11

Missile Cooperative Engagement Formation Configuration Control Method

Changzhu Wei[1,2], Yi Shen[1], Xiaoxiao Ma[3],
Naigang Cui[1] and Jifeng Guo[1]
[1]*Harbin Institute of Technology, Harbin,*
[2]*Yonsei University Observatory, Seoul,*
[3]*Xuzhou Air Force College, Xuzhou,*
[1,3]*China*
[2]*Republic of Korea*

1. Introduction

In traditional engagement model, there is only one single weapon or platform which resists to another single one, and the connections between members are few, so the complete and global information in engagement space can't be utilized sufficiently, which directly results in traditional model is confronting with more and more disadvantages especially as the high-tech and information-tech are developing faster and faster. In view of those issues, the concepts and technology of missile formation cooperative are presented, developed and expanded recently(Cui et al., 2009). Compared to traditional model, weapon or platform with cooperative manner manifests great advantages in aspects of ability of penetration, electronic countermeasures and ability of searching moving targets etc, furthermore the synthetical engagement efficacy is developed greatly.

Many new cooperative weapon systems are established and developed fast recently, such as Cooperative Engagement Capability (CEC) system, Net Fire System and LOw Cost Autonomous Attack System (LOCAAS) etc, wherein the LOCAAS is most relevant to our topic, so we will introduce it more detailed.

In order to meet the requirements of future aerial warfare, United States Force has developed a series of high technical and high accurate airborne guided weapon systems, such as Joint Common Missile (JCM), Joint Direct Attack Missile (JDAM), Wind Corrected Munitions Dispenser (WCMD) and LOw Cost Autonomous Attack System (LOCAAS). These weapons are paid much attention because their great capabilities of high-precision, all-weather engagement and attacking beyond defence area etc. Especially, LOCAAS also has other great advantages besides those aspects mentioned above, such as low cost, general utilization and attacking multi-targets simultaneously, so it is paid more attention than other weapons, and it becomes an outstanding representative of high-tech weapon and new engagement model.

LOCAAS is developed from an previous weapon named as Low Cost Anti-Armor Sub-munitions (also called LOCAAS for short) which was a kind of short-range unpowered airborne and air-to-ground guided weapon developed in 1998. At a later time, researchers added thrust system to this old LOCAAS, so it had capability of launching beyond defence area and searching moving targets within large scope. This weapon was added bidirectional data link once again in 2003, so it can implement Man-in-the-Loop control and command, which meant that this weapon system can attack targets autonomously and manifest a kind of smart engagement capability. After those improvements, this old anti-armor weapon was re-named to LOw Cost Autonomous Attack System (LOCAAS). LOCAAS adopts INS and mid-course guidance, and it also equips a smart fuze and a sensor that can be used to search moving targets, so LOCAAS can not only monitor targets within large range but also can re-locate, recognize and aim at them autonomously. The most outstanding character is that LOCAAS can also make intelligent decision for choosing the optimal orientation and sequence to achieve optimal attack, and it also has the capability of on-line planning mission.

At present, United State Force sets the engagement schemes for LOCAAS as: there will be some LOCAASs flying in the battle field to cruise or put on standby, and they can connect with each other by data link. While one of them finding the targets and can't destroy them alone, it will send out the signals to require for cooperative attacking, however if it can destroy the targets by itself, this LOCAAS will attack the targets and the other LOCAASs will continue searching targets after receiving the instruction signals from the mentioned LOCAAS which has gone into the battle. The sketch map Fig.1 shows the main concepts of how LOCAASs take part in engagement cooperatively. Authors who are interested in LOCAAS can get more detailed information in the website of Lockheed Martin.

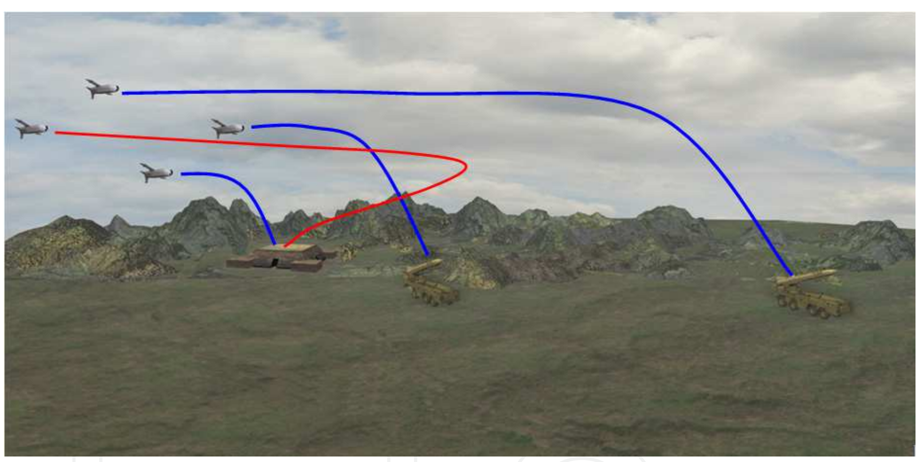


Fig. 1. The sketch map of LOCAAS engagement.

The background and significance were presented first, and then the technique frame of missile formation cooperative control system is showed, which can clearly elaborate how missiles in a formation work together in a cooperative manner. The specific relationships between each loop are analyzed, which are very necessary to clearly explain how to design missile formation control system. Followed are the main contents of this chapter, which are the detailed processes of establishing and designing missile formation control system. In this part, we will only consider the external loop of missile formation control system because the design method on individual missile inner controller is easy to be found in many literatures, and then we just assumed it is closed-loop and stable. The detailed process is divided into two steps, first is using proportion-differential control method to design missile formation keeping controller, the other explains how to design optimal keeping controller of missile formation. Some simulations are made to compare these two formation control systems proposed in this chapter at last.

2. Frame of missile formation control system

The missile formation control system mentioned in this chapter only has one leader missile (showed as Fig.2), and it mainly consists of the following subsystems: cooperative engagement mission planning subsystem, formation configuration describing subsystem, leader inner-loop control subsystem, follower inner-loop control subsystem and formation configuration control subsystem. The sketch map illustrates relationships between each subsystem in one missile formation system is showed in Fig.3.

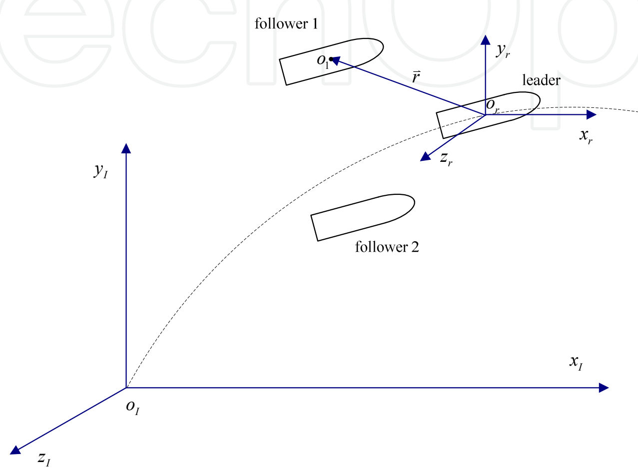


Fig. 2. Sketch map of missile formation consisted of two followers and only one leader.

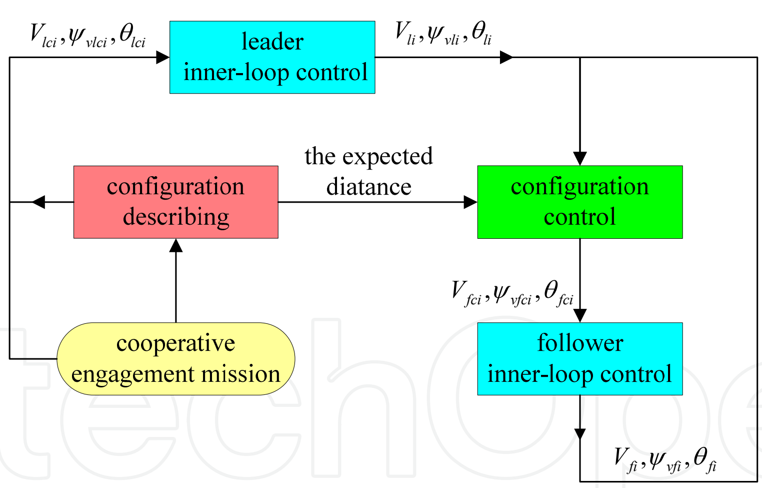


Fig. 3. Relationships between subsystems of missile formation control system.

The cooperative mission planning subsystem supplies real-time status of mission space and distribution situation of targets, and restricts the flight status of leader at the same time; the formation configuration describing system receives the information supplied by cooperative engagement subsystem and establishes associated function of engagement efficacy according to different missile formation configurations, and then makes the decision set about formation configurations based on the returned value of this efficacy function. After optimizing the values of efficacy function according to the specific mission space, the optimal formation configuration can be calculated within this decision set. The optimal formation configuration will be different when the missile formation is flying in different segments of mission space, that is, the optimal formation configuration will be decided by specific engagement requirement. The formation configuration describing subsystem will also restrict flight status of leader together with cooperative mission planning subsystem at the same time; The leader inner-loop control subsystem receives the information sent from the cooperative engagement planning subsystem and controls the leader to fly stably, and then the real flight states of leader can be obtained; The required states of follower missile can be calculated through the states in relative spatial dimension i.e. the distance between missiles which is obtained from formation configuration describing subsystem; The follower inner-loop control subsystem receives the command states, which is similar to that of leader mentioned above, and then control follower to fly stably. The actual flight states of follower missiles are fed back to the formation configuration control subsystem and used to achieve the expected formation configuration which is decided by the specific cooperative engagement mission.

This chapter focus on missile formation keeping control problems. First, we consider the actual flight states of leader as perturbation variables acting on the controller, and assume the follower inner-controller is closed and stable loop, that is, followers can track the required commands of velocity, flight path angle and flight deflection angle rapidly and stably. We will make further assumption that those three channels referred above are one-order systems expressed as (Wei et al., 2010):

$$\begin{aligned}\dot{V}_f &= -\frac{1}{\tau_{vf}}(V_f - V_{fc})\\ \dot{\theta}_f &= -\frac{1}{\tau_{\theta f}}(\theta_f - \theta_{fc})\\ \dot{\psi}_{vf} &= -\frac{1}{\tau_{\psi_{vf}}}(\psi_{vf} - \psi_{vfc})\end{aligned} \tag{1}$$

where V_f, θ_f and ψ_{vf} is the velocity, flight path angle and flight deflection angle of follower respectively; V_{fc}, θ_{fc} and ψ_{vfc} are commands of velocity, flight path angle and flight deflection angle of follower respectively; τ_{vf}, $\tau_{\theta f}$ and $\tau_{\psi_{vf}}$ are the inertial time constants of velocity, flight path angle and flight deflection angle channel of follower respectively. These values can be calculated by analyzing four-dimensional guidance and control system referrd in the literature of Cui et al., 2010, they are:

$$\begin{aligned}\tau_v &= 3.51\text{s}\\ \tau_\theta &= 2.25\text{s}\\ \tau_{\psi_v} &= 2.37\text{s}\end{aligned} \tag{2}$$

These inertial time constants will be used in the subsequent sections for designing and simulating missile formation keeping controller.

After missile formation achieving four-dimensionally rendezvous, this formation should carry out some subsequent missions, such as relative navigation and location, cooperative searching targets, locating and recognizing targets and cooperative penetration etc., however, these missions require missile formation keeping its configuration or relative

dimensional states for some periods of time. Next sections will present some further and detailed researches on how to keep missile formation optimally and robustly.
The main thought of the following parts is showed as below: basing on the kinematics relationships between missiles in inertial coordinate frame, the missile formation proportional-derivative (PD) controller via feeding back full states will be present firstly; Second, the relative motion will be established in relative coordinates frame, which can indicate the characteristics of relative motion directly. Based on this direct relative motion, the optimal controller that has non-zero given point and restrains slowly variant perturbations is designed in the third part.

3. PD controller of missile formation keeping

3.1 Definition of coordinate frame system

1. Relative coordinate frame $o_r - x_r y_r z_r$

The origin o_r of this coordinate coincides with the mass centre of leader, and axis $o_r x_r$ points to the velocity direction of leader, $o_r y_r$ is perpendicular to $o_r x_r$ and points to up direction, $o_r z_r$ composes right-hand coordinate frame together with other two axes mentioned above.

2. Inertial coordinate frame system $O_I - X_I Y_I Z_I$

The origin O_I of this coordinate frame is fixed to an arbitrary point on ground, axis $O_I X_I$ lies in horizontal plane and points to target, axis $O_I Y_I$ is perpendicular to $O_I X_I$ and points to up direction, the last axis $O_I Z_I$ also composes right-hand coordinate frame system together with those two axes mentioned above.
The relationship between these two coordinate frames is showed in Fig.4.

