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# PI/PID Control for Nonlinear Systems via Singular Perturbation Technique

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## 1. Introduction

The problem of output regulation for nonlinear time-varying control systems under uncertainties is one of particular interest for real-time control system design. There is a broad set of practical problems in the control of aircraft, robotics, mechatronics, chemical industry, electrical and electro-mechanical systems where control systems are designed to provide the following objectives: (i) robust zero steady-state error of the reference input realization; (ii) desired output performance specifications such as overshoot, settling time, and system type of reference model for desired output behavior; (iii) insensitivity of the output transient behavior with respect to unknown external disturbances and varying parameters of the system.

In spite of considerable advances in the recent control theory, it is common knowledge that PI and PID controllers are most widely and successfully used in industrial applications (Morari & Zafiriou, 1999). A great attention of numerous researchers during the last few decades was devoted to turning rules (Åström & Hägglund, 1995; O'Dwyer, 2003; Ziegler & Nichols, 1942), identification and adaptation schemes (Li et al., 2006) in order to fetch out the best PI and PID controllers in accordance with the assigned design objectives. The most recent results have concern with the problem of PI and PID controller design for linear systems. However, various design technics of integral controllers for nonlinear systems were discussed as well (Huang & Rugh, 1990; Isidori & Byrnes, 1990; Khalil, 2000; Mahmoud & Khalil, 1996). The main disadvantage of existence design procedures of PI or PID controllers is that the desired transient performances in the closed-loop system can not be guaranteed in the presence of nonlinear plant parameter variations and unknown external disturbances. The lack of clarity with regard to selection of sampling period and parameters of discrete-time counterparts for PI or PID controllers is the other disadvantage of the current state of this question.

The output regulation problem under uncertainties can be successfully solved via such advanced technics as control systems with sliding motions (Utkin, 1992; Young & Özgüner, 1999), control systems with high gain in feedback (Meerov, 1965; Young et al., 1977). A set of examples can be found from mechanical applications and robotics where acceleration feedback control is successfully used (Krutko, 1988; 1991; 1995; Lun et al., 1980; Luo et al., 1985; Studenny & Belanger, 1984; 1986). The generalized approach to nonlinear control system design based on control law with output derivatives and high gain in feedback, where integral action can be incorporated in the controller, is developed as well and one is used

effectively under uncertainties (Błachuta et al., 1997; 1999; Czyba & Błachuta, 2003; Yurkevich, 1995; 2004). The distinctive feature of such advanced technics of control system design is the presence of two-time-scale motions in the closed-loop system. Therefore, a singular perturbation method (Kokotović et al., 1976; 1999; Kokotović & Khalil, 1986; Naidu & Calise, 2001; Naidu, 2002; Saksena et al., 1984; Tikhonov, 1948; 1952) should be used for analysis of closed-loop system properties in such systems.

The goal of the chapter is to give an overview in tutorial manner of the newest unified design methodology of PI and PID controllers for continuous-time or discrete-time nonlinear control systems which guarantees desired transient performances in the presence of plant parameter variations and unknown external disturbances. The chapter presents the up-to-date coverage of fundamental issues and recent research developments in singular perturbation technique of nonlinear control system design. The discussed control law structures are an extension of PI/PID control scheme. The proposed design methodology allows to provide effective control of nonlinear systems on the assumption of uncertainty, where a distinctive feature of the designed control systems is that two-time-scale motions are artificially forced in the closed-loop system. Stability conditions imposed on the fast and slow modes, and a sufficiently large mode separation rate, can ensure that the full-order closed-loop system achieves desired properties: the output transient performances are as desired, and they are insensitive to parameter variations and external disturbances. PI/PID control design methodology for continuous-time control systems, as well as corresponding discrete-time counterpart, is discussed in the paper. The method of singular perturbations is used to analyze the closed-loop system properties throughout the chapter.

The chapter is organized as follows. First, some preliminary results concern with properties of singularly perturbed systems are discussed. Second, the application of the discussed design methodology for a simple model of continuous-time single-input single-output nonlinear system is presented and main steps of the design method are explained. The relationship of the presented design methodology with problem of PI and PID controllers design for nonlinear systems is explained. Third, the discrete-time counterpart of the discussed design methodology for sampled-data control systems design is highlighted. Numerical examples with simulation results are included as well.

The main impact of the chapter is the presentation of the unified approach to continuous as well as digital control system design that allows to guarantee the desired output transient performances in the presence of plant parameter variations and unknown external disturbances. The discussed design methodology may be used for a broad class of nonlinear time-varying systems on the assumption of incomplete information about varying parameters of the plant model and unknown external disturbances. The advantage of the discussed singular perturbation technique for closed-loop system analysis is that analytical expressions for parameters of PI, PID, or PID controller with additional lowpass filtering can be found for nonlinear systems, where controller parameters depend explicitly on the specifications of the desired output behavior.

## **2. Singularly perturbed systems**

### **2.1 Continuous-time singularly perturbed systems**

The singularly perturbed dynamical control systems arise in various applications mainly due to two reasons. The first one is that fast dynamics of actuators or sensors leads to the plant

model in the form of singularly perturbed system (Kokotović et al., 1976; Naidu & Calise, 2001; Naidu, 2002; Saksena et al., 1984). The second one is that the singularly perturbed dynamical systems can also appear as the result of a high gain in feedback (Meerov, 1965; Young et al., 1977). In accordance with the second one, a distinctive feature of the discussed control systems in this chapter is that two-time-scale motions are artificially forced in the closed-loop control system due to an application of a fast dynamical control law or high gain parameters in feedback.

The main notions of singularly perturbed systems can be considered based on the following continuous-time system:

$$\dot{X} = f(X, Z), \quad (1)$$

$$\mu \dot{Z} = g(X, Z), \quad (2)$$

where  $\mu$  is a small positive parameter,  $X \in \mathbb{R}^n$ ,  $Z \in \mathbb{R}^m$ , and  $f$  and  $g$  are continuously differentiable functions of  $X$  and  $Z$ . The system (1)–(2) is called the standard singularly perturbed system (Khalil, 2002; Kokotović et al., 1976; 1999; Kokotović & Khalil, 1986).

From (1)–(2) we can get the fast motion subsystem (FMS) given by

$$\mu \frac{dZ}{dt} = g(X, Z) \quad (3)$$

as  $\mu \rightarrow 0$  where  $X(t)$  is the frozen variable. Assume that

$$\det \left\{ \frac{\partial g(X, Z)}{\partial Z} \right\} \neq 0 \quad (4)$$

for all  $Z \in \Omega_Z$  where  $\Omega_Z$  is the specified bounded set  $\Omega_Z \subset \mathbb{R}^m$ .

From (4) it follows that the function  $\bar{Z} = \psi(X)$  exists such that  $g(X(t), \bar{Z}(t)) = 0 \forall t$  holds where  $\bar{Z}$  is an isolated equilibrium point of (3). Assume that the equilibrium point  $\bar{Z}$  is unique and one is stable (exponentially stable).

After the fast damping of transients in the FMS (3), the state space vector of the system (1)–(2) belong to slow-motion manifold (SMM) given by

$$M_{smm} = \{(X, Z) : g(X, Z) = 0\}.$$

By taking  $\mu = 0$ , from (1)–(2), the slow motion subsystem (SMS) (or a so-called reduced system) follows in the form

$$\dot{X} = f(X, \psi(X)).$$

## 2.2 Discrete-time singularly perturbed systems

Let us consider the system of difference equations given by

$$X_{k+1} = \{I_n + \mu A_{11}\} X_k + \mu A_{12} Y_k, \quad (5)$$

$$Y_{k+1} = A_{21} X_k + A_{22} Y_k, \quad (6)$$

where  $\mu$  is the small positive parameter,  $X \in \mathbb{R}^n$ ,  $Y \in \mathbb{R}^m$ , and the  $A_{ij}$  are matrices with appropriate dimensions.

If  $\mu$  is sufficiently small, then from (5)–(6) the FMS equation

$$Y_{k+1} = A_{21}X_k + A_{22}Y_k \quad (7)$$

results, where  $X_{k+1} - X_k \approx 0$  (that is  $X_k \approx \text{const}$ ) during the transients in the system (7). Assume that the FMS (7) is stable. Then the steady-state of the FMS is given by

$$Y_k = \{I_m - A_{22}\}^{-1} A_{21}X_k. \quad (8)$$

Substitution of (8) into (5) yields the SMS

$$X_{k+1} = \{I_n + \mu[A_{11} + A_{12}(I_m - A_{22})^{-1}A_{21}]\}X_k.$$

The main qualitative property of the singularly perturbed systems is that: if the equilibrium point of the FMS is stable (exponentially stable), then there exists  $\mu^* > 0$  such that for all  $\mu \in (0, \mu^*)$ , the trajectories of the singularly perturbed system approximate to the trajectories of the SMS (Hoppensteadt, 1966; Klimushchev & Krasovskii, 1962; Litkouhi & Khalil, 1985; Tikhonov, 1948; 1952). This property is important both from a theoretical viewpoint and for practical applications in control system analysis and design, in particular, that will be used throughout the discussed below design methodology for continuous-time or sampled-data nonlinear control systems.

### 3. PI controller of the 1-st order nonlinear system

#### 3.1 Control problem statement

Consider a nonlinear system of the form

$$\frac{dx}{dt} = f(x, w) + g(x, w)u, \quad (9)$$

where  $t$  denotes time,  $t \in [0, \infty)$ ,  $y = x$  is the measurable output of the system (9),  $x \in \mathbb{R}^1$ ,  $u$  is the control,  $u \in \Omega_u \subset \mathbb{R}^1$ ,  $w$  is the vector of unknown bounded external disturbances or varying parameters,  $w \in \Omega_w \subset \mathbb{R}^l$ ,  $\|w(t)\| \leq w_{max} < \infty$ , and  $w_{max} > 0$ .

We assume that  $dw/dt$  is bounded for all its components,

$$\|dw/dt\| \leq \bar{w}_{max} < \infty,$$

and that the conditions

$$0 < g_{min} \leq g(x, w) \leq g_{max} < \infty, \quad |f(x, w)| \leq f_{max} < \infty \quad (10)$$

are satisfied for all  $(x, w) \in \Omega_{x,w}$ , where  $f(x, w)$ ,  $g(x, w)$  are unknown continuous bounded functions of  $x(t)$ ,  $w(t)$  on the bounded set  $\Omega_{x,w}$  and  $\bar{w}_{max} > 0$ ,  $g_{min} > 0$ ,  $g_{max} > 0$ ,  $f_{max} > 0$ . Note,  $g(x, w)$  is the so called a high-frequency gain of the system (9).

A control system is being designed so that

$$\lim_{t \rightarrow \infty} e(t) = 0, \quad (11)$$

where  $e(t)$  is an error of the reference input realization,  $e(t) := r(t) - y(t)$ ,  $r(t)$  is the reference input, and  $y = x$ . Moreover, the output transients should have the desired performance indices. These transients should not depend on the external disturbances and varying parameters of the system (9).

Throughout the chapter a controller is designed in such a way that the closed-loop system is required to be close to some given reference model, despite the effects of varying parameters and unknown external disturbances  $w(t)$  in the plant model. So, the destiny of the controller is to provide an appropriate reference input-controlled output map of the closed-loop system as shown in Fig. 1, where the reference model is selected based on the required output transient performance indices.

