use of Smith charts in a lot of commercial software about antennas and microwave devices, to display the simulation results. The importance of the chart is enhanced by the global information that is possible to obtain simultaneously.

The authors believe that the use of the Smith chart by undergraduate students and engineers is an important pedagogical tool, since many aspects of the voltage, current, impedance, Voltage Standing Wave Ratio (VSWR), referred commonly as SWR, reflection coefficient and matching design problems can be easily interpreted and well visualized using the Smith chart.

The chapter will be organized as follows:

- History and use of the Smith chart and its importance in the resolution of classical transmission line problems. Justification of its current use despite the present generalization of personal computers and powerful calculators.
- Construction of this chart from basic equations and concepts. Explanation of the main parameters that can be obtained from the chart. All these aspects will be supported by *MATLAB* scripts that display, step by step, the graphical procedure involved in the process.
- How to use the Smith chart. How to mark a normalized impedance and from then on to get several related parameters such as, the corresponding admittance, the VSWR, the reflection coefficient, the concept of travelling toward the generator or toward the load, the impedance at a given distance etc. Again, all these transmission line concepts will be explained through step by step procedures based on *MATLAB* scripts.
- Presentation of some examples that integrates all these transmission line concepts. One example is the single stub matching. The authors developed a *MATLAB* script that display, step by step, the graphical procedure that must be used to solve this problem. Others examples will be presented, because we believe they are important so students can learn on their own.

Throughout the chapter, when explaining the step by step procedure, several displays will be shown to illustrate the use of the Smith chart.

2. History of the Smith chart

Phillip H. Smith, inventor of the well known Smith chart, was born in Lexington, Massachusetts, on April 29, 1905 and died in Berkeley Heights, New Jersey on August 29, 1987, at the age of 82. In 1928, after graduating from Tufts College (now Tufts University) at the top of his class with B.S. degree in electrical communications, he was offered a job at Bell Telephone Laboratories. After 42 years in this company, Phillip Smith retired in 1970 and started a small company –Analog Instruments Company in New Providence, New Jersey – which initially sold navigational instruments for light aircraft. In his lifetime Smith held 21 U.S. patents and published over 35 technical papers on antennas and transmission lines.

In 1931, by modifying J. A. Fleming's 1911 telephone equations in an effort to simplify the solution of the transmission line problem, Smith developed his first graphical solution in the form of a rectangular chart. Even though the rectangular chart was very useful, Smith knew it had some limitations, namely the amount of data that could be accommodated. In 1936, Smith constructed a new type of transmission line chart that eliminated most of the limitations in his first diagram. The new chart was a special polar coordinate diagram which

could show all values of impedance and is essentially the Smith chart used today. Smith approached a number of technical magazines for publications of his transmission line diagram, however acceptance was slow. Finally, after two years Smith's article describing his chart was published in January 1939 issue of Electronics magazine. In a second article, published in the January 1944 issue of Electronics, Smith incorporated further improvements into his chart, including its usage as an impedance chart or an admittance chart, (Inan, 2005).

An interesting historical and theoretical background of the Smith chart can be found in the article written by Aleksandar Marinčić, (Marinčić, 1997).

3. Construction of the Smith chart

The Smith chart is constructed based on the voltage reflection coefficient and can be considered as parameterized plot, on polar coordinates, of the generalized voltage reflection coefficient $\rho\left(|\rho|, \varphi\right) = |\rho| e^{j\varphi}$, within a circle of unit radius $\left(|\rho| \le 1\right)$.

It is well known from transmission line theory, that the voltage reflection coefficient at the load is given by:

$$\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{1}$$

where Z_L is the load impedance and Z_0 is the characteristics impedance of the line. According to the transmission line theory, Z_0 is a real value but in general Z_L is a complex value. Equation 1 can be written as:

$$\rho_L = |\rho_L| \angle \varphi = \rho_r + j\rho_i \tag{2}$$

where ρ_r and ρ_i are respectively the real and imaginary parts of the reflection coefficient. Instead of having separate Smith chart for transmission lines with different characteristics impedances, it is preferable to have just one that can be used for any line. This is achieved using a normalized chart in which all impedances are normalized to the characteristic impedance Z_0 of the particular line under consideration. For example, for the load impedance Z_L, the normalized impedance z_L is given by,

$$z_L = \frac{Z_L}{Z_0} = r + jx \tag{3}$$

where r and x are respectively the real and imaginary parts of the normalized impedance. Substituting equations 2 and 3 into equation 1 gives,

$$\rho_r + j\rho_i = \frac{z_L - 1}{z_L + 1} \tag{4}$$

or

$$r + jx = \frac{(1 + \rho_r) + j\rho_i}{(1 - \rho_r) - j\rho_i} \tag{5}$$

Equating real and imaginary parts, we obtain

$$r = \frac{1 - \rho_r^2 - \rho_i^2}{(1 - \rho_r)^2 + \rho_i^2} \tag{6}$$

$$x = \frac{2\rho_i}{(1 - \rho_r)^2 + \rho_i^2} \tag{7}$$

rearranging the terms in equation 6 leads to:

$$\left(\rho_r - \frac{r}{1+r}\right)^2 + \rho_i^2 = \left(\frac{1}{1+r}\right)^2 \tag{8}$$

rearranging the terms in equation 7 leads to:

$$(\rho_r - 1)^2 + \left(\rho_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \tag{9}$$

Each of equations 8 and 9 is similar to the circle equation. Equation 8 is a r-circle (resistance circle) with center at $\left(\frac{r}{1+r}, 0\right)$ and radius equal to $\frac{1}{1+r}$. Several of these circles for various values of normalized resistance r, are plotted in Figure 1a). From the Figure 1a), we see that all circles pass the point (1,0).

a)   b)
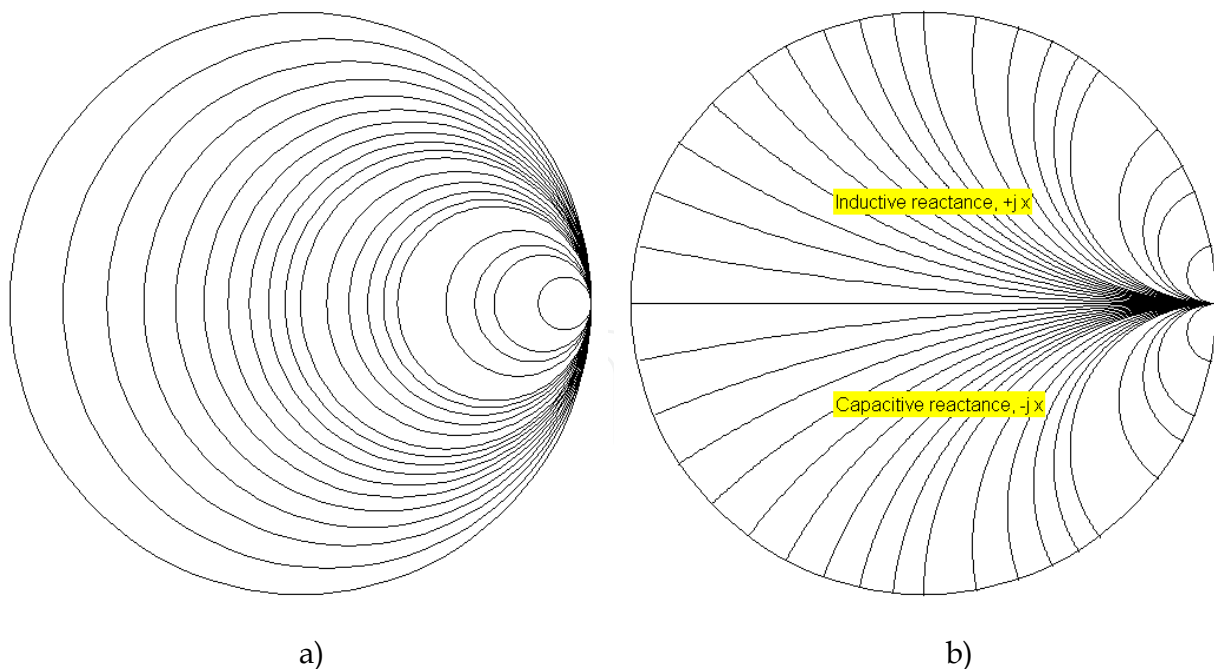
Fig. 1. Basic Smith chart. a)- normalized resistance circles. b)- normalized reactance curves.

Similarly, equation 9 is an x circle (reactance circle) with center at $\left(1, \frac{1}{x}\right)$ and radius equal to $\frac{1}{x}$. Several of these circles are plotted in Figure 1b), this time for positive and negative

values of the normalized reactance x. Notice that while r is always positive, x can be positive (inductive impedance) or negative (capacitive impedance). From this figure we see that there are symmetry about the horizontal central axis of the chart. Only the portion of the circles inside the central circle of radius one is shown, since the maximum value of $|\rho|$ for any passive loaded line is one. The circles given by equations 8 and 9 are orthogonal circles that make a conform mapping chart.

If the r circles and the x circles are superimposed, the result is the Smith chart shown in the Figure 2.
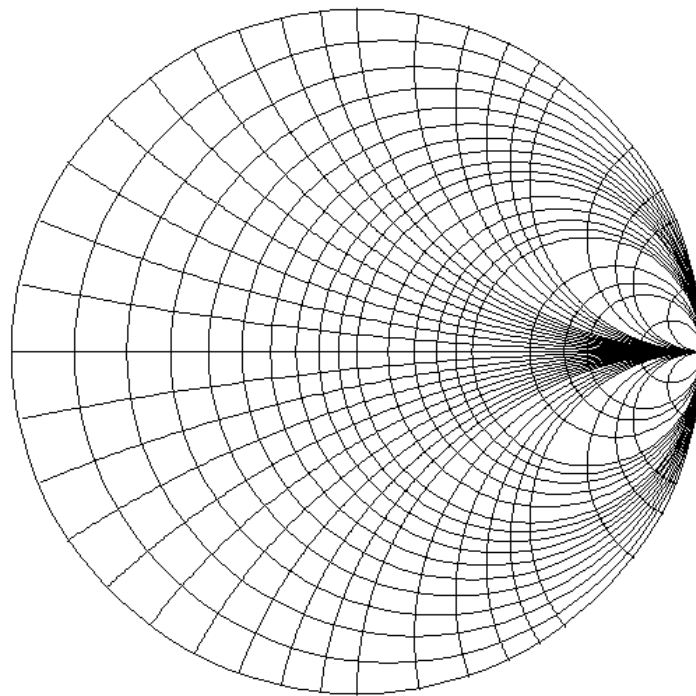
Fig. 2. Basic Smith chart.

