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## Polarization Losses in Optical Fibers

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### 1. Introduction

Very long span optical communications are mainly limited by the chromatic dispersion (CD) or group velocity dispersion (GVD), fiber nonlinearities, and optical amplifier noise (Agrawal 2005). Different frequencies of a pulse travel with their own velocities, which involves a pulse spreading. In a fiber-optic communication system, information is transmitted within a fiber by using a coded sequence of optical pulses whose width is determined by the bit rate of the system. The CD induced broadening of pulses is undesirable phenomenon since it interferes with the detection process leading to errors in the received bit pattern (Kogelnik & Jopson 2002; Mechels et al. 1997). Clearly GVD will limit the bit rate and the transmission distance of a fiber-optic communication system. GVD is basically constant over time, and compensation can be set once and forgotten (Karlsson 1994).

When the signal channel bit rates reached beyond 10 Gb/s, polarization mode dispersion (PMD) becomes interesting to a larger technical community. PMD is now regarded as a major limitation in optical transmission systems in general, and an ultimate limitation for ultra-high speed signal channel systems based on standard single mode fibers (Mahgerftech & Menyuk 1999). PMD arises in optical fibers when the cylindrical symmetry is broken due to noncircular symmetric stress. The loss of such symmetry destroys the degeneracy of the two eigen-polarization modes in fiber, which will cause different GVD parameters for these modes. In standard single mode fibers, PMD is random, i.e. it varies from fiber to fiber. Moreover, at the same fiber PMD will vary randomly with respect to wavelength and ambient temperature (Lin & Agrawal 2003b; Sunnerud et al. 2002). The differential group delay (DGD) between two orthogonal states of polarization called the principal states of polarization (PSP's) causes the PMD (Tan et al. 2002; Wang et al. 2001). As a pulse propagates through a light-wave transmission system with a PMD, the pulse is split into a fast and slow one, and therefore becomes broadened. This kind of PMD is commonly known as first-order PMD. Under first-order PMD, a pulse at the input of a fiber can be decomposed into two pulses with orthogonal states of polarization (SOP). Both pulses will arrive at the output of the fiber undistorted and polarized along different SOP's, the output SOP's being orthogonal (Chertkov et al. 2004; Foshchini & Poole 1991). Both the PSP's and the DGD are assumed to be frequency independent when only first-order PMD is being considered (Lin & Agrawal 2003c; Gordon & Kogelnik 2000).

Second-order PMD effects account for the frequency dependence of the DGD and the PSP's. The frequency dependence of the DGD introduces an effective chromatic dispersion of opposite sign on the signals polarized along the output PSP's (Elbers et al. 1997; Ibragimov &

Shtenge 2002). Fiber PMD causes a variety of impairments in optical fiber transmission systems. First of all there is the inter-symbol interference (ISI) impairment of a single digital transmission channel. The ISI impairment is caused by the DGD between the two pulses propagating in the fiber when the input polarization of the signal does not match one of the PSP's of the fiber PMD impairments due to inter-channel effects that occur in polarization-multiplexed transmission systems (Agrawal 2005; Yang et al. 2001).

There are two polarization effects that lead to impairments in the long-haul optical fiber transmission systems: PMD and polarization dependent loss (PDL) (Chen et al. 2003; Chen et al. 2007). The WDM systems whose channels are spread over a large bandwidth rapidly change their state of polarizations (SOP's) due to PMD so that the overall DOP of the system is nearly zero (Agrawal 2005; Kogelnik & Jopson 2002). At the same time different channels experience different amounts of PDL, and since the amplifiers maintain the total signal power nearly constant, individual channels undergo a kind of random walk so that it is possible for some channels to fade (Shtaif & Rosenberg 2005; Menyuk et al. 1997). Calculating the impairments due to the combination of PMD and PDL in WDM systems is a formidable theoretical challenge (Phua & Ippen 2005). Physically, light pulses polarized along these PSP's propagate without polarization-induced distortion. When there is no PDL, the two PSP's are orthogonal and correspond to the fastest and slowest pulses, which can propagate in the fiber (Yasser 2010; Yaman et al. 2006). They thus constitute a convenient basis for polarization modes. When the system includes PDL, the Jones formalism is still applicable, but several of the above facts are not valid anymore. The notion of PSP's is still correct, but the two PSP's are not orthogonal nor do they represent the fastest and slowest pulses (Yoon & Lee 2004).

In this chapter, the analysis of Jones and Stokes vectors and the relation between them were discussed in section 2. The statistics of PMD are presented in section 3. The pulse broadening in presence of PMD and CD were illustrated in section 4. In section 5, the principal comparison between PMD and birefringence vector will be obtained. The combined effects of PMD and PDL are presented in section 6. Finally, section 7 will summarize the effects of nonlinearity on the effective birefringence vector.

## 2. Polarization dynamics

The representation of polarization in Jones and Stokes spaces and the connection between the two spaces will be presented in this section. Throughout this chapter, it is assumed that the usual loss term of the fiber has been factored out so that one can deal with unitary transmission matrices. Light in optical fibers can be treated as transverse electromagnetic waves. Considering the two perpendicular and linearly polarized light waves propagating through the same region of space in the  $z$ -direction, the two fields can be represented in complex notation as (Azzam & Bashara 1989)

$$\vec{E}_x(z, t) = \hat{x} E_{x0} e^{i(kz - \omega t + \varphi_x)} \quad (1a)$$

$$\vec{E}_y(z, t) = \hat{y} E_{y0} e^{i(kz - \omega t + \varphi_y)} \quad (1b)$$

where  $\varphi_x$  and  $\varphi_y$  are the phases of the two field components, and  $k$  is the propagation constant. The resultant optical field is the vector sum of these two perpendicular waves, i.e.

$$\vec{E}(z, t) = \vec{E}_x(z, t) + \vec{E}_y(z, t) \quad (2)$$

The polarization state can be represented in terms of Jones vectors as

$$\hat{A} = \begin{bmatrix} a_x e^{i\varphi_x} \\ a_y e^{i\varphi_y} \end{bmatrix} \quad (3)$$

where  $a_x = E_{x0} / \sqrt{E_{x0}^2 + E_{y0}^2}$ ,  $a_y = E_{y0} / \sqrt{E_{x0}^2 + E_{y0}^2}$ , and  $\sqrt{a_x^2 + a_y^2} = 1$ . Here  $E_{x0}$  and  $E_{y0}$  are the initial amplitude components of the light. The familiar form of Jones vector is denoted as ket vector as (Gordon & Kogelnik 2000)

$$|s\rangle = \begin{bmatrix} s_x \\ s_y \end{bmatrix} = \begin{bmatrix} a_x e^{i\varphi_x} \\ a_y e^{i\varphi_y} \end{bmatrix} \quad (4)$$

whereas the bra  $\langle s|$  indicates the corresponding complex conjugate row vector, i.e.  $\langle s| = [s_x^* \ s_y^*]$ , where  $*$  indicates complex conjugation. The bra-ket notation is used to distinguish Jones vectors from another type of vectors that will be used in this chapter which is called the Stokes vectors. Partial correlation yields partial polarization and total correlation gives total polarization (Karlsson 1994; Sunnerud et al. 2002). When the light is coherent, Jones vectors are all of unit magnitude, i.e.  $\langle s|s\rangle = s_x s_x^* + s_y s_y^* = 1$ . Given the Jones vector, the values of the azimuth angle,  $\psi$ , and the ellipticity angle,  $\eta$ , can be found by solving the equations (Rogers 2008)

$$\tan 2\psi = \frac{2 \operatorname{Re}(s_y / s_x)}{1 - |s_y / s_x|^2} \quad (5a)$$

$$\sin 2\eta = \frac{2 \operatorname{Im}(s_y / s_x)}{1 + |s_y / s_x|^2} \quad (5b)$$

where Re and Im denote the real and imaginary parts, respectively. Fig.(1 a) illustrates Jones representation of polarization vector.

The Poincare sphere is a graphical tool in real three dimensional space that allows convenient description of polarized signals and polarization transformations caused by propagation through devices. Any SOP can be represented uniquely by a point on or within a unit sphere centered on a rectangular coordinates system. The coordinates of a point are the three normalized Stokes parameters describing the state of polarization (Azzam & Bashara 1989; Rogers 2008). Partially polarized light can be considered as a combination of purely polarized light and un-polarized light. Orthogonal polarizations are located diametrically opposite to the sphere. As shown in Fig.(1 b), linear polarizations are located on the equator. Circular states are located at the poles, with intermediate elliptical states continuously distributed between the equator and the poles (Karlsson 1994; Kogelnik & Jopson 2002). There are two angles (or degrees of freedom, i.e.  $\psi$  and  $\eta$ ) describing an arbitrary Jones vector. These angles can be interpreted as coordinates in a spherical coordinates system, and each polarization state can then correspond to a point, represented

by a Stokes vector,  $\hat{s} = (s_1, s_2, s_3)^t$  on the Poincare sphere, where  $t$  represents the transpose. The three Cartesian components can be defined as (Gordon & Kogelnik 2000)

$$\begin{aligned} s_1 &= |E_x|^2 - |E_y|^2 = \cos 2\psi \cos 2\eta \\ s_2 &= E_x E_y^* + E_y E_x^* = \sin 2\psi \cos 2\eta \\ s_3 &= i(E_y E_x^* - E_x E_y^*) = \sin 2\eta \end{aligned} \quad (6)$$

Therefore, the angle  $2\psi$  is the angle from the direction of  $s_1$  to the projection of  $\hat{s}$  on the  $s_1 - s_2$  plane, and  $2\eta$  is the angle from  $s_1 - s_2$  plane to the vector  $\hat{s}$ , see Fig. (1 b). Given Stokes vector, the values of  $\psi$  and  $\eta$  are obtained by solving the equations  $s_2 / s_1 = \tan 2\psi$ , and  $s_3 = \sin 2\eta$ .