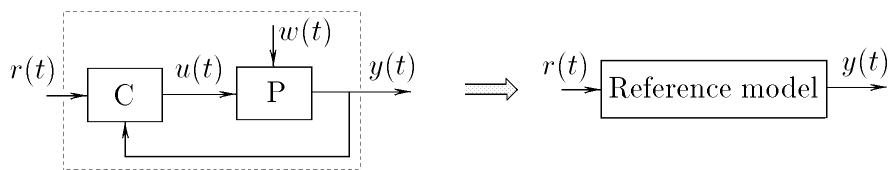


Fig. 1. Block diagram of the closed-loop control system

### 3.2 Insensitivity condition

Let us consider the reference equation of the desired behavior for (9) in the form of the 1st order stable differential equation given by

$$\frac{dx}{dt} = \frac{1}{T}(r - x), \quad (12)$$

which corresponds to the desired transfer function

$$G^d(s) = \frac{1}{Ts + 1},$$

where  $y = x = r$  at the equilibrium point for  $r = \text{const}$  and the time constant  $T$  is selected in accordance with the desired settling time of output transients.

Let us denote  $F(x, r) := (r - x)/T$  and rewrite (12) as

$$\frac{dx}{dt} = F(x, r), \quad (13)$$

where  $F(x, r)$  is the desired value of  $\dot{x}$  for (9),  $\dot{x} := dx/dt$ . Hence, the deviation of the actual behavior of (9) from the desired behavior prescribed by (12) can be defined as the difference

$$e_F := F(x, r) - \frac{dx}{dt}. \quad (14)$$

Accordingly, if the condition

$$e_F = 0 \quad (15)$$

holds, then the behavior of  $x(t)$  with prescribed dynamics of (13) is fulfilled. The expression (15) is an insensitivity condition for the behavior of the output  $x(t)$  with respect to the external disturbances and varying parameters of the system (9).

Substitution of (9), (13), and (14) into (15) yields

$$F(x, r) - f(x, w) - g(x, w)u = 0. \quad (16)$$

So, the requirement (11) has been reformulated as a problem of finding a solution of the equation  $e_f(u) = 0$  when its varying parameters are unknown. From (16) we get  $u = u^{id}$ , where

$$u^{id} = [g(x, w)]^{-1}[F(x, r) - f(x, w)] \quad (17)$$

and  $u^{id}(t)$  is the analytical solution of (16). The control function  $u(t) = u^{id}(t)$  is called a solution of the nonlinear inverse dynamics (id) (Boyчук, 1966; Porter, 1970; Slotine & Li, 1991). It is clear that the control law in the form of (17) is useless in practice under uncertainties, as far as one may be used only if complete information is available about the disturbances, model parameters, and state of the system (9).

Note, the nonlinear inverse dynamics solution is used in such known control design methodologies as exact state linearization method, dynamic inversion, the computed torque control in robotics, etc (Qu et al., 1991; Slotine & Li, 1991).

### 3.3 PI controller

The subject of our consideration is the problem of control system design given that the functions  $f(x, w)$ ,  $g(x, w)$  are unknown and the vector  $w(t)$  of bounded external disturbances or varying parameters is unavailable for measurement. In order to reach the discussed control goal and, as a result, to provide desired dynamical properties of  $x(t)$  in the specified region of the state space of the uncertain nonlinear system (9), consider the following control law:

$$\mu \frac{du}{dt} = k_0 \left\{ \frac{1}{T}(r - x) - \frac{dx}{dt} \right\}, \quad (18)$$

where  $\mu$  is a small positive parameter. The discussed control law (18) may be expressed in terms of transfer functions, that is the structure of the conventional PI controller

$$u(s) = \frac{k_0}{\mu Ts} [r(s) - x(s)] - \frac{k_0}{\mu} x(s). \quad (19)$$

For purposes of numerical simulation or practical implementation, let us rewrite the control law (18) in the state-space form. Denote

$$b_1 = -\frac{k_0}{\mu}, \quad b_0 = -\frac{k_0}{\mu T}, \quad c_0 = \frac{k_0}{\mu T}.$$

Then, (18) can be rewritten as  $u^{(1)} = b_1 x^{(1)} + b_0 x + c_0 r$ . Hence, the following expression  $u^{(1)} - b_1 x^{(1)} = b_0 x + c_0 r$  results. Denote  $u_1^{(1)} = b_0 x + c_0 r$ . Finally, we obtain the equations of the controller given by

$$\begin{aligned} \dot{u}_1 &= b_0 x + c_0 r, \\ u &= u_1 + b_1 x. \end{aligned} \quad (20)$$

The block diagram of PI controller (20) is shown in Fig. 2(a).

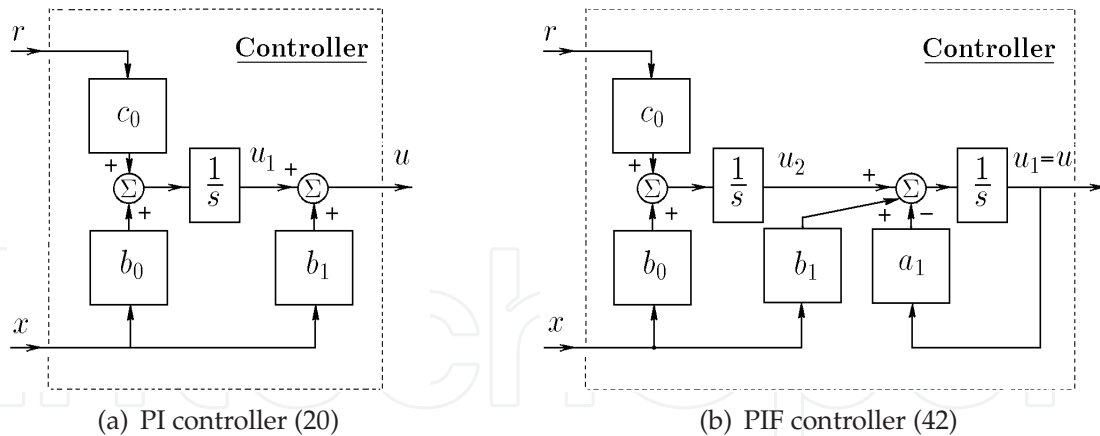


Fig. 2. Block diagrams of PI and PIF controllers

**3.4 Two-time-scale motion analysis**

In accordance with (9) and (18), the equations of the closed-loop system are given by

$$\frac{dx}{dt} = f(x, w) + g(x, w)u, \tag{21}$$

$$\mu \frac{du}{dt} = k_0 \left\{ \frac{1}{T}(r - x) - \frac{dx}{dt} \right\}. \tag{22}$$

Substitution of (21) into (22) yields the closed-loop system equations in the form

$$\frac{dx}{dt} = f(x, w) + g(x, w)u, \tag{23}$$

$$\mu \frac{du}{dt} = -k_0 g(x, w)u + k_0 \left\{ \frac{1}{T}(r - x) - f(x, w) \right\}. \tag{24}$$

Since  $\mu$  is the small positive parameter, the closed-loop system equations (23)–(24) have the standard singular perturbation form given by (1)–(2). If  $\mu \rightarrow 0$ , then fast and slow modes are artificially forced in the system (23)–(24) where the time-scale separation between these modes depends on the parameter  $\mu$ . Accordingly, the singular perturbation method (Kokotović et al., 1976; 1999; Kokotović & Khalil, 1986; Naidu & Calise, 2001; Naidu, 2002; Saksena et al., 1984; Tikhonov, 1948; 1952) may be used to analyze the closed-loop system properties.

From (23)–(24), we obtain the FMS given by

$$\mu \frac{du}{dt} + k_0 g(x, w)u = k_0 \left[ \frac{1}{T}(r - x) - f(x, w) \right], \tag{25}$$

where  $x(t)$  and  $w(t)$  are treated as the frozen variables during the transients in (25).

In accordance with the assumption (10), the gain  $k_0$  can be selected such that the condition  $g(x, w)k_0 > 0$  holds for all  $(x, w) \in \Omega_{x,w}$ , then the FMS is stable and, after the rapid decay of transients in (25), we have the steady state (more precisely, quasi-steady state) for the FMS (25), where  $u(t) = u^{id}(t)$  and  $u^{id}(t)$  is given by (17). Hence, if the steady state of the FMS (25) takes place, then the closed-loop system equations (23)–(24) imply that

$$\frac{dx}{dt} = \frac{1}{T}(r - x)$$



is the equation of the SMS, which is the same as the reference equation (12).

So, if a sufficient time-scale separation between the fast and slow modes in the closed-loop system and exponential convergence of FMS transients to equilibrium are provided, then after the damping of fast transients the desired output behavior prescribed by (12) is fulfilled despite that  $f(x, w)$  and  $g(x, w)$  are unknown complex functions of  $x(t)$  and  $w(t)$ . Thus, the output transient performance indices are insensitive to parameter variations of the nonlinear system and external disturbances, by that the solution of the discussed control problem (11) is maintained.

### 3.5 Selection of PI controller parameters

The time constant  $T$  of the reference equation (12) is selected in accordance with the desired settling time of output transients. Take the gain  $k_0 \approx g^{-1}(x, w)$ . Then, in accordance with (25), the FMS characteristic polynomial is given by  $\mu s + 1$ . The time constant  $\mu$  is selected as  $\mu = T/\eta$  where  $\eta$  is treated as the degree of time-scale separation between the fast and slow modes in the closed-loop system, for example,  $\eta \geq 10$ .

### 3.6 Example 1

Consider the nonlinear system given by

$$\dot{x} = x^3 - (2 + x^2)u, \quad (26)$$

which is accompanied by the discussed PI controller (18). Substitution of (26) into (18) yields the singularly perturbed differential equations of the closed-loop system

$$\dot{x} = x^3 - (2 + x^2)u, \quad (27)$$

$$\mu \dot{u} = k_0[(r - x)/T - x^3 + (2 + x^2)u], \quad (28)$$

where fast and slow modes are forced as  $\mu \rightarrow 0$ . From (27)-(28), the FMS

$$\mu \dot{u} - k_0(2 + x^2)u = k_0[(r - x)/T - x^3] \quad (29)$$

follows, where  $x$  is treated as the frozen parameter during the transients in (29).

Take  $k_0 = -0.5 < 0$ , then the transients of (29) are exponentially stable and the unique exponentially stable isolated equilibrium point  $u_{id}$  of the FMS (29) is given by

$$u_{id} = (2 + x^2)^{-1}[(r - x)/T - x^3]. \quad (30)$$

Substitution of  $\mu = 0$  into (27)-(28) yields the equation of the SMS which is the same as the reference equation (12).

Note, at the equilibrium point of the FMS (29), the state of the closed-loop system (27)-(28) belongs to the slow-motion manifold (SMM) given by

$$M_{smm} = \{(x, u) : (r - x)/T - x^3 + (2 + x^2)u = 0\}, \quad (31)$$

which is the attractive manifold when the FMS (29) is stable and the behavior of  $x(t)$  on the SMM is described by (12).