3.1 Important features on a Smith chart

On a Smith chart there are some important points, lines and contours that should be mentioned. In Figure 3 some of these important features are indicated. The outer circle is the locus of the pure reactive impedances, that is, those with zero resistance. The horizontal axis is the locus of the real impedances. The left radius is the locus of the resistances less than Z_0 for which the reflection coefficient has a phase of 180º. The left extreme of this radius is the zero resistance and zero reactance point, that is, the short circuit point (SC). The right radius is the locus of the resistances greater than Z_0 for which the reflection coefficient has a phase of 0º. The right extreme of this radius is the infinite resistance and infinite reactance point, that is, the open circuit point (OC).

For a lossless transmission line terminated in a load with a reflection coefficient ρ_L, the circle with radius $|\rho_L|$ (known as the ρ circle or S circle), is the locus of all impedances appearing along the line, normalized to the characteristic impedance Z_0 of the line. These impedances can be obtained moving along the line either toward the load (counter

clockwise) or toward the generator (clockwise). When moving along the ρ circle, every crossing of the left horizontal radius corresponds to a voltage minimum (therefore a current maximum) in the line and to a real impedance less than Z_0. Similarly, every crossing of the right horizontal radius corresponds to a voltage maximum (therefore a current minimum) in the line and to a real impedance greater than Z_0.

In Figure 3a) some important features of when the Smith chart is used as an impedance chart are pointed out.

In the resolution of some problems, it is more convenient to work with admittances than with impedances. In these cases the Smith chart can be effectively used as an admittance chart. In this case the r circles and the x curves should be seen as g circles and b curves respectively. Also the upper half of the chart now corresponds to capacitive susceptances given by positive values of b and, the lower half of the chart corresponds to inductive susceptances given by negative values of b. In Figure 3b) some important features are indicated when the Smith chart is used as an admittance chart.

It is easy to transform a normalized impedance z in the corresponding admittance y. It will be located on the ρ circle in the opposite side of the diameter that passes through z.
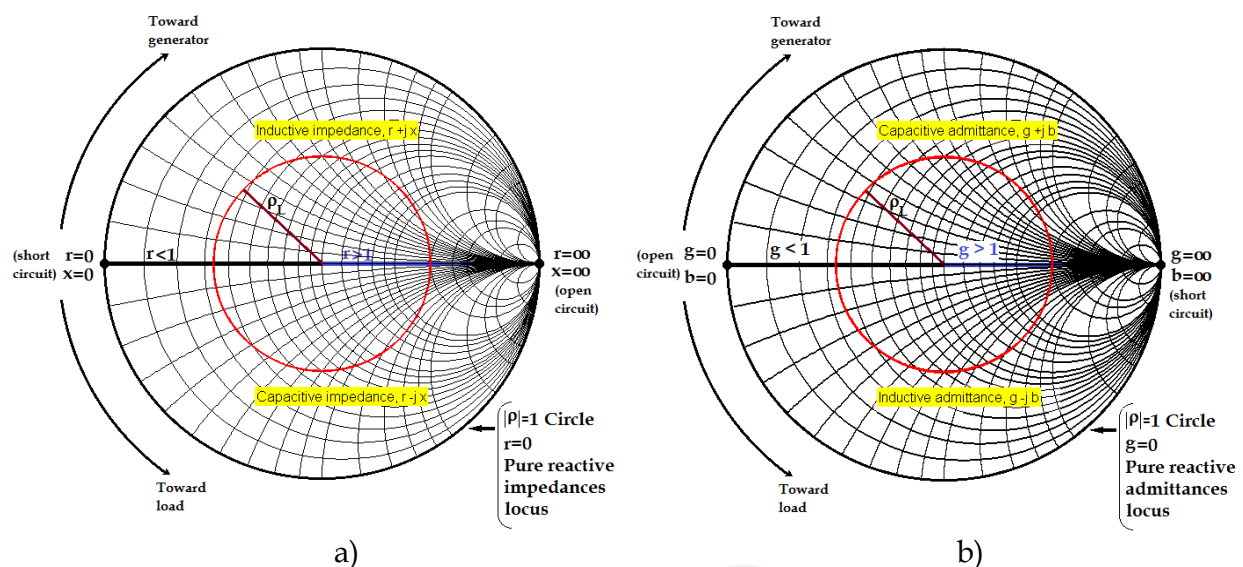

Fig. 3. Some important features of a Smith chart. a)- When used as an impedance chart. b)- When used as an admittance chart.

Nowadays, the Smith chart appears in several different types. One of them is shown in Figure 4. The main difference between this chart and the basic Smith chart shown in figure 2 is the existence of three scales around the periphery.

The outermost scale is used to determine distances in wavelengths toward the generator and the next scale is used to determine distances in wavelengths toward the load. The innermost scale is a protractor (in degrees) and is primarily used to determine the phase of the reflection coefficient and the phase of the transmission coefficient. It can also be used to determine distances, toward the load or toward the generator, expressed in degrees bearing in mind that a distance of λ/2 corresponds to 360°.

The Smith chart illustrated in figure 4 has also other auxiliary scales useful for the determination of some parameters like for example, the VSWR, the amplitudes of the reflection and transmission coefficients, the return loss in dB etc.

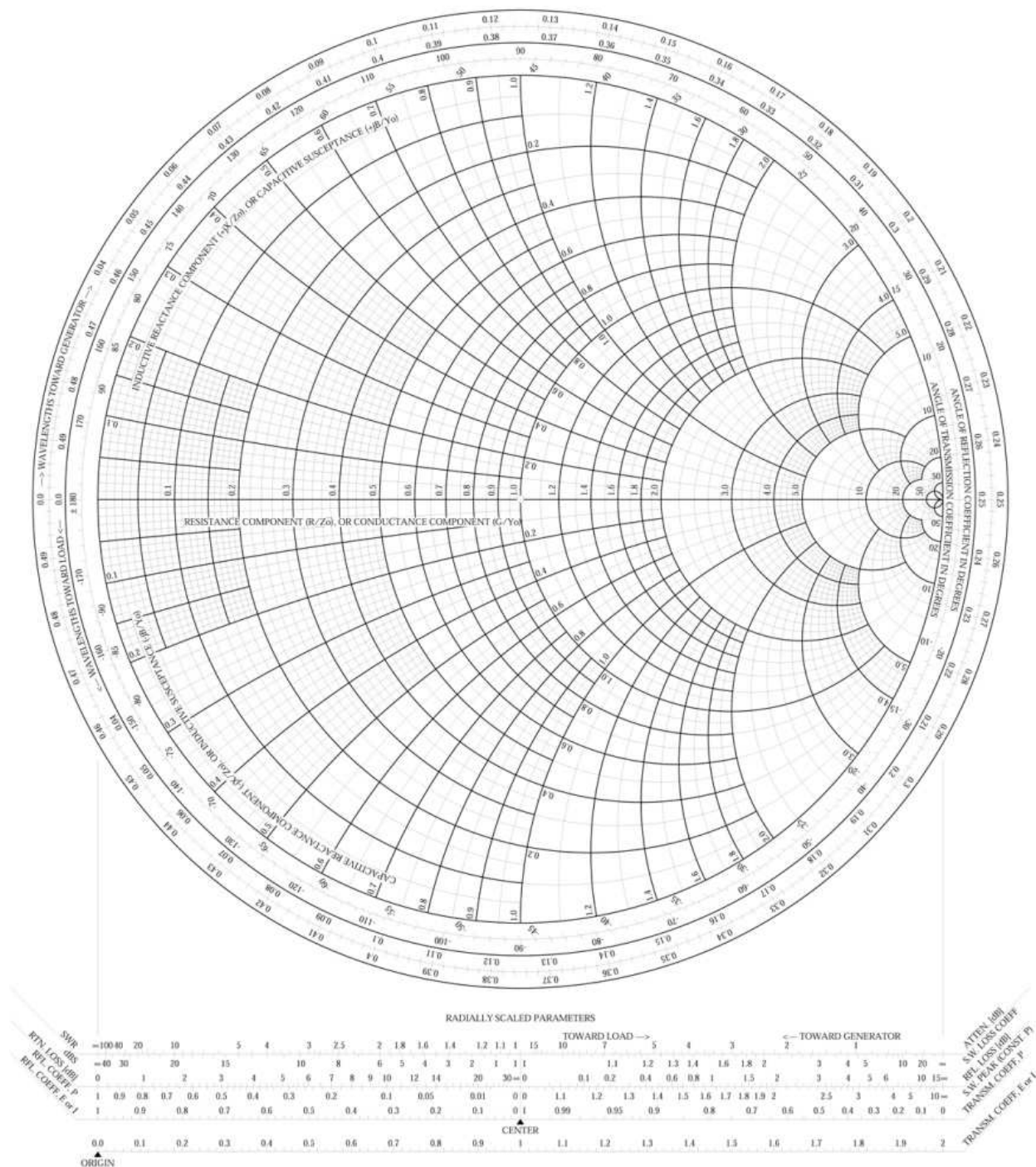## The Complete Smith Chart

Black Magic Design

Fig. 4. Typical Smith chart. With permission of Spread Spectrum Scene, http://www.sss-mag.com/pdf/smithchart.pdf.

The Smith chart simplifies transmission line analysis, and is still used today in most modern textbooks and courses in electrical engineering. It promotes a better understanding of the problem being solved. And such an understanding might be relevant for the interpretation

of the simulation results given by commercial software about antennas and microwave devices. Most modern computer based automatic network analyzers rely on the Smith chart for data display.

This chart is a unique diagram which has been used nearly for ninety years and we believe that it will be in use for many years to come not only as a pedagogically perfect analogue display, but also as an aid to professionals in obtaining quick answers to many line problems which they meet.

