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Generalized PI Control of Active Vehicle Suspension Systems with MATLAB

Esteban Chávez Conde¹, Francisco Beltrán Carbajal²
Antonio Valderrábano González³ and Ramón Chávez Bracamontes⁴

¹*Universidad del Papaloapan, Campus Loma Bonita*

²*Universidad Autónoma Metropolitana, Plantel Azcapotzalco, Departamento de Energía*

³*Universidad Politécnica de la Zona Metropolitana de Guadalajara*

⁴*Instituto Tecnológico de Cd. Guzmán
México*

1. Introduction

The main objective on the active vibration control problem of vehicles suspension systems is to get security and comfort for the passengers by reducing to zero the vertical acceleration of the body of the vehicle. An actuator incorporated to the suspension system applies the control forces to the vehicle body of the automobile for reducing its vertical acceleration in active or semi-active way.

The topic of active vehicle suspension control system has been quite challenging over the years. Some research works in this area propose control strategies like LQR in combination with nonlinear backstepping control techniques (Liu et al., 2006) which require information of the state vector (vertical positions and speeds of the tire and car body). A reduced order controller is proposed in (Yousefi et al., 2006) to decrease the implementation costs without sacrificing the security and the comfort by using accelerometers for measurements of the vertical movement of the tire and car body. In (Tahboub, 2005), a controller of variable gain that considers the nonlinear dynamics of the suspension system is proposed. It requires measurements of the vertical position of the car body and the tire, and the estimation of other states and of the profile of the ride.

This chapter proposes a control design approach for active vehicle suspension systems using electromagnetic or hydraulic actuators based on the Generalized Proportional Integral (GPI) control design methodology, sliding modes and differential flatness, which only requires vertical displacement measurements of the vehicle body and the tire. The profile of the ride is considered as an unknown disturbance that cannot be measured. The main idea is the use of integral reconstruction of the non-measurable state variables instead of state observers. This approach is quite robust against parameter uncertainties and exogenous perturbations. Simulation results obtained from Matlab are included to show the dynamic performance and robustness of the proposed active control schemes for vehicles suspension systems.

GPI control for the regulation and trajectory tracking tasks on time invariant linear systems was introduced by Fliess and co-workers in (Fliess et al., 2002). The main objective is to avoid the explicit use of state observers. The integral reconstruction of the state variables is carried out by means of elementary algebraic manipulations of the system model along with suitable

invocation of the system model observability property. The purpose of integral reconstructors is to get expressions for the unmeasured states in terms of inputs, outputs, and sums of a finite number of iterated integrals of the measured variables. In essence, constant errors and iterated integrals of such constant errors are allowed on these reconstructors. The current states thus differ from the integrally reconstructed states in time polynomial functions of finite order, with unknown coefficients related to the neglected, unknown, initial conditions. The use of these integral reconstructors in the synthesis of a model-based computed stabilizing state feedback controller needs suitable counteracting the effects of the implicit time polynomial errors. The destabilizing effects of the state estimation errors can be compensated by additively complementing a pure state feedback controller with a linear combination of a sufficient number of iterated integrals of the output tracking error, or output stabilization error. The closed loop stability is guaranteed by a simple characteristic polynomial assignment to the higher order compensated controllable and observable input-output dynamics. Experimental results of the GPI control obtained in a platform of a rotational mechanical system with one and two degrees of freedom are presented in (Chávez-Conde et al., 2006). Sliding mode control of a differentially flat system of two degrees of freedom, with vibration attenuation, is shown in (Enríquez-Zárate et al., 2000). Simulation results of GPI and sliding mode control techniques for absorption of vibrations of a vibrating mechanical system of two degrees of freedom were presented in (Beltrán-Carbajal et al., 2003).

This chapter is organized as follows: Section 2 presents the linear mathematical models of suspension systems of a quarter car. The design of the controllers for the active suspension systems are introduced in Sections 3 and 4. Section 5 divulges the use of sensors for measuring the variables required by the controller while the simulation results are shown in Section 6. Finally, conclusions are brought out in Section 7.