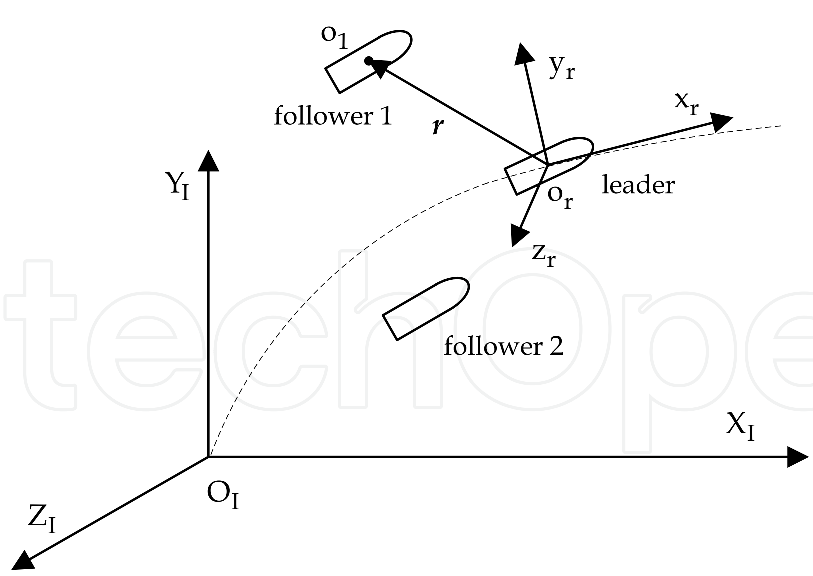


Fig. 4. Relationship between relative and inertial coordinate frame.

3. Trajectory coordinate frame system $o_1 - x_2 y_2 z_2$

The origin of this coordinate o_1 coincides with the mass centre of missile, and axis $o_1 x_2$ points to the velocity direction of missile, $o_1 y_2$ is perpendicular to $o_1 x_2$ and points to up direction, o_1z_2 composes right-hand coordinate frame together with other two axes mentioned above. The spatial directions are parallel to those in relative coordinate frame system.

3.2 Model of relative motion between missiles

3.2.1 Basic assumptions

It is assumed that the control system of missile self is closed-loop and stable, that is, missile can track the required velocity, flight path angle and flight deflection angle rapidly and stably. Further, we consider these tracking channels are one-order inertial loops described as section 2. In order to express the relationships conveniently, we will change the one-order inertial loops showed in Equ.(1) to the following forms:

$$\begin{cases} \dot{V}_i = -\lambda_v (V_i - V_{ic}) \\ \dot{\theta}_i = -\lambda_\theta (\theta_i - \theta_{ic}) \\ \dot{\psi}_{vi} = -\lambda_{\psi_v} (\psi_{vi} - \psi_{vic}) \end{cases} \tag{3}$$

where i is the number of missile, V_i , θ_i and ψ_{vi} represent the actual velocity, flight path angle and flight deflection angle of i[th] missile respectively; V_{ic} , θ_{ic} and ψ_{vic} are desired velocity, desired flight path angle and desired flight deflection angle of i[th] missile respectively; λ_v , λ_θ and λ_{ψ_v} are the reciprocals of inertial time constants τ_v , τ_θ and τ_{ψ_v} in channels of veloctiy, flight path angle and flight deflection angle respectively.

3.2.2 Establishing the control model

The kinematics equations of missile in inertial coordinate frame can be expressed as:

$$\begin{cases} \dot{X}_i = V_i \cos\theta_i \cos\psi_{vi} \\ \dot{Y}_i = V_i \sin\theta_i \\ \dot{Z} = -V_i \cos\theta_i \sin\psi_{vi} \end{cases} \tag{4}$$

The kinematics relationship of two missiles in inertial and relative coordinate frame is showed in Fig.5, so the relative positions of two missiles can be obtained from these sketch maps:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \boldsymbol{T}_2(\psi_{v1}) \boldsymbol{T}_1(\theta_1) \begin{bmatrix} x^* \\ y^* \\ z^* \end{bmatrix} \tag{5}$$

where x^* , y^* and z^* are the distances relative to leader in relative coordinate frame, and the transformation matrices in Equ.(5) are:

$$\boldsymbol{T}_1(.) = \begin{bmatrix} \cos(.) & -\sin(.) & 0 \\ \sin(.) & \cos(.) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \boldsymbol{T}_2(.) = \begin{bmatrix} \cos(.) & 0 & \sin(.) \\ 0 & 1 & 0 \\ -\sin(.) & 0 & \cos(.) \end{bmatrix}$$

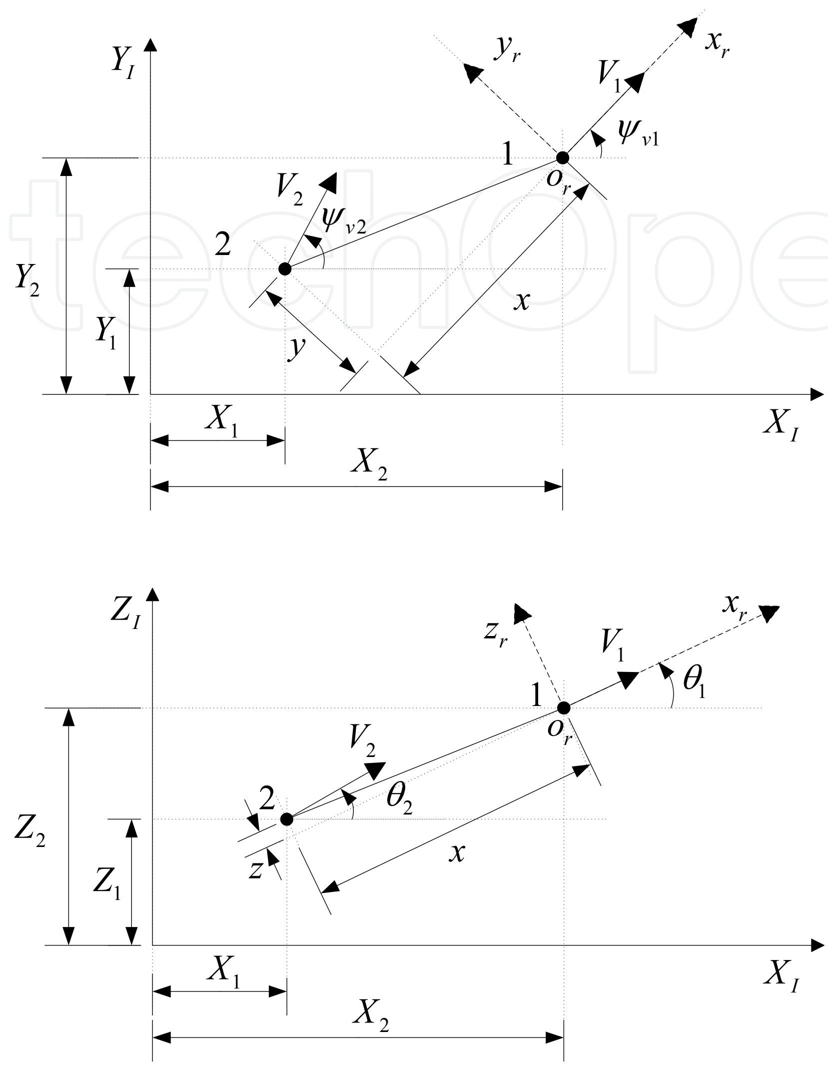


Fig. 5. Relative position relations between two missiles.

Further, the deviations of relative positions can be expressed from Equ.(5) to:

$$\boldsymbol{e} = \begin{bmatrix} X_2 - X_1 \\ Y_2 - Y_1 \\ Z_2 - Z_1 \end{bmatrix} - \boldsymbol{T}_2(\psi_{v1})\boldsymbol{T}_1(\theta_1)\begin{bmatrix} x^* \\ y^* \\ z^* \end{bmatrix} \tag{6}$$

and making derivatives to last equation yields:

$$\dot{\boldsymbol{e}} = \begin{bmatrix} \dot{X}_2 - \dot{X}_1 \\ \dot{Y}_2 - \dot{Y}_1 \\ \dot{Z}_2 - \dot{Z}_1 \end{bmatrix} - \frac{d\boldsymbol{T}_2(\psi_{v1})}{d\psi_{v1}}\dot{\psi}_{v1}\boldsymbol{T}_1(\theta_1)\begin{bmatrix} x^* \\ y^* \\ z^* \end{bmatrix} - \boldsymbol{T}_2(\psi_{v1})\frac{d\boldsymbol{T}_1(\theta_1)}{d\theta_1}\dot{\theta}_1\begin{bmatrix} x^* \\ y^* \\ z^* \end{bmatrix} \tag{7}$$

making further derivatives gets:

$$\ddot{\boldsymbol{e}}=\begin{bmatrix}\ddot{X}_2-\ddot{X}_1\\ \ddot{Y}_2-\ddot{Y}_1\\ \ddot{Z}_2-\ddot{Z}_1\end{bmatrix}-\frac{d\boldsymbol{T}_2^2(\psi_{v1})}{d\psi_{v1}^2}\dot{\psi}_{v1}^2\boldsymbol{T}_1(\theta_1)\begin{bmatrix}x^*\\ y^*\\ z^*\end{bmatrix}-\frac{d\boldsymbol{T}_2(\psi_{v1})}{d\psi_{v1}}\ddot{\psi}_{v1}\boldsymbol{T}_1(\theta_1)\begin{bmatrix}x^*\\ y^*\\ z^*\end{bmatrix}$$
$$-2\frac{d\boldsymbol{T}_2(\psi_{v1})}{d\psi_{v1}}\dot{\psi}_{v1}\frac{d\boldsymbol{T}_1(\theta_1)}{d\theta_1}\dot{\theta}_1\begin{bmatrix}x^*\\ y^*\\ z^*\end{bmatrix}-\boldsymbol{T}_2(\psi_{v1})\frac{d\boldsymbol{T}_1{}^2(\theta_1)}{d\theta_1^2}\dot{\theta}_1^2\begin{bmatrix}x^*\\ y^*\\ z^*\end{bmatrix}-\boldsymbol{T}_2(\psi_{v1})\frac{d\boldsymbol{T}_1(\theta_1)}{d\theta_1}\ddot{\theta}_1\begin{bmatrix}x^*\\ y^*\\ z^*\end{bmatrix}\tag{8}$$

where

$$\ddot{\theta}_1=-\lambda_\theta(\dot{\theta}_1-\dot{\theta}_{1c}),\quad \ddot{\psi}_{v1}=-\lambda_{\psi_v}(\dot{\psi}_{v1}-\dot{\psi}_{v1c})$$

$$\frac{d\boldsymbol{T}_1(.)}{d(.)}=\begin{bmatrix}-\sin(.)&-\cos(.)&0\\ \cos(.)&-\sin(.)&0\\ 0&0&0\end{bmatrix},\quad \frac{d\boldsymbol{T}_2(.)}{d(.)}=\begin{bmatrix}-\sin(.)&0&\cos(.)\\ 0&0&0\\ -\cos(.)&0&-\sin(.)\end{bmatrix}$$

$$\frac{d^2\boldsymbol{T}_1(.)}{d^2(.)}=\begin{bmatrix}-\cos(.)&\sin(.)&0\\ -\sin(.)&-\cos(.)&0\\ 0&0&0\end{bmatrix},\quad \frac{d^2\boldsymbol{T}_2(.)}{d^2(.)}=\begin{bmatrix}-\cos(.)&0&-\sin(.)\\ 0&0&0\\ \sin(.)&0&-\cos(.)\end{bmatrix}$$

besides, there is:

$$\begin{aligned}\ddot{X}_i&=\dot{V}_i\cos\theta_i\cos\psi_{vi}-V_i\sin\theta_i\dot{\theta}_i\cos\psi_{vi}-V_i\cos\theta_i\sin\psi_{vi}\dot{\psi}_{vi}\\ \ddot{Y}_i&=\dot{V}_i\sin\theta_i+V_i\cos\theta_i\dot{\theta}_i\\ \ddot{Z}_i&=-\dot{V}_i\cos\theta_i\sin\psi_{vi}+V_i\sin\theta_i\dot{\theta}_i\sin\psi_{vi}-V_i\cos\theta_i\cos\psi_{vi}\dot{\psi}_{vi}\end{aligned}\tag{9}$$

so we can get the following vector equation:

$$\ddot{\boldsymbol{e}}=\boldsymbol{f}_1+\boldsymbol{G}_r\boldsymbol{u}\tag{10}$$

