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Partially Decentralized Design Principle in Large-Scale System Control

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1. Introduction

A number of problems that arise in state control can be reduced to a handful of standard convex and quasi-convex problems that involve matrix inequalities. It is known that the optimal solution can be computed by using interior point methods (Nesterov & Nemirovsky (1994)) which converge in polynomial time with respect to the problem size, and efficient interior point algorithms have recently been developed for and further development of algorithms for these standard problems is an area of active research. For this approach, the stability conditions may be expressed in terms of linear matrix inequalities (LMI), which have a notable practical interest due to the existence of powerful numerical solvers. Some progres review in this field can be found e.g. in Boyd et al. (1994), Hermann et al. (2007), Skelton et al. (1998), and the references therein.

Over the past decade, H_∞ norm theory seems to be one of the most sophisticated frameworks for robust control system design. Based on concept of quadratic stability which attempts to find a quadratic Lyapunov function (LF), H_∞ norm computation problem is transferred into a standard LMI optimization task, which includes bounded real lemma (BRL) formulation (Wu et al. (2010)). A number of more or less conservative analysis methods are presented to assess quadratic stability for linear systems using a fixed Lyapunov function. The first version of the BRL presents simple conditions under which a transfer function is contractive on the imaginary axis of the complex variable plain. Using it, it was possible to determine the H_∞ norm of a transfer function, and the BRL became a significant element to shown and prove that the existence of feedback controllers (that results in a closed loop transfer matrix having the H_∞ norm less than a given upper bound) is equivalent to the existence of solutions of certain LMIs. Linear matrix inequality approach based on convex optimization algorithms is extensively applied to solve the above mentioned problem (Jia (2003), Kozáková & Veselý (2009)), Pipeleers et al. (2009).

For time-varying parameters the quadratic stability approach is preferable utilized (see. e.g. Feron et al. (1996)). In this approach a quadratic Lyapunov function is used which is independent of the uncertainty and which guarantees stability for all allowable uncertainty values. Setting Lyapunov function be independent of uncertainties, this approach guarantees uniform asymptotic stability when the parameter is time varying, and, moreover, using a parameter-dependent Lyapunov matrix quadratic stability may be established by LMI tests over the discrete, enumerable and bounded set of the polytope vertices, which define the uncertainty domain. To include these requirements the equivalent LMI representations of

BRL for continuous-time, as well as discrete-time uncertain systems were introduced (e.g. see Wu and Duan (2006), and Xie (2008)). Motivated by the underlying ideas a simple technique for the BRL representation can be extended to state feedback controller design, performing system H_∞ properties of quadratic performance. When used in robust analysis of systems with polytopic uncertainties, they can reduce conservatism inherent in the quadratic methods and the parameter-dependent Lyapunov function approach. Of course, the conservativeness has not been totally eliminated by this approach.

In recent years, modern control methods have found their way into design of interconnected systems leading to a wide variety of new concepts and results. In particular, paradigms of LMIs and H_∞ norm have appeared to be very attractive due to their good promise of handling systems with relative high dimensions, and design of partly decentralized schemes substantially minimized the information exchange between subsystems of a large scale system. With respect to the existing structure of interconnections in a large-scale system it is generally impossible to stabilize all subsystems and the whole system simultaneously by using decentralized controllers, since the stability of interconnected systems is not only dependent on the stability degree of subsystems, but is closely dependent on the interconnections (Jamshidi (1997), Lunze (1992), Mahmoud & Singh (1981)). Including into design step the effects of interconnections, a special view point of decentralized control problem (Filasová & Krokavec (1999), Filasová & Krokavec (2000), Leros (1989)) can be such adapted for large-scale systems with polytopic uncertainties. This approach can be viewed as pairwise-autonomous partially decentralized control of large-scale systems, and gives the possibility establish LMI-based design method as a special problem of pairwise autonomous subsystems control solved by using parameter dependent Lyapunov function method in the frames of equivalent BRL representations.

The chapter is devoted to studying partially decentralized control problems from above given viewpoint and to presenting the effectiveness of parameter-dependent Lyapunov function method for large-scale systems with polytopic uncertainties. Sufficient stability conditions for uncertain continuous-time systems are stated as a set of linear matrix inequalities to enable the determination of parameter independent Lyapunov matrices and to encompass quadratic stability case. Used structures in the presented forms enable potentially to design systems with the reconfigurable controller structures.

The chapter is organized as follows. In section 2 basis preliminaries concerning the H_∞ norm problems are presented along with results on BRL, improved BRLs representations and modifications, as well as with quadratic stability. To generalize properties of non-expansive systems formulated as H_∞ problems in BRL forms, the main motivation of section 3 was to present the most frequently used BRL structures for system quadratic performance analyzes. Starting work with such introduced formalism, in section 4 the principle of memory-less state control design with quadratic performances which performs H_∞ properties of the closed-loop system is formulated as a feasibility problem and expressed over a set of LMIs. In section 5, the BRL based design method is outlined to posse the sufficient conditions for the pairwise decentralized control of one class of large-scale systems, where Lyapunov matrices are separated from the matrix parameters of subsystem pairs. Exploring such free Lyapunov matrices, the parameter-dependent Lyapunov method is adapted for pairwise decentralized controller design method of uncertain large-scale systems in section 6, namely quadratic stability conditions and the state feedback stabilizability problem based on these conditions. Finally, some concluding remarks are given in the end. However, especially in sections 4-6,

numerical examples are given to illustrate the feasibility and properties of different equivalent BRL representations.

2. Basic preliminaries

2.1 System model
The class of the systems considering in this section can be formed as follows

$$\dot{q}(t) = Aq(t) + Bu(t) \tag{1}$$

$$y(t) = Cq(t) + Du(t) \tag{2}$$

where $q(t) \in R^n$, $u(t) \in R^r$, and $y(t) \in R^m$ are vectors of the state, input and measurable output variables, respectively, nominal system matrices $A \in R^{n \times n}$, $B \in R^{n \times r}$, $C \in R^{m \times n}$ and $D \in R^{m \times r}$ are real matrices.

2.2 Schur complement
Proposition 1. . *Let $Q > 0$, $R > 0$, S are real matrices of appropriate dimensions, then the next inequalities are equivalent*

$$\begin{bmatrix} Q & S \\ S^T & -R \end{bmatrix} < 0 \; \Leftrightarrow \; \begin{bmatrix} Q + SR^{-1}S^T & 0 \\ 0 & -R \end{bmatrix} < 0 \; \Leftrightarrow \; Q + SR^{-1}S^T < 0, \; \mathbf{R} > 0 \tag{3}$$

Proof. Let the linear matrix inequality takes the starting form in (3), $\det R \neq 0$ then using Gauss elimination principle it yields

$$\begin{bmatrix} I & SR^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} Q & S \\ S^T & -R \end{bmatrix} \begin{bmatrix} I & 0 \\ R^{-1}S^T & I \end{bmatrix} = \begin{bmatrix} Q + SR^{-1}S^T & 0 \\ 0 & -R \end{bmatrix} \tag{4}$$

Since

$$\det \begin{bmatrix} I & SR^{-1} \\ 0 & I \end{bmatrix} = 1 \tag{5}$$

and it is evident that (4) implies (3). This concludes the proof. □

Note that in the next sections the matrix notations Q, R, S, can be used in another context, too.

2.3 Bounded real lemma
Proposition 2. *System (1), (2) is stable with quadratic performance $\|C(sI - A)^{-1}B + D\|_\infty^2 \leq \gamma$ if there exist a symmetric positive definite matrix $P > 0$, $P \in R^{n \times n}$ and a positive scalar $\gamma > 0$, $\gamma \in R$ such that*

$$i. \; \begin{bmatrix} A^TP + PA & PB & C^T \\ * & -\gamma I_r & D^T \\ * & * & -I_m \end{bmatrix} < 0$$

$$ii. \; \begin{bmatrix} PA^T + AP & PC^T & B \\ * & -\gamma I_m & D \\ * & * & -I_r \end{bmatrix} < 0$$

$$iii. \quad \begin{bmatrix} P^{-1}A^T+AP^{-1} & B & P^{-1}C^T \\ * & -\gamma I_r & D^T \\ * & * & -I_m \end{bmatrix} < 0$$

$$iv. \quad \begin{bmatrix} A^TP^{-1}+P^{-1}A & C^T & P^{-1}B \\ * & -\gamma I_m & D \\ * & * & -I_r \end{bmatrix} < 0 \tag{6}$$

where $I_r \in R^{r \times r}$, $I_m \in R^{m \times m}$ are identity matrices, respectively.

Hereafter, $*$ denotes the symmetric item in a symmetric matrix.

Proof. i. Defining Lyapunov function as follows (Gahinet et al. (1996))

$$v(q(t)) = q^T(t)Pq(t) + \int_0^t (y^T(r)y(r) - \gamma u^T(r)u(r))\mathrm{d}r > 0 \tag{7}$$

where $P = P^T > 0$, $P \in R^{n \times n}$, $\gamma > 0 \in R$, and evaluating the derivative of $v(q(t))$ with respect to t along a system trajectory then it yields

$$\dot{v}(q(t)) = \dot{q}^T(t)Pq(t) + q^T(t)P\dot{q}(t) + y^T(t)y(t) - \gamma u^T(t)u(t) < 0 \tag{8}$$

Thus, substituting (1), (2) into (8) gives

$$\dot{v}(q(t)) = (Aq(t) + Bu(t))^T Pq(t) + q^T(t)P(Aq(t)+Bu(t)) - \gamma u^T(t)u(t) + \\ + (Cq(t)+Du(t))^T(Cq(t)+Du(t)) < 0 \tag{9}$$

and with the next notation

$$q_c^T(t) = \begin{bmatrix} q^T(t) & u^T(t) \end{bmatrix} \tag{10}$$

it is obtained

$$\dot{v}(q(t)) = q_c^T(t)P_c q_c(t) < 0 \tag{11}$$

where

$$P_c = \begin{bmatrix} A^TP + PA & PB \\ * & -\gamma I_r \end{bmatrix} + \begin{bmatrix} C^TC & C^TD \\ * & D^TD \end{bmatrix} < 0 \tag{12}$$

Since

$$\begin{bmatrix} C^TC & C^TD \\ * & D^TD \end{bmatrix} = \begin{bmatrix} C^T \\ D^T \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix} \geq 0 \tag{13}$$

Schur complement property implies

$$\begin{bmatrix} 0 & 0 & C^T \\ * & 0 & D^T \\ * & * & -I_m \end{bmatrix} \geq 0 \tag{14}$$

and using (14) the LMI condition (12) can be written compactly as *i.* of (2).

ii. Since H_∞ norm is closed with respect to complex conjugation and matrix transposition (Petersen et al. (2000)), then

$$\|C(sI-A)^{-1}B + D\|_\infty^2 \leq \gamma \quad \Leftrightarrow \quad \|B^T(sI-A^T)^{-1}C^T + D^T\|_\infty^2 \leq \gamma \tag{15}$$

and substituting the dual matrix parameters into *i.* of (2) implies *ii.* of (2).
iii. Defining the congruence transform matrix

$$L_1 = \text{diag} \left[P^{-1} \; I_r \; I_m \right] \tag{16}$$

and pre-multiplying left-hand side and right-hand side of *i.* of (2) by (16) subsequently gives *ii.* of (16).

iii. Analogously, substituting the matrix parameters of the dual system description form into *iii.* of (2) implies *iv.* of (2). □

Note, to design the gain matrix of memory-free control law using LMI principle only the condition *ii.* and *iii.* of (2) are suitable.

Preposition 2 is quite attractive giving a representative result of its type to conclude the asymptotic stability of a system which H_∞ norm is less than a real value $\gamma > 0$, and can be employed in the next for comparative purposes. However, its proof is technical, which more or less, can brings about inconvenience in understanding and applying the results. Thus, in this chapter, some modifications are proposed to directly reach applicable solutions.

2.4 Improved BRL representation

As soon as the representations (2) of the BRL is given, the proof of improvement BRL representation is rather easy as given in the following.

