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Higher Dimensional Cosmological Model of the Universe with Variable Equation of State Parameter in the Presence of G and Λ

G S Khadekar[1], Vaishali Kamdi[1] and V G Miskin[2]

[1]*Department of Mathematics, Rashtrasant Tukadoji Maharaj Nagpur University, Mahatma Jyotiba Phule Educational Campus, Amravati Road, Nagpur-440033*

[2]*Department of Mathematics, Yeshwantrao Chavan College of Engineering (YCCE), Hingna Road, Wanadongri, Nagpur- 441110*

India

1. Introduction

The Kaluza-Klein theory has a long and venerable history. However, the original Kaluza version of this theory suffered from the assumption that the 5-dimensional metric does not depend on the extra coordinate (the cylinder condition). Hence the proliferation in recent years of various versions of Kaluza-Klein theory, supergravity and superstrings. The number of authors (Wesson (1992), Chatterjee et al. (1994a), Chatterjee (1994b), Chakraborty and Roy (1999)) have considered multi dimensional cosmological model. Kaluza-Klein achievements is shown that five dimensional general relativity contains both Einstein's four-dimensional theory of gravity and Maxwell's theory of electromagnetism.

Chatterjee and Banerjee (1993) and Banerjee et al. (1995) have studied Kaluza-Klein inhomogeneous cosmological model with and without cosmological constants respectively. So far there has been many cosmological solution dealing with higher dimensional model containing a variety of matter field. However, there is a few work in a literature where variable G and Λ have been consider in higher dimension.

Beesham (1986a, 1986b) and Abdel-Rahman (1990) used a theory of gravitation using G and Λ as no constant coupling scalars. Its motivation was to include a G-coupling 'constant' of gravity as pioneered by Dirac (1937). Since the similar papers by Dirac (1938), a possible variation of G has been investigated with no success by several teams, through geophysical and astronomical observations, at the scale of solar system and with binary systems (Uzan (2003)). However, it should be stressed that we are talking here about time variations at a cosmological scale and cosmological observations still can not put strong limits on such a variation, specially at the late times of the evolution. In any case the strongest constraints are the presently observed G_0 value and observational limits of Λ_0. Sistero (1991) found exact solution for zero pressure models satisfying $G = G_0(\frac{R}{R_0})^m$. Barrow (1996) formulated and studied the problem of varying G in Newtonian Gravitation and Cosmology. Exact solutions and all asymptotic cosmological behaviour are found for universe with $G \propto t^m$.

A key object in dark energy investigation is the equation of state parameter ω, which relates pressure and density through an equation of state of the form $p = \omega\rho$. Due to lack of observational evidence in making a distinction between constant and variable ω, usually the equation of state parameter is considered as a constant (Kujat et al. (2002), Bartelmann et al. (2005)) with values $0, \frac{1}{3}, -1$ and $+1$ for dust, radiation, vacuum fluid and stiff fluid dominated Universe respectively. But in general, ω is a function of time or redshift (Chevron and Zhuravlev (2000), Zhuravlev (2001), Peebles and Ratra (2003), Das et al. (2005)). For instance, quintessence models involving scalar fields give rise to time-dependent ω (Ratra and Peebles (1988), Turner and White (1997), Caldwell et al. (1998), Liddle and Scherrer (1999), Steinhardt et al. (1999)). So, there is enough ground for considering ω as time-dependent for a better understanding of the cosmic evolution.

A number of authors have argued in favor of the dependence $\Lambda \sim t^{-2}$ first expressed by Bertolami (1986) and later by several authors (Berman (1990), Beesham (1986b), Singh et al. (1998), Gasperini (1987), Khadekar et al. (2006)) in different context. Motivation with the work of Ibotombi (2007) and Mukhopadhyay et al. (arXiv:0711.4800v1, (2010)), in this work we have studied 5D Kaluza-Klein type metric with perfect fluid and variable G and Λ.

Recently the cosmological implication of a variable speed of light (VSL) during the early evolution of the universe have been considered by [Belincho and Chakrabarty (2003), Belincho (2004)]. Varying speed of light (VSL) model proposed by Moffat (1993) and Albrecht and Maguejio (1999) in which light was traveling faster in the early periods of the existence of the universe, might solve the same problems as inflation. Einstein's field equations for Friedmann-Roberton-Walker (FRW) space time in the VSL theory have been solved by Barrow (1999), who also obtained the rate of variation of speed of light required to solve the flatness and cosmological constant problem for a review of these theories.