where

$$\boldsymbol{f}_1=-\begin{bmatrix}\ddot{X}_1\\ \ddot{Y}_1\\ \ddot{Z}_1\end{bmatrix}-\boldsymbol{G}_r\begin{bmatrix}V_2\\ \theta_2\\ \psi_{v2}\end{bmatrix}-\frac{d\boldsymbol{T}_2^2(\psi_{v1})}{d\psi_{v1}^2}\dot{\psi}_{v1}^2\boldsymbol{T}_1(\theta_1)\begin{bmatrix}x^*\\ y^*\\ z^*\end{bmatrix}-\frac{d\boldsymbol{T}_2(\psi_{v1})}{d\psi_{v1}}\ddot{\psi}_{v1}\boldsymbol{T}_1(\theta_1)\begin{bmatrix}x^*\\ y^*\\ z^*\end{bmatrix}$$

$$-2\frac{d\boldsymbol{T}_2(\psi_{v1})}{d\psi_{v1}}\dot{\psi}_{v1}\frac{d\boldsymbol{T}_1(\theta_1)}{d\theta_1}\dot{\theta}_1\begin{bmatrix}x^*\\ y^*\\ z^*\end{bmatrix}-\boldsymbol{T}_2(\psi_{v1})\frac{d\boldsymbol{T}_1{}^2(\theta_1)}{d\theta_1^2}\dot{\theta}_1^2\begin{bmatrix}x^*\\ y^*\\ z^*\end{bmatrix}-\boldsymbol{T}_2(\psi_{v1})\frac{d\boldsymbol{T}_1(\theta_1)}{d\theta_1}\ddot{\theta}_1\begin{bmatrix}x^*\\ y^*\\ z^*\end{bmatrix}$$

$$\boldsymbol{G}_r = \begin{bmatrix} \lambda_v \cos\theta_2 \cos\psi_{v2} & -\lambda_\theta V_2 \sin\theta_2 \cos\psi_{v2} & -\lambda_{\psi_v} V_2 \cos\theta_2 \sin\psi_{v2} \\ \lambda_v \sin\theta_2 & \lambda_\theta V_2 \cos\theta_2 & 0 \\ -\lambda_v \cos\theta_2 \sin\psi_{v2} & \lambda_\theta V_2 \sin\theta_2 \sin\psi_{v2} & -\lambda_{\psi_v} V_2 \cos\theta_2 \cos\psi_{v2} \end{bmatrix} \quad (11)$$

The control variables of follower are:

$$\boldsymbol{u} = \begin{bmatrix} V_{2c} \\ \theta_{2c} \\ \psi_{v2c} \end{bmatrix} \quad (12)$$

As for the expression of $\boldsymbol{f}_1$, we also need to know:

$$\begin{bmatrix} \ddot{X}_1 \\ \ddot{Y}_1 \\ \ddot{Z}_1 \end{bmatrix} = \begin{bmatrix} \dot{V}_1 \cos\theta_1 \cos\psi_{v1} - V_1 \sin\theta_1 \dot{\theta}_1 \cos\psi_{v1} - V_1 \cos\theta_1 \sin\psi_{v1} \dot{\psi}_{v1} \\ \dot{V}_1 \sin\theta_1 + V_1 \cos\theta_1 \dot{\theta}_1 \\ -\dot{V}_1 \cos\theta_1 \sin\psi_{v1} + V_1 \sin\theta_1 \dot{\theta}_1 \sin\psi_{v1} - V_1 \cos\theta_1 \cos\psi_{v1} \dot{\psi}_{v1} \end{bmatrix}$$

In order to eliminate the deviations of relative positions, that is, make the deviation $\boldsymbol{e}$ be asymptotically equivalent to zero, we will choose the following PD control laws:

$$\ddot{\boldsymbol{e}} + k_1 \dot{\boldsymbol{e}} + k_2 \boldsymbol{e} = \boldsymbol{0} \quad (13)$$

so there will be:

$$-k_1 \dot{\boldsymbol{e}} - k_2 \boldsymbol{e} = \boldsymbol{f}_1 + \boldsymbol{G}_r \boldsymbol{u}$$

further, the required control quantity can be expressed as below:

$$\boldsymbol{u} = \boldsymbol{G}_r^{-1}(-\boldsymbol{f}_1 - k_1 \dot{\boldsymbol{e}} - k_2 \boldsymbol{e}) \quad (14)$$

What should be done next is to analyze the existence conditions of expression (14). The condition is: iff matrix $\boldsymbol{G}_r$ is non-singular, the control quantity exists. The determinant of $\boldsymbol{G}_r$ can be calculated by Equ.(11), it is:

$$\det \boldsymbol{G}_r = -\lambda_\theta \lambda_{\psi_v} \lambda_v V_2^2 \cos\theta \quad (15)$$

Through checking this expression, it is obvious that $\boldsymbol{G}_r$ is non-singular when the missile formation is flying, that is, $V_2 \neq 0$, so we can choose proper coefficients k_1, k_2 to guarantee expression (13) to be converged, which is $\lim_{t\to\infty} \boldsymbol{e} = \boldsymbol{0}$.

4. Optimal controller of missile formation keeping

4.1 Establishing model of relative motion

Because relative coordinate frame is rotating, the relationship between relative derivative and absolute derivative should be considered during the process of establishing relative motion equations in relative coordinate frame. The relationship between these two derivatives is:

$$\boldsymbol{V}_{fr} - \boldsymbol{V}_{lr} = \boldsymbol{V}_r + \boldsymbol{\omega} \times \boldsymbol{r} \tag{16}$$

where $\boldsymbol{V}_{fr}$ and $\boldsymbol{V}_{lr}$ are the absolute velocities of follower and leader in relative coordinate frame respectively; $\boldsymbol{V}_r$ is the relative velocity from follower to leader in relative coordinate frame; $\boldsymbol{\omega}$ represents the rotating angular velocity of relative coordinate frame relative with respect to inertial space, and it is described in relative coordinate frame; $\boldsymbol{r}$ is the position vector of follower relative to leader in relative coordinate frame.
We can get the absolute velocities of follower and leader in relative coordinate frame by the transformations as below:

$$\begin{aligned} \boldsymbol{V}_{fr} &= \boldsymbol{\Phi}_I^r \boldsymbol{\Phi}_2^I \boldsymbol{V}_{f2} \\ \boldsymbol{V}_{lr} &= \boldsymbol{V}_{l2} \end{aligned}$$

where $\boldsymbol{\Phi}_I^r$ is the transformation matrix from inertial coordinate frame to relative coordinate frame:

$$\boldsymbol{\Phi}_I^r = \begin{bmatrix} \cos\theta_l \cos\psi_{vl} & \sin\theta_l & -\cos\theta_l \sin\psi_{vl} \\ -\sin\theta_l \cos\psi_{vl} & \cos\theta_l & \sin\theta_l \sin\psi_{vl} \\ \sin\psi_{vl} & 0 & \cos\psi_{vl} \end{bmatrix}$$

$\boldsymbol{\Phi}_2^I$ is the transformation matrix from trajectory coordinate frame of follower $O_{1f} - x_2 y_2 z_2$ to inertial coordinate frame:

$$\boldsymbol{\Phi}_2^I = \begin{bmatrix} \cos\theta_f \cos\psi_{vf} & -\sin\theta_f \cos\psi_{vf} & \sin\psi_{vf} \\ \sin\theta_f & \cos\theta_f & 0 \\ -\cos\theta_f \sin\psi_{vf} & \sin\theta_f \sin\psi_{vf} & \cos\psi_{vf} \end{bmatrix}$$

$\boldsymbol{V}_{f2}$ and $\boldsymbol{V}_{l2}$ represent the velocity vectors of follower and leader in themselves trajectory coordinate frames respectively, and the components of them are:

$$\boldsymbol{V}_{f2} = \begin{bmatrix} V_f \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{V}_{l2} = \begin{bmatrix} V_l \\ 0 \\ 0 \end{bmatrix}$$

Besides, V_l is the velocity of leader in inertial coordinate frame; θ_l is the flight path angle of leader; ψ_{vl} is flight deflection angle of leader.
Further, the difference of absolute velocities between two missiles in the relative coordinate frame is:

$$\boldsymbol{V}_{fr} - \boldsymbol{V}_{lr} = \begin{bmatrix} V_f \cos\theta_f \cos\theta_l \cos(\psi_{vl} - \psi_{vf}) + V_f \sin\theta_f \sin\theta_l - V_l \\ -V_f \cos\theta_f \sin\theta_l \cos(\psi_{vl} - \psi_{vf}) + V_f \sin\theta_f \cos\theta_l \\ V_f \cos\theta_f \sin(\psi_{vl} - \psi_{vf}) \end{bmatrix} \tag{17}$$

at the same time, the relative velocity between two missiles in relative coordinate frame:

$$V_r = [\dot{x}, \dot{y}, \dot{z}]^T \tag{18}$$

where x, y and z are components of relative position vector $\boldsymbol{r}$ in relative coordinate frame, and the rotating angular velocity of relative coordinate frame with respect to inertial space can be expressed as:

$$\boldsymbol{\omega} = \begin{bmatrix} \dot{\psi}_{vl} \sin\theta_l \\ \dot{\psi}_{vl} \cos\theta_l \\ \dot{\theta}_l \end{bmatrix} \tag{19}$$

so, as for the expression $\boldsymbol{\omega} \times \boldsymbol{r}$, there will be:

$$\boldsymbol{\omega} \times \boldsymbol{r} = \begin{bmatrix} z\dot{\psi}_{vl} \cos\theta_l - y\dot{\theta}_l \\ x\dot{\theta}_l - z\dot{\psi}_{vl} \sin\theta_l \\ y\dot{\psi}_{vl} \sin\theta_l - x\dot{\psi}_{vl} \cos\theta_l \end{bmatrix} \tag{20}$$

further, we can get the following expression directly from equation (16):

$$\boldsymbol{V}_r = (\boldsymbol{V}_{fr} - \boldsymbol{V}_{lr}) - \boldsymbol{\omega} \times \boldsymbol{r} \tag{21}$$

After expanding all the terms from Equ.(17) to Equ.(20) finally, we can obtain the missile relative kinematics model in 3-dimension space:

$$\begin{cases} \dot{x} = V_f \cos\theta_f \cos\theta_l \cos\psi_e + V_f \sin\theta_f \sin\theta_l - V_l - z\dot{\psi}_{vl} \cos\theta_l + y\dot{\theta}_l \\ \dot{y} = -V_f \cos\theta_f \sin\theta_l \cos\psi_e + V_f \sin\theta_f \cos\theta_l - x\dot{\theta}_l + z\dot{\psi}_{vl} \sin\theta_l \\ \dot{z} = V_f \cos\theta_f \sin\psi_e - y\dot{\psi}_{vl} \sin\theta_l + x\dot{\psi}_{vl} \cos\theta_l \\ \psi_e = \psi_{vl} - \psi_{vf} \end{cases} \tag{22}$$

4.2 Establishing the optimal control model of missile formation keeping
4.2.1 Linearized method

The formation motion equations (22) are nonlinear, we can treat them by linearized method to get linear forms that can be utilized and analyzed more conveniently. During the process of missile formation flight, some variables can be considered as small quantities, such as θ_f, θ_l and $\psi_e = \psi_l - \psi_f$, and the states of leader can be considered as inputs, then Equ.(22) can be transformed to:

$$\begin{cases} \dot{x} = V_f + V_f\theta_f\theta_l - V_l - z\dot{\psi}_{vl} + y\dot{\theta}_l \\ \dot{y} = -V_f\theta_l + V_f\theta_f - x\dot{\theta}_l + z\dot{\psi}_{vl}\theta_l \\ \dot{z} = V_f(\psi_{vl} - \psi_{vf}) - y\dot{\psi}_{vl}\theta_l + x\dot{\psi}_{vl} \end{cases}$$

Dealing with this expression by small perturbation linearized method yields:

$$\begin{cases} \dot{x} = \dot{\theta}_l y - \dot{\psi}_{vl} z + (1 + \theta_{fb}\theta_l)V_f + V_{fb}\theta_l\theta_f - V_l \\ \dot{y} = -\dot{\theta}_l x + \theta_l\dot{\psi}_{vl} z + (\theta_{fb} - \theta_l)V_f + V_{fb}\theta_f \\ \dot{z} = \dot{\psi}_{vl} x - \dot{\psi}_{vl}\theta_l y + (\psi_{vl} - \psi_{vfb})V_f - V_{fb}\psi_{vf} \end{cases} \tag{23}$$

where V_{fb}, θ_{fb} and ψ_{vfb} are feature points of linearized equations. We can describe this model by the following states space form:

$$\begin{aligned} \dot{\boldsymbol{X}} &= \boldsymbol{A}\boldsymbol{X} + \boldsymbol{B}\boldsymbol{U} + \tilde{\boldsymbol{B}}\boldsymbol{W} \\ \boldsymbol{Y} &= \boldsymbol{C}\boldsymbol{X} \end{aligned} \tag{24}$$

where $\boldsymbol{X} = [x, y, z]^T$ are the state variables; the control variables of formation controller are motion states of follower, that is, $\boldsymbol{U} = [V_f, \theta_f, \psi_{vf}]^T$; outputs are $\boldsymbol{Y} = [x, y, z]^T$; perturbation variables are the velocities of leader $\boldsymbol{W} = [V_l, 0, 0]^T$, which can be considered as slowly variant variables; the system matrix $\boldsymbol{A}$ is described as:

$$\boldsymbol{A} = \begin{bmatrix} 0 & \dot{\theta}_l & -\dot{\psi}_{vl} \\ -\dot{\theta}_l & 0 & \theta_l \dot{\psi}_{vl} \\ \dot{\psi}_{vl} & -\dot{\psi}_{vl}\theta_l & 0 \end{bmatrix}$$

control matrix $\boldsymbol{B}$ is:

$$\boldsymbol{B} = \begin{bmatrix} 1 + \theta_{f0}\theta_l & V_{f0}\theta_l & 0 \\ \theta_{f0} - \theta_l & V_{f0} & 0 \\ \psi_{vl} - \psi_{vf0} & 0 & -V_{f0} \end{bmatrix}$$

and output matrix $\boldsymbol{C}$ is:

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

effect matrix of perturbations $\tilde{\boldsymbol{B}}$ is:

$$\tilde{\boldsymbol{B}} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

4.2.2 Method of substituting variables

As analyzed in last section, we can get linear control model showed as Equ.(24) via small perturbation linearized method, however, this linear model requires the actual flight path is closely around feature points. Although we can design the missile formation keeping controller at those points within the whole flight scope by gain-scheduled method, it will increase amount of work greatly. Next we will re-deal with the nonlinear equations of relative kinematics showed in Equ.(22). If we transform the control variables from $\boldsymbol{U} = [V_f, \theta_f, \psi_{vf}]^T$ to $\bar{\boldsymbol{U}}$ with the following form:

$$\bar{\boldsymbol{U}} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} V_f \cos\theta_f \cos\psi_{vf} \\ V_f \sin\theta_f \\ V_f \cos\theta_f \sin\psi_{vf} \end{bmatrix} \tag{25}$$

then an indirectly linear control model of relation motion can be obtained:

$$\begin{cases} \dot{x} = \dot{\theta}_l y - \dot{\psi}_{vl} \cos\theta_l z + \cos\theta_l \cos\psi_{vl} u_1 + \sin\theta_l u_2 + \cos\theta_l \sin\psi_{vl} u_3 - V_l \\ \dot{y} = -\dot{\theta}_l x + \dot{\psi}_{vl} \sin\theta_l z - \sin\theta_l \cos\psi_{vl} u_1 + \cos\theta_l u_2 - \sin\theta_l \sin\psi_{vl} u_3 \\ \dot{z} = \dot{\psi}_{vl} \cos\theta_l x - \dot{\psi}_{vl} \sin\theta_l y + \sin\psi_{vl} u_1 - \cos\psi_{vl} u_3 \end{cases} \tag{26}$$

further, the direct control variables $\boldsymbol{U}$ which act on followers can be calculated by the following expressions:

$$\psi_{vf} = \arctan\left(\frac{u_3}{u_1}\right), \theta_f = \arctan\left[\frac{u_2 \cos(\psi_{vf})}{u_1}\right], V_f = \frac{u_2}{\sin\theta_f}$$

at last, we can tranform equation (26) to the states space form showed as below:

$$\begin{aligned} \dot{\boldsymbol{X}} &= \boldsymbol{AX} + \boldsymbol{B}\bar{\boldsymbol{U}} + \tilde{\boldsymbol{B}}\boldsymbol{W} \\ \boldsymbol{Y} &= \boldsymbol{CX} \end{aligned} \tag{27}$$

where system matrix $\boldsymbol{A}$ is:

$$\boldsymbol{A} = \begin{bmatrix} 0 & \dot{\theta}_l & -\dot{\psi}_{vl} \cos\theta_l \\ -\dot{\theta}_l & 0 & \dot{\psi}_{vl} \sin\theta_l \\ \dot{\psi}_{vl} \cos\theta_l & -\dot{\psi}_{vl} \sin\theta_l & 0 \end{bmatrix}$$

control matrix $\boldsymbol{B}$ is:

$$\boldsymbol{B} = \begin{bmatrix} \cos\theta_l \cos\psi_{vl} & \sin\theta_l & \cos\theta_l \sin\psi_{vl} \\ -\sin\theta_l \cos\psi_{vl} & \cos\theta_l & -\sin\theta_l \sin\psi_{vl} \\ \sin\psi_{vl} & 0 & -\cos\psi_{vl} \end{bmatrix}$$

the meanings of other variables are same as those in Equ.(24).

It can be stated through analyzing this section that the indirect control model of relative motion based on the relationships of relative motion between missiles has a direct mapping relationship between inputs and outputs. However, this mapping just can be indicated by a mathematical form rather than some intuitive physical meanings, so we can't give the linear expressions just through modelling directly. As mentioned above, we should transform the coordinate space of input variables and then establish the indirect linear model (27).

After the missile formation controller system is described by the Equ.(27), we also need to consider the completely controllable ability of those indirect variables $\bar{\boldsymbol{U}}$, that is, we should analyze the relationships between rank of $[\boldsymbol{B} \vdots \boldsymbol{AB} \vdots \boldsymbol{A}^2\boldsymbol{B}]$ and the dimensions of this system. Because matrix $\boldsymbol{B}$ is full rank in the feasible flight scope, that is, $\text{rank}[\boldsymbol{B} \vdots \boldsymbol{AB} \vdots \boldsymbol{A}^2\boldsymbol{B}]=3$, system(27) should be completely controllable.

4.3 Establishing the optimal control model of missile formation keeping

As for the missile formation keeping control problem, the aim is to keep the distances between members within one formation on a non-zero states. After considering equation (27), we can further describe this formation keeping problem to another problem that how to regulate non-zero given value of output affected by slowly variant perturbations. So we can design this optimal controller by two steps: first, we need to design an optimal output regulator that can overcome slowly variant perturbations; second, we should further design optimal controller based on the first step which can maintain the missile formation on a non-zero desired relative states.

4.3.1 Optimal proportional-integral (PI) controller of missile formation

As for the system with invariant or slowly variant perturbations showed in expression (27), we can choose PI control law to overcome these perturbations, which is similar to classical control method.

In order to clearly explain the principle of how this integral feed-back controller eliminates stable errors, we will make a hypothesis that the stable outputs of system are zeros. Because there has an integrator in the controller, outputs can be constants although inputs are zeros, if these values are just rightly equivalent to the perturbations that are effecting on the input points but the signs are inverse, then the inputs of this control system will be eliminated to zeros, so the final outputs of system can be kept to zeros (Xie, 1986).

First, we transform the perturbations of system to control inputting ports, and then system (27) can be changed to:

$$\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{B}(\boldsymbol{U} + \tilde{\boldsymbol{W}}) \tag{28}$$

where $\tilde{\boldsymbol{W}}$ is transformation of initial perturbations, it has following form after comparing to the Equ.(27):

$$\tilde{\boldsymbol{W}} = \boldsymbol{B}^{-1}\tilde{\boldsymbol{B}}\boldsymbol{W}$$

Until now, the problem has been transformed to how to design PI optimal controller for system(28). This new system is augmented to:

$$\begin{aligned} &\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{B}(\boldsymbol{U} + \tilde{\boldsymbol{W}}) \\ &\dot{\boldsymbol{U}} + \dot{\tilde{\boldsymbol{W}}} = \boldsymbol{U}_1 \qquad \boldsymbol{U}(\mathrm{t}_0) + \tilde{\boldsymbol{W}}(\mathrm{t}_0) \text{is given} \end{aligned} \tag{29}$$

This augmented system can be further marked as:

$$\begin{aligned} &\dot{\boldsymbol{X}}_1 = \boldsymbol{A}_1\boldsymbol{X}_1 + \boldsymbol{B}_1\boldsymbol{U}_1 \\ &\boldsymbol{Y} = \boldsymbol{C}_1\boldsymbol{X}_1 \end{aligned} \tag{30}$$

where states variables of augmented system are $\boldsymbol{X}_1 = [\boldsymbol{X}, \boldsymbol{U} + \tilde{\boldsymbol{W}}]^{\mathrm{T}}$; system matrix $\boldsymbol{A}_1$ of this augmented system is:

$$\boldsymbol{A}_1 = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix}$$

control matrix of this augmented system is $\boldsymbol{B}_1 = [\boldsymbol{0}_{3\times3}, \boldsymbol{I}_{3\times3}]^T$, and output matrix is $\boldsymbol{C}_1 = [\boldsymbol{I}_{3\times3}, \boldsymbol{0}_{3\times3}]$.
The quadratic optimal performance index for system (30) is appointed to:

$$J = \int_{t_0}^{t_f} [\boldsymbol{X}_1^T \boldsymbol{Q}_1 \boldsymbol{X}_1 + \boldsymbol{U}_1^T \boldsymbol{R}_1 \boldsymbol{U}_1] dt \tag{31}$$

where $\boldsymbol{Q}_1$ is state regulating weight matrix of augmented system; $\boldsymbol{R}_1$ is control energy weight matrix of augmented system. When augmented system (30) is controllable, the optimal control quantities for minimizing the performance index (31) should be:

$$\boldsymbol{U}_1^* = -\boldsymbol{R}_1^{-1} \boldsymbol{B}_1^T \overline{\boldsymbol{P}} \boldsymbol{X}_1 \tag{32}$$

where $\overline{\boldsymbol{P}}$ is the solution of Riccati Equation:

$$\dot{\overline{\boldsymbol{P}}} = -\overline{\boldsymbol{P}} \boldsymbol{A}_1 - \boldsymbol{A}_1^T \overline{\boldsymbol{P}} + \overline{\boldsymbol{P}} \boldsymbol{B}_1 \boldsymbol{R}_1^{-1} \boldsymbol{B}_1^T \overline{\boldsymbol{P}} - \boldsymbol{Q}_1$$

Here we have to make further analysis on performance index expression(31). $\boldsymbol{Q}_1$ can be decomposed to the following form based on the expansions of state variables $\boldsymbol{X}_1$:

$$\boldsymbol{Q}_1 = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R} \end{bmatrix}$$

and then, there will be:

$$\boldsymbol{X}_1^T \boldsymbol{Q}_1 \boldsymbol{X}_1 = \boldsymbol{X}^T \boldsymbol{Q} \boldsymbol{X} + (\boldsymbol{U} + \tilde{\boldsymbol{W}})^T \boldsymbol{R} (\boldsymbol{U} + \tilde{\boldsymbol{W}}) \tag{33}$$

where $\boldsymbol{Q}$ is the states regulating weight matrix of original system(27), $\boldsymbol{R}$ is the control energy weight matrix of original system. From original system(27), there will be expression showed as below:

$$\boldsymbol{X}^T \boldsymbol{Q} \boldsymbol{X} = \boldsymbol{X}^T \boldsymbol{C}^T \boldsymbol{Q}_Y \boldsymbol{C} \boldsymbol{X} = \boldsymbol{Y}^T \boldsymbol{Q}_Y \boldsymbol{Y}$$

where $\boldsymbol{Q}_Y$ is the outputs regulating weight matrix, then expression(31) that describes the problem of quadratic optimal states regulating can be transformed to the problem of quadratic optimal output regulating, which is:

$$J = \int_{t_0}^{t_f} [\boldsymbol{Y}^T \boldsymbol{Q}_Y \boldsymbol{Y} + (\boldsymbol{U} + \tilde{\boldsymbol{W}})^T \boldsymbol{R} (\boldsymbol{U} + \tilde{\boldsymbol{W}}) + \boldsymbol{U}_1^T \boldsymbol{R}_1 \boldsymbol{U}_1] dt \tag{34}$$

besides, because perturbations $\tilde{\boldsymbol{W}}$ is slowly variant variables, following expression can be noted:

$$\dot{\tilde{\boldsymbol{W}}} = 0$$

and then

$$\boldsymbol{U}_1 = \dot{\tilde{\boldsymbol{U}}} \tag{35}$$

At last the quadratic optimal performance index of outputs regulating can be expressed as:

$$\mathrm{J} = \int_{\mathrm{t}_0}^{\mathrm{t_f}} [\boldsymbol{Y}^{\mathrm{T}} \boldsymbol{Q}_{\mathrm{Y}} \boldsymbol{Y} + (\tilde{\boldsymbol{U}} + \tilde{\boldsymbol{W}})^{\mathrm{T}} \boldsymbol{R} (\tilde{\boldsymbol{U}} + \tilde{\boldsymbol{W}}) + \dot{\tilde{\boldsymbol{U}}}^{\mathrm{T}} \boldsymbol{R}_1 \dot{\tilde{\boldsymbol{U}}}]\mathrm{dt} \tag{36}$$

As for the optimal control variables(32), it can be transformed from Equ.(35) to:

$$\boldsymbol{U}_1^{\,*} = \dot{\tilde{\boldsymbol{U}}}^* = -\boldsymbol{R}_1^{\,-1} \boldsymbol{B}_1^{\mathrm{T}} \overline{\boldsymbol{P}} \boldsymbol{X}_1 \tag{37}$$

Expanding equation(37) yields:

$$\begin{aligned} \dot{\tilde{\boldsymbol{U}}}^* &= -\boldsymbol{R}_1^{\,-1} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \overline{\boldsymbol{P}}_{11} & \overline{\boldsymbol{P}}_{12} \\ \overline{\boldsymbol{P}}_{21} & \overline{\boldsymbol{P}}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{X} \\ \dot{\tilde{\boldsymbol{U}}}^* + \tilde{\boldsymbol{W}} \end{bmatrix} \\ &= -\boldsymbol{R}_1^{\,-1} \overline{\boldsymbol{P}}_{21} \boldsymbol{X} - \boldsymbol{R}_1^{\,-1} \overline{\boldsymbol{P}}_{22} (\tilde{\boldsymbol{U}}^* + \tilde{\boldsymbol{W}}) \end{aligned} \tag{38}$$

which is the expression of optimal control quantities that can minimize the performance index (34) for zero-given points.