Theorem 1. *System (1), (2) is stable with quadratic performance $\|C(sI-A)^{-1}B+D\|_\infty^2 \leq \gamma$ if there exist a symmetric positive definite matrix $P > 0$, $P \in R^{n \times n}$, matrices S_1, $S_2 \in R^{n \times n}$, and a scalar $\gamma > 0$, $\gamma \in R$ such that*

$$i. \quad \begin{bmatrix} -S_1A - A^TS_1^T & -S_1B & P+S_1 - A^TS_2^T & C^T \\ * & -\gamma I_r & -B^TS_2^T & D^T \\ * & * & S_2+S_2^T & 0 \\ * & * & * & -I_m \end{bmatrix} < 0$$

$$ii. \quad \begin{bmatrix} -S_1A^T - AS_1^T & -S_1C^T & P+S_1 - AS_2^T & B \\ * & -\gamma I_m & -CS_2^T & D \\ * & * & S_2+S_2^T & 0 \\ * & * & * & -I_r \end{bmatrix} < 0 \tag{17}$$

Proof. i. Since (1) implies

$$\dot{q}(t) - Aq(t) - Bu(t) = 0 \tag{18}$$

then with arbitrary square matrices $S_1, S_2 \in R^{n \times n}$ it yields

$$(q^T(t)S_1 + \dot{q}^T(t)S_2)(\dot{q}(t) - Aq(t) - Bu(t)) = 0 \tag{19}$$

Thus, adding (19), as well as its transposition to (8) and substituting (2) it yields

$$\dot{v}(q(t)) =$$
$$= \dot{q}^T(t)Pq(t) + q^T(t)P\dot{q}(t) - \gamma u^T(t)u(t) + (Cq(t)+Du(t))^T(Cq(t)+Du(t)) +$$
$$+ (q^T(t)S_1 + \dot{q}^T(t)S_2)(\dot{q}(t) - Aq(t) - Bu(t)) +$$
$$+ (\dot{q}^T(t) - q^T(t)A^T - u^T(t)B^T)(S_1^Tq(t) + S_2^T\dot{q}(t)) < 0 \tag{20}$$

and using the notation

$$q_c^T(t) = \begin{bmatrix} q^T(t) \ u^T(t) \ \dot{q}^T(t) \end{bmatrix} \tag{21}$$

it can be obtained

$$\dot{v}(q(t)) = q_c^T(t) P_c^\circ q_c(t) < 0 \tag{22}$$

where

$$P_c^\circ = \begin{bmatrix} C^TC & C^TD & 0 \\ * & D^TD & 0 \\ * & * & 0 \end{bmatrix} + \begin{bmatrix} -S_1A - A^TS_1^T & -S_1B & P + S_1 - A^TS_2^T \\ * & -\gamma I_m & -B^TS_2^T \\ * & * & S_2 + S_2^T \end{bmatrix} < 0 \tag{23}$$

Thus, analogously to (13), (14) it then follows the inequality (23) can be written compactly as *i.* of (17).

ii. Using duality principle, substituting the dual matrix parameters into *i.* of (17) implies *ii.* of (17). □

2.5 Basic modifications
Obviously, the aforementioned proof for Theorem 1 is rather simple, and connection between Theorem 1 and the existing results of Preposition 2 can be established. To convert it into basic modifications the following theorem yields alternative ways to describe the H$_\infty$-norm.

Theorem 2. *System (1), (2) is stable with quadratic performance $\|C(sI - A)^{-1}B + D\|_\infty^2 \le \gamma$ if there exist a symmetric positive definite matrix $P > 0$, $P \in R^{n \times n}$, a matrix $S_2 \in R^{n \times n}$, and a scalar $\gamma > 0$, $\gamma \in R$ such that*

$$i. \quad \begin{bmatrix} P^{-1}A^T + AP^{-1} & B & P^{-1}A^T & P^{-1}C^T \\ * & -\gamma I_r & B^T & D^T \\ * & * & -S_2^{-1} - S_2^{-T} & 0 \\ * & * & * & -I_m \end{bmatrix} < 0$$

$$ii. \quad \begin{bmatrix} PA^T + AP & PC^T & A & B \\ * & -\gamma I_m & C & D \\ * & * & -S_2^{-1} - S_2^{-T} & 0 \\ * & * & * & -I_r \end{bmatrix} < 0 \tag{24}$$

Proof. i. Since S_1, S_2 are arbitrary square matrices selection of S_1 can now be made in the form $S_1 = -P$, and it can be supposed that $\det(S_2) \ne 0$. Thus, defining the congruence transform matrix

$$L_2 = \mathrm{diag} \begin{bmatrix} P^{-1} & I_r & -S_2^{-1} & I_m \end{bmatrix} \tag{25}$$

and pre-multiplying right-hand side of *i.* of (17) by L_2, and left-hand side of *i.* of (17) by L_2^T leads to *i.* of (24).

ii. Analogously, selecting $S_1 = -P$, and considering $\det(S_2) \ne 0$ the next congruence transform matrix can be introduced

$$L_3 = \mathrm{diag} \begin{bmatrix} I_n & I_m & -S_2^{-1} & I_n \end{bmatrix} \tag{26}$$

and pre-multiplying right-hand side of *ii.* of (17) by L_3, and left-hand side of *ii.* of (17) by L_3^T leads to *ii.* of (24). □

2.6 Associate modifications

Since alternate conditions of a similar type are also available, similar to the proof of Theorem 2 the following conclusions can be given.

Corollary 1. *Similarly, setting* $S_2 = -\delta P$, *where* $\delta > 0$, $\delta \in \mathbf{R}$ *the inequality ii. given in (24) reduces to*

$$\begin{bmatrix} PA^T + AP & PC^T & A & B \\ * & -\gamma I_m & C & D \\ * & * & -2\delta^{-1}P^{-1} & 0 \\ * & * & * & -I_r \end{bmatrix} < 0 \tag{27}$$

$$\begin{bmatrix} PA^T + AP & PC^T & AP & B \\ * & -\gamma I_m & CP & D \\ * & * & -2\delta^{-1}P & 0 \\ * & * & * & -I_r \end{bmatrix} < 0 \tag{28}$$

respectively, and using Schur complement property then (28) can now be rewritten as

$$\Lambda_1 + 0.5\,\delta\Lambda_2 < 0 \tag{29}$$

where

$$\Lambda_1 = \begin{bmatrix} AP + PA^T & PC^T & B \\ * & -\gamma I_m & D \\ * & * & -I_r \end{bmatrix} < 0 \tag{30}$$

$$\Lambda_2 = \begin{bmatrix} AP \\ CP \\ 0 \end{bmatrix} P^{-1} \begin{bmatrix} PA^T & PC^T & 0 \end{bmatrix} = \begin{bmatrix} APA^T & APC^T & 0 \\ CPA^T & CPC^T & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{31}$$

Choosing δ *as a sufficiently small scalar, where*

$$0 < \delta < 2\lambda_1/\lambda_2 \tag{32}$$

$$\lambda_1 = \lambda_{max}(-\Lambda_1), \qquad \lambda_2 = \lambda_{min}(\Lambda_2) \tag{33}$$

(28) be negative definite for a feasible P *of ii. of (2).*   ∎

Remark 1. *Associated with the second statement of the Theorem 2, setting* $S_2 = -\delta I_n$, *then ii. of (24) implies*

$$\begin{bmatrix} AP + PA^T & PC^T & A & B \\ * & -\gamma I_m & C & D \\ * & * & -2\delta^{-1}I_n & 0 \\ * & * & * & -I_r \end{bmatrix} < 0 \tag{34}$$

and (34) can be written as (29), with (30) and with

$$\Lambda_2 = \begin{bmatrix} AA^T & AC^T & 0 \\ CA^T & CC^T & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{35}$$

Thus, satisfying (32), (33) then (34) be negative definite for a feasible P *of iii. of (2).*   ∎

Note, the form (34) is suitable to optimize a solution with respect to both LMI variables γ, δ in an LMI structure. Conversely, the form (28) behaves LMI structure only if δ is a prescribed constant design parameter, and only γ can by optimized as an LMI variable if possible, or to formulate design task as BMI problem.

Corollary 2. *By the same way, setting $S_2 = -\delta P$, where $\delta > 0$, $\delta \in R$ the inequality i. given in (24) be reduced to*

$$\begin{bmatrix} P^{-1}A^T + AP^{-1} & B & P^{-1}A^T & P^{-1}C^T \\ * & -\gamma I_r & B^T & D^T \\ * & * & -2\delta^{-1}P^{-1} & 0 \\ * & * & * & -I_m \end{bmatrix} < 0 \tag{36}$$

Then (36) can be written as (29), with

$$\Lambda_1 = \begin{bmatrix} P^{-1}A^T + AP^{-1} & B & P^{-1}C^T \\ * & -\gamma I_r & D^T \\ * & * & -I_m \end{bmatrix} \tag{37}$$

$$\Lambda_2 = \begin{bmatrix} P^{-1}A^T P A P^{-1} & P^{-1}A^T P B & 0 \\ B^T P A P^{-1} & B^T P B & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{38}$$

Thus, satisfying (32), (33) then (36) be negative definite for a feasible P of iii. of (2). ∎

Remark 2. *By a similar procedure, setting $S_2 = -\delta I_n$, where $\delta > 0$, $\delta \in R$ then i. of (24) implies the following*

$$\begin{bmatrix} P^{-1}A^T + AP^{-1} & B & P^{-1}A^T & P^{-1}C^T \\ * & -\gamma I_r & B^T & D^T \\ * & * & -2\delta^{-1}I_n & 0 \\ * & * & * & -I_m \end{bmatrix} < 0 \tag{39}$$

It is evident that (39) yields with the same Λ_1 as given in (37) and

$$\Lambda_2 = \begin{bmatrix} P^{-1}A^T A P^{-1} & P^{-1}A^T B & 0 \\ B^T A P^{-1} & B^T B & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{40}$$

Thus, this leads to the equivalent results as presented above, but with possible different interpretation.

∎

3. Control law parameter design

3.1 Problem description

Through this section the task is concerned with the computation of a state feedback $u(t)$, which control the linear dynamic system given by (1), (2), i.e.

$$\dot{q}(t) = Aq(t) + Bu(t) \tag{41}$$

$$y(t) = Cq(t) + Du(t) \tag{42}$$

Problem of the interest is to design stable closed-loop system with quadratic performance $\gamma > 0$ using the linear memoryless state feedback controller of the form

$$u(t) = -Kq(t) \tag{43}$$

where matrix $K \in R^{r \times n}$ is a gain matrix.

Then the unforced system, formed by the state controller (43), can be written as

$$\dot{q}(t) = (A - BK)q(t) \tag{44}$$

$$y(t) = (C - DK)q(t) \tag{45}$$

The state-feedback control problem is to find, for an optimized (or prescribed) scalar $\gamma > 0$, the state-feedback gain K such that the control law guarantees an upper bound of $\sqrt{\gamma}$ to H_∞ norm of the closed-loop transfer function. Thus, Theorem 2 can be reformulated to solve this state-feedback control problem for linear continuous time systems.

Theorem 3. *Closed-loop system (44), (45) is stable with performance $\|C_c(sI - A_c)^{-1}B\|_\infty^2 \leq \gamma$, $A_c = A - BK$, $C_c = C - DK$ if there exist regular square matrices $T, U, V \in R^{n \times n}$, a matrix $W \in R^{r \times n}$, and a scalar $\gamma > 0$, $\gamma \in R$ such that*

$$T = T^T > 0, \quad \gamma > 0 \tag{46}$$

$$\begin{bmatrix} VA^T - W^TB^T + AV^T - BW & -B & T - U^T + VA^T - W^TB^T & -VC^T + W^TD^T \\ * & -\gamma I_r & -B^T & D^T \\ * & * & -U - U^T & 0 \\ * & * & * & -I_m \end{bmatrix} < 0 \tag{47}$$

The control law gain matrix is now given as

$$K = WV^{-T} \tag{48}$$

Proof. Considering that $\det S_1 \neq 0$, $\det S_2 \neq 0$ the congruence transform L_4 can be defined as follows

$$L_4 = \text{diag}\left[S_1^{-1} \ I_r \ S_2^{-1} \ I_m \right] \tag{49}$$

and multiplying left-hand side of $i.$ of (17) by L_4, and right-hand side of (17) by L_4^T gives

$$\begin{bmatrix} -AS_1^{-T} - S_1^{-1}A^T & -B & S_1^{-1}PS_2^{-T} + S_2^{-T} - S_1^{-1}A^T & S_1^{-1}C^T \\ * & -\gamma I_r & -B^T & D^T \\ * & * & S_2^{-1} + S_2^{-T} & 0 \\ * & * & * & -I_m \end{bmatrix} < 0 \tag{50}$$

Inserting $A \leftarrow A_c$, $C \leftarrow C_c$ into (50) and denoting

$$S_1^{-1}PS_2^{-T} = T, \qquad S_1^{-1} = -V, \qquad S_2^{-1} = -U \tag{51}$$

(50) takes the form

$$\begin{bmatrix} (A - BK)V^T + V(A - BK)^T & -B & T - U^T + V(A - BK)^T & -V(C - DK)^T \\ * & -\gamma I_r & -B^T & D^T \\ * & * & -U - U^T & 0 \\ * & * & * & -I_m \end{bmatrix} < 0 \tag{52}$$

and with

$$W = KV^T \tag{53}$$
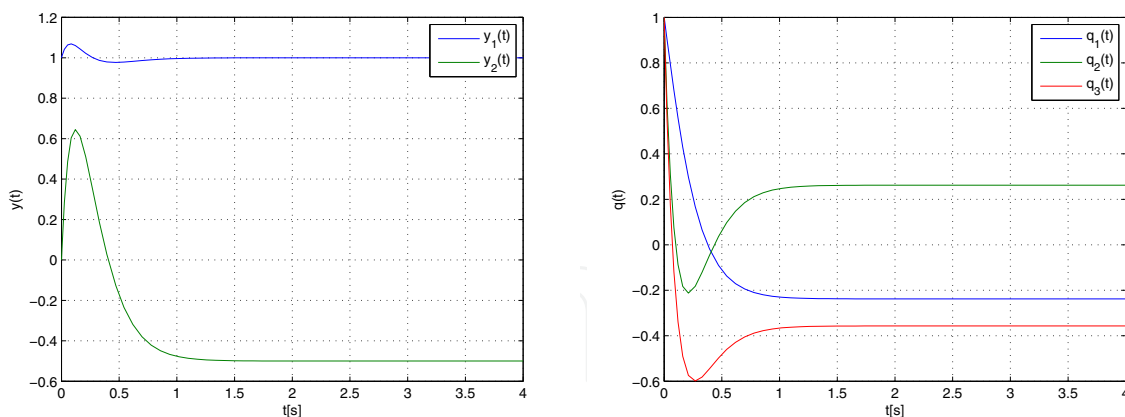
(50) implies (47).   □