4. Getting started

As pointed out above, the aim of this chapter is to help the reader understand and learn how to use the Smith chart describing step by step procedures based in *MATLAB* scripts that should teach the students to solve different kinds of transmission line problems by themselves in a paper chart using a pencil, a ruler and a compass. One of the first things they should know how to do, is how to mark a given reflection coefficient in a chart and read related data associated to this reflection coefficient, such the transmission coefficient, the normalized impedance, the normalized admittance and the voltage standing wave ratio. The authors developed a *MATLAB* script called *SmithChart_InputRho_Eng_FV.m* that display, step by step, how to do this exercise.

The graphical solution given by this script, is shown in Figure 5, for a reflection coefficient $\rho = 0.6\angle120^{\circ}$.

Figure 5a) shows the first 2 steps:
1.   Marking ρ_L, from the amplitude and phase;
2.   Drawing the ρ_L constant circle;

Figure 5b) shows the last 3 steps:
3.   Getting the transmission coefficient $\rho_t = 1 + \rho_L = |\rho_t|e^{j\psi}$;
4.   Getting the normalized impedance z_L;
5.   Getting the normalized admittance, by inverting z_L to y_L;
6.   Getting the associated SWR.

Another basic thing students should know how to do, is to mark a given normalized impedance in a chart and read related data associated to it, such as the reflection coefficient, the normalized admittance and the voltage standing wave ratio. The authors developed a *MATLAB* script called *SmithChart_InputZ_Eng_FV.m* that displays step by step, how to do this exercise.

The graphical solution given by this script, is shown in Figure 6, for a normalized impedance $z_L = 0.3 + j0.5$.

Figure 6a) shows the first 2 steps:
1.   Highlighting the curves r_L and x_L;
2.   Marking the normalized impedance z_L;

Figure 6b) shows the last 3 steps:
3.   Getting the normalized admittance, by inverting z_L to y_L;
4.   Getting the corresponding reflection coefficient $\rho_L = |\rho_L|e^{j\varphi}$;
5.   Getting the corresponding transmission coefficient $\rho_t = 1 + \rho_L = |\rho_t|e^{j\psi}$;
6.   Getting the associated SWR.

The given reflection coefficient is:
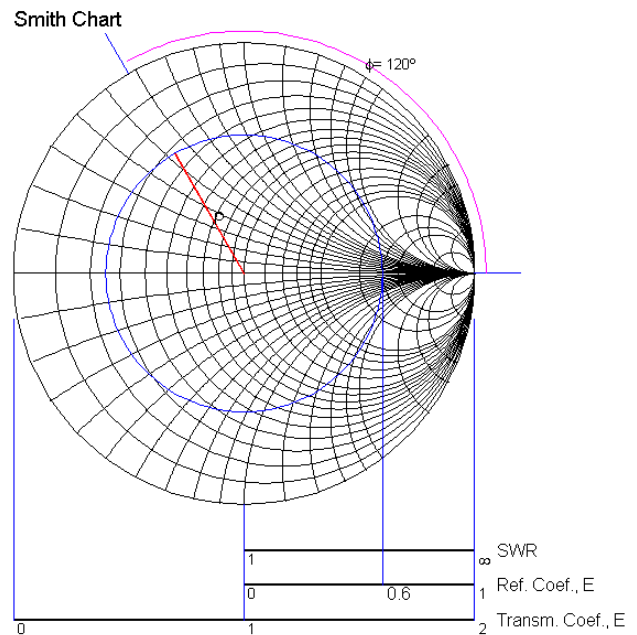
$\rho = -0.3 + j\,0.51962;$ $\rho = 0.6\ /\!\!\!\underline{\ \ }\ 120\,°$

a)

The given reflection coefficient is:

$\rho = -0.3 + j\,0.51962;$ $\rho = 0.6\ /\!\!\!\underline{\ \ }\ 120\,°$

From the chart some results associated to this reflection coefficient can be obtained:

Transmission coefficient

$\rho_t = 0.7 + j\,0.51962;$ $\rho_t = 0.87178\ /\!\!\!\underline{\ \ }\ 36.5868\,°$

Normalized impedance

$z_L = 0.32653 + j\,0.53022;$ $z_L = 0.6227\ /\!\!\!\underline{\ \ }\ 58.3736\,°$

Normalized admitance

$y_L = 0.84211 - j\,1.3674;$ $y_L = 1.6059\ /\!\!\!\underline{\ \ }\ -58.3736\,°$

Voltage Standing Wave Ratio

SWR= 4
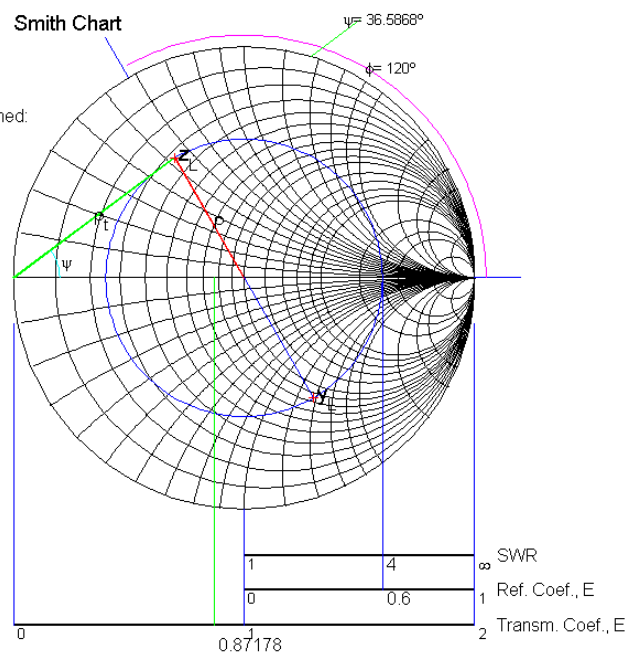
b)

Fig. 5. Inputting a reflection coefficient. Display given by *SmithChart_InputRho_Eng_FV.m.*

The normalized load impedance is:

$z_L = 0.3 + j\,0.5;$ $\qquad z_L = 0.5831\ /\!\!\underline{\ \ 59.0362}\,°$

Smith Chart
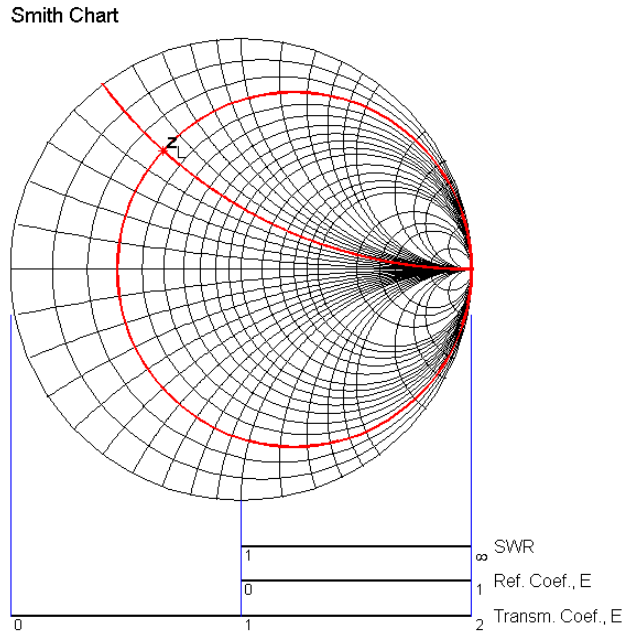
a)

The normalized load impedance is:

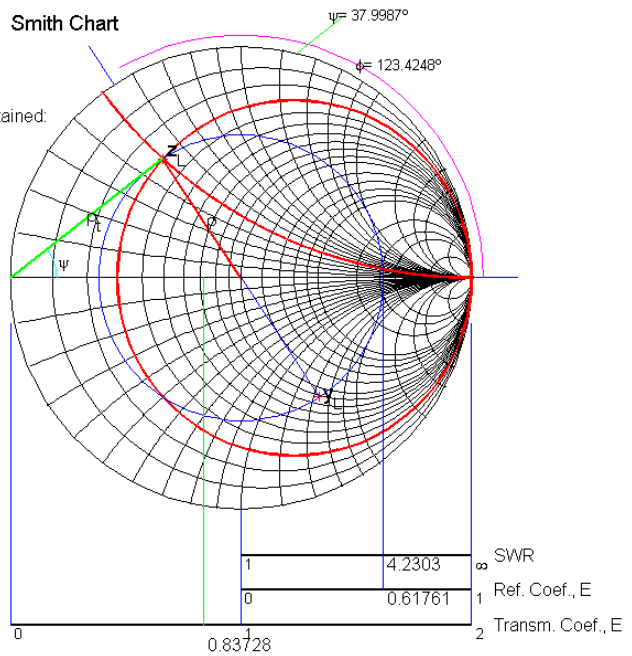$z_L = 0.3 + j\,0.5;$ $\qquad z_L = 0.5831\ /\!\!\underline{\ \ 59.0362}\,°$

From the chart some results associated to this normalized impedance can be obtained:

Normalized admitance

$y_L = 0.88235 - j\,1.4706;$ $\qquad y_L = 1.715\ /\!\!\underline{\ \ -59.0362}\,°$

Reflection Coefficient

$\rho = -0.34021 + j\,0.51546;$ $\quad \rho = 0.61761\ /\!\!\underline{\ \ 123.4248}\,°$

Transmission coefficient

$\rho_t = 0.65979 + j\,0.51546;$ $\quad \rho_t = 0.83728\ /\!\!\underline{\ \ 37.9987}\,°$

Voltage Standing Wave Ratio

SWR= 4.2303

Smith Chart

b)

Fig. 6. Inputting a normalized impedance. Display given by the script *SmithChart_InputZ_Eng_FV.m*.

It is also important to know how to locate the voltage maxima and minima along the line, given a normalized impedance using the chart. The *MATLAB* script called *SmithChart_InputZ_FindVmin_FindVmax_Eng_FV.m* displays, step by step, how to locate the first voltage maximum and minimum.