We have obtained exact solutions for Zeldovich fluid models satisfying $G = G_0(\frac{R}{R_0})^m$ with global equation of state of the form $p = \frac{1}{3}\phi\rho$, where ϕ is a function of scale factor R. In section 2 and 3 of the chapter we have studied two variable Λ model of the form $\Lambda \sim (\frac{\dot{R}}{R})^2$ and $\Lambda \sim \rho$ under the assumption that the equation of state parameter ω is a function of time. It is shown that possibility of signature flip of the deceleration parameter q. In section 4 of the chapter we have examined the perfect fluid cosmological model by considering the equation of state parameter ω is constant with varying G, c and Λ by using Lie method given by Ibrabimov (1999) and find the possible forms of the constants G, Λ and c that integrable the field equations in the framework of Kaluza-Klein theory of gravitation.

2. Field equations

We consider the 5D Robertson-Walker metric

$$ds^2 = c^2(t)dt^2 - R^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] + A^2(t)d\psi^2, \tag{1}$$

where $R(t)$ is the scale factor, $A(t) = R^n$ and $k = 0,\ -1\ \ or + 1$ is the curvature parameter for flat, open and closed universe, respectively. The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor of a perfect fluid

$$T_{ij} = (p+\rho)u_i u_j - pg_{ij}, \tag{2}$$

where, ρ is the energy density of the cosmic matter and p is its pressure and u_i is the five velocity vector such that $u_i u^j = 1$.

The Einstein field equations are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi G(t)\left[\frac{T_{ij}}{c^4} - \frac{\Lambda(t)}{8\pi G}g_{ij}\right], \tag{3}$$

where the cosmological term Λ is time-dependent and c, the velocity of light in vacuum. In the follwing two section we have assumed the velocity of light is unity i.e. $c = 1$. The conservation equation for variable G and Λ is given by

$$\dot{\rho} + (3+n)\frac{\dot{R}}{R}(p+\rho) = -\left(\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G}\right). \tag{4}$$

Using co-moving co-ordinates $u^i = (1,0,0,0,0)$ in (2) and with metric (1), the Einstein field equations become

$$8\pi G\rho = 3\left[(n+1)\frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right] - \Lambda(t), \tag{5}$$

$$8\pi Gp = -(n+2)\frac{\ddot{R}}{R} - (n^2+n+1)\frac{\dot{R}^2}{R^2} - \frac{k}{R^2} + \Lambda(t), \tag{6}$$

$$8\pi Gp = -3\left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right) + \Lambda(t). \tag{7}$$

where dot $(\cdot)$ denotes derivative with respective to t.
The usual conservation law yields (i.e. $T^{ij}_{;j} = 0$)

$$\dot{\rho} + (3+n)(\rho+p)\frac{\dot{R}}{R} = 0. \tag{8}$$

Using Eq.(8) in Eq.(4)we have,

$$8\pi\dot{G}\rho + \dot{\Lambda} = 0. \tag{9}$$

Equations (5), (6) and (9) are the fundamental equations and they reduce to standard Friedmann cosmology when G and Λ are constants. Equations (5) and (6) may be written as

$$3(n+2)\ddot{R} = -8\pi GR(3p+\rho) - 3n^2\frac{\dot{R}^2}{R} + 2\Lambda R, \tag{10}$$

$$3(n+1)\dot{R}^2 = 8\pi GR^2\left[\rho + \frac{\Lambda}{8\pi G}\right] - 3k. \tag{11}$$

Eq.(8) can also be expressed as

$$\frac{d}{dt}(\rho R^{n+3}) + p\frac{d}{dt}(R^{n+3}) = 0. \tag{12}$$

Equations (5), (9) and (12) are independent and they will be used as fundamental. Once the problem is determined, the integration constants are characterized by the observable parameters

$$H_0 = \frac{\dot{R}_0}{R_0}, \tag{13}$$

$$\sigma_0 = \frac{4\pi}{3}\frac{G_0\rho_0}{H_0^2}, \tag{14}$$

$$q_0 = -\frac{\ddot{R}_0}{R_0 H_0^2}, \tag{15}$$

$$\epsilon_0 = \frac{p_0}{\rho_0}, \tag{16}$$

which must satisfy Einstein's equations at present cosmic time t_0:

$$\Lambda_0 = 3H_0^2\left[\sigma_0(3\epsilon_0+1) - \frac{(n+2)}{2}q_0 + \frac{n^2}{2}\right], \tag{17}$$

$$\frac{k}{R_0^2} = H_0^2\left[3(1+\epsilon_0)\sigma_0 - \frac{(n+2)}{2}q_0 + \frac{(n^2-2n-2)}{2}\right], \tag{18}$$

and the conservation Eq. (9) can be written as

$$\dot{\Lambda}_0 G_0 + 6\dot{G}_0 H_0^2\sigma_0 = 0. \tag{19}$$

3. Solutions of field equations

We find out the solutions of the field equations for two different equation of state: (i) $p = \frac{1}{3}\rho\phi$ and (ii) $p = \omega(t)\rho$

3.1 Case (I):

We assume the global equation of state

$$p = \frac{1}{3}\rho\phi, \tag{20}$$

where ϕ is a function of the scale factor R.
From Eq.(12) and Eq.(20) we obtain

$$\frac{1}{\psi}\frac{d\psi}{dR} + \frac{(n+3)}{3}\frac{\phi}{R} = 0, \tag{21}$$

where

$$\psi = \rho R^{n+3}. \tag{22}$$

Equation (21) be the first condition to determine the problem; either ϕ or ψ may be in term of arbitrary function. If ϕ is a given explicit function of R, then Eq.(20) is determined and ψ follows from Eq.(21)

$$\psi = \psi_0 \; exp\left[-\int \frac{(n+3)}{3}\frac{\phi}{R}dR\right]. \tag{23}$$

If ψ is given function, from Eq.(20) we get ϕ as

$$\phi = -\frac{3}{(n+3)}\frac{R}{\psi}\frac{d\psi}{dR}. \tag{24}$$

Substitute the value of ψ from Eq.(22) in the Friedmann's Eq.(5) we get

$$3(n+1)\dot{R}^2 = 8\pi G\psi R^{-(n+1)} + \Lambda R^2 - 3k. \tag{25}$$

Eqs. (9) and (22) with $\frac{d}{dt} = (\dot{R}\frac{d}{dR})$ give

$$8\pi\frac{dG}{dR} + \psi^{-1}R^{n+3}\frac{d\Lambda}{dR} = 0. \tag{26}$$

If $G(R)$ is given then after integrating from Eq.(26) we get $\Lambda(R)$ and from Eq.(25) we get $R = R(t)$ and the problem is solved. Similarly if $\Lambda(R)$ may be given instead of $G(R)$ derives from Eq.(26) we get $G(R)$ and then from Eq. (25) we get $R = R(t)$.

3.1.1 Zeldovich fluid satisfying $G = G_0(\frac{R}{R_0})^m$

To solve Eq.(26) for Zeldovich fluid with $\phi = 3$. In this case (23) gives,

$$\psi = \rho_0\left(\frac{R_0}{R}\right)^{n+3}. \tag{27}$$

Substituting ψ from (26) into (25), we have

$$\Lambda = \Lambda_0 + B_m\left[1 - \left(\frac{R}{R_0}\right)^{[m-2(n+3)]}\right]R_0^{-(n+3)}, \tag{28}$$

where,

$$B_m = \frac{6m}{[m-2(n+3)]}\sigma_0 H_0^2, \tag{29}$$

for $m \neq 2(n+3)$, B_m is a parameter related to the integration constant of Eq.(25). From Eq.(17),

$$\Lambda_0 = 3H_0^2\left[4\sigma_0 - \frac{(n+2)}{2}q_0 + \frac{n^2}{2}\right]. \tag{30}$$

Taking into account Eqs.(26 & 28), Friedmann's Eq. (24) takes the form

$$\dot{R}^2 = \alpha_n R^{m-2(n+2)} + \beta_n R^2 - \frac{1}{(n+1)}k, \tag{31}$$

where

$$\alpha_n = \frac{-4(n+3)}{(n+1)(m-2(n+3))}H_0^2\sigma_0 R_0^{(n+3)-m}, \tag{32}$$

$$\beta_n = \frac{H_0^2}{(n+1)}\left[\left(4 + \frac{2m}{(m-2(n+3))}R_0^{-(n+3)}\right)\sigma_0 - \frac{(n+2)}{2}q_0\right]. \tag{33}$$