Fig. 1. System output and state response

3.2 Basic modification

Corollary 3. *Following the same lines of that for Theorem 2 it is immediate by inserting $A \leftarrow A_c$, $C \leftarrow C_c$ into i. of (24) and denoting*

$$P^{-1} = X, \qquad S_2 = Z \tag{54}$$

that

$$\begin{bmatrix} AX+XA^T-BKX-XK^TB^T & B & -XA^T+XK^TB^T & XC^T-XK^TD^T \\ * & -\gamma I_r & -B^T & D^T \\ * & * & -Z-Z^T & 0 \\ * & * & * & -I_m \end{bmatrix} < 0 \tag{55}$$

Thus, using Schur complement equivalency, and with

$$Y = KX \tag{56}$$

(58) implies

$$X = X^T > 0, \qquad \gamma > 0 \tag{57}$$

$$\begin{bmatrix} AX+XA^T-BY-Y^TB^T & B & XA^T-Y^TB^T & XC^T-Y^TD^T \\ * & -\gamma I_r & B^T & D^T \\ * & * & -Z-Z^T & 0 \\ * & * & * & -I_m \end{bmatrix} < 0 \tag{58}$$

∎

Illustrative example

The approach given above is illustrated by an example where the parameters of the (41), (42) are

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -9 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 5 \end{bmatrix}, \quad C^T = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 0 \end{bmatrix}, \quad D = 0$$

Solving (57), (58) with respect to the next LMI variables X, Y, Z, and δ using SeDuMi (Self-Dual-Minimization) package for Matlab (Peaucelle et al. (1994)) given task was feasible with

$$X = \begin{bmatrix} 0.6276 & -0.3796 & -0.0923 \\ -0.3796 & 0.7372 & 0.3257 \\ -0.0923 & 0.3257 & 0.9507 \end{bmatrix}, \quad Z = \begin{bmatrix} 5.0040 & 0.1209 & 0.4891 \\ 0.1209 & 4.9512 & 0.4888 \\ 0.4891 & 0.4888 & 5.2859 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0.4917 & 3.2177 & 0.7775 \\ 0.6100 & -1.5418 & -0.3739 \end{bmatrix}, \qquad \gamma = 8.4359$$

and results the control system parameters

$$K = \begin{bmatrix} 5.1969 & 7.6083 & -1.2838 \\ -0.5004 & -2.5381 & 0.4276 \end{bmatrix}, \quad \rho(A_c) = \{-5.5999, \, -8.3141 \pm 1.6528\,i\}$$

The example is shown of the closed-loop system response in the forced mode, where in the Fig. 1 the output response, as well as state variable response are presented, respectively. The desired steady-state output variable values were set as $[y_1 \, y_2] = [1 \, -0.5]$.

3.3 Associate modifications

Remark 3. *Inserting $A \leftarrow A_c$, $C \leftarrow C_c$ into (39) and setting $X = P^{-1}$, $Y = KX$, $\delta^{-1} = \xi$, as well as inserting the same into (34) and setting $X = P$, $Y = KX$, $\delta^{-1} = \xi$ gives*

$$X = X^T > 0, \quad \gamma > 0, \quad \xi > 0 \tag{59}$$

$$i. \quad \begin{bmatrix} AX+XA^T-BY-Y^TB^T & B & XA^T-Y^TB^T & XC^T-Y^TD^T \\ * & -\gamma I_r & B^T & D^T \\ * & * & -2\xi I_n & 0 \\ * & * & * & -I_m \end{bmatrix} < 0 \tag{60}$$

$$ii. \quad \begin{bmatrix} AX+XA^T-BY-Y^TB^T & XC^T-Y^TD^T & AX-BY & B \\ * & -\gamma I_m & CX-DY & D \\ * & * & -2\xi I_n & 0 \\ * & * & * & -I_r \end{bmatrix} < 0$$

where feasible X, Y, γ, ξ implies the gain matrix (48). ∎

Illustrative example

Considering the same parameters of (41), (42) and desired output values as is given above then solving (59), (59) with respect to LMI variables X, Y, and γ given task was feasible with

$i.$ $\gamma = 8.3659$ $ii.$ $\gamma = 35.7411$

$\xi = 5.7959$ $\xi = 30.0832$

$$X = \begin{bmatrix} 0.6402 & -0.3918 & -0.1075 \\ -0.3918 & 0.7796 & 0.3443 \\ -0.1075 & 0.3443 & 0.9853 \end{bmatrix} \qquad X = \begin{bmatrix} 8.7747 & -4.7218 & -1.2776 \\ -4.7218 & 5.8293 & 0.4784 \\ -1.2776 & 0.4784 & 8.4785 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0.5451 & 3.3471 & 0.6650 \\ 0.6113 & -1.6481 & -0.3733 \end{bmatrix} \qquad Y = \begin{bmatrix} 2.7793 & 14.7257 & 5.1591 \\ 3.3003 & -6.8347 & -1.8016 \end{bmatrix}$$

$$K = \begin{bmatrix} 5.2296 & 7.5340 & -1.3870 \\ -0.5590 & -2.6022 & 0.4694 \end{bmatrix} \qquad K = \begin{bmatrix} 3.1145 & 4.9836 & 0.7966 \\ -0.4874 & -1.5510 & -0.1984 \end{bmatrix}$$

$\rho(A_c) = \{-6.3921, \, -7.7931 \pm 1.8646\,i\}$ $\rho(A_c) = \{-2.3005, \, -3.8535, \, -8.7190\}$

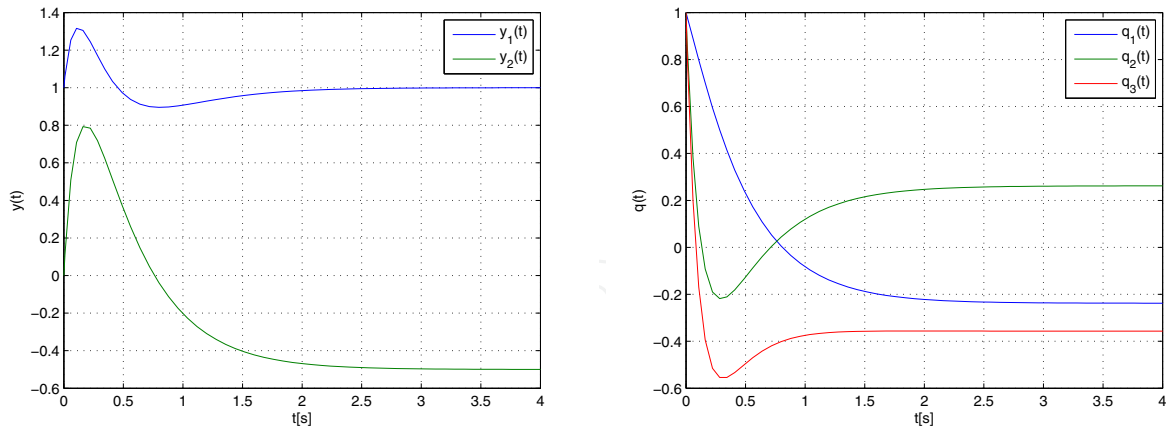The closed-loop system response concerning *ii.* of (60) is in the Fig. 2.
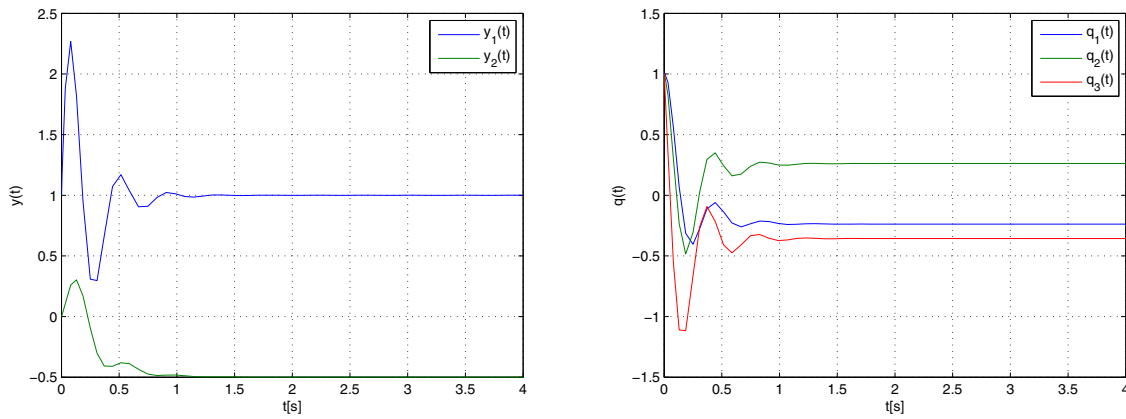
Fig. 2. System output and state response

Fig. 3. System output and state response

Remark 4. *The closed-loop system (44), (45) is stable with quadratic performance* $\gamma > 0$ *and the inequalities (15) are true if and only if there exists a symmetric positive definite matrix* $X > 0$, $X \in \mathbb{R}^{n \times n}$, *a matrix* $Y \in \mathbb{R}^{r \times n}$, *and a scalar* $\gamma > 0$, $\gamma \in \mathbb{R}$ *such that*

$$X = X^T > 0, \quad \gamma > 0, \quad \xi > 0 \tag{61}$$

$$i. \begin{bmatrix} AX + XA^T - BY - Y^TB^T & B & XC^T - Y^TD^T \\ * & -\gamma I_r & 0 \\ * & * & -I_m \end{bmatrix} < 0 \tag{62}$$

$$ii. \begin{bmatrix} AX + XA^T - BY - Y^TB^T & XC^T - Y^TD^T & B \\ * & -\gamma I_m & D \\ * & * & -I_r \end{bmatrix} < 0$$

∎

Illustrative example

Using the same example consideration as are given above then solving (61), (62) with respect to LMI variables X, Y, and γ given task was feasible with

$i.\ \gamma = 6.8386$ $\qquad\qquad\qquad ii.\ \gamma = 17.6519$

$$X = \begin{bmatrix} 1.1852 & 0.1796 & 0.6494 \\ 0.1796 & 1.4325 & 1.1584 \\ 0.6494 & 1.1584 & 2.1418 \end{bmatrix} \qquad X = \begin{bmatrix} 6.0755 & -0.9364 & 1.0524 \\ -0.9364 & 5.1495 & 2.4320 \\ 1.0524 & 2.4320 & 7.2710 \end{bmatrix}$$

$$Y = \begin{bmatrix} 2.0355 & 3.7878 & -3.2286 \\ 0.6142 & -2.1847 & -3.0636 \end{bmatrix} \qquad Y = \begin{bmatrix} 6.3651 & 9.9547 & -8.7603 \\ 2.2941 & -5.3741 & -6.2975 \end{bmatrix}$$

$$K = \begin{bmatrix} 4.4043 & 7.8029 & -7.0627 \\ 1.5030 & -0.3349 & -1.7049 \end{bmatrix} \qquad K = \begin{bmatrix} 2.0688 & 3.5863 & -2.7038 \\ 0.4033 & -0.6338 & -0.7125 \end{bmatrix}$$

$\rho(A_c) = \{-4.3952,\ -4.6009 \pm 14.8095\,\mathrm{i}\} \quad \rho(A_c) = \{-2.2682,\ -3.1415 \pm 9.634\,\mathrm{i}\}$

The simulation results are shown in Fig. 3, and are concerning with $i.$ of (62).

It is evident that different design conditions implying from the equivalent, but different, bounded lemma structures results in different numerical solutions.

3.4 Dependent modifications

Similar extended LMI characterizations can be derived by formulating LMI in terms of product ξP, where ξ is a prescribed scalar to overcome BMI formulation (Veselý & Rosinová (2009)).

Theorem 4. *Closed-loop system (1), (2) is stable with quadratic performance $\|C_c(sI - A_c)^{-1}B\|_\infty^2 \leq \gamma$, $A_c = A - BK$, $C_c = C - DK$ if for given $\xi > 0$ there exist a symmetric positive definite matrix $X > 0$, $X \in R^{n \times n}$, a regular square matrix $Z \in R^{n \times n}$, a matrix $Y \in R^{r \times n}$, and a scalar $\gamma > 0$, $\gamma \in R$ such that*

$$X = X^T > 0, \quad \gamma > 0 \tag{63}$$

$$i.\ \begin{bmatrix} AX + XA^T - BY - Y^TB^T & B & XA^T - Y^TB^T & XC^T - Y^TD^T \\ * & -\gamma I_r & B^T & D^T \\ * & * & -2\xi X & 0 \\ * & * & * & -I_m \end{bmatrix} < 0$$

$$ii.\ \begin{bmatrix} AX + XA^T - BY - Y^TB^T & XC^T - Y^TD^T & AX - BY & B \\ * & -\gamma I_m & CX - DY & D \\ * & * & -2\xi X & 0 \\ * & * & * & -I_r \end{bmatrix} < 0 \tag{64}$$

where K is given in (48).

