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Adaptive Gain PID Control for Mechanical Systems

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1. Introduction

The design and use of PID controllers is a part of what has been denominated Classical Control, which as the name implies, has been studied for many years (DiStefano et al. 1996), however it continues to be a source for research (Alvarez et al. 2008), (Ang et al. 2008), (Su et al. 2010).

The structure of the controller contains a differential term to aid in the reduction of system friction and an integral term to attenuate steady state error. The drawbacks of this control scheme, particularly for nonlinear mechanical systems, include the difficulty in selecting adecuate controller gains, a process usually refered to as tuning. The difficulty usually lies in the fact that if the controller gains are set too small, the control objective may never be reached, whereas the selection of excesively large controller gains may result in system instability.

Many approaches have been proposed to properly tune PID gains (Ang et al. 2008), (Chang & Jung 2009), (Su et al. 2010), others have tried to improve upon the performance of the PID controller by including modern control techniques such as neural networks, fuzzy logic or variable structure control (Guerra et al. 2005).

Among these, variable structure control, specifically sliding mode control, has shown to possess certain desirable properties, such as disturbance rejection and finite time convergence; however it also presents unwanted behaviors mainly high frequency switching, a phenomenon refered to as chattering, which is undesirable in mechanical systems because it can cause accelerated wear of the mechanical components as well as activate unmodeled dynamics. One solution presented is to include an adaptive gain in the high frequency term so that the desirable properties may be exploited, and the undesirable effects minimized, achieving an enhanced performance (Guerra et al. 2005).

2. Background

The control of mechanical systems is subject to many difficulties, as evidenced by the research devoted to such aspects of mechanical systems as dead zone (Zhang & Gen 2009), and friction (Canudas de Wit et al. 1995).

Consider a first order mechanical system given by (Canudas de Wit et al. 1995)

$$\ddot{x} = \frac{u - f(\dot{x})}{m} \tag{1}$$

where x is the position variable, m is the mass, u is the control input and the function $f(\dot{x})$ denotes the nonlinear friction force. The PID control law is given by (Canudas de Wit et al. 1995):

$$u = K_p e + K_i \int_0^t e(\tau)d\tau + K_d \dot{e} \tag{2}$$

where K_p, K_d and K_i, are the proportional, derivative and integral gains, respectively and the error term is given by $e = x - x_d$, where x_d is the constant desired value.

Mechanical systems under integral control action have been known to present limit cycles, due in part to the complex nature of the friction force. This results in the system never reaching the desired position (Canudas de Wit et al. 1995).

The authors in (Guerra et al. 2005) present an approach considering a PD controller which is modified by the inclusion of a neural networks chattering controller that allows the high frequency swithching when the system is away from the desired position, but tends to vanish once the desired position is reached. In this chapter we will build upon that result and apply a similar stragegy to a PID controller.

3. Controller design

Consider the system (1) with unit mass and friction force given by (Makkar et al. 2005):

$$f(\dot{x}) = \gamma_1 \left[\tanh(\gamma_2 \dot{x}) - \tanh(\gamma_3 \dot{x})\right] + \gamma_4 \tanh(\gamma_5 \dot{x}) + \gamma_6 \dot{x} \tag{3}$$

The objective is for the error e to reach zero, *i.e.,*:

$$\lim_{t \to \infty} e(t) = 0 \tag{4}$$

where

$$e = x - x_d \tag{5}$$

to achieve this, the controller (2) is modified to:

$$u = -K_p e - K_i \zeta - [2\varepsilon + \delta K_d]\dot{e} \tag{6}$$

where

$$\dot{\zeta} = e \tag{7}$$

$$\dot{\delta} = -\alpha \ln(\delta + 1) + K_r \frac{[\delta + 1]}{\ln(\delta + 1) + 1} e^2 \tag{8}$$

where $\varepsilon > 0$, $\alpha > 0$ and $K_r > 0$ are constant parameters. The term ζ is used for simplicity in place of the term $\int_0^t e(\tau)d\tau$. It should be noted that for an intnitial condition $\delta(t_0) = \delta_0 \geq 0$, $\delta(t) \geq 0$, for all $t \geq t_0$ (Hench, 1999). In addtion, the adaptive gain can be considered to be bound by $\delta \leq \delta_M$ by taking into account that a practical controller is subject to saturation.

4. Closed loop system

To analyze the stability of the closed loop system, the following variable change is introduced:

$$\omega = \varepsilon \zeta + e \tag{9}$$

which is used to form the vector $\xi = [\omega \ e \ \dot{e}]^T$. Using equations (1), (3), (5), (6) and (7) the dynamic of the closed loop system is given by:

$$\dot{\xi} = A(\delta)\xi + B(\dot{e}) \tag{10}$$

where

$$A(\delta) = \begin{bmatrix} 0 & \varepsilon & 1 \\ 0 & 0 & 1 \\ -\varepsilon^{-1}K_i & -\left[K_p - \varepsilon^{-1}K_i\right] & -\left[2\varepsilon + \delta K_d\right] \end{bmatrix} \tag{11}$$

$$B(\dot{e}) = [0 \ \ 0 \ \ -f(\dot{e})]^T \tag{12}$$

The state δ contained in $A(\delta)_{3,3}$ is governed by the dynamic adaptation law (8). By setting (8) and (10) to zero, it can be seen that the origin of the state space ($\xi = 0$, $\delta = 0$) is the unique equillibrium for the system which, when applied to equation (9), implies $\zeta = 0$.