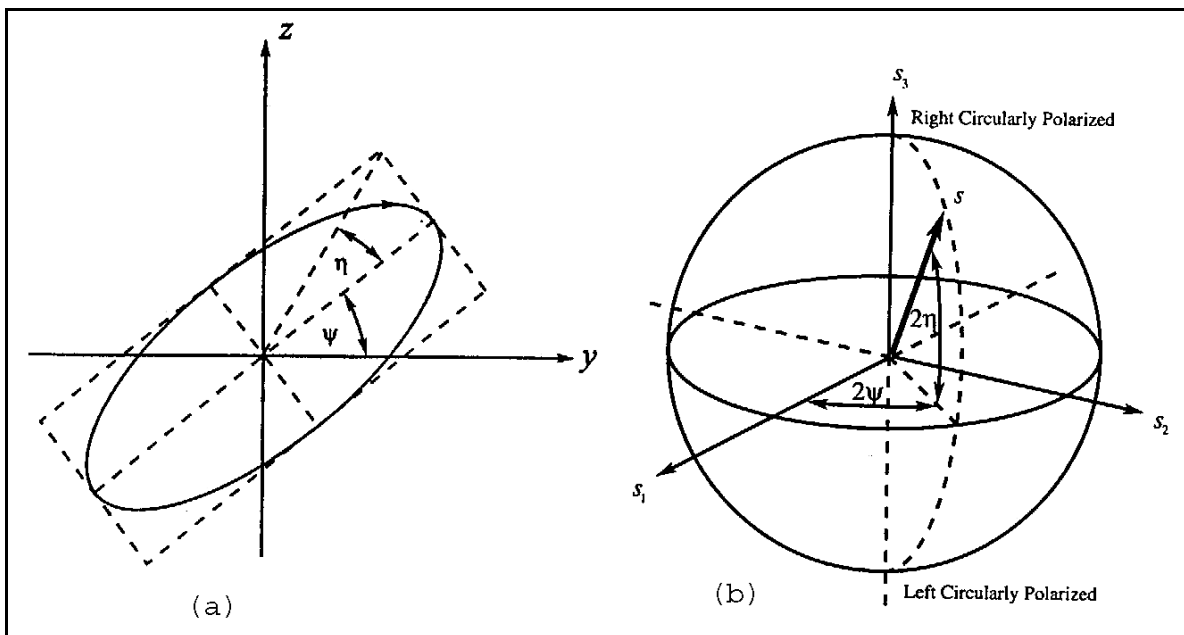


Fig. 1. Illustration of: a) Jones representation, b) Stokes representation.

Any Stokes vector  $\hat{s}$  is related to another one  $|s\rangle$  in Jones space as  $\hat{s} = \langle s | \vec{\sigma} | s \rangle$ , where  $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$  is the Pauli spin vector whose components are defined as (Levent et al. 2003)

$$\sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad (7)$$

It is important to note that if the angle between  $\hat{p}$  and  $\hat{s}$  in Stokes vector is  $\theta$ , then the angle between  $|p\rangle$  and  $|s\rangle$  in Jones space is  $\theta/2$ . That is; if two vectors are perpendicular in Jones space then the corresponding two vectors in Stokes space are antiparallel. Each of these two spaces gives certain illustrations according to the case of study. For totally polarization, the value of polarization vector is unity, elsewhere, the value differs from unity. In general, the three components of  $\hat{s}$  are not zero for elliptical polarization. The third component of  $\hat{s}$  equals zero for linear polarization, whereas the first two components of  $\hat{s}$  are zero for circular polarization. There is a unitary matrix,  $T$ , in Jones space which

relates output to input via  $|s\rangle = T|t\rangle$ , where  $|s\rangle$  and  $|t\rangle$  are the output and input Jones vectors, respectively. On the other hand, a transformation matrix (Muller),  $R$ , in Stokes space relates output to input via  $\hat{s} = R\hat{t}$ , where  $\hat{s}$  and  $\hat{t}$  are the output and input Stokes vectors, respectively. The transmission matrices are related as  $R\vec{\sigma} = T^\dagger\vec{\sigma}T$ , where  $\dagger$  denotes the transpose of the complex conjugate (Agrawal 2007; Chen et al. 2007).

### 3. Statistical managements

The effects of PMD are usually treated by means of the three-dimensional PMD vector that is defined as  $\vec{\tau} = \tau_{pmd}\hat{p}$ , where  $\hat{p}$  is a unit vector pointing in the direction of slow PSP and  $\tau_{pmd}$  is the DGD between the fast and slow components which is defined as (Mahgerftech & Menyuk 1999)

$$\tau_{pmd} = |\vec{\tau}| = \sqrt{\tau_1^2 + \tau_2^2 + \tau_3^2} \quad (8)$$

The PMD vector  $\vec{\tau}$  in Stokes space gives the relation between the output SOP,  $\hat{s}$ , and the frequency derivative of the output SOP:  $d\hat{s}(w)/dw = \vec{\tau}(w) \times \hat{s}(w)$ . The PSP's are defined as the states that  $\vec{\tau}(w) \times \hat{s}(w) = 0$ , so that no changes in output polarization can be observed close to these states at first order in  $w$ . To the first order, the impulse response of an optical fiber with PMD is defined as (Karlsson 1994)

$$h_{pmd}(T) = \gamma_+ \delta(T - \tau_{pmd}/2) |p_+\rangle + \gamma_- \delta(T + \tau_{pmd}/2) |p_-\rangle \quad (9)$$

where  $\gamma_\pm$  are the splitting ratios and  $|p_\pm\rangle$  are the PSP's vectors. The factors  $\gamma_\pm$  and  $\tau_{pmd}$  vary depending on the particular fiber and its associated stresses, where the splitting ratios can range from zero to one. Note that, the function  $h_{pmd}(T)$  is normalized in the range  $(-\infty$  to  $\infty)$ .

#### 3.1 Splitting ratios

Consider that the PSP's occur with a uniform distribution over the Poincare sphere, and that  $\hat{s}$  is aligned with the north pole of the sphere as shown in Fig.(2). The probability density of PSP's which is found in the range  $d\theta$  about the angle  $\theta$  relative to  $\hat{s}$  is proportional to the differential area  $2\pi \sin\theta d\theta$  sketched in the figure. As there is north/south symmetry in the differential area, the ranges  $(0$  to  $\pi/2)$  and  $(\pi/2$  to  $\pi)$  of  $\theta$  are combined to obtain the combined probability density  $p_\theta(\theta) = \sin\theta$ . For the effective range  $(0$  to  $\pi/2)$  describing the occurrence of PSP's with angle  $\theta$  (and  $\pi - \theta$ ) relative to  $\pm\hat{s}$ , the distribution  $p_\theta(\theta)$  is properly normalized through the range  $(0$  to  $\pi/2)$ . The analyses of splitting ratios have led to a number of important fundamental advances as well as the technical point of view (Rogers 2008; Kogelnik & Jopson 2002). The splitting ratios  $\gamma_\pm$  can be determined from the polarization vectors. In other words  $\gamma_\pm$  represent the projection of  $|p_\pm\rangle$  and  $|p_-\rangle$  onto  $|s\rangle$ . Formally,  $\gamma_\pm^2 = \langle s | p_\pm \rangle^2$ , where  $|s\rangle$  and  $|p_\pm\rangle$  are the input SOP and the two PSP's vectors.

If the PSP's are defined as  $|p_\pm\rangle = [p_{\pm x} \ p_{\pm y}]^t$ , then

$$|p_\pm\rangle \langle p_\pm| = \begin{bmatrix} p_{\pm x} \\ p_{\pm y} \end{bmatrix} \begin{bmatrix} p_{\pm x}^* & p_{\pm y}^* \end{bmatrix} = \begin{bmatrix} |p_{\pm x}|^2 & p_{\pm x} p_{\pm y}^* \\ p_{\pm y} p_{\pm x}^* & |p_{\pm y}|^2 \end{bmatrix} \quad (10)$$

where  $\langle p_{\pm} |$  are the transpose conjugation of  $|p_{\pm}\rangle$ . Now, it is straightforward to show that

$$\pm \hat{p} = \langle p_{\pm} | \vec{\sigma} | p_{\pm} \rangle = \begin{bmatrix} \pm p_1 \\ \pm p_2 \\ \pm p_3 \end{bmatrix} = \begin{bmatrix} \langle p_{\pm} | \sigma_1 | p_{\pm} \rangle \\ \langle p_{\pm} | \sigma_2 | p_{\pm} \rangle \\ \langle p_{\pm} | \sigma_3 | p_{\pm} \rangle \end{bmatrix} = \begin{bmatrix} |p_{\pm x}|^2 - |p_{\pm y}|^2 \\ p_{\pm x} p_{\pm y}^* + p_{\pm y} p_{\pm x}^* \\ i(p_{\pm y} p_{\pm x}^* - p_{\pm x} p_{\pm y}^*) \end{bmatrix} \quad (11)$$

Comparing Eqs.(10) and (11),  $|p_{\pm}\rangle \langle p_{\pm}| = (I_2 \pm \hat{p} \cdot \vec{\sigma}) / 2$  can be extracted. In turn, the splitting ratios can be calculated by using Eq.(11) and the fact that  $\langle a | \hat{p} \cdot \vec{\sigma} | a \rangle = \hat{p} \cdot \hat{a}$  as follows

$$\gamma_+^2 = \langle s | p_+ \rangle \langle p_+ | s \rangle = \langle s | (I_2 + \hat{p} \cdot \vec{\sigma}) | s \rangle / 2 = (1 + \hat{p} \cdot \hat{s}) / 2 = \cos^2(\theta / 2) \quad (12a)$$

$$\gamma_-^2 = \langle s | p_- \rangle \langle p_- | s \rangle = \langle s | (I_2 - \hat{p} \cdot \vec{\sigma}) | s \rangle / 2 = (1 - \hat{p} \cdot \hat{s}) / 2 = \sin^2(\theta / 2) \quad (12b)$$

Until now, the relationship between the splitting ratios and elevation angle was calculated, where the ratios  $\gamma_{\pm}$  are identical only for  $\theta = \pi / 2$ .

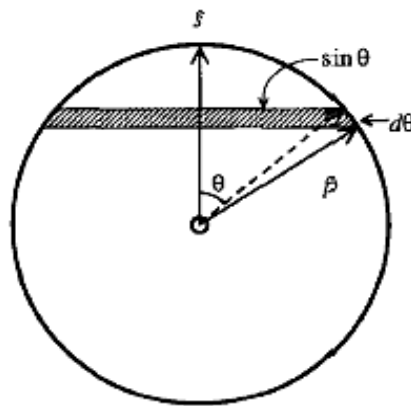


Fig. 2. Sketch of differential area on Poincaré sphere as a function of elevation angle  $\theta$ .

