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Electroweak Interactions in a Chiral Effective Lagrangian for Nuclei

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1. Introduction

The understanding of electroweak (EW) interactions in nuclei has played an important role in nuclear and particle physics. Previously, the electromagnetic (EM) interaction has provided valuable information about nuclear structure. On the other hand, weak interactions, which are intrinsically correlated with the EM interaction, can be complementary to the EM probe. Moreover, a good knowledge of (anti)neutrino–nucleus scattering cross sections is needed in other processes, including neutrino-oscillation experiments, neutrino astrophysics, and others.

To understand EW interactions in nuclei, we need to deal with the strong interaction that binds nucleons together. The fundamental theory of the strong interaction is quantum chromodynamics (QCD), which is a relativistic field theory with local gauge invariance, whose elementary constituents are colored quarks and gluons. In principle, QCD should provide a complete description of nuclear structure and dynamics. Unfortunately, QCD predictions at nuclear length scales with the precision of existing (and anticipated) experimental data are not available, and this state of affairs will probably persist for some time. Even if it becomes possible to use QCD to describe nuclei directly, this description is likely to be cumbersome and inefficient, since quarks cluster into hadrons at low energies.

How can we make progress towards understanding the EW interactions of nuclei? We will employ a framework based on Lorentz-covariant, effective quantum field theory and density functional theory. Effective field theory (EFT) embodies basic principles that are common to many areas of physics, such as the natural separation of length scales in the description of physical phenomena. In EFT, the long-range dynamics is included explicitly, while the short-range dynamics is parameterized generically; all of the dynamics is constrained by the symmetries of the interaction. When based on a local, Lorentz-invariant lagrangian (density), EFT is the most general way to parameterize observables consistent with the principles of quantum mechanics, special relativity, unitarity, gauge invariance, cluster decomposition, microscopic causality, and the required internal symmetries.

Covariant meson–baryon effective field theories of the nuclear many-body problem (often called quantum hadrodynamics or QHD) have been known for many years to provide a realistic description of the bulk properties of nuclear matter and heavy nuclei. [See Refs. (Furnstahl, 2003; Serot & Walecka, 1986; 1997; Serot, 2004), for example.] Some time

ago, a QHD effective field theory (EFT) was proposed (Furnstahl et al., 1997) that includes all of the relevant symmetries of the underlying QCD. In particular, the spontaneously broken $SU(2)_L \otimes SU(2)_R$ chiral symmetry is realized nonlinearly. The motivation for this EFT and illustrations of some calculated results are discussed in Refs. (Furnstahl et al., 1997; Hu et al., 2007; Huertas, 2002; 2003; 2004; McIntire et al., 2007; McIntire, 2008; Serot, 2007; 2010), for example. This QHD EFT has also been applied to a discussion of the isovector axial-vector current in nuclei (Ananyan et al., 2002).

This QHD EFT has three desirable features: (1) It uses the same degrees of freedom to describe the currents and the strong-interaction dynamics; (2) It respects the same internal symmetries, both discrete and continuous, as the underlying QCD; and (3) Its parameters can be calibrated using strong-interaction phenomena, like π N scattering and the properties of finite nuclei (as opposed to EW interactions with nuclei).

In this work, we focus on the introduction of EW interactions in the QHD EFT, with the Delta (1232) resonance (Δ) included as manifest degrees of freedom. To realize the symmetries of QCD in QHD EFT, including both chiral symmetry $SU(2)_L \otimes SU(2)_R$ and discrete symmetries, we apply the background-field technique (Gasser & Leutwyler, 1984; Serot, 2007). Based on the EW synthesis in the Standard Model, a proper substitution of background fields in terms of EW gauge bosons in the lagrangian, as constrained by the EW interactions of quarks (Donoghue et al., 1992), leads to EW interactions of hadrons at low energy. This lagrangian has a linear realization of the $SU(2)_V$ isospin symmetry and a nonlinear realization of the spontaneously broken $SU(2)_L \otimes SU(2)_R$ (modulo $SU(2)_V$) chiral symmetry (when the pion mass is zero). It was shown in Ref. (Furnstahl et al., 1997) that by using Georgi's naive dimensional analysis (NDA) (Georgi, 1993) and the assumption of naturalness (namely, that all appropriately defined, dimensionless couplings are of order unity), it is possible to truncate the lagrangian at terms involving only a few powers of the meson fields and their derivatives, at least for systems at normal nuclear densities (Müller & Serot, 1996). It was also shown that a mean-field approximation to the lagrangian could be interpreted in terms of density functional theory (Kohn, 1999; Müller & Serot, 1996; Serot & Walecka, 1997), so that calibrating the parameters to observed bulk and single-particle nuclear properties (approximately) incorporates many-body effects that go beyond Dirac–Hartree theory. Explicit calculations of closed-shell nuclei provided such a calibration and verified the naturalness assumption. This approach therefore embodies the three desirable features needed for a description of electroweak interactions in the nuclear many-body problem.

Moreover, the technical issues involving spin-3/2 degrees of freedom in relativistic quantum field theory are also discussed here (Krebs et al., 2010; Pascalutsa, 2008). Following the construction of the lagrangian, we apply it to calculate certain matrix elements to illustrate the consequences of chiral symmetries in this theory, including the conservation of vector current (CVC) and the partial conservation of axial-vector current (PCAC). To explore the discrete symmetries, we talk about the manifestation of G parity in these current matrix elements.

This chapter is organized as follows: After a short introduction, we discuss chiral symmetry and discrete symmetries in QCD in the framework of background fields. The EW interactions of quarks are also presented, and this indicates the relation between the EW bosons and background fields. Then we consider the nonlinear realization of chiral symmetry and other symmetries in QHD EFT, as well as the EW interactions. Following that, we outline the lagrangian with the Δ included. Subtleties concerning the number of degrees of freedom

and redundant interaction terms are discussed. Finally, some concrete calculations of matrix elements serve as examples and manifestations of symmetries in the theory. We also briefly touch on how this formalism can be used to study neutrino–nucleus scattering (Serot & Zhang, 2010; 2011a;b; Zhang, 2012).

2. QCD, symmetries, and electroweak synthesis

In this section, we talk about various symmetries in QCD including Lorentz-invariance, C, P, and T symmetries, and approximate $SU(2)_L \otimes SU(2)_R$ chiral symmetry (together with baryon number conservation). The last one is the major focus. Here, we consider only u and d quarks, and their antiquarks, while others are chiral singlets. Moreover, the EW interactions, realized in the electroweak synthesis of the Standard Model, are also discussed with limited scope.

2.1 Symmetries

To consider the symmetries, we apply the background-field technique (Gasser & Leutwyler, 1984). First we introduce background fields into the QCD lagrangian, including $\mathsf{v}^\mu \equiv \mathsf{v}^{i\mu}\tau_i/2$ (isovector vector), $\mathsf{v}^\mu_{(s)}$ (isoscalar vector), $\mathsf{a}^\mu \equiv \mathsf{a}^{i\mu}\tau_i/2$ (isovector axial-vector), $\mathsf{s} \equiv \mathsf{s}^i\tau_i/2$ (isovector scalar), and $\mathsf{p} \equiv \mathsf{p}^i\tau_i/2$ (isovector pseudoscalar), where $i = x, y, z$ or $+1, 0, -1$ (the convention about $i = \pm 1,\ 0$ will be shown in Sec. 3.1):

$$\mathcal{L} = \mathcal{L}_{QCD} + \overline{q}\gamma_\mu(\mathsf{v}^\mu + B\mathsf{v}^\mu_{(s)} + \gamma_5\mathsf{a}^\mu)q - \overline{q}(\mathsf{s} - i\gamma_5\mathsf{p})q$$

$$= \mathcal{L}_{QCD} + \overline{q}_L\gamma_\mu(l^\mu + B\mathsf{v}^\mu_{(s)})q_L + \overline{q}_R\gamma_\mu(r^\mu + B\mathsf{v}^\mu_{(s)})q_R$$

$$- \overline{q}_L(\mathsf{s} - i\mathsf{p})q_R - \overline{q}_R(\mathsf{s} + i\mathsf{p})q_L$$

$$\equiv \mathcal{L}_{QCD} + \mathcal{L}_{\text{ext}} . \tag{1}$$

Here, $r^\mu = \mathsf{v}^\mu + \mathsf{a}^\mu$, $l^\mu = \mathsf{v}^\mu - \mathsf{a}^\mu$, $q_L = \frac{1}{2}(1 - \gamma_5)q$, $q_R = \frac{1}{2}(1 + \gamma_5)q$, $q = (u,d)^T$ and $B = 1/3$ is the baryon number. To preserve C, P, and T invariance of $\mathcal{L}$, the change of background fields under these discrete symmetry transformations are determined by the the properties of the currents coupled to them. The details are in Tabs. 1 and 2. Inside the tables, $\mathcal{P}^\mu_\nu = \text{diag}(1, -1, -1, -1)_{\mu\nu}$ and $\mathcal{T}^\mu_\nu = \text{diag}(-1, 1, 1, 1)_{\mu\nu}$. Moreover, the Lorentz-invariance is manifest, considering the definition of these background fields.

	v^μ	$\mathsf{v}^\mu_{(s)}$	a^μ	s	p
C	$-\mathsf{v}^{T\mu}$	$-\mathsf{v}^\mu_{(s)}$	$\mathsf{a}^{T\mu}$	s^T	p^T
P	$\mathcal{P}^\mu_\nu \mathsf{v}^\nu$	$\mathcal{P}^\mu_\nu \mathsf{v}^\nu_{(s)}$	$-\mathcal{P}^\mu_\nu \mathsf{a}^\nu$	s	$-\mathsf{p}$
T	$-\mathcal{T}^\mu_\nu \mathsf{v}^\nu$	$-\mathcal{T}^\mu_\nu \mathsf{v}^\nu_{(s)}$	$-\mathcal{T}^\mu_\nu \mathsf{a}^\nu$	s	$-\mathsf{p}$

Table 1. Transformations of background fields under C, P, and T operations. The transformations of spacetime arguments are not shown here.

	r^μ	l^μ	$f_{R\mu\nu}$	$f_{L\mu\nu}$	$f_{s\mu\nu}$
C	$-l^{T\mu}$	$-r^{T\mu}$	$-f_{L\mu\nu}^T$	$-f_{R\mu\nu}^T$	$-f_{s\mu\nu}^T$
P	$\mathcal{P}_\nu^\mu l^\nu$	$\mathcal{P}_\nu^\mu r^\nu$	$\mathcal{P}_\mu^\lambda \mathcal{P}_\nu^\sigma f_{L\lambda\sigma}$	$\mathcal{P}_\mu^\lambda \mathcal{P}_\nu^\sigma f_{R\lambda\sigma}$	$\mathcal{P}_\mu^\lambda \mathcal{P}_\nu^\sigma f_{s\lambda\sigma}$
T	$-\mathcal{T}_\nu^\mu r^\nu$	$-\mathcal{T}_\nu^\mu l^\nu$	$-\mathcal{T}_\mu^\lambda \mathcal{T}_\nu^\sigma f_{R\lambda\sigma}$	$-\mathcal{T}_\mu^\lambda \mathcal{T}_\nu^\sigma f_{L\lambda\sigma}$	$-\mathcal{T}_\mu^\lambda \mathcal{T}_\nu^\sigma f_{s\lambda\sigma}$

Table 2. Continuation of Tab. 1.