The phase portrait of (27),(28) in case of  $r(t) \equiv 1$  and the output response of (20),(26) are shown in Fig. 3, where the simulation has been done for  $T = 1$ ,  $\mu = 0.05$  s,  $k_0 = -0.5$ . It is

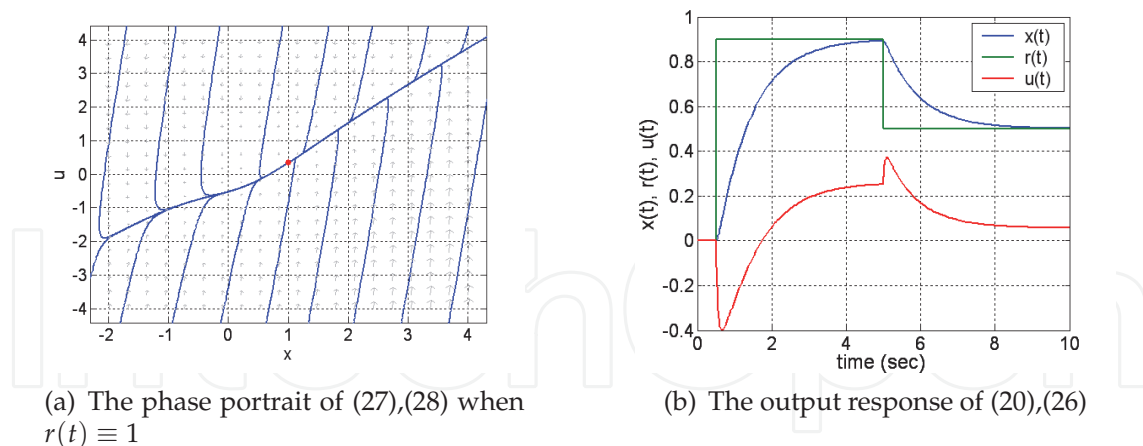


Fig. 3. The phase portrait and output response of the closed-loop system in Example 1

easy to see from Fig. 3(a), there is fast transition of the closed-loop system state trajectories on the SMM (31) where the motions along this manifold correspond to the SMS given by (12). Hence, after the damping of fast transients, the condition  $x(t) \rightarrow r = \text{const}$  holds due to (12) for arbitrary initial conditions, that is the output stabilization of (26), where the desired settling time is defined by selection of the parameter  $T$ . The output response of the closed-loop system (20),(26) provided for initial conditions at origin reveals the transients behavior of the reference equation given by (12) as shown in Fig. 3(b).

#### 4. PIF controller of the 1-st order nonlinear system

##### 4.1 High-frequency sensor noise attenuation

Consider the nonlinear system (9) in presence of high-frequency sensor noise  $n_s(t)$ , that is

$$\frac{dx}{dt} = f(x, w) + g(x, w)u, \quad \hat{y} = x + n_s, \quad y = x, \tag{32}$$

where the sensor output  $\hat{y}(t)$  is corrupted by a zero-mean, high-frequency measurement noise  $n_s(t)$ . Hence, instead of (21)-(22), we get of the closed-loop system given by

$$\frac{dx}{dt} = f(x, w) + g(x, w)u, \quad \hat{y} = x + n_s, \tag{33}$$

$$\mu \frac{du}{dt} = k_0 \left\{ \frac{1}{T}(r - \hat{y}(t)) - \frac{d\hat{y}(t)}{dt} \right\}. \tag{34}$$

The main disadvantage of the sensor noise  $n_s(t)$  in the closed-loop system is that it leads to high-frequency chattering in the control variable  $u(t)$ . At the same time, the effect of the high-frequency noise  $n_s(t)$  on the behavior of the output variable  $y(t)$  is much smaller since the system (32) rejects high frequencies.

From the closed-loop system equations given by (33)-(34), the FMS equation

$$\mu \dot{u} + k_0 g(x, w)u = k_0 \left\{ \frac{1}{T}(r - x) - f(x, w) - \frac{1}{T}n_s - \dot{n}_s \right\} \tag{35}$$

results, where  $x(t)$  and  $w(t)$  are treated as the frozen variables during the transients in (35). From (35), we obtain the transfer function  $G_{un_s}(s) = u(s)/n_s(s)$ , that is

$$G_{un_s}(s) = -\frac{k_0}{T} \frac{Ts + 1}{\mu s + k_0 g},$$

where

$$\lim_{\omega \rightarrow \infty} |G_{un_s}(j\omega)| = k_0/\mu. \quad (36)$$

The transfer function  $G_{un_s}(s)$  determines the sensitivity of the plant input  $u(t)$  to the sensor noise signal  $n_s(t)$  in the closed-loop system. In other words,  $G_{un_s}(s)$  is an input sensitivity function with respect to noise  $n_s(t)$  in the closed-loop system. The requirement on high-frequency sensor noise attenuation can be expressed by the following inequality:

$$|G_{un_s}(j\omega)| \leq \varepsilon_{un_s}(\omega), \quad \forall \quad \omega \geq \omega_{min}^{n_s}, \quad (37)$$

where  $\varepsilon_{un_s}(\omega)$  is an upper bound on the amplitude of the input sensitivity function with respect to noise for high frequencies.

In order to provide a high-frequency measurement noise attenuation assigned by (37), we can consider, instead of (18), the control law given by

$$\mu^2 \ddot{u} + d_1 \mu \dot{u} = k_0 \left\{ \frac{1}{T} (r - \hat{y}) - \dot{\hat{y}} \right\}, \quad (38)$$

which can also be expressed in terms of transfer functions as

$$u(s) = \frac{k_0}{\mu(\mu s + d_1)} \left\{ \frac{1}{Ts} [r(s) - \hat{y}(s)] - \hat{y}(s) \right\}.$$

that is, in compare with (19), the structure of PI controller with additional lowpass filtering (PIF controller).

The way for two-time-scale motion analysis in the closed-loop system is the same as it was shown above. Hence, from the closed-loop system equations given by (32) and (38), the FMS equation

$$\mu^2 \ddot{u} + d_1 \mu \dot{u} + k_0 g(x, w) u = k_0 \left\{ \frac{1}{T} (r - x) - f(x, w) - \frac{1}{T} n_s - \dot{n}_s \right\} \quad (39)$$

results, where  $x(t)$  and  $w(t)$  are treated as the frozen variables during the transients in (39). Accordingly, from (39), the transfer function

$$G_{un_s}(s) = -\frac{k_0}{T} \frac{Ts + 1}{\mu^2 s^2 + d_1 \mu s + k_0 g}$$

results, where

$$|G_{un_s}(j\omega)| = \frac{|k_0|}{T} \frac{\sqrt{(T\omega)^2 + 1}}{\sqrt{(k_0 g - \mu^2 \omega^2)^2 + (d_1 \mu \omega)^2}}. \quad (40)$$

Note, in contrast to (36), we have

$$\lim_{\omega \rightarrow \infty} |G_{un_s}(j\omega)| = 0.$$

So, the high-frequency measurement noise attenuation is provided in case of control law given by (38). The amplitude of high-frequency oscillations induced in behavior of the control variable  $u(t)$  due to effect of the high-frequency harmonic measurement noise can be calculated by (40).

#### 4.2 Selection of PIF controller parameters

The time constant  $T$  of the reference equation (12) is selected in accordance with the desired settling time of output transients. Take the gain  $k_0 \approx g^{-1}(x, w)$  and parameter  $d_1 = 2$ . Then, in accordance with (39), the FMS characteristic polynomial is given by  $(\mu s + 1)^2$ . The time constant  $\mu$  is selected as  $\mu = T/\eta$  where  $\eta$  is treated as the degree of time-scale separation between the fast and slow modes in the closed-loop system, for example,  $\eta \geq 10$ .

#### 4.3 Implementation of PIF controller

The discussed PIF controller (38) can be rewritten in the form given by

$$u^{(2)} + \frac{d_1}{\mu} u^{(1)} = -\frac{k_0}{\mu^2} x^{(1)} - \frac{k_0}{\mu^2 T} x + \frac{k_0}{\mu^2 T} r, \quad (41)$$

where  $\hat{y}$  is replaced by  $x$ . Denote

$$a_1 = \frac{d_1}{\mu}, \quad b_1 = -\frac{k_0}{\mu^2}, \quad b_0 = -\frac{k_0}{\mu^2 T}, \quad c_0 = \frac{k_0}{\mu^2 T}.$$

From (41), we have  $u^{(2)} + a_1 u^{(1)} = b_1 x^{(1)} + b_0 x + c_0 r$  and, thereafter,  $u^{(2)} + a_1 u^{(1)} - b_1 x^{(1)} = b_0 x + c_0 r$ . Denote  $u_2^{(1)} = b_0 x + c_0 r$ . Then we get  $u^{(1)} + a_1 u - b_1 x = u_2$ . Denote  $u_1^{(1)} = u_2 - a_1 u + b_1 x$ . Hence,  $u = u_1$ . Finally, the state space equations of the PIF controller are given by

$$\begin{aligned} \dot{u}_1 &= u_2 - a_1 u_1 + b_1 x, \\ \dot{u}_2 &= b_0 x + c_0 r, \\ u &= u_1. \end{aligned} \quad (42)$$

The block diagram of the PIF controller (42) is shown in Fig. 2(b).

## 5. PID controller of the 2-nd order nonlinear system

### 5.1 Control problem and insensitivity condition

Consider a nonlinear system of the 2-nd order given by

$$\ddot{x} = f(X, w) + g(X, w)u, \quad (43)$$

where  $x$  is the measurable output of the system (43),  $y = x$ , and  $\dot{x}$  is the unmeasurable variable of the state  $X = [x, \dot{x}]^T$ . Assume that the inequalities

$$0 < g_{min} \leq g(X, w) \leq g_{max} < \infty, \quad |f(X, w)| \leq f_{max} < \infty \quad (44)$$

are satisfied for all  $(X, w) \in \Omega_{X,w}$ , where  $f(X, w)$ ,  $g(X, w)$  are unknown continuous bounded functions of  $X(t)$ ,  $w(t)$  on the bounded set  $\Omega_{X,w}$ .

The control objective is given by (11), where the desired settling time and overshoot have to be provided for  $x(t)$  regardless the presence of the external disturbances and varying parameters  $w(t)$  of the system (43).