4.3.2 Optimal controller for non-zero given points

In order to keep the output variables $\boldsymbol{Y} = [x, y, z]^T$ on non-zero points, the final system states and control inputs should also be non-zero, and then the optimal control quantities will be transformed from Equ.(37) to the following form:

$$\dot{\tilde{\boldsymbol{U}}}^* = -\boldsymbol{R}_1^{\,-1} \boldsymbol{B}_1^T \overline{\boldsymbol{P}} \boldsymbol{X}_1 + \boldsymbol{U}_0' = -\boldsymbol{K} \boldsymbol{X}_1 + \boldsymbol{U}_0' \tag{39}$$

where $\boldsymbol{U}_0'$ is additional control quantities for non-zero states.
Considering the output equations of augmented system:

$$\boldsymbol{Y} = \boldsymbol{C}_1 \boldsymbol{X}_1 \tag{40}$$

and the expansions of augmented states $\boldsymbol{X}_1$:

$$\boldsymbol{X}_1 = [x, y, z, V_f + \tilde{\boldsymbol{W}}(1), \theta_f + \tilde{\boldsymbol{W}}(2), \psi_{vf} + \tilde{\boldsymbol{W}}(3)]^T$$

we should choose the following output matrix of augmented system to make sure the outputs of augmented system accord with those of original system:

$$\boldsymbol{C}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Substituting the control values(39) into the state equation(30) of augmented system yields:

$$\dot{\boldsymbol{X}}_1 = (\boldsymbol{A}_1 - \boldsymbol{B}_1 \boldsymbol{K}) \boldsymbol{X}_1 + \boldsymbol{B}_1 \boldsymbol{U}_0' \tag{41}$$

Because the closed-loop system(41) is asymptotically stable, there will be:

$$\lim_{t\to\infty}\dot{\boldsymbol{X}}_1(t)=0$$

and then the asymptotically stable system can be expressed as:

$$\boldsymbol{0}=(\boldsymbol{A}_1-\boldsymbol{B}_1\boldsymbol{K})\boldsymbol{X}_{10}+\boldsymbol{B}_1\boldsymbol{U}_0' \tag{42}$$

where $\boldsymbol{X}_{10}$ is the stable value of state $\boldsymbol{X}_1$. If all the eigenvalues of matrix $\boldsymbol{A}_1-\boldsymbol{B}_1\boldsymbol{K}$ lie in the left complex plane, then matrix is $\boldsymbol{A}_1-\boldsymbol{B}_1\boldsymbol{K}$ non-singular, and further we can get the expression showed as below from Equ.(42):

$$\boldsymbol{X}_{10}=-(\boldsymbol{A}_1-\boldsymbol{B}_1\boldsymbol{K})^{-1}\boldsymbol{B}_1\boldsymbol{U}_0' \tag{43}$$

and the relationship between non-zero points and stable value of states will meet:

$$\boldsymbol{Y}_{10}^*=\boldsymbol{C}_1\boldsymbol{X}_{10}$$

further the relationship required between non-zero states $\boldsymbol{Y}_{10}^*$ and the stable value $\boldsymbol{X}_{10}$ of state $\boldsymbol{X}_1$ should be expressed as:

$$\boldsymbol{U}_0'=[\boldsymbol{C}_1(\boldsymbol{B}_1\boldsymbol{K}-\boldsymbol{A}_1)^{-1}\boldsymbol{B}_1]^{-1}\boldsymbol{Y}_{10}^* \tag{44}$$

At last we can implement optimal control to system(27) that describes the relative kinematics of missiles by the following optimal control quantity:

$$\dot{\tilde{\boldsymbol{U}}}^*=-\boldsymbol{R}_1^{-1}\boldsymbol{B}_1^T\bar{\boldsymbol{P}}\boldsymbol{X}_1+[\boldsymbol{C}_1(-\boldsymbol{B}_1\boldsymbol{R}_1^{-1}\boldsymbol{B}_1^T\bar{\boldsymbol{P}}-\boldsymbol{A}_1)^{-1}\boldsymbol{B}_1]^{-1}\boldsymbol{Y}_{10}^* \tag{45}$$

and this control quantity can keep missile formation on the desired relative dimensional states.

4.4 Stability analysis

In order to analyze the stability of optimal controller of missile formation, we need to transform the model of missile formation relative motion to tracking error model. The tracking error of state is chosen by state equation(27) as:

$$\hat{\boldsymbol{X}}=\boldsymbol{X}-\boldsymbol{X}_d \tag{46}$$

where is $\boldsymbol{X}$ the actual flight states of missile formation; $\boldsymbol{X}_d$ is the expected states. Substituting last equation into Equ.(27) obtains the error state equation:

$$\dot{\hat{\boldsymbol{X}}}=\boldsymbol{A}\hat{\boldsymbol{X}}+\boldsymbol{B}\boldsymbol{U}+\hat{\boldsymbol{W}} \tag{47}$$

where $\hat{\boldsymbol{W}}$ is the invariant perturbations of tracking error state equation, the value of this term is:

$$\hat{\boldsymbol{W}}=\boldsymbol{W}+\boldsymbol{A}\boldsymbol{X}_d$$

Because invariant perturbations do not affect the dynamic performance of control system no more than affect the stable tracking performance of this system, we can ignore this term when just analyzing the stability of control system, that is, we can just analyze the following control system:

$$\dot{\hat{\boldsymbol{X}}} = \boldsymbol{A}\hat{\boldsymbol{X}} + \boldsymbol{B}\boldsymbol{U} \tag{48}$$

Taking Lyapunov function to this system gets:

$$\Phi = \frac{1}{2}(\hat{x}^2 + \hat{y}^2 + \hat{z}^2) \tag{49}$$

The derivative of this Lyapunov function is:

$$\dot{\Phi} = \hat{x}\dot{\hat{x}} + \hat{y}\dot{\hat{y}} + \hat{z}\dot{\hat{z}} \tag{50}$$

Substituting Equ.(26) to expression(50) yields:

$$\dot{\Phi} = (\boldsymbol{B}\boldsymbol{U})^T \hat{\boldsymbol{X}} \tag{51}$$

and the optimal control quantity $\boldsymbol{U}$ for this error state equation is:

$$\boldsymbol{U} = -\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}\hat{\boldsymbol{X}}$$

substituting it into Equ.(50) gets:

$$\dot{\Phi} = (-\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}\hat{\boldsymbol{X}})^T \hat{\boldsymbol{X}}$$

Because the control energy weight matrix $\boldsymbol{R}$ in this chapter is unit matrix, there should have:

$$\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}\hat{\boldsymbol{X}} = \boldsymbol{P}\hat{\boldsymbol{X}}$$

and then the derivative of Lyapunov function can be changed to:

$$\dot{\Phi} = -\hat{\boldsymbol{X}}^T\boldsymbol{P}^T\hat{\boldsymbol{X}} \tag{52}$$

where $\boldsymbol{P}$ is the solution of Riccati Equation, which has the following characters (Xie, 1986):

1. for every $t \in [t_0, T]$, $\boldsymbol{P}$ is symmetrical matrix;
2. for every $t \in [t_0, T]$, $\boldsymbol{P}$ is non-negative matrix.

So Equ.(52) is non-negative, and it also states that we can't get the conclusion that this system is asymptotical until now, and need some more stronger conditions. Original system is time-variant system, so we can complement the stability conditions of system(48) through Barbalat Lemma.

The contents of Barbalat Lemma are: if a scalar function $\Theta(x,t)$ is satisfied with the following series of conditions:

1. $\Theta(x,t)$ has lower bounded;
2. $\dot{\Theta}(x,t)$ is semi-negative;
3. $\dot{\Theta}(x,t)$ is uniformly continuous with respect to time.

then $\dot{\Theta}(x,t) \to 0$ when $t \to \infty$ (Slotine et al, 1991).

After analyzing that Lyapunov function appointed in the previous section, we can find that this function satisfies the former two aspects of Barbalat Lemma. If we want to prove expression(52) also satisfies the third point, we just need to prove $\ddot{\Phi}$ exists and is bounded. The expression of $\ddot{\Phi}$ is:

$$\ddot{\Phi} = -\dot{\hat{\boldsymbol{X}}}^T \boldsymbol{P}^T \hat{\boldsymbol{X}} - \hat{\boldsymbol{X}}^T \boldsymbol{P}^T \dot{\hat{\boldsymbol{X}}}$$

at the same time, from Equ.(52) we can know:

$$\dot{\Phi} \leq 0$$

so, according to Equ.(49), it can be concluded that system state $\hat{\boldsymbol{X}}$ is bounded, further $\ddot{\Phi}$ exists and is bounded, and then Lyapunov function meets Barbalat Lamme, that is, when $t \to \infty$,there will be $\dot{\Phi}(x,t) \to 0$,and then further $\hat{\boldsymbol{X}} \to \boldsymbol{0}$, which indicates that the system(48) is asymptotically stable.

5. Simulations

5.1 Simulations for PD controller of missile formation keeping

5.1.1 Initial conditions

We will choose the following conditions for the simulations of PD controller:

1. missile formation is consisted of three missiles;
2. flight time of missile formation is 150s;
3. three inertial time constants of each channel of follower are $\tau_v = 3.51\text{s}$, $\tau_\theta = 2.25\text{s}$ and $\tau_{\psi_v} = 2.37\text{s}$;
4. the flight states of leader are:
   - initial positions in inertial coordinate frame are: $X_{l0} = 500\text{m}$, $Y_{l0} = 800\text{m}$ and $Z_{l0} = 0\text{m}$; intial velocitiy is $V_{l0} = 240\text{m/s}$; initial flight path angle is $\theta_0 = 10^\circ$;initial flight deflection angle is $\psi_{v0} = 20^\circ$.
   - the change rules of veloctity, fligh path angle and flight deflection angle of leader are:

$$\begin{aligned} V_l &= 240 + 20\sin(0.15t) \\ \theta_l &= 0.2\sin(0.05t + 0.4515) \\ \psi_{vl} &= 0.4\sin(0.06t + 0.5511) \end{aligned}$$

initial distances between leader and follower 1 and follower 2 are:

$$\begin{bmatrix} x_{f1} \\ y_{f1} \\ z_{f1} \end{bmatrix} = \begin{bmatrix} -500 \\ -550 \\ -350 \end{bmatrix}\text{m}, \begin{bmatrix} x_{f2} \\ y_{f2} \\ z_{f2} \end{bmatrix} = \begin{bmatrix} -400 \\ -450 \\ 350 \end{bmatrix}\text{m}$$

desired distances between leader and follower 1 and follower 2 are:

$$\begin{bmatrix} x_{f1}^* \\ y_{f1}^* \\ z_{f1}^* \end{bmatrix} = \begin{bmatrix} -450 \\ -500 \\ -400 \end{bmatrix} \text{m}, \begin{bmatrix} x_{f2}^* \\ y_{f2}^* \\ z_{f2}^* \end{bmatrix} = \begin{bmatrix} -450 \\ -500 \\ 400 \end{bmatrix} \text{m}$$

intial states of follower 1 and follower 2 are:

- velocity: $V_{f10} = 180\text{m/s}$, $V_{f20} = 240\text{m/s}$;
- flight path angle: $\theta_{f10} = 5^\circ$, $\theta_{f20} = -5^\circ$;
- flight deflection angle: $\psi_{vf10} = -10^\circ$, $\psi_{vf20} = 10^\circ$.

satuaration of control quantities

Considering the feasible fight scope of missile formation, we should limit the control quantities of formation controller, that is, the command states of follower should be limilted. Here we set the satuatations are:

$$150\text{m/s} \le V_{fc} \le 300\text{m/s} \text{ , -15}^\circ \le \theta_{fc} \le 35^\circ \text{ , -40}^\circ \le \psi_{vfc} \le 40^\circ$$

5.1.2 Simulation results and analysis

After simulating, we can get the result curves Fig.6-Fig.12, where Fig.6 is the 3-dimensional motions of missile formation; Fig.7 and Fig.8 are the position components of follower 1 and follower 2 in relative coordinate frame respectively; Fig.9 is the distances between members in missile formation; Fig.10-Fig.11 are the control quantities in velocity, flight path angle and flight deflection angle channels of followers respectively.