By definition, the reflection coefficient is the ratio of the phasors of the reverse and forward voltage waves. A voltage maximum occurs when these waves are in phase adding together constructively. If these waves are in opposite phase, a voltage minimum results. Therefore, travelling along the line, from the load toward the generator, if the right horizontal axis is reached first this corresponds to a reflection coefficient with a 0° phase, which means a voltage maximum. If, however the left horizontal axis is reached first, this corresponds to a reflection coefficient with a 180° phase, which means a voltage minimum.

The graphical solution given by this script, is shown in Figure 7, for the normalized impedance $z_L = 1.4 + j1.6$.
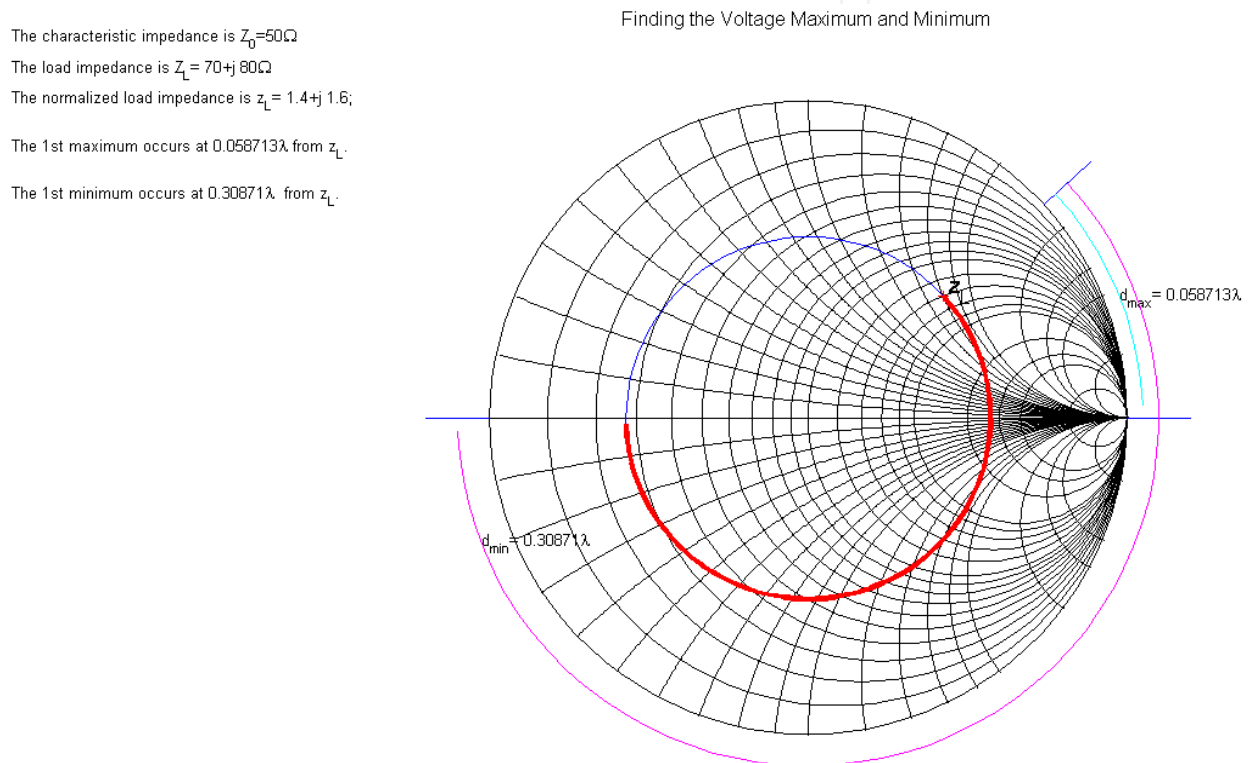

Fig. 7. Display given by SmithChart_InputZ_FindVmin_FindVmax_Eng_FV.m.

From Figure 7, one can perceive that travelling along the line, from the load toward the generator, the first particular point is a voltage maximum that occurs at a distance d=0.0587λ. This happens because the load is inductive. Continuing to travel along the line, after λ/4 a voltage minimum is found. As it is well known, the maxima are separated by λ/2. The same applies to the minima. This implies that a minimum is separated from the consecutive maxima by λ/4.

For a lossless line the absolute value of the reflection coefficient remains constant along the line, however its phase changes and therefore the impedance along the line also changes. Since a complete turn on the chart corresponds to travel λ/2, this means that after traveling λ/2, from the load to the generator, the load point is reached. This means that the impedance at a distance of λ/2 from the load is equal to the load. Therefore, for a lossless line, all the points separated by λ/2 have the same characteristics.

The authors developed a *MATLAB* script called *LosslessLine_Eng_FV.m* that graphically illustrates how the impedance along the line changes, as shown in figure 8. In this example a lossless line 0.35λ long with a characteristic impedance Z_0=50Ω, terminated with the load Z_L=100+j60 Ω, has an input impedance Z_{in}=21.88+j17.43 Ω. It is important to note the changes in nature of the impedance along the line when one moves from the load to the generator. At the load the impedance is inductive, then became real greater than Z_0. After that, and for the next λ/4, became capacitive and then again inductive until it reached the generator plane.

The load impedance is Z_L = 100 + j50 Ω
The SWR at the load is 2.618

The input impedance is Z_{in} = 21.8862 + j17.4334 Ω
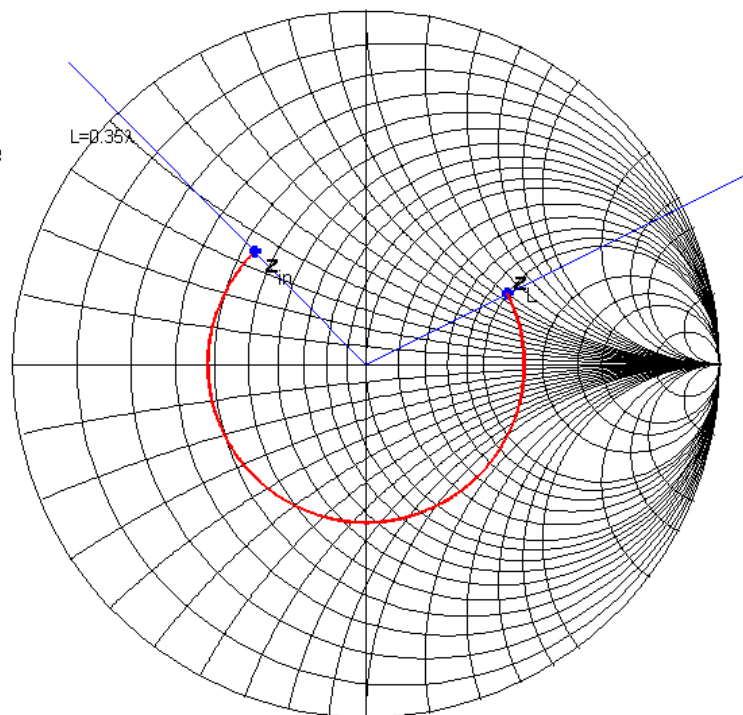The SWR at the input of the line is 2.618

L=0.35λ

Fig. 8. Impedance variation along a lossless line 0.35λ long. Graphical solution given by the *LosslessLine_Eng_FV.m* script.

5. Applications examples

In this section, some examples that integrate transmission line concepts are presented. The following examples are explained: (1) Single stub matching, (2) Bandwidth of a single-stub impedance matching system, (3) Quarter wavelength impedance matching, (4) Analysis of lossy lines. The authors developed *MATLAB* scripts that display, step by step, the graphical procedures used to solve these problems.

An example illustrating the double-stub impedance matching problem was also developed by the authors, (Pereira & Pinho, 2010).

5.1 Single-stub impedance matching

In any Transmission Line course, the concept of impedance matching is a topic that must be addressed. In transmission line context, impedance matching occurs when the characteristic impedance Z_0 of the line is equal to the load impedance Z_L. When this happens, the

characteristic impedance and load impedance are said to be matched. In this situation, the reflection coefficient is zero and no standing waves exist. In transmission line applications, it is desirable to achieve the matching condition.

There are several methods to achieve impedance matching. One of the simplest methods to match a transmission line to a given load is to connected a reactive element in parallel with the line at a point where the real part of the line admittance is equal to the characteristic admittance. This reactive element can be realized by a short piece of line, called stub. That is why this method is known by the single-stub impedance matching.

Although the location of the stub and its length can be found analytically using a computer or even a calculator, the authors believe that the use of the Smith chart to solve graphically this problem will give to the undergraduate students a much better insight of the aspects involved in this problem. The authors developed a *MATLAB* script called *SingleStubMatching_Eng_FV.m* that displays step by step, this graphical procedure. This script is available for download and should be used when reading this section.

The basic layout of the single-stub impedance matching is illustrated in Figure 9. The parameters to be evaluated are the distance *d*, measured from the load, at which the stub must be placed and, the stub length *Ls*. The stub is connected in parallel with the line. The stub can be short-circuited terminated or open-circuited terminated. Since the stub is connected in parallel with the line, the solution of this problem must be approached in terms of admittance. If the load is inputted as an impedance, then it is necessary to transform it in an admittance, using the graphical procedure explained before.
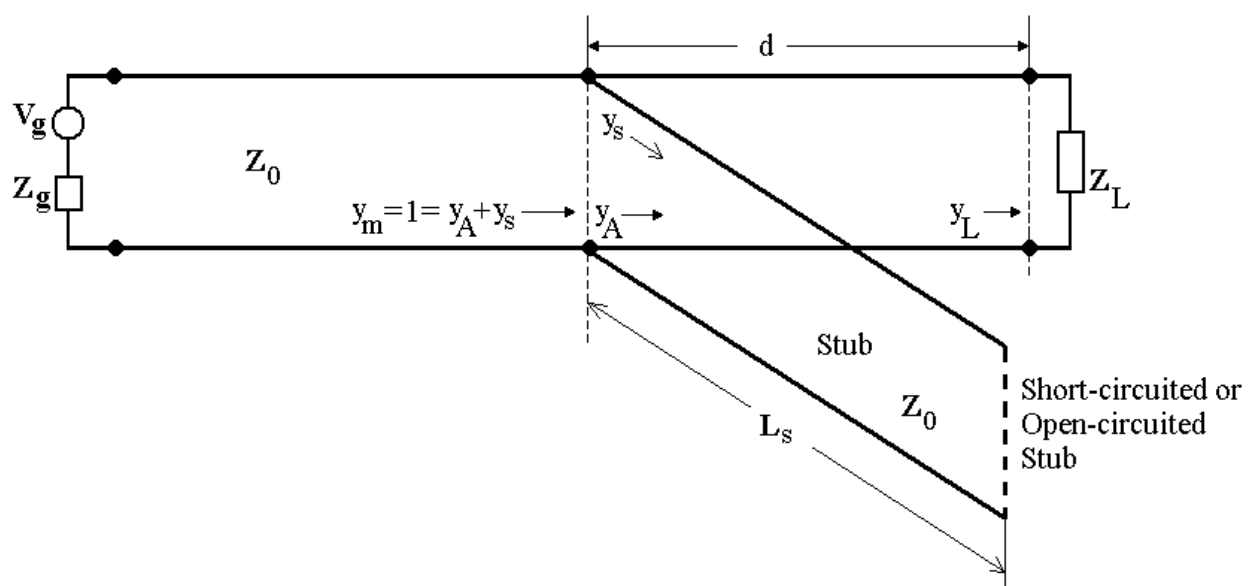

Fig. 9. Basic layout of the single-stub impedance matching.