Proof. $i.$ Inserting $A \leftarrow A_c$, $C \leftarrow C_c$ into (36) and setting $X = P^{-1}$, $Y = KX$, and $\xi = \delta^{-1}$ then (36) implies $ii.$ of (64).

$ii.$ Inserting $A \leftarrow A_c$, $C \leftarrow C_c$ into (28) and setting $X = P$, $Y = KX$, and $\xi = \delta^{-1}$ then (28) implies $i.$ of (64). $\qquad\square$
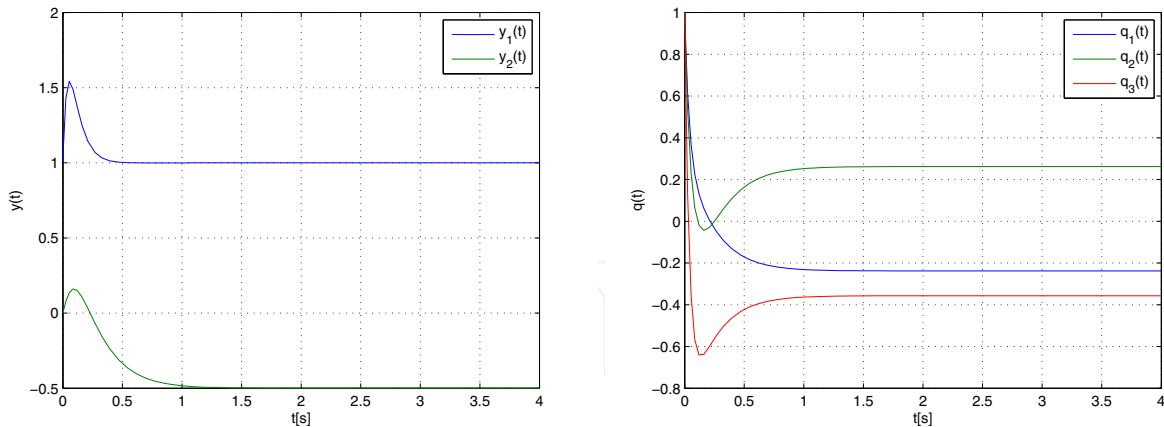
Fig. 4. System output and state response

Note, other nontrivial solutions can be obtained using different setting of S_l, $l = 1, 2$.

Illustrative example
Considering the same system parameters of (1), (2), and the same desired output values as are given above then solving (63), (64) with respect to LMI variables X, Y, and γ with prescribed $\tilde{\xi} = 10/xi = 30$, respectively, given task was feasible with

$i.$ $\gamma = 8.3731$

$\xi = 10$

$$X = \begin{bmatrix} 0.5203 & -0.2338 & 0.0038 \\ -0.2338 & 0.7293 & 0.2359 \\ 0.0038 & 0.2359 & 0.7728 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0.8689 & 3.2428 & 0.6068 \\ 0.3503 & -1.6271 & -0.1495 \end{bmatrix}$$

$$K = \begin{bmatrix} 4.4898 & 6.2565 & -1.1462 \\ -0.4912 & -2.5815 & 0.5968 \end{bmatrix}$$

$\rho(A_c) = \{-8.3448, -5.7203 \pm 3.6354\,\mathrm{i}\}$

$ii.$ $\gamma = 17.6519$

$\xi = 30$

$$X = \begin{bmatrix} 0.8926 & -0.2332 & 0.0489 \\ -0.2332 & 1.2228 & 0.3403 \\ 0.0489 & 0.3403 & 1.3969 \end{bmatrix}$$

$$Y = \begin{bmatrix} 3.0546 & 8.8611 & 0.2482 \\ 2.0238 & -2.8097 & 3.0331 \end{bmatrix}$$

$$K = \begin{bmatrix} 5.8920 & 8.9877 & -2.2185 \\ 1.3774 & -2.8170 & 2.8094 \end{bmatrix}$$

$\rho(A_c) = \{-4.6346, -12.3015, -25.0751\}$

The same simulation study as above was carried out, and the simulation results concerning $ii.$ of (64) for the states and output variables of the system are shown in Fig. 4.

It also should be noted, the cost value γ will not be a monotonously decreasing function with the decreasing of ξ, if $\delta = \xi^{-1}$ is chosen.

4. Uncertain continuous-time systems

The importance of Theorem 3 is that it separates T from A, B, C, and D, i.e. there are no terms containing the product of T and any of them. This enables to derive other forms of bonded real lemma for a system with polytopic uncertainties by using a parameter-dependent Lyapunov function.

4.1 Problem description

Assuming that the matrices A, B, C, and D of (1), (2) are not precisely known but belong to a polytopic uncertainty domain $\mathcal{O}$,

$$\mathcal{O} := \left\{ (A, B, C, D)(a) : (A, B, C, D)(a) = \sum_{i=1}^{s} a_i (A_i, B_i, C_i, D_i), \quad a \in \mathcal{Q} \right\} \tag{65}$$

$$\mathcal{Q} = \left\{ (a_1, a_2, \cdots, a_s) : \sum_{i=1}^{s} a_i = 1; \quad a_i > 0, \ i = 1, 2, \ldots, s \right\} \tag{66}$$

where $\mathcal{Q}$ is the unit simplex, A_i, B_i, C_i, and D_i are constant matrices with appropriate dimensions, and a_i, $i = 1, 2, \ldots, s$ are time-invariant uncertainties.

Since a is constrained to the unit simplex as (66) the matrices $(A, B, C, D)(a)$ are affine functions of the uncertain parameter vector $a \in R^s$ described by the convex combination of the vertex matrices (A_i, B_i, C_i, D_i), $i = 1, 2, \ldots, s$.

The state-feedback control problem is to find, for a $\gamma > 0$, the state-feedback gain matrix K such that the control law of

$$u(t) = -Kq(t) \tag{67}$$

guarantees an upper bound of $\sqrt{\gamma}$ to H_∞ norm.

By virtue of the property of convex combinations, (48) can be readily used to derive the robust performance criterion.

Theorem 5. *Given system (65), (66) the closed-loop H_∞ norm is less than a real value $\sqrt{\gamma} > 0$, if there exist positive matrices $T_i \in R^{n \times n}$, $i = 1, 2, \ldots, s$, real square matrices $U, V \in R^{n \times n}$, and a real matrix $W \in R^{r \times n}$ such that*

$$\gamma > 0 \tag{68}$$

$$\begin{bmatrix} VA_i^T - W^T B_i^T + A_i V^T - B_i W & -B_i & T_i - U^T + VA_i^T - W^T B_i^T & -VC_i^T + W^T D_i^T \\ * & -\gamma I_r & -B_i^T & D_i^T \\ * & * & -U - U^T & 0 \\ * & * & * & -I_m \end{bmatrix} < 0 \tag{69}$$

If the existence is affirmative, the state-feedback gain K is given by

$$K = WV^{-T} \tag{70}$$

Proof. It is obvious that (47), (48) implies directly (69), (70). □

Remark 5. *Thereby, robust control performance of uncertain continuous-time systems is guaranteed by a parameter-dependent Lyapunov matrix, which is constructed as*

$$T(a) = \sum_{i=1}^{s} a_i T_i \tag{71}$$

4.2 Dependent modifications

Theorem 6. *Given system (65), (66) the closed-loop* H_∞ *norm is less than a real value* $\sqrt{\gamma} > 0$, *if there exist positive symmetric matrices* $T_i \in R^{n \times n}$, $i = 1, 2, \ldots, n$, *a real square matrices* $V \in R^{n \times n}$, *a real matrix* $W \in R^{r \times n}$, *and a positive scalar* $\delta > 0$, $\delta \in R$ *such that*

$$T_i > 0, \; i = 1, 2, \ldots, n, \quad \gamma > 0 \tag{72}$$

i.
$$\begin{bmatrix} VA_i^T + A_iV^T - W^TB_i^T - B_iW & -B_i & T_i - \delta V^T + VA_i^T - W^TB_i^T & -VC_i^T + W^TD_i^T \\ * & -\gamma I_r & -B_i^T & D_i^T \\ * & * & -\delta(V + V^T) & 0 \\ * & * & * & -I_m \end{bmatrix} < 0 \tag{73}$$

ii.
$$\begin{bmatrix} VA_i^T + A_iV^T - W^TB_i^T - B_iW & VC_i^T - W^TD_i^T & T_i - V^T + \delta A_iV - \delta B_iW & B_i \\ * & -\gamma I_m & \delta C_iV - \delta D_iW & D_i \\ * & * & -\delta(V + V^T) & 0 \\ * & * & * & -I_r \end{bmatrix} < 0$$

If the existence is affirmative, the state-feedback gain K *is given by*

$$K = WV^{-T} \tag{74}$$

Proof. i. Setting $U = \delta V$ then (69) implies *i.* of (73).
ii. Setting $S_1 = -V$, and $S_2 = -\delta V$ then *ii.* of (17) implies *ii.* of (73).   □

Illustrative example

The approach given above is illustrated by the numerical example yielding the matrix parameters of the system $D(t) = D = 0$

$$A(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -6r(t) & -5r(t) \end{bmatrix}, \quad B(t) = B = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 5 \end{bmatrix}, \quad C^T(t) = C^T = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 0 \end{bmatrix}$$

where the time varying uncertain parameter $r(t)$ lies within the interval $\langle 0.5, 1.5 \rangle$.
In order to represent uncertainty on $r(t)$ it is assumed that the matrix parameters belongs to the polytopic uncertainty domain $\mathcal{O}$,

$$\mathcal{O} := \left\{ (A, B, C, D)(a) : (A, B, C, D)(a) = \sum_{i=1}^{2} a_i (A_i, B_i, C_i, D_i), \quad a \in \mathcal{Q} \right\}$$

$$\mathcal{Q} = \{(a_1, a_2) : a_2 = 1 - a_1; \quad 0 < a_1 < 1\}$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -3 & -2.5 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -9 & -7.5 \end{bmatrix}$$

$$B_1 = B_2 = B, \quad C_1^T = C_2^T = C^T, \quad D_1 = D_2 = 0$$

$$A = a_1A_1 + (1 - a_1)A_2, \quad A_c = A - BK \quad A_{c0} = A_0 - BK$$

Thus, solving (72) and *i.* of (73) with respect to the LMI variables T_1, T_2, V, W, and δ given task was feasible for $a_1 = 0.2$, $\delta = 20$. Subsequently, with
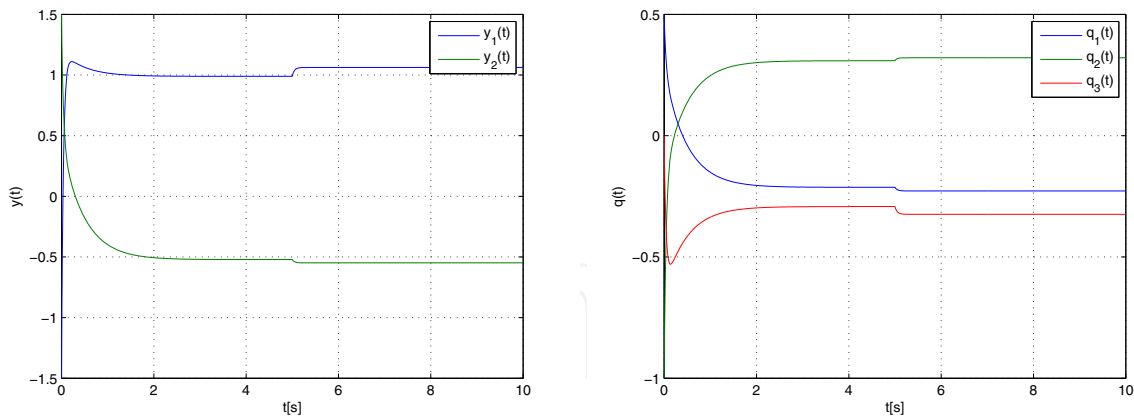
$$\gamma = 10.5304$$