Consider the reference equation of the desired behavior for (43) in the form of the 2nd order stable differential equation given by

$$T^2 \ddot{x} + a_1^d T \dot{x} + x = b_1^d T \dot{r} + r.$$

Hence, we have

$$\ddot{x} = \frac{1}{T} [b_1^d \dot{r} - a_1^d \dot{x}] + \frac{1}{T^2} [r - x]. \quad (45)$$

Let us rewrite (45) in the form

$$\ddot{x} = F(X, R),$$

where  $R = [r, \dot{r}]^T$  and the parameters  $T$ ,  $a_1^d$ , and  $b_1^d$  are selected in accordance with the desired system type, settling time, and overshoot for  $x(t)$ . Denote

$$e_f := F(X, R) - \ddot{x}.$$

Hence, the behavior of  $x(t)$  with prescribed dynamics of (45) is fulfilled in presence of the external disturbances and varying parameters of (43), if the insensitivity condition  $e_f = 0$  holds. Similar to the above, the nonlinear inverse dynamics solution is given by

$$u^{id} = [g(X, w)]^{-1} [F(X, R) - f(X, w)]. \quad (46)$$

## 5.2 PID controller

Consider the control law in the form

$$\mu^2 \ddot{u} + d_1 \mu \dot{u} = k_0 [F(X, R) - \ddot{x}], \quad (47)$$

where  $\mu$  is a small positive parameter. In accordance with (45), the controller (47) can be represented as

$$\mu^2 \ddot{u} + d_1 \mu \dot{u} = k_0 \left\{ -\ddot{x} + \frac{1}{T} [b_1^d \dot{r} - a_1^d \dot{x}] + \frac{1}{T^2} [r - x] \right\}. \quad (48)$$

The discussed control law (48) can also be expressed in terms of transfer functions

$$u(s) = \frac{k_0}{\mu(\mu s + d_1)} \left\{ \frac{1}{T} [b_1^d r(s) - a_1^d x(s)] + \frac{1}{T^2 s} [r(s) - x(s)] - s x(s) \right\}, \quad (49)$$

which corresponds to the PID controller and (49) is implemented without an ideal differentiation of  $x(t)$  or  $r(t)$  due to the presence of the term  $k_0/[\mu(\mu s + d_1)]$ . Note, PID controller with additional lowpass filtering (PIDF controller)

$$\mu^q u^{(q)} + d_{q-1} \mu^{q-1} u^{(q-1)} + \dots + d_1 \mu u^{(1)} = k_0 [F(X, R) - x^{(2)}] \quad (50)$$

can be considered as well, where  $q > 2$ .

### 5.3 Two-time-scale motion analysis

Consider the closed-loop system equations (43),(47), that are

$$\ddot{x} = f(X, w) + g(X, w)u, \quad (51)$$

$$\mu^2 \ddot{u} + d_1 \mu \dot{u} = k_0 [F(X, R) - \ddot{x}]. \quad (52)$$

Substitution of (51) into (52) yields

$$\ddot{x} = f(X, w) + g(X, w)u, \quad (53)$$

$$\mu^2 \ddot{u} + d_1 \mu \dot{u} + k_0 g(x, w)u = k_0 [F(X, R) - f(X, w)]. \quad (54)$$

Denote  $u_1 = u$  and  $u_2 = \mu u$ . Hence, the system (53)–(54) can be represented as a standard singular perturbation system, that is

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = f(x_1, x_2, w) + g(x_1, x_2, w)u_1,$$

$$\mu \dot{u}_1 = u_2,$$

$$\mu \dot{u}_2 = -k_0 g(x, w)u_1 - d_1 u_2 + k_0 [F(x_1, x_2, R) - f(x_1, x_2, w)].$$

From the above system, the fast-motion subsystem (FMS) equation

$$\mu^2 \ddot{u} + d_1 \mu \dot{u} + k_0 g(x, w)u = k_0 [F(X, R) - f(X, w)] \quad (55)$$

follows, where  $X(t)$  and  $w(t)$  are frozen variables during the transients in (55).

By selection of  $\mu$ ,  $d_1$ , and  $k_0$ , we can provide the FMS stability as well as the desired degree of time-scale separation between fast and slow modes in the closed-loop system. Then, after the rapid decay of transients in (55) (or, by taking  $\mu = 0$  in (55)), we obtain the steady state (more precisely, quasi-steady state) for the FMS (55), where  $u(t) = u^{id}(t)$ . Hence, from (53)–(54), we get the slow-motion subsystem (SMS) equation, which is the same as (45) in spite of unknown external disturbances and varying parameters of (43) and by that the desired behavior of  $x(t)$  is provided.

### 5.4 Selection of PID controller parameters

The time constant  $T$  of the reference equation (45) is selected in accordance with the desired settling time of output transients. The parameter  $a_1^d$  is defined by permissible overshoot of the output step response. Take, for example  $a_1^d = 2$ . Take  $b_1^d = 0$  if the reference model given by (45) is a system of type 1. Take  $b_1^d = a_1^d$  if the reference model given by (45) is a system

of type 2. Take the gain  $k_0 \approx g^{-1}(X, w)$ , and parameter  $d_1 = 2$ . Then, in accordance with (55), the FMS characteristic polynomial is given by  $(\mu s + 1)^2$ . The time constant  $\mu$  is selected as  $\mu = T/\eta$  where  $\eta$  is the desired degree of time-scale separation between the fast and slow modes in the closed-loop system, for example,  $\eta \geq 10$ .

Note, in case of PIDF controller given by (50), the FMS characteristic polynomial has the form  $(\mu s + 1)^q$  when the parameters  $d_{q-1}, \dots, d_2, d_1$  are selected as the coefficients of the binomial polynomial, that is

$$(s + 1)^q = s^q + d_{q-1}s^{q-1} + \dots + d_2s^2 + d_1s + 1.$$

The more detailed results and procedures for selection of controller parameters can be found in (Yurkevich, 2004).

### 5.5 Implementation of PID controller

The discussed control law (48) can be rewritten in the form given by

$$u^{(2)} + \frac{d_1}{\mu}u^{(1)} = -\frac{k_0}{\mu^2}x^{(2)} - \frac{k_0a_1^d}{\mu^2T}x^{(1)} - \frac{k_0}{\mu^2T^2}x + \frac{k_0b_1^d}{\mu^2T}r^{(1)} + \frac{k_0}{\mu^2T^2}r,$$

that is

$$u^{(2)} + a_1u^{(1)} = b_2x^{(2)} + b_1x^{(1)} + b_0x + c_1r^{(1)} + c_0r, \quad (56)$$

where

$$a_1 = \frac{d_1}{\mu}, \quad b_2 = -\frac{k_0}{\mu^2}, \quad b_1 = -\frac{k_0a_1^d}{\mu^2T}, \quad b_0 = -\frac{k_0}{\mu^2T^2}, \quad c_1 = \frac{k_0b_1^d}{\mu^2T}, \quad c_0 = \frac{k_0}{\mu^2T^2}.$$

The block diagram representation of the discussed control law (56) can be obtained based on the following derivations:

$$\begin{aligned} u^{(2)} - b_2x^{(2)} + a_1u^{(1)} - b_1x^{(1)} - c_1r^{(1)} &= \underbrace{b_0x + c_0r}_{= \dot{u}_2} \implies u^{(1)} - b_2x^{(1)} + a_1u - b_1x - c_1r = u_2 \\ \implies u^{(1)} - b_2x^{(1)} &= \underbrace{u_2 - a_1u + b_1x + c_1r}_{= \dot{u}_1} \implies u = u_1 + b_2x. \end{aligned}$$

Hence, we obtain the equations of the controller given by

$$\begin{aligned} \dot{u}_1 &= u_2 - a_1u + b_1x + c_1r, \\ \dot{u}_2 &= b_0x + c_0r, \\ u &= u_1 + b_2x. \end{aligned} \quad (57)$$

From (57), the block diagram of the controller follows as shown in Fig. 4(a).

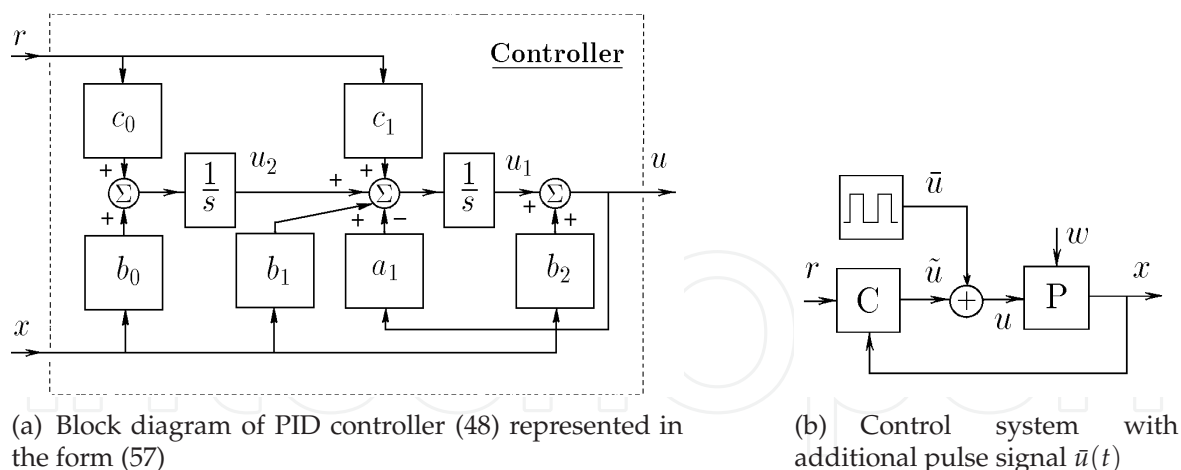


Fig. 4. Control system with PID controller

### 6. On-line tuning of controller parameters

Let us consider the closed-loop system with an additional pulse signal  $\bar{u}(t)$  as shown in Fig. 4(b). Then, instead of (51)–(52), we get

$$\begin{aligned} \ddot{x} &= f(X, w) + g(X, w)[\tilde{u} + \bar{u}], \\ \mu^2 \tilde{u}^{(2)} + d_1 \mu \tilde{u}^{(1)} &= k_0 [F(X, R) - x^{(2)}]. \end{aligned}$$

From the above system, the FMS equation

$$\mu^2 \tilde{u}^{(2)} + d_1 \mu \tilde{u}^{(1)} + k_0 g(x, w) \tilde{u} = k_0 [F(X, R) - f(X, w) - g(x, w) \bar{u}] \tag{58}$$

results, where  $X(t)$  and  $w(t)$  are frozen variables during the transients in (58). In accordance with (58) and  $u = \tilde{u} + \bar{u}$ , the input sensitivity function with respect to pulse signal  $\bar{u}(t)$  can be defined as the following transfer function  $G_{u\bar{u}}(s) = u(s)/\bar{u}(s)$ , that is

$$G_{u\bar{u}}(s) = \frac{\mu^2 s^2 + d_1 \mu s}{\mu^2 s^2 + d_1 \mu s + k_0 g'}$$

or we may consider sensitivity function defined as  $G_{\tilde{u}\bar{u}}(s) = \tilde{u}(s)/\bar{u}(s)$ , that is

$$G_{\tilde{u}\bar{u}}(s) = -\frac{k_0 g}{\mu^2 s^2 + d_1 \mu s + k_0 g}$$

For example, if  $d_1 = 2$  and  $k_0 g = 1$ , then the shape of the fast-motion transients excited by  $\bar{u}(t)$  in behavior of  $u(t)$  and  $\tilde{u}(t)$  is easily predictable one. Therefore, on-line tuning of controller parameters can be provided based on direct observations of the fast-motion transients that are excited by the pulse signal  $\bar{u}(t)$ . In particular, if  $d_1 = 2$  and the high-frequency gain  $g(x, w)$  is unknown, then the gain  $k_0$  can be manually adjusted such that to provide acceptable small oscillations of FMS transients excited by  $\bar{u}(t)$ .



### 6.1 Example 2

Consider a SISO nonlinear continuous-time system in the form

$$x^{(2)} = x^3 + |x^{(1)}| - (2 + x^2)u + w, \quad (59)$$

where the reference equation of the desired behavior for the output  $x(t)$  is assigned by (45) and the control law structure is given by (48).