5. Stability analysis

Consider the candidate Lyapunov function:

$$V(\xi, \delta) = \xi^T P_c \xi + (\delta + 1)\ln(\delta + 1) \tag{13}$$

where

$$P_c = \frac{1}{2}\left(P + P^T\right) \tag{14}$$

$$P = \begin{bmatrix} \beta\varepsilon^{-1}K_i & 0 & 0 \\ 0 & \beta\left[K_p - \varepsilon^{-1}K_i\right] & 0 \\ 0 & 2\beta\varepsilon & \beta \end{bmatrix} \tag{15}$$

It should be noted that $V > 0$ implies that $P_c > 0$, which by applying Sylvester's Theorem (Kelly et al. 2005) requires that $\beta > 0$, the complete analysis to ensure positivity of matrix P_c is presented in the next section. To simplify stability analysis, the equality $\xi^T P_c \xi = \xi^T P \xi$ is considered so that expression (13) can be restated as

$$V(\xi, \delta) = \xi^T P \xi + (\delta + 1)\ln(\delta + 1) \tag{16}$$

The time derivative of (16) along the closed loop system (8) and (10) yields:

$$\dot{V} = -\xi^T Q(\delta)\xi - R(\xi) - \alpha\ln(\delta + 1)[\ln(\delta + 1) + 1] \tag{17}$$

where

$$R(\xi) = -B(\dot{e})^T P \xi - \xi^T P B(\dot{e}) = 2\beta\varepsilon e \, f(\dot{e}) + 2\beta\dot{e}f(\dot{e}) \tag{18}$$

$$W(\delta) = - \left[PA(\delta) + A(\delta)^T P \right] \tag{19}$$

$$Q(\delta) = \frac{1}{2} \left[W(\delta) + W(\delta)^T \right] - \hat{e}_2 K_r (\delta + 1) \hat{e}_2^T$$
$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\beta\varepsilon \left[K_p - \varepsilon^{-1} K_i \right] - K_r (\delta + 1) & \beta\varepsilon \left[2\varepsilon + \delta K_d \right] \\ 0 & \beta\varepsilon \left[2\varepsilon + \delta K_d \right] & 2\beta \left[\varepsilon + \delta K_d \right] \end{bmatrix} \tag{20}$$

where $\hat{e}_2 = [0 \ \ 1 \ \ 0]^T$. Regarding equation (3) used in (18), every term in the expression can be bound by $b \tanh(c) \leq |b||c| \forall b, c \in \Re$. It can be stated that equation (3) satisfies:

$$- ef(\dot{e}) \leq K_\gamma |e||\dot{e}| \tag{21}$$

where

$$K_\gamma = \gamma_1 |\gamma_2 - \gamma_3| + \gamma_4 \gamma_5 + \gamma_6 \tag{22}$$

it should be remembered that all the parameters γ_ι for $\iota = 1 \ldots 6$ are positive constants. Regarding the term $\dot{e} f(\dot{e})$ in equation (18), it can be seen that this term is positive for $\gamma_2 \geq \gamma_3 > 0$ by using the properties of hiperbolic functions in equation (3) and considering $\dot{e} \to \Theta \geq 0$ (first quadrant) we find that:

$$\tanh \left([\gamma_2 - \gamma_3] \Theta \right) \left[1 - \tanh (\gamma_2 \Theta) \tanh (\gamma_3 \Theta) \right] \geq 0 \tag{23}$$

given that Θ, γ_2 and γ_3 are considered to be positive, the second term will always be non negative, whereas the first will be non negative if $\gamma_2 \geq \gamma_3 > 0$ (as was previously stated). These considerations apply also when $\dot{e} \to \Theta \leq 0$ (third quadrant). By applying (21), (22) and (23) in (18), along with the previously stated $\delta \leq \delta_M$ equation (17) can be bounded by:

$$\dot{V} \leq - \begin{bmatrix} |e| \\ |\dot{e}| \end{bmatrix}^T Q_c \begin{bmatrix} |e| \\ |\dot{e}| \end{bmatrix} - 2\beta \dot{e} f(\dot{e}) - \alpha \ln(\delta + 1) \left[\ln(\delta + 1) + 1 \right] \tag{24}$$

where

$$Q_c = \begin{bmatrix} 2\beta\varepsilon \left(K_p - \varepsilon^{-1} K_i \right) - K_r (\delta_M + 1) & \beta\varepsilon (2\varepsilon + \delta_M K_d - K_\gamma) \\ \beta\varepsilon (2\varepsilon + \delta_M K_d - K_\gamma) & 2\beta\varepsilon \end{bmatrix} \tag{25}$$

In the follwing section, a process for tuning the controller gains will be introduced, this will also be useful in provinding sufficient conditions to guarantee the positiviy of matrices P_c and Q_c.