Fig. 5. System output and state response

$$T_1 = \begin{bmatrix} 7.0235 & 2.4579 & 2.6301 \\ 2.4579 & 7.4564 & -0.4037 \\ 2.6301 & -0.4037 & 5.3152 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 6.6651 & 2.6832 & 2.0759 \\ 2.6832 & 7.4909 & -0.2568 \\ 2.0759 & -0.2568 & 6.2386 \end{bmatrix}$$

$$V = \begin{bmatrix} 0.2250 & -0.0758 & -0.0350 \\ 0.0940 & 0.1801 & -0.0241 \\ 0.1473 & 0.0375 & 0.1992 \end{bmatrix}, \quad W = \begin{bmatrix} 0.7191 & 3.0209 & 0.2881 \\ 0.1964 & -0.7401 & 0.7382 \end{bmatrix}$$

the control law parameters were computed as

$$K = \begin{bmatrix} 6.5392 & 12.5891 & -5.7581 \\ 0.2809 & -3.6944 & 4.1922 \end{bmatrix}, \quad \|K\| = 16.3004$$

and including into the state control law the were obtained the closed-loop system matrix eigenvalues set

$$\rho(A_{c0}) = \{-2.0598, \ -22.2541, \ -24.7547\}$$

Solving (72) and ii. of (73) with respect to the LMI variables T_1, T_2, V, W, and δ given task was feasible for $a_1 = 0.2$, $\delta = 20$, too, and subsequently, with

$$\gamma = 10.5304$$

$$T_1 = \begin{bmatrix} 239.1234 & 108.9248 & 250.1206 \\ 108.9248 & 307.9712 & 13.8497 \\ 250.1206 & 13.8497 & 397.1333 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 222.8598 & 121.9115 & 251.6458 \\ 121.9115 & 341.0193 & 63.4202 \\ 251.6458 & 63.4202 & 445.9279 \end{bmatrix}$$

$$V = \begin{bmatrix} 6.5513 & -2.0718 & -0.2451 \\ 2.1635 & 2.2173 & 0.1103 \\ 0.2448 & 0.2964 & 0.4568 \end{bmatrix}, \quad W = \begin{bmatrix} 4.6300 & 6.6167 & -2.6780 \\ 1.7874 & -0.7898 & 4.3214 \end{bmatrix}$$

the closed-loop parameters were computed as

$$K = \begin{bmatrix} 1.1296 & 2.2771 & -7.9446 \\ 0.2888 & -1.1375 & 10.0427 \end{bmatrix}, \quad \|K\| = 13.1076$$

$$\rho(A_c) = \{-50.4633, \ -1.1090 \pm 2.1623\,\mathrm{i}\}$$

It is evident, that the eigenvalues spectrum $\rho(A_{c0})$ of the closed control loop is stable in both cases. However, taking the same values of γ, the solutions differ especially in the
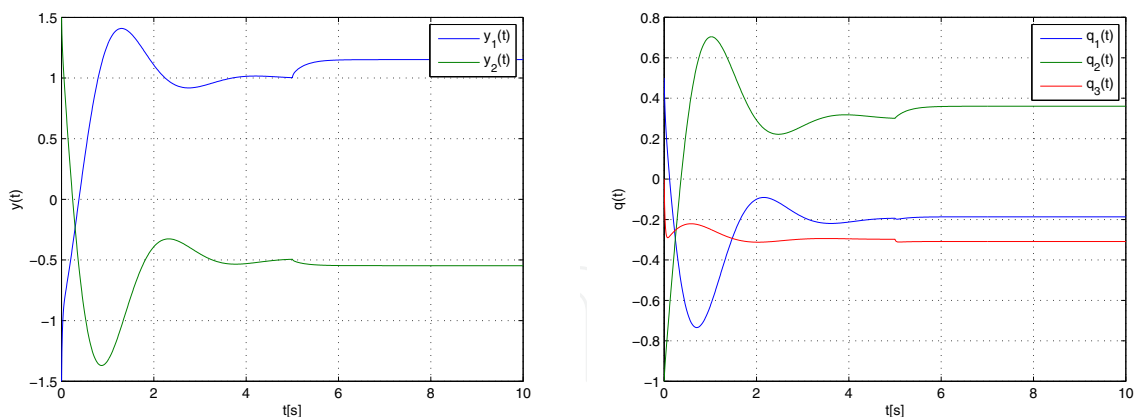
Fig. 6. System output and state response

closed-loop dominant eigenvalues, as well as in the control law gain matrix norm, giving together closed-loop system matrix eigenstructure. To prefer any of them is not as so easy as it seems at the first sight, and the less gain norm may not be the best choice.

Fig. 5 illustrates the simulation results with respect to a solution of i. of (73) and (72). The initial state of system state variable was setting as $[q_1 \, q_2 \, q_3]^T = [0.5 \, 1 \, 0]^T$, the desired steady-state output variable values were set as $[y_1 \, y_2]^T = [1 \, {-}0.5]^T$, and the system matrix parameter change from $p = 1$ to $p = 0.54$ was realized 5 seconds after the state control start-up.

The same simulation study was carried out using the control parameter obtained by solving ii. of (73), (72), and the simulation results are shown in Fig. 6. It can be seen that the presented control scheme partly eliminates the effects of parameter uncertainties, and guaranteed the quadratic stability of the closed-loop system.

5. Pairwise-autonomous principle in control design

5.1 Problem description
Considering the system model of the form (1), (2), i.e.

$$\dot{q}(t) = Aq(t) + Bu(t) \tag{75}$$

$$y(t) = Cq(t) + Du(t) \tag{76}$$

but reordering in such way that

$$A = \begin{bmatrix} A_{i,l} \end{bmatrix}, \; C = \begin{bmatrix} C_{i,l} \end{bmatrix}, \; B = \text{diag} \begin{bmatrix} B_i \end{bmatrix}, \; D = 0 \tag{77}$$

where $i, l = 1, 2, \ldots, p$, and all parameters and variables are with the same dimensions as it is given in Subsection 2.1. Thus, respecting the above give matrix structures it yields

$$\dot{q}_h(t) = A_{hh}q_h(t) + \sum_{l=1,\, l \neq h}^{p} \left(A_{hl}q_l(t) + B_h u_h(t) \right) \tag{78}$$

$$y_h(t) = C_{hh}q_h(t) + \sum_{l=1,\, l \neq h}^{p} C_{hl}q_l(t) \tag{79}$$

where $q_h(t) \in R^{n_h}$, $u_h(t) \in R^{r_h}$, $y_h(t) \in R^{m_h}$, $A_{hl} \in R^{n_h \times n_l}$, $B_h \in R^{r_h \times n_h}$, and $C_{hl} \in R^{m_h \times n_h}$, respectively, and $n = \sum_{l=1}^{p} n_l$, $r = \sum_{l=1}^{p} r_l$, $m = \sum_{l=1}^{p} m_l$.

Problem of the interest is to design closed-loop system using a linear memoryless state feedback controller of the form

$$u(t) = -Kq(t) \tag{80}$$

in such way that the large-scale system be stable, and

$$K = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1p} \\ K_{21} & K_{22} & \dots & K_{2p} \\ & & \vdots & \\ K_{p1} & K_{p2} & \dots & K_{pp} \end{bmatrix}, \qquad K_{hh} = \sum_{l=1,\, l \neq h}^{p} K_h^l \tag{81}$$

$$u_h(t) = -K_{hh}q_h(t) - \sum_{l=1,\, l \neq h}^{p} K_{hl}q_l(t), \quad h = 1, 2, \dots, p \tag{82}$$

Lemma 1. *Unforced (autonomous) system (75)-(77) is stable if there exists a set of symmetric matrices*

$$P_{hk}^{\circ} = \begin{bmatrix} P_h^k & P_{hk} \\ P_{kh} & P_k^h \end{bmatrix} \tag{83}$$

such that

$$\sum_{h=1}^{p-1} \sum_{k=h+1}^{p} \left(\dot{q}_{hk}^T(t) \begin{bmatrix} P_h^k & P_{hk} \\ P_{kh} & P_k^h \end{bmatrix} q_{hk}(t) + q_{hk}^T(t) \begin{bmatrix} P_h^k & P_{hk} \\ P_{kh} & P_k^h \end{bmatrix} \dot{q}_{hk}(t) \right) < 0 \tag{84}$$

where

$$\dot{q}_{hk}(t) = \begin{bmatrix} A_{hh} & A_{hk} \\ A_{kh} & A_{kk} \end{bmatrix} q_{hk}(t) + \sum_{l=1,\, l \neq h,k}^{p} \begin{bmatrix} A_{hl} \\ A_{kl} \end{bmatrix} q_l(t) \tag{85}$$

$$q_{hk}^T(t) = \begin{bmatrix} q_h^T(t) & q_k^T(t) \end{bmatrix} \tag{86}$$

Proof. Defining Lyapunov function as follows

$$v(q(t)) = q^T(t)Pq(t) > 0 \tag{87}$$

where $P = P^T > 0$, $P \in R^{n \times n}$, then the time rate of change of $v(q(t))$ along a solution of the system (75), (77) is

$$\dot{v}(q(t)) = \dot{q}^T(t)Pq(t) + q^T(t)P\dot{q}(t) < 0 \tag{88}$$

Considering the same form of P with respect to K, i.e.

$$P = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1M} \\ P_{21} & P_{22} & \dots & P_{2M} \\ & & \vdots & \\ P_{M1} & P_{M2} & \dots & P_{MM} \end{bmatrix}, \qquad P_{hh} = \sum_{l=1,\, l \neq h}^{p} P_h^l \tag{89}$$

then the next separation is possible

$$
P = \left(\left(\left(\begin{bmatrix} P_1^2 & P_{12} & 0 & \dots & 0 \\ P_{21} & P_2^1 & 0 & \dots & 0 \\ & & \vdots & & \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} + \dots + \begin{bmatrix} P_1^p & 0 & \dots & 0 & P_{1p} \\ 0 & 0 & \dots & 0 & 0 \\ & & \vdots & & \\ P_{p1} & 0 & \dots & 0 & P_p^1 \end{bmatrix} \right) + \right. \right.
$$
$$
\left. \left. + \dots + \begin{bmatrix} 0 & \dots & 0 & 0 & 0 \\ & & \vdots & & \\ 0 & \dots & 0 & P_{p-1}^p & P_{p-1,p} \\ 0 & \dots & 0 & P_{p,p-1} & P_p^{p-1} \end{bmatrix} \right) \right). \tag{90}
$$

Writing (78) as

$$
\dot{q}_{hk}(t) = \begin{bmatrix} A_{hh} & A_{hk} \\ A_{kh} & A_{kk} \end{bmatrix} q_{hk}(t) + \sum_{l=1,\, l \neq h,k}^{p} \begin{bmatrix} A_{hl} \\ A_{kl} \end{bmatrix} q_l(t) + \begin{bmatrix} B_h & 0 \\ 0 & B_k \end{bmatrix} \begin{bmatrix} u_h(t) \\ u_k(t) \end{bmatrix} \tag{91}
$$

and considering that for unforced system there are $u_l(t) = 0$, $l = 1, \dots p$ then (91) implies (85). Subsequently, with (90), (91) the inequality (88) implies (84). $\square$

5.2 Pairwise system description

Supposing that there exists the partitioned structure of K as is defined in (81), (82) then it yields

$$
u_h(t) = - \sum_{l=1,\, l \neq h}^{p} \begin{bmatrix} K_h^l & K_{hl} \end{bmatrix} \begin{bmatrix} q_h(t) \\ q_l(t) \end{bmatrix} =
$$
$$
= - \begin{bmatrix} K_h^k & K_{hk} \end{bmatrix} \begin{bmatrix} q_h(t) \\ q_k(t) \end{bmatrix} - \sum_{l=1,\, l \neq h,k}^{p} \begin{bmatrix} K_h^l & K_{hl} \end{bmatrix} \begin{bmatrix} q_h(t) \\ q_l(t) \end{bmatrix} = u_h^k(t) + \sum_{l=1,\, l \neq h,k}^{p} u_h^l(t) \tag{92}
$$

where for $l = 1, 2, \dots, p, i \neq h, k$

$$
u_h^l(t) = - \begin{bmatrix} K_h^l & K_{hl} \end{bmatrix} \begin{bmatrix} q_h(t) \\ q_l(t) \end{bmatrix} \tag{93}
$$

Defining with $h = 1, 2 \dots, p-1, k = h+1, h+2 \dots, p$

$$
\begin{bmatrix} u_h^k(t) \\ u_k^h(t) \end{bmatrix} = - \begin{bmatrix} K_h^k & K_{hk} \\ K_{kh} & K_k^h \end{bmatrix} \begin{bmatrix} q_h(t) \\ q_k(t) \end{bmatrix} = -K_{hk}^{\circ} \begin{bmatrix} q_h(t) \\ q_k(t) \end{bmatrix} \tag{94}
$$

$$
K_{hk}^{\circ} = \begin{bmatrix} K_h^k & K_{hk} \\ K_{kh} & K_k^h \end{bmatrix} \tag{95}
$$

and combining (92) for h and k it is obtained

$$
\begin{bmatrix} u_h(t) \\ u_k(t) \end{bmatrix} = - \begin{bmatrix} K_h^k & K_{kh} \\ K_{hk} & K_k^h \end{bmatrix} \begin{bmatrix} q_h(t) \\ q_k(t) \end{bmatrix} - \begin{bmatrix} \sum_{l=1,\, l \neq h,k}^{p} \begin{bmatrix} K_h^l & K_{hl} \end{bmatrix} \begin{bmatrix} q_h(t) \\ q_l(t) \end{bmatrix} \\ \sum_{l=1,\, l \neq h,k}^{p} \begin{bmatrix} K_k^l & K_{kl} \end{bmatrix} \begin{bmatrix} q_k(t) \\ q_l(t) \end{bmatrix} \end{bmatrix} \tag{96}
$$

$$\begin{bmatrix} \boldsymbol{u}_h(t) \\ \boldsymbol{u}_k(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{u}_h^k(t) \\ \boldsymbol{u}_k^h(t) \end{bmatrix} + \sum_{l=1,\, l\neq h,k}^{p} \begin{bmatrix} \boldsymbol{u}_h^l(t) \\ \boldsymbol{u}_k^l(t) \end{bmatrix} \tag{97}$$

respectively. Then substituting (97) in (91) gives

$$\dot{\boldsymbol{q}}_{hk}(t) =$$

$$= \left[\begin{bmatrix} \boldsymbol{A}_{hh} & \boldsymbol{A}_{hk} \\ \boldsymbol{A}_{kh} & \boldsymbol{A}_{kk} \end{bmatrix} - \begin{bmatrix} \boldsymbol{B}_h & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_k \end{bmatrix} \begin{bmatrix} \boldsymbol{K}_h^k & \boldsymbol{K}_{hk} \\ \boldsymbol{K}_{kh} & \boldsymbol{K}_k^h \end{bmatrix} \right] \boldsymbol{q}_{hk}(t) + \sum_{l=1,\, l\neq h,k}^{p} \begin{bmatrix} \boldsymbol{B}_h \boldsymbol{u}_h^l(t) + \boldsymbol{A}_{hl}\boldsymbol{q}_l(t) \\ \boldsymbol{B}_k \boldsymbol{u}_k^l(t) + \boldsymbol{A}_{kl}\boldsymbol{q}_l(t) \end{bmatrix} \tag{98}$$