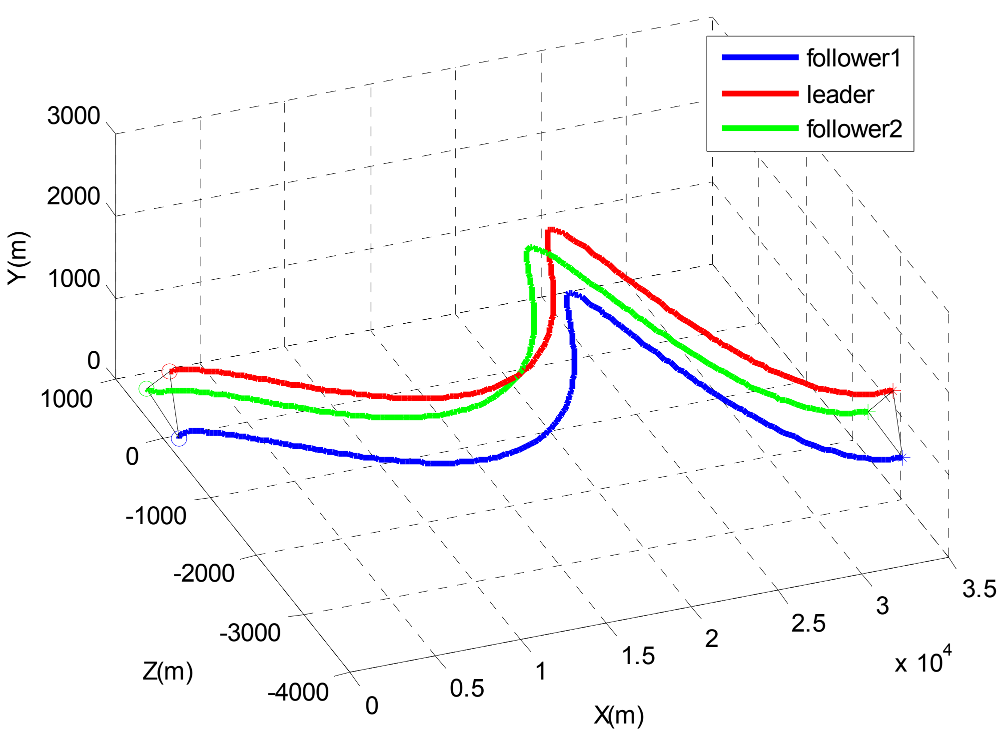


Fig. 6. Three-dimensional trajectories of missile formation under PD controlling.

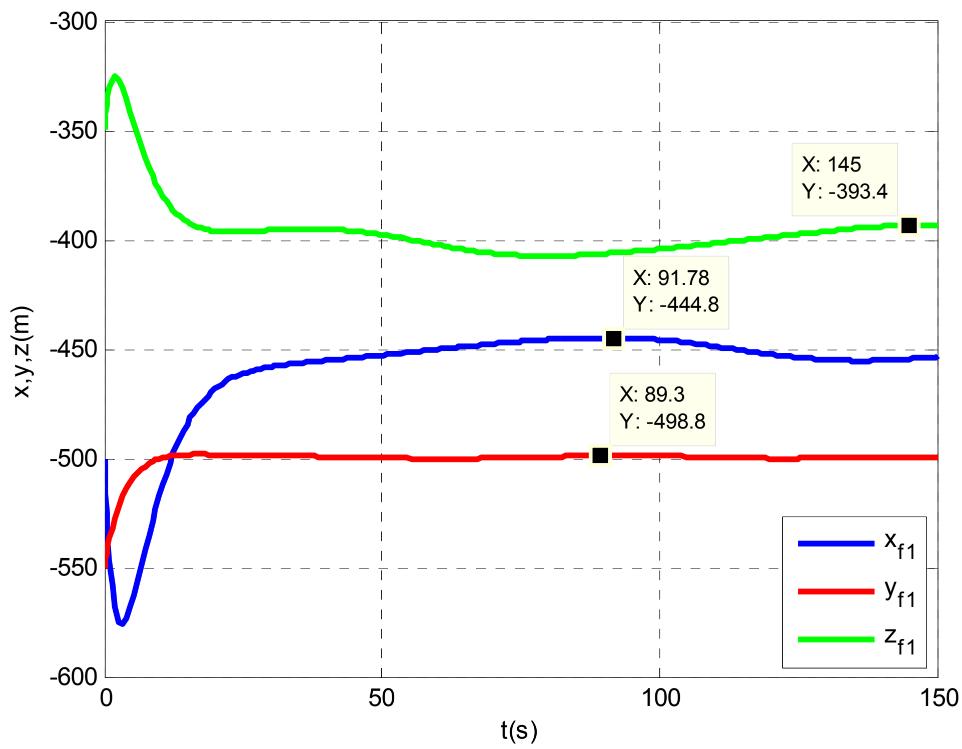


Fig. 7. Distances between follower 1 and leader in relative coordinate frame.

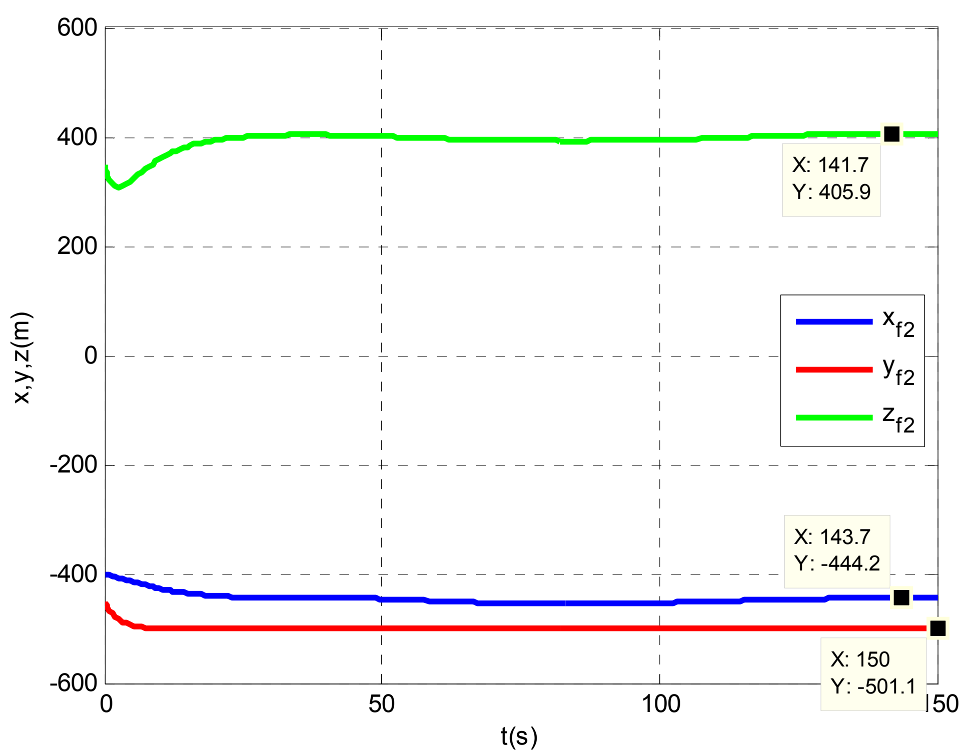


Fig. 8. Distances between follower 2 and leader in relative coordinate frame.

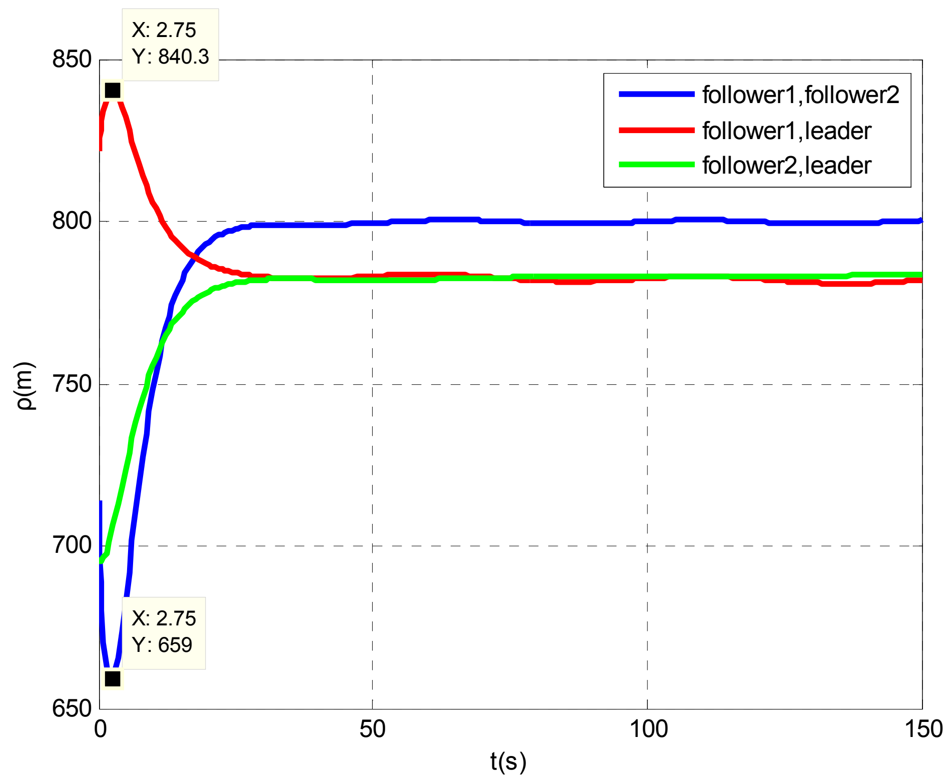


Fig. 9. Distances between members of missile formation in relative coordinate frame.

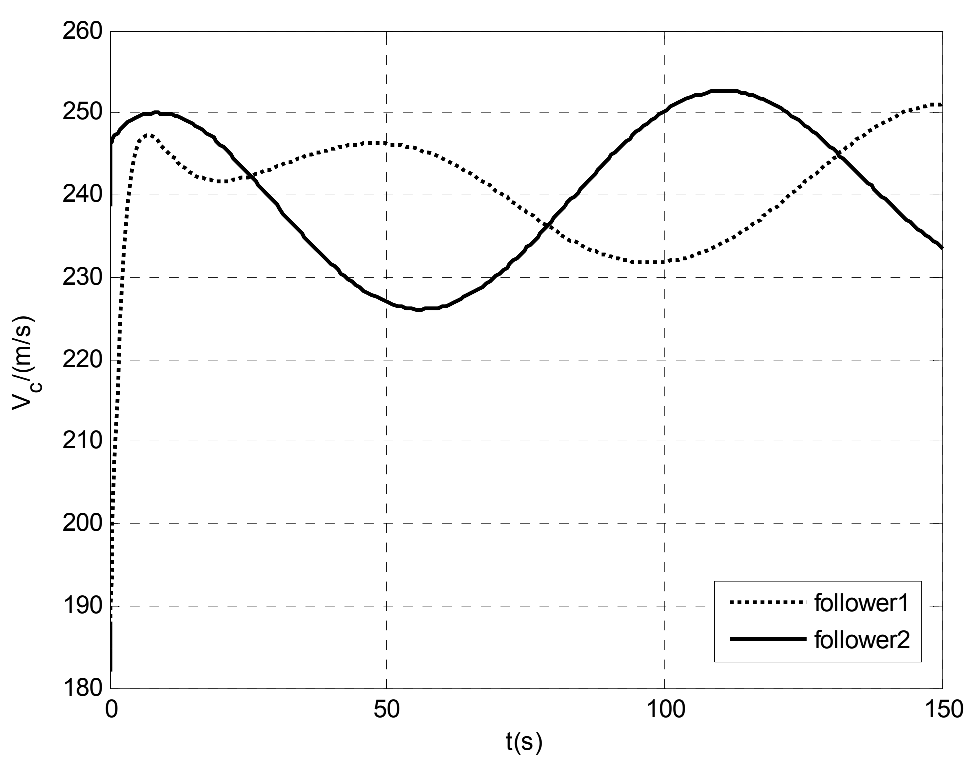


Fig. 10. Velocity commands of followers.