6. Controller tuning

In order to establish bounds on the controller gains, we first analyze the matrix P_c defined in expression (14). To find the roots of this symmetric matrix, we apply Sylvester's Theorem (Kelly et al. 2005), which generates a cubic polynomial of the form $\varepsilon^3 - 3b\varepsilon + 2a < 0$ with $a = \frac{K_i}{2}$ and $b = \frac{K_p}{3}$ which is satisfied for $b^3 > a^2$. Using $\exp(\bullet)$ to denote the exponential function, we define the terms $v_{1,2} = -a \pm ic = r \exp \left[\mp i (\theta - \pi) \right]$, $c = \sqrt{b^3 - a^2}$, $r = b^{\frac{3}{2}}$, $\theta = \arctan \left(\frac{c}{a} \right)$, $\vartheta_{1,2} = 2a + v_{1,2} = r \exp (\pm i\theta)$ and using Euler's formula the roots are:

$$\varepsilon_1 = -(p_1 + p_2) = -2r^{\frac{1}{3}} \cos\left(\frac{\theta}{3}\right) \tag{26}$$

$$\varepsilon_2 = \sqrt{y_1 y_2} = 2r^{\frac{1}{3}} \sin\left(\frac{\pi - 2\theta}{6}\right) \tag{27}$$

$$\varepsilon_1 = q_1 + q_2 = 2r^{\frac{1}{3}} \cos\left(\frac{\pi - \theta}{3}\right) \tag{28}$$

considering that $p_\iota = \vartheta_\iota^{\frac{1}{3}}$, $q_\iota = v_\iota^{\frac{1}{3}}$, and $y_\iota = p_\iota - q_\iota$ for $\iota = 1, 2$. Given that $c \in \Re^+$, $\theta \in \left(0, \frac{\pi}{2}\right]$. Taking then $\varepsilon_1 < 0$, and $0 < \varepsilon_2 \leq \varepsilon_3$. The polynomial $\varepsilon^3 - 3b\varepsilon + 2a = (\varepsilon - \varepsilon_1)(\varepsilon - \varepsilon_2)(\varepsilon - \varepsilon_3) < 0$ is satisfied for all $\varepsilon_2 < \varepsilon < \varepsilon_3$. We propose the definition $b^3 = (\sigma^2 + 1)a^2$ with $\sigma \gg 0$, in other words, the proportional gain in equation (6) is tuned as

$$K_p = \left[\frac{27K_i^2}{4}\left(\sigma^2 + 1\right)\right]^{\frac{1}{3}} \tag{29}$$

Returning to Q_c defined in expression (25), this matrix can be defined as positive by applying Sylvester's Theorem (Kelly et al. 2005) and tuning the derivitave gain in (6) as

$$K_d = \frac{K_\gamma - 2\varepsilon}{\delta_M} \tag{30}$$

the numerator in this equation must be positive, specifically, the constant bound from equation (22) must satisfy $K_\gamma > 2\varepsilon$, so from equations (26)-(30) the positivity of matrices P_c and Q_c is restricted to

$$\max\left\{\frac{K_i}{K_p}, \frac{2\beta K_i + K_r(\delta_M + 1)}{2\beta K_p}, \varepsilon_2\right\} < \varepsilon < \min\left\{\frac{K_\gamma}{2}, \varepsilon_3\right\} \tag{31}$$

By establishing conditions to satisfy (31), which include the values of K_r and K_i selected to generate a valid range for ε, we can conclude that expression (13) is positive definite and that expression (17) is locally negative semi-definite, consequently the system (8) and (10) has a stable equilibrium at the origin. Moreover, by restricting $\eta = \left[\xi^T \delta\right]^T$ by the bounds $\eta_{min} \leq \eta \leq \eta_{max}$ and applying LaSalle's Principle (Kelly et al. 2005) to expression (24) a closed set can be defined as:

$$\Omega = \{\eta \in \Re^4 : \dot{V}(\eta) = 0\} = \{\omega \in \Re, [e \ \dot{e} \ \delta]^T = 0\} \tag{32}$$

Solving (32) along (8) and (10) it can be seen that

$$\lim_{t \to \infty} \omega(t) = 0 \tag{33}$$

and by invoking the variable change (9) that

$$\lim_{t \to \infty} \zeta(t) = 0 \tag{34}$$

therefore the origin of the system defined by (8) and (10) is locally asymptotically stable.

7. Simulation results

In order to test the performance of the proposed controller simulations were carried out using the friction model (3) with the parameters set to $\gamma_1 = 1.25$, $\gamma_2 = 100$, $\gamma_3 = 10$, $\gamma_4 = \gamma_5 = 1.$, $\gamma_6 = 0.1$, $\alpha = 10$, $\beta = 1$, $\delta_M = 1$, $\sigma = 100$, $K_i = K_r = 10$ and the mass is considered to be unitary.