Using the next notations

$$\boldsymbol{A}_{hkc}^{\circ} = \begin{bmatrix} \boldsymbol{A}_{hh} & \boldsymbol{A}_{hk} \\ \boldsymbol{A}_{kh} & \boldsymbol{A}_{kk} \end{bmatrix} - \begin{bmatrix} \boldsymbol{B}_h & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_k \end{bmatrix} \begin{bmatrix} \boldsymbol{K}_h^k & \boldsymbol{K}_{hk} \\ \boldsymbol{K}_{kh} & \boldsymbol{K}_k^h \end{bmatrix} = \boldsymbol{A}_{hk}^{\circ} - \boldsymbol{B}_{hk}^{\circ}\boldsymbol{K}_{hk}^{\circ} \tag{99}$$

$$\boldsymbol{\omega}_{hk}^{\circ}(t) = \sum_{l=1,\, l\neq h,k}^{p} \begin{bmatrix} \boldsymbol{B}_h \boldsymbol{u}_h^l(t) + \boldsymbol{A}_{hl}\boldsymbol{q}_l(t) \\ \boldsymbol{B}_k \boldsymbol{u}_k^l(t) + \boldsymbol{A}_{kl}\boldsymbol{q}_l(t) \end{bmatrix} = \sum_{l=1,\, l\neq h,k}^{p} \left(\boldsymbol{B}_{hk}^{\circ} \begin{bmatrix} \boldsymbol{u}_h^l(t) \\ \boldsymbol{u}_k^l(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{A}_{hl} \\ \boldsymbol{A}_{kl} \end{bmatrix} \boldsymbol{q}_l(t) \right) = \tag{100}$$

$$= \boldsymbol{B}_{hk}^{\circ}\boldsymbol{\omega}_{hk}(t) + \sum_{l=1,\, l\neq h,k}^{p} \boldsymbol{A}_{hk}^{l\circ}\boldsymbol{q}_l(t)$$

where

$$\boldsymbol{\omega}_{hk}(t) = \sum_{l=1,\, l\neq h,k}^{p} \begin{bmatrix} \boldsymbol{u}_h^l(t) \\ \boldsymbol{u}_k^l(t) \end{bmatrix}, \qquad \boldsymbol{A}_{hk}^{l\circ} = \begin{bmatrix} \boldsymbol{A}_{hl} \\ \boldsymbol{A}_{kl} \end{bmatrix} \tag{101}$$

$$\boldsymbol{A}_{hk}^{\circ} = \begin{bmatrix} \boldsymbol{A}_{hh} & \boldsymbol{A}_{hk} \\ \boldsymbol{A}_{kh} & \boldsymbol{A}_{kk} \end{bmatrix}, \quad \boldsymbol{B}_{hk}^{\circ} = \begin{bmatrix} \boldsymbol{B}_h & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_k \end{bmatrix}, \quad \boldsymbol{K}_{hk}^{\circ} = \begin{bmatrix} \boldsymbol{K}_h^k & \boldsymbol{K}_{hk} \\ \boldsymbol{K}_{kh} & \boldsymbol{K}_k^h \end{bmatrix} \tag{102}$$

(98) can be written as

$$\dot{\boldsymbol{q}}_{hk}(t) = \boldsymbol{A}_{hkc}^{\circ}\boldsymbol{q}_{hk}(t) + \sum_{l=1,\, l\neq h,k}^{p} \boldsymbol{A}_{hk}^{l\circ}\boldsymbol{q}_l(t) + \boldsymbol{B}_{hk}^{\circ}\boldsymbol{\omega}_{hk}(t) \tag{103}$$

where $\boldsymbol{\omega}_{hk}(t)$ can be considered as a generalized auxiliary disturbance acting on the pair h,k of the subsystems.

On the other hand, if

$$\boldsymbol{C}_{hh} = \sum_{l=1,\, l\neq h}^{p} \boldsymbol{C}_h^l, \quad \boldsymbol{C}_{hk}^{\circ} = \begin{bmatrix} \boldsymbol{C}_h^k & \boldsymbol{C}_{hk} \\ \boldsymbol{C}_{kh} & \boldsymbol{C}_k^h \end{bmatrix}, \quad \boldsymbol{C}_{hk}^{l\circ} = \begin{bmatrix} \boldsymbol{C}_{hl} \\ \boldsymbol{C}_{kl} \end{bmatrix} \tag{104}$$

then

$$\boldsymbol{y}(t) = \sum_{h=1}^{p-1} \sum_{k=h+1}^{p} \left(\boldsymbol{C}_{hk}^{\circ}\boldsymbol{q}_{hk}(t) + \sum_{l=1,\, l\neq h}^{p} \boldsymbol{C}_h^l\boldsymbol{q}_l(t) \right) \tag{105}$$

$$\boldsymbol{y}_{hk}(t) = \boldsymbol{C}_{hk}^{\circ}\boldsymbol{q}_{hk}(t) + \sum_{l=1,\, l\neq h}^{p} \boldsymbol{C}_{hk}^{l\circ}\boldsymbol{q}_l(t) + \boldsymbol{0}\,\boldsymbol{\omega}_{hk}(t) \tag{106}$$

Now, taking (103), (106) considered pair of controlled subsystems is fully described as

$$\dot{\boldsymbol{q}}_{hk}(t) = \boldsymbol{A}_{hkc}^{\circ}\boldsymbol{q}_{hk}(t) + \sum_{l=1,\, l\neq h,k}^{p} \boldsymbol{A}_{hk}^{l\circ}\boldsymbol{q}_l(t) + \boldsymbol{B}_{hk}^{\circ}\boldsymbol{\omega}_{hk}(t) \tag{107}$$

$$\boldsymbol{y}_{hk}(t) = \boldsymbol{C}_{hk}^{\circ}\boldsymbol{q}_{hk}(t) + \sum_{l=1,\, l\neq h}^{p} \boldsymbol{C}_{hk}^{l\circ}\boldsymbol{q}_l(t) + \boldsymbol{0}\,\boldsymbol{\omega}_{hk}(t) \tag{108}$$

5.3 Controller parameter design

Theorem 7. *Subsystem pair (91) in system (75), (77), controlled by control law (97) is stable with quadratic performances* $\|C_{hk}^\circ(sI - A_{hkc}^\circ)^{-1}B_{hk}^\circ\|_\infty^2 \leq \gamma_{hk}$, $\|C_{hk}^{l\circ}(sI - A_{hkc}^\circ)^{-1}B_{hk}^{l\circ}\|_\infty^2 \leq \varepsilon_{hkl}$ *if for* $h = 1, 2 \ldots, p-1$, $k = h+1, h+2 \ldots, p$, $l = 1, 2 \ldots, p$, $l \neq h, k$, *there exist a symmetric positive definite matrix* $X_{hk}^\circ \in R^{(n_h+n_k) \times (n_h+n_k)}$, *matrices* $Z_{hk}^\circ \in R^{(n_h+n_k) \times (n_h+n_k)}$, $Y_{hk}^\circ \in R^{(r_h+r_k) \times (n_h+n_k)}$, *and positive scalars* γ_{hk}, $\varepsilon_{hkl} \in R$ *such that*

$$X_{hk}^\circ = X_{hk}^{\circ T} > 0, \quad \varepsilon_{hkl} > 0, \quad \gamma_{hk} > 0, \quad h, l = 1, \ldots, p, \; l \neq h, k, \; h < k \leq p \tag{109}$$

$$\begin{bmatrix} \Phi_{hk}^\circ & A_{hk}^{1\circ} & \cdots & A_{hk}^{p\circ} & B_{hk}^\circ & X_{hk}^\circ A_{hk}^{\circ T} - Y_{hk}^{\circ T} B_{hk}^{\circ T} & X_{hk}^\circ C_{hk}^{\circ T} \\ * & -\varepsilon_{hk1} I_{n_1} & \cdots & 0 & 0 & A_{hk}^{1\circ T} & C_{hk}^{1\circ T} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & \cdots -\varepsilon_{hkp} I_{n_p} & 0 & A_{hk}^{p\circ T} & C_{hk}^{p\circ T} \\ * & * & \cdots & * & -\gamma_{hk} I_{(r_h+r_k)} & B_{hk}^{\circ T} & 0 \\ * & * & \cdots & * & * & -Z_{hk}^\circ - Z_{hk}^{\circ T} & 0 \\ * & * & \cdots & * & * & * & -I_{(m_h+m_k)} \end{bmatrix} < 0 \tag{110}$$

where A_{hk}°, B_{hk}°, $A_{hk}^{l\circ}$, C_{hk}°, $C_{hk}^{l\circ}$ *are defined in (99), (101), (104), respectively,*

$$\Phi_{hk}^\circ = X_{hk}^\circ A_{hk}^{\circ T} + A_{hk}^\circ X_{hk}^\circ - B_{hk}^\circ Y_{hk}^\circ - Y_{hk}^{\circ T} B_{hk}^{\circ T} \tag{111}$$

and where $A_{hk}^{h\circ}$, $A_{hk}^{k\circ}$, *as well as* $C_{hk}^{h\circ}$, $C_{hk}^{k\circ}$ *are not included into the structure of (110). Then* K_{hk}° *is given as*

$$K_{hk}^\circ = Y_{hk}^\circ X_{hk}^{\circ -1} \tag{112}$$

Note, using the above given principle based on the the pairwise decentralized design of control, the global system be stable. The proof can be find in Filasová & Krokavec (2011).

Proof. Considering $\omega_{hk}^\circ(t)$ given in (100) as an generalized input into the subsystem pair (107), (108) then using (83) - (86), and (107) it can be written

$$\sum_{h=1}^{p-1} \sum_{k=h+1}^{p} (\dot{q}_{hk}^T(t) P_{hk}^\circ q_{hk}(t) + q_{hk}^T(t) P_{hk}^\circ \dot{q}_{hk}(t)) < 0 \tag{113}$$

$$\sum_{h=1}^{p-1} \sum_{k=h+1}^{p} \left(\begin{array}{l} \left(A_{hkc}^\circ q_{hk}(t) + \sum_{l=1,\, l\neq h,k}^{p} A_{hk}^{l\circ} q_l(t) + B_{hk}^\circ \omega_{hk}(t)\right)^T P_{hk}^\circ q_{hk}(t) + \\ + q_{hk}^T(t) P_{hk}^\circ \left(A_{hkc}^\circ q_{hk}(t) + \sum_{l=1,\, l\neq h,k}^{p} A_{hk}^{l\circ} q_l(t) + B_{hk}^\circ \omega_{hk}(t)\right) \end{array} \right) < 0 \tag{114}$$

respectively. Introducing the next notations

$$B_{hk}^{l\circ} = \left[\left\{A_{hk}^{l\circ}\right\}_{l=1,\, l\neq h,k}^{p} \quad B_{hk}^\circ \right], \qquad \omega_{hk}^{l\circ T} = \left[\left\{q_l^T\right\}_{l=1,\, l\neq h,k}^{p} \quad \omega_{hk}^T(t) \right] \tag{115}$$

(114) can be written as

$$\sum_{h=1}^{p-1} \sum_{k=h+1}^{p} \left((A_{hkc}^\circ q_{hk}(t) + B_{hk}^{l\circ} \omega_{hk}^{l\circ})^T P_{hk}^\circ q_{hk}(t) + q_{hk}^T(t) P_{hk}^\circ (A_{hkc}^\circ q_{hk}(t) + B_{hk}^{l\circ} \omega_{hk}^{l\circ}) \right) < 0 \tag{116}$$

Analogously, (106) can be rewritten as

$$y_{hk}(t) = C^{\circ}_{hk}q_{hk}(t) + \sum_{l=1,\, l \neq h}^{p} C^{l\circ}_{hk}q_l(t) + 0\,\omega_{hk}(t) = C^{\circ}_{hk}q_{hk}(t) + D^{l\circ}_{hk}\omega^{l\circ}_{hk} \tag{117}$$

where

$$D^{l\circ}_{hk} = \left[\{C_{hk}\}^{p}_{l=1,\, l \neq h,k} \ 0 \right] \tag{118}$$