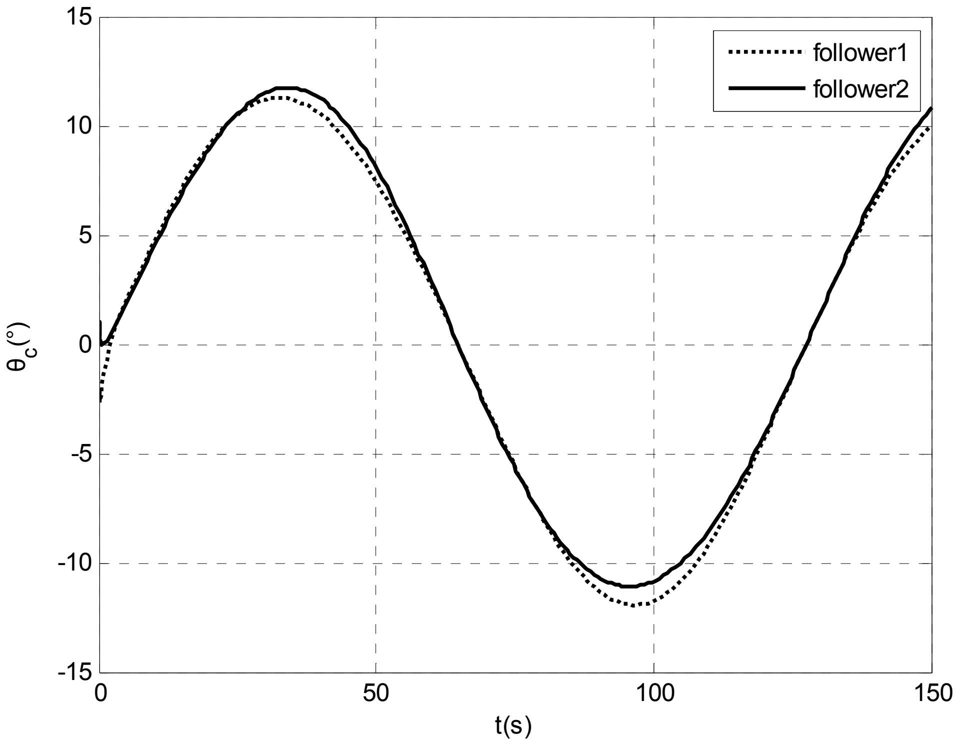


Fig. 11. Flight path angle commands of followers.

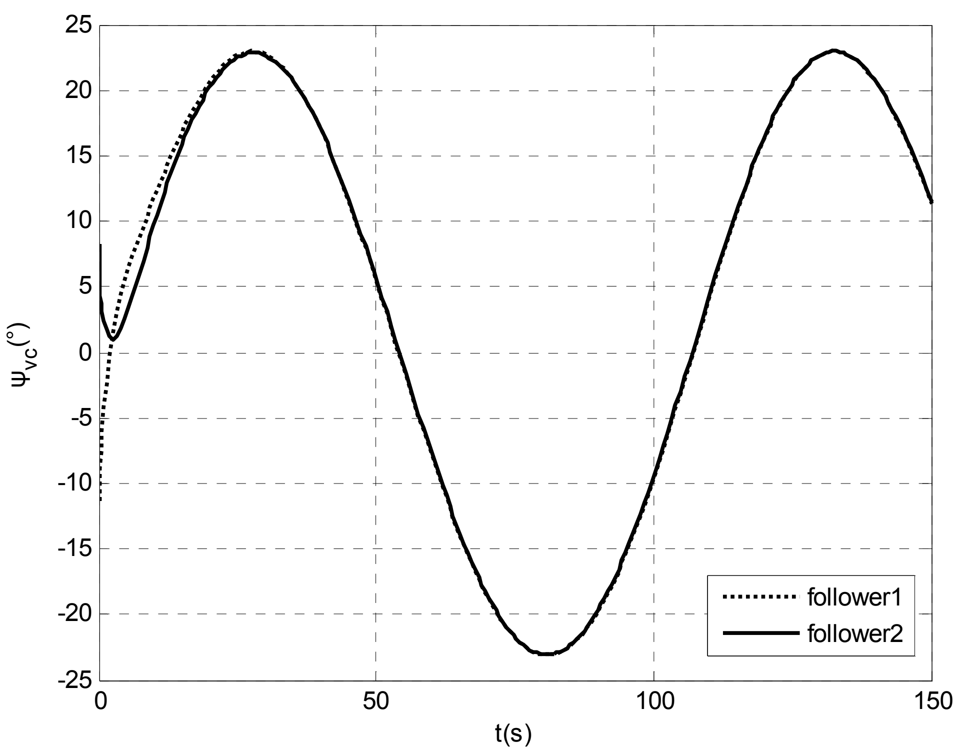


Fig. 12. Flight deflection angle commands of followers.

It can be found from those result curves showed above that PD keeping controller of missile formation can implement keeping control at about 25s, and control quantities are feasible; the stable error in x_r direction is about 5m, and those in y_r and z_r directions are 1m and 6m respectively.
Because three channels of PD controller are coupled seriously, it is very difficult to find the obvious relationships between the inertial time constants of each channel and control performance; From the change of each curve, we can find that the maneuver of leader affect the formation keeping control distinctly; Besides, we can find that there are evident regulating processes from initial states to expected states, and the maximum control quantity exists in two followers, its value is about 60m.

5.2 Simulations for Optimal Controller of missile formation keeping
5.2.1 Initial Conditions
We will choose the following conditions for the simulations to optimal controller:
1. weight matrices of optimal controller are:
- output regulating weight matrix: $\boldsymbol{Q}_y = \mathrm{diag}(4.0, 6.0, 6.0)$;
- control energy weight matrix: $\boldsymbol{R} = \mathrm{diag}(1.0, 1.0, 1.0)$;
- weight matrix of control energy changing: $\boldsymbol{R}_1 = \mathrm{diag}(1.0, 1.0, 1.0)$.
2. other conditions are same as section 5.1.

5.2.2 Simulation results and analysis
After simulating, we can get the result curves Fig.13-Fig.18, where Fig.13 is the 3-dimensional motion of missile formation; Fig.14 and Fig.15 are the position components of follower 1 and follower 2 in relative coordinate frame respectively; Fig.16-Fig.18 are the control quantities in velocity, flight path angle and flight deflection angle channels of followers respectively.
There are following conclusions after analyzing above result curves:
1. Optimal keeping controller of missile formation can implement keeping on desired relative states at about 20s, it has faster response speed than PD controller;
2. The maneuver motion of leader also disturbs missile formation keeping control, and there also exists regulation process when the states of missile formation changing from initial states to final states, but the disturbance amplitude is smaller that of PD controller;
3. The stable error of relative motion in x_r direction is about 0.5m, and those in other two directions are 3m and 4m respectively, which are decreased by comparing to PD controller. Especially the improvement in x_r direction is more evident, which is because the velocity of leader is considered as slowly variant perturbation that will affect evidently to the relative motion in x_r direction, however, the optimal controller designed in this chapter can restrict this perturbation well, so there will be more higher tracking precision;
4. We can further decrease the stable errors by enhancing output regulating weight matrix $\boldsymbol{Q}_y$, but this manner will increase the changing rate at the same time, so we should coordinate the values of weight matrix $\boldsymbol{Q}_y$, $\boldsymbol{R}$ and $\boldsymbol{R}_1$ to obtain proper control quantities, and achieve the minimum stable tracking error.

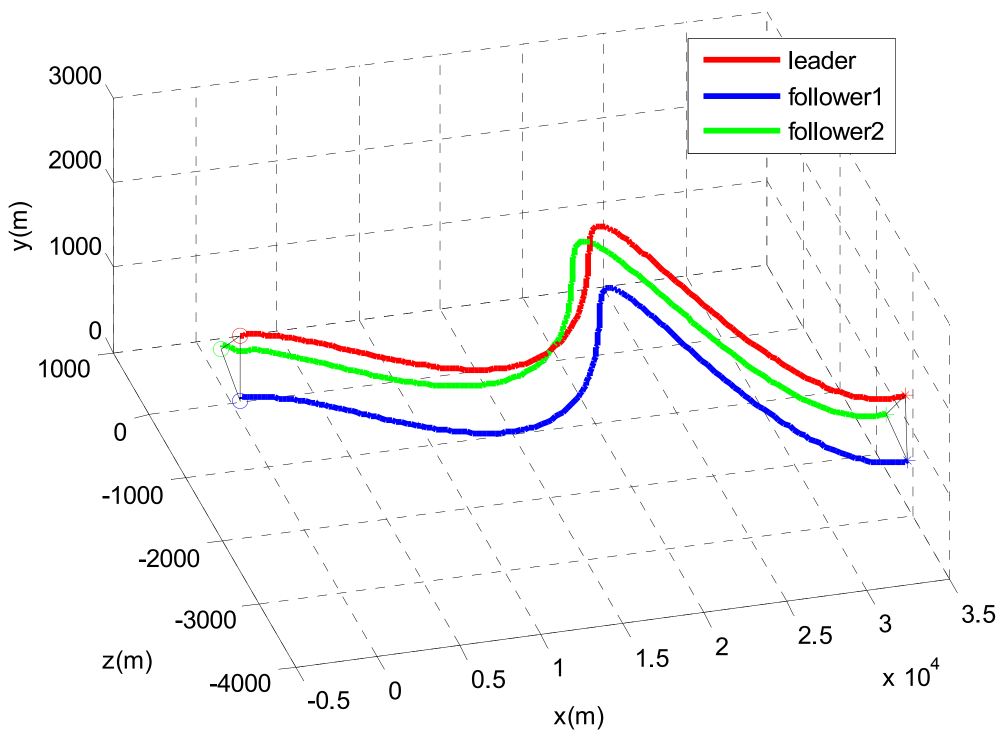


Fig. 13. Three-dimensional trajectories of missile formation under optimal controlling.

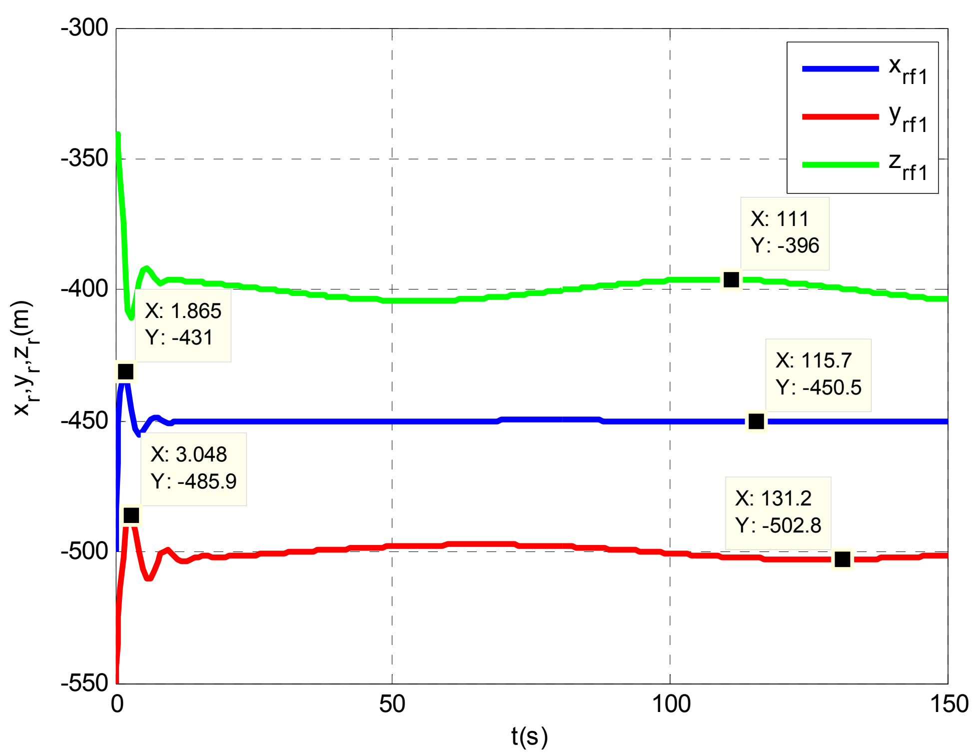


Fig. 14. Distances between follower 1 and leader in relative coordinate frame.