The goal is to match the load Z_L to the line with characteristic impedance Z_0. Therefore the normalized admittance y_m, at the generator side of the stub, must be equal 1. On the other hand, this normalized admittance is equal to the sum of y_A and y_s. Since by definition, assuming lossless transmission lines, the input admittance of a stub has no real part, that is, $y_s = \pm j b_s$, that implies that the admittance y_A must be $y_A = 1 \pm j b_A$. Furthermore, the value of b_s must be the symmetrical of b_A in order to cancel each other out.

5.1.1 Graphical procedure

As explained above, the admittance at the load side of the stub must be $y_A = 1 \pm jb_A$. This means that the locus of y_A in the Smith chart is the $g = 1$ circle. Since lossless transmission lines are assumed, when travelling from the load toward the generator the absolute value of the reflection coefficient remains constant, that is, $|\rho| = |\rho_L|$.

Therefore, the possible values of y_A must be given by the intersection of the two circles: the $g = 1$ circle, and the ρ_L constant circle. Except for loads with real part equal to zero, there are two intersection points and therefore two solutions.

The solutions are found starting at the value of y_L in the Smith chart and moving toward the generator (clockwise), along the corresponding constant ρ_L circle, until the intersection with the $g = 1$ circle is obtained.

After choosing a solution for y_A, since $y_A = 1 \pm jb_A$ it is possible to get the corresponding value of $y_s = \mp jb_s = \pm jb_A$.

To determine the length of a short-circuited terminated stub, with an input admittance y_s, one should move, from the admittance SC point in the Smith chart, toward the generator (clockwise) to the point corresponding to the input admittance, along the $g = 0$ circle and read the required distance in wavelengths. To determine the length of an open-circuited terminated stub, with an input admittance y_s, one should move from the admittance OC point in the Smith chart, toward the generator (clockwise) to the point corresponding to the input admittance, along the $g = 0$ circle and read the required distance in wavelengths.

The authors developed a *MATLAB* script called *SingleStubMatching_Eng_FV.m* that displays step by step, the graphical procedure described above. Four cases are studied:

- Intersection with the upper half of the $g = 1$ circle and a short-circuited stub;
- Intersection with the lower half of the $g = 1$ circle, and a short-circuited stub;
- Intersection with the upper half of the $g = 1$ circle, and a open-circuited stub;
- Intersection with the lower half of the $g = 1$ circle, and a open-circuited stub;

The graphical solution given by this script for the first case, is shown in Figure 10, for a line with a characteristic impedance Z_0=50Ω and the load Z_L=100+j60 Ω.

Figure 10a) shows the first 7 steps:

1. Marking the normalized impedance z_L;
2. Drawing the ρ_L constant circle;
3. Transforming the normalized impedance in admittance, by inverting z_L to y_L;
4. Drawing the $g = 1$ constant circle;
5. Choosing one of the intersection points of the ρ_L constant circle with the $g = 1$ circle, (point A);
6. Finding the admittance y_A from the chosen intersection point;
7. Finding the distance *d*, in wavelengths, moving from y_L, toward the generator (clockwise) along the ρ_L constant circle, until y_A.

Figure 10b) shows the last 2 steps:

8. Getting the value of y_s from the value of y_A;
9. Determining the length *Ls* of the stub, in wavelengths, moving, from the admittance SC point in the Smith chart, toward the generator (clockwise) along the $g = 0$ circle to the point corresponding to the input admittance of the stub (point B**).**

One stub matching using a shorted terminated stub - 1th Solution

$d_1 = 0.20459\lambda$

$y_L = 0.36765 - j0.22059$
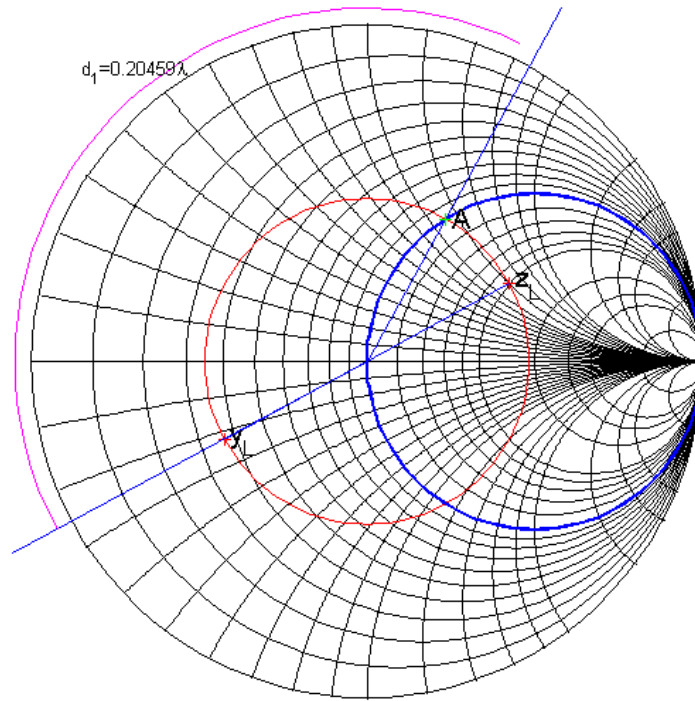
$y_A = 1 + j1.1045$

a)

One stub matching using a shorted terminated stub - 1th Solution

$d_1 = 0.20459\lambda$

$y_L = 0.36765 - j0.22059$

$y_A = 1 + j1.1045$

$y_s = -j1.1045$

S.C.

$L_{s1} = 0.1171\lambda$
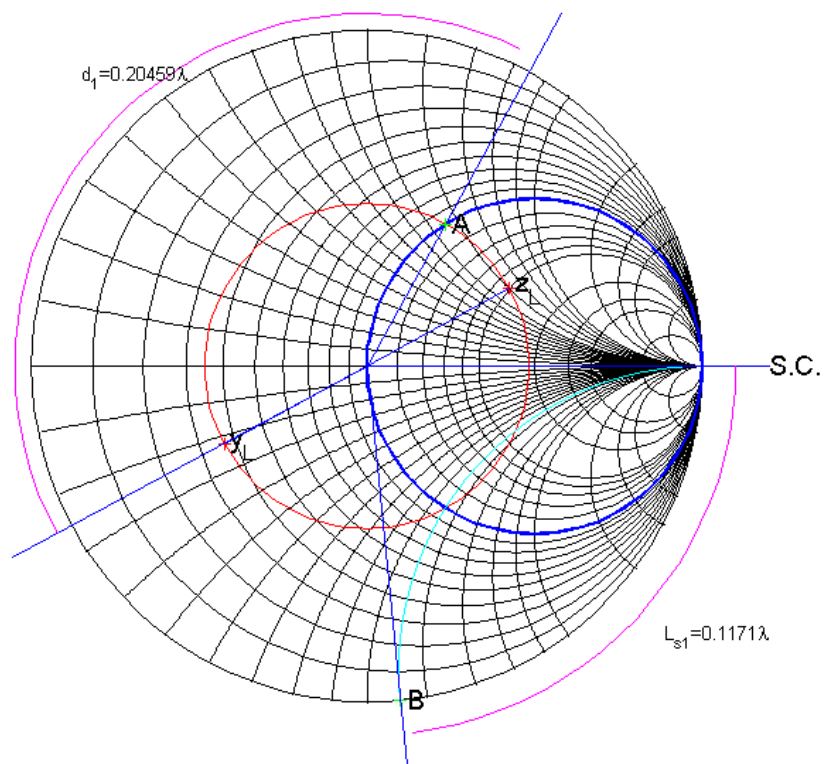
b)

Fig. 10. Graphical solution given by the *SingleStubMatching_Eng_FV.m* script.

Following the display produced by this script, students should be able to solve in a paper chart any single-stub impedance matching problem using a ruler and a compass.

The other cases given by script *SingleStubMatching_Eng_FV.m* can be explored by the reader.

5.2 Bandwidth of a single-stub impedance matching system

Another important topic that should be explained to the undergraduate students is the concept of bandwidth of a system. The majority of the systems based in transmission lines are perfectly matched at just one frequency. However, since the system should be able to operate over a frequency band, it is important to find the frequency band for which the system is consider to be acceptable matched. This frequency band is called the bandwidth of the system. Usually the criterion used to consider a system matched is when the VSWR is less or equal to 2.

The lower and the upper frequencies of the bandwidth can be found analytically. However, the authors believe that using the Smith chart to find graphically the bandwidth, will give to the students a much better understanding of the effects on the admittance of the line, and therefore on the matching conditions, when the frequency changes. The authors developed two *MATLAB* scripts called *SingleStubMatching_Eng_BW_FV2a.m* and *SingleStubMatching_Eng_BW_FV2b.m* that displays graphically, step by step, the effects on the matching conditions, when the frequency changes. Both are for systems using short-circuited stubs. One of the scripts is for one of the two possible solutions and the other for the other solution. It is important for the students to verify that the bandwidth is not the same for the two solutions. These scripts is available for download and should be explored when reading this section.