Take  $T = 0.3$  s,  $a_1^d = 2$ ,  $\mu = 0.03$  s,  $k_0 = -0.5$ , and  $d_1 = 2$ , where the control law (48) is represented in the form (57). The simulation results of the system (59) controlled by the algorithm (57) are displayed in Figs. 5–9, where the initial conditions are zero. The output response of the system (59) with controller (57) for a ramp reference input  $r(t)$ , in case where  $b_1^d = 0$  (the reference model is a system of type 1) reveals the large value of a velocity error as shown in Fig. 6. The velocity error can be significantly reduced by taking  $b_1^d = a_1^d$  (the reference model is a system of type 2) as shown in Fig. 8. Note, the high pulse in control variable, as shown in Fig. 7(b), is caused by discrepancy between relative degree of the system (59) and relative degree of (45) when  $b_1^d = a_1^d$ . This high pulse can be eliminated by the use of a smooth reference input function  $r(t)$  as shown in Fig. 9.

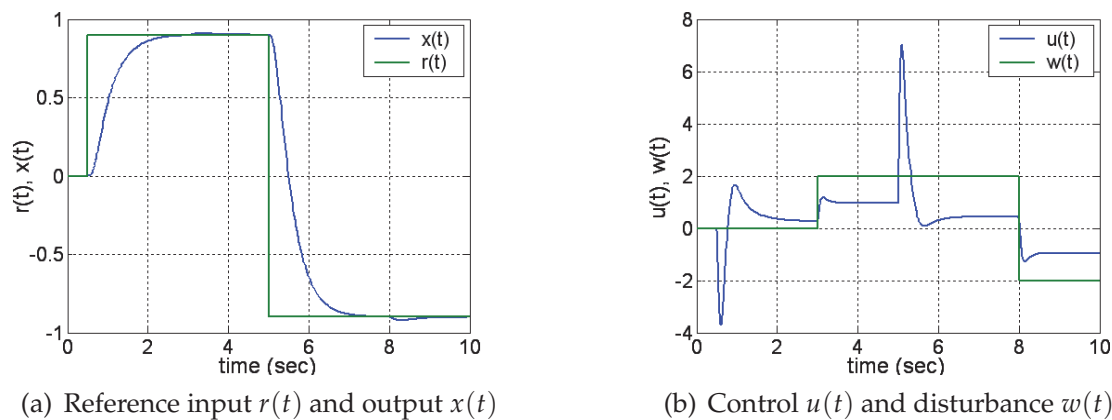


Fig. 5. Output response of the system (59) with controller (57) for a step reference input  $r(t)$  and a step disturbance  $w(t)$ , where  $b_1^d = 0$  (the reference model is a system of type 1)

## 7. Sampled-data nonlinear system of the 1-st order

### 7.1 Control problem and insensitivity condition

In this section the discrete-time counterpart of the above singular perturbation design methodology is discussed. Let us consider the backward difference approximation of the nonlinear system (9) preceded by a zero-order hold (ZOH) with the sampling period  $T_s$ , that is

$$x_k = x_{k-1} + T_s [f(x_{k-1}, w_{k-1}) + g(x_{k-1}, w_{k-1})u_{k-1}], \quad (60)$$

where  $x_k$ ,  $w_k$ , and  $u_k$  represent samples of  $x(t)$ ,  $w(t)$ , and  $u(t)$  at  $t = kT_s$ , respectively. The objective is to design a control system having

$$\lim_{k \rightarrow \infty} e_k = 0. \quad (61)$$

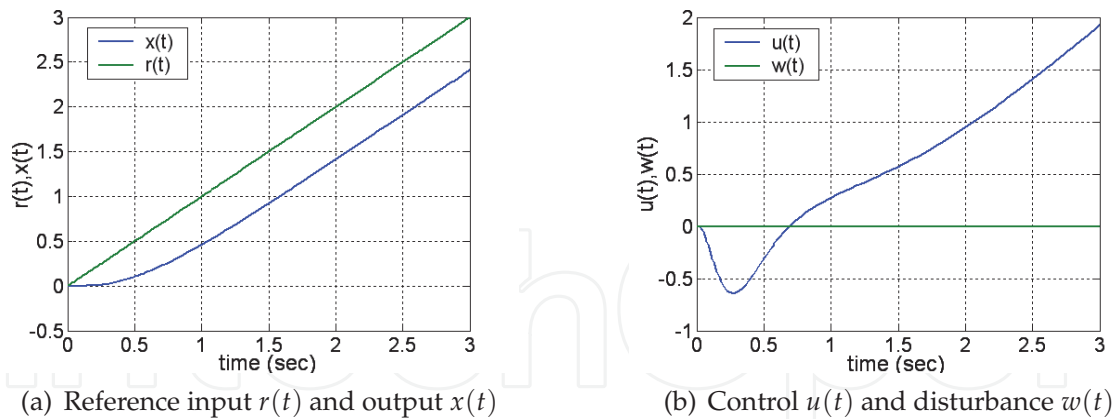


Fig. 6. Output response of the system (59) with controller (57) for a ramp reference input  $r(t)$ , where  $b_1^d = 0$  and  $w(t) = 0$  (the reference model is a system of type 1)

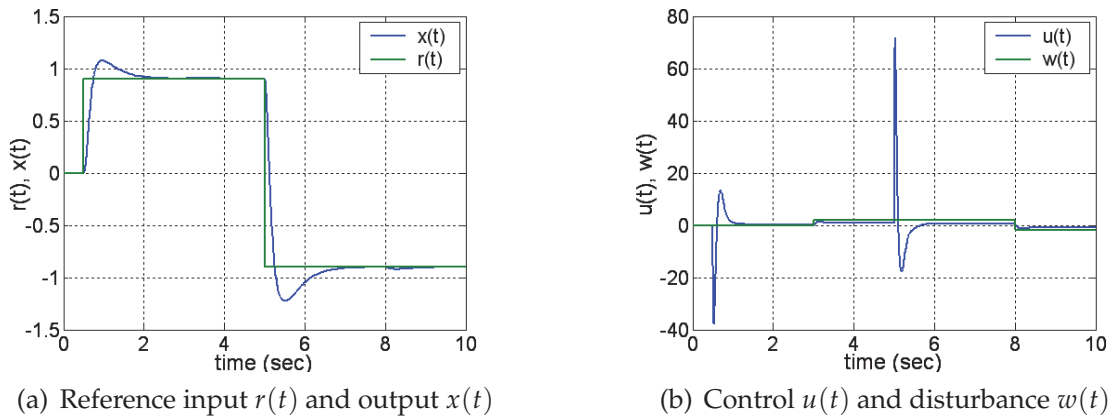


Fig. 7. Output response of the system (59) with controller (57) for a step reference input  $r(t)$  and a step disturbance  $w(t)$ , where  $b_1^d = a_1^d$  (the reference model is a system of type 2)

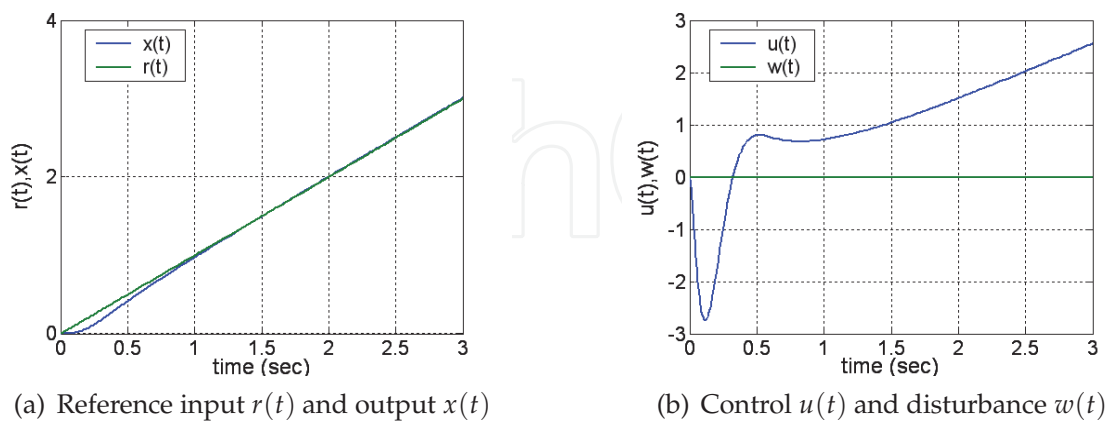


Fig. 8. Output response of the system (59) with controller (57) for a ramp reference input  $r(t)$ , where  $b_1^d = a_1^d$  and  $w(t) = 0$  (the reference model is a system of type 2)

Here  $e_k := r_k - x_k$  is the error of the reference input realization,  $r_k$  being the samples of the reference input  $r(t)$ , where the control transients  $e_k \rightarrow 0$  should meet the desired performance

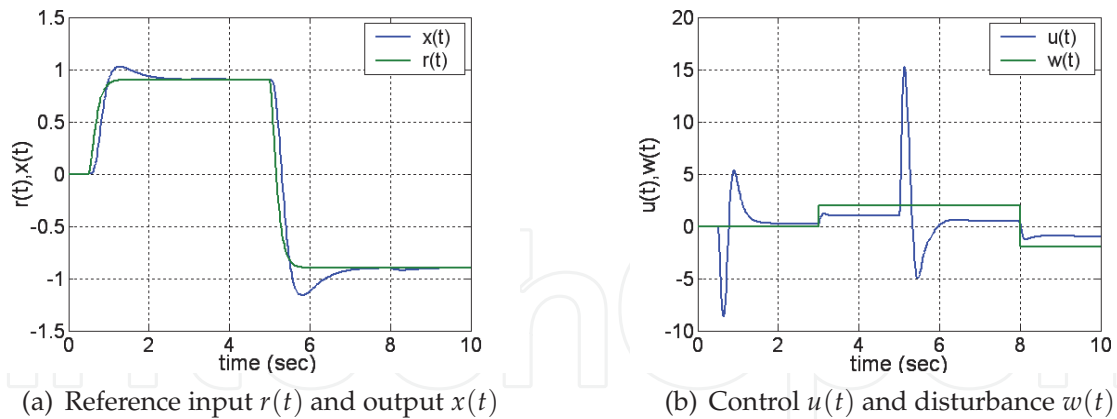


Fig. 9. Output response of the system (59) with controller (57) for a smooth reference input  $r(t)$  and a step disturbance  $w(t)$ , where  $b_1^d = a_1^d$  (the reference model is a system of type 2)

specifications given by (12).

By a  $\mathcal{Z}$ -transform of (12) preceded by a ZOH, the desired pulse transfer function

$$H_{xr}^d(z) = \frac{z-1}{z} \mathcal{Z} \left\{ \mathcal{L}^{-1} \left[ \frac{1/T}{s(s+1/T)} \right] \Big|_{t=kT_s} \right\} = \frac{1 - e^{-T_s/T}}{z - e^{-T_s/T}} \quad (62)$$

follows. Hence, from (62), the desired stable difference equation

$$x_k = x_{k-1} + T_s a(T_s) [r_{k-1} - x_{k-1}] \quad (63)$$

results, where

$$a(T_s) = \frac{1 - e^{-T_s/T}}{T_s}, \quad \lim_{T_s \rightarrow 0} a(T_s) = \frac{1}{T},$$

and the output response of (63) corresponds to the assigned output transient performance indices.