To understand $SU(2)_L \otimes SU(2)_R \otimes U(1)_B$ symmetry, we can see that the $\mathcal{L}$ defined in Eq. (1) has this symmetry with the following *local* transformation rules:

$$q_{LA} \to \exp\left[-i\frac{\theta(x)}{3}\right] \left(\exp\left[-i\theta_{Li}(x)\frac{\tau^i}{2}\right]\right)_A^B q_{LB} \equiv \exp\left[-i\frac{\theta(x)}{3}\right](L)_A^B q_{LB} , \qquad (2)$$

$$q_R \to \exp\left[-i\frac{\theta(x)}{3}\right] \exp\left[-i\theta_{Ri}(x)\frac{\tau^i}{2}\right] q_R \equiv \exp\left[-i\frac{\theta(x)}{3}\right] R q_R , \qquad (3)$$

$$l^\mu \to L\, l^\mu L^\dagger + iL\, \partial^\mu L^\dagger , \qquad (4)$$

$$r^\mu \to R\, r^\mu R^\dagger + iR\, \partial^\mu R^\dagger , \qquad (5)$$

$$\mathsf{v}_{(s)}^\mu \to \mathsf{v}_{(s)}^\mu - \partial^\mu\theta , \qquad (6)$$

$$\mathsf{s} + i\mathsf{p} \to R(\mathsf{s}+i\mathsf{p})L^\dagger . \qquad (7)$$

We can also construct field strength tensors that transform homogeneously:

$$f_{L\mu\nu} \equiv \partial_\mu l_\nu - \partial_\nu l_\mu - i\left[l_\mu, l_\nu\right] \to L f_{L\mu\nu} L^\dagger , \qquad (8)$$

$$f_{R\mu\nu} \equiv \partial_\mu r_\nu - \partial_\nu r_\mu - i\left[r_\mu, r_\nu\right] \to R f_{R\mu\nu} R^\dagger , \qquad (9)$$

$$f_{s\mu\nu} \equiv \partial_\mu \mathsf{v}_{(s)\nu} - \partial_\nu \mathsf{v}_{(s)\mu} \to f_{s\mu\nu} . \qquad (10)$$

2.2 Electroweak synthesis

Now we can discuss the electroweak synthesis ($SU_L(2) \otimes U_Y(1)$) of the Standard Model, which is mostly summarized in Tab. 3 (electric charge $Q = Y/2 + T_L^3$) (Donoghue et al., 1992; Itzykson & Zuber, 1980). We ignore the Higgs fluctuations and gauge boson self-interactions:

$$\begin{aligned}
\mathcal{L}_I &= -\overline{q_L}\gamma^\mu(g\frac{\tau_i}{2}W_\mu^i + g'\frac{Y}{2}B_\mu)q_L - \overline{q_R}\gamma^\mu(g'\frac{Y}{2}B_\mu)q_R \\
&= -\overline{q_L}\gamma^\mu g(\frac{\tau_{+1}}{2}W_\mu^{+1} + \frac{\tau_{-1}}{2}W_\mu^{-1})q_L - \overline{q_L}\gamma^\mu(g\frac{\tau_0}{2}W_\mu^0 + g'\frac{Y}{2}B_\mu)q_L \\
&\quad - \overline{q_R}\gamma^\mu(g'\frac{Y}{2}B_\mu)q_R .
\end{aligned} \qquad (11)$$

	T_L	T_L^3	Q	Y	B
u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
u_R	0	0	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{1}{3}$
d_R	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$

Table 3. Multiplets in electroweak synthesis.

Here g, g' and e are the $SU(2)_L$, $U(1)_Y$ and $U(1)_{EM}$ charges. To make sure that $U_{EM}(1)$ is preserved, we impose the following redefinition of excitations relative to the vacuum (θ_w is the weak mixing angle):

$$B^\mu = \cos\theta_w A^\mu - \sin\theta_w Z^\mu \,, \tag{12}$$

$$W^{0\mu} = \cos\theta_w Z^\mu + \sin\theta_w A^\mu \,, \tag{13}$$

$$g\sin\theta_w = g'\cos\theta_w \equiv e \,. \tag{14}$$

After substituting Eqs. (12) to (14) into Eq. (11), we have the right coupling for the EM interaction. Let's compare Eq. (11) with Eq. (1); we deduce the following (V_{ud} describes u and d mixing):

$$l_\mu = -e\frac{\tau^0}{2}A_\mu + \frac{g}{\cos\theta_w}\sin^2\theta_w\frac{\tau^0}{2}Z_\mu$$
$$- \frac{g}{\cos\theta_w}\frac{\tau^0}{2}Z_\mu - gV_{ud}\left(W_\mu^{+1}\frac{\tau_{+1}}{2} + W_\mu^{-1}\frac{\tau_{-1}}{2}\right) \,, \tag{15}$$

$$r_\mu = -e\frac{\tau^0}{2}A_\mu + \frac{g}{\cos\theta_w}\sin^2\theta_w\frac{\tau^0}{2}Z_\mu \,, \tag{16}$$

$$\mathsf{v}_{(s)\mu} = -e\frac{1}{2}A_\mu + \frac{g}{\cos\theta_w}\sin^2\theta_w\frac{1}{2}Z_\mu \,. \tag{17}$$

Furthermore,

$$f_{L\mu\nu} = -e\frac{\tau^0}{2}A_{[\nu,\mu]} + \frac{g}{\cos\theta_w}\sin^2\theta_w\frac{\tau^0}{2}Z_{[\nu,\mu]} - \frac{g}{\cos\theta_w}\frac{\tau^0}{2}Z_{[\nu,\mu]}$$
$$- gV_{ud}\frac{\tau^{+1}}{2}W_{+1[\nu,\mu]} - gV_{ud}\frac{\tau^{-1}}{2}W_{-1[\nu,\mu]}$$
$$+ \text{ interference terms including }(WZ),(WA),(WW),\text{ but no }(ZA)\,, \tag{18}$$

$$f_{R\mu\nu} = -e\frac{\tau^0}{2}A_{[\nu,\mu]} + \frac{g}{\cos\theta_w}\sin^2\theta_w\frac{\tau^0}{2}Z_{[\nu,\mu]} \quad \text{(no interference terms)}\,, \tag{19}$$

$$f_{s\mu\nu} = -e\frac{1}{2}A_{[\nu,\mu]} + \frac{g}{\cos\theta_w}\sin^2\theta_w\frac{1}{2}Z_{[\nu,\mu]}\,. \tag{20}$$

Here $A_{[\nu,\mu]} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and so are the indices of other fields. If we define [see Eq. (1)]

$$\mathcal{L}_{\text{ext}} \equiv \mathsf{v}_{i\mu} V^{i\mu} - \mathsf{a}_{i\mu} A^{i\mu} + \mathsf{v}_{(s)\mu} J^{B\mu}$$

$$= J^L_{i\mu} l^{i\mu} + J^R_{i\mu} r^{i\mu} + \mathsf{v}_{(s)\mu} J^{B\mu} , \tag{21}$$

$$\mathcal{L}_I = -e J^{EM}_\mu A^\mu - \frac{g}{\cos\theta_w} J^{NC}_\mu Z^\mu - g V_{ud} J^L_{+1\mu} W^{+1\mu} - g V_{ud} J^L_{-1\mu} W^{-1\mu} , \tag{22}$$

and use Eqs. (15) to (17), we can discover

$$J^L_{i\mu} \equiv \frac{1}{2} \left(V_{i\mu} + A_{i\mu} \right) , \tag{23}$$

$$J^R_{i\mu} \equiv \frac{1}{2} \left(V_{i\mu} - A_{i\mu} \right) , \tag{24}$$

$$J^{EM}_\mu = V^0_\mu + \frac{1}{2} J^B_\mu , \tag{25}$$

$$J^{NC}_\mu = J^{L0}_\mu - \sin^2\theta_w J^{EM}_\mu . \tag{26}$$

Here J^B_μ is the baryon current, defined to be coupled to $\mathsf{v}^\mu_{(s)}$. These relations are consistent with the charge algebra $Q = T^0 + B/2$ (B is the baryon number). $V^{i\mu}$ and $A^{i\mu}$ are the isovector vector current and the isovector axial-vector current, respectively. J^{NC}_μ, $J^L_{\pm1\mu}$ are the conventional neutral current (NC) and charged current (CC) up to normalization factors.

3. QHD EFT, symmetries, and electroweak interactions

Here we present parallel discussions about QHD EFT's symmetries and EW interactions. The QHD EFT, as an EFT of QCD at low energy, should respect all the symmetries of QCD. Moreover, the approximate global chiral symmetry $SU(2)_L \otimes SU(2)_R \otimes U(1)_B$ in two flavor QCD is spontaneously broken to $SU(2)_V \otimes U(1)_B$, and is also manifestly broken due to the small masses of the quarks. To implement such broken global symmetry in the phenomenological lagrangian using hadronic degrees of freedom, it was found that there exists a general nonlinear realization of such symmetry (Callan et al., 1969; Coleman et al., 1969; Weinberg, 1968). Here, we follow the procedure in Ref. (Gasser & Leutwyler, 1984). The discussion about the conventions is presented first.

3.1 Conventions

In this work, the metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)_{\mu\nu}$, and for the Levi–Civita symbol $\epsilon^{\mu\nu\alpha\beta}$, the convention is $\epsilon^{0123} = 1$. Since we are going to talk about the Δ, which is the lowest isospin $I = 3/2$ nucleon resonance, we define the conventions for isospin indices. The following example, which shows the relation between two isospin representations for Δ, may help explain the convention:

$$\Delta^{*a} \equiv T^a_{iA} \Delta^{*iA} . \tag{27}$$

Here $a = \pm3/2, \pm1/2$, $i = \pm1, 0$, and $A = \pm1/2$. The upper components labeled as 'a', 'i', and 'A' furnish $\mathcal{D}^{(3/2)}$, $\mathcal{D}^{(1)}$, and $\mathcal{D}^{(1/2)}$ representations of the isospin $SU(2)$ group. (We

work with spherical vector components for $I = 1$ isospin indices, which requires some care with signs.) We can immediately realize that $T_{iA}^a = \langle 1, \frac{1}{2}; i, A | \frac{3}{2}; a \rangle$, which are CG coefficients. It is well known that the complex conjugate representation of $SU(2)$ is equivalent to the representation itself, so we introduce a metric linking the two representations to raise or lower the indices a, i, and A. For example, $\Delta_a \equiv (\Delta^{*a})^* = T_a^{\dagger\,iA} \Delta_{iA}$, where $T_a^{\dagger\,iA} = \langle \frac{3}{2}; a | 1, \frac{1}{2}; i, A \rangle$, should also be able to be written as

$$\Delta_a = T_a^{iA} \Delta_{iA} \equiv T_{jB}^b \, \widetilde{\delta}_{ba} \, \widetilde{\delta}^{ji} \, \widetilde{\delta}^{BA} \, \Delta_{iA} \,. \tag{28}$$

Here, $\widetilde{\delta}$ denotes a metric for one of the three representations. It can be shown that in this convention, $T_a^{\dagger\,iA} = T_a^{iA}$. Details about the conventions are given in Appendix 7.A.