Using the mentioned values in equations (22), (29) and (30) we obtain $K_p = 188.9945$, $K_\gamma = 122.6$ and applying the obtained values to equation (31) we arrive at max $\{0.53, 0.106, 0.52\} < \varepsilon < $ min $\{61.3, 13.7\}$ such that the value chosen was $\varepsilon = 6.808$ and hence $K_d = 108.9848$.

Figure 1 shows the performance of the position regulation. It should be noted that there is a very small overshot and that no limit cycles are present. The asymptotic stability can be easily seen in Figure 2 where the error is presented, it is clear that the error is still decreasing, achieving an accuracy within a micrometer after 200 seconds.
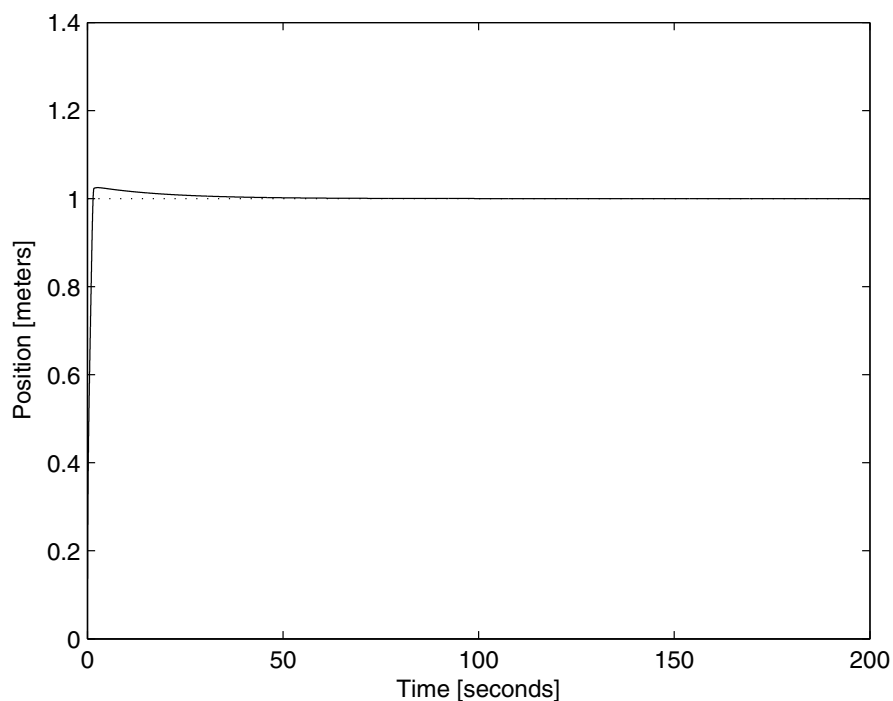
Fig. 1. Controller Performance: Achieved Position.

Figure 3 shows the evolution of the adaptive gain δ, it is clear that as the error approaches zero, so too does the value of the adaptive gain, and consequently so does the value of the control variable, shown in Figure 4.

The control variable initially presents a large value which then decreases. It can be inferred from the asymptotic stability that the control variable decreses asymptotically with time as shown in Figures 4 and 5. Figure 5 shows the control variable in more detail. During the first ten seconds a small oscillation can be seen but it is eliminated after approximately 3 seconds.