The single-stub impedance matching system explained in the previous section, gives a perfect matching at one frequency. Once constructed, if the frequency changes so does the electric length of the distance *d* and length *Ls*, and therefore a perfect matching is no longer achieved.

It is important to study the evolution of the matching values for a single-stub matching system, when the frequency varies around the central value for which a perfect matching was achieved. To observe this evolution, the authors developed *MATLAB* scripts that graphically display this evolution and give the VSWR 2:1 bandwidth. After choosing short-circuited or open-circuited stubs, there are still two solutions for the single-stub matching system, the perfect matching can be achieved with two pairs of values of *d* and *Ls* and therefore two different evolution of the matching values are also obtained. The script called *SingleStubMatching_Eng_BW_FV2a.m* is intended for one of the solutions and script called *SingleStubMatching_Eng_BW_FV2b.m* is intended for the other. Both use short-circuited stubs. It is important to compare the VSWR 2:1 bandwidth obtained for both solutions.

Figure 11 shows the graphical evolution of the matching values given by the script *SingleStubMatching_Eng_BW_FV2a.m* for the example illustrated in Figure 10. In this script the chosen solution is the one that corresponds to the intersection with the upper half of the $g = 1$ circle.
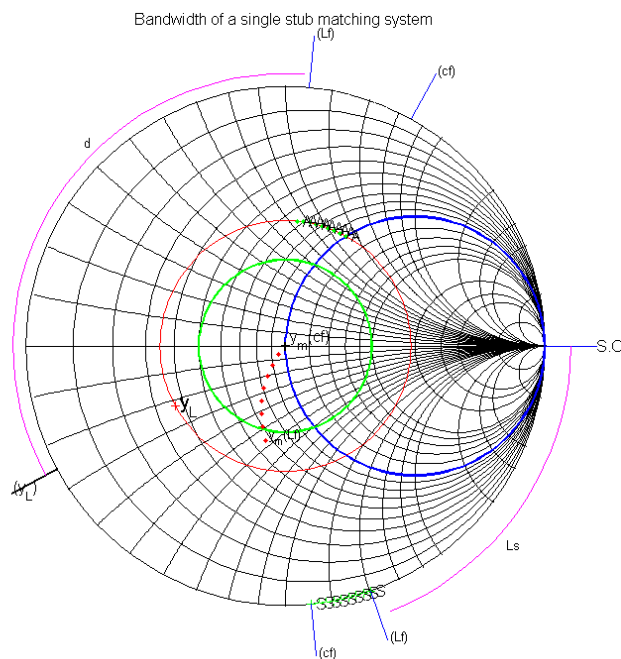
Figure 11a) shows the graphical evolution of the matching values when the frequency decreases. When the frequency decreases, then the electrical sizes of *d* and *Ls* also decreases. Point A moves toward the load and point S toward SC. y_m moves away from the center of chart (perfect matching).

Figure 11b) shows the graphical evolution of the matching values when the frequency increases. When the frequency increases, then the electrical sizes of *d* and *Ls* also increases.

Point A moves away from the load and point S away from SC. y_m moves away from the center of chart (perfect matching). With this solution a VSWR 2:1 bandwidth of 38% is obtained.

y_L=0.36765-j0.22059

y_A=1+j1.1045; y_s= -j1.1045 ==> y_m=y_A+y_s=1-j0

For the central frequency a perfect matching is achived.
The stub must be placed at a distance 0.20459λ from the load and its length should be 0.1171λ

If the frequency decreases, then the electrical sizes of d and Ls also decreases.
Point A moves towards Load and point S towards SC.
ym moves away from the center of chart (perfect matching)

Watch the chart...

f=0.98fc => y_{sn}= -j1.1377 y_m=yA_n+y_{sn}=0.94556-j0.065077   ==> SWR=1.0911

....

f=0.84fc => y_{sn}= -j1.4065 y_m=yA_n+y_{sn}=0.67-j0.56482   ==> SWR=2.18

a)

y_L=0.36765-j0.22059

y_A=1+j1.1045; y_s= -j1.1045 ==> y_m=y_A+y_s=1-j0

For the central frequency a perfect matching is achived.
The stub must be placed at a distance 0.20459λ from the load and its length should be 0.1171λ

If the frequency decreases, then the electrical sizes of d and Ls also decreases.
Point A moves towards Load and point S towards SC.
ym moves away from the center of chart (perfect matching)

Watch the chart...

f=0.98fc => y_{sn}= -j1.1377 y_m=yA_n+y_{sn}=0.94556-j0.065077   ==> SWR=1.0911

....

f=0.84fc => y_{sn}= -j1.4065 y_m=yA_n+y_{sn}=0.67-j0.56482   ==> SWR=2.18

If the frequency increases, then the electrical sizes of d and Ls also increases.
Point A moves away from the Load and point S away from SC.
ym moves away from the center of chart (perfect matching)

Watch the chart...

f=1.02fc => y_{sn}= -j1.0724 y_m=yA_n+y_{sn}=1.0593+j0.062868   ==> SWR=1.0876

....

f=1.22fc => y_{sn}= -j0.79741 y_m=yA_n+y_{sn}=1.9879+j0.40646   ==> SWR=2.0972

Uf=1.22fc   Lf=0.84fc
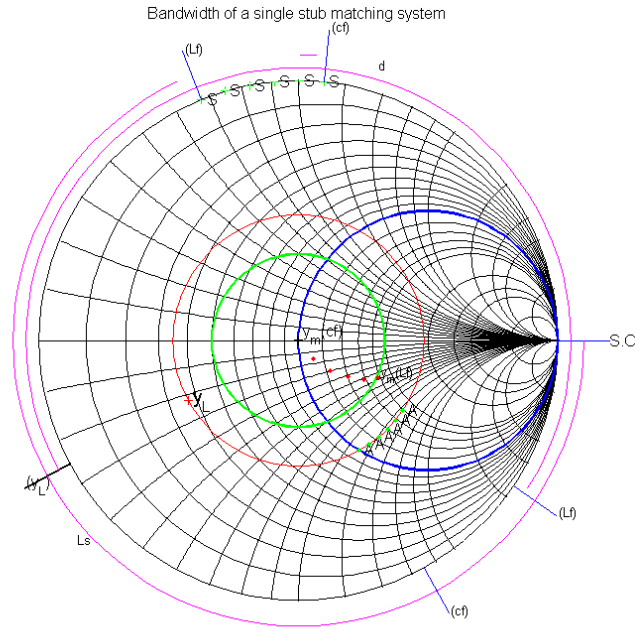BW=38 %   for the swr= 2   criterion
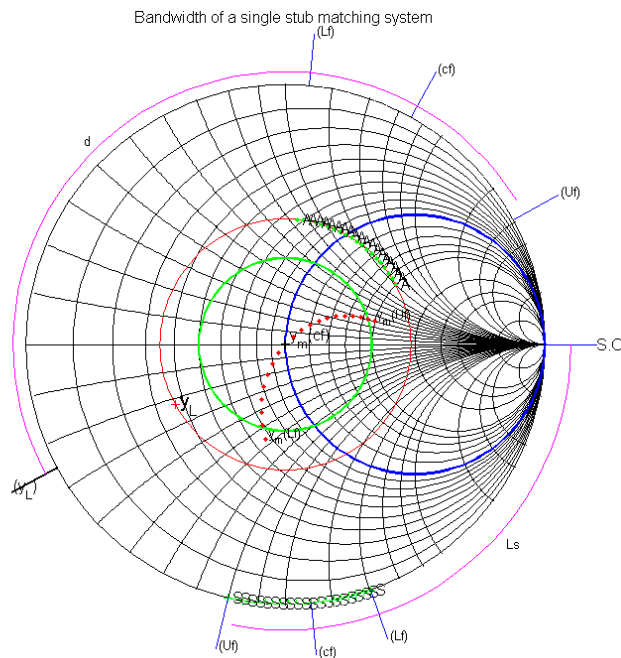
b)

Fig. 11. Graphical solution given by the *SingleStubMatching_Eng_BW_FV2a.m* script.

Figure 12 shows the graphical evolution of the matching values given by the script *SingleStubMatching_Eng_BW_FV2b.m* for the example illustrated in Figure 10. In this script

y_L=0.36765-j0.22059

y_A=1-j1.1045; y_s= +j1.1045 ==> y_m=y_A+y_s=1-j0

For the central frequency a perfect matching is achived.
The stub must be placed at a distance 0.37428λ from the load and its length should be 0.3829λ.

If the frequency decreases, then the electrical sizes of d and Ls also decreases.
Point A moves towards Load and point S towards SC.
ym moves away from the center of chart (perfect matching)

Watch the chart...

f=0.98fc => y_{sn}= j1.003 y_m=yA_n+y_{sn}=1.1124-j0.15649   ==> SWR=1.2001

....

f=0.9fc => y_{sn}= j0.67601 y_m=yA_n+y_{sn}=1.7678-j0.57586   ==> SWR=2.028

a)

y_L=0.36765-j0.22059

y_A=1-j1.1045; y_s= +j1.1045 ==> y_m=y_A+y_s=1-j0

For the central frequency a perfect matching is achived.
The stub must be placed at a distance 0.37428λ from the load and its length should be 0.3829λ.