### 3.2 Statistics of DGD

Throughout this subsection, the PMD statistics have been carefully analyzed since it causes a variation in the pulse properties. A proper measure of pulse width for pulses of arbitrary shapes is the root-mean square (rms) width of the pulse defined as  $\tau_{rms} = \sqrt{\langle T^2 \rangle - \langle T \rangle^2}$ . The PMD induced pulse broadening is characterized by the rms value of  $\tau_{pmd}$ . The  $\tau_{rms}$  is obtained after averaging over random birefringence changes. The second moment of  $\tau_{pmd}$  is given by (Fushchini & Poole 1991)

$$\langle \tau_{pmd}^2 \rangle = \tau_{rms}^2 = 2(\Delta\beta_1)^2 \ell_c^2 [L / \ell_c + e^{-L/\ell_c} - 1] \quad (13)$$

where  $\ell_c$  is the correlation length that is defined as the length over which two polarization components remain correlated,  $\Delta\beta_1 = v_{gx}^{-1} - v_{gy}^{-1}$  is related to the difference in group velocities along the two PSP's. For distances  $L \gg 1 \text{ km}$ , a reasonable estimate of pulse broadening was obtained by taking the limit  $L \gg \ell_c$  in Eq.(13). The result is given by  $\tau_{rms} \approx \Delta\beta_1 \sqrt{2L\ell_c} = D_p \sqrt{L}$ , where  $D_p$  is known as the PMD parameter that takes the values (0.01-10)ps/ $\sqrt{\text{km}}$ . The variable  $\tau_{pmd}$  has been determined to obey a Maxwellian distribution of the form (Agrawal 2005)

$$p(\tau_{pmd}) = \sqrt{\frac{54}{\pi}} \frac{\tau_{pmd}^2}{\tau_{rms}^3} e^{-3\tau_{pmd}^2/2\tau_{rms}^2} \quad (14)$$

The mean of  $\tau_{pmd}$  is done simply as  $\bar{\tau}_{pmd} = \sqrt{8/3\pi} \tau_{rms}$ . Using this result, the Maxwellian distribution will take the form

$$p(\tau_{pmd}) = \frac{32}{\pi^2} \frac{\tau_{pmd}^2}{\bar{\tau}_{pmd}^3} e^{-4\tau_{pmd}^2/\pi\bar{\tau}_{pmd}^2} \quad (15)$$

A cursory inspection of Eq.(15) reveals that the  $p(\tau_{pmd})$  can be found if  $\bar{\tau}_{pmd}$  is known. Here, a relationship for  $\tau_{pmd}$  that will maximize  $p(\tau_{pmd})$  can be found. The distribution  $p(\tau_{pmd})$  has a maximum value at  $\tau_{pmd} = \tau_{pmd}^{\max} = \sqrt{\pi}\bar{\tau}_{pmd}/2$ . This conclusion provides a method for calculating the maximum likelihood value of  $\tau_{pmd}$  if  $\bar{\tau}_{pmd}$  is known.

### 3.3 Statistics of impulse response

The rms width of the impulse response,  $\tau_{eff}$ , can be readily calculated by substituting Eq.(9) into  $\tau_{rms} = \sqrt{\langle T^2 \rangle - \langle T \rangle^2}$  to yield

$$\tau_{eff} = \sqrt{\int_{-\infty}^{\infty} T^2 h_{pmd}(T) dT - \left[ \int_{-\infty}^{\infty} T h_{pmd}(T) dT \right]^2} = \sin \theta \tau_{pmd} / 2 \quad (16)$$

Using the result  $p_{\theta}(\theta) = \sin \theta$  and Eq.(16), the density distribution for  $\theta$  can be transformed to the density for  $\tau_{eff}$  as follows

$$p_{\tau_{eff}}(\tau_{eff}) = p_{\theta}(\theta(\tau_{eff})) \frac{d\theta}{d\tau_{eff}} = \frac{4\tau_{eff}}{\tau_{pmd} \sqrt{\tau_{pmd}^2 - 4\tau_{eff}^2}} \quad (17)$$

It is important to note that the probability density is a function of  $\tau_{eff}$  and  $\tau_{pmd}$ . As a consequence of this dependence, Eq.(17) can not be integrated to determine  $\tau_{eff}$  due to the presence of the other variable  $\tau_{pmd}$ . So, the next step is to seek about  $p_{\tau_{eff}}(\tau_{eff})$  in order to determine the statistical properties of output pulses. The joint probability distribution  $p(\tau_{eff}, \tau_{pmd})$  can be illustrated using Eqs.(14) and (17) as follows

$$p(\tau_{eff}, \tau_{pmd}) = \frac{64\tau_{eff}}{\pi\bar{\tau}_{pmd}^3} \frac{\tau_{pmd}}{\sqrt{\tau_{pmd}^2 - 4\tau_{eff}^2}} e^{-4\tau_{pmd}^2/\pi\bar{\tau}_{pmd}^2} \quad (18)$$

Recalling Eq.(16), it may be written as  $\tau_{pmd} = 2\tau_{eff}/\sin\theta$ . Since  $0 \leq \sin\theta \leq 1$ , such that  $2\tau_{eff} \leq \tau_{pmd} < \infty$ . The probability distribution  $p(\tau_{eff})$  can be found by integrating Eq.(18) about  $\tau_{pmd}$  through the range  $2\tau_{eff} \leq \tau_{pmd} < \infty$  to obtain

$$p(\tau_{eff}) = \frac{32\tau_{eff}}{\pi\bar{\tau}_{pmd}^2} e^{-16\tau_{eff}^2/\pi\bar{\tau}_{pmd}^2} \quad (19)$$



At a basic level, Eq.(17) is the same as Eq.(19) but the latter is a function of  $\tau_{eff}$  only, which can be integrated to obtain  $\tau_{eff}$ . However, both equations are normalized properly. The mean value of  $\tau_{eff}$  is determined as  $\theta = \pi / 2$ . So, Eq.(19) may be written as

$$p(\tau_{eff}) = \frac{\pi \tau_{eff}}{2 \tau_{eff}^2} e^{-\pi \tau_{eff}^2 / 4 \tau_{eff}^2} \quad (20)$$

The distribution  $p(\tau_{eff})$  has a maximum value at  $\tau_{eff} = \tau_{eff}^{\max} = \sqrt{\pi / 32 \tau_{pmd}}$ . This is equivalent to find the maximum likelihood value of  $\tau_{eff}$  if  $\tau_{pmd}$  is known.

### 3.4 Pulse characteristics

Using the PSP's as an orthogonal basis set, any input or output polarization can be expressed as the vector sum of two components, each aligned with a PSP. Within the realm of the first-order PMD, the output electric field from a fiber with PMD has the form (Rogers 2008)

$$|A_{out}(T)\rangle = \gamma_+ A_{in}(T - \tau_{pmd} / 2) |p_+\rangle + \gamma_- A_{in}(T + \tau_{pmd} / 2) |p_-\rangle \quad (21)$$

where  $A_{in}(T)$  is the input electric field. To determine the output power  $P_{out}(T) = \langle A_{out}(T) | A_{out}(T) \rangle$ , it is important to point out the orthogonality properties of Jones vectors, that is;  $\langle p_{\pm} | p_{\mp} \rangle = 0$  and  $\langle p_{\pm} | p_{\pm} \rangle = 1$ . Note that, we perform the derivation using a normalized Gaussian pulse that takes the form  $A_{in}(T) = D \exp(-T^2 / 2T_0^2)$ , where  $D = \sqrt{E_{in} / T_0 \sqrt{\pi}}$ ,  $T_0$  is the initial pulse width, and  $E_{in}$  is the input pulse energy. For normalized power, we make  $D^2 = 1$ . Therefore, according to Eq.(21), the shifted pulses will reshape as

$$A_{in}(T \pm \tau_{pmd} / 2) = D \exp\left[-\frac{(T \pm \tau_{pmd} / 2)^2}{2T_0^2}\right] \quad (22)$$

Substituting Eqs.(12) and (22) into (21), using the output power definition, using the orthogonality properties of Jones vectors, and simplified the result, we obtain the following expression

$$P_{out}(T) = \left[ \cos^2(\theta / 2) e^{-T \tau_{pmd} / T_0^2} + \sin^2(\theta / 2) e^{T \tau_{pmd} / T_0^2} \right] e^{-(4T^2 + \tau_{pmd}^2) / 4T_0^2} \quad (23)$$

The width of the output pulse  $T_1$  can be determined as follows

$$T_1 = \sqrt{\int_{-\infty}^{\infty} T^2 P_{out}(T) dT - \left[ \int_{-\infty}^{\infty} T P_{out}(T) dT \right]^2} = \sqrt{T_0^2 + (\tau_{pmd} / 2)^2 \sin^2 \theta} \quad (24)$$

The time jittering of the pulse can be found by determining the maximum value of  $P_{out}(T)$ . This maximum value will happen at  $T = T_{peak} = \tau_{pmd} \cos(\theta) / 2$ . The peak power, as a function of DGD and an angle  $\theta$ , at the pulse center can be determined by substituting the latter result into Eq.(23) to get

$$P_{peak}(\tau_{pmd}, \theta) = \cos^2(\theta / 2) e^{-\sin^2(\theta / 2) \tau_{pmd}^2 / T_0^2} + \sin^2(\theta / 2) e^{-\cos^2(\theta / 2) \tau_{pmd}^2 / T_0^2} \quad (25)$$

At this point, we derive formulas for the output power form, final width, time jittering (shifting), and peak power as functions of the random physical variables  $\theta$  and  $\tau_{pmd}$ .

Fig.(3) illustrates the simulation with the parameters:  $L = 50 \text{ km}$ ,  $D_p = 0.5 \text{ ps} / \sqrt{\text{km}}$ , and  $T_0 = 5 \text{ ps}$ . The solid line represents the original pulse while the discrete lines represent the resulted pulses with different values of  $\tau_{pmd}$  ranging from 0 to  $8 \text{ ps}$ , where the closest to  $T = 0$  is the pulse that has least value of  $\tau$ . At the angle  $\theta = 0$ , one note that the pulse is faced only by a displacement to the right at  $T_{peak} = \tau_{pmd} / 2$ . Increasing  $\theta$ , the pulse width and distortion will be increased, while the power and shifting will be decreased. These variations are the greatest at  $\theta = \pi / 2$ . After  $\theta = \pi / 2$ , the effects are reversed. At  $\theta = \pi$ , again the pulse is faced only by a displacement but to the left at  $T_{peak} = -\tau_{pmd} / 2$ . It is clear that the penalty could be greater if  $\theta = \pi / 2$  and will be zero at  $\theta = 0$  or  $\pi$ .

