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# Robust Control of Nonlinear Time-Delay Systems via Takagi-Sugeno Fuzzy Models

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## 1. Introduction

Robust control theory is an interdisciplinary branch of engineering and applied mathematics literature. Since its introduction in 1980's, it has grown to become a major scientific domain. For example, it gained a foothold in Economics in the late 1990 and has seen increasing numbers of Economic applications in the past few years. This theory aims to design a controller which guarantees closed-loop stability and performances of systems in the presence of system uncertainty. In practice, the uncertainty can include modelling errors, parametric variations and external disturbance. Many results have been presented for robust control of linear systems. However, most real physical systems are nonlinear in nature and usually subject to uncertainties. In this case, the linear dynamic systems are not powerful to describe these practical systems. So, it is important to design robust control of nonlinear models. In this context, different techniques have been proposed in the literature (Input-Output linearization technique, backstepping technique, Variable Structure Control (VSC) technique, ...).

These two last decades, fuzzy model control has been extensively studied; see (Zhang & Heng, 2002)-(Chadli & ElHajjaji, 2006)-(Kim & Lee, 2000)-(Boukas & ElHajjaji, 2006) and the references therein because T-S fuzzy model can provide an effective representation of complex nonlinear systems. On the other hand, time-delay are often occurs in various practical control systems, such as transportation systems, communication systems, chemical processing systems, environmental systems and power systems. It is well known that the existence of delays may deteriorate the performances of the system and can be a source of instability. As a consequence, the T-S fuzzy model has been extended to deal with nonlinear systems with time-delay. The existing results of stability and stabilization criteria for this class of T-S fuzzy systems can be classified into two types: delay-independent, which are applicable to delay of arbitrary size (Cao & Frank, 2000)-(Park et al., 2003)-(Chen & Liu, 2005b), and delay-dependent, which include information on the size of delays, (Li et al., 2004) - (Chen & Liu, 2005a). It is generally recognized that delay-dependent results are usually less conservative than delay-independent ones, especially when the size of delay

is small. We notice that all the results of analysis and synthesis delay-dependent methods cited previously are based on a single LKF that bring conservativeness in establishing the stability and stabilization test. Moreover, the model transformation, the conservative inequalities and the so-called Moon's inequality (Moon et al., 2001) for bounding cross terms used in these methods also bring conservativeness. Recently, in order to reduce conservatism, the weighting matrix technique was proposed originally by He and al. in (He et al., 2004)-(He et al., 2007). These works studied the stability of linear systems with time-varying delay. More recently, Huai-Ning et al. (Wu & Li, 2007) treated the problem of stabilization via PDC (Parallel Distributed Compensation) control by employing a fuzzy LKF combining the introduction of free weighting matrices which improves existing ones in (Li et al., 2004) - (Chen & Liu, 2005a) without imposing any bounding techniques on some cross product terms. In general, the disadvantage of this new approach (Wu & Li, 2007) lies in that the delay-dependent stabilization conditions presented involve three tuning parameters. Chen et al. in (Chen et al., 2007) and in (Chen & Liu, 2005a) have proposed delay-dependent stabilization conditions of uncertain T-S fuzzy systems. The inconvenience in these works is that the time-delay must be constant. The designing of observer-based fuzzy control and the introduction of performance with guaranteed cost for T-S with input delay have discussed in (Chen, Lin, Liu & Tong, 2008) and (Chen, Liu, Tang & Lin, 2008), respectively.

In this chapter, we study the asymptotic stabilization of uncertain T-S fuzzy systems with time-varying delay. We focus on the delay-dependent stabilization synthesis based on the PDC scheme (Wang et al., 1996). Different from the methods currently found in the literature (Wu & Li, 2007)-(Chen et al., 2007), our method does not need any transformation in the LKF, and thus, avoids the restriction resulting from them. Our new approach improves the results in (Li et al., 2004)-(Guan & Chen, 2004)-(Chen & Liu, 2005a)-(Wu & Li, 2007) for three great main aspects. The first one concerns the reduction of conservatism. The second one, the reduction of the number of LMI conditions, which reduce computational efforts. The third one, the delay-dependent stabilization conditions presented involve a single fixed parameter. This new approach also improves the work of B. Chen et al. in (Chen et al., 2007) by establishing new delay-dependent stabilization conditions of uncertain T-S fuzzy systems with time varying delay. The rest of this chapter is organized as follows. In section 2, we give the description of uncertain T-S fuzzy model with time varying delay. We also present the fuzzy control design law based on PDC structure. New delay dependent stabilization conditions are established in section 3. In section 4, numerical examples are given to demonstrate the effectiveness and the benefits of the proposed method. Some conclusions are drawn in section 5.

*Notation:*  $\mathbb{R}^n$  denotes the  $n$ -dimensional Euclidian space. The notation  $P > 0$  means that  $P$  is symmetric and positive definite.  $W + W^T$  is denoted as  $W + (*)$  for simplicity. In symmetric bloc matrices, we use  $*$  as an ellipsis for terms that are induced by symmetry.

## 2. Problem formulation

Consider a nonlinear system with state-delay which could be represented by a T-S fuzzy time-delay model described by

Plant Rule  $i$  ( $i = 1, 2, \dots, r$ ): If  $\theta_1$  is  $\mu_{i1}$  and  $\dots$  and  $\theta_p$  is  $\mu_{ip}$  THEN

$$\begin{aligned} \dot{x}(t) &= (A_i + \Delta A_i)x(t) + (A_{\tau i} + \Delta A_{\tau i})x(t - \tau(t)) + (B_i + \Delta B_i)u(t) \\ x(t) &= \psi(t), t \in [-\bar{\tau}, 0], \end{aligned} \quad (1)$$

where  $\theta_j(x(t))$  and  $\mu_{ij}(i = 1, \dots, r, j = 1, \dots, p)$  are respectively the premise variables and the fuzzy sets;  $\psi(t)$  is the initial conditions;  $x(t) \in \mathfrak{R}^n$  is the state;  $u(t) \in \mathfrak{R}^m$  is the control input;  $r$  is the number of IF-THEN rules; the time delay,  $\tau(t)$ , is a time-varying continuous function that satisfies

$$0 \leq \tau(t) \leq \bar{\tau}, \dot{\tau}(t) \leq \beta \quad (2)$$

The parametric uncertainties  $\Delta A_i, \Delta A_{\tau i}, \Delta B_i$  are time-varying matrices that are defined as follows

$$\Delta A_i = M_{A_i} F_i(t) E_{A_i}; \Delta A_{\tau i} = M_{A_{\tau i}} F_i(t) E_{A_{\tau i}}; \Delta B_i = M_{B_i} F_i(t) E_{B_i} \quad (3)$$

where  $M_{A_i}, M_{A_{\tau i}}, M_{B_i}, E_{A_i}, E_{A_{\tau i}}, E_{B_i}$  are known constant matrices and  $F_i(t)$  is an unknown matrix function with the property

$$F_i(t)^T F_i(t) \leq I \quad (4)$$

Let  $\bar{A}_i = A_i + \Delta A_i; \bar{A}_{\tau i} = A_{\tau i} + \Delta A_{\tau i}; \bar{B}_i = B_i + \Delta B_i$