Figures 6 and 7 show that the term $\zeta = \int_0^t e(\tau)d\tau$ also approaches zero. It can be clearly seen, especially in Figure 7 that ζ asymptotically approaches zero.
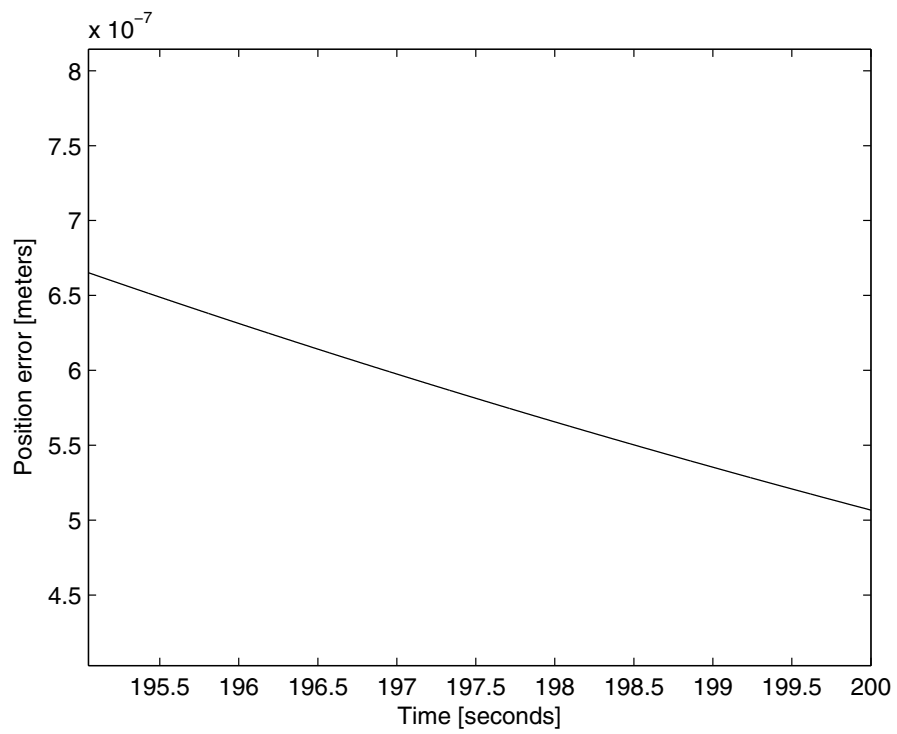
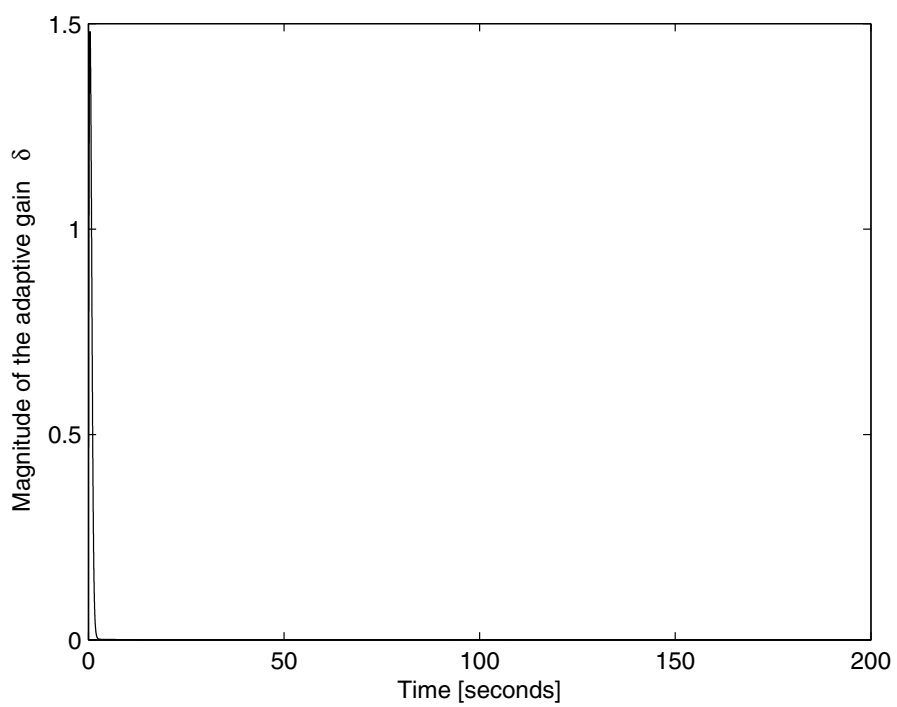Fig. 2. Controller Performance: Position Error.

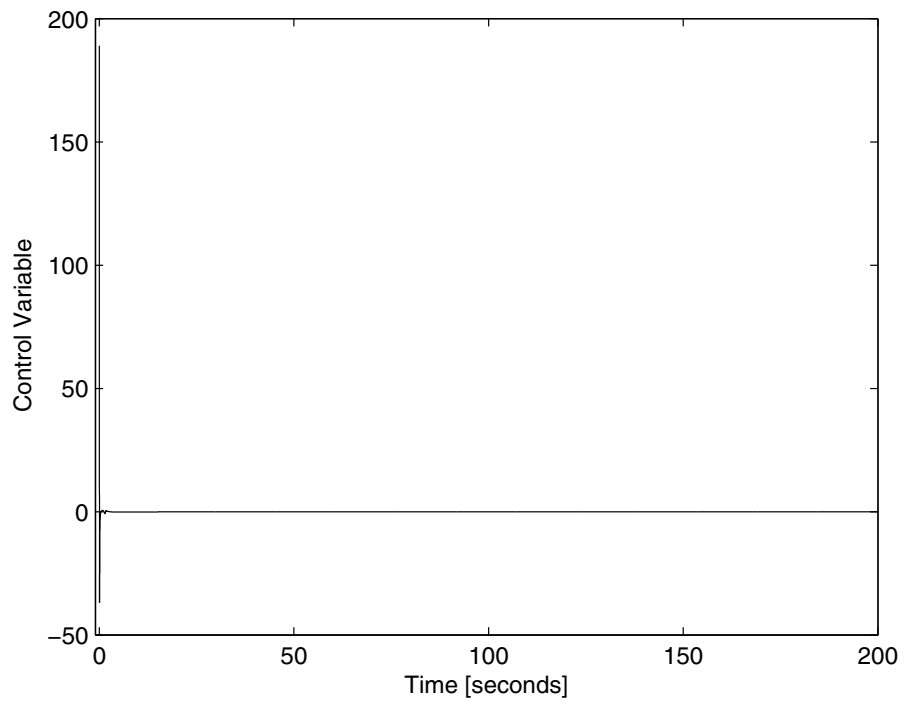Fig. 3. Controller Performance: Adaptive Gain.
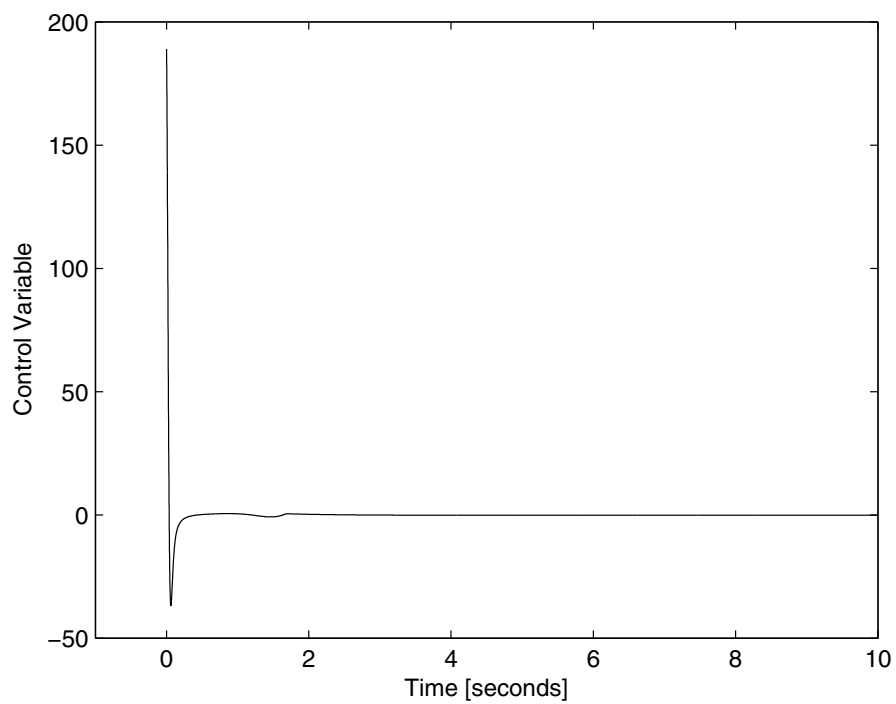
Fig. 4. Controller Performance: Control Variable.