3.2 QHD's symmetry realizations

Now we proceed to discuss a low-energy lagrangian involving N^A, Δ^a, π^i, ρ_μ^i, and the chiral singlets V_μ and ϕ (Furnstahl et al., 1997; Serot & Walecka, 1997). Under the transformations shown in Eqs. (2) to (7), the symmetry is realized nonlinearly in terms of hadronic degrees of freedom (Gasser & Leutwyler, 1984):

$$U \equiv \exp\left[2i \frac{\pi_i(x)}{f_\pi} t^i \right] \to L U R^\dagger \,, \tag{29}$$

$$\xi \equiv \sqrt{U} = \exp\left[i \frac{\pi_i}{f_\pi} t^i \right] \to L \xi h^\dagger = h \xi R^\dagger \,, \tag{30}$$

$$\widetilde{v}_\mu \equiv \frac{-i}{2} [\xi^\dagger(\partial_\mu - i l_\mu)\xi + \xi(\partial_\mu - i r_\mu)\xi^\dagger] \equiv \widetilde{v}_{i\mu} t^i \to h \widetilde{v}_\mu h^\dagger - i h \, \partial_\mu h^\dagger \,, \tag{31}$$

$$\widetilde{a}_\mu \equiv \frac{-i}{2} [\xi^\dagger(\partial_\mu - i l_\mu)\xi - \xi(\partial_\mu - i r_\mu)\xi^\dagger] \equiv \widetilde{a}_{i\mu} t^i \to h \widetilde{a}_\mu h^\dagger \,, \tag{32}$$

$$\widetilde{\partial}_\mu U \equiv \partial_\mu U - i l_\mu U + i U r_\mu \to L \widetilde{\partial}_\mu U R^\dagger \,, \tag{33}$$

$$(\widetilde{\partial}_\mu \psi)_\alpha \equiv (\partial_\mu + i \widetilde{v}_\mu - i v_{(s)\mu} B)_\alpha^{\ \beta} \psi_\beta \to \exp\left[-i\theta(x) B \right] h_\alpha^{\ \beta} (\widetilde{\partial}_\mu \psi)_\beta \,, \tag{34}$$

$$\widetilde{v}_{\mu\nu} \equiv -i[\widetilde{a}_\mu, \widetilde{a}_\nu] \to h \widetilde{v}_{\mu\nu} h^\dagger \,, \tag{35}$$

$$F_{\mu\nu}^{(+)} \equiv \xi^\dagger f_{L\mu\nu} \xi + \xi f_{R\mu\nu} \xi^\dagger \to h F_{\mu\nu}^{(+)} h^\dagger \,, \tag{36}$$

$$F_{\mu\nu}^{(-)} \equiv \xi^\dagger f_{L\mu\nu} \xi - \xi f_{R\mu\nu} \xi^\dagger \to h F_{\mu\nu}^{(-)} h^\dagger \,, \tag{37}$$

$$\widetilde{\partial}_\lambda F_{\mu\nu}^{(\pm)} \equiv \partial_\lambda F_{\mu\nu}^{(\pm)} + i[\widetilde{v}_\lambda, F_{\mu\nu}^{(\pm)}] \to h \widetilde{\partial}_\lambda F_{\mu\nu}^{(\pm)} h^\dagger \,. \tag{38}$$

In the preceding equations, t^i are the generators of reducible representations of $SU(2)$. Specifically, they could be generators of $\mathcal{D}_N^{(1/2)} \oplus \mathcal{D}_\rho^{(1)} \oplus \mathcal{D}_\Delta^{(3/2)}$, which operate on non-Goldstone isospin multiplets including the nucleon, ρ meson, and Δ. We generically label these fields by $\psi_\alpha = (N_A, \rho_i, \Delta_a)_\alpha$. Most of the time, the choice of t^i is clear from the context. B is the baryon number of the particle. The transformations of the isospin and chiral singlets V_μ and ϕ are trivial ($\phi \to \phi$, $V_\mu \to V_\mu$). h is generally a local $SU(2)_V$ matrix. We also make

use of the dual field tensors, for example, $\overline{F}^{(\pm)\,\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} F^{(\pm)}_{\alpha\beta}$, which have the same chiral transformations as the ordinary field tensors. Here we do not include the background fields s and p mentioned in Eq. (1), which are the source of manifest chiral-symmetry breaking in the Standard Model.

The C, P, and T transformation rules are summarized in Tabs. 4 and 5. A plus sign means normal, while a minus sign means abnormal, i.e., an extra minus sign exists in the transformation. The convention for *Dirac matrices* sandwiched by nucleon and/or Δ fields are

$$
C\overline{N}\,\Gamma NC^{-1} = \begin{cases} -N^T\,\Gamma^T\,\overline{N}^T \,, & \text{normal}\,; \\ N^T\,\Gamma^T\,\overline{N}^T \,, & \text{abnormal}\,. \end{cases} \tag{39}
$$

$$
C(\overline{\Delta}\,\Gamma N + \overline{N}\,\Gamma\Delta)C^{-1} = \begin{cases} -\Delta^T\,\Gamma^T\,\overline{N}^T - N^T\,\Gamma^T\,\overline{\Delta}^T \,, & \text{normal}\,; \\ +\Delta^T\,\Gamma^T\,\overline{N}^T + N^T\,\Gamma^T\,\overline{\Delta}^T \,, & \text{abnormal}\,. \end{cases} \tag{40}
$$

$$
Ci(\overline{\Delta}\,\Gamma N - \overline{N}\,\Gamma\Delta)C^{-1} = \begin{cases} +i\Delta^T\,\Gamma^T\,\overline{N}^T - iN^T\,\Gamma^T\,\overline{\Delta}^T \,, & \text{normal}\,; \\ -i\Delta^T\,\Gamma^T\,\overline{N}^T + iN^T\,\Gamma^T\,\overline{\Delta}^T \,, & \text{abnormal}\,. \end{cases} \tag{41}
$$

Here, in Eqs. (39), (40), and (41), the extra minus sign arises because the fermion fields anticommute. The factor of i in Eq. (41) is due to the requirement of Hermiticity of the lagrangian. To make the analysis easier for $\overline{\Delta}\,\Gamma N + C.C.$, we can just attribute a minus sign to an i under the C transformation. Whenever an i exists, the lagrangian takes the form $i(\overline{\Delta}\,\Gamma N - \overline{N}\,\Gamma\Delta)$. When no i exists, the lagrangian is like $\overline{\Delta}\,\Gamma N + \overline{N}\,\Gamma\Delta$.

For P and T transformations, the conventions are the same for N and Δ fields, except for an extra minus sign in the parity assignment for each Δ field (Weinberg, 1995a), so we list only the N case:

$$
P\overline{N}\,\Gamma_\mu NP^{-1} = \begin{cases} \overline{N}\,\mathcal{P}^\nu_\mu\,\Gamma_\nu\,N \,, & \text{normal}\,; \\ -\overline{N}\,\mathcal{P}^\nu_\mu\,\Gamma_\nu\,N \,, & \text{abnormal}\,. \end{cases} \tag{42}
$$

$$
T\overline{N}\,\Gamma_\mu NT^{-1} = \begin{cases} \overline{N}\,\mathcal{T}^\nu_\mu\,\Gamma_\nu\,N \,, & \text{normal}\,; \\ -\overline{N}\,\mathcal{T}^\nu_\mu\,\Gamma_\nu\,N \,, & \text{abnormal}\,. \end{cases} \tag{43}
$$

It is easy to generalize these results to $\Gamma_{\mu\nu}$, etc.

Now a few words about isospin structure are in order. Suppose an isovector object is denoted as $O_\mu \equiv O_{i\mu}t^i$, then the conventions are explained below:

$$
CO_\mu C^{-1} = \begin{cases} O_\mu^T \,, & \text{normal}\,; \\ -O_\mu^T \,, & \text{abnormal}\,. \end{cases} \tag{44}
$$

$$
PO_\mu P^{-1} = \begin{cases} \mathcal{P}^\nu_\mu O_\nu \,, & \text{normal}\,; \\ -\mathcal{P}^\nu_\mu O_\nu \,, & \text{abnormal}\,. \end{cases} \tag{45}
$$

$$
TO_\mu T^{-1} = \begin{cases} \mathcal{T}^\nu_\mu O_\nu \,, & \text{normal}\,; \\ -\mathcal{T}^\nu_\mu O_\nu \,, & \text{abnormal}\,. \end{cases} \tag{46}
$$

	γ^μ	$\sigma^{\mu\nu}$	1	$\gamma^\mu\gamma_5$	$i\gamma_5$	i	$i\overset{\leftrightarrow}{\partial}$	$\epsilon^{\mu\nu\alpha\beta}$
C	−	−	+	+	+	−	−	+
P	+	+	+	−	−	+	+	−
T	−	−	+	−	−	−	−	−

Table 4. Transformation properties of objects under C, P, and T. Here '+' means normal and '−' means abnormal.

	$\widetilde{a}_\mu$	$\widetilde{v}_\mu$	$\widetilde{v}_{\mu\nu}$	ρ_μ	$\rho_{\mu\nu}$	$\overline{\rho}_{\mu\nu}$	V_μ	$V_{\mu\nu}$	$\overline{V}_{\mu\nu}$	$F^{(\pm)}_{\mu\nu}$	$f_{s\mu\nu}$	$\overline{F}^{(\pm)}_{\mu\nu}$	$\overline{f}_{s\mu\nu}$
C	+	−	−	−	−	−	−	−	−	∓	−	∓	−
P	−	+	+	+	+	−	+	+	−	±	+	∓	−
T	−	−	−	−	−	+	−	−	+	−	−	+	+

Table 5. Continuation of Tab. 4.

The same convention applies to the isovector (pseudo)tensors. For isovector (pseudo)scalars, the $\mathcal{P}$ and $\mathcal{T}$ should be changed to $\mathbf{1}$. For the C transformation, O^T means transposing both isospin and Dirac matrices in the definition of O, if necessary.

3.3 QHD EFT lagrangian (without Δ) and electroweak interactions

Now we begin to discuss the QHD EFT lagrangian. Based on the symmetry transformation rules discussed above, we can construct the lagrangian as an invariant of these transformations by using the building blocks shown in Eqs. (29) to (38). In principle, there are an infinite number of possible interaction terms in this lagrangian. However, power counting (Furnstahl et al., 1997; Hu et al., 2007; McIntire et al., 2007) and Naive Dimensional Analysis (NDA) (Georgi & Manohar, 1984; Georgi, 1993) enable us to truncate this series of interactions to achieve a good approximation. Following the discussion in Ref. (Furnstahl et al., 1997), we associate with each interaction term a power-counting index:

$$\hat{\nu} \equiv d + \frac{n}{2} + b . \tag{47}$$

Here d is the number of derivatives (small momentum transfer) in the interaction, n is the number of fermion fields, and b is the number of heavy meson fields.

The QHD theory has been developed for some time. Details can be found in Refs. (Furnstahl et al., 1997; Serot & Walecka, 1997; Serot, 2007). *Here, we give a complete treatment of electroweak interactions in this theory.* (However, we do not discuss "seagull" terms of higher order in the couplings.) We begin with

$$\mathcal{L}_{N(\hat{\nu}\leqslant 3)} = \overline{N}(i\gamma^\mu[\widetilde{\partial}_\mu + ig_\rho\rho_\mu + ig_v V_\mu] + g_A\gamma^\mu\gamma_5\,\widetilde{a}_\mu - M + g_s\phi)N$$

$$- \frac{f_\rho g_\rho}{4M}\,\overline{N}\rho_{\mu\nu}\sigma^{\mu\nu}N - \frac{f_v g_v}{4M}\,\overline{N}V_{\mu\nu}\sigma^{\mu\nu}N - \frac{\kappa_\pi}{M}\,\overline{N}\,\widetilde{v}_{\mu\nu}\sigma^{\mu\nu}N$$

$$+ \frac{4\beta_\pi}{M}\,\overline{N}N\,\mathrm{Tr}(\widetilde{a}_\mu\widetilde{a}^\mu) + \frac{i\kappa_1}{2M^2}\,\overline{N}\gamma_\mu\overset{\leftrightarrow}{\widetilde{\partial}}_\nu N\,\mathrm{Tr}\,(\widetilde{a}^\mu\widetilde{a}^\nu)$$

$$+ \frac{1}{4M}\,\overline{N}\sigma^{\mu\nu}(2\lambda^{(0)}f_{s\mu\nu} + \lambda^{(1)}F^{(+)}_{\mu\nu})N , \tag{48}$$

where $\widetilde{\partial}_\mu$ is defined in Eq. (34), $\overset{\leftrightarrow}{\widetilde{\partial}_\nu} \equiv \widetilde{\partial}_\nu - (\overset{\leftarrow}{\partial_\nu} - i\widetilde{v}_\nu + iv_{(s)\nu})$, and the new field tensors are $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$ and

$$\rho_{\mu\nu} \equiv \partial_{[\mu}\rho_{\nu]} + i\overline{g}_\rho[\rho_\mu, \rho_\nu] + i([\widetilde{v}_\mu, \rho_\nu] - \mu \leftrightarrow \nu) \to h\,\rho_{\mu\nu}h^\dagger \,. \tag{49}$$

The superscripts $^{(0)}$ and $^{(1)}$ denote the isospin. In Appendix 7.B, details about the tilde objects (which are defined exactly above) are shown explicitly in terms of pion and background fields.