By using the common used center-average defuzzifier, product inference and singleton fuzzifier, the T-S fuzzy systems can be inferred as

$$\dot{x}(t) = \sum_{i=1}^r h_i(\theta(x(t))) [\bar{A}_i x(t) + \bar{A}_{\tau i} x(t - \tau(t)) + \bar{B}_i u(t)] \quad (5)$$

where  $\theta(x(t)) = [\theta_1(x(t)), \dots, \theta_p(x(t))]$  and  $v_i(\theta(x(t))) : \mathfrak{R}^p \rightarrow [0, 1], i = 1, \dots, r$ , is the membership function of the system with respect to the  $i$ th plan rule. Denote  $h_i(\theta(x(t))) = v_i(\theta(x(t))) / \sum_{i=1}^r v_i(\theta(x(t)))$ . It is obvious that

$$h_i(\theta(x(t))) \geq 0 \text{ and } \sum_{i=1}^r h_i(\theta(x(t))) = 1$$

the design of state feedback stabilizing fuzzy controllers for fuzzy system (5) is based on the Parallel Distributed Compensation.

Controller Rule  $i(i = 1, 2, \dots, r)$ : If  $\theta_1$  is  $\mu_{i1}$  and  $\dots$  and  $\theta_p$  is  $\mu_{ip}$  THEN

$$u(t) = K_i x(t) \quad (6)$$

The overall state feedback control law is represented by

$$u(t) = \sum_{i=1}^r h_i(\theta(x(t))) K_i x(t) \quad (7)$$

In the sequel, for brevity we use  $h_i$  to denote  $h_i(\theta(x(t)))$ . Combining (5) with (7), the closed-loop fuzzy system can be expressed as follows

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j [\hat{A}_{ij} x(t) + \bar{A}_{\tau i} x(t - \tau(t))] \quad (8)$$

with  $\hat{A}_{ij} = \bar{A}_i + \bar{B}_i K_j$

In order to obtain the main results in this chapter, the following lemmas are needed

**Lemma 1.** (Xie & DeSouza, 1992)-(Oudghiri et al., 2007) (Guerra et al., 2006) Considering  $\Pi < 0$  a matrix  $X$  and a scalar  $\lambda$ , the following holds

$$X^T \Pi X \leq -2\lambda X - \lambda^2 \Pi^{-1} \quad (9)$$

**Lemma 2.** (Wang et al., 1992) Given matrices  $M, E, F(t)$  with compatible dimensions and  $F(t)$  satisfying  $F(t)^T F(t) \leq I$ .

Then, the following inequalities hold for any  $\epsilon > 0$

$$MF(t)E + E^T F(t)^T M^T \leq \epsilon MM^T + \epsilon^{-1} E^T E \quad (10)$$

### 3. Main results

#### 3.1 Time-delay dependent stability conditions

First, we derive the stability condition for unforced system (5), that is

$$\dot{x}(t) = \sum_{i=1}^r h_i [\bar{A}_i x(t) + \bar{A}_{\tau i} x(t - \tau(t))] \quad (11)$$

**Theorem 1.** System (11) is asymptotically stable, if there exist some matrices  $P > 0, S > 0, Z > 0, Y$  and  $T$  satisfying the following LMIs for  $i = 1, 2, \dots, r$

$$\begin{bmatrix} \varphi_i + \epsilon_{A_i} E_{A_i}^T E_{A_i} & PA_{\tau i} - Y + T^T & A_{\tau i}^T Z & -Y & PM_{A_i} & PM_{A_{\tau i}} \\ * & -(1 - \beta)S - T - T^T + \epsilon_{A_{\tau i}} E_{\tau i}^T E_{\tau i} & A_{\tau i}^T Z & -T & 0 & \\ * & * & -\frac{1}{\tau} Z & 0 & ZM_{A_i} & ZM_{A_{\tau i}} \\ * & * & * & -\frac{1}{\tau} Z & 0 & \\ * & * & * & * & -\epsilon_{A_i} I & 0 \\ * & * & * & * & * & -\epsilon_{A_{\tau i}} I \end{bmatrix} < 0 \quad (12)$$

where  $\varphi_i = PA_i + A_i^T P + S + Y + Y^T$ .

**Proof 1.** Choose the LKF as

$$V(x(t)) = x(t)^T P x(t) + \int_{t-\tau(t)}^t x(\alpha)^T S x(\alpha) d\alpha + \int_{-\bar{\tau}}^0 \int_{t+\sigma}^t \dot{x}(\alpha)^T Z \dot{x}(\alpha) d\alpha d\sigma \quad (13)$$

the time derivative of this LKF (13) along the trajectory of system (11) is computed as

$$\begin{aligned} \dot{V}(x(t)) = & 2x(t)^T P \dot{x}(t) + x(t)^T S x(t) - (1 - \dot{\tau}(t))x(t - \tau(t))^T S x(t - \tau(t)) \\ & + \bar{\tau} \dot{x}(t)^T Z \dot{x}(t) - \int_{t-\bar{\tau}}^t \dot{x}(s)^T Z \dot{x}(s) ds \end{aligned} \quad (14)$$

Taking into account the Newton-Leibniz formula

$$x(t - \tau(t)) = x(t) - \int_{t-\tau(t)}^t \dot{x}(s) ds \quad (15)$$

We obtain equation (16)

$$\begin{aligned} \dot{V}(x(t)) = & \sum_{i=1}^r h_i [2x(t)^T P \bar{A}_i x(t) + 2x(t)^T P \bar{A}_{\tau i} x(t - \tau(t))] \\ & + x(t)^T S x(t) - (1 - \beta)x(t - \tau(t))^T S x(t - \tau(t)) \\ & + \bar{\tau} \dot{x}(t)^T Z \dot{x}(t) - \int_{t-\bar{\tau}}^t \dot{x}(s)^T Z \dot{x}(s) ds \\ & + 2[x(t)^T Y + x(t - \tau(t))^T T] \times [x(t) - x(t - \tau(t)) - \int_{t-\tau(t)}^t \dot{x}(s) ds] \end{aligned} \quad (16)$$

As pointed out in (Chen & Liu, 2005a)

$$\dot{x}(t)^T Z \dot{x}(t) \leq \sum_{i=1}^r h_i \eta(t)^T \begin{bmatrix} \bar{A}_i^T Z \bar{A}_i & \bar{A}_i^T Z \bar{A}_{\tau i} \\ \bar{A}_{\tau i}^T Z \bar{A}_i & \bar{A}_{\tau i}^T Z \bar{A}_{\tau i} \end{bmatrix} \eta(t) \quad (17)$$

where  $\eta(t)^T = [x(t)^T, x(t - \tau(t))^T]$ .