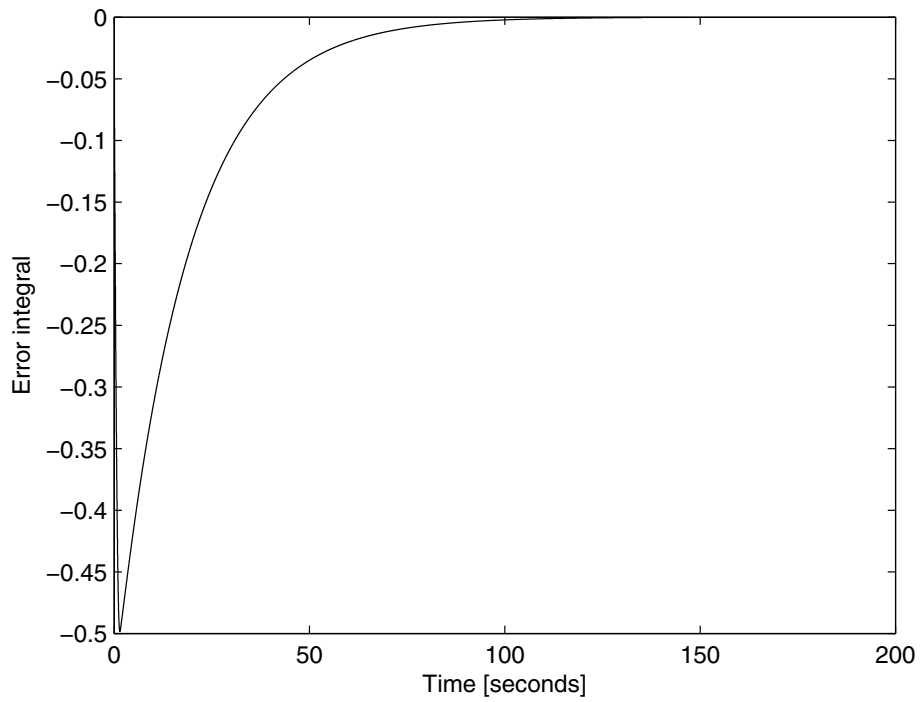Fig. 5. Controller Performance: Control Variable (detail).