Next is a purely mesonic lagrangian:

$$\begin{aligned}
\mathcal{L}_{\text{meson}(\hat{v}\leqslant 4)} = {}& \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{4}f_\pi^2\,\text{Tr}[\widetilde{\partial}_\mu U(\widetilde{\partial}^\mu U)^\dagger] + \frac{1}{4}f_\pi^2\,m_\pi^2\,\text{Tr}(U + U^\dagger - 2) \\[6pt]
& - \frac{1}{2}\,\text{Tr}(\rho_{\mu\nu}\rho^{\mu\nu}) - \frac{1}{4}\,V^{\mu\nu}V_{\mu\nu} \\[6pt]
& + \frac{1}{2}\left(1 + \eta_1\frac{g_s\phi}{M} + \frac{\eta_2}{2}\frac{g_s^2\phi^2}{M^2}\right)m_v^2\,V_\mu V^\mu + \frac{1}{4!}\,\zeta_0\,g_v^2(V_\mu V^\mu)^2 \\[6pt]
& + \left(1 + \eta_\rho\frac{g_s\phi}{M}\right)m_\rho^2\,\text{Tr}(\rho_\mu\rho^\mu) - \left(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{g_s\phi}{M} + \frac{\kappa_4}{4!}\frac{g_s^2\phi^2}{M^2}\right)m_s^2\phi^2 \\[6pt]
& + \frac{1}{2g_\gamma}\left(\text{Tr}(F^{(+)\mu\nu}\rho_{\mu\nu}) + \frac{1}{3}f_s^{\mu\nu}V_{\mu\nu}\right) .
\end{aligned} \tag{50}$$

The $\nu = 3$ and $\nu = 4$ terms in $\mathcal{L}_{\text{meson}(\hat{v}\leqslant 4)}$ are important for describing the bulk properties of nuclear many-body systems (Furnstahl et al., 1995; 1996; 1997). The only manifest chiral-symmetry breaking is through the nonzero pion mass. It is well known that there are other $\hat{v} = 4$ terms involving pion-pion interactions. Since multiple pion interactions and chiral-symmetry-violating terms other than the pion mass term are not considered, this additional lagrangian is not shown here.

Finally, we have

$$\begin{aligned}
\mathcal{L}_{N,\pi(\hat{v}=4)} = {}& \frac{1}{2M^2}\,\overline{N}\gamma_\mu(2\beta^{(0)}\partial_\nu f_s^{\mu\nu} + \beta^{(1)}\widetilde{\partial}_\nu F^{(+)\mu\nu} + \beta_A^{(1)}\gamma_5\widetilde{\partial}_\nu F^{(-)\mu\nu})N \\[6pt]
& - \omega_1\,\text{Tr}(F_{\mu\nu}^{(+)}\widetilde{v}^{\mu\nu}) + \omega_2\,\text{Tr}(\widetilde{a}_\mu\widetilde{\partial}_\nu F^{(-)\mu\nu}) + \omega_3\,\text{Tr}\left(\widetilde{a}_\mu i\left[\widetilde{a}_\nu, F^{(+)\mu\nu}\right]\right) \\[6pt]
& - g_{\rho\pi\pi}\frac{2f_\pi^2}{m_\rho^2}\,\text{Tr}(\rho_{\mu\nu}\widetilde{v}^{\mu\nu}) \\[6pt]
& + \frac{c_1}{M^2}\,\overline{N}\gamma^\mu N\,\text{Tr}\left(\widetilde{a}^\nu\,\overline{F}_{\mu\nu}^{(+)}\right) + \frac{e_1}{M^2}\,\overline{N}\gamma^\mu\,\widetilde{a}^\nu N\,\overline{f}_{s\mu\nu} \\[6pt]
& + \frac{c_{1\rho}g_\rho}{M^2}\,\overline{N}\gamma^\mu N\,\text{Tr}\left(\widetilde{a}^\nu\,\overline{\rho}_{\mu\nu}\right) + \frac{e_{1v}g_v}{M^2}\,\overline{N}\gamma^\mu\,\widetilde{a}^\nu N\,\overline{V}_{\mu\nu} \,.
\end{aligned} \tag{51}$$

Note that $\mathcal{L}_{N,\pi(\hat{v}=4)}$ is *not* a complete list of all possible $\hat{v} = 4$ interaction terms. However, $\beta^{(0)}$ and $\beta^{(1)}$ are used in the form factors of the nucleon's vector current, $\omega_{1,2,3}$ contribute to the form factor of the pion's vector current, and $g_{\rho\pi\pi}$ is used in the form factors that incorporate

vector meson dominance (VMD).[1] Special attention should be given to the $c_1, e_1, c_{1\rho}$, and e_{1v} couplings, since they are the only relevant $\hat{v} = 4$ terms for NC photon production (Serot & Zhang, 2011a;b).

The construction of these high-order terms, $\mathcal{L}_{N,\pi(\hat{v}=4)}$ for example, is carried out by exhaustion. Based on the various symmetry transformation rules, at a given order there are a finite number of interaction terms, although the number can be big. For example, the interaction terms involving two pions and only one nucleon at $\hat{v} = 4$ *without chiral symmetry breaking* are (Ellis & Tang, 1998)

$$\overline{N}\sigma^{\mu\nu}i\overset{\leftrightarrow}{\widetilde{\partial}}{}^{\lambda}N\,\mathrm{Tr}\left(\widetilde{\partial}_{\lambda}\widetilde{a}_{\mu}\widetilde{a}_{\nu}\right) \qquad \text{and other contractions of Lorentz indices ,}$$

$$\overline{N}\gamma_{\mu}i\left[\widetilde{\partial}^{\mu}\widetilde{a}^{\nu}\,,\,\widetilde{a}_{\nu}\right]N \qquad \text{and other contractions of Lorentz indices .}$$

3.4 Introducing Δ resonances

The pathologies of relativistic field theory with spin-3/2 particles have been investigated in the canonical quantization framework for some time. There are two kinds of problems: one is the so-called Johnson–Sudarshan problem (Capri & Kobes, 1980; Hagen, 1971; Johnson & Sudarshan, 1961); the other one is the Velo–Zwanzinger problem (Capri & Kobes, 1980; Singh, 1973; Velo & Zwanziger, 1969). It was realized in (Kobayashi & Takahashi, 1987) that the two problems may both be related to the fact that the classical equation of motion, as the result of minimizing the action, fails to eliminate redundant spin components, because the invertibility condition of the constraint equation is not satisfied all the time. For example, in the Rarita–Schwinger formalism, the representation of the field is ψ^{μ}: $\left(\frac{1}{2}, \frac{1}{2}\right) \otimes \left(\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)\right) = \left(1, \frac{1}{2}\right) \oplus \left(\frac{1}{2}, 1\right) \oplus \left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)$ (Weinberg, 1995b). It can be shown that the spin-1/2 components are not dynamical in the free theory, which is generally not true after introducing interactions. Another issue is about the so-called *off-shell couplings*, which have the form $\gamma_{\mu}\psi^{\mu}, \partial_{\mu}\psi^{\mu}, \overline{\psi}^{\mu}\gamma_{\mu}$, and $\partial_{\mu}\overline{\psi}^{\mu}$ (still in the Rarita–Schwinger representation).

Recently, the problem has been investigated in a path-integral formalism in Ref. (Pascalutsa, 1998), where a gauge invariance is required for interactions. But this constraint conflicts with the manifest nonlinear chiral-symmetry realization in chiral EFT. Subsequently, in (Krebs et al., 2009; Pascalutsa, 2001), the authors realized that the commonly used non-invariant interactions are related to gauge-invariant interactions by field redefinitions, up to some contact interaction terms. Moreover, from the modern chiral EFT viewpoint, it has been concluded (Krebs et al., 2010; Tang & Ellis, 1996) that the off-shell couplings are redundant, since they lead to contributions to contact interactions without spin-3/2 degrees of freedom. Furthermore, it has been proved that off-shell couplings with ∂_{μ} changed to $\widetilde{\partial}_{\mu}$ are also redundant, which makes the manifest realization of chiral symmetry possible with a spin-3/2 particle.

However, the modern argument, which makes use of field redefinitions and gauge invariance for the EFT, looks abstract. The whole argument is that the field redefinitions, constructed to transform non-invariant terms to gauge-invariant terms, is applicable here, which requires us

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[1] VMD in QHD EFT has been discussed in detail in Ref. (Serot, 2007). We will discuss VMD for the form factor of the transition current involving Δ and N.

to be far away from the singularities of these transformations, i.e., to stay at low-energy and in weak-field regions (Krebs et al., 2009). This leads us to give another interesting argument, based directly on this assumption. In the Hamiltonian formalism, these two issues are somewhat clarified,[2] however the quantization of the EFT and hence Lorentz-invariance are not straightforward. So we use the path-integral approach.

Let's focus on the spin-3/2 propagator in the Rarita–Schwinger representation. First, we can decompose the free propagator into different spin components:

$$S_F^{0\mu\nu}(p) = \frac{-(\not{p}+m)}{p^2 - m^2 + i\epsilon}\left[g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{p^\mu\gamma^\nu - p^\nu\gamma^\mu}{3m} - \frac{2}{3m^2}p^\mu p^\nu\right]$$

$$\equiv -\frac{1}{\not{p}-m+i\epsilon}P^{(\frac{3}{2})\mu\nu} - \frac{1}{\sqrt{3}m}P_{12}^{(\frac{1}{2})\mu\nu} - \frac{1}{\sqrt{3}m}P_{21}^{(\frac{1}{2})\mu\nu}$$

$$+\frac{2}{3m^2}(\not{p}+m)P_{22}^{(\frac{1}{2})\mu\nu}\,,\tag{52}$$

$$P^{(\frac{3}{2})\mu\nu} = g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{1}{3p^2}\gamma^{[\mu}p^{\nu]}\,\not{p} - \frac{2}{3p^2}p^\mu p^\nu\,,\tag{53}$$

$$P_{11}^{(\frac{1}{2})\mu\nu} = \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{1}{3p^2}\gamma^{[\mu}p^{\nu]}\,\not{p} - \frac{1}{3p^2}p^\mu p^\nu\,,\tag{54}$$

$$P_{12}^{(\frac{1}{2})\mu\nu} = \frac{1}{\sqrt{3}p^2}(-p^\mu p^\nu + \gamma^\mu p^\nu\,\not{p})\,,\tag{55}$$

$$P_{21}^{(\frac{1}{2})\mu\nu} = \frac{1}{\sqrt{3}p^2}(p^\mu p^\nu - \gamma^\nu p^\mu\,\not{p})\,,\tag{56}$$

$$P_{22}^{(\frac{1}{2})\mu\nu} = \frac{1}{p^2}p^\mu p^\nu\,.\tag{57}$$

By using the identities shown in Eqs. (114) to (119) in Appendix 7.C, we can immediately write down

$$S_F^0(p) = P^{(\frac{3}{2})}\frac{-1}{\not{p}-m+i\epsilon}P^{(\frac{3}{2})}$$

$$+P^{(\frac{3}{2}\perp)}\left[-\frac{1}{\sqrt{3}m}P_{12}^{(\frac{1}{2})} - \frac{1}{\sqrt{3}m}P_{21}^{(\frac{1}{2})} + P_{22}^{(\frac{1}{2})}\frac{2}{3m^2}(\not{p}+m)P_{22}^{(\frac{1}{2})}\right]P^{(\frac{3}{2}\perp)}$$

$$\equiv S_F^{0(\frac{3}{2})} + S_F^{0(\frac{3}{2}\perp)}\,.\tag{58}$$

---

[2] In the perturbative calculation of EFT, the time-ordered free propagator defined in the Hamiltonian formalism for a spin-3/2 particle always satisfies the constraint on the degrees of freedom. (Assume we have a well defined Hamiltonian for the EFT.) For finite sums of the series of diagrams involving this propagator, the constraint is always satisfied. Moreover, those off-shell terms when either contracted to external legs or to the internal propagator of spin-3/2 degrees of freedom, give zero value. We may conclude that they are redundant. However, it is not clear whether the two conclusions hold for infinite sums. Moreover, as we know, the time-ordered propagator is not covariant, and leads to the difficulty of understanding Lorentz-invariance.