Allowing  $W^T = [Y^T, T^T]$ , we obtain equation (18)

$$\begin{aligned} \dot{V}(x(t)) \leq & \sum_{i=1}^r h_i \eta(t)^T [\tilde{\Phi}_i + \bar{\tau} W Z^{-1} W^T] \eta(t) \\ & - \int_{t-\tau(t)}^t [\eta^T(t) W + \dot{x}(s)^T Z] Z^{-1} [\eta^T(t) W + \dot{x}(s)^T Z]^T ds \end{aligned} \quad (18)$$

where

$$\tilde{\Phi}_i = \begin{bmatrix} P \bar{A}_i + \bar{A}_i^T P + S + \bar{\tau} \bar{A}_i^T Z \bar{A}_i + Y + Y^T & P \bar{A}_{\tau i} + \bar{\tau} \bar{A}_i^T Z \bar{A}_{\tau i} - Y + T^T \\ * & -(1 - \beta) S + \bar{\tau} \bar{A}_{\tau i}^T Z \bar{A}_{\tau i} - T - T^T \end{bmatrix} \quad (19)$$

By applying Schur complement  $\tilde{\Phi}_i + \bar{\tau} W Z^{-1} W^T < 0$  is equivalent to

$$\tilde{\Phi}_i = \begin{bmatrix} \bar{\varphi}_i & P \bar{A}_{\tau i} - Y + T^T & \bar{A}_i^T Z & -Y \\ * & -(1 - \beta) S - T - T^T & \bar{A}_{\tau i}^T Z & -T \\ * & * & -\frac{1}{\bar{\tau}} Z & 0 \\ * & * & * & -\frac{1}{\bar{\tau}} Z \end{bmatrix} < 0$$

The uncertain part is represented as follows

$$\begin{aligned} \Delta \tilde{\Phi}_i = & \begin{bmatrix} P \Delta A_i + \Delta A_i^T P & P \Delta A_{\tau i} & \Delta A_i^T Z & 0 \\ * & 0 & \Delta A_{\tau i}^T Z & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \\ = & \begin{bmatrix} P M_{Ai} \\ 0 \\ Z M_{Ai} \\ 0 \end{bmatrix} F(t) [E_{Ai} \ 0 \ 0 \ 0] + (*) + \begin{bmatrix} P M_{A\tau i} \\ 0 \\ Z M_{A\tau i} \\ 0 \end{bmatrix} F(t) [0 \ E_{A\tau i} \ 0 \ 0] + (*) \end{aligned} \quad (20)$$

By applying lemma 2, we obtain

$$\begin{aligned} \Delta\bar{\Phi}_i \leq & \epsilon_{Ai}^{-1} \begin{bmatrix} PM_{Ai} \\ 0 \\ ZM_{Ai} \\ 0 \end{bmatrix} [M_{Ai}^T P \ 0 \ M_{Ai}^T Z \ 0] + \epsilon_{Ai} \begin{bmatrix} E_{Ai}^T \\ 0 \\ 0 \\ 0 \end{bmatrix} [E_{Ai} \ 0 \ 0 \ 0] \\ & + \epsilon_{A\tau i}^{-1} \begin{bmatrix} PM_{A\tau i} \\ 0 \\ ZM_{A\tau i} \\ 0 \end{bmatrix} [M_{A\tau i}^T P \ 0 \ M_{A\tau i}^T Z \ 0] + \epsilon_{A\tau i} \begin{bmatrix} 0 \\ E_{A\tau i}^T \\ 0 \\ 0 \end{bmatrix} [0 \ E_{A\tau i} \ 0 \ 0] \end{aligned} \quad (21)$$

where  $\epsilon_{Ai}$  and  $\epsilon_{A\tau i}$  are some positive scalars.

By using Schur complement, we obtain theorem 1.

### 3.2 Time-delay dependent stabilization conditions

**Theorem 2.** System (8) is asymptotically stable if there exist some matrices  $P > 0$ ,  $S > 0$ ,  $Z > 0$ ,  $Y$ ,  $T$  satisfying the following LMIs for  $i, j = 1, 2, \dots, r$  and  $i \leq j$

$$\bar{\Phi}_{ij} + \bar{\Phi}_{ji} \leq 0 \quad (22)$$

where  $\bar{\Phi}_{ji}$  is given by

$$\bar{\Phi}_{ij} = \begin{bmatrix} P\hat{A}_{ij} + \hat{A}_{ij}^T P + S + Y + Y^T & P\bar{A}_{\tau i} - Y + T^T & \hat{A}_{ij}^T Z & -Y \\ * & -(1-\beta)S - T - T^T & \bar{A}_{\tau i}^T Z & -T \\ * & * & -\frac{1}{\tau}Z & 0 \\ * & * & * & -\frac{1}{\tau}Z \end{bmatrix} \quad (23)$$

**Proof 2.** As pointed out in (Chen & Liu, 2005a), the following inequality is verified.

$$\dot{x}(t)^T Z \dot{x}(t) \leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \eta(t)^T \begin{bmatrix} \frac{(\hat{A}_{ij} + \hat{A}_{ji})^T}{2} Z \frac{(\hat{A}_{ij} + \hat{A}_{ji})}{2} & \frac{(\hat{A}_{ij} + \hat{A}_{ji})^T}{2} Z \frac{(\bar{A}_{\tau i} + \bar{A}_{\tau j})}{2} \\ \frac{(\bar{A}_{\tau i} + \bar{A}_{\tau j})^T}{2} Z \frac{(\hat{A}_{ij} + \hat{A}_{ji})}{2} & \frac{(\bar{A}_{\tau i} + \bar{A}_{\tau j})^T}{2} Z \frac{(\bar{A}_{\tau i} + \bar{A}_{\tau j})}{2} \end{bmatrix} \eta(t) \quad (24)$$