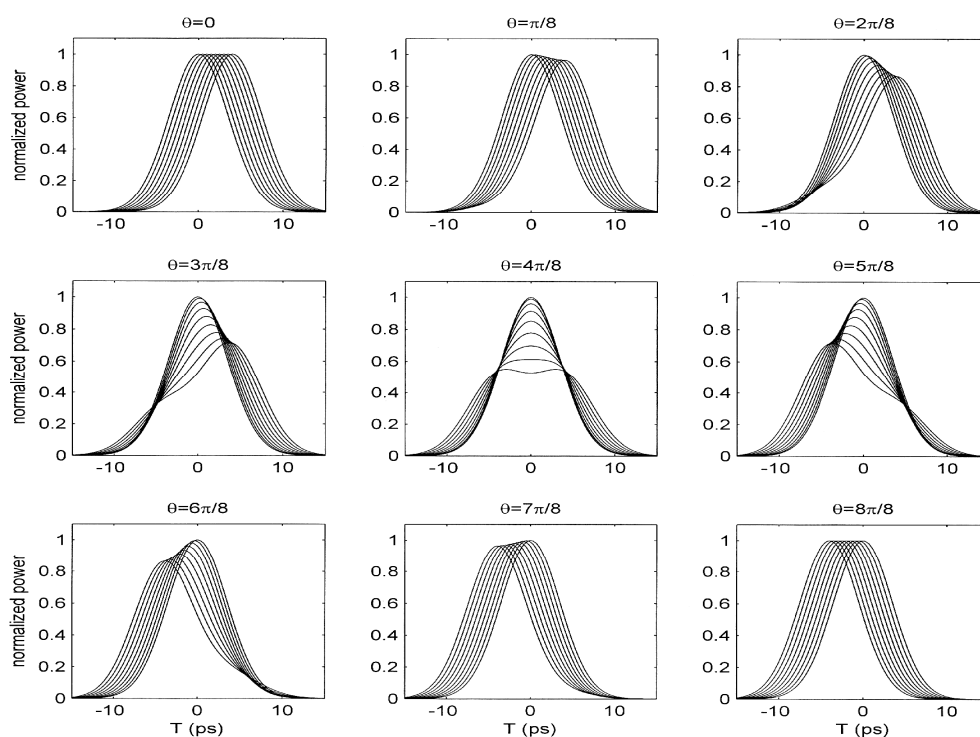


Fig. 3. Pulse shape with different values of  $\tau_{pmd}$  and  $\theta$  for different values of  $\tau_{pmd}$ ; the lower value of  $\tau_{pmd}$  is the closest to the pulse center.

#### 4. Polarization mode dispersion and chromatic dispersion

The pulses that propagate through single mode fiber (SMF) are affected by two types of dispersion which are CD and PMD. Notice that the effects of the two types of dispersion happen at the same time, so to give a distinct sense of the two types of dispersion we decided to obtain the effects in the frequency domain. The initial pulse,  $\tilde{A}(0, \omega) = \mathfrak{F}\{A(0, T)\}$ , first faces the affect of CD (the transfer function  $H_1(\omega)$ ) to obtain  $H_1(\omega)\tilde{A}(0, \omega)$ . The CD does not depend on SOP therefore the input SOP (the Jones vector  $|a\rangle$ ) will not change. Next, the pulse divides into two orthogonal components towards PSP's ( $|a^+\rangle$  and  $|a^-\rangle$ ) under the effects of PMD. The component in the direction  $|a^+\rangle$  will face the effects of the function  $H_{2s}(\omega)$  to obtain the pulse  $H_{2s}(\omega)H_1(\omega)\tilde{A}(0, \omega)$  and at the same time the SOP will

change from  $|a^+ \rangle$  to  $|b^+ \rangle$ . On the other hand, the pulse in the direction  $|a^- \rangle$  faces the effects of the function  $H_{2f}(w)$  to yield  $H_{2f}(w)H_1(w)\tilde{A}(0,w)$  and also the SOP will change from  $|a^- \rangle$  to  $|b^- \rangle$ . The input or output PSP's does remain orthogonal when the PDL is absent. Finally, the vector sum of the two components will produce the final pulse  $H_2(w)H_1(w)\tilde{A}(0,w)$ . The transfer function of the CD of lossless fiber in frequency domain is  $H_1(w) = \exp(i w^2 \beta_2 L / 2)$ , where  $\beta_2 = -\lambda^{-2} d(\lambda) / 2\pi c$ ,  $d(\lambda)$  is fiber chromatic dispersion parameter,  $L$  is the fiber length, and  $\lambda$  is light wavelength. Now, assume that there is negligible PDL, so that we can use the principal states model (Lin & Agrawal 2003b; Ibragimov & Shtenge 2002; Foshchini & Poole 1991) to characterize first-order PMD. Under this model, there exist a pair of orthogonal input PSP's,  $|a^+ \rangle$  and  $|a^- \rangle$ , and a pair of orthogonal output PSP's,  $|b^+ \rangle$  and  $|b^- \rangle$ , where all of PSP's are expressed as Jones vectors. If an arbitrary polarized field  $\tilde{A}_a(t) = A_a(t) |a \rangle$  is input to the fiber, this input field can be projected onto the two PSP's as

$$\tilde{A}_a(T) = \gamma_+ A_a(T) |a^+ \rangle + \gamma_- A_a(T) |a^- \rangle \quad (26)$$

In terms of first-order PMD, the output field of the fiber takes the form

$$\tilde{A}_b(T) = \gamma_+ A_b(T - \tau_{pmd} / 2) |b^+ \rangle + \gamma_- A_b(T + \tau_{pmd} / 2) |b^- \rangle \quad (27)$$

According to Eq.(9), the fiber transfer functions for first-order in the time and frequency domains are given by

$$h_2(t) = \gamma_+ \delta(T - \tau_{pmd} / 2) |b^+ \rangle + \gamma_- \delta(T + \tau_{pmd} / 2) |b^- \rangle \quad (28a)$$

$$H_2(w) = \gamma_+ e^{i w \tau_{pmd} / 2} |b^+ \rangle + \gamma_- e^{-i w \tau_{pmd} / 2} |b^- \rangle \quad (28b)$$

The root mean square width of this impulse response which can be calculated as

$$\begin{aligned} \langle T \rangle &= \int_{-\infty}^{\infty} T h_2(T) dT = (\gamma_+ \tau_{pmd} |b^+ \rangle - \gamma_- \tau_{pmd} |b^- \rangle) / 2 \\ \langle T^2 \rangle &= \int_{-\infty}^{\infty} T^2 h_2(T) dT = (\gamma_+ \tau_{pmd}^2 |b^+ \rangle + \gamma_- \tau_{pmd}^2 |b^- \rangle) / 4 \\ \tau_{rms}^{\pm} &= [\sqrt{\langle T^2 \rangle - \langle T \rangle^2}]_{\pm} = 0 \end{aligned} \quad (29)$$

where the signs (+, -) mean that the impulse response in directions of  $|b^+ \rangle$  or  $|b^- \rangle$ , respectively. That is; the width of an impulse response in the direction of PSP's will be zero, while the width in the direction of  $|b \rangle$  will be  $\tau_{rms} = \sin \theta \tau_{pmd} / 2$ . This represents the extra width that results due to the effects of PMD on the propagated signal. It is clear that, if the input SOP is in direction of PSP's, then the pulse will not suffer any broadening.

The Fourier transformation of the initial pulse takes the form  $\tilde{A}(0,w) = D\sqrt{2\pi}T_0 \exp(-w^2 T_0^2 / 2)$ . The total effects on the pulse shape can be obtained by using the convolution of the transfer functions of the combined PMD and CD with the input Gaussian signal in the time domain, or equivalently by using the inverse Fourier transform as follows

$$A(z,T) = \mathfrak{F}^{-1}\{\tilde{A}(0,w) \cdot H_1(w) \cdot H_2(w)\} = \cos(\theta / 2) A_+(z,T) |b^+ \rangle + \sin(\theta / 2) A_-(z,T) |b^- \rangle \quad (30)$$

where

$$A_{\pm}(z, T) = D \frac{T_0}{\chi} \exp\left(-\frac{T_{\pm}^2}{2 T_1^2}\right) \exp(i \phi_{\pm}(z, T))$$

$$\chi = \sqrt[4]{T_0^4 + (\beta_2 z)^2}$$

$$T_{\pm} = T \pm \tau_{pmd} / 2$$

$$T_1 = \sqrt{T_0^2 + (\beta_2 z / T_0)^2}$$

$$\phi_{\pm}(z, T) = -\frac{\beta_2 z}{2 T_1^2} \frac{T_{\pm}^2}{T_0^2} + \frac{1}{2} \tan^{-1}(\beta_2 z / T_0^2)$$

The parameter  $T_1$  represents the pulse width including CD effects where it is the same for the two orthogonal components. The width of each component will not increase under the effects of PMD, but the pulse which results from the vector sum of the two orthogonal components will face a broadening that can be determined by  $\tau_{rms}$ . The parameters  $\phi_{\pm}(z, T)$  represent the nonlinear phases that generate through the propagation in optical fiber. The nonlinear phase as a function of time differs from one component to another by the amount  $\tau_{pmd}$ , but in the frequency domain they remain the same and add the same value of noise to both components. The frequency chirp can be written as

$$\delta w_{\pm}(T) = -\frac{\partial \phi_{\pm}(z, T)}{\partial T} = \frac{\beta_2 z}{T_1^2} \frac{T_{\pm}}{T_0^2} = \frac{\beta_2 z}{2 T_1^2} \frac{T \pm \Delta \tau / 2}{T_0^2} \quad (31)$$

This means that the new frequencies generated are similar for the two components and the difference lies in  $T \pm \tau_{pmd} / 2$  only, which means that one of the components advances the other by time  $\tau_{pmd}$ . Eq.(30) explains that the pulse amplitude will decrease by increasing the propagation distance, which will be converted to the same equations as in reference (Agrawal 2007) by ignoring the effects of PMD. The Jones vectors  $|b^+ \rangle$  and  $|b^- \rangle$  are orthogonal, i.e.  $\langle b^+ | b^- \rangle = 0$ . That is enough to assume a random form to one of them to find the other. For example, if  $|b^+ \rangle = [x \quad iy]^t$  then  $|b^- \rangle = [iy \quad x]^t$  keeping in mind that all the polarization vectors have unit values.