In principle, the decomposition shown in Eq. (58) should be obvious in the beginning, because Lorentz-invariance is preserved. However the key is that only the spin-3/2 component has pole structure, while the spin-1/2 components resemble contact vertices.

Furthermore, given certain interaction terms, we can carry out the calculation of the self-energy insertion, as done in Ref. (Ellis & Tang, 1998), for example. Based on the same argument as given above, the self-energy for renormalization should also be decomposed into a diagonal form for the spin. The details are as follows. The self-energy of the Δ can be defined as $\Sigma_{\mu\nu} = \Sigma^\Delta g_{\mu\nu} + \delta\Sigma_{\mu\nu}$. We see immediately that $\delta\Sigma_{\mu\nu}$'s indices can only have a structure like the products of $(\gamma_\mu, p_\mu) \times (\text{Dirac matrices}) \times (\gamma_\nu, p_\nu)$. Then we find

$$\Sigma = \Sigma^\Delta g + \delta\Sigma \tag{59}$$

$$= P^{(\frac{3}{2})}\Sigma^\Delta g P^{(\frac{3}{2})} + P^{(\frac{3}{2}\perp)}\Sigma P^{(\frac{3}{2}\perp)}$$

$$+ P^{(\frac{3}{2})}(\Sigma^\Delta g + \delta\Sigma)P^{(\frac{3}{2}\perp)} + P^{(\frac{3}{2}\perp)}(\Sigma^\Delta g + \delta\Sigma)P^{(\frac{3}{2})} \,. \tag{60}$$

So, we can conclude that $\Sigma = P^{(3/2)}\Sigma^\Delta P^{(3/2)} + P^{(3/2\perp)}\Sigma P^{(3/2\perp)} \equiv \Sigma^{(3/2)} + \Sigma^{(3/2\perp)}$. In the proof, we make use of $\left[P^{(3/2)}, \Sigma^\Delta\right] = 0$, $\left[P^{(3/2\perp)}, \Sigma^\Delta\right] = 0$, because the only possible spin structures of Σ^Δ are $\mathbf{1}$, $\not{p}$ and γ_5 (parity violation), which commute with the two projection operators. Then $P^{(3/2)}\Sigma^\Delta g P^{(3/2\perp)} = 0$ and $P^{(3/2)\perp}\Sigma^\Delta g P^{(3/2)} = 0$. Also we make use of Eqs. (115) and (116), so we get $P^{(3/2)}\delta\Sigma P^{(3/2\perp)} = 0$ and $P^{(3/2\perp)}\delta\Sigma P^{(3/2)} = 0$.

Based on previous discussions, we can have the following renormalization of the spin-3/2 propagator:

$$S_F = (S_F^{0(\frac{3}{2})} + S_F^{0(\frac{3}{2}\perp)}) + (S_F^{0(\frac{3}{2})} + S_F^{0(\frac{3}{2}\perp)})(\Sigma^{(\frac{3}{2})} + \Sigma^{(\frac{3}{2}\perp)})(S_F^{0(\frac{3}{2})} + S_F^{0(\frac{3}{2}\perp)}) + \dots$$

$$= S_F^{0(\frac{3}{2})} + S_F^{0(\frac{3}{2})}\Sigma^{(\frac{3}{2})}S_F^{0(\frac{3}{2})} + \dots \tag{61}$$

$$+ S_F^{0(\frac{3}{2}\perp)} + S_F^{0(\frac{3}{2}\perp)}\Sigma^{(\frac{3}{2}\perp)}S_F^{0(\frac{3}{2}\perp)} + \dots \,. \tag{62}$$

So the renormalized propagator is decomposed into two different components: $S_F \equiv S_F^{(3/2)} + S_F^{(3/2\perp)}$. The resonant contribution is $S_F^{(3/2)} = S_F^{0(3/2)} + S_F^{0(3/2)}\Sigma^{(3/2)}S_F^{(3/2)}$. The background contribution is $S_F^{(3/2\perp)} = S_F^{0(3/2\perp)} + S_F^{0(3/2\perp)}\Sigma^{(3/2\perp)}S_F^{(3/2\perp)}$. The renormalization shifts the pole position of the resonant part. For the nonresonant part, as long as power counting is valid, i.e., $O(\Sigma/m) \ll 1$, we are away from any unphysical pole in the renormalized nonresonant part; $[1 - O(\Sigma/m)]^{-1}$ never diverges. This also suggests that we will not see the unphysical pole in the renormalized propagator, when working in the low-energy perturbative region. Meanwhile, the argument helps to clarify the redundancy of the off-shell couplings. We have seen that the self-energy due to these couplings does not contribute in the renormalization of $S_F^{(3/2)}$. But it indeed changes the nonresonant part. However, the effect is power expandable. So essentially it is the same as higher-order contact terms without the Δ. This justifies the redundancy of these couplings. To ignore them in a way which does not break chiral symmetry on a term-by-term basis, we can always associate the ∂^μ with π fields so that it becomes $\widetilde{\partial}^\mu$. *This indicates that those couplings having $\widetilde{\partial}^\mu$ or γ^μ contracted with Δ_μ can be ignored without breaking manifest chiral symmetry.*

A few words on the singularity of $1/p^2$ are in order here. [See Eqs. (52) to (57).] The whole calculation is only valid in the low-energy limit, and in this limit we should not find any diagrams with Δ's that are far "off shell". Take pion scattering for example; we assume the pion energy to be small, and hence p^2 is always roughly equal to the incoming nucleon's invariant mass. So the singularity in $1/p^2$ should not be a problem in the low-energy theory from a very general perspective.

3.5 QHD with Δ

Consider first $\mathcal{L}_\Delta$ ($\hat{v} \leqslant 3$), which is essentially a copy of the corresponding lagrangian for the nucleon as shown in Eq. (48):

$$\mathcal{L}_\Delta = \frac{-i}{2} \overline{\Delta}_\mu^{\,a} \{\sigma^{\mu\nu} , (i\,\widetilde{\partial} - h_\rho\,\slashed{\rho} - h_v\,\slashed{V} - m + h_s\phi)\}_a^b \Delta_{b\nu} + \widetilde{h}_A \overline{\Delta}_\mu^{\,a}\,\widetilde{a}_a^b\gamma_5\Delta_b^\mu$$

$$- \frac{\widetilde{f}_\rho h_\rho}{4m}\,\overline{\Delta}_\lambda\,\rho_{\mu\nu}\sigma^{\mu\nu}\Delta^\lambda - \frac{\widetilde{f}_v h_v}{4m}\,\overline{\Delta}_\lambda V_{\mu\nu}\sigma^{\mu\nu}\Delta^\lambda$$

$$- \frac{\widetilde{\kappa}_\pi}{m}\,\overline{\Delta}_\lambda\widetilde{v}_{\mu\nu}\sigma^{\mu\nu}\Delta^\lambda + \frac{4\widetilde{\beta}_\pi}{m}\,\overline{\Delta}_\lambda\Delta^\lambda \operatorname{Tr}(\widetilde{a}^\mu\,\widetilde{a}_\mu) \ . \tag{63}$$

Here the sub- and superscripts $a, b = (\pm 3/2, \pm 1/2)$, and the isospin conventions and T matrix have been discussed in Sec. 3.1.

To produce the $N \leftrightarrow \Delta$ transition currents, we construct the following lagrangians ($\hat{v} \leqslant 4$):

$$\mathcal{L}_{\Delta,N,\pi} = h_A \overline{\Delta}^{\,a\mu}\, T_a^{\dagger\,iA}\,\widetilde{a}_{i\mu} N_A + \text{C.C.} \ , \tag{64}$$

$$\mathcal{L}_{\Delta,N,\text{background}} = \frac{ic_{1\Delta}}{M}\,\overline{\Delta}_\mu^{\,a}\gamma_v\gamma_5\,T_a^{\dagger\,iA}F_i^{(+)\mu\nu}N_A + \frac{ic_{3\Delta}}{M^2}\,\overline{\Delta}_\mu^{\,a}\,i\gamma_5\,T_a^{\dagger\,iA}(\widetilde{\partial}_\nu F^{(+)\mu\nu})_i N_A$$

$$+ \frac{c_{6\Delta}}{M^2}\,\overline{\Delta}_\lambda^{\,a}\sigma_{\mu\nu}T_a^{\dagger\,iA}(\widetilde{\partial}^\lambda \overline{F}^{(+)\mu\nu})_i N_A$$

$$- \frac{d_{2\Delta}}{M^2}\,\overline{\Delta}_\mu^{\,a}\,T_a^{\dagger\,iA}(\widetilde{\partial}_\nu F^{(-)\mu\nu})_i N_A - \frac{id_{4\Delta}}{M}\,\overline{\Delta}_\mu^{\,a}\gamma_v\,T_a^{\dagger\,iA}F_i^{(-)\mu\nu}N_A$$

$$- \frac{id_{7\Delta}}{M^2}\,\overline{\Delta}_\lambda^{\,a}\sigma_{\mu\nu}T_a^{\dagger\,iA}(\widetilde{\partial}^\lambda F^{(-)\mu\nu})_i N_A + \text{C.C.} \ , \tag{65}$$

$$\mathcal{L}_{\Delta,N,\rho} = \frac{ic_{1\Delta\rho}}{M}\,\overline{\Delta}_\mu^{\,a}\,\gamma_v\gamma_5\,T_a^{\dagger\,iA}\rho_i^{\mu\nu}N_A + \frac{ic_{3\Delta\rho}}{M^2}\,\overline{\Delta}_\mu^{\,a}\,i\gamma_5\,T_a^{\dagger\,iA}(\widetilde{\partial}_\nu\rho^{\mu\nu})_i N_A$$

$$+ \frac{c_{6\Delta\rho}}{M^2}\,\overline{\Delta}_\lambda^{\,a}\sigma_{\mu\nu}T_a^{\dagger\,iA}(\widetilde{\partial}^\lambda\overline{\rho}^{\mu\nu})_i N_A + \text{C.C.} \ . \tag{66}$$

It can be checked that the interaction terms respect all of the required symmetries. Terms omitted from these lagrangians are either redundant or are not relevant to the transition interaction involving N and Δ (at tree level). The construction of terms is by means of exhausting all the possibilities. Here we give an example:

$$\overline{\Delta}_\mu\gamma_v N\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}^{(+)} = 2i\overline{\Delta}_\mu\gamma_v\gamma_5 N F^{(+)\mu\nu} + iF_{\alpha\beta}^{(+)}\overline{\Delta}_\mu\gamma_5(\gamma^\mu\gamma^\alpha\gamma^\beta - g^{\alpha\beta}\gamma^\mu)N \ . \tag{67}$$

The preceding identity indicates that $\overline{\Delta}^a_\mu \gamma_\nu T_a^{\dagger iA} \overline{F}_i^{(+)\mu\nu} N_A$ differs from the $c_{1\Delta}$ coupling in Eq. (65) by off-shell terms, which can be ignored.