Following a similar development to that for theorem 1, we obtain

$$\begin{aligned} \dot{V}(x(t)) \leq & \sum_{i=1}^r \sum_{j=1}^r h_i h_j \eta(t)^T [\bar{\Phi}_{ij} + \tau W Z^{-1} W^T] \eta(t) \\ & - \int_{t-\tau(t)}^t [\eta(t)^T W + \dot{x}(s)^T Z] Z^{-1} [\eta(t)^T W + \dot{x}(s)^T Z]^T ds \end{aligned} \quad (25)$$

where  $\bar{\Phi}_{ij}$  is given by

$$\bar{\Phi}_{ij} = \begin{bmatrix} P\hat{A}_{ij} + \hat{A}_{ij}^T P + S & P\bar{A}_{\tau i} + \bar{\tau} \frac{(\hat{A}_{ij} + \hat{A}_{ji})^T}{2} Z \frac{(\bar{A}_{\tau i} + \bar{A}_{\tau j})}{2} \\ + \bar{\tau} \frac{(\hat{A}_{ij} + \hat{A}_{ji})^T}{2} Z \frac{(\hat{A}_{ij} + \hat{A}_{ji})}{2} + Y + Y^T & -Y + T^T \\ * & -(1-\beta)S + \bar{\tau} \frac{(\bar{A}_{\tau i} + \bar{A}_{\tau j})^T}{2} Z \frac{(\bar{A}_{\tau i} + \bar{A}_{\tau j})}{2} \\ & -T - T^T \end{bmatrix} \quad (26)$$

By applying Schur complement  $\sum_{i=1}^r \sum_{j=1}^r h_i h_j \tilde{\Phi}_{ij} + \bar{\tau} W Z^{-1} W^T < 0$  is equivalent to

$$\begin{aligned} \sum_{i=1}^r \sum_{j=1}^r h_i h_j \hat{\Phi}_{ij} &= \frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\hat{\Phi}_{ij} + \hat{\Phi}_{ji}) \\ &= \frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\tilde{\Phi}_{ij} + \tilde{\Phi}_{ji}) < 0 \end{aligned} \tag{27}$$

where  $\hat{\Phi}_{ij}$  is given by

$$\hat{\Phi}_{ij} = \begin{bmatrix} P\hat{A}_{ij} + \hat{A}_{ij}^T P + S + Y + Y^T & P\bar{A}_{\tau i} - Y + T^T & \frac{(\hat{A}_{ij} + \hat{A}_{ji})^T}{2} Z & -Y \\ * & -(1 - \beta)S - T - T^T & \frac{(\bar{A}_{\tau i} + \bar{A}_{\tau j})^T}{2} Z & -T \\ * & * & -\frac{1}{\bar{\tau}} Z & 0 \\ * & * & * & -\frac{1}{\bar{\tau}} Z \end{bmatrix} \tag{28}$$

Therefore, we get  $\dot{V}(x(t)) \leq 0$ .

Our objective is to transform the conditions in theorem 2 in LMI terms which can be easily solved using existing solvers such as LMI TOOLBOX in the Matlab software.

**Theorem 3.** For a given positive number  $\lambda$ . System (8) is asymptotically stable if there exist some matrices  $P > 0, S > 0, Z > 0, Y, T$  and  $N_i$  as well as positives scalars  $\epsilon_{Aij}, \epsilon_{A\tau ij}, \epsilon_{Bij}, \epsilon_{Ci}, \epsilon_{C\tau i}, \epsilon_{Di}$  satisfying the following LMIs for  $i, j = 1, 2, \dots, r$  and  $i \leq j$

$$\Xi_{ij} + \Xi_{ji} \leq 0 \tag{29}$$

where  $\Xi_{ij}$  is given by

$$\Xi_{ij} = \begin{bmatrix} \begin{bmatrix} \xi_{ij} + \epsilon_{Aij} M_{Ai} M_{Ai}^T \\ + \epsilon_{Bij} M_{Bi} M_{Bi}^T \end{bmatrix} & P A_{\tau i}^T - Y + T^T & A_i P + B_i N_j & -Y \\ * & \begin{bmatrix} -(1 - \beta)S - T - T^T \\ + \epsilon_{A\tau ij} M_{A\tau ij} M_{A\tau ij}^T \end{bmatrix} & A_{\tau i} P & \\ * & * & \frac{1}{\bar{\tau}} (-2\lambda P + \lambda^2 Z) & 0 \\ * & * & * & -\frac{1}{\bar{\tau}} Z \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \tag{30}$$

$$\begin{bmatrix} P E_{Ai}^T & N_j^T E_{Bi}^T & P E_{A\tau i}^T \\ -T & 0 & 0 \\ P E_{Ai}^T & N_j^T E_{Bi}^T & P E_{A\tau i}^T \\ 0 & 0 & 0 \\ -\epsilon_{Aij} I & 0 & 0 \\ * & -\epsilon_{Bij} I & 0 \\ * & * & -\epsilon_{A\tau ij} I \end{bmatrix}$$



in which  $\xi_{ij} = PA_i^T + N_j^T B_i^T + A_i P + B_i N_j + S + Y + Y^T$ . If this is the case, the  $K_i$  local feedback gains are given by

$$K_i = N_i P^{-1}, i = 1, 2, \dots, r \quad (31)$$

**Proof 3.** Starting with pre-and post multiplying (22) by  $\text{diag}[I, I, Z^{-1}P, I]$  and its transpose, we get

$$\Xi_{ij}^1 + \Xi_{ji}^1 \leq 0, \quad 1 \leq i \leq j \leq r \quad (32)$$

where

$$\Xi_{ij}^1 = \begin{bmatrix} P\hat{A}_{ij} + \hat{A}_{ij}^T P + S + Y + Y^T & P\bar{A}_{\tau i} - Y + T^T & \hat{A}_{ij}^T P & -Y \\ * & -(1-\beta)S - T - T^T & \bar{A}_{\tau i}^T P & -T \\ * & * & -\frac{1}{\tau} P Z^{-1} P & 0 \\ * & * & * & -\frac{1}{\tau} Z \end{bmatrix} \quad (33)$$

As pointed out by Wu et al. (Wu et al., 2004), if we just consider the stabilization condition, we can replace  $\hat{A}_{ij}$ ,  $A_{\tau i}$  with  $\hat{A}_{ij}^T$  and  $A_{\tau i}^T$ , respectively, in (33).