Now, the reconstructed width after including the effects of CD is  $T_1$ . Next, the input pulse has a width  $T_1$  which will be increased by the amount  $\tau_{rms}$  due to the PMD. Such that, the final width will be

$$T_f = \sqrt{T_1^2 + \tau_{pmd}^2 \sin^2 \theta / 4} \quad (32)$$

Fig.(4 a) illustrates the shape of pulse for various values of  $\beta_2$ , assuming  $\tau_{pmd} = 2 \text{ ps}$ ,  $\theta = \pi / 2$ ,  $T_0 = 10 \text{ ps}$ , and  $L = 60 \text{ km}$ . Since  $\tau_{pmd}$  is constant for all cases, this implies that the time separation between the orthogonal components remains the same. The width of both components increases (under the effects of CD) by increasing  $\beta_2$ . Consequently, the width of the final pulse increases by increasing  $\beta_2$ , but the amplitude is decreased. The existence of CD causes a broadening factor (BR) of value  $T_1 / T_0$ , and the existence of PMD adds a BR of value  $\tau_{rms} / T_1$ . That is; the width of pulse will increase due to the existence of

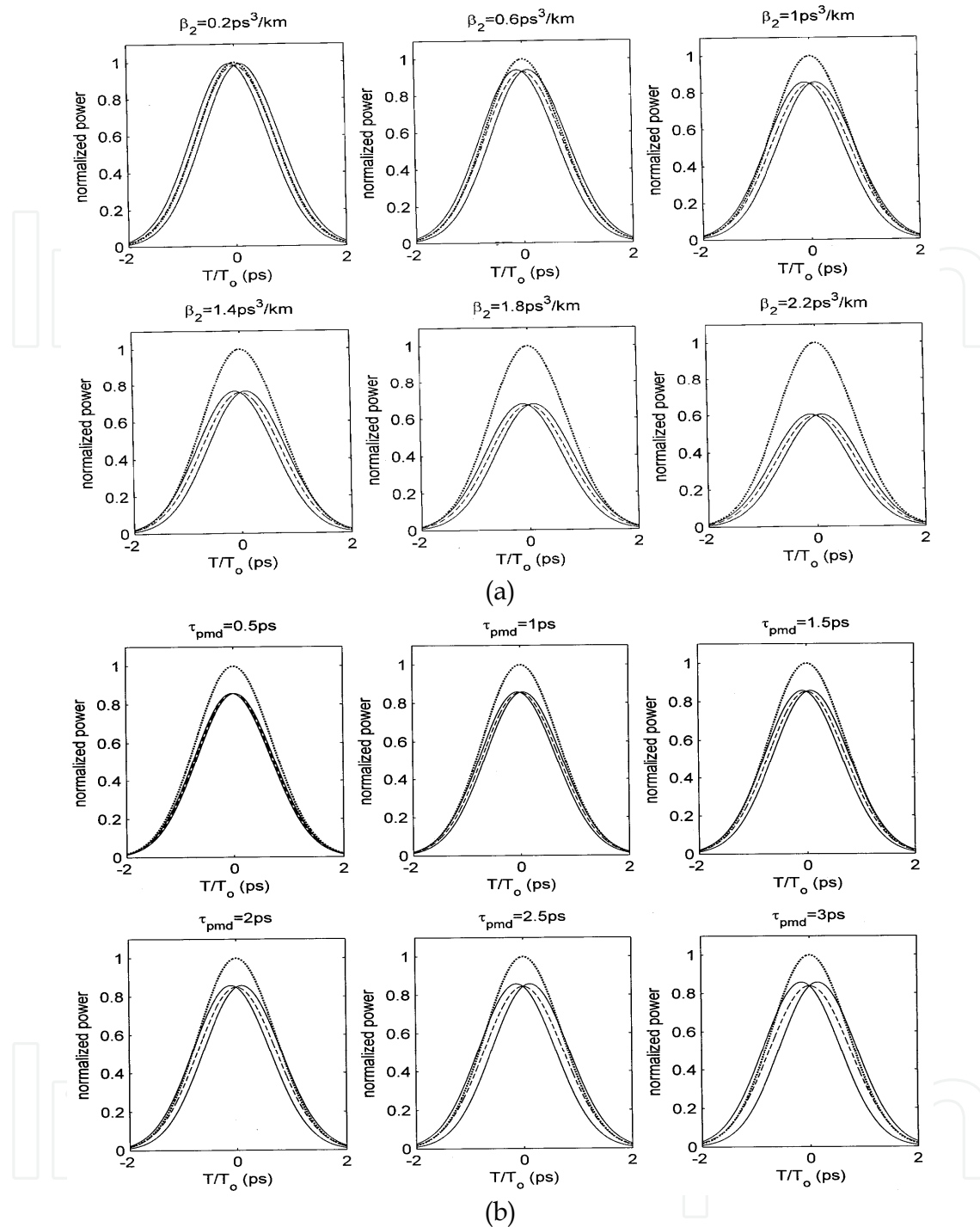


Fig. 4. Evolution of the pulse shape at  $\theta = \pi / 2$ ,  $T_0 = 10$  ps, and  $L = 60$  km : a) for various values of  $\beta_2$  and  $\tau_{pmd} = 2$  ps, b) for various values of  $\tau_{pmd}$  and  $\beta_2 = 1$  ps<sup>3</sup> / km. The dotted, continuous, and discrete lines refer to the initial pulse, two orthogonal components, and final pulse, respectively.

the two types of dispersion. In other words, the time separation between the two orthogonal components will be fixed, both amplitude and width of the pulse will change under the effects of CD. Fig.(4 b) illustrates the shape of pulse for various values of  $\tau_{pmd}$ , assuming  $\beta_2 = 1$  ps<sup>3</sup> / km,  $\theta = \pi / 2$ ,  $T_0 = 10$  ps, and  $L = 60$  km. Since  $\beta_2$  and  $L$  are

constants this implies that  $T_1$  is constant also. That is; the width of both orthogonal components are similar for all  $\tau_{pmd}$  values, but the difference appears as a time increase separation between the two components. This leads to adding a BR of value  $\tau_{rms} / T_1$  to the reconstructed pulse.

## 5. Polarization mode dispersion and birefringence

In the optical fibers, the birefringence vector  $\vec{\beta}$  may be defined in two forms as (Schuh et al. 1995)

$$\vec{\beta}_L = \begin{bmatrix} \Delta\beta \cos 2\alpha \\ \Delta\beta \sin 2\alpha \\ 0 \end{bmatrix} \quad \text{or} \quad \vec{\beta}_{NL} = \begin{bmatrix} \Delta\beta \cos 2\alpha \\ \Delta\beta \sin 2\alpha \\ \zeta T \end{bmatrix} \quad (33)$$

where  $\alpha$  is the angle of birefringence in Jones space,  $\Delta\beta$  is the magnitude of linear birefringence, i.e.  $\Delta\beta = |\vec{\beta}_L|$ ,  $\zeta$  is the photo-elastic coefficient of glass, and  $T$  is the twist rate in (rad/m). The angle  $\alpha$  is not constant along the fiber; also,  $\Delta\beta$  and  $T$ . This means that each segment of fiber has a birefringence vector differs from another position randomly, depending on the values of  $\alpha$ ,  $\Delta\beta$ , and  $T$ . If  $\Delta\beta = |\vec{\beta}_L|$ , then  $\vec{\beta}_L = \Delta\beta \hat{r}$ , where  $\hat{r}$  represents a unit vector in Stokes space. The vector  $\hat{r}$  represents a rotation axis of the polarization vector, which differs from one section to another randomly.

Consequently, the PMD vector can be defined as a function of  $\hat{r}$  and  $\phi$  (Gordon & Kogelnik 2000)

$$\vec{\tau} = \phi_w \hat{r} + \hat{r}_w \sin \phi + \hat{r}_w \times \hat{r} (\cos \phi - 1) \quad (34)$$

where  $\phi = \Delta\beta \Delta z$  represents the rotation angle of the polarization state vector  $\hat{s}$  around the birefringence vector  $\vec{\beta}$ , and  $\phi_w$  and  $\hat{r}_w$  represent their first derivatives of frequency. Eq.(34) obtains that the angle and direction of rotation control the resultant vector  $\vec{\tau}$ . Substituting the first definition in Eq.(33) into (34), yields

$$\vec{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} \varepsilon \Delta z \cos(2\alpha) \\ \varepsilon \Delta z \sin(2\alpha) \\ 0 \end{bmatrix} + 2 \frac{d\alpha}{dw} \begin{bmatrix} -\sin(\phi) \sin(2\alpha) \\ \sin(\phi) \cos(2\alpha) \\ 1 - \cos(\phi) \end{bmatrix} \quad (35)$$

where  $\varepsilon = d\Delta\beta / dw$  represents PMD parameter, and  $\Delta z$  is the fiber segment length. On the other hand,  $\vec{\tau}$  is a function of  $w$ , which may be written as a Taylor series around the central frequency  $w_0$  as follows (Agrawal 2005)

$$\vec{\tau}(w) = \vec{\tau}(w_0) + \Delta w \frac{d\vec{\tau}}{dw} \Big|_{w=w_0} + \frac{\Delta w^2}{2} \frac{d^2\vec{\tau}}{dw^2} \Big|_{w=w_0} + \dots \quad (36)$$

Comparing Eqs.(35) and (36), the first term on the right hand side of Eq.(35) will represent the first order of PMD vector, while the second term indicates all higher orders of PMD vector. Accounting that the higher orders depend on the value of  $d\alpha / dw$ . For a very small variations of  $\alpha$  with frequency, the second term on the right hand side of Eq.(35) may be

neglected. Elsewhere, the higher order effects must be included through the determination of PMD vector.

### 5.1 Linear birefringence

Neglecting the higher order effects makes the PMD vector as follows

$$\vec{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} \varepsilon \Delta z \cos(2\alpha) \\ \varepsilon \Delta z \sin(2\alpha) \\ 0 \end{bmatrix} = \varepsilon \Delta z \hat{r} = \frac{\varepsilon \Delta z}{\Delta \beta} \vec{\beta} = \text{const. } \vec{\beta} \quad (37)$$

This means,  $\vec{\tau}$  coincides with the birefringence vector  $\vec{\beta}$  if the intrinsic birefringence is linear and the higher order PMD effects are neglected. Elsewhere, the two vectors are never coincided. Using Eq.(37), we can obtain DGD of the fiber segment as

$$DGD_1 = \tau_{pmd}^{(1)} = |\vec{\tau}| = \varepsilon \Delta z \quad (38)$$

The value of  $DGD_1$  represents the delay time between the two components of polarization in a single segment of the optical fiber. Since the DGD's of the fiber segments are random, so that  $DGD_1$  can be calculated as  $\langle \tau_{pmd} \rangle = \frac{1}{N} \sum_{i=1}^N \tau_{pmd}^i$ . For the case of wide frequency band, the higher order effects of the PMD must be included. The  $DGD_2$  of this case can be obtained using Eq.(35) as follows

$$DGD_2 = \tau_{pmd}^{(2)} = \sqrt{(\varepsilon \Delta z)^2 + 8(1 - \cos \phi)^2 \alpha_w^2} \quad (39)$$

Clearly, the  $DGD_2$  is related to the change of  $\alpha$  with respect to frequency, and  $\tau_{pmd}^{(1)} < \tau_{pmd}^{(2)}$ . This means that the higher order effects increase the DGD. The angle between the two vectors  $\vec{\tau}$  and  $\vec{\beta}$  is determined as:  $\psi = \cos^{-1}(\tau_{pmd}^{(1)} / \tau_{pmd}^{(2)})$ . This means that the two vectors in the same direction if the higher order PMD is neglected, i.e.  $\tau_{pmd}^{(1)} = \tau_{pmd}^{(2)}$ .