Moreover, the terms in the lagrangian in Eq. (66) and the $1/g_\gamma$ coupling in Eq. (50) are necessary for the realization of transition form factors using VMD. First, we make the following definitions:

$$\langle \Delta, a, p_\Delta | V^{i\mu}(A^{i\mu}) | N, A, p_N \rangle \equiv T_a^{\dagger iA} \, \overline{u}_{\Delta\alpha}(p_\Delta) \, \Gamma^{\alpha\mu}_{V(A)}(q) \, u_N(p_N) \, . \tag{68}$$

Based on the lagrangians given previously, formulas shown in Appendix 7.B, and the definitions of currents in Eq. (21), we find (note that $\sigma_{\mu\nu}\epsilon^{\mu\nu\alpha\beta} \propto i\sigma^{\alpha\beta}\gamma_5$)

$$\Gamma^{\alpha\mu}_V = \frac{2c_{1\Delta}(q^2)}{M} (q^\alpha \gamma^\mu - \not{q}g^{\alpha\mu})\gamma_5 + \frac{2c_{3\Delta}(q^2)}{M^2} (q^\alpha q^\mu - g^{\alpha\mu}q^2)\gamma_5$$

$$- \frac{8c_{6\Delta}(q^2)}{M^2} q^\alpha \sigma^{\mu\nu} iq_\nu \gamma_5 \, ,$$

$$c_{i\Delta}(q^2) \equiv c_{i\Delta} + \frac{c_{i\Delta\rho}}{2g_\gamma} \frac{q^2}{q^2 - m_\rho^2} \qquad i = 1, 3, 6, \tag{69}$$

$$\Gamma^{\alpha\mu}_A = -h_A \left(g^{\alpha\mu} - \frac{q^\alpha q^\mu}{q^2 - m_\pi^2} \right) + \frac{2d_{2\Delta}}{M^2} (q^\alpha q^\mu - g^{\alpha\mu}q^2) - \frac{2d_{4\Delta}}{M} (q^\alpha \gamma^\mu - g^{\alpha\mu} \not{q})$$

$$- \frac{4d_{7\Delta}}{M^2} q^\alpha \sigma^{\mu\nu} iq_\nu \, , \tag{70}$$

where h_A is from Eq. (64). Quite similar to the $c_{i\Delta}(q^2)$, we can introduce axial-vector meson [$a_1(1260)$] exchange into the axial transition current, which leads to a structure for the $d_{i\Delta}(q^2)$ that is similar to the vector transition current form factors. There is one subtlety associated with the realization of $h_A(q^2)$: with our lagrangian, we have the pion-pole contribution associated only with the h_A coupling, and all the higher-order terms contained in $\delta h_A(q^2) \equiv h_A(q^2) - h_A$ conserve the axial transition current. With the limited information about manifest chiral-symmetry breaking, we ignore this subtlety and still use the form similar to the $c_{1\Delta}(q^2)$ to parameterize $h_A(q^2)$. The axial-vector meson couplings $h_{\Delta a_1}$ and $d_{i\Delta a_1}$ are the combinations of g_{a_1} (a_1 and isovector axial-vector external field coupling strength) and the coupling strength of the $\overline{\Delta} a_1 N$ interaction. m_{a_1} is the 'mass' of the meson. So we have

$$h_A(q^2) \equiv h_A + h_{\Delta a_1} \frac{q^2}{q^2 - m_{a_1}^2} \, , \tag{71}$$

$$d_{i\Delta}(q^2) \equiv d_{i\Delta} + d_{i\Delta a_1} \frac{q^2}{q^2 - m_{a_1}^2} \qquad i = 2, 4, 7. \tag{72}$$

To determine the coefficients in the transition form factors shown in Eqs. (69), (71), and (72), we need to compare ours with the conventional ones used in the literature. In

Refs. (Graczyk et al., 2009; Hernández et al., 2007) for example, the definition is

$$\langle \Delta, \tfrac{1}{2} | j^{\mu}_{cc+} | N, -\tfrac{1}{2} \rangle \equiv \overline{u}_{\alpha}(p_{\Delta}) \left\{ \left[\frac{C^V_3}{M} \left(g^{\alpha\mu} \slashed{q} - q^{\alpha}\gamma^{\mu} \right) + \frac{C^V_4}{M^2} \left(q \cdot p_{\Delta} g^{\alpha\mu} - q^{\alpha} p^{\mu}_{\Delta} \right) \right. \right.$$

$$\left. + \frac{C^V_5}{M^2} \left(q \cdot p_N g^{\alpha\mu} - q^{\alpha} p^{\mu}_N \right) \right] \gamma_5$$

$$+ \left[\frac{C^A_3}{M} \left(g^{\alpha\mu} \slashed{q} - q^{\alpha}\gamma^{\mu} \right) + \frac{C^A_4}{M^2} \left(q \cdot p_{\Delta} g^{\alpha\mu} - q^{\alpha} p^{\mu}_{\Delta} \right) \right.$$

$$\left. \left. + C^A_5 g^{\alpha\mu} + \frac{C^A_6}{M^2} q^{\mu} q^{\alpha} \right] \right\} u(p_N) . \tag{73}$$

The basis given above is known to be complete. The determination of the couplings through comparing our results with the conventional ones has been given in Ref. (Serot & Zhang, 2010). There we find that our meson dominance form factors are accurate up to $Q^2 \approx 0.3\,\mathrm{GeV}^2$. Moreover, CVC and PCAC can be easily checked for the transition currents. The details can be found in Ref. (Serot & Zhang, 2010).

4. Application

In this section, we briefly discuss the weak production of pions from nucleons. We focus only on two properties of the Feynman diagrams in this problem, including the G parity and the current's Hermiticity. Then we talk about the production from nuclei, in which Δ dynamics is the key component (for both the interaction mechanism and the final state interaction of the pion). This points out the importance of understanding the strong interaction, associated with nuclear structure and Δ dynamics, in the study of the electroweak response of nuclei. So it is necessary to have a framework that includes the two and also provides for efficient calculations. The details of these subjects are presented in Refs. (Serot & Zhang, 2010) and (Serot & Zhang, 2011a;b).

4.1 Weak production of pions from free nucleons

The relevant Feynman diagrams are shown in Fig. 1 for weak production of pions due to (anti)neutrino scattering off free nucleons. The 'C' in the figure stands for various currents including the vector current, axial current, and baryon current, of which both CC and NC are composed according to Sec. 2.2. The details about these diagrams can be found in (Serot & Zhang, 2010). Here we begin with G parity. We use $\langle N, B, \pi, j | J^{\mu} | N, A \rangle$ to represent the contribution of diagrams, where 'A' and 'B' denote isospin-1/2 projections. From G parity, we have

$$G A^{i\mu} G^{-1} = -A^{i\mu} ,$$

$$G V^{i\mu} (J^{\mu}_B) G^{-1} = V^{i\mu} (J^{\mu}_B) .$$

By applying this to the current's matrix elements, we get

$$\langle N, B, \pi, j | A^{i\mu} | N, A \rangle = \langle \overline{N}, B, \pi, j | A^{i\mu} | \overline{N}, A \rangle , \tag{74}$$
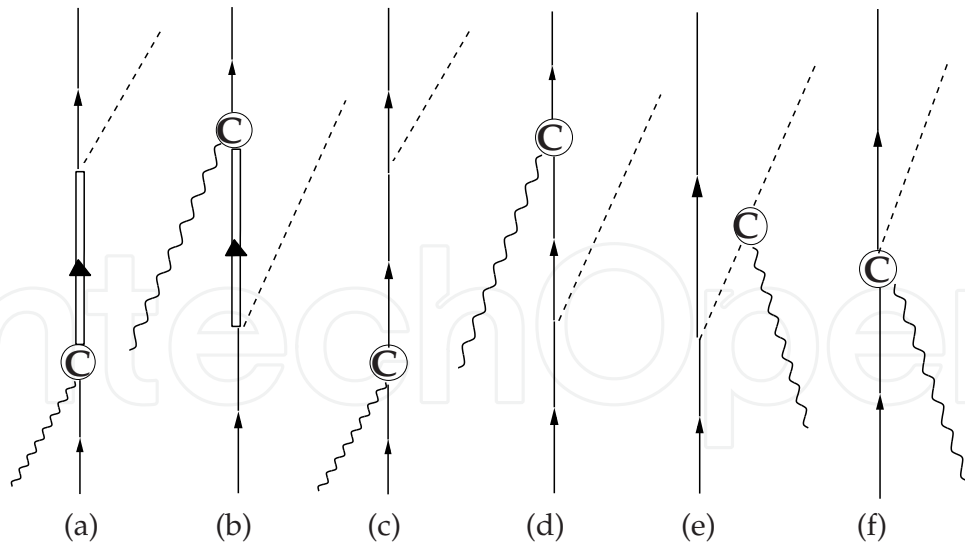
Fig. 1. Feynman diagrams for pion production. Here, **C** stands for various types of currents including vector, axial-vector, and baryon currents. Some diagrams may be zero for some specific type of current. For example, diagrams (a) and (b) will not contribute for the (isoscalar) baryon current. Diagram (e) will be zero for the axial-vector current. The pion-pole contributions to the axial current in diagrams (a) (b) (c) (d) and (f) are included in the vertex functions of the currents.

$$\langle N, B, \pi, j | V^{i\mu}(J^{\mu}_B) | N, A \rangle = -\langle \overline{N}, B, \pi, j | V^{i\mu}(J^{\mu}_B) | \overline{N}, A \rangle \,. \tag{75}$$

Eqs. (74) and (75) give a relation between a current's matrix element involving nucleon states and a matrix element involving antinucleon states. Because of the isospin symmetry, we can define

$$\langle N, B, p_f; \pi, j, k_\pi | \ A^{i\mu} \ | N, A, p_i \rangle$$

$$\equiv \ \delta^i_j \delta^A_B \, \overline{u}(p_f) \Gamma^{\mu}_{sym}(p_f, k_\pi; p_i, q) u(p_i)$$

$$+ i\epsilon^i_{\ jk} \left(\frac{\tau^k}{2} \right)^A_B \overline{u}(p_f) \Gamma^{\mu}_{asym}(p_f, k_\pi; p_i, q) u(p_i) \,. \tag{76}$$

Vector currents can be decomposed in the same way. From crossing symmetry, we can see

$$\langle \overline{N}, B, p_f; \pi, j, k_\pi | \ A^{i\mu} \ | \overline{N}, A, p_i \rangle$$

$$= \ -\delta^i_j \delta^A_B \, \overline{v}(p_i) \Gamma^{\mu}_{sym}(-p_i, k_\pi; -p_f, q) v(p_f)$$

$$- i\epsilon^i_{\ jk} \left(-\frac{\tau^{k\,T}}{2} \right)^B_A \overline{v}(p_i) \Gamma^{\mu}_{asym}(-p_i, k_\pi; -p_f, q) v(p_f)$$

$$= \ \delta^i_j \delta^A_B \, \overline{u}(p_f) \left(-\mathcal{C} \Gamma^{T\mu}_{sym}(-p_i, k_\pi; -p_f, q) \mathcal{C} \right) u(p_i)$$

$$- i\epsilon^i_{\ jk} \left(\frac{\tau^k}{2} \right)^A_B \overline{u}(p_f) \left(-\mathcal{C} \Gamma^{T\mu}_{asym}(-p_i, k_\pi; -p_f, q) \mathcal{C} \right) u(p_i) \,. \tag{77}$$

In Eq. (77), the $-\frac{1}{2}\tau^{k^T}$ appears because antiparticles furnish the complex conjugate representation. (It is equivalent to the original representation.) $\mathcal{C}$ is the charge conjugation matrix applied to a Dirac spinor, i.e., $\psi^C(x) = \mathcal{C}(\overline{\psi}(x))^T$. By comparing Eq. (77) with Eq. (74), we have the following constraint on the axial current's matrix element:

$$-\mathcal{C}\Gamma^{T\mu}_{(a)sym}(-p_i, k_\pi; -p_f, q)\mathcal{C} = \underset{(-)}{+} \Gamma^\mu_{(a)sym}(p_f, k_\pi; p_i, q) \, . \tag{78}$$

Similarly, we have the following constraint on vector current's matrix element:

$$-\mathcal{C}\Gamma^{T\mu}_{(a)sym}(-p_i, k_\pi; -p_f, q)\mathcal{C} = \underset{(+)}{-} \Gamma^\mu_{(a)sym}(p_f, k_\pi; p_i, q) \, . \tag{79}$$

For the baryon current $\langle N, B', \pi, j | J^\mu_B | N, A \rangle \equiv (\frac{1}{2}\tau_j)^A_{B'} \overline{u}(p_f)\Gamma^\mu_B(p_f, k_\pi; p_i, q)u(p_i)$, G parity indicates

$$-\mathcal{C}\Gamma^{T\mu}_B(-p_i, k_\pi; -p_f, q)\mathcal{C} = -\Gamma^\mu_B(p_f, k_\pi; p_i, q) \, . \tag{80}$$

Now we can see how adding a crossed diagram involving the Δ is necessary to satisfy G parity. For example, let's talk about the vector current's matrix element. If we define it for diagrams (a) and (b) in Fig. 1 as follows:

$$\langle V^{i\mu} \rangle_a \equiv T^a_{Bj} T^{\dagger iA}_a \overline{u}_f \Gamma^\mu_{dir}(p_f, k_\pi; p_i, q)u_i \, , \tag{81}$$

$$\langle V^{i\mu} \rangle_b \equiv T^{ai}_B T^{\dagger A}_{ja} \overline{u}_f \Gamma^\mu_{cross}(p_f, k_\pi; p_i, q)u_i \, . \tag{82}$$

Then by using Eq. (98), we get [here we include only diagram (a) and (b) contributions]

$$\Gamma^\mu_{(a)sym} = \frac{2}{3}\left(\Gamma^\mu_{dir} \underset{(-)}{+} \Gamma^\mu_{cross} \right) \, . \tag{83}$$

By calculating the diagrams, it is straightforward to prove that

$$-\mathcal{C}\Gamma^{T\mu}_{cross}(-p_i, k_\pi; -p_f, q)\mathcal{C} = -\Gamma^\mu_{dir}(p_f, k_\pi; p_i, q) \, . \tag{84}$$

This equation justifies the G parity of the vector current's matrix elements. Other currents' matrix elements can be justified in a similar way.