If the frequency decreases, then the electrical sizes of d and Ls also decreases.
Point A moves towards Load and point S towards SC.
ym moves away from the center of chart (perfect matching)

Watch the chart...

f=0.98fc => y_{sn}= j1.003 y_m=yA_n+y_{sn}=1.1124-j0.15649   ==> SWR=1.2001

....

f=0.9fc => y_{sn}= j0.67601 y_m=yA_n+y_{sn}=1.7678-j0.57586   ==> SWR=2.028

If the frequency increases, then the electrical sizes of d and Ls also increases.
Point A moves away from the Load and point S away from SC.
ym moves away from the center of chart (perfect matching)

Watch the chart...

f=1.02fc => y_{sn}= j1.2174 y_m=yA_n+y_{sn}=0.9038+j0.1718   ==> SWR=1.2297

....

f=1.06fc => y_{sn}= j1.489 y_m=yA_n+y_{sn}=0.75091+j0.5648   ==> SWR=2.0099

Uf=1.06fc   Lf=0.9fc

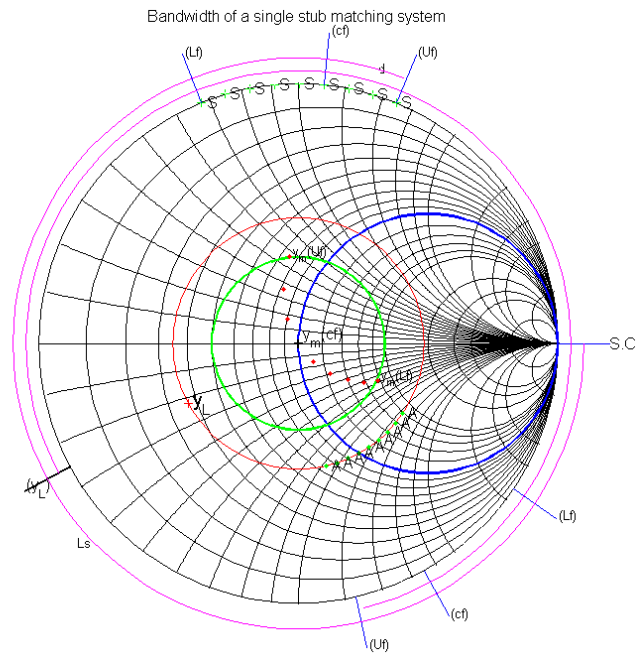BW=16 %   for the swr= 2   criterion

Fig. 12. Graphical solution given by the *SingleStubMatching_Eng_BW_FV2b.m* script.

the chosen solution is the one that corresponds to the intersection with the lower half of the *g* = 1 circle.
Figure 12a) shows the graphical evolution of the matching values when the frequency decreases. When the frequency decreases, the electrical sizes of *d* and *Ls* also decreases. Point A moves toward the load and point S toward SC. y_m moves away from the center of chart (perfect matching).
Figure 12b) shows the graphical evolution of the matching values when the frequency increases. When the frequency increases, the electrical sizes of *d* and *Ls* also increases. Point

A moves away from the load and point S away from SC. y_m moves away from the center of chart (perfect matching). With this solution a VSWR 2:1 bandwidth of 16% is obtained. So, as mentioned before, different bandwidth are obtained depending on the chosen for pair of values of d and Ls.

5.3 Quarter wavelength impedance matching system

As pointed out before, there are several methods to achieve impedance matching. One well known method consists in the insertion of a transmission line with a length of a quarter wavelength and an appropriate characteristic impedance, in a position where the impedance is real. This matching technique is known as a quarter wavelength impedance matching system or as the quarter wavelength transformer.

The characteristic impedance of this quarter wavelength transformer is given by $Z_1 = \sqrt{Z_{R1}Z_{R2}}$, where Z_{R1} is the impedance at the right side of the transformer and Z_{R2} is the impedance at the left side of the transformer. Since a common transmission line has a real characteristic impedance, Z_{R1} and Z_{R2} must both be real. If a line is terminated in a complex load, the quarter wavelength transformer cannot be inserted at the load plane. It is then necessary to move along the line, a distance d, toward the generator till a real impedance Z_{R1} is obtained. This is illustrated in Figure 13. Two values for Z_{R1} are possible. One greater than Z_0 and another less than Z_0, separated by $\lambda/4$. Once Z_{R1} is chosen, Z_1 can be calculated bearing in mind that Z_{R2} must be equal Z_0 to achieve a perfect matching.
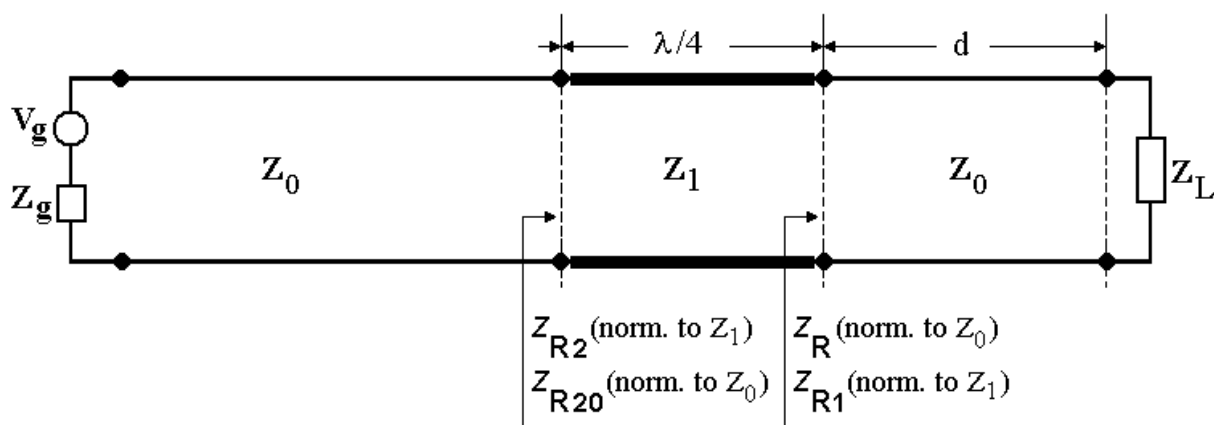

Fig. 13. Basic layout of the quarter wavelength impedance matching.

In general the students learn this method in a analytical way, by the direct computation of the required characteristic impedance of the quarter wavelength line and the location for its insertion. However, the authors believe that the use of the Smith chart to solve graphically this problem will give to the students a much better insight of the several impedance transformations involved in this problem to achieve an impedance equal to the characteristic impedance of the main line.

The authors developed a *MATLAB* script called *QuarterWavelengthTransformer_Eng_FV.m* that graphically explains, the quarter wavelength matching mechanism.

This is shown in Figure 14, for a line with a characteristic impedance Z_0=50Ω and a load Z_L=100+j60 Ω.

The characteristic impedance is Z_0=50Ω

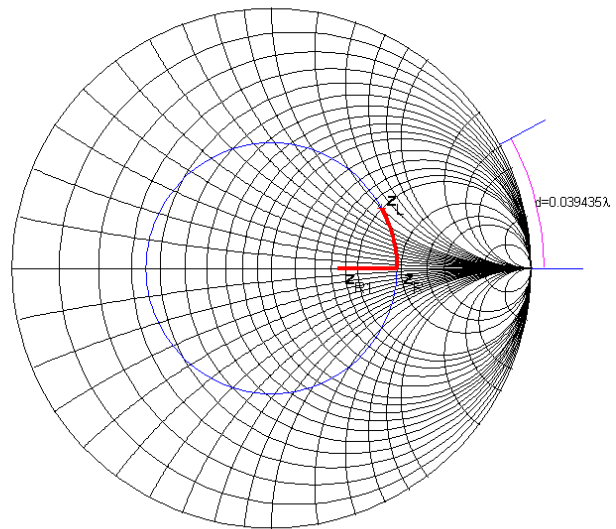The load impedance is Z_L=100+j60Ω

The normalized load impedance is z_L=2+j1.2;

After moving from z_L, towards the generator, the distance d, a real impedance Z_R= 143.5892 Ω is achived.
This means that the Quarter Wavelength Transformer must be placed at a distance 0.039435λ from the load.

The characteristic impedance of this Quarter Wavelength Transformer should be $Z_1=\sqrt{(Z_R*Z_0)}$=84.7317 Ω.

After obtaining Z_R, it is necessary to renormalize it in relation to the characteristic impedance Z_1,
The new normalized impedance is z_{R1} and its value is 1.6946.

How the Quarter Wavelength Transformer matching procedure works

d=0.039435λ

a)

The characteristic impedance is Z_0=50Ω

The load impedance is Z_L=100+j60Ω

The normalized load impedance is z_L=2+j1.2;

After moving from z_L, towards the generator, the distance d, a real impedance Z_R= 143.5892 Ω is achived.
This means that the Quarter Wavelength Transformer must be placed at a distance 0.039435λ from the load.

The characteristic impedance of this Quarter Wavelength Transformer should be $Z_1=\sqrt{(Z_R*Z_0)}$=84.7317 Ω.

After obtaining Z_R, it is necessary to renormalize it in relation to the characteristic impedance Z_1,
The new normalized impedance is z_{R1} and its value is 1.6946.

After obtaining z_{R1}, it is necessary to move a quarter wavelength from z_{R1}, towards the generator
to get the impedance z_{R2}.
The normalized impedance z_{R2} is= 0.5901.
After obtaining z_{R2}, it is necessary to denormalize it in relation to the characteristic impedance Z_1
in order to get the value of Z_{R2}. Its value is 50 Ω.
As we can verify, its value is equal to Z_0. So a perfect mathing is achieved as desired.

Normalizing Z_{R2} in relation to the characteristic Z_0, the center of the chart is reached
which means that a perfect matching is achieved.

How the Quarter Wavelength Transformer matching procedure works

d=0.039435λ
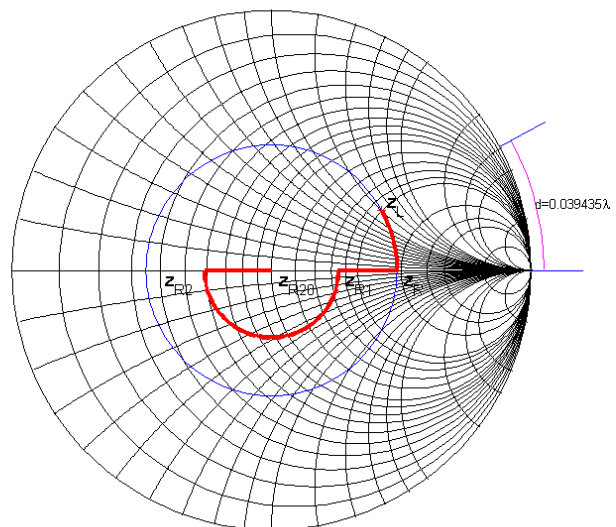
b)

Fig. 14. Graphical solution given by the *QuarterWavelengthTransformer_Eng_FV.m* script.