### 5.2 Nonlinear birefringence

For the nonlinear intrinsic birefringence,  $\vec{\tau}$  can be calculated using the second definition in Eq.(33) and (34) as follows

$$\vec{\tau} = \begin{bmatrix} (a_1 + a_3 \sin \phi) \cos(2\alpha) + a_6(\cos \phi - 1) \sin(2\alpha) \\ (a_1 + a_3 \sin \phi) \sin(2\alpha) - a_6(\cos \phi - 1) \cos(2\alpha) \\ a_2 + a_5 \sin \phi \end{bmatrix} + \frac{d\alpha}{d\omega} \begin{bmatrix} -a_4 \sin \phi \sin(2\alpha) + a_7(\cos \phi - 1) \cos(2\alpha) \\ a_4 \sin \phi \cos(2\alpha) + a_7(\cos \phi - 1) \sin(2\alpha) \\ a_8(\cos \phi - 1) \end{bmatrix} \quad (40)$$

where the parameters  $a_1$  into  $a_8$  are defined as

$$\begin{aligned} a_1 &= \frac{\Delta \beta \varepsilon \Delta z}{\Delta \beta_{NL}} & a_2 &= -\frac{\Delta \beta \varepsilon \zeta T}{\Delta \beta_{NL}^2} \\ a_3 &= a_5 = -\frac{\zeta T \Delta \beta \varepsilon}{K} & a_4 &= \frac{2 \Delta \beta}{\Delta \beta_{NL}} \\ a_7 &= \frac{\zeta T \Delta \beta^2 \varepsilon}{\Delta \beta_{NL}^2} & a_8 &= -\frac{2b_L}{\Delta \beta_{NL}^2} \end{aligned}$$

$$\Delta\beta_{NL} = \sqrt{\Delta\beta^2 + (\zeta T)^2} \quad K = (\Delta\beta^2 + \zeta^2 T^2)^{3/2}$$

$$a_6 = \frac{2\zeta T \Delta\beta}{\Delta\beta_{NL}} - \frac{(\zeta T)^3 \varepsilon}{K}$$

Eq.(40) represents a new formula of the PMD vector demonstrating the difficulties to compensate the noise that arises due to PMD when the pulse propagates through optical fibers. Many approaches have been proposed (McCurdy et al. 2004; Lima et al. 2001; Vanwiggeren & Ray 1999; Ibragimov & Shtenge 2002; Schuh et al. 1995), which deal only with the first order of PMD. This means that the compensation depends on the first term presented in the right hand side of Eq.(40) and assuming that the birefringence vector is linear.

The vector  $\vec{\tau}$  can be found from  $\vec{\beta}$ . Ignoring the higher orders of the vector  $\vec{\tau}$ , the vector  $\vec{\tau}$  is linear only if  $\vec{\beta}$  is linear, otherwise they are different. When the distance is changed this implies to rotate  $\hat{s}$  around  $\vec{\beta}$  by an angle  $\phi$ . On the other hand, the change of frequency causes to rotate  $\hat{s}$  around  $\vec{\tau}$  by an angle  $\theta$ . Fig.(5 a) illustrates the relation among the three vectors  $\hat{s}$ ,  $\vec{\beta}$ , and  $\vec{\tau}$  where the polarization vector  $\hat{s}$  is rotating around  $\vec{\beta}$  and  $\vec{\tau}$ . Adding the higher orders of  $\vec{\tau}$ , the vector  $\vec{\tau}$  is now nonlinear which does not coincide with the vector  $\vec{\beta}$  as illustrated in Fig.(5 b). The general case considers the birefringence vector is nonlinear and assuming all orders of  $\vec{\tau}$  as illustrated in Fig.(5 c), which shows that each vector rotates in Stokes space.

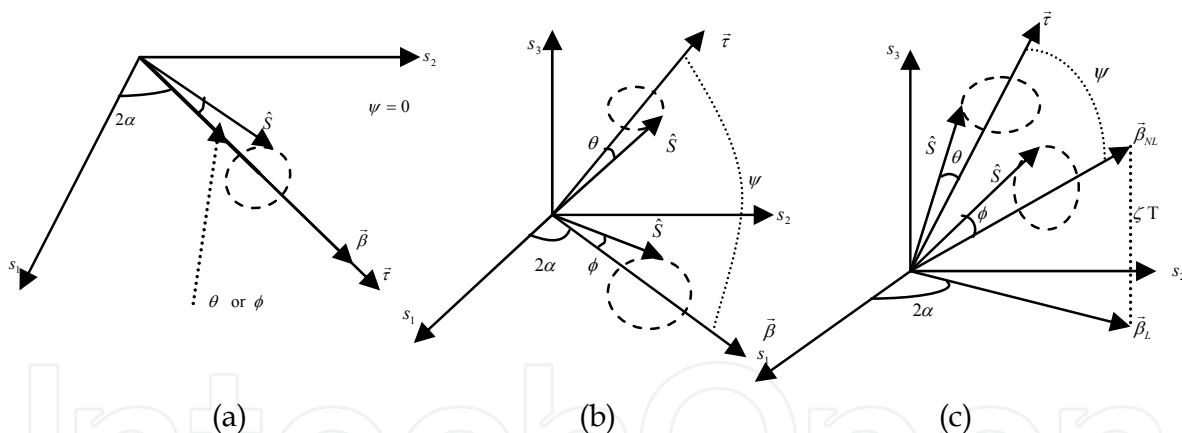


Fig. 5. Rotation of SOP around  $\vec{\beta}$  and  $\vec{\tau}$ : a)  $\vec{\beta}$  and  $\vec{\tau}$  are linear, b)  $\vec{\beta}$  is linear and  $\vec{\tau}$  is nonlinear, c)  $\vec{\beta}$  and  $\vec{\tau}$  are nonlinear.

## 6. Combined PMD and PDL effects

As far as the continuum limit at the end is set, the following simple arrangement are considered: each PMD element (having  $\vec{\tau}_i$  vector) is followed by a PDL element (having  $\vec{\alpha}_i$ ) leading to the following transmission Jones matrix (Yasser 2010)

$$T = T_{PDL} T_{PMD} = \exp\left(\frac{1}{2} \vec{\alpha}_j \cdot \vec{\sigma}\right) \exp\left(-\frac{i\omega}{2} \vec{\tau}_j \cdot \vec{\sigma}\right) \quad (41)$$

where



$$\exp\left(\frac{1}{2} \vec{\alpha}_j \cdot \vec{\sigma}\right) = [\cosh(\alpha_j / 2) + (\hat{\alpha}_j \cdot \vec{\sigma}) \sinh(\alpha_j / 2)]$$

$$\exp\left(-\frac{i\omega}{2} \vec{\tau}_j \cdot \vec{\sigma}\right) = [\cos(\omega\tau_{pmd}^{(j)} / 2) - i(\hat{\tau}_j \cdot \vec{\sigma}) \sin(\omega\tau_{pmd}^{(j)} / 2)]$$

Here  $\vec{\tau}_j = \tau_{pmd}^{(j)} \hat{p}_j$  represents the j-th PMD segment having DGD  $\tau_{pmd}^{(j)}$  and the fast polarization axis is expressed by the unit vector  $\hat{p}_j$  in the Stokes space. The PMD vector  $\vec{\tau}$  is, generally, frequency dependent; the first term in the Taylor expansion of  $\vec{\tau}(w)$  is conventionally referred to as the first-order PMD (Agrawal 2005). To clarify the notation used in this section, we attempt to keep the notation simple and transparent while linking to the notation already established as much as possible. The following is an abbreviated group of present notation: The letters C, c, S, and s represent  $\cos(\omega\tau_{pmd}^{(j)} / 2)$ ,  $\cosh(\alpha_j / 2)$ ,  $\sin(\omega\tau_{pmd}^{(j)} / 2)$ , and  $\sinh(\alpha_j / 2)$ , respectively.

Notice that, in this representation PDL matrix, the polarization component of the field that is parallel to  $\vec{\alpha}_j$  experiences a gain  $e^\alpha$ , but the anti-parallel component is attenuated by  $e^{-\alpha}$ . The expressions  $e^{\pm\alpha_j/2}$  represent the eigenvalues  $\lambda_1, \lambda_2$  of PDL matrix. The vector  $\vec{\alpha}_j = \alpha_j \hat{\alpha}_j$  stands for the j-th PDL segment with value expressed in dB by

$$PDL(dB) = 10 \log_{10} \left( \frac{\lambda_1}{\lambda_2} \right)^2 = 20 |\alpha_j| \log_{10}(e) \quad (42)$$

The action of an optical component exhibiting PDL and PMD on a field can be described by (Chen et al. 2007)

$$|s\rangle = T |t\rangle = T_{PDL} T_{PMD} |t\rangle \quad (43)$$

where  $|s\rangle$  and  $|t\rangle$  are output and input SOP, respectively. The eigenvalues of the matrix  $T = T_{PDL} T_{PMD}$  are (Yasser 2010)

$$\lambda = [cC - i(\hat{\alpha} \cdot \hat{p})sS] \pm \sqrt{[cC - i(\hat{\alpha} \cdot \hat{p})sS]^2 - 1} \quad (44)$$

It was evident from Eq.(44) that the eigenvalues are complex, where the real part will control the new rotation angle of  $\hat{s}$  around the PSP vector, and imaginary part can be used into Eq.(42) to obtain the PDL value in presence of PMD. Obviously, the new eigenvalues in presence of the combined PMD and PDL effects are different from that obtained for each effect separately.

### 6.1 Special cases

1. In presenting PDL only, the eigenstates of the PDL matrix are orthogonal, the output Stokes vector can be obtained as follows: combining the relations  $|s\rangle = T_{PDL} |t\rangle$ , and  $\langle s| = \langle t| T_{PDL}^\dagger$  into  $\hat{s} = \langle s| \vec{\sigma} |s\rangle$ , and using the facts (Yasser 2010; Gordon & Kogelnik 2000)

$$(\vec{\alpha} \cdot \vec{\sigma})(\vec{\beta} \cdot \vec{\sigma}) = \vec{\alpha} \cdot \vec{\beta} + i\vec{\alpha} \times \vec{\beta} \cdot \vec{\sigma} \quad (45a)$$

$$(\vec{\beta} \cdot \vec{\sigma})(\vec{\alpha} \cdot \vec{\sigma}) = \vec{\alpha} \cdot \vec{\beta} - i\vec{\alpha} \times \vec{\beta} \cdot \vec{\sigma} \quad (45b)$$

$$(\vec{\beta} \cdot \vec{\sigma})\vec{\sigma} = \vec{\beta} - i\vec{\beta} \times \vec{\sigma} \quad (45c)$$

$$\vec{\sigma}(\vec{\beta} \cdot \vec{\sigma}) = \vec{\beta} + i\vec{\beta} \times \vec{\sigma} \quad (45d)$$

$$(\vec{\beta} \cdot \vec{\sigma})\vec{\sigma}(\vec{\beta} \cdot \vec{\sigma}) = 2\vec{\beta}(\vec{\beta} \cdot \vec{\sigma}) - \beta^2 \vec{\sigma} \quad (45e)$$

$$T_{PDL}^\dagger = T_{PDL} = c + (\vec{\alpha} \cdot \vec{\sigma})s \quad (45f)$$

a useful relation can be deduced

$$\hat{s} = c^2 \hat{t} - s^2 \hat{t} + 2sc\hat{\alpha} + 2s^2 \hat{\alpha}(\hat{\alpha} \cdot \hat{t}) \quad (46)$$

The output SOP which is a combination of the vectors  $\hat{t}$  and  $\hat{\alpha}$ , i.e.  $\hat{t}$  does not rotate around  $\hat{\alpha}$ . If the input SOP is parallel or anti-parallel to PDL then the output SOP takes the form  $e^{-\alpha \hat{t}}$  or  $e^{\alpha \hat{t}}$ . The first component, that is parallel to PDL vector, experience a gain  $e^\alpha$  and the other, that is anti-parallel to PDL vector, is attenuated by  $e^{-\alpha}$ .