Finally the equation for the parameter (18) reduces to

$$\frac{k}{R_0^2} = H_0^2\left[6\sigma_0 - \frac{(n+2)}{2}q_0 + \frac{(n^2-2n-2)}{2}\right]. \tag{34}$$

and (19) is also satisfied.

$$\Lambda_0 G_0 + 6\dot{G}_0 H_0^2 \sigma_0, \tag{35}$$

The model is characterized by the set of parameters $(H_0, G_0, \sigma_0, q_0, m)$ with $m \neq 2(n+3)$. The case $m < 2(n+3)$ implies $B_m < 0$ in Eq.(29) and $\alpha_n > 0$ in Eq.(32) and vice-versa; $\beta_n < (\geq)0$ according to m n, σ_0 and q_0 combine in Eq.(33); $\Lambda_0 < (\geq)0$ as $\sigma_0 < (\geq)(\frac{(n+2)}{2}q_0 - \frac{n^2}{2})$ as given by Eq. (30). From Eq.(34) it is observed that for the curvature parameter $k = +1, \ 0, -1$ we get $[6\sigma_0 - \frac{(n+2)}{2}q_0 + \frac{(n^2-2n-2)}{2}] < (\geq)0$. The models are completely characterized by the set of parameters $(H_0, G_0, \sigma_0, q_0, m)$ with $m \neq 2(n+3)$.

3.2 Case (II):

Let us choose the barotropic equation of state

$$p = \omega\rho. \tag{36}$$

Here, we assume that the equation of state parameter ω is time-dependent i.e. $\omega = \omega(t)$ such that $\omega = (\frac{t}{\tau})^a - 1$ where τ is a constant having dimension of time.
Field equations (5-7) can also be expressed as

$$3(n+1)H^2 + \frac{3k}{R^2} = 8\pi G\rho + \Lambda(t), \tag{37}$$

$$3(n+1)H^2 + 3(n+1)\dot{H} = -8\pi G[(n+1)p + \rho] - \frac{3nk}{R^2} + n\Lambda(t). \tag{38}$$

From Eq. (37), for flat universe ($k = 0$), we get

$$\rho = \frac{3(n+1)H^2 - \Lambda(t)}{8\pi G}. \tag{39}$$

Using Eq. (37) and Eq.(38) with Eq. (36) we get the differential equation of the form

$$\frac{dH}{dt} = \frac{(1+\omega)\Lambda}{3} + [(n+1)\omega - 2]H^2. \tag{40}$$

To solve Eq. (40) we assume two variable Λ model: $\Lambda = 3\alpha H^2$ and $\Lambda = 8\pi G\gamma\rho$.

3.2.1 Case (i): $\Lambda = 3\alpha H^2$

For this case Eq. (40) reduces to

$$\frac{dH}{H^2} = \left[\frac{(n+\alpha+1)t^a}{\tau^a} - (n+3)\right] dt. \tag{41}$$

After solving equation (41) we get,

$$H = \frac{(a+1)\tau^a}{[(n+3)(a+1)t\tau^a - (n+\alpha+1)t^{(a+1)}]}, \tag{42}$$

writing $H = \frac{\dot{R}}{R}$ in Eq. (42) and integrating it further we get the solution set as

$$R(t) = C_2 \left[(n+3)(a+1)\tau^a t^{-a} - (n+\alpha+1)\right]^{-\frac{1}{a(n+3)}}, \tag{43}$$

$$\rho(t) = \frac{3(n-\alpha+1)(a+1)^2\tau^{2a}}{8\pi G}\left[(n+3)(a+1)\tau^a t - (n+\alpha+1)t^{(a+1)}\right]^{-2}, \tag{44}$$

$$\Lambda(t) = \frac{3\alpha(a+1)^2\tau^{2a}}{\left[(n+3)(a+1)\tau^a t - (n+\alpha+1)t^{(a+1)}\right]^2}, \tag{45}$$

where C_2 is an integration constant.
If $a = 0$ then $\omega = 0$ and $\tau = 1$ but Eq. (42) indicates that a can not be equal to zero for physical validity.
Again, using Eqs. (39) and (42) we get

$$\frac{\alpha}{(n+1)} = 1 - \Omega_m = \Omega_\Lambda, \tag{46}$$

where, in absence of any curvature, matter density Ω_m and dark energy density Ω_Λ are related by the equation

$$\Omega_m + \Omega_\Lambda = 1. \tag{47}$$

3.2.2 Case (ii): $\Lambda = 8\pi G\gamma\rho$

For this case Eq. (40) can be written as

$$\frac{dH}{dt} = \left[\left(\frac{1+2\gamma}{1+\gamma}\right)\left(\frac{t}{\tau}\right)^a (n+1) - (n+3)\right]H^2. \tag{48}$$