Figure 14a) shows the first 5 steps:

1. Marking the normalized impedance z_L;
2. Finding the distance d, in wavelengths, moving from z_L, toward the generator (clockwise) along the ρ_L constant circle, until the real normalized impedance z_R is obtained. In this example z_R greater than 1 was chosen;
3. Renormalize z_R impedance with reference to Z_0 in order to get Z_R;
4. Calculate $Z_1 = \sqrt{Z_R Z_0}$;
5. Normalize Z_R with reference to Z_1 obtaining z_{R1};

Figure 14b) shows the last 3 steps:

6. Moving from z_{R1}, toward the generator (clockwise) along the ρ constant circle, until the real normalized impedance z_{R2} is obtained;
7. Renormalizing z_{R2} impedance with reference to Z_1 an impedance $Z_{R2} = Z_0$ is obtained, meaning that a perfect matching is achieved;
8. Normalizing $Z_{R2} = Z_0$ with reference to Z_0 a normalized impedance of 1 is obtained, which means that the center of the chart is reached, confirming a perfect matching;

5.4 Analysis of lossy lines

In all the above examples, lossless transmission lines have been used. However all lines have some losses and this changes the results. One of the main influences of the losses is in the amplitude of the reflection coefficient and therefore in the impedance along the line. For a lossy line the reflection coefficient is given by equation 10.

$$\rho(d) = \rho_L\, e^{-2\alpha d}\, e^{-j2\beta d} \tag{10}$$

being d the distance measured from the load toward the generator, α the attenuation constant in Np/m and β the propagation constant in rad/m.

From equation 10, we notice that the phase changing of the reflection coefficient is equal to a lossless line, however the amplitude decreases from the load to the generator according with the equation 11.

$$\left|\rho(d)\right| = \left|\rho_L\right| e^{-2\alpha d} \tag{11}$$

Due to this amplitude decreasing, when travelling from the load to the generator the locus of ρ is a spiral approaching the center of the chart instead of a circle like in a lossless line. This means that at the input of a lossy line there is a better matching than at the load. Due to the loss of energy in the line, at the generator there is less returned energy to the generator and therefore a better matching. The authors developed a *MATLAB* script called *LossyLine_Eng_FV.m* that graphically explains these effects as illustrated in Figures 15 and 16. As shown in Figure 16 a long line terminated in a load with the high VSWR of 10.4, has at the input a very acceptable VSWR of 1.7.

Characteristics of the line
Characteristic impedance, Z_0= 300 Ω
Length, L= 2 m
Velocity factor, vf= 0.7

Frequency of operation, f= 145 MHz
Attenuation for this frequency, α = 1 dB/m
Equivalent to 1.4483 dB/λ
Length, L= 1.381λ
The total line attenuation is 2 dB

The load impedance is Z_L = 70 + j30 Ω
The SWR at the load is 4.331

The input impedance is Z_{in}= 183.9445 - j164.7848 Ω.
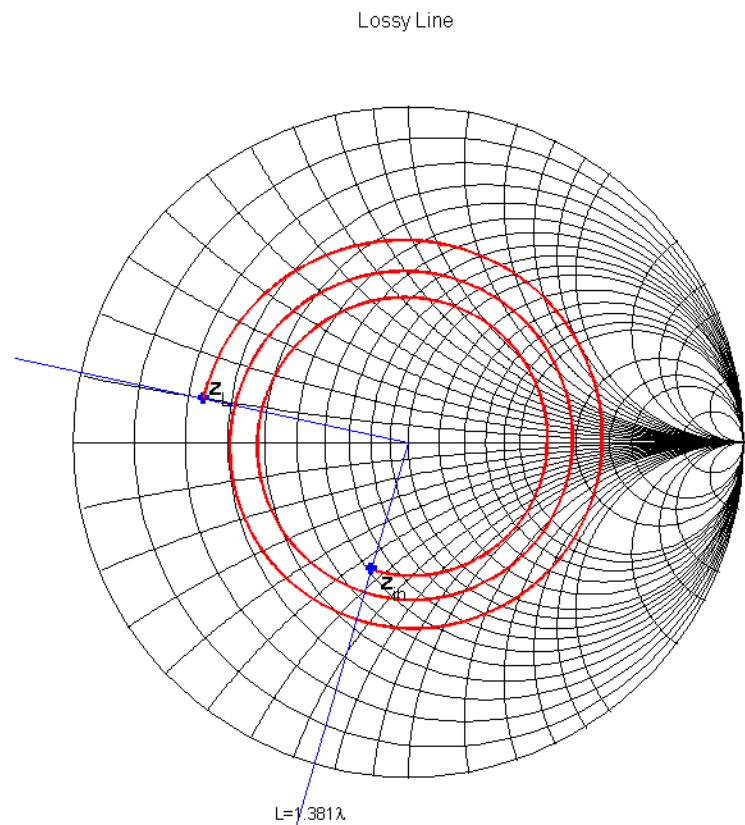The SWR at the input of the line is 2.3017

Fig. 15. Graphical solution given by the *LossyLine_Eng_FV.m* script for a short line.

Characteristics of the line
Characteristic impedance, Z_0= 50 Ω
Length, L= 5 m
Velocity factor, vf= 0.6

Frequency of operation, f= 400 MHz
Attenuation for this frequency, α = 1 dB/m
Equivalent to 0.45 dB/λ
Length, L= 11.1111λ
The total line attenuation is 5 dB

The load impedance is Z_L = 200 - j250 Ω
The SWR at the load is 10.4039

The input impedance is Z_{in}= 42.1828 - j23.5469 Ω.
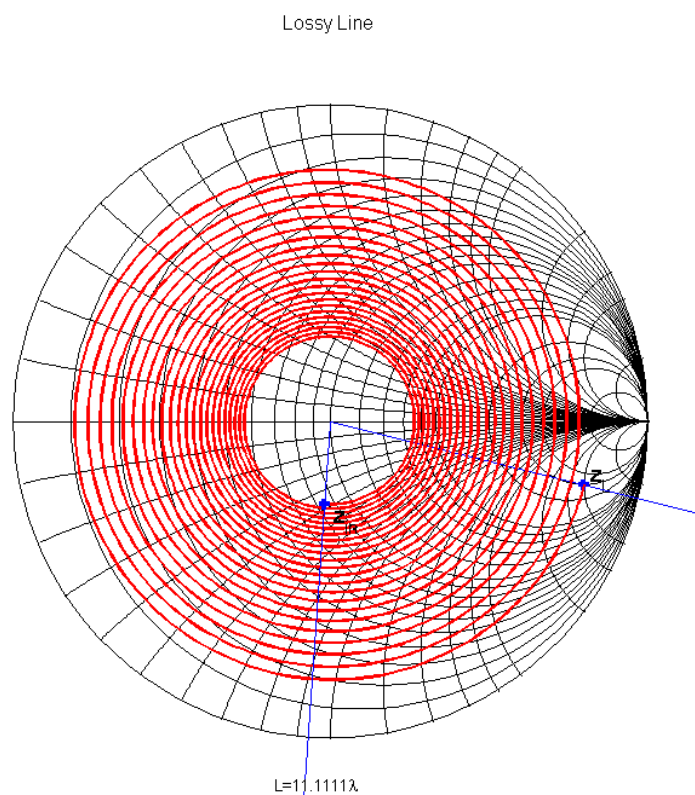The SWR at the input of the line is 1.7055

Fig. 16. Graphical solution given by the *LossyLine_Eng_FV.m* script for a long line.

If a long line is to be considered, it is important to know its attenuation constant and to evaluate its implications in the problem being considered.

6. Conclusion

It is well known that many transmission line problems can easily be solved using graphical procedures based on the Smith chart. The authors still believe that the use of the Smith chart by the students is an important pedagogical tool even knowing that personal computers and calculators are commonly available nowadays.

Since the main topic of this book is concerned with *e-learning*, the aim of this chapter is to help the reader understand and learn how to use the Smith chart following a step by step procedure based on *MATLAB* scripts, that should be used when reading this chapter. This approach should teach the students to solve several kinds of transmission line problems by themselves in a paper chart using a pencil, a ruler and a compass.

To exemplify this concept, the authors developed *MATLAB* scripts that display, step by step, the graphical procedure used in several applications. Using these scripts, many aspects of the transmission line theory such as: the voltage, current, impedance, Voltage Standing Wave Ratio (VSWR), reflection coefficient and matching design problems can be easily interpreted and well visualized using the Smith chart.

The chapter was organized as follows:

- History and use of the Smith chart and its importance in the resolution of classical transmission line problems.
- Construction of this chart from the basic equations and concepts.
- How to use the Smith chart.
- Presentation of some examples that integrates the transmission line concepts.

The authors developed a *MATLAB* scripts that display, step by step, the graphical procedure that must be used to solve these examples.

All the *MATLAB* scripts can be download from the link:

http://www.av.it.pt/rochap/MatlabScripts.zip

7. References

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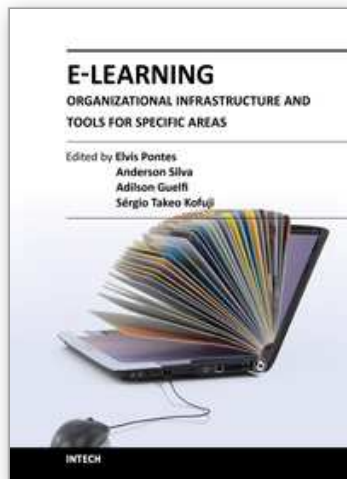
E-Learning-Organizational Infrastructure and Tools for Specific Areas

Edited by Prof. Adilson Guelfi

Technology development, mainly for telecommunications and computer systems, was a key factor for the interactivity and, thus, for the expansion of e-learning. This book is divided into two parts, presenting some proposals to deal with e-learning challenges, opening up a way of learning about and discussing new methodologies to increase the interaction level of classes and implementing technical tools for helping students to make better use of e-learning resources. In the first part, the reader may find chapters mentioning the required infrastructure for e-learning models and processes, organizational practices, suggestions, implementation of methods for assessing results, and case studies focused on pedagogical aspects that can be applied generically in different environments. The second part is related to tools that can be adopted by users such as graphical tools for engineering, mobile phone networks, and techniques to build robots, among others. Moreover, part two includes some chapters dedicated specifically to e-learning areas like engineering and architecture.

How to reference

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