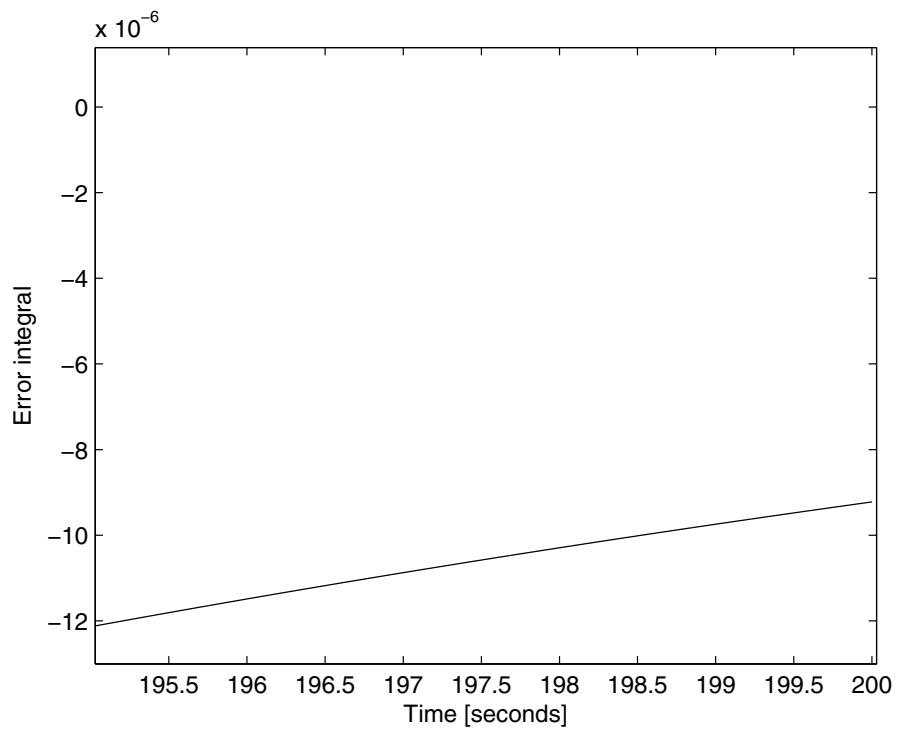Fig. 6. Controller Performance: Error Integral.

Fig. 7. Controller Performance: Error Integral (detail).

Figure 8 shows how with increasing time, the value of the adaptive gain draws even closer to zero. The same can be said of the error in Figure 9 and of ζ in Figure 10.
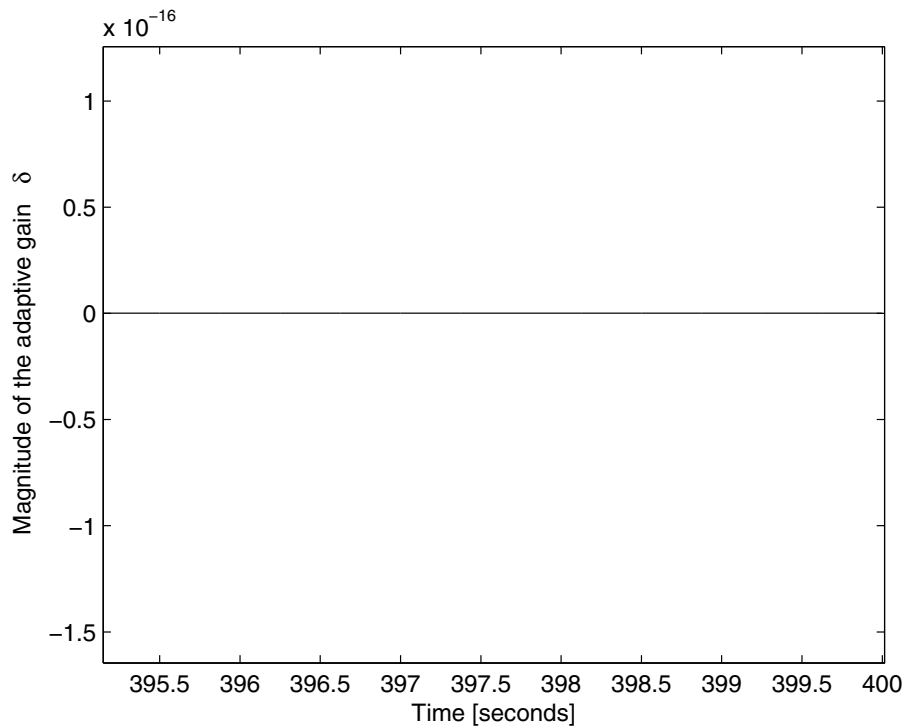