2. Similarly, in presence of PMD only, the eigenstates of the PMD matrix are also orthogonal and the output SOP can be determined as follows: combining the relations  $\langle s | = \langle t | T_{PMD}^\dagger$  and  $|s \rangle = T_{PMD} |s \rangle$  into  $\hat{s} = \langle s | \vec{\sigma} |s \rangle$ , using Eqs.(45) with the facts that  $T_{PMD} = C - i(\hat{p} \cdot \vec{\sigma})S$  and  $T_{PMD}^\dagger = T_{PMD}^{-1} = C + i(\hat{p} \cdot \vec{\sigma})S$  to yield

$$\hat{s} = C^2 \hat{t} - S^2 \hat{t} + 2SC(\hat{p} \times \hat{t}) + 2S^2 \hat{p}(\hat{p} \cdot \hat{t}) \quad (47)$$

This equation refers to the input SOP that are parallel or anti-parallel to PMD vector which experiences no change, i.e.  $\hat{s} = \hat{t}$  along the optical fiber. Notice that, the PMD causes a rotation of the SOP around  $\vec{\tau}$ , which is presented through the third term.

3. Finally, in presenting the combined PDL-PMD effects, determining  $\hat{s}$  as a function of  $\hat{t}$ ,  $\hat{\alpha}$ , and  $\hat{p}$  which is very complicated, is beyond the scope of this chapter.

## 6.2 The output power

The normalized Gaussian pulse before entering the PMD and PDL components has the form

$$|A_{in}(T)\rangle = D e^{-T^2/2T_0^2} |a\rangle \quad (48)$$

where  $T_0$  is the initial pulse width, and  $|a\rangle$  is the Jones vector of the signal. Clearly, the normalized input power is found to be  $\vec{P}_{in}(T) = \langle A_{in}(T) | \vec{\sigma} | A_{in}(T) \rangle = e^{-T^2/T_0^2} \hat{s}$ , where  $\hat{s}$  is the input Stokes vector. The Fourier transform of Eq.(48) is

$$|A_{in}(w)\rangle = \mathfrak{F}\{|A_{in}(T)\rangle\} = D \frac{T_0}{\sqrt{2\pi}} e^{-w^2 T_0^2/2} |a\rangle \quad (49)$$

As far  $|A_{out}(w)\rangle = T_{PDL} T_{PMD}(w) |A_{in}(w)\rangle$ , the output field which can be illustrated by the inverse Fourier transformation as follows

$$|A_{out}(T)\rangle = D \frac{T_0}{\sqrt{2\pi}} T_{PDL} \mathfrak{F}^{-1}\{e^{-w^2 T_0^2/2} e^{-i(w/2)(\vec{\tau} \cdot \vec{\sigma})}\} |a\rangle = e^{-\frac{T^2 + \tau_{pmd}^2}{2T_0^2}} e^{\vec{\alpha} \cdot \vec{\sigma}} e^{-T \vec{\tau} \cdot \vec{\sigma} / 2T_0^2} |a\rangle \quad (50)$$

In order to compute the output power from this equation. The vector  $\vec{n}$  was set to equal  $\vec{n} = (\vec{\alpha} - \vec{\tau} T / T_0^2) / 2$ , such that

$$|A_{out}(T)\rangle = e^{-\frac{T^2 + \tau_{pmd}^2}{2T_o^2}} e^{\vec{n} \cdot \vec{\sigma}} |a\rangle \quad (51)$$

The new vector  $\vec{n}$  is a random. Its value is  $n = \sqrt{\alpha^2 + T^2 \tau_{pmd}^2 / T_o^4 - 2T\alpha\tau_{pmd} \cos\theta / T_o^2} / 2$ , where  $\theta$  is the angle between  $\vec{\alpha}$  and  $\vec{\tau}$ , while the direction is  $\hat{n} = (\vec{\alpha} - T\vec{\tau} / T_o^2) / n$ . Substituting Eq.(51) into the definition  $\vec{P}_{out}(T) = \langle A_{out} | \vec{\sigma} | A_{out} \rangle$  and introducing the fact  $(\vec{n} \cdot \vec{\sigma})^\dagger = \vec{n} \cdot \vec{\sigma}$ , yields

$$\vec{P}_{out}(T) = e^{-\frac{T^2 + \tau_{pmd}^2}{T_o^2}} \langle a | (\cosh n + \hat{n} \cdot \vec{\sigma} \sinh n) \vec{\sigma} (\cosh n + \hat{n} \cdot \vec{\sigma} \sinh n) | a \rangle \quad (52)$$

Considering Eqs.(45), the last equation may be written as

$$\vec{P}_{out}(t) = e^{-\frac{T^2 + \tau_{pmd}^2}{T_o^2}} [\hat{s} + 2\hat{n}(\sinh n \cosh n + \sinh^2 n \cos\phi_1)] \quad (53)$$

where  $\phi_1$  is the angle between the random vector  $\hat{n}$  and the input SOP,  $\hat{s}$ . To visualize the situation more easily, Eq.(53) was written as

$$\vec{P}_{out}(T) = e^{-\frac{T^2 + \tau_{pmd}^2}{T_o^2}} \cdot f(T, \tau_{pmd}, \alpha) \cdot \hat{s}_{out} \quad (54)$$

where  $f(T, \tau_{pmd}, \alpha)$  and  $\hat{s}_{out}$  are the value and direction of the expression inside the square brackets. Eq.(54) represents the output power in presenting of PMD and PDL, which may be written in certain cases as in the following subsections.

### 6.2.1 PMD only

In this case,  $\vec{n} = -T\vec{\tau} / 2T_o^2$  and  $\hat{n} = -\hat{\tau}$ , hence, Eq.(53) can be simplified as

$$\vec{P}_{out}(T) = e^{-\frac{T^2 + \tau_{pmd}^2}{T_o^2}} [\hat{s} - 2\hat{\tau}(\sinh n \cosh n + \sinh^2 n \cos\phi_2)] = f(T, \tau_{pmd}) e^{-\frac{T^2 + \tau_{pmd}^2}{T_o^2}} \hat{s}_{out} \quad (55)$$

Here  $\phi_1$  is replaced by  $\phi_2$  which represents the angle between  $\hat{\tau}$  and  $\hat{s}$ . If  $\vec{\tau} = 0$ , then  $\vec{P}_{in} = \vec{P}_{out}$ . That is; the power and SOP are not affected in absence of PMD. The PSP's are the states that are parallel or antiparallel to  $\hat{\tau}$ , so the powers in the PSP's direction are  $\vec{P}_{out}(T)_{PSP} = \exp(-T^2 \pm \tau_{pmd}^2 / T_o^2) \hat{s}$ . The parallel or antiparallel SOP to  $\hat{\tau}$  will not be changed through the propagation, but the position of the pulse components will be shifted by  $\pm \tau_{pmd} / 2$ .

### 6.2.2 PDL only

Here,  $\vec{n} = \vec{\alpha} / 2$  and  $\hat{n} = \hat{\alpha}$ , hence, Eq.(53) will be

$$\vec{P}_{out}(T) = e^{-\frac{T^2}{T_o^2}} [\hat{s} + 2\hat{\alpha}(\sinh n \cosh n + \sinh^2 n \cos\phi_3)] = f(\alpha) e^{-\frac{T^2}{T_o^2}} \hat{s}_{out} \quad (56)$$

Here  $\phi_1$  is replaced by  $\phi_3$  which represents the angle between  $\hat{\alpha}$  and  $\hat{s}$ . If  $\vec{\alpha} = 0$ , then  $\vec{P}_{in} = \vec{P}_{out}$ . That is; the power and SOP are not affected in absence of PDL. There are two

important SOP's that are parallel or antiparallel to  $\hat{\alpha}$ . For these SOP's, Eq.(56) will be reduced to  $\vec{P}_{out}(T) = e^{\pm\alpha} e^{-T^2/T_0^2} \hat{s}$ . This means that, the power will be affected by the factor  $e^{\pm\alpha}$  but the pulse shape and SOP will not change.

### 6.3 The complex PSP vector

Before discussing the impact of PMD and PDL on the dynamical equation of SOP, we notice: First, without including PDL, the transmission matrix of the fiber is always unitary. However, when the fiber PMD is intertwined with PDL elements, the transmission matrix loses its unitary property. Nevertheless, by the polar decomposition theorem (Kogelnik & Jopson 2002), a complex  $2 \times 2$  matrix can be decomposed into  $T = T_{PDL} T_{PMD}$ , where  $T_{PDL}$  is a positive definite Hermitian matrix, i.e.  $T_{PDL}^\dagger = T_{PDL}$ , and  $T_{PMD}$  is a unitary matrix, i.e.  $T_{PMD}^\dagger T_{PMD} = I$ . Second, the PDL vector may be frequency dependent. This will influence the PDL induced waveform distortion effect in an optic link. Considering that such frequency dependent waveform distortion is not so important in a system with realistic parameters (Shtaif & Rosenberg 2005; Phua & Ippen 2005), the PDL vector was approximated as a frequency independent.

As pulses are described by wave packets with a finite frequency band, the frequency dependence of  $|s\rangle$  should be considered now. A fixed input polarization was assumed, i.e.  $|t\rangle_w = 0$  hence  $\hat{t}_w = 0$ , as is appropriate for a pulse entering the fiber at zero time. Now, by differentiating Eq.(43) with respect to frequency and eliminating  $|t\rangle$ , the change of the output Jones vector was obtained

$$\frac{d|s\rangle}{dw} = T_{PDL} T'_{PMD} T_{PMD}^{-1} T_{PDL}^{-1} |s\rangle \quad (57)$$

where  $T'_{PMD}$  represents the derivative of  $T_{PMD}$  with respect to frequency. Eq.(57) tell us that for most input polarizations, the output polarization will change with frequency in the first order. Notice that, if  $|s\rangle$  either of the two eigenstates of the operator  $T_{PDL} T'_{PMD} T_{PMD}^{-1} T_{PDL}^{-1}$  then  $|s\rangle_w = 0$ . The dynamical equation of SOP in Stokes space can be obtained by using Eq.(57) as, see (Yasser 2010)

$$\hat{s}_w = [(c^2 + s^2)\vec{\tau} - 2s^2(\vec{\tau} \cdot \hat{\alpha})\hat{\alpha} + 2isc(\vec{\tau} \times \hat{\alpha})] \times \hat{s} \quad (58)$$

Many published studies (Chen et al. 2007; Wang & Menyuk 2001; Shtaif & Rosenberg 2005) related to the theoretical treatment of the combined effects of PMD and PDL, which are introduced in many forms of the frequency derivative of Stokes vector, but all these forms may be considered as a partial form of Eq.(58) above.