Assuming  $N_j = K_j P$ , we get

$$\Xi_{ij}^2 + \Xi_{ji}^2 \leq 0, \quad 1 \leq i \leq j \leq r \quad (34)$$

$$\Xi_{ij}^2 = \begin{bmatrix} \bar{\xi}_{ij} & P\bar{A}_{\tau i}^T - Y + T^T & \bar{A}_i P + \bar{B}_i N_j & -Y \\ * & \begin{bmatrix} -(1-\beta)S \\ -T - T^T \end{bmatrix} & \bar{A}_{\tau i} P & -T \\ * & * & -\frac{1}{\tau} P Z^{-1} P & 0 \\ * & * & * & -\frac{1}{\tau} Z \end{bmatrix} \quad (35)$$

It follows from lemma 1 that

$$-PZ^{-1}P \leq -2\lambda P + \lambda^2 Z \quad (36)$$

We obtain

$$\Xi_{ij}^3 + \Xi_{ji}^3 \leq 0, \quad 1 \leq i \leq j \leq r \quad (37)$$

where

$$\Xi_{ij}^3 = \begin{bmatrix} \bar{\xi}_{ij} & P\bar{A}_{\tau i}^T - Y + T^T & \bar{A}_i P + \bar{B}_i N_j & -Y \\ * & \begin{bmatrix} -(1-\beta)S \\ -T - T^T \end{bmatrix} & \bar{A}_{\tau i} P & -T \\ * & * & \begin{bmatrix} \frac{1}{\tau}(-2\lambda P \\ +\lambda^2 Z) \end{bmatrix} & 0 \\ * & * & * & -\frac{1}{\tau} Z \end{bmatrix} \quad (38)$$

The uncertain part is given by

$$\begin{aligned}
 \Delta \bar{\Xi}_{ij} &= \begin{bmatrix} P\Delta A_i^T + N_j^T \Delta B_i^T + \Delta A_i P + \Delta B_i N_j & P\Delta A_{\tau i}^T & \Delta A_i P + \Delta B_i N_j & 0 \\ * & 0 & \Delta A_{\tau i} P & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \\
 &= \begin{bmatrix} M_{Ai} \\ 0_{3 \times 1} \end{bmatrix} F(t) \begin{bmatrix} E_{Ai} P & 0 & E_{Ai} P & 0 \end{bmatrix} + (*) \\
 &\quad + \begin{bmatrix} M_{Bi} \\ 0_{3 \times 1} \end{bmatrix} F(t) \begin{bmatrix} E_{Bi} N_j & 0 & E_{Bi} N_j & 0 \end{bmatrix} + (*) \\
 &\quad + \begin{bmatrix} 0 \\ M_{A\tau i} \\ 0_{2 \times 1} \end{bmatrix} F(t) \begin{bmatrix} E_{A\tau i} P & 0 & E_{A\tau i} P & 0 \end{bmatrix} + (*) \tag{39}
 \end{aligned}$$

By using lemma 2, we obtain

$$\begin{aligned}
 \Delta \bar{\Xi}_{ij} &\leq \epsilon_{Aij} \begin{bmatrix} M_{Ai} \\ 0_{3 \times 1} \end{bmatrix} \begin{bmatrix} M_{Ai}^T & 0_{1 \times 3} \end{bmatrix} + \epsilon_{Aij}^{-1} \begin{bmatrix} P E_{Ai}^T \\ 0 \\ P E_{Ai}^T \\ 0 \end{bmatrix} \begin{bmatrix} E_i P & 0 & E_i P & 0 \end{bmatrix} \\
 &\quad + \epsilon_{Bij} \begin{bmatrix} M_{Bi} \\ 0_{3 \times 1} \end{bmatrix} \begin{bmatrix} M_{Bi}^T & 0_{1 \times 3} \end{bmatrix} + \epsilon_{Bij}^{-1} \begin{bmatrix} N_j^T E_{Bi}^T \\ 0 \\ N_j^T E_{Bi}^T \\ 0 \end{bmatrix} \begin{bmatrix} E_{Bi} N_j & 0 & E_{Bi} N_j & 0 \end{bmatrix} \\
 &\quad + \epsilon_{A\tau ij} \begin{bmatrix} 0 \\ M_{A\tau i} \\ 0_{2 \times 1} \end{bmatrix} \begin{bmatrix} 0 & M_{A\tau i}^T & 0_{1 \times 2} \end{bmatrix} + \epsilon_{A\tau ij}^{-1} \begin{bmatrix} P E_{A\tau i}^T \\ 0 \\ P E_{A\tau i}^T \\ 0 \end{bmatrix} \begin{bmatrix} E_{A\tau i} P & 0 & E_{A\tau i} P & 0 \end{bmatrix} \tag{40}
 \end{aligned}$$

where  $\epsilon_{Aij}$ ,  $\epsilon_{A\tau ij}$  and  $\epsilon_{Bij}$  are some positive scalars.

By applying Schur complement and lemma 2, we obtain theorem 3.

**Remark 1.** It is noticed that (Wu & Li, 2007) and theorem (3) contain, respectively,  $r^3 + r^3(r - 1)$  and  $\frac{1}{2}r(r + 1)$  LMIs. This reduces the computational complexity. Moreover, it is easy to see that the requirements of  $\beta < 1$  are removed in our result due to the introduction of variable  $T$ .

**Remark 2.** It is noted that Wu et al. in (Wu & Li, 2007) have presented a new approach to delay-dependent stabilization for continuous-time fuzzy systems with time varying delay. The disadvantages of this new approach is that the LMIs presented involve three tuning parameters. However, only one tuning parameter is involved in our approach.

**Remark 3.** Our method provides a less conservative result than other results which have been recently proposed (Wu & Li, 2007), (Chen & Liu, 2005a), (Guan & Chen, 2004). In next paragraph, a numerical example is given to demonstrate numerically this point.

#### 4. Illustrative examples

In this section, three examples are used to illustrate the effectiveness and the merits of the proposed results.

The first example is given to compare our result with the existing one in the case of constant delay and time-varying delay.

##### 4.1 Example 1

Consider the following T-S fuzzy model

$$\dot{x}(t) = \sum_{i=1}^2 h_i(x_1(t)) [(A_i + \Delta A_i)x(t) + (A_{\tau i} + \Delta A_{\tau i})x(t - \tau(t)) + B_i u(t)] \quad (41)$$

where

$$A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, A_{\tau 1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Delta A_i = MF(t)E_i, \Delta A_{\tau i} = MF(t)E_{\tau i}$$

$$M = \begin{bmatrix} -0.03 & 0 \\ 0 & 0.03 \end{bmatrix}$$

$$E_1 = E_2 = \begin{bmatrix} -0.15 & 0.2 \\ 0 & 0.04 \end{bmatrix}$$

$$E_{\tau 1} = E_{\tau 2} = \begin{bmatrix} -0.05 & -0.35 \\ 0.08 & -0.45 \end{bmatrix}$$

The membership functions are defined by

$$h_1(x_1(t)) = \frac{1}{1 + \exp(-2x_1(t))}$$

$$h_2(x_1(t)) = 1 - h_1(x_1(t)) \quad (42)$$

For the case of delay being constant and unknown and no uncertainties ( $\Delta A_i = 0, \Delta A_{\tau i} = 0$ ), the existing delay-dependent approaches are used to design the fuzzy controllers.