Fig. 8. Controller Performance: Adaptive Gain (detail).
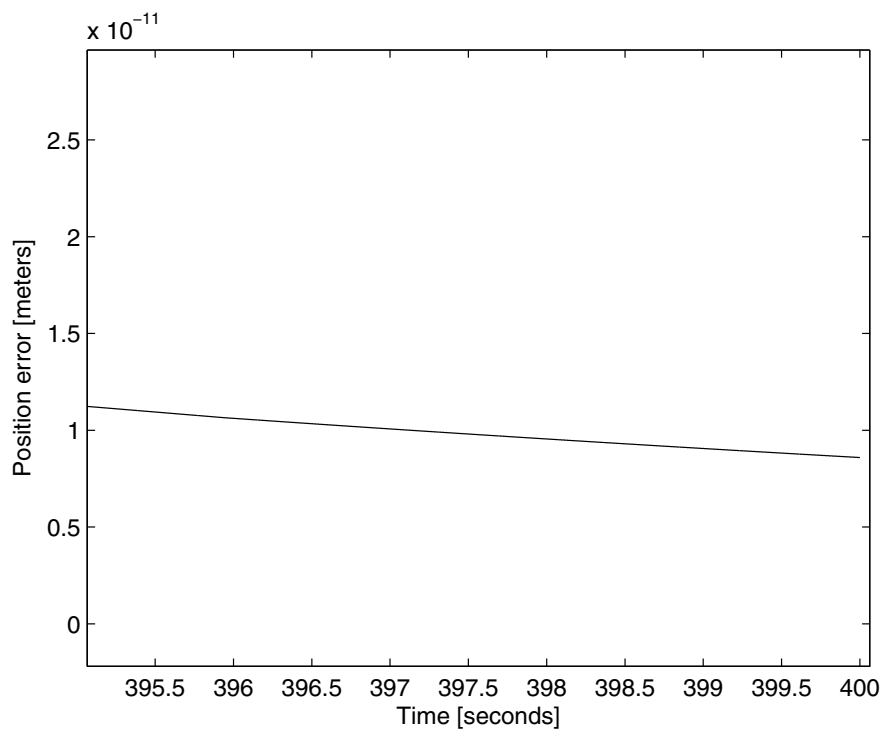
Fig. 9. Controller Performance: Position Error (detail).
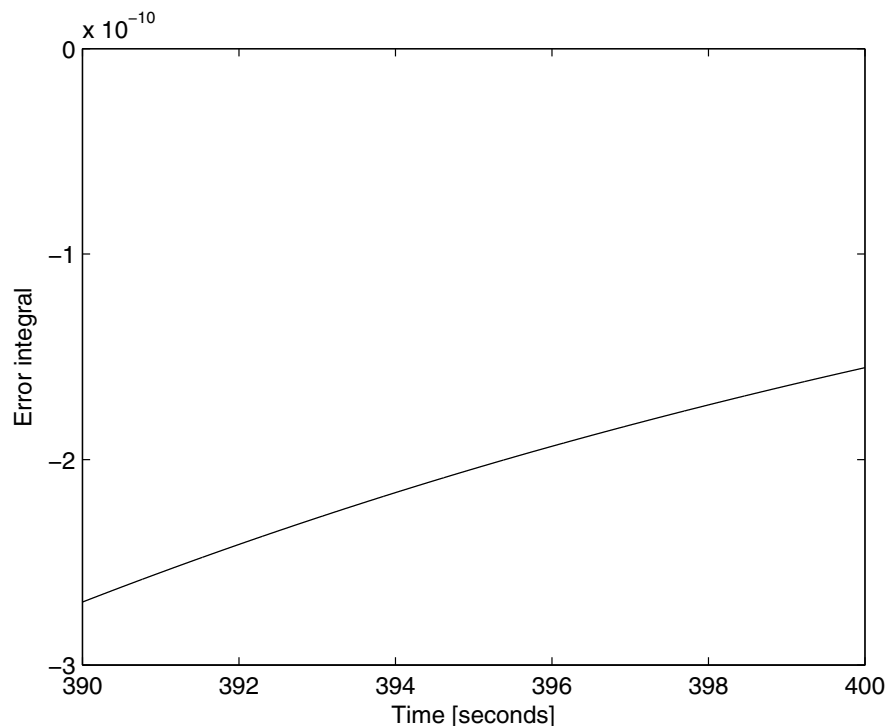
Fig. 10. Controller Performance: Error Integral (detail).

8. Conclusions

An extension to the traditional PID controller has been presented that incorporates an adaptive gain. The adaptive gain PID controller presented is demonstrated to asymptotically stabilize the system, this is shown in the simulations where the position error converges to zero.

In the presented analysis, considerations using known bounds of the system (such as friction coefficients) are used to show the stability of the system as well as to tune the controller gains K_p and K_d.

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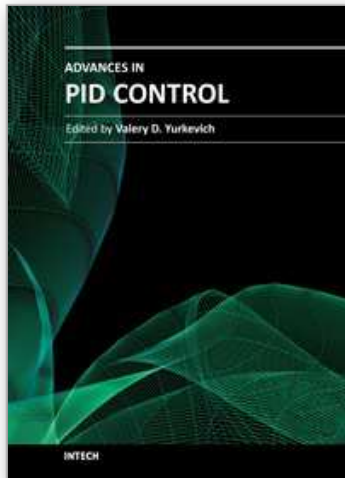
Advances in PID Control

Edited by Dr. Valery D. Yurkevich

Since the foundation and up to the current state-of-the-art in control engineering, the problems of PID control steadily attract great attention of numerous researchers and remain inexhaustible source of new ideas for process of control system design and industrial applications. PID control effectiveness is usually caused by the nature of dynamical processes, conditioned that the majority of the industrial dynamical processes are well described by simple dynamic model of the first or second order. The efficacy of PID controllers vastly falls in case of complicated dynamics, nonlinearities, and varying parameters of the plant. This gives a pulse to further researches in the field of PID control. Consequently, the problems of advanced PID control system design methodologies, rules of adaptive PID control, self-tuning procedures, and particularly robustness and transient performance for nonlinear systems, still remain as the areas of the lively interests for many scientists and researchers at the present time. The recent research results presented in this book provide new ideas for improved performance of PID control applications.

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