After solving Eq. (48) we get,

$$H = \frac{(1+\gamma)(a+1)\tau^a}{\left[(n+3)(1+\gamma)(a+1)\tau^a t - (1+2\gamma)(n+1)t^{a+1}\right]}. \tag{49}$$

Using $H = \frac{\dot{R}}{R}$ in Eq. (49) and integrating we get

$$R(t) = C_3\left[(n+3)(1+\gamma)(a+1)\tau^a t^{-a} - (1+2\gamma)(n+1)\right]^{-\frac{1}{a(n+3)}}, \tag{50}$$

$$\rho(t) = \frac{3(n+1)}{8\pi G}\frac{(1+\gamma)(a+1)^2\tau^{2a}}{\left[(n+3)(1+\gamma)(a+1)\tau^a t - (1+2\gamma)(n+1)t^{(a+1)}\right]^2}, \tag{51}$$

$$\Lambda(t) = \frac{3(n+1)\gamma(1+\gamma)(a+1)^2\tau^{2a}}{\left[(n+3)(1+\gamma)(a+1)\tau^a t - (1+2\gamma)(n+1)t^{(a+1)}\right]^2}, \tag{52}$$

where C_3 is an integration constant.
Eq. (50) shows that for physical validity $a \neq 0$. Again from the field equations we can easily find that γ is related to Ω_m and Ω_Λ through the relation

$$\gamma = \frac{\Omega_\Lambda}{\Omega_m}. \tag{53}$$

4. Solution of the field equations by using Lie method

The Einstein's filed equations (3) with varying G, c and Λ for the flat model (1) when for $R = A$ i.e. $n = 1$ and $k = 0$ can be written as

$$8\pi G\rho + \Lambda c^2 = 6H^2, \tag{54}$$

$$-8\pi G p + \Lambda c^2 = 3\left(\frac{\ddot{R}}{R} + H^2\right), \tag{55}$$

$$\dot{\rho} + 4(\rho + p)H = -\frac{\dot{\Lambda} c^4}{8\pi G\rho} - \frac{\dot{G}}{G} + 4\frac{\dot{c}}{c}. \tag{56}$$

We assume that div $(T^i_j) = 0$, then with $p = \omega\rho$, where $\omega = constant$ then Eq. (56) reduces to

$$\dot{\rho} + 4(1+\omega)\rho H = 0, \tag{57}$$

$$-\frac{\dot{\Lambda} c^4}{8\pi G\rho} - \frac{\dot{G}}{G} + 4\frac{\dot{c}}{c} = 0. \tag{58}$$

In this section we shall study the Kaluza-Klein type cosmological model through the method of Lie group symmetries, showing that under the assumed hypothesis there are other solutions of the field equations. We shall show how the Lie method allow us to obtain different solutions for the field equations.
In order to use the Lie method, we can write the field equations: from Eqs. (54)-(55) we obtain

$$\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} = -\frac{8\pi G}{3c^2}(\omega+1)\rho, \tag{59}$$

and therefore,

$$\dot{H} = -\frac{8\pi G}{3c^2}(\omega+1)\rho. \tag{60}$$

From equation (57), we can obtain

$$H = -\frac{1}{4(\omega+1)}\frac{\dot{\rho}}{\rho}. \tag{61}$$

Hence

$$\dot{H} = -\frac{1}{4(\omega+1)}(\dot{\frac{\dot{\rho}}{\rho}}). \tag{62}$$

Hence from Eq. (60)

$$(\dot{\frac{\dot{\rho}}{\rho}}) = \frac{16\pi G}{3c^2}(\omega+1)^2\rho. \tag{63}$$