Based on theorem 3, for  $\lambda = 5$ , the largest delay is computed to be  $\bar{\tau} = 0.4909$  such that system (41) is asymptotically stable. Based on the results obtained in (Wu & Li, 2007), we get this table

Methods	Maximum allowed $\tau$
Theorem of Chen and Liu (Chen & Liu, 2005a)	0.1524
Theorem of Guan and Chen (Guan & Chen, 2004)	0.2302
Theorem of Wu and Li (Wu & Li, 2007)	0.2664
Theorem 3	0.4909

Table 1. Comparison Among Various Delay-Dependent Stabilization Methods

It appears from this table that our result improves the existing ones. Letting  $\bar{\tau} = 0.4909$ , the state-feedback gain matrices are

$$K_1 = [5.5780 \quad -16.4347], K_2 = [4.0442 \quad -15.4370]$$

Fig 1 shows the control results for system (41) with constant time-delay via fuzzy controller (7) with the previous gain matrices under the initial condition  $x(t) = [2 \ 0]^T, t \in [-0.4909 \ 0]$ .

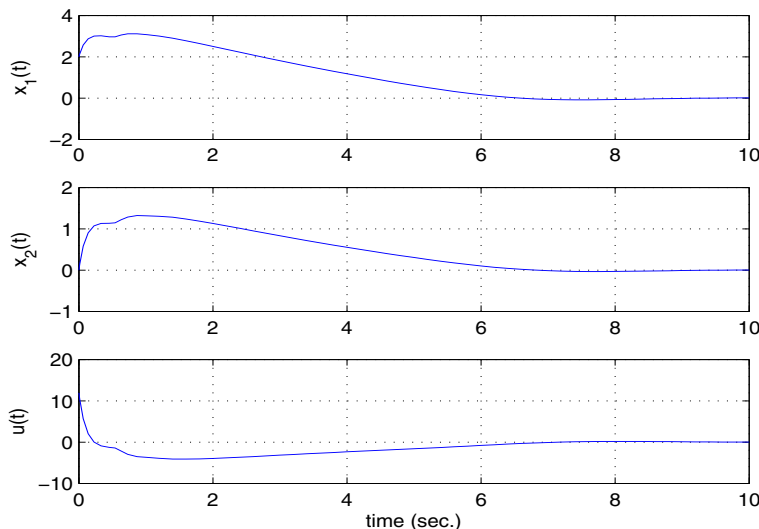


Fig. 1. Control results for system (41) without uncertainties and with constant time delay  $\tau = 0.4909$ .

It is clear that the designed fuzzy controller can stabilize this system.

For the case of  $\Delta A_i \neq 0, \Delta A_{\tau i} \neq 0$  and constant delay, the approaches in (Guan & Chen, 2004) (Wu & Li, 2007) (Lin et al., 2006) cannot be used to design feedback controllers as the system contains uncertainties. The method in (Chen & Liu, 2005b) and theorem 3 with  $\lambda = 5$  can be used to design the fuzzy controllers. The corresponding results are listed below.

Methods	Maximum allowed $\tau$
Theorem of Chen and Liu (Chen & Liu, 2005a)	0.1498
Theorem 3	0.4770

Table 2. Comparison Among Various Delay-Dependent Stabilization Methods With uncertainties

It appears from Table 2 that our result improves the existing ones in the case of uncertain T-S fuzzy model with constant time-delay.

For the case of uncertain T-S fuzzy model with time-varying delay, the approaches proposed in (Guan & Chen, 2004) (Chen & Liu, 2005a) (Wu & Li, 2007) (Chen et al., 2007) and (Lin et al., 2006) cannot be used to design feedback controllers as the system contains uncertainties and time-varying delay. By using theorem 3 with the choice of  $\lambda = 5, \tau(t) = 0.25 + 0.15 \sin(t) (\bar{\tau} = 0.4, \beta = 0.15)$ , we can obtain the following state-feedback gain matrices:

$$K_1 = [4.7478 \quad -13.5217], K_2 = [3.1438 \quad -13.2255]$$

The simulation was tested under the initial conditions  $x(t) = [2 \ 0]^T$ ,  $t \in [-0.4 \ 0]$  and uncertainty  $F(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}$ .

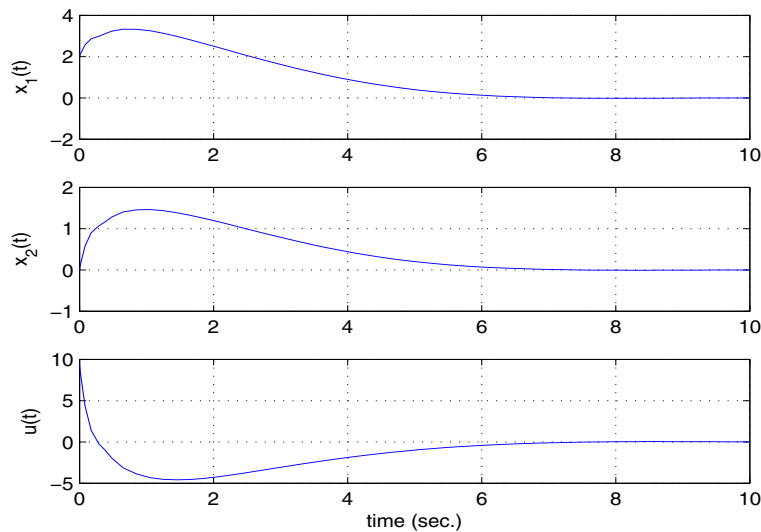


Fig. 2. Control results for system (41) with uncertainties and with time varying-delay  $\tau(t) = 0.25 + 0.15\sin(t)$

From the simulation results in figure 2, it can be clearly seen that our method offers a new approach to stabilize nonlinear systems represented by uncertain T-S fuzzy model with time-varying delay.

The second example illustrates the validity of the design method in the case of slow time varying delay ( $\beta < 1$ )

#### 4.2 Example 2: Application to control a truck-trailer

In this example, we consider a continuous-time truck-trailer system, as shown in Fig. 3.