Therefore, defining

$$\Gamma^{\circ}_{hk} = \mathrm{diag} \left[\{\varepsilon_{hkl}I_{n_l}\}^{p}_{l=1,\, l \neq h,k} \ \gamma_{hk}I_{(r_h+r_k)} \right] \tag{119}$$

and inserting appropriate into (57), (58) then (109), (110) be obtained. □

Illustrative example

To demonstrate properties of this approach a simple system with four-inputs and four-outputs is used in the example. The parameters of (75)-(77) are

$$A = \begin{bmatrix} 3 & 1 & 2 & -1 \\ -1 & 2 & 0 & 1 \\ 1 & -1 & 1 & 3 \\ 1 & -2 & -2 & 2 \end{bmatrix}, \ C = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 0 & 6 & 1 & 0 \\ 2 & -1 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}, \ B = \mathrm{diag}\,[1\ 1\ 1\ 1],$$

To solve this problem the next separations were done

$$B_{hk} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ h = 1,2,3, \ k = 2,3,4, \ h < k$$

$$A^{\circ}_{12} = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, \ A^{3\circ}_{12} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \ A^{4\circ}_{12} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \ C^{\circ}_{12} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \ C^{3\circ}_{12} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \ C^{4\circ}_{12} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A^{\circ}_{13} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}, \ A^{2\circ}_{13} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \ A^{4\circ}_{13} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \ C^{\circ}_{13} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \ C^{2\circ}_{13} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \ C^{4\circ}_{13} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A^{\circ}_{14} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}, \ A^{2\circ}_{14} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \ A^{3\circ}_{14} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}, \ C^{\circ}_{14} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \ C^{2\circ}_{14} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ C^{3\circ}_{14} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$A^{\circ}_{23} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}, \ A^{1\circ}_{23} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \ A^{4\circ}_{23} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \ C^{\circ}_{23} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}, \ C^{1\circ}_{23} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \ C^{4\circ}_{23} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A^{\circ}_{24} = \begin{bmatrix} 2 & 1 \\ -2 & 2 \end{bmatrix}, \ A^{1\circ}_{24} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \ A^{3\circ}_{24} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}, \ C^{\circ}_{24} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \ C^{1\circ}_{24} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ C^{3\circ}_{24} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^{\circ}_{34} = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}, \ A^{1\circ}_{34} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ A^{2\circ}_{34} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}, \ C^{\circ}_{34} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \ C^{1\circ}_{34} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \ C^{2\circ}_{34} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Solving e.g. with respect to X°_{23}, Y°_{23}, Z°_{23}, ε_{231}, ε_{234} δ_{23} it means to rewrite (109)-(111) as

$$X^{\circ}_{23} = X^{\circ T}_{23} > 0, \quad \varepsilon_{231} > 0, \quad \varepsilon_{234} > 0, \quad \gamma_{23} > 0$$

$$\begin{bmatrix} \Phi^{\circ}_{23} & A^{1\circ}_{23} & A^{4\circ}_{23} & B^{\circ}_{23} & X^{\circ}_{23}A^{\circ T}_{23} - Y^{\circ T}_{23}B^{\circ T}_{23} & X^{\circ}_{23}C^{\circ T}_{23} \\ * & -\varepsilon_{231} & 0 & 0 & A^{1\circ T}_{23} & C^{1\circ T}_{23} \\ * & * & -\varepsilon_{234} & 0 & A^{4\circ T}_{23} & C^{4\circ T}_{23} \\ * & * & * & -\gamma_{23}I_2 & B^{\circ T}_{23} & 0 \\ * & * & * & * & -Z^{\circ}_{23}-Z^{\circ T}_{23} & 0 \\ * & * & * & * & * & -I_2 \end{bmatrix} < 0$$

$$\boldsymbol{\Phi}_{23}^{\circ} = \boldsymbol{X}_{23}^{\circ}\boldsymbol{A}_{23}^{\circ T} + \boldsymbol{A}_{23}^{\circ}\boldsymbol{X}_{23}^{\circ} - \boldsymbol{B}_{23}^{\circ}\boldsymbol{Y}_{23}^{\circ} - \boldsymbol{Y}_{23}^{\circ T}\boldsymbol{B}_{23}^{\circ T}$$

Using SeDuMi package for Matlab given task was feasible with

$$\varepsilon_{231} = 9.3761, \qquad \varepsilon_{234} = 6.7928, \qquad \gamma_{23} = 6.2252$$

$$\boldsymbol{X}_{23}^{\circ} = \begin{bmatrix} 0.5383 & -0.0046 \\ -0.0046 & 0.8150 \end{bmatrix}, \ \boldsymbol{Y}_{23}^{\circ} = \begin{bmatrix} 4.8075 & -0.0364 \\ -0.4196 & 5.1783 \end{bmatrix}, \ \boldsymbol{Z}_{23}^{\circ} = \begin{bmatrix} 4.2756 & 0.1221 \\ 0.1221 & 4.5297 \end{bmatrix}$$

$$\boldsymbol{K}_{23}^{\circ} = \begin{bmatrix} 1.1255 & -0.0384 \\ -0.1309 & 1.1467 \end{bmatrix}$$

By the same way computing the rest gain matrices the gain matrix set is

$$\boldsymbol{K}_{12}^{\circ} = \begin{bmatrix} 7.3113 & 3.8869 \\ 1.4002 & 10.0216 \end{bmatrix}, \ \boldsymbol{K}_{13}^{\circ} = \begin{bmatrix} 7.9272 & 4.0712 \\ 4.2434 & 8.8245 \end{bmatrix}, \ \boldsymbol{K}_{14}^{\circ} = \begin{bmatrix} 7.4529 & 1.5651 \\ 1.6990 & 5.6584 \end{bmatrix}$$

$$\boldsymbol{K}_{24}^{\circ} = \begin{bmatrix} 7.2561 & 0.7243 \\ -2.7951 & 4.4839 \end{bmatrix}, \qquad \boldsymbol{K}_{34}^{\circ} = \begin{bmatrix} 6.3680 & 4.1515 \\ 0.8099 & 5.2661 \end{bmatrix}$$

Note, the control laws are realized in the partly-autonomous structure (94), (95), where every subsystem pair is stable, and the large-scale system be stable, too. To compare, an equivalent gain matrix (81) to centralized control can be constructed

$$\boldsymbol{K} = \begin{bmatrix} 22.6914 & 3.8869 & 4.0712 & 1.5651 \\ 1.4002 & 18.4032 & -0.0384 & 0.7243 \\ 4.2434 & -0.1309 & 16.3393 & 4.1515 \\ 1.6990 & -2.7951 & 0.8099 & 15.4084 \end{bmatrix}$$

Thus, the resulting closed-loop eigenvalue spectrum is

$$\rho(\boldsymbol{A} - \boldsymbol{BK}) = \{-13.0595 \pm 0.4024\,\mathrm{i} \ -16.2717 \ -22.4515\}$$

Matrix $\boldsymbol{K}$ structure implies evidently that the control gain is diagonally dominant.

6. Pairwise decentralized design of control for uncertain systems

Consider for the simplicity that only the system matrix blocks are uncertain, and one or none uncertain function is associated with a system matrix block. Then the structure of the pairwise system description implies

$$\boldsymbol{A}_{hk}r(t) \in \begin{cases} \boldsymbol{A}_{hk}^{\circ} \cup \{\boldsymbol{A}_{lh}^{k\circ}\}_{l=1}^{h-1} \cup \{\boldsymbol{A}_{hl}^{k\circ}\}_{l=h+1}^{p} & ; \text{ upper triagonal blocks } (h < k) \\ \{\boldsymbol{A}_{lh}^{\circ}\}_{l=1}^{h-1} \cup \{\boldsymbol{A}_{hl}^{\circ}\}_{l=h}^{p} & ; \text{ diagonal blocks } (h = k) \\ \boldsymbol{A}_{kh}^{\circ} \cup \{\boldsymbol{A}_{lk}^{h\circ}\}_{l=1}^{k-1} \cup \{\boldsymbol{A}_{kl}^{h\circ}\}_{l=k+1}^{p} & ; \text{ lower triagonal blocks } (h > k) \end{cases} \qquad (120)$$

Analogously it can be obtained equivalent expressions with respect to $\boldsymbol{B}_{hk}r(t)$, $\boldsymbol{C}_{hk}r(t)$, respectively. Thus, it is evident already in this simple case that a single uncertainty affects $p-1$ from $q = \binom{p}{2}$ linear matrix inequalities which have to be included into design. Generally, the next theorem can be formulated.

Theorem 8. *Uncertain subsystem pair (91) in system (75), (77), controlled by control law (97) is stable with quadratic performances* $\|C_{hk}^\circ(sI - A_{hkc}^\circ)^{-1}B_{hk}^\circ\|_\infty^2 \leq \gamma_{hk}$, $\|C_{hk}^{l\circ}(sI - A_{hkc}^\circ)^{-1}B_{hk}^{l\circ}\|_\infty^2 \leq \varepsilon_{hkl}$ *if for* $\delta > 0$, $\delta \in R$, $h = 1, 2\ldots, p-1$, $k = h+1, h+2\ldots, p$, $l = 1, 2\ldots, p$, $l \neq h, k$, *there exist symmetric positive definite matrices* $T_{hki}^\circ \in\in R^{(n_h+n_k)\times(n_h+n_k)}$, *matrices* $V_{hk}^\circ \in R^{(n_h+n_k)\times(n_h+n_k)}$, $W_{hk}^\circ \in R^{(r_h+r_k)\times(n_h+n_k)}$, *and positive scalars* γ_{hk}, $\varepsilon_{hkl} \in R$ *such that for* $i = 1, 2\ldots, s$

$$T_{hki}^\circ = T_{hki}^{\circ T} > 0, \ \varepsilon_{hkl} > 0, \ \gamma_{hk} > 0, \ h, l = 1, \ldots, p, \ l \neq h, k, \ h < k \leq p, \ i = 1, 2, \ldots, s \quad (121)$$

$$\begin{bmatrix} \Phi_{hki}^\circ & A_{hki}^{1\circ} & \cdots & A_{hki}^{p\circ} & B_{hki}^\circ & T_{hki}^\circ - \delta V_{hk}^{\circ T} + V_{hk}^\circ A_{hki}^{\circ T} - W_{hk}^{\circ T} B_{hki}^{\circ T} & V_{hk}^\circ C_{hki}^{\circ T} \\ * & -\varepsilon_{hk1}I_{n_1} & \cdots & 0 & 0 & A_{hki}^{1\circ T} & C_{hki}^{1\circ T} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & \cdots -\varepsilon_{hkp}I_{n_p} & 0 & A_{hki}^{p\circ T} & C_{hki}^{p\circ T} \\ * & * & \cdots & * & -\gamma_{hk}I_{(r_h+r_k)} & B_{hki}^{\circ T} & 0 \\ * & * & \cdots & * & * & -\delta(V_{hk}^\circ + V_{hk}^{\circ T}) & 0 \\ * & * & \cdots & * & * & * & -I_{(m_h+m_k)} \end{bmatrix} < 0 \quad (122)$$

where A_{hk}°, B_{hk}°, $A_{hk}^{l\circ}$, C_{hk}°, $C_{hk}^{l\circ}$ *are equivalently defined as in (99), (101), (104), respectively,*

$$\Phi_{hki}^\circ = V_{hk}^\circ A_{hki}^{\circ T} + A_{hki}^\circ V_{hk}^{\circ T} - B_{hki}^\circ W_{hk}^\circ - W_{hk}^{\circ T} B_{hki}^{\circ T} \quad (123)$$

and where $A_{hki}^{h\circ}$, $A_{hki}^{k\circ}$, *as well as* $C_{hki}^{h\circ}$, $C_{hki}^{k\circ}$ *are not included into the structure of (122). Then* K_{hk}° *is given as*

$$K_{hk}^\circ = W_{hk}^\circ V_{hk}^{\circ T-1} \quad (124)$$

Proof. Considering (109)-(112) and inserting these appropriate into (72), *i* of(73), and (74) then (121)-(124) be obtained. □