By taking $A_0 = -\frac{16\pi}{3}(\omega+1)^2$, we get

$$(\dot{\frac{\dot{\rho}}{\rho}}) = \frac{A_0 G}{3c^2}\rho. \tag{64}$$

After expanding Eq. (64) we get

$$\ddot{\rho} = \frac{\dot{\rho}^2}{\rho} + \frac{AG}{3c^2}\rho. \tag{65}$$

We are going now to apply the standard procedure of Lie group analysis to this equation [see Ibragimov (1999) for details and notation].
A vector field X

$$X = \zeta(t,\rho)\partial_t + \eta(t,\rho)\partial_\rho, \tag{66}$$

is a symmetry of equation (65) iff

$$-\zeta f_t - \eta f_\rho + \eta_{tt} + (2\eta_{t\rho} - \zeta_{tt}) + (\eta_{\rho\rho} - 2\zeta_{t\rho})\dot{\rho}^{-2} - \zeta_{\rho\rho}\dot{\rho}^3$$
$$+ (\eta_\rho - 2\zeta_t - 3\dot{\rho}\zeta_\rho)f - \left[\eta t + (\eta_\rho - \zeta_t)\dot{\rho} - \dot{\rho}^2\zeta_\rho\right] f_{\dot{\rho}} = 0, \tag{67}$$

where $f(t,\rho,\dot{\rho}) = \frac{\dot{\rho}^2}{\rho} + \frac{A_0 G}{3c^2}\rho$.
By expanding and separating (67) with respect to power of $\dot{\rho}$ we obtain the overdetermined system:

$$\zeta_{\rho\rho} + \rho^{-1}\zeta_\rho = 0, \tag{68}$$

$$\eta_{\rho\rho} - 2\zeta_{t\rho} + \rho^{-2}\eta - \rho^{-1}\eta_\rho = 0, \tag{69}$$

$$2\eta_{t\rho} - \zeta_{tt} - 3A\frac{G}{c^2}\rho^2\zeta_\rho - 2\rho^{-1}\eta_t = 0, \tag{70}$$

$$\eta_{tt} - A(\frac{\dot{G}}{c^2} - 2G\frac{\dot{c}}{c^3})\rho^2\zeta - 2A\eta\frac{G}{c^2} + (\eta_\rho - 2\zeta_t)A\frac{G}{c^2}\rho^2 = 0. \tag{71}$$

Solving (68) - (71), we find that

$$\zeta(t,\rho) = 2et + a, \qquad \eta(t,\rho) = (bt + d_0)\rho, \tag{72}$$

subject to the constrain

$$\frac{\dot{G}}{G} = 2\frac{\dot{c}}{c} + \frac{bt + d_0 - 4e}{2et - a}, \tag{73}$$

where a, b, e, d_0 are all constants.
In order to solve Eq. (73) we consider the case $b = 0$ and $d_0 - 4e = 0$. In this case the solution (73) reduces to

$$\frac{\dot{G}}{G} = 2\frac{\dot{c}}{c} \Rightarrow \frac{G}{c^2} = B = Constant, \tag{74}$$

which means that constant G and c vary but in such a way that the relation $\frac{G}{c^2}$ is constant.
The solution of the type

$$\frac{dt}{\zeta(t,\rho)} = \frac{d\rho}{\eta(t,\rho)}, \tag{75}$$

is called invariant solution, therefore, from (72) with $b = 0$ and $d_0 - 4e = 0$, the energy density is obtained as:

$$\frac{dt}{-2et + a} = \frac{d\rho}{4e\rho}, \tag{76}$$

$$\Rightarrow \rho = \frac{\rho_0}{(2et - a)^2}, \tag{77}$$

for simplicity we adopt

$$\Rightarrow \rho = \rho_0 t^{-2}, \tag{78}$$

where ρ_0 is constant of integration.
From the value of ρ we can easily obtained the scale factor R as: from (61) after integration we get

$$\rho = A_\omega R^{-4(\omega+1)}, \tag{79}$$

$$\Rightarrow R = \left(A^*_\omega t\right)^{1/2(\omega+1)}, \tag{80}$$

where A_ω and A^*_ω are constants.
From this value of R we can easily find H and from equation (54) we obtain the behaviour of cosmological constant Λ.

$$c^2\Lambda = 6H^2 - \frac{8\pi G\rho}{c^2}\rho, \tag{81}$$

we get

$$\Lambda = (3\beta^2 - 8\pi B\rho_0)\frac{1}{t^2c^2} = \frac{L_0}{t^2c^2}, \tag{82}$$

where $L_0 = (3\beta^2 - 8\pi B\rho_0)$.
Put all the above results in (58) we get the exact behaviour for c:

$$\frac{\dot{c}}{c} + \lambda(\frac{\dot{c}}{c} + \frac{1}{t}) = 0, \tag{83}$$

where $\lambda = \frac{L_0}{8\pi B\rho_0}$ with $\lambda \in R^+$ i.e. a positive real number and thus we get from (83) after integration

$$c = c_0 t^{-\alpha}, \tag{84}$$

where $\alpha = \frac{\lambda}{1+\lambda}$.