We will use the delayed model given by (Chen & Liu, 2005a). It is assumed that  $\tau(t) = 1.10 + 0.75 \sin(t)$ . Obviously, we have  $\bar{\tau} = 1.85$ ,  $\beta = 0.75$ . The time-varying delay model with uncertainties is given by

$$\dot{x}(t) = \sum_{i=1}^2 h_i(x_1(t)) [(A_i + \Delta A_i)x(t) + (A_{\tau i} + \Delta A_{\tau i})x(t - \tau(t)) + (B_i + \Delta B_i)u(t)] \quad (43)$$

where

$$A_1 = \begin{bmatrix} -a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{v^2\bar{t}^2}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix}, A_{\tau 1} = \begin{bmatrix} -(1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{dv^2\bar{t}^2}{2Lt_0} & \frac{dv\bar{t}}{t_0} & 0 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} -(1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{dv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}$$

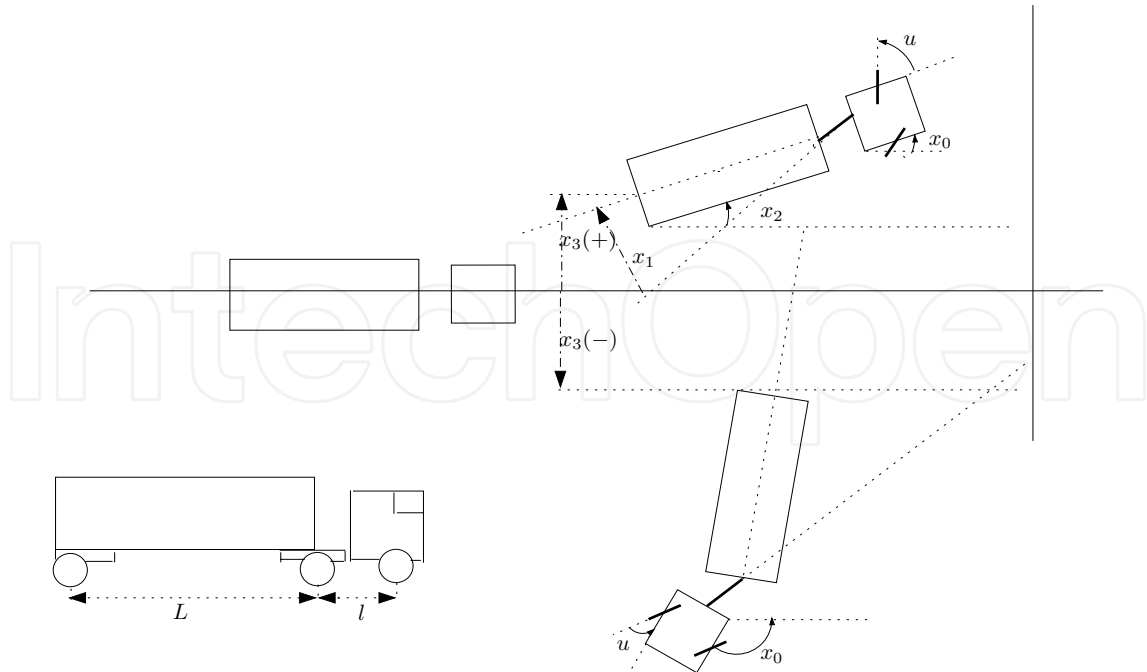


Fig. 3. Truck-trailer system

$$B_1 = B_2 = \left[ \frac{v\bar{t}}{lt_0} \ 0 \ 0 \right]^T$$

$$\Delta A_1 = \Delta A_2 = \Delta A_{\tau 1} = \Delta A_{\tau 2} = MF(t)E$$

with

$$M = [0.255 \ 0.255 \ 0.255]^T, \ E = [0.1 \ 0 \ 0]$$

$$\Delta B_1 = \Delta B_2 = M_b F(t) E_b$$

with

$$M_b = [0.1790 \ 0 \ 0]^T, \ E_{b1} = 0.05, \ E_{b2} = 0.15$$

where

$$l = 2.8, \ L = 5.5, \ v = -1, \ \bar{t} = 2, \ t_0 = 0.5, \ a = 0.7, \ d = \frac{10t_0}{\pi}$$

The membership functions are defined as

$$h_1(\theta(t)) = \left(1 - \frac{1}{1 + \exp(-3(\theta(t) - 0.5\pi))}\right) \times \left(\frac{1}{1 + \exp(-3(\theta(t) + 0.5\pi))}\right)$$

$$h_2(\theta(t)) = 1 - h_1$$

where

$$\theta(t) = x_2(t) + a(v\bar{t}/2L)x_1(t) + (1 - a)(v\bar{t}/2L)x_1(t - \tau(t))$$

By using theorem 3, with the choice of  $\lambda = 5$ , we can obtain the following feasible solution:

$$P = \begin{bmatrix} 0.2249 & 0.0566 & -0.0259 \\ 0.0566 & 0.0382 & 0.0775 \\ -0.0259 & 0.0775 & 2.7440 \end{bmatrix}, \ S = \begin{bmatrix} 0.2408 & -0.0262 & -0.1137 \\ -0.0262 & 0.0236 & 0.0847 \\ -0.1137 & 0.0847 & 0.3496 \end{bmatrix}$$

$$Z = \begin{bmatrix} 0.0373 & 0.0133 & -0.0052 \\ 0.0133 & 0.0083 & 0.0202 \\ -0.0052 & 0.0202 & 1.0256 \end{bmatrix}, T = \begin{bmatrix} 0.0134 & 0.0053 & 0.0256 \\ 0.0075 & 0.0038 & -0.0171 \\ 0.0001 & 0.0014 & 0.0642 \end{bmatrix}$$

$$Y = \begin{bmatrix} -0.0073 & -0.0022 & 0.0192 \\ -0.0051 & -0.0031 & 0.0096 \\ 0.0012 & -0.0012 & -0.0804 \end{bmatrix}$$

$$\epsilon_{A1} = 0.1087, \epsilon_{A2} = 0.0729, \epsilon_{A12} = 0.1184$$

$$\epsilon_{A\tau1} = 0.0443, \epsilon_{A\tau2} = 0.0369, \epsilon_{A\tau12} = 0.0432$$

$$\epsilon_{B1} = 0.3179, \epsilon_{B2} = 0.3383, \epsilon_{B12} = 0.3250$$

$$K_1 = [3.7863 \quad -5.7141 \quad 0.1028]$$

$$K_2 = [3.8049 \quad -5.8490 \quad 0.0965]$$

The simulation was carried out for an initial condition  $x(t) = [-0.5\pi \quad 0.75\pi \quad -5]^T$ ,  $t \in [-1.85 \quad 0]$ .