Let us rewrite, for short, the desired difference equation (63) as

$$x_k = F(x_{k-1}, r_{k-1}), \quad (64)$$

where we have  $r_k = x_k$  at the equilibrium of (64) for  $r_k = \text{const}$ ,  $\forall k$ . Denote

$$e_k^F := F(x_{k-1}, r_{k-1}) - x_k, \quad (65)$$

where  $e_k^F$  is the realization error of the desired dynamics assigned by (64). Accordingly, if for all  $k = 0, 1, \dots$  the condition

$$e_k^F = 0 \quad (66)$$

holds, then the desired behavior of  $x_k$  with the prescribed dynamics of (64) is fulfilled. The expression (66) is the insensitivity condition for the output transient performance with respect to the external disturbances and varying parameters of the plant model given by (60). In other words, the control design problem (61) has been reformulated as the requirement (66).

The insensitivity condition given by (66) is the discrete-time counterpart of (15) which was introduced for the continuous-time system (9).

### 7.2 Discrete-time counterpart of PI controller

Let us consider the following control law:

$$u_k = u_{k-1} + \lambda_0 [F(x_{k-1}, r_{k-1}) - x_k], \quad (67)$$

where  $\lambda_0 = T_s^{-1} \tilde{\lambda}$  and the reference model of the desired output behavior is given by (63). In accordance with (63) and (65), the control law (67) can be rewritten as the difference equation

$$u_k = u_{k-1} + \tilde{\lambda} \left\{ a(T_s) [r_{k-1} - x_{k-1}] - \frac{x_k - x_{k-1}}{T_s} \right\}. \quad (68)$$

The control law (68) is the discrete-time counterpart of the conventional continuous-time PI controller given by (18).

### 7.3 Two-time-scale motion analysis

Denote  $f_{k-1} = f(x_{k-1}, w_{k-1})$  and  $g_{k-1} = g(x_{k-1}, w_{k-1})$  in the expression (60). Hence, the closed-loop system equations have the following form:

$$x_k = x_{k-1} + T_s [f_{k-1} + g_{k-1} u_{k-1}], \quad (69)$$

$$u_k = u_{k-1} + \tilde{\lambda} \left\{ a(T_s) [r_{k-1} - x_{k-1}] - \frac{x_k - x_{k-1}}{T_s} \right\}. \quad (70)$$

Substitution of (69) into (70) yields

$$x_k = x_{k-1} + T_s [f_{k-1} + g_{k-1} u_{k-1}], \quad (71)$$

$$u_k = [1 - \tilde{\lambda} g_{k-1}] u_{k-1} + \tilde{\lambda} \{ a(T_s) [r_{k-1} - x_{k-1}] - f_{k-1} \}. \quad (72)$$

The sampling period  $T_s$  can be treated as a small parameter, then the closed-loop system equations (71)–(72) have the standard singular perturbation form given by (5)–(6). First, the stability and the rate of the transients of  $u_k$  in (71)–(72) depend on the controller parameter  $\tilde{\lambda}$ . Second, note that  $x_k - x_{k-1} \rightarrow 0$  as  $T_s \rightarrow 0$ . Hence, we have a slow rate of the transients of  $x_k$  as  $T_s \rightarrow 0$ . Thus, if  $T_s$  is sufficiently small, the two-time-scale transients are artificially induced in the closed-loop system (71)–(72), where the FMS is governed by

$$u_k = [1 - \tilde{\lambda} g_{k-1}] u_{k-1} + \tilde{\lambda} \{ a(T_s) [r_{k-1} - x_{k-1}] - f_{k-1} \} \quad (73)$$

and  $x_k = x_{k-1}$ , i.e.,  $x_k = \text{const}$  (hence,  $x_k$  is the frozen variable) during the transients in the FMS (73).

Let  $g = g_k \forall k$ . From (73), the FMS characteristic polynomial

$$z - 1 + \tilde{\lambda} g \quad (74)$$

results, where its root lies inside the unit disk (hence, the FMS is stable) if  $0 < \tilde{\lambda} < 2/g$ . To ensure stability and fastest transient processes of  $u_k$ , let us take the controller parameter

$\tilde{\lambda} = 1/g$ , then the root of (74) is placed at the origin. Hence, the deadbeat response of the FMS (73) is provided. We may take  $T_s \leq T/\eta$ , where  $\eta \geq 10$ .

Third, assume that the FMS (73) is stable and consider its steady state (quasi-steady state), i.e.,

$$u_k - u_{k-1} = 0. \quad (75)$$

Then, from (73) and (75), we get  $u_k = u_k^{id}$ , where

$$u_k^{id} = g^{-1} \{a(T_s)[r_{k-1} - x_{k-1}] - f_{k-1}\}. \quad (76)$$

Substitution of (75) and (76) into (71) yields the SMS of (71)–(72), which is the same as the desired difference equation (63) in spite of unknown external disturbances and varying parameters of (60) and by that the desired behavior of  $x_k$  is provided.

## 8. Sampled-data nonlinear system of the 2-nd order

### 8.1 Approximate model

The above approach to approximate model derivation can also be used for nonlinear system of the 2-nd order, which is preceded by ZOH with high sampling rate. For instance, let us consider the nonlinear system given by (43)

$$x^{(2)} = f(X, w) + g(X, w)u, \quad y = x,$$

which is preceded by ZOH, where  $y \in \mathbb{R}^1$  is the output, available for measurement;  $u \in \mathbb{R}^1$  is the control;  $w$  is the external disturbance, unavailable for measurement;  $X = \{x, x^{(1)}\}^T$  is the state vector.

We can obtain the state-space equations of (43) given by

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= f(\cdot) + g(\cdot)u, \\ y &= x_1. \end{aligned}$$

Let us introduce the new time scale  $t_0 = t/T_s$ . We obtain

$$\begin{aligned} \frac{d}{dt_0} x_1 &= T_s x_2, \\ \frac{d}{dt_0} x_2 &= T_s \{f(\cdot) + g(\cdot)u\}, \\ y &= x_1, \end{aligned} \quad (77)$$

where  $dX/dt_0 \rightarrow 0$  as  $T_s \rightarrow 0$ . From (77) it follows that

$$\frac{d^2 y}{dt_0^2} = T_s^2 \{f(\cdot) + g(\cdot)u\}. \quad (78)$$

Assume that the sampling period  $T_s$  is sufficiently small such that the conditions  $X(t) = \text{const}$ ,  $g(X, w) = \text{const}$  hold for  $kT_s \leq t < (k+1)T_s$ . Then, by taking the  $\mathcal{Z}$ -transform of (78), we get

$$y(z) = \frac{\mathcal{E}_2(z)}{2!(z-1)^2} T_s^2 \{f(z) + \{gu\}(z)\}, \quad (79)$$

where  $\mathcal{E}_2(z) = z + 1$ . Denote  $\mathcal{E}_2(z) = \epsilon_{2,1}z + \epsilon_{2,2}$  and  $z^2 - a_{2,1}z - a_{2,2} = (z-1)^2$ , where  $\epsilon_{2,1} = \epsilon_{2,2} = 1$ ,  $a_{2,1} = 2$ , and  $a_{2,2} = -1$ . From (79) we get the difference equation

$$y_k = \sum_{j=1}^2 a_{2,j} y_{k-j} + T_s^2 \sum_{j=1}^2 \frac{\epsilon_{2,j}}{2!} \{f_{k-j} + g_{k-j} u_{k-j}\} \quad (80)$$

given that the high sampling rate takes place, where  $g_k = g(X(t), w(t))|_{t=kT_s}$ ,  $f_k = f(X(t), w(t))|_{t=kT_s}$ , and

$$y_k - y_{k-j} \rightarrow 0, \quad \forall j = 1, 2 \text{ as } T_s \rightarrow 0. \quad (81)$$

## 8.2 Reference equation and insensitivity condition

Denote  $e_k := r_k - y_k$  is the error of the reference input realization, where  $r_k$  being the reference input. Our objective is to design a control system having

$$\lim_{k \rightarrow \infty} e_k = 0. \quad (82)$$

Moreover, the control transients  $e_k \rightarrow 0$  should have desired performance indices such as overshoot, settling time, and system type. These transients of  $y_k$  should not depend on the external disturbances and varying parameters of the nonlinear system (43).

Let us consider the continuous-time reference model for the desired behavior of the output  $y(t) = x(t)$  in the form given by (45), which can be rewritten as

$$y(s) = G^d(s)r(s),$$

where the parameters of the 2nd-order stable continuous-time transfer function  $G^d(s)$  are selected based on the required output transient performance indices and such that

$$G^d(s) \Big|_{s=0} = 1.$$

By a  $\mathcal{Z}$ -transform of  $G^d(s)$  preceded by a ZOH, the desired pulse transfer function

$$H_{yr}^d(z) = \frac{z-1}{z} \mathcal{Z} \left\{ \mathcal{L}^{-1} \left[ \frac{G_{yr}^d(s)}{s} \right] \Big|_{t=kT_s} \right\} = \frac{B^d(z)}{A^d(z)} \quad (83)$$

can be found, where

$$H_{yr}^d(z) \Big|_{z=1} = 1.$$

Hence, from (83), the desired stable difference equation

$$y_k = \sum_{j=1}^2 a_j^d y_{k-j} + \sum_{j=1}^2 b_j^d r_{k-j} \quad (84)$$

results, where

$$1 - \sum_{j=1}^2 a_j^d = \sum_{j=1}^2 b_j^d, \quad \sum_{j=1}^2 b_j^d \neq 0,$$

and the parameters of (84) correspond to the assigned output transient performance indices. Let us rewrite, for short, the desired difference equation (84) as

$$y_k = F(Y_k, R_k), \quad (85)$$

where  $Y_k = \{y_{k-2}, y_{k-1}\}^T$ ,  $R_k = \{r_{k-2}, r_{k-1}\}^T$ , and  $r_k = y_k$  at the equilibrium of (85) for  $r_k = \text{const}$ ,  $\forall k$ . By definition, put  $F_k = F(Y_k, R_k)$  and denote

$$e_k^F := F_k - y_k, \quad (86)$$

where  $e_k^F$  is the realization error of the desired dynamics assigned by (85). Accordingly, if for all  $k = 0, 1, \dots$  the condition

$$e_k^F = 0 \quad (87)$$

holds, then the desired behavior of  $y_k$  with the prescribed dynamics of (85) is fulfilled. The expression (87) is the insensitivity condition for the output transients with respect to the external disturbances and varying parameters of the plant model (80). In other words, the control design problem (82) has been reformulated as the requirement (87). The insensitivity condition (87) is the discrete-time counterpart of the condition  $e_F = 0$  for the continuous-time system (43).