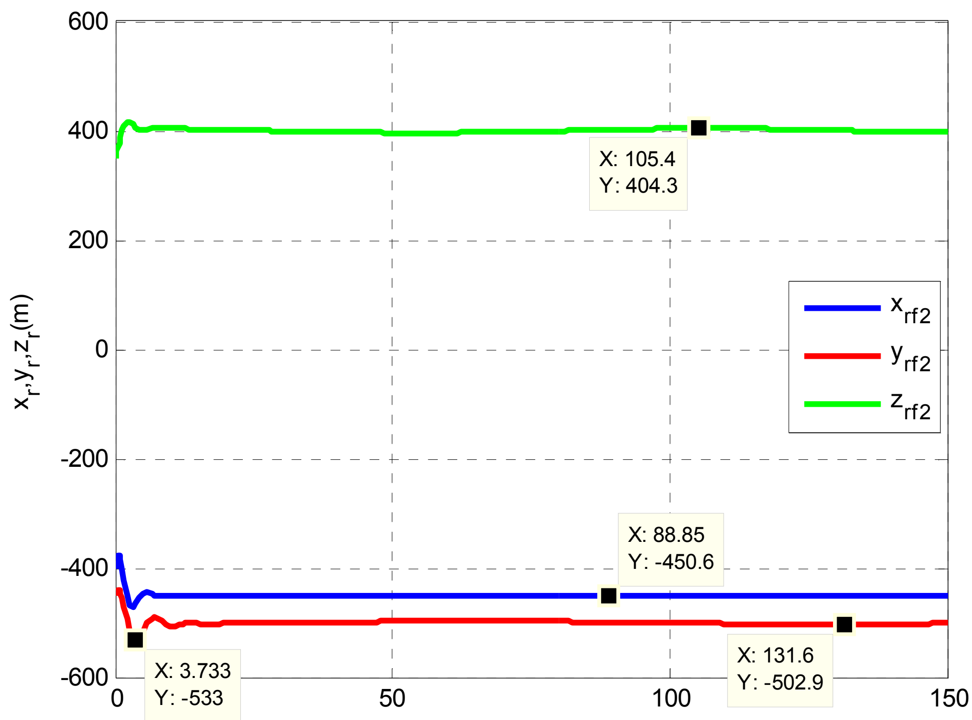


Fig. 15. Distances between follower 2 and leader in relative coordinate frame.

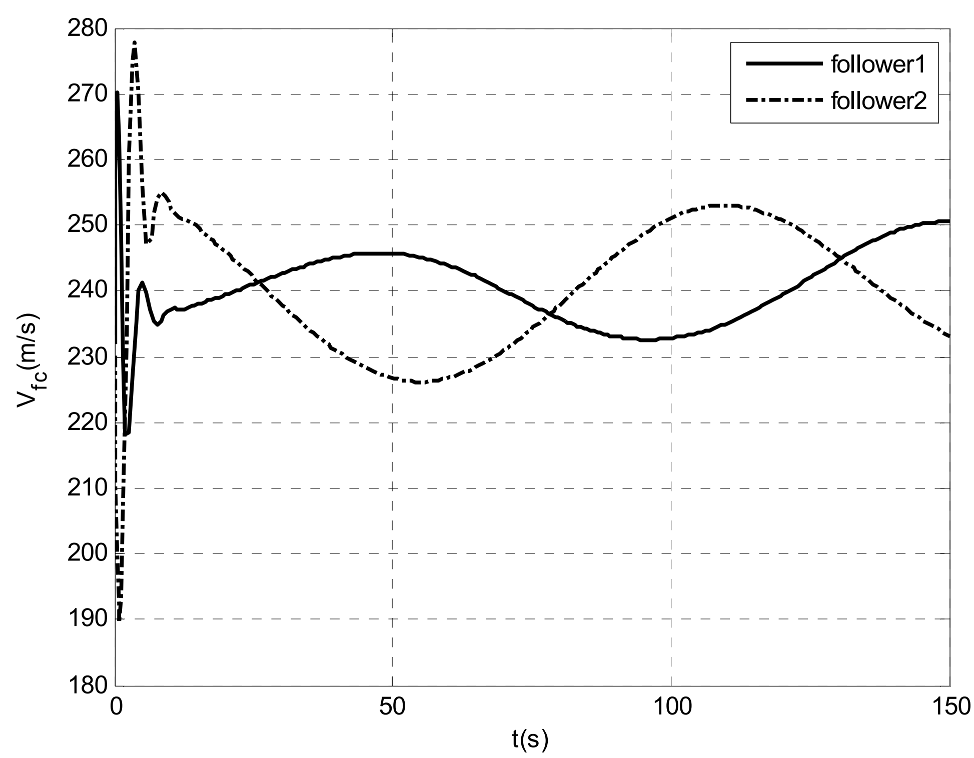


Fig. 16. Velocity commands of followers.

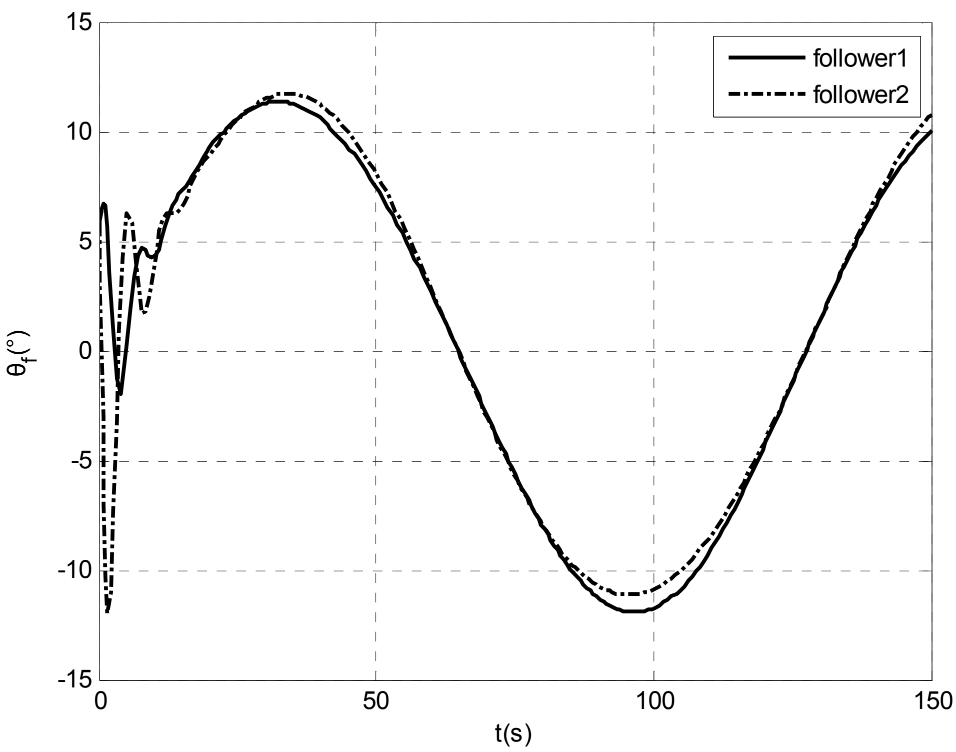


Fig. 17. Flight path angle commands of followers.

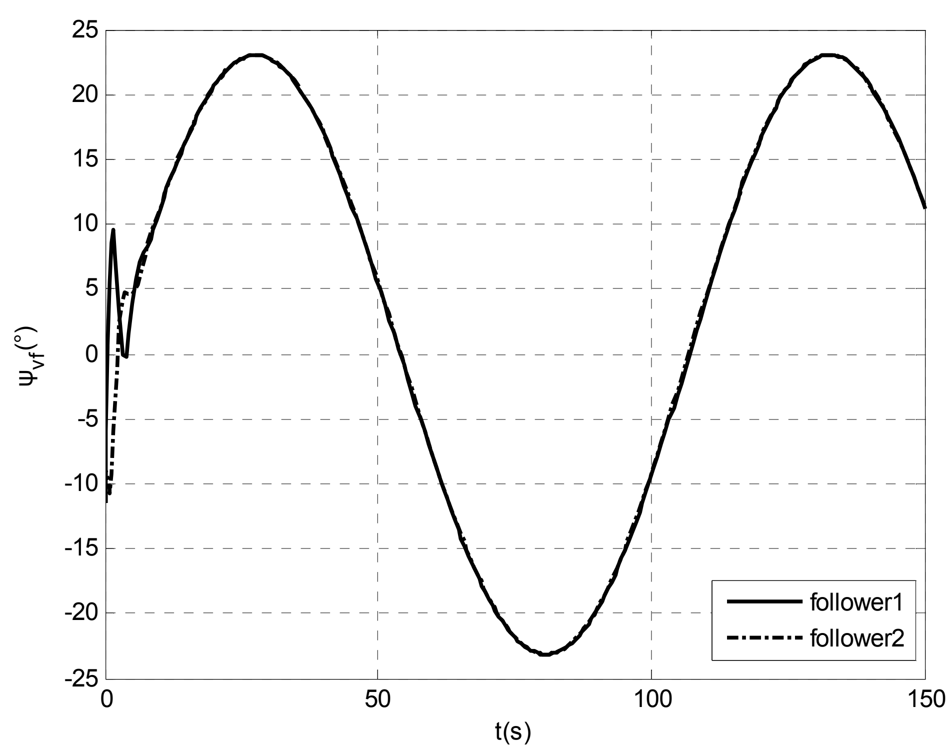


Fig. 18. Flight deflection angle commands of followers.

6. Conclusion

In this chapter, we mainly focus on some control problems of missile formation engagement. Significance of cooperative engagement was first presented by taking LOCAAS as instance, which showed the synthetical efficacy can be greatly increased by adopting cooperative engagement manner. Following the significance was analysis of frame of cooperative engagement system, which supplied the train of thought of how to research on missile formation control problem. Missile formation keeping control system design is the main content of this chapter. In this part, we established the model of relative motion in two ways firstly, and then designed missile formation control system based on PD control law and optimal control method respectively. Finally, the comparisons were made by a series of simulations, the conclusion that optimal controller is better from the points of view of stability and rapidity can be obtained from the result curves.

7. Acknowledgment

Thank Prof. Sang-Young Park and Korean Industry-Academic Cooperation Foundation very much. They supply plenteous financial support and good working conditions.

8. References

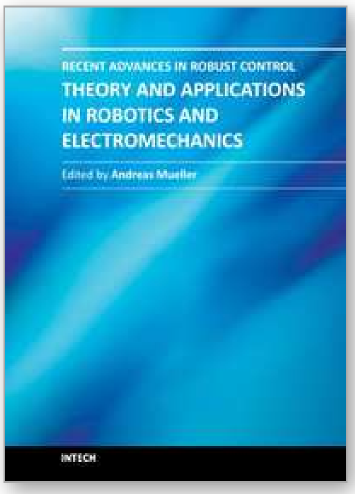
Cui, Naigang ; Wei, Changzhu; Guo, Jifeng & Zhao, Biao (2010). Research on the missile formation control system. *Proceeding of the 2009 IEEE International Conference on Mechatronics and Automation*, pp. 4197-4198, Changchun, China, August 9-12, 2009

Wei, Changzhu; Guo, Jifeng & Cui, Naigang (2010). Research on the missile formation keeping optimal control for cooerative engagement. *Journal of Astronautics*, Vol.31, No.4, (2010), pp. 1043-1044, ISBN 1000-1328

Cui, Naigang; Wei, Changzhu & Guo, Jifeng (2010). Research on four-dimensional guidance and control for missile cooperative engagement. *Flight Dynamics*, Vol.28, No.2, (2010), pp. 63-66, ISBN 1002-0853

Xie, Xueshu (1986). *Optimal control theroty and application* (1St edition), Tsinghua University Press, Beijing

Slotine, Jean Jacques & Li, Weiping (1991). *Applied nonlinear control* (1St edition), Prentice-Hall International (UK) Limited, London, ISBN: 0130408905



Recent Advances in Robust Control - Theory and Applications in Robotics and Electromechanics
Edited by Dr. Andreas Mueller

ISBN 978-953-307-421-4
Hard cover, 396 pages
Publisher InTech
Published online 21, November, 2011
Published in print edition November, 2011

Robust control has been a topic of active research in the last three decades culminating in H_2/H_\infty and \mu design methods followed by research on parametric robustness, initially motivated by Kharitonov's theorem, the extension to non-linear time delay systems, and other more recent methods. The two volumes of Recent Advances in Robust Control give a selective overview of recent theoretical developments and present selected application examples. The volumes comprise 39 contributions covering various theoretical aspects as well as different application areas. The first volume covers selected problems in the theory of robust control and its application to robotic and electromechanical systems. The second volume is dedicated to special topics in robust control and problem specific solutions. Recent Advances in Robust Control will be a valuable reference for those interested in the recent theoretical advances and for researchers working in the broad field of robotics and mechatronics.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

Changzhu Wei, Yi Shen, Xiaoxiao Ma, Naigang Cui and Jifeng Guo (2011). Missile Cooperative Engagement Formation Configuration Control Method, Recent Advances in Robust Control - Theory and Applications in Robotics and Electromechanics, Dr. Andreas Mueller (Ed.), ISBN: 978-953-307-421-4, InTech, Available from: http://www.intechopen.com/books/recent-advances-in-robust-control-theory-and-applications-in-robotics-and-electromechanics/missile-cooperative-engagement-formation-configuration-control-method

INTECH
open science | open minds

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821