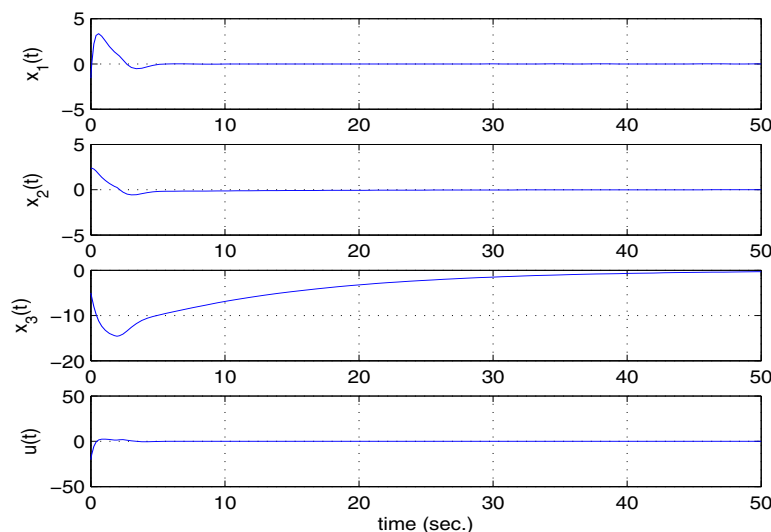


Fig. 4. Control results for the truck-trailer system (41)

The third example is presented to illustrate the effectiveness of the proposed main result for fast time-varying delay system.

#### 4.3 Example 3: Application to an inverted pendulum

Consider the well-studied example of balancing an inverted pendulum on a cart (Cao et al., 2000).

$$\dot{x}_1 = x_2 \tag{44}$$

$$\dot{x}_2 = \frac{g \sin(x_1) - amlx_2^2 \sin(2x_1)/2 - a \cos(x_1)u}{4l/3 - aml \cos^2(x_1)} \tag{45}$$

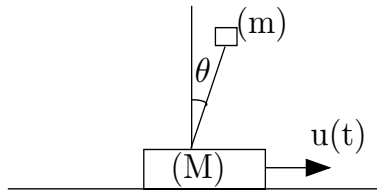


Fig. 5. Inverted pendulum

where  $x_1$  is the pendulum angle (represented by  $\theta$  in Fig. 5), and  $x_2$  is the angular velocity ( $\dot{\theta}$ ).  $g = 9.8m/s^2$  is the gravity constant,  $m$  is the mass of the pendulum,  $M$  is the mass of the cart,  $2l$  is the length of the pendulum and  $u$  is the force applied to the cart.  $a = 1/(m + M)$ . The nonlinear system can be described by a fuzzy model with two IF-THEN rules:

Plant Rule 1: IF  $x_1$  is about 0, Then

$$\dot{x}(t) = A_1x(t) + B_1u(t) \quad (46)$$

Plant rule 2: IF  $x_1$  is about  $\pm \frac{\pi}{2}$ , Then

$$\dot{x}(t) = A_2x(t) + B_2u(t) \quad (47)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 12.6305 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix}$$

The membership functions are

$$h_1 = \left(1 - \frac{1}{1 + \exp(-7(x_1 - \pi/4))}\right) \times \left(1 + \frac{1}{1 + \exp(-7(x_1 + \pi/4))}\right)$$

$$h_2 = 1 - h_1$$

In order to illustrate the use of theorem (3), we assume that the delay terms are perturbed along values of the scalar  $s \in [0, 1]$ , and the fuzzy time-delay model considered here is as follows:

$$\dot{x}(t) = \sum_{i=1}^r h_i [((1-s)A_i + \Delta A_i)x(t) + (sA_{\tau i} + \Delta A_{\tau i})x(t - \tau(t)) + B_i u(t)] \quad (48)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 12.6305 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix}$$

$$\Delta A_1 = \Delta A_2 = \Delta A_{\tau 1} = \Delta A_{\tau 2} = MF(t)E$$



with

$$M = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}^T, E = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}$$

Let  $s = 0.1$  and uncertainty  $F(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}$ . We consider a fast time-varying delay  $\tau(t) = 0.2 + 1.2 |\sin(t)|$  ( $\beta = 1.2 > 1$ ).

Using LMI-TOOLBOX, there is a set of feasible solutions to LMIs (29).

$$K_1 = [159.7095 \ 30.0354], K_2 = [347.2744 \ 78.5552]$$

Fig. 4 shows the control results for the system (48) with time-varying delay  $\tau(t) = 0.2 + 1.2 |\sin(t)|$  under the initial condition  $x(t) = [2 \ 0]^T, t \in [-1.40 \ 0]$ .

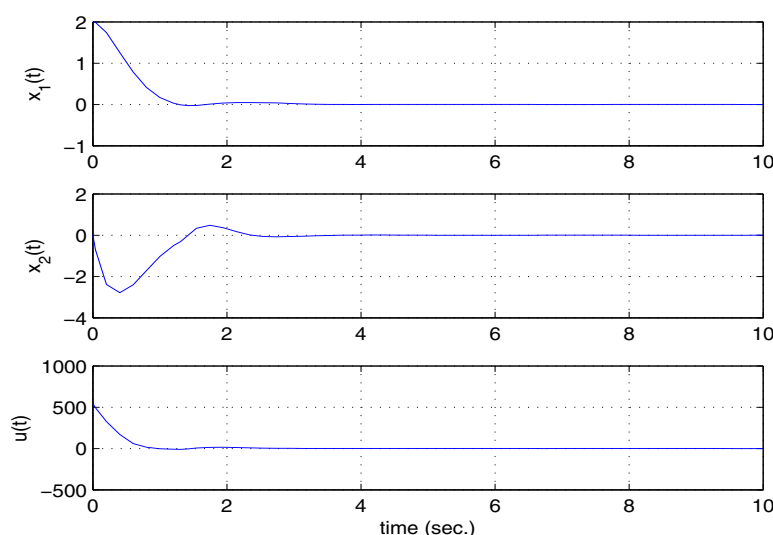


Fig. 6. Control results for the system (48) with time-varying delay  $\tau(t) = 0.2 + 1.2 |\sin(t)|$ .

## 5. Conclusion

In this chapter, we have investigated the delay-dependent design of state feedback stabilizing fuzzy controllers for uncertain T-S fuzzy systems with time varying delay. Our method is an important contribution as it establishes a new way that can reduce the conservatism and the computational efforts in the same time. The delay-dependent stabilization conditions obtained in this chapter are presented in terms of LMIs involving a single tuning parameter. Finally, three examples are used to illustrate numerically that our results are less conservative than the existing ones.

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Robust control has been a topic of active research in the last three decades culminating in  $H_2/H_\infty$  and  $\mu$  design methods followed by research on parametric robustness, initially motivated by Kharitonov's theorem, the extension to non-linear time delay systems, and other more recent methods. The two volumes of Recent Advances in Robust Control give a selective overview of recent theoretical developments and present selected application examples. The volumes comprise 39 contributions covering various theoretical aspects as well as different application areas. The first volume covers selected problems in the theory of robust control and its application to robotic and electromechanical systems. The second volume is dedicated to special topics in robust control and problem specific solutions. Recent Advances in Robust Control will be a valuable reference for those interested in the recent theoretical advances and for researchers working in the broad field of robotics and mechatronics.

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