### 8.3 Discrete-time counterpart of PIDF controller

In order to fulfill (87), let us construct the control law as the difference equation

$$u_k = \sum_{j=1}^{q \geq 2} d_j u_{k-j} + \lambda_0 [F_k - y_k], \quad (88)$$

where

$$d_1 + d_2 + \dots + d_q = 1, \quad \text{and} \quad \lambda_0 \neq 0. \quad (89)$$

From (89) it follows that the equilibrium of (88) corresponds to the insensitivity condition (87). In accordance with (84) and (86), the control law (88) can be rewritten as the difference

equation

$$u_k = \sum_{j=1}^{q \geq 2} d_j u_{k-j} + \lambda_0 \left\{ -y_k + \sum_{j=1}^2 a_j^d y_{k-j} + \sum_{j=1}^2 b_j^d r_{k-j} \right\}. \tag{90}$$

The control law (90) is the discrete-time counterpart of the continuous-time PIDF controller (50). In particular, if  $q = 2$ , then (90) can be rewritten in the following state-space form:

$$\begin{aligned} \bar{u}_{1,k} &= \bar{u}_{2,k-1} + d_1 \bar{u}_{1,k-1} + \lambda_0 [a_1^d - d_1] y_{k-1} + \lambda_0 b_1^d r_{k-1}, \\ \bar{u}_{2,k} &= d_2 \bar{u}_{1,k-1} + \lambda_0 [a_2^d - d_2] y_{k-1} + \lambda_0 b_2^d r_{k-1}, \\ u_k &= \bar{u}_{1,k} - \lambda_0 y_k. \end{aligned} \tag{91}$$

Then, from (91), we get the block diagram of the controller as shown in Fig. 10.

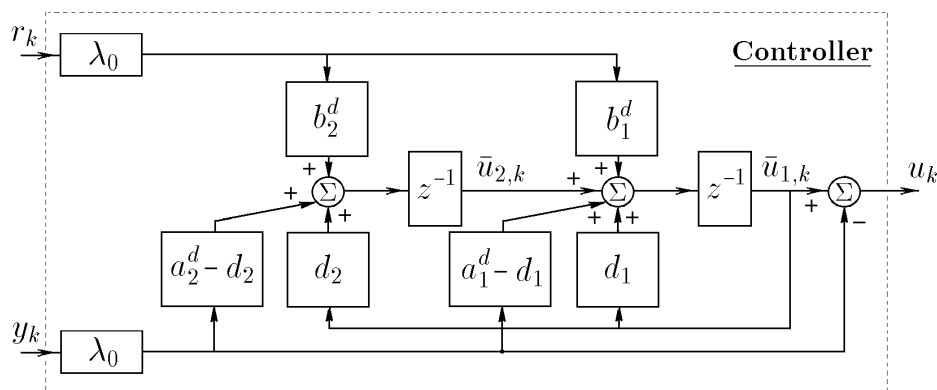


Fig. 10. Block diagram of the control law (90), where  $q = 2$ , represented in the form (91)

### 8.4 Two-time-scale motion analysis

The closed-loop system equations have the following form:

$$y_k = \sum_{j=1}^2 a_{2,j} y_{k-j} + T_s^2 \sum_{j=1}^2 \frac{\epsilon_{2,j}}{2!} [f_{k-j} + g_{k-j} u_{k-j}], \tag{92}$$

$$u_k = \sum_{j=1}^{q \geq 2} d_j u_{k-j} + \lambda_0 [F_k - y_k]. \tag{93}$$

Substitution of (92) into (93) yields

$$y_k = \sum_{j=1}^2 a_{2,j} y_{k-j} + T_s^2 \sum_{j=1}^2 \frac{\epsilon_{2,j}}{2!} [f_{k-j} + g_{k-j} u_{k-j}], \tag{94}$$

$$u_k = \sum_{j=n+1}^{q > 2} d_j u_{k-j} + \sum_{j=1}^2 [d_j - \lambda_0 T_s^2 \frac{\epsilon_{2,j}}{2!} g_{k-j}] u_{k-j} + \lambda_0 \left\{ F_k - \sum_{j=1}^2 \left\{ a_{2,j} y_{k-j} - T_s^2 \frac{\epsilon_{2,j}}{2!} f_{k-j} \right\} \right\}. \tag{95}$$

First, note that the rate of the transients of  $u_k$  in (94)–(95) depends on the controller parameters  $\lambda_0, d_1, \dots, d_q$ . At the same time, in accordance with (81), we have a slow rate of the transients



of  $y_k$ , because the sampling period  $T_s$  is sufficiently small one. Therefore, by choosing the controller parameters it is possible to induce two-time scale transients in the closed-loop system (94)–(95), where the rate of the transients of  $y_k$  is much smaller than that of  $u_k$ . Then, as an asymptotic limit, from the closed-loop system equations (94)–(95) it follows that the FMS is governed by

$$u_k = \sum_{j=3}^{q>2} d_j u_{k-j} + \sum_{j=1}^2 [d_j - \lambda_0 T_s^2 \frac{\epsilon_{2,j}}{2!} g_{k-j}] u_{k-j} + \lambda_0 \left\{ F_k - \sum_{j=1}^2 \left\{ a_{2,j} y_{k-j} - T_s^2 \frac{\epsilon_{2,j}}{2!} f_{k-j} \right\} \right\}, \quad (96)$$

where  $y_k - y_{k-j} \approx 0, \forall 1, \dots, q$ , i.e.,  $y_k = \text{const}$  during the transients in the system (96).

Second, assume that the FMS (96) is exponentially stable (that means that the unique equilibrium point of (96) is exponentially stable), and  $g_k - g_{k-j} \rightarrow 0, \forall j = 1, 2, \dots, q$  as  $T_s \rightarrow 0$ . Then, consider steady state (or more exactly quasi-steady state) of (96), i.e.,

$$u_k - u_{k-j} = 0, \quad \forall j = 1, \dots, q. \quad (97)$$

Then, from (89), (96), and (97) we get  $u_k = u_k^{id}$ , where

$$u_k^{id} = [T_s^2 g_k]^{-1} \left\{ F_k - \sum_{j=1}^2 \left\{ a_{2,j} y_{k-j} + T_s^2 \frac{\epsilon_{2,j}}{2!} f_{k-j} \right\} \right\}. \quad (98)$$

The discrete-time control function  $u_k^{id}$  given by (98) corresponds to the insensitivity condition (87), that is,  $u_k^{id}$  is the discrete-time counterpart of the nonlinear inverse dynamics solution (46). Substitution of (97) into (94)–(95) yields the SMS of (94)–(95), which is the same as the desired difference equation (85) and by that the desired behavior of  $y_k$  is provided.

### 8.5 Selection of discrete-time controller parameters

Let, the sake of simplicity,  $q = 2, \bar{g} = g_k = \text{const} \forall k$ , and take

$$\lambda_0 = \{T_s^2 \bar{g}\}^{-1}, \quad d_j = \frac{\epsilon_{2,j}}{2!}, \quad \forall i = 1, 2. \quad (99)$$

Then all roots of the characteristic polynomial of the FMS (96) are placed at the origin. Hence, the deadbeat response of the FMS (96) is provided. This, along with assumption that the sampling period  $T_s$  is sufficiently small, justifies two-time-scale separation between the fast and slow motions. So, if the degree of time-scale separation between fast and slow motions in the closed-loop system (94)–(95) is sufficiently large and the FMS transients are stable, then after the fast transients have vanished the behavior of  $y_k$  tends to the solution of the reference equation given by (85). Accordingly, the controlled output transient process meets the desired performance specifications. The deadbeat response of the FMS (96) has a finite settling time given by  $t_{s,FMS} = 2T_s$  when  $q = 2$ . Then the relationship

$$T_s \leq \frac{t_{s,SMS}}{2\eta} \quad (100)$$

may be used to estimate the sampling period in accordance with the required degree of time-scale separation between the fast and slow modes in the closed-loop system. Here  $t_{s,SMS}$  is the settling time of the SMS and  $\eta$  is the degree of time-scale separation,  $\eta \geq 10$ .

The advantage of the presented above method is that knowledge of the high-frequency gain  $g$  suffices for controller design; knowledge of external disturbances and other parameters of the system is not needed. Note that variation of the parameter  $g$  is possible within the domain where the FMS (96) is stable and the fast and slow motion separation is maintained.

### 8.6 Example 3

Let us consider the system (59). Assume that the specified region of  $x(t)$  is given by  $x(t) \in [-2, 2]$ . Hence, the range of high-frequency gain variations has the following bounds  $g(x) \in [2, 6]$ . We have that  $\mathcal{E}_2(z) = z + 1$ . Let the desired output behavior is described by the reference equation (45) where  $a_1^d = 2$ . Therefore, from (45), the desired transfer function

$$G^d(s) = \frac{b_1^d T_s + 1}{T^2 s^2 + a_1^d T_s + 1} = \frac{b_1^d T_s + 1}{T^2 (s + \bar{\alpha})^2} \quad (101)$$

results, where  $\bar{\alpha} = 1/T$ . The pulse transfer function  $H^d(z)$  of a series connection of a zero-order hold and the system of (101) is the function given by

$$H^d(z) = \frac{\bar{b}_1^d z + \bar{b}_2^d}{z^2 - \bar{a}_1^d z - \bar{a}_2^d}, \quad (102)$$

where  $\bar{a}_1^d = 2d$ ,  $\bar{a}_2^d = -d^2$ ,  $\bar{b}_1^d = T^{-2}[1 - d + (b_1^d T - \bar{\alpha})dT_s]$ , and  $\bar{b}_2^d = T^{-2}d[d - 1 + (\bar{\alpha} - b_1^d T)T_s]$ . Take, for simplicity,  $q = 2$ . Hence, in accordance with (90) and (99), the discrete-time controller has been obtained

$$u_k = d_1 u_{k-1} + d_2 u_{k-2} + [T_s^2 \bar{g}]^{-1} \{-y_k + \bar{a}_1^d y_{k-1} + \bar{a}_2^d y_{k-2} + \bar{b}_1^d r_{k-1} + \bar{b}_2^d r_{k-2}\}, \quad (103)$$

where  $d_1 = d_2 = 0.5$ . The controller given by (103) is the discrete-time counterpart of PID controller (48). Let the sampling period  $T_s$  is so small that the degree of time-scale separation between fast and slow motions in the closed-loop system is large enough, then  $g_k = g_{k-1} = g_{k-2}$ ,  $\forall k$ . From (96) and (99), the FMS characteristic equation

$$z^2 + 0.5 \left[ \frac{g}{\bar{g}} - 1 \right] z + 0.5 \left[ \frac{g}{\bar{g}} - 1 \right] = 0 \quad (104)$$

results, where the parameter  $g$  is treated as a constant value during the transients in the FMS. Take  $\bar{g} = 4$ , then it can be easily verified, that  $\max\{|z_1|, |z_2|\} \leq 0.6404$  for all  $g \in [2, 6]$ , where  $z_1$  and  $z_2$  are the roots of (104). Hence, the stability of the FMS is maintained for all  $g \in [2, 6]$ . Let  $T = 0.3$  s. and  $\eta = 10$ . Take  $T_s = T/\eta = 0.03$  s. The simulation results for the output of the system (59) controlled by the algorithm (103) are displayed in Figs. 11–15, where the initial conditions are zero. Note, the simulation results shown in Figs. 11–15 approach ones shown in Figs. 5–9 when  $T_s$  becomes smaller.