Also from

$$\frac{\dot{G}}{G} = 2\frac{\dot{c}}{c} \Rightarrow G = G_0 t^{-2\alpha}. \tag{85}$$

Hence we get the following solutions in the framework of Kaluza-Klien theory of gravitation.

$$G = G_0 t^{-2\alpha} \quad c = c_0 t^{-\alpha}, \quad \Lambda = \Lambda_0 t^{-2(1-\alpha)},$$
$$R = \left(A^*_\omega t\right)^{1/2(\omega+1)}, \quad \rho = \rho_0 t^{-2}. \tag{86}$$

This type of solutions are obtained by Belinchon (2004) in the context of general theory of relativity.

5. Conclusion

In this chapter by considering the gravity with G and Λ a coupling constant of Einstein field equations with usual conservation laws ($T^{ij}_{;j} = 0$), we obtained the exact solution of the field equations. It is shown that the field equations for perfect fluid cosmology are identical to Eisenstein equations for G and Λ including Eq. (12). It is also observed that, the additional conservation Eq. (9) gives the coupling of scalar field with matter.
In the case (I) by introducing the general method of solving the cosmological field equations using a global equation of state of the form $p = \frac{1}{3}\rho\phi$, without loss of generality, we find the exact solutions for Zeldovich matter distribution. It is observed that from Eq. (29) $B_m < 0$ for

the case $m < 2(n+3)$ and $\Lambda_0 < (\geq)0$ as $\sigma_0 < (\geq)(\frac{(n+2)}{2}q_0 - \frac{n^2}{2})$. Similarly $[6\sigma_0 - \frac{(n+2)}{2}q_0 + \frac{(n^2-2n-2)}{2}] < (\geq)0$ depends on the value of curvature parameter k.
In the case (II), by using equation of state of the form $p = \omega(t)\rho$, we again find out the exact solutions of the field equations for two different cases: $\Lambda = 3\alpha H^2$ and $\Lambda = 8\pi G\gamma\rho$. By selecting a simple power law expression of t for the equation of state parameter ω, equivalence of model $\Lambda \sim (\frac{\dot{R}}{R})^2$ and $\Lambda \sim \rho$ have been established in the frame work of Kaluza-Klein theory of gravitation. With the help of Eqs. (45) and (52) it is easy to show that Eqs. (42) and (49) are differ by constant while Eqs. (43) and (44) become identical with the Eqs. (50) and (51) respectively. This implies that $\Lambda \sim (\frac{\dot{R}}{R})^2$ and $\Lambda \sim \rho$ are equivalent for five dimensional space time.
Using Eq. (42) and Eq. (46), we obtain

$$q = -\left[1 - \left[(n+3) - (n+1)(2-\Omega_m)\left(\frac{t}{\tau}\right)^a\right]\right]. \tag{87}$$

Eq. (87) shows that q is time dependent and hence may be change its sign during cosmic evolution. It has also been possible to show that the sought for signature flipping of deceleration parameter q can be obtained by a suitable choice of a.
In the last section of the chapter we have studied the behaviour of time varying constants G, c and Λ in a perfect fluid model. To obtain the solution we imposed the assumption, $div(T^i_j) = 0$, from which we obtained the dimensional constant A_ω that relates $\rho \propto R^{-4(\omega+1)}$and the relationship $\frac{G}{c^2} = B = constant$ for all value of t, i.e. G and c vary but in such a way that $\frac{G}{c^2}$ remain constant. It is also observed that G, c and Λ are decreasing function of t. The Lie method maybe the most powerful but has drawbacks, it is very complicate.

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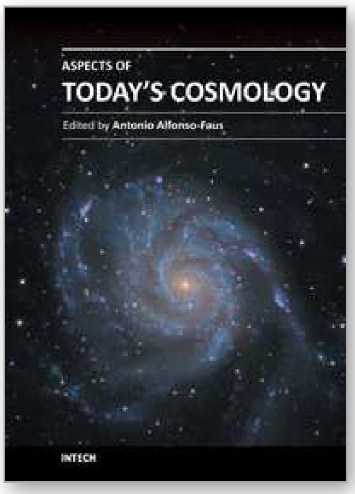

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InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821