Illustrative example
Considering the same system parameters as were those given in the example presented in Subsection 5.3 but with $A_{34}r(t)$, and $r(t)$ lies within the interval $\langle 0.8, 1.2 \rangle$ then the next matrix parameter have to be included into solution

$$A_{131}^{4\circ} = \begin{bmatrix} -1 \\ 2.4 \end{bmatrix}, \ A_{132}^{4\circ} = \begin{bmatrix} -1 \\ 3.6 \end{bmatrix}, \ A_{231}^{4\circ} = \begin{bmatrix} 1 \\ 2.4 \end{bmatrix}, \ A_{23}^{4\circ} = \begin{bmatrix} 1 \\ 3.6 \end{bmatrix}$$

$$A_{341}^\circ = \begin{bmatrix} 1 & 2.4 \\ -2 & 2 \end{bmatrix}, \ A_{342}^\circ = \begin{bmatrix} 1 & 3.6 \\ -2 & 2 \end{bmatrix},$$

i.e. a solution be associated with T_{13i}°, T_{23i}°, and T_{34i}°, $i = 1, 2$, and in other cases only one matrix inequality be computed (T_{12}°, T_{14}°, T_{24}°).
The task is feasible, the Lyapunov matrices are computed as follows

$$T_{131}^\circ = \begin{bmatrix} 5.7244 & -0.3591 \\ 0.1748 & 5.6673 \end{bmatrix}, \ T_{132}^\circ = \begin{bmatrix} 5.0484 & 0.0232 \\ 0.0232 & 5.0349 \end{bmatrix}, \ T_{12}^\circ = \begin{bmatrix} 6.3809 & 0.5280 \\ -0.6811 & 6.3946 \end{bmatrix}$$

$$T_{231}^\circ = \begin{bmatrix} 6.1360 & 0.0841 \\ 0.0090 & 6.2377 \end{bmatrix}, \ T_{232}^\circ = \begin{bmatrix} 5.5035 & 0.0258 \\ 0.0258 & 5.5252 \end{bmatrix}, \ T_{14}^\circ = \begin{bmatrix} 7.2453 & 0.9196 \\ -1.0352 & 7.5124 \end{bmatrix}$$

$$T_{341}^\circ = \begin{bmatrix} 2.4585 & 3.9935 \\ -3.7569 & 1.5487 \end{bmatrix}, \; T_{342}^\circ = \begin{bmatrix} 5.7297 & \\ & 5.7249 \end{bmatrix}, \; T_{24}^\circ = \begin{bmatrix} 2.5560 & 2.1220 \\ -1.9076 & 2.9055 \end{bmatrix}$$

the control law matrices takes form

$$K_{12}^\circ = \begin{bmatrix} 13.2095 & 0.7495 \\ 2.2753 & 14.1033 \end{bmatrix}, \; K_{13}^\circ = \begin{bmatrix} 14.2051 & 4.4679 \\ 1.9440 & 13.4616 \end{bmatrix}, \; K_{14}^\circ = \begin{bmatrix} 12.6360 & -1.6407 \\ 2.9881 & 10.6109 \end{bmatrix}$$

$$K_{23}^\circ = \begin{bmatrix} 14.3977 & -0.4237 \\ -1.0494 & 12.3509 \end{bmatrix}, \; K_{24}^\circ = \begin{bmatrix} -2.9867 & 5.9950 \\ -6.8459 & -2.6627 \end{bmatrix}, \; K_{34}^\circ = \begin{bmatrix} 5.3699 & 2.7480 \\ -0.6542 & 6.1362 \end{bmatrix}$$

and with the common $\delta = 10$ the subsystem interaction transfer functions H_∞-norm upper-bound squares are

$\varepsilon_{123} = 10.9960, \; \varepsilon_{124} = 7.6712, \; \gamma_{12} = 7.1988, \; \varepsilon_{132} = 7.7242, \; \varepsilon_{134} = 8.7654, \; \gamma_{13} = 6.4988$

$\varepsilon_{142} = 8.9286, \; \varepsilon_{143} = 12.1338, \; \gamma_{14} = 8.1536, \; \varepsilon_{231} = 10.3916, \; \varepsilon_{234} = 8.2081, \; \gamma_{23} = 7.0939$

$\varepsilon_{241} = 5.3798, \; \varepsilon_{243} = 6.6286, \; \gamma_{24} = 5.4780, \; \varepsilon_{341} = 16.1618, \; \varepsilon_{342} = 15.0874, \; \gamma_{34} = 9.0965$

In the same sense as given above, the control laws are realized in the partly-autonomous structure (94), (95), too, and as every subsystem pair as the large-scale system be stable.

Only for comparison reason, the composed gain matrix (defined as in (81)), and the resulting closed-loop system matrix eigenvalue spectrum, realized using the nominal system matrix parameter A_n and the robust and the nominal equivalent gain matrices K, A_n, respectively, were constructed using the set of gain matrices K_{hk}, $k = 1, 2, 3$, $h = 2, 3, 4$, $h \neq k$. As it can see

$$K = \begin{bmatrix} 40.0507 & 0.7495 & 4.4679 & -1.6407 \\ 2.2753 & 25.5144 & -0.4237 & 5.9950 \\ 1.9440 & -1.0494 & 31.1824 & 2.7480 \\ 2.9881 & -6.8459 & -0.6542 & 14.0844 \end{bmatrix}, \; \rho(A_n - BK) = \begin{bmatrix} -15.0336 \\ -20.6661 \\ -29.8475 \\ -37.2846 \end{bmatrix}$$

$$K_n = \begin{bmatrix} 39.6876 & 0.7495 & 4.2372 & -1.6407 \\ 2.2753 & 24.8764 & -0.4500 & 5.9950 \\ 2.3218 & -1.0008 & 30.3905 & 3.2206 \\ 2.9881 & -6.8459 & -0.6666 & 14.0725 \end{bmatrix}, \; \rho(A_n - BK_n) = \begin{bmatrix} -15.3818 \\ -19.6260 \\ -29.0274 \\ -36.9918 \end{bmatrix}$$

and the resulted structures of both gain matrices imply that by considering parameter uncertainties in design step the control gain matrix K is diagonally more dominant then K_n reflecting only the system nominal parameters. ∎

It is evident that Lyapunov matrices T_{hki}° are separated from A_{hki}°, $A_{hki}^{l\circ}$, B_{hki}°, C_{hki}°, and $C_{hki}^{l\circ}$ $h = 1, 2 \ldots, p-1$, $k = h+1, h+2 \ldots, p$, $l = 1, 2 \ldots, p$, $l \neq h, k$, i.e. there are no terms containing the product of T_{hki}° and any of them. By introducing a new variable V_{hk}°, the products of type $P_{hki}^\circ A_{hki}^\circ$ and $A_{hki}^{\circ T} P_{hki}^\circ$ are relaxed to new products $A_{hki}^\circ V_{hk}^{\circ T}$ and $V_{hk}^\circ A_{hki}^{\circ T}$ where V_{hk}° needs not be symmetric and positive definite. This enables a robust BRL can be obtained for a system with polytopic uncertainties by using a parameter-dependent Lyapunov function, and to deal with linear systems with parametric uncertainties.

Although no common Lyapunov matrices are required the method generally leads to a larger number of linear matrix inequalities, and so more computational effort be needed to provide robust stability. However, used conditions are less restrictive than those obtained via a quadratic stability analysis (i.e. using a parameter-independent Lyapunov function), and are more close to necessity conditions. It is a very useful extension to control performance synthesis problems.

7. Concluding remarks

The main difficulty of solving the decentralized control problem comes from the fact that the feedback gain is subject to structural constraints. At the beginning study of large scale system theory, some people thought that a large scale system is decentrally stabilizable under controllability condition by strengthening the stability degree of subsystems, but because of the existence of decentralized fixed modes, some large scale systems can not be decentrally stabilized at all. In this chapter the idea to stabilize all subsystems and the whole system simultaneously by using decentralized controllers is replaced by another one, to stabilize all subsystems pairs and the whole system simultaneously by using partly decentralized control. In this sense the final scope of this chapter are quadratic performances of one class of uncertain continuous-time large-scale systems with polytopic convex uncertainty domain. It is shown how to expand the Lyapunov condition for pairwise control by using additive matrix variables in LMIs based on equivalent BRL formulations. As mentioned above, such matrix inequalities are linear with respect to the subsystem variables, and does not involve any product of the Lyapunov matrices and the subsystem ones. This enables to derive a sufficient condition for quadratic performances, and provides one way for determination of parameter-dependent Lyapunov functions by solving LMI problems. Numerical examples demonstrate the principle effectiveness, although some computational complexity is increased.

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9. References

Boyd, D.; El Ghaoui, L.; Peron, E. & Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*. SIAM, ISBN 0-89871-334-X, Philadelphia.

Duan, Z.; Wang, J.Z. & Huang, L. (1994). Special decentralized control problems and effectiveness of parameter-dependent Lyapunov function method. In *Proceedings of the American Control Conference 2005*, pp. 1697-1702, Portland, June 2005.

Feron, E.; Apkarian, P. & Gahinet, P. (1996). Analysis and synthesis of robust control systems via parameter-dependent Lyapunov functions, *IEEE Transactions on Automatic Control*, Vol. 41, No. 7, 1996, 1041-1046, ISSN 0018-9286

Filasová, A. & Krokavec, D. (1999). Pair-wise decentralized control of large-scale systems. *Journal of Electrical Engineering* Vol. 50, No. 3-4, 1999, 1-10, ISSN 1335-3632

Filasová, A. & Krokavec, D. (2000). Pair-wise partially decentralized Kalman estimator. In *Control Systems Design (CSD 2000): A Proceedings Volume from IFAC Conference*, Kozák, Š., Huba, M. (Ed.), pp. 125-130, ISBN 00-08-043546-7, Bratislava, June, 2000, Elsevier, Oxford.

Filasová, A. & Krokavec, D. (2011). Pairwise control principle in large-scale systems. *Archives of Control Sciences*, 21, 2011 (in print).

Gahinet, P.; Nemirovski, A.; Laub, A.J. & Chilali, M. (1995). *LMI Control Toolbox User's Guide*. The MathWorks, Inc., Natick, 1995.

Gahinet, P.; Apkarian, P. & Mahmoud Chilali, M. (1996)   Affine parameter-dependent Lyapunov functions and real parametric uncertainty, *IEEE Transactions on Automatic Control*, Vol. 41, No. 3, 1996, 436-442, ISSN 0018-9286

Herrmann, G.; Turner, M.C. & Postlethwaite, I. (2007). Linear matrix inequalities in control, In *Mathematical Methods for Robust and Nonlinear Control. EPRSC Summer School.*, Turner, M.C., Bates, D.G. (Ed.), pp. 123-142, Springer–Verlag, ISBN 978-1-84800-024-7, Berlin.

Jamshidi, M. (1997). *Large-Scale Systems: Modeling, Control and Fuzzy Logic*, Prentice Hall, ISBN 0-13-125683-1, Upper Saddle River.

Jia, Y. (2003). Alternative proofs for improved LMI representations for the analysis and the design of continuous-time systems with poolytopic type uncertainty: A predictive approach, *IEEE Transactions on Automatic Control*, Vol. 48, No. 8, 2003, 1413-1416, ISSN 0018-9286

Kozáková, A. & V. Veselý, V. (2009). Design of robust decentralized controllers using the $M - \Delta$ structures. Robust stability conditions. *International Journal of System Science*, Vol. 40, No. 5, 2009, 497-505, ISSN 0020-7721

Krokavec, D. & Filasová, A. (2008). *Discrete-Time Systems*, Elfa, ISBN 978-80-8086-101-8, Košice. (in Slovak)

Leros, A.P. (1989).   LSS linear regulator problem. A partially decentralized approach. *International Journal of Control*, Vol. 49, No. 4, 1989, 1377-1399, ISSN 0020-7179

Lunze, J. (1992). *Feedback Control of Large-Scale Systems*, Prentice Hall, ISBN 0-13-318353-X, London.

Mahmoud, M.S. & Singh, M.G. (1981). *Large-Scale Systems Modelling*, Pergamon Press, ISBN 00-08-027313-0, Oxford.

Nesterov, Y. & Nemirovsky, A. (1994). *Interior Point Polynomial Methods in Convex Programming. Theory and Applications*. SIAM, ISBN 0-89871-319-6, Philadelphia.

Peaucelle, D.; Henrion, D.; Labit, Y. & Taitz, K. (1994). *User's Guide for SeDuMi Interface 1.04*, LAAS-CNRS, Toulouse.

Petersen, I.R.; Ugrinovskii, V.A. & Savkin A.V. (2000). *Robust Control Design Using H_∞ Methods*, Springer-Verlag, ISBN 1-85233-171-2, London.

Pipeleers, G.; Demeulenaerea, B.; Sweversa, J. & Vandenbergheb, L. (2009). Extended LMI characterizations for stability and performance of linear systems, *Systems & Control Letters*, Vol.58, 2009, 510-518, ISSN 0167-6911

Skelton, E.E.; Iwasaki, T. & Grigoriadis, K. (1998). *A Unified Algebraic Approach to Linear Control Design*, Taylor & Francis, ISBN 0-7484-0592-5, London.

Veselý, V. & Rosinová, D. (2009). Robust output model predictive control design. BMI approach, *International Journal of Innovative Computing, Information and Control*, Vol. 5, No. 4, 2009, 1115-1123, ISBN 1751-648X

Wang, Q.G. (2003). *Decoupling Control*, Springer-Verlag, ISBN 978-3-54044-128-1, Berlin.

Wu, A.I. & Duan, G.R. (2006). Enhanced LMI representations for H_2 performance of polytopic uncertain systems: Continuous-time case, *International Journal of Automation and Computing*, Vol. 3, 2006, 304-308, ISSN 1476-8186

Wu, M.; He, Y. & She, J.H. (2010). *Stability Analysis and Robust Control of Time-Delay Systems*, Springer–Verlag, ISBN 3-64203-036-X, Berlin.

Xie, W. (2008). An equivalent LMI representation of bounded real lemma for continous-time systems, *Journal of Inequalities and Applications*, Vol. 5, 2010, ISSN 1025-5834

Zhou, K.; Doyle, J.C. & Glover, K. (1995). *Robust and Optimal Control*, Prentice Hall, ISBN 0-13-456567-3, Englewood Cliffs

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