Now we discuss the Hermiticity of the current. Let's consider $\langle N, \pi\, out | J^\mu | N, in \rangle$:

$$\langle N, p_f, \pi, k_\pi, out | J^\mu | N, p_i, in \rangle^* = \langle N, p_i, in | J^{\dagger\mu} | N, p_f, \pi, k_\pi, out \rangle$$

$$\neq \langle N, p_i, out | J^{\dagger\mu} | N, p_f, \pi, k_\pi, in \rangle \, . \tag{85}$$

But how do we generally understand $\langle i, in | O | f, out \rangle$? Naively, we would have the following:

$$\langle i, in | O | f, out \rangle = \langle i | U(-\infty, 0)U(0, t)o(t)U(t, 0)U(0, +\infty) | f \rangle$$

$$= \langle i | \overline{T}o(t) \exp[i \int dt H_I(t)] | f \rangle \, . \tag{86}$$

Here O and $o(t)$ are the operators in the Heisenberg and interaction pictures. $\overline{T}$ is another type of time ordering: $\overline{T}H_I(t_1)H_I(t_2) = \theta(t_2 - t_1)H_I(t_1)H_I(t_2) + \theta(t_1 - t_2)H_I(t_2)H_I(t_1)$. It is easy to realize that in momentum space, if we mirror the pole of the T defined Green's function, and apply $(-)$ to the overall Green's function, we get the $\overline{T}$ defined Green's function. Second, each interaction vertex in the $\langle i, in|O|f, out\rangle$ calculation differs from that of $\langle i, out|O|f, in\rangle$ by a $(-)$ sign. Third, since now all the poles are in the first and third quadrants in the complex momentum plane, the corresponding loop integration differs from the normal loop integration by a $(-)$ sign! So, without a rigorous proof, we have that after calculating $\langle i, out|O|f, in\rangle$, if we mirror all the poles relative to the real axis for the propagator and apply a phase $(-)^{(V-V_o)+I+L} = (-)^{V_o-1}$ to it, then we get the corresponding $\langle i, in|O|f, out\rangle$. Here $V, V_o, I,$ and L are the number of vertices in the graph, vertices in the operator O, internal lines, and loops. For the current operator J^μ, $V_o = 1$ and hence the phase is $(+)$.

Now let's proceed to see the consequence of the Hermiticity of $J^\mu(x = 0)$, i.e., $J^{i\mu\dagger} = J_i^\mu$:

$$\langle N, B, p_f, \pi, j, k_\pi, out| J^{i\mu} |N, A, p_i, in\rangle^*$$

$$= \langle N, A, p_i, in|J_i^\mu|N, B, p_f, \pi, j, k_\pi, out\rangle$$

$$= \langle N, A, p_i, out|J_i^\mu|N, B, p_f, \pi, j, k_\pi, in\rangle|_{pm}$$

$$= \delta_{ii'}\delta^{jj'} \langle N, A, p_i, \pi, j', -k_\pi, out|J^{i'\mu}|N, B, p_f, in\rangle|_{pm} . \tag{87}$$

Here $|_{pm}$ indicates poles are mirrored with respect to the real axis. In the following, we decompose the general current matrix element into symmetric and antisymmetric parts, as we did in in Eq. (76):

$$\langle N, B, p_f, \pi, j, k_\pi, out|J^{i\mu}|N, A, p_i, in\rangle^*$$

$$= \delta_i^j \delta_A^B \, \overline{u}(p_i)\overline{\Gamma}_{sym}^\mu(p_f, k_\pi; p_i, q)u(p_f) - i\epsilon_i^{jk}\left(\frac{\tau_k}{2}\right)_A^B \overline{u}(p_i)\overline{\Gamma}_{asym}^\mu(p_f, k_\pi; p_i, q)u(p_f) . \tag{88}$$

Here, $\overline{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$. Meanwhile, Eq. (87) can be rewritten as

$$\langle N, A, p_i, \pi, j', -k_\pi, out| J^{i'\mu} |N, B, p_f, in\rangle|_{pm} \, \delta_{ii'}\delta^{jj'}$$

$$= \delta_{ii'}\delta^{jj'}\Big[\delta_{j'}^{i'}\delta_A^B \, \overline{u}(p_i)\Gamma_{sym}^\mu(p_i, -k_\pi; p_f, -q)u(p_f)$$

$$+ i\epsilon_{j'k}^{i'}\left(\frac{\tau^k}{2}\right)_A^B \overline{u}(p_i)\Gamma_{asym}^\mu(p_i, -k_\pi; p_f, -q)u(p_f)\Big]_{pm}$$

$$= \delta_i^j\delta_A^B \, \overline{u}(p_i)\Gamma_{sym}^\mu(p_i, -k_\pi; p_f, -q)u(p_f)|_{pm}$$

$$+ i\epsilon_i^{jk}\left(\frac{\tau_k}{2}\right)_A^B \overline{u}(p_i)\Gamma_{asym}^\mu(p_i, -k_\pi; p_f, -q)u(p_f)|_{pm} . \tag{89}$$

If we compare Eq. (88) with Eq. (89), we see the Hermiticity constraint is

$$\gamma^0[\Gamma_{(a)sym}^\mu(p_f, k_\pi; p_i, q)]^\dagger\gamma^0 = \genfrac{}{}{0pt}{}{+}{(-)} \Gamma_{(a)sym}^\mu(p_i, -k_\pi; p_f, -q)|_{pm} . \tag{90}$$

Now let's focus on the constraint on diagrams (a) and (b) in Fig. 1. We can check by calculating diagrams:

$$\overline{\Gamma}^{\mu}_{dir}(p_f, k_\pi; p_i, q) = \Gamma^{\mu}_{cross}(p_i, -k_\pi; p_f, -q)|_{pm} .$$

(91)

We can choose kinematics where no poles and cuts arise, i.e., there is no phase shift, and then test the constraint without $|_{pm}$. The preceding observation, with Eq. (83) taken into account, leads to the satisfaction of the constraint in Eq. (90). The Hermiticity of the baryon current can be studied in a similar way, and hence is not shown explicitly here. Moreover, it is interesting to see that the higher-order contact terms satisfy the requirements due to G parity and Hermiticity on a term-by-term basis.

4.2 Weak production of pions from nuclei, Δ dynamics

With the development of neutrino-oscillation experiments, precise knowledge about the neutrino (antineutrino)-nuclei scattering cross sections is needed for the understanding of the experiments' background. Take MiniBooNE (Aguilar-Arevalo et al., 2009; 2010), for example; the median energy of the neutrino (antineutrino) beam is around 0.6 (0.5) GeV, and the high-energy tail extends up to 2 GeV. In this regime, the Δ is the most important resonance for the interaction mechanism, except in the *very* low-energy region. Therefore, to understand pion production, we need to study Δ dynamics in the nucleus. This subject has been extensively discussed in the nonrelativistic framework (Hirata et al., 1976; Horikawa et al., 1980; Oset & Salcedo, 1987), and it has also been initiated in the relativistic framework in (Herbert et al., 1992; Wehrberger et al., 1989; Wehrberger & Wittman, 1990; Wehrberger, 1993). It is shown that the Δ width increases in the normal nuclear medium, since new decay channels are opened, like $\Delta N \rightarrow NN$, for example. The real part of the Δ's self-energy has also been studied. From the lagrangian in Eq. (63), we can see that the two parameters h_s and h_v in the lagrangian are important.[3] However, the information in (Boguta, 1982; Kosov et al., 1998; Wehrberger et al., 1989; Wehrberger, 1993) is still limited. In (Serot & Zhang, 2011a;b), we have realized that these Δ-meson couplings are responsible for the Δ's spin-orbit coupling in the nucleus, and based on this we provide some information about the couplings from this new perspective.

Meanwhile, the Δ dynamics is also strongly correlated with the pion dynamics in the nuclear medium, and hence is important for understanding the pion's final state interactions, especially in the energy regime of these neutrino-oscillation experiments.

5. Summary

In this work, we have studied EW interactions in QHD EFT. First, we discuss the EW interactions at the quark level. Then we include EW interactions in QHD EFT by using the background-field technique. The completed QHD EFT has a nonlinear realization of $SU(2)_L \otimes SU(2)_R \otimes U(1)_B$ (chiral symmetry and baryon number conservation), as well as realizations of other symmetries including Lorentz-invariance, C, P, and T. Meanwhile, as we know, chiral symmetry is manifestly broken due to the nonzero quark masses; the P and C symmetries are also broken because of weak interactions. All these breaking patterns are parameterized in a general way in the EFT. Moreover, we have included the

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[3] h_ρ should not play an important role in normal nuclei with small asymmetry.

Δ resonance as manifest degrees of freedom in our QHD EFT. This enables us to discuss physics at the kinematics where the resonance becomes important. As a result, the effective theory uses hadronic degrees of freedom, satisfies the constraints due to QCD (symmetries and their breaking pattern), and is calibrated to strong-interaction phenomena. (The EW interaction of *individual hadrons*, like the transition currents discussed in this work, need to be parameterized.) So this effective field theory satisfies the three listed points laid out in the Introduction.

The technical issues that arise when introducing the Δ in the EFT need to be emphasized here. It has been proven that the general EFT with conventional interactions has no redundant degrees of freedom (Krebs et al., 2009). (Unphysical degrees of freedom have been considered in the canonical quantization scheme as the reason for pathologies in field theory with high-spin fields.) However, the proof rests on the work of (Pascalutsa, 1998), which claims that gauge invariance could eliminate the redundant degrees of freedom. Here, we have provided another perturbative argument about this issue, which indicates that as long as we work in the low-energy and weak-field limit, the unphysical degrees of freedom do not show up. This condition is satisfied in the EFT. Throughout the argument, we do not need to make use of the gauge-invariance requirement. And in this way, we can easily see the redundancy of *off-shell* interactions, which has also been rigorously addressed in (Krebs et al., 2010). Moreover, the argument can be easily generalized to other high-spin fields.

To appreciate the importance of the symmetries realized in QHD EFT, we have discussed the currents' matrix elements in pion production from nucleons. The calculation and results are detailed in (Serot & Zhang, 2010). Here, we first briefly mention the consequence of chiral symmetry (and its breaking), i.e., CVC and PCAC. These two principles provide important constraints on the EW interactions at the hadronic level. The *G* parity is then studied for pion production. This provides another constraint on the analytical structure of matrix elements. Meanwhile, it also points out the importance of including cross diagrams involving the Δ. When combining the Δ's contribution in the *s* and *u* channels, the full result respects *G* parity. Moreover, the constraint due to the Hermiticity of current operators is explored. It is important to notice that other contact terms respect all these constraints. So, it is necessary to have a theoretical framework that satisfies these constraints. The QHD EFT, with symmetries included, clearly provides such a framework.