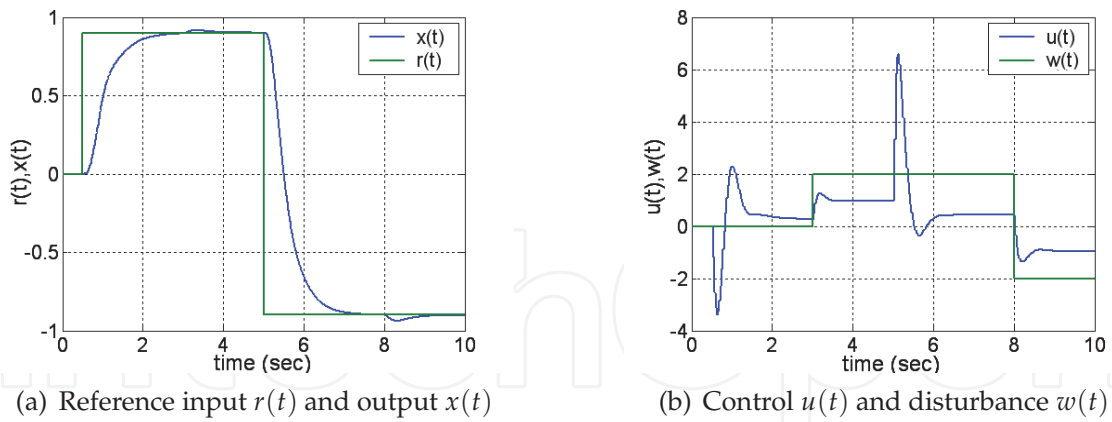


Fig. 11. Output response of the system (59) with controller (103) for a step reference input  $r(t)$  and a step disturbance  $w(t)$ , where  $b_1^d = 0$  (the reference model is a system of type 1)

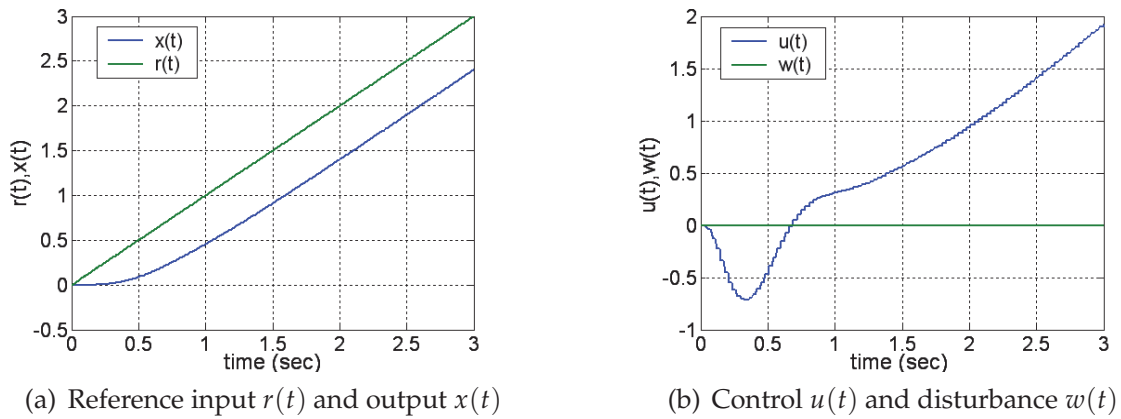


Fig. 12. Output response of the system (59) with controller (103) for a ramp reference input  $r(t)$ , where  $b_1^d = 0$  and  $w(t) = 0$  (the reference model is a system of type 1)

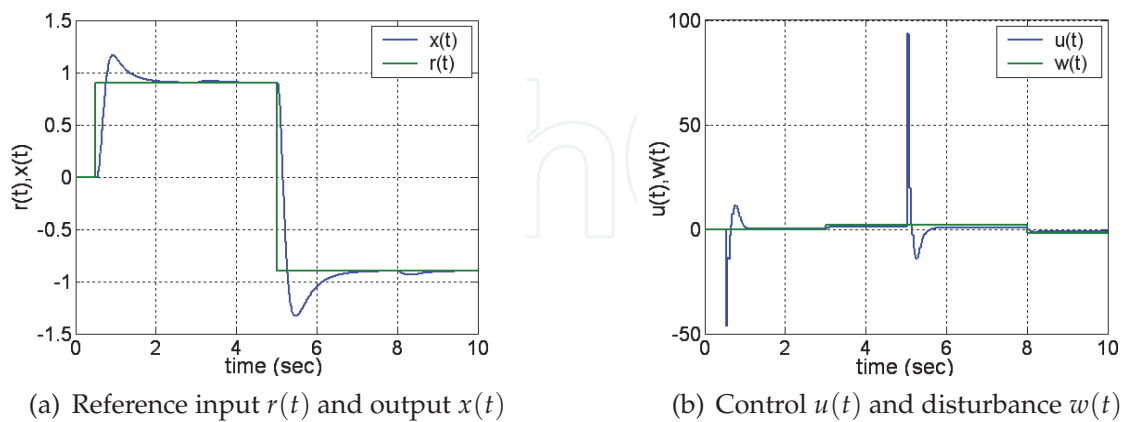


Fig. 13. Output response of the system (59) with controller (103) for a step reference input  $r(t)$  and a step disturbance  $w(t)$ , where  $b_1^d = a_1^d$  (the reference model is a system of type 2)

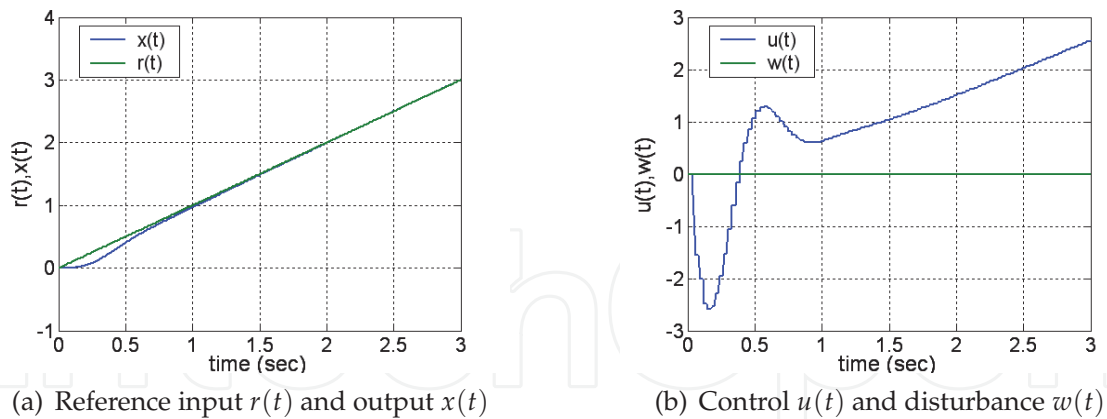


Fig. 14. Output response of the system (59) with controller (103) for a ramp reference input  $r(t)$ , where  $b_1^d = a_1^d$  and  $w(t) = 0$  (the reference model is a system of type 2)

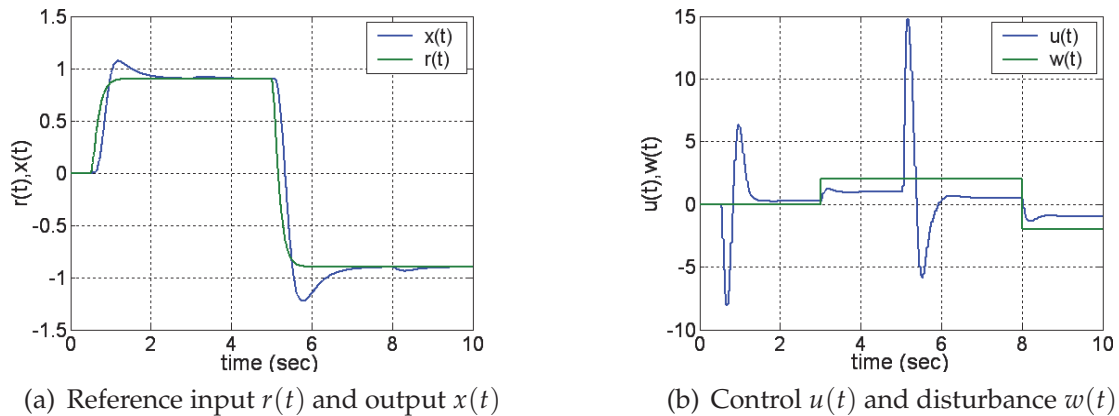


Fig. 15. Output response of the system (59) with controller (103) for a smooth reference input  $r(t)$  and a step disturbance  $w(t)$ , where  $b_1^d = a_1^d$  (the reference model is a system of type 2)

## 9. Conclusion

In accordance with the presented above approach the fast motions occur in the closed-loop system such that after fast ending of the fast-motion transients, the behavior of the overall singularly perturbed closed-loop system approaches that of the SMS, which is the same as the reference model. The desired dynamics realization accuracy and an acceptable level of disturbance rejection can be provided by increase of time-scale separation degree between slow and fast motions in the closed-loop system. However, it should be emphasized that the time-scale separation degree is bounded above in practice due to the presence of unmodeled dynamics or time delay in feedback loop. So, the effect of unmodeled dynamics and time delay on FMS transients stability should be taken in to account in order to proper selection of controller parameters (Yurkevich, 2004). This effect puts the main restriction on the practical implementation of the discussed control design methodology via singular perturbation technique. The presented design methodology may be used for a broad class of nonlinear time-varying systems, where the main advantage is the unified approach to continuous as well as digital control system design that allows to guarantee the desired output transient performances in the presence of plant parameter variations and unknown external

disturbances. The other advantage, caused by two-time-scale technique for closed-loop system analysis, is that analytical expressions for parameters of PI, PID, or PID controller with additional lowpass filtering for nonlinear systems can be found, where controller parameters depend explicitly on the specifications of the desired output behavior. The presented design methodology may be useful for real-time control system design under uncertainties and illustrative examples can be found in (Czyba & Błachuta, 2003; Khorasani et al., 2005).

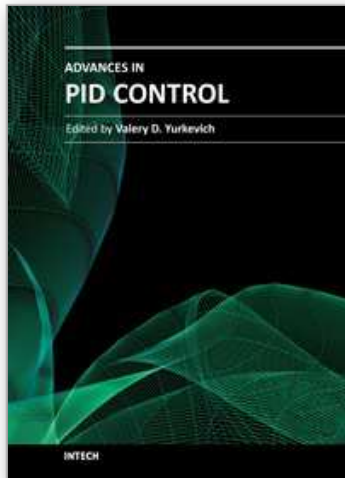
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## **Advances in PID Control**

Edited by Dr. Valery D. Yurkevich

ISBN 978-953-307-267-8

Hard cover, 274 pages

**Publisher** InTech

**Published online** 06, September, 2011

**Published in print edition** September, 2011

Since the foundation and up to the current state-of-the-art in control engineering, the problems of PID control steadily attract great attention of numerous researchers and remain inexhaustible source of new ideas for process of control system design and industrial applications. PID control effectiveness is usually caused by the nature of dynamical processes, conditioned that the majority of the industrial dynamical processes are well described by simple dynamic model of the first or second order. The efficacy of PID controllers vastly falls in case of complicated dynamics, nonlinearities, and varying parameters of the plant. This gives a pulse to further researches in the field of PID control. Consequently, the problems of advanced PID control system design methodologies, rules of adaptive PID control, self-tuning procedures, and particularly robustness and transient performance for nonlinear systems, still remain as the areas of the lively interests for many scientists and researchers at the present time. The recent research results presented in this book provide new ideas for improved performance of PID control applications.

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