The expression between brackets in the right hand side of the last equation represents the complex PSP vector which can be decomposed as real and imaginary parts as follows

$$\vec{W} = \vec{\Omega} + i\vec{\Lambda} \quad (59)$$

where  $\vec{\Omega}$  and  $\vec{\Lambda}$  represent the new vectors in presenting of PMD and PDL. The two new vectors take the forms

$$\vec{\Omega} = (c^2 + s^2)\vec{\tau} - 2s^2(\vec{\tau} \cdot \hat{\alpha})\hat{\alpha} \quad (60a)$$

$$\vec{\Lambda} = 2sc(\vec{\tau} \times \hat{\alpha}) \quad (60b)$$

There are many features that can be deduced from Eq.(59): if  $\vec{\tau}$  is parallel or anti-parallel to  $\hat{\alpha}$  then  $\vec{\Omega} = \vec{\tau}$ , i.e. the old and new PMD vectors are identical, and  $\vec{\Lambda} = 0$ , i.e. the PDL effects will disappear. If  $\vec{\tau}$  is perpendicular on  $\hat{\alpha}$  then  $\vec{\Omega} = (c^2 + s^2)\vec{\tau}$ , i.e. the old and new PMD vector have the same direction but distinct values, and  $\vec{\Lambda} = 2sc\vec{\xi}$  (where  $\vec{\xi} = \vec{\tau} \times \hat{\alpha}$ ) that means the new PDL vector is perpendicular to the plane that contains  $\vec{\tau}$  and  $\hat{\alpha}$ . If  $\vec{\tau} = 0$  then both vectors  $\vec{\Omega}$  and  $\vec{\Lambda}$  are zero. Remembering that, the absence of PMD will not permit the emergence of two components, as a result there is no PDL but the reverse is not correct. Since the PSP vector is complex, then the fast and slow PSP's are not orthogonal. If  $\hat{\alpha} = 0$ , i.e. no PDL, then  $\vec{\Omega} = \vec{\tau}$ . The new DGD takes the form  $\tau_{pmd}^{new} = \text{Re}\sqrt{\vec{W} \cdot \vec{W}} = \tau_{pmd}^{old}$ , where the meaning of DGD over infinite frequency is called the scalar PMD. Thereafter, the SOP rotates around the PSP vector by an angle  $\tau_{pmd}^{new}w$ . The new DAS takes the form  $\alpha_{new} = \text{Im}\sqrt{\vec{W} \cdot \vec{W}}$ . Accordingly, the new PDL value is  $20 |\alpha_{new}| \log_{10}(e)$ .

## 7. Birefringence and nonlinearity

To formulate the birefringence effects more precisely, considering the nonlinear Helmholtz equation (Agrawal 2007)

$$\nabla^2 \vec{E} + \frac{w^2 \tilde{\epsilon}_s}{c^2} \vec{E} = -\frac{w^2}{c^2 \epsilon_0} \vec{P}_{NL} \quad (61)$$

where the tilde denotes the Fourier transformation,  $\epsilon_0$  is the vacuum permittivity, and  $\tilde{\epsilon}_s$  is the linear part of the dielectric constant. Notice that the tensorial nature is important to account for the PMD effects that have their origin in the birefringence of silica fibers, while its frequency dependence leads to chromatic dispersion. Assuming that the instantaneous electronic response dominates and neglecting Raman contribution (Lin & Agrawal 2003 a), the third order nonlinear polarization in a medium as silica glass is found to be

$$\vec{P}_{NL}(w) = \frac{\epsilon_0 \chi_{xxxx}^{(3)}}{4} \left[ (\vec{E} \cdot \vec{E}) \vec{E}^* + 2(\vec{E}^* \cdot \vec{E}) \vec{E} \right] \quad (62)$$

The electric field vector evolves along the fiber length and its SOP changes because of the birefringence. It is assumed here that the z-axis is directed along the fiber length and The electric field vector lies in the x-y plane. This assumption amounts to neglect the longitudinal component of the vector and is justified in practice as long as the spatial size of the fiber mode is longer than the optical wavelength. In Jones-matrix notation, the field at any point  $r$  inside the fiber can be written as (Kogelnik & Jopson 2002)

$$\vec{E}(r, w) = F(x, y) |A(z, w)\rangle e^{ikz} \quad (63)$$

where  $F(x, y)$  represents the fiber mode profile,  $k$  is the propagation constant, and Jones vector  $|A\rangle$  is a two-dimensional column vector representing the two components of the electric field in the x-y plane. Since  $F(x, y)$  does not change with  $z$ , one needs to consider only the evolution of  $|A\rangle$  along the fiber.

Substituting Eq.(63) into Eq.(62), inserting the result into Eq.(61), and integrate over the transverse mode distribution in the x-y plane, assuming  $|A\rangle$  to be slowly varying function of  $z$  so that neglecting their second-order derivative with respect to  $z$ . With these simplifications, the equation governing the evolution of  $|A\rangle$  takes the form

$$\frac{d|A\rangle}{dz} + \left( \frac{w^2 \vec{\epsilon}_s}{2ikc^2} + i \frac{k\sigma_o}{2} \right) |A\rangle = \frac{i\gamma}{3} [2\langle A|A\rangle + |A^*\rangle\langle A^*|] |A\rangle \quad (64)$$

where  $\sigma_o$  is a unit matrix. To proceed Eq.(64) further, the dielectric constant tensor  $\vec{\epsilon}_s$  may be represented in the basis of Pauli matrices as (Lin & Agrawal 2003 a)

$$\frac{w^2 \vec{\epsilon}_s}{c^2} = \left[ k + i \frac{\alpha}{2} \right]^2 \sigma_o - k \vec{\beta} \cdot \vec{\sigma} \quad (65)$$

The vector  $\vec{\beta}$  accounts for the fiber birefringence and its frequency dependence produces PMD. The vector  $\vec{\sigma}$  is formed as  $\vec{\sigma} = \hat{e}_1\sigma_1 + \hat{e}_2\sigma_2 + \hat{e}_3\sigma_3$ , where  $\hat{e}_1$ ,  $\hat{e}_2$ , and  $\hat{e}_3$  are a three unit vectors in the Stokes space. Substituting Eq.(65) into (64) leads to the following vector equation

$$\frac{d|A\rangle}{dz} + \frac{\alpha}{2} \sigma_o |A\rangle = -\frac{i}{2} \vec{\beta} \cdot \vec{\sigma} |A\rangle + \frac{i\gamma}{3} [2\langle A|A\rangle + |A^*\rangle\langle A^*|] |A\rangle \quad (66)$$

Eq.(66) can be put in simplified form by neglecting the second term on the left hand side, by proposing that the medium is lossless; then, using the following identity

$$|A^*\rangle\langle A^*| = [\langle A|A\rangle + \langle A|\vec{\sigma}|A\rangle \cdot \vec{\sigma}] / 2 - \langle A|\sigma_3|A\rangle \sigma_3 \quad (67)$$

into Eq.(66) yields the following elegant equation that describes the evolution of Jones vector through the optical fiber

$$\frac{d|A\rangle}{dz} = \left( -\frac{i}{2} \vec{\beta} \cdot \vec{\sigma} + \frac{i\gamma}{6} [\hat{s} \cdot \vec{\sigma}]^t \right) |A\rangle \quad (68)$$

where the proportionality term  $|A\rangle$  affects only the global phase and can be neglected,  $\hat{s} = \langle A|\vec{\sigma}|A\rangle$  is the normalized power (Stokes vector). Using Eq.(68), it is not difficult to obtain

$$\frac{d\hat{s}}{dz} = (\vec{\beta} + 2\gamma(0,0,s_3)^t / 3) \times \hat{s} \quad (69)$$

Eq.(69) presents the effect of nonlinearity. Introducing  $\gamma$  effect is considered as the main contribution of this section, because it is a phenomenon that can not be neglected in the study of the evolution of polarization through the optical fibers. However, the rotation axis in presence of nonlinearity is  $\vec{\beta} + 2\gamma(0,0,s_3)^t / 3$  instead of  $\vec{\beta}$ . The simplest case, without nonlinearity effect, has been studied by many researches using different approaches, see for example (Gordon & Kogelnik 2000; Agrawal 2005; Vanwiggeren and Roy 1999).

## 8. Conclusions

In conclusion, we have achieved the following: an important mathematical relationship between PMD and birefringence are presented and all possible assumptions are discussed. The statistics of PMD are simply analyzed. The combined effect of PMD and chromatic dispersion causes an additional amount of pulse broadening. Interaction of PMD and PDL makes the two PSP's are not orthogonal nor do they represent the fastest and slowest pulses, which causes a change in DGD and PDL compared with the impact of each individual. Nonlinearity causes a change in the rotation axis and therefore it changes the properties of polarization state during the propagation. Finally, all results are generally subject to random changes as long as most of the causes random.

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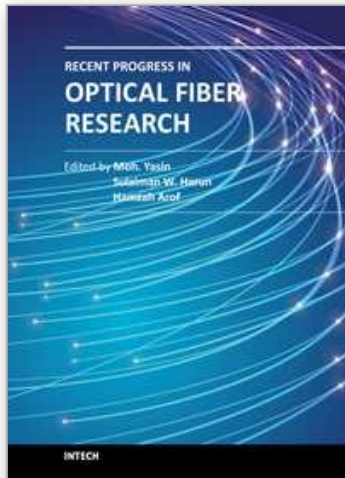
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## **Recent Progress in Optical Fiber Research**

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This book presents a comprehensive account of the recent progress in optical fiber research. It consists of four sections with 20 chapters covering the topics of nonlinear and polarisation effects in optical fibers, photonic crystal fibers and new applications for optical fibers. Section 1 reviews nonlinear effects in optical fibers in terms of theoretical analysis, experiments and applications. Section 2 presents polarization mode dispersion, chromatic dispersion and polarization dependent losses in optical fibers, fiber birefringence effects and spun fibers. Section 3 and 4 cover the topics of photonic crystal fibers and a new trend of optical fiber applications. Edited by three scientists with wide knowledge and experience in the field of fiber optics and photonics, the book brings together leading academics and practitioners in a comprehensive and incisive treatment of the subject. This is an essential point of reference for researchers working and teaching in optical fiber technologies, and for industrial users who need to be aware of current developments in optical fiber research areas.

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