However, the calibration of a model on the hadronic level does not guarantee its success at the nuclear level. To study EW interactions in nuclei, we clearly have to understand how the nucleons are bound together to form nuclei. QHD has been applied extensively to this kind of problem (Serot & Walecka, 1986; 1997), and the recently developed chiral QHD EFT has also been tested in the nuclear many-body problem (Furnstahl et al., 1997). The mean-field approximation is understood in terms of density functional theory (Kohn, 1999), and hence the theory calibrated to nuclear properties includes many-body correlations beyond the Hartree approximation. Moreover, the power counting of diagrams in terms of $O(k/M)$ (*k* can be the Fermi momentum, mean-field strength, or other dimensional quantities) in the many-body calculations has also been studied in this framework with the justification that fitted parameters are *natural* (Hu et al., 2007; McIntire et al., 2007). This enables us to discuss the EW interactions order-by-order in the nuclear many-body system using QHD EFT.

As mentioned before, we have initiated the study of weak production of pions due to neutrino and antineutrino scattering off nuclei in this framework (Serot & Zhang, 2010; 2011a;b).

Moreover, we also studied the production of photons, in which the conservation of the EM current is clearly crucial. The discussion of power counting has been presented in these references. Furthermore, we should also anticipate the importance of Δ dynamics modified in nuclei. It has been studied in the nonrelativistic framework, but just started in the relativistic framework. The study indicates that the Δ decay width increases at normal nuclear density because the reduced pion-decay phase space is more than compensated by the opening of other decay channels. But a detailed discussion on this is still needed. The real part of the Δ self-energy is still unclear. As we pointed out, the h_s and h_v couplings in Eq. (63) play important roles, but there are still limited constraints on them. (Some constraints have been gained from an equation of state perspective, and others come from electron scattering.) As we realized in (Serot & Zhang, 2011a;b), the phenomenologically fitted spin-orbit coupling of the Δ in the nucleus may shed some light on this issue. Clearly, more efforts are needed to study Δ dynamics, which in the meantime is closely related to pion dynamics in the nuclear many-body system.

6. Acknowledgements

7. Appendix

A. Isospin indices, T matrices

Suppose $\vec{t}$ are the generators of some (ir)reducible representation of $SU(2)$; then it is easy to prove that $(\widetilde{\underline{\delta}} \equiv -\underline{e}^{-i\pi t^y})$

$$(-\vec{t}^{\,T})^{\,i}_{\,j} = \widetilde{\delta}^{ik}\,\vec{t}_k^{\,l}\,\widetilde{\delta}_{lj} \equiv \vec{t}^{\,i}_{\,j} \quad ,\text{i.e.,} \quad -\underline{\vec{t}}^{\,T} = \widetilde{\underline{\delta}}\,\underline{\vec{t}}\,\widetilde{\underline{\delta}}^{-1}\,. \tag{92}$$

Here the superscript T denotes transpose. This equation justifies the use of $\widetilde{\delta}$ as a metric linking the representation and the equivalent complex-conjugate representation. One easily finds for $\mathcal{D}^{(3/2)}$, $\mathcal{D}^{(1)}$, and $\mathcal{D}^{(1/2)}$,

$$\widetilde{\delta}^{ab} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad \widetilde{\delta}_{ab} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \tag{93}$$

$$\widetilde{\delta}^{ij} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad \widetilde{\delta}_{ij} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \tag{94}$$

$$\widetilde{\delta}^{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \widetilde{\delta}_{AB} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{95}$$

We now turn to the T matrices. As discussed in Sec. 3.1,

$$T_a^{\dagger\ iA} = \langle \frac{3}{2}; a|1, \frac{1}{2}; i, A \rangle \,,$$
$$T_{iA}^a = \langle 1, \frac{1}{2}; i, A|\frac{3}{2}; a \rangle \,. \tag{96}$$

It is easy to prove the following relations (here τ^i is a Pauli matrix):

$$\tau^i \tau_j = \widetilde{\delta}_j^i + i\,\widetilde{\epsilon}_{jk}^i \tau^k \,, \tag{97}$$

$$\left(P_i^j\right)_A^B \equiv T_{iA}^a\, T_a^{\dagger\ jB} = \widetilde{\delta}_i^j \widetilde{\delta}_A^B - \frac{1}{3}(\tau_i \tau^j)_A^B \,, \tag{98}$$

$$T_a^{\dagger\ iA}\, T_{iA}^b = \widetilde{\delta}_a^b \,. \tag{99}$$

Here P_i^j is a projection operator that projects $\mathcal{H}^{(1/2)} \otimes \mathcal{H}^{(1)}$ onto $\mathcal{H}^{(3/2)}$.

A few words about $\widetilde{\epsilon}_{jk}^i$ are in order here. We have the following transformations of pion fields:

$$\pi^i = \pi^I u_I^i \qquad \text{here, } i = +1, 0, -1; \quad I = x, y, z \,,$$

$$(\pi^{+1},\ \pi^0,\ \pi^{-1}) = (\pi^x,\ \pi^y,\ \pi^z) \begin{pmatrix} \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & 0 & \frac{-i}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix} \,. \tag{100}$$

Under such transformations,

$$\widetilde{\epsilon}^{ijk} \equiv u_I^i u_J^j u_K^k \epsilon^{IJK} = \det(u_I^i)\epsilon^{ijk} = -i\,\epsilon^{ijk}$$
$$\implies \quad \widetilde{\epsilon}^{ijk} = \begin{cases} -i, & \text{if } ijk = +1, 0, -1 \,; \\ -i\,\delta_{\mathcal{P}}, & \text{if } ijk = \mathcal{P}(+1, 0, -1) \,. \end{cases} \tag{101}$$

Here $\delta_{\mathcal{P}}$ is the phase related with the $\mathcal{P}$ permutation. It is $+\,(-)$ with an even (odd) number of permutations. To simplify the notation, we will ignore the *tilde* on $\widetilde{\delta}$ and $\widetilde{\epsilon}$ in other places.

B. Expansion of tilde objects

Here we show some details about $\widetilde{v}_\mu, \widetilde{a}_\mu, F_{\mu\nu}^{(\pm)}$ and others, which are needed for understanding electroweak interactions in QHD EFT. The pion-decay constant is $f_\pi \approx 93$ MeV.

$$\text{Tr}(\frac{\tau^i}{2}[U, \partial^\mu U^\dagger]) \approx 2i\epsilon^{ijk} \frac{\pi_j}{f_\pi} \frac{\partial_\mu \pi_k}{f_\pi} \,, \tag{102}$$

$$\text{Tr}(\frac{\tau^i}{2}\{U, \partial^\mu U^\dagger\}) \approx -2i \frac{\partial_\mu \pi^i}{f_\pi} \,, \tag{103}$$

$$\xi^\dagger \frac{\tau^i}{2}\xi + \xi\frac{\tau^i}{2}\xi^\dagger \approx \tau^i \,, \tag{104}$$

$$\xi^\dagger \frac{\tau^i}{2}\xi - \xi\frac{\tau^i}{2}\xi^\dagger \approx -\epsilon^{ijk}\frac{\pi_j}{f_\pi}\tau_k \,, \tag{105}$$

$$\widetilde{v}_\mu \approx \frac{1}{2f_\pi^2}\epsilon^{ijk}\pi_j\partial_\mu\pi_k\frac{\tau_i}{2} - \mathsf{v}_{i\mu}\frac{\tau^i}{2} - \epsilon^{ijk}\frac{\pi_j}{f_\pi}\frac{\tau_k}{2}\mathsf{a}_{i\mu} \,, \tag{106}$$

$$\widetilde{a}_\mu \approx \frac{1}{f_\pi}\partial_\mu\pi^i\frac{\tau_i}{2} + \mathsf{a}_{i\mu}\frac{\tau^i}{2} + \epsilon^{ijk}\frac{\pi_j}{f_\pi}\frac{\tau_k}{2}\mathsf{v}_{i\mu} \,, \tag{107}$$

$$\widetilde{v}_{\mu\nu} \approx \frac{1}{f_\pi^2}\epsilon^{ijk}\partial_\mu\pi_j\partial_\nu\pi_k\frac{\tau_i}{2}$$
$$- \left(i\left[\frac{1}{f_\pi}\partial_\mu\pi^i\frac{\tau_i}{2}\,,\,\mathsf{a}_\nu + \epsilon^{ijk}\frac{\pi_j}{f_\pi}\frac{\tau_k}{2}\mathsf{v}_{i\nu}\right] - (\mu \leftrightarrow \nu)\right)$$
$$+ \text{ background interference terms}, \tag{108}$$

$$\rho_{\mu\nu} = \partial_{[\mu}\rho_{\nu]} + i\overline{g}_\rho[\rho_\mu\,,\,\rho_\nu] + i([\widetilde{v}_\mu\,,\,\rho_\nu] - \mu \leftrightarrow \nu)\,, \tag{109}$$

$$f_{L\mu\nu} + f_{R\mu\nu} = 2\partial_{[\mu}\mathsf{v}_{\nu]} - 2i[\mathsf{v}_\mu\,,\,\mathsf{v}_\nu] - 2i[\mathsf{a}_\mu\,,\,\mathsf{a}_\nu]\,, \tag{110}$$

$$f_{L\mu\nu} - f_{R\mu\nu} = -2\partial_{[\mu}\mathsf{a}_{\nu]} + 2i[\mathsf{v}_\mu\,,\,\mathsf{a}_\nu] + 2i[\mathsf{a}_\mu\,,\,\mathsf{v}_\nu]\,, \tag{111}$$

$$F_{\mu\nu}^{(+)} \approx 2\partial_{[\mu}\mathsf{v}_{\nu]} + 2\epsilon^{ijk}\frac{\pi_j}{f_\pi}\frac{\tau_k}{2}\partial_{[\mu}\mathsf{a}_{i\nu]} + \text{background interference}, \tag{112}$$

$$F_{\mu\nu}^{(-)} \approx -2\partial_{[\mu}\mathsf{a}_{\nu]} - 2\epsilon^{ijk}\frac{\pi_j}{f_\pi}\frac{\tau_k}{2}\partial_{[\mu}\mathsf{v}_{i\nu]} + \text{background interference}. \tag{113}$$

C. Properties of projection operators in the spin-3/2 propagator

We have properties about these spin projectors:

$$(P_{ij}^{(I)})^{\mu\nu}(P_{kl}^{(J)})_{\nu\lambda} = \delta_{IJ}\delta_{jk}(P_{il}^{(I)})_\lambda^\mu \,, \tag{114}$$

$$\gamma^\mu P_{\mu\nu}^{(\frac{3}{2})} = P_{\mu\nu}^{(\frac{3}{2})}\gamma^\nu = 0 \,, \tag{115}$$

$$p^\mu P_{\mu\nu}^{(\frac{3}{2})} = P_{\mu\nu}^{(\frac{3}{2})}p^\nu = 0 \,. \tag{116}$$

Based on the above identities, we can prove that

$$P^{(\frac{3}{2})} + P_{11}^{(\frac{1}{2})} + P_{22}^{(\frac{1}{2})} = \mathbf{1} \,, \tag{117}$$

$$P_{11}^{(\frac{1}{2})} + P_{22}^{(\frac{1}{2})} \equiv P^{(\frac{3}{2}\perp)} \,, \tag{118}$$

$$\left[P^{(\frac{3}{2})} \,, \, \not{p} \right] = \left[P_{11}^{(\frac{1}{2})} \,, \, \not{p} \right] = \left[P_{22}^{(\frac{1}{2})} \,, \, \not{p} \right] = 0 \,. \tag{119}$$

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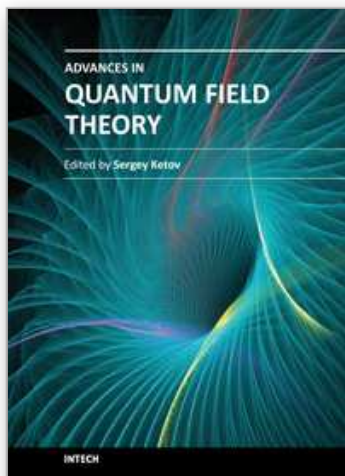
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