2. Quarter-car suspension systems

2.1 Mathematical model of passive suspension system

A schematic diagram of a quarter-vehicle suspension system is shown in Fig. 1(a). The mathematical model of passive suspension system is described by

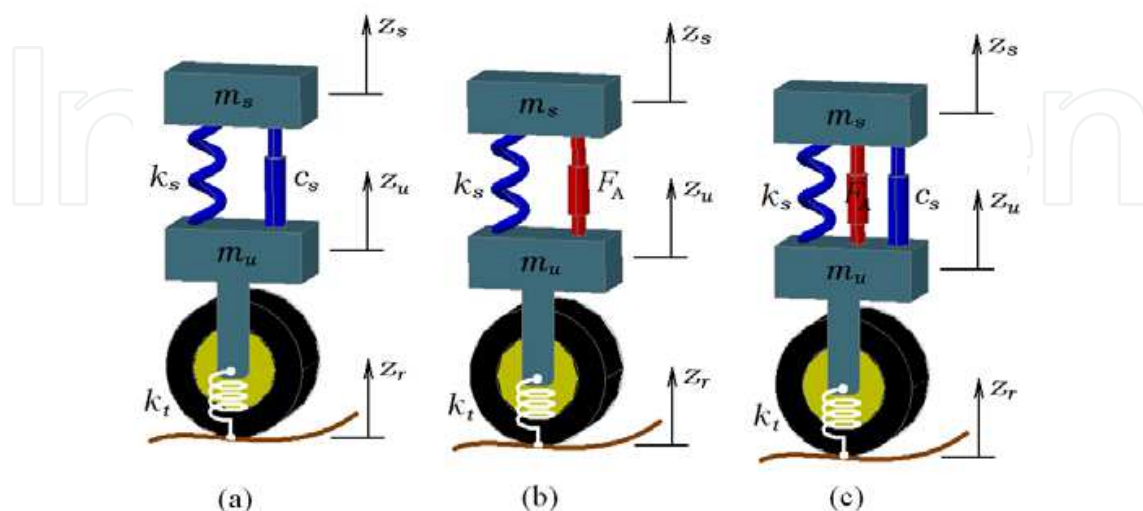


Fig. 1. Quarter-car suspension systems: (a) Passive Suspension System, (b) Active Electromagnetic Suspension System and (c) Active Hydraulic Suspension System.

$$m_s \ddot{z}_s + c_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) = 0 \quad (1)$$

$$m_u \ddot{z}_u - c_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) + k_t (z_u - z_r) = 0 \quad (2)$$

where m_s represents the sprung mass, m_u denotes the unsprung mass, c_s is the damper coefficient of suspension, k_s and k_t are the spring coefficients of suspension and the tire, respectively, z_s is the displacements of the sprung mass, z_u is the displacements of the unsprung mass and z_r is the terrain input disturbance.

2.2 Mathematical model of active electromagnetic suspension system

A schematic diagram of a quarter-car active electromagnetic suspension system is illustrated in Fig.1 (b). The electromagnetic actuator replaces the damper, forming a suspension with the spring (Martins et al., 2006). The friction force of an electromagnetic actuator is neglected. The mathematical model of electromagnetic active suspension system is given by

$$m_s \ddot{z}_s + k_s (z_s - z_u) = F_A \quad (3)$$

$$m_u \ddot{z}_u - k_s (z_s - z_u) + k_t (z_u - z_r) = -F_A \quad (4)$$

where m_s , m_u , k_s , k_t , z_s , z_u and z_r represent the same parameters and variables as ones described for the passive suspension system. The electromagnetic actuator force is represented here by F_A , which is considered as the control input.

2.3 Mathematical model of hydraulic active suspension system

Fig. 1(c) shows a schematic diagram of a quarter-car active hydraulic suspension system. The mathematical model of this active suspension system is given by

$$m_s \ddot{z}_s + c_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) = -F_f + F_A \quad (5)$$

$$m_u \ddot{z}_u - c_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) + k_t (z_u - z_r) = F_f - F_A \quad (6)$$

where m_s , m_u , k_s , k_t , z_s , z_u and z_r represent the same parameters and variables shown for the passive suspension system. The hydraulic actuator force is represented by F_A , while F_f represents the friction force generated by the seals of the piston with the cylinder wall inside the actuator. This friction force has a significant magnitude ($> 200N$) and cannot be ignored (Martins et al., 2006; Yousefi et al., 2006). The net force given by the actuator is the difference between the hydraulic force F_A and the friction force F_f .

3. Control of electromagnetic suspension system

The mathematical model of the active electromagnetic suspension system, illustrated in Fig. 1(b) is given by the equations (3) and (4). Defining the state variables $x_1 = z_s$, $x_2 = \dot{z}_s$, $x_3 = z_u$ and $x_4 = \dot{z}_u$, the representation in the state-space is,

$$\dot{x}(t) = Ax(t) + Bu(t) + Ez_r(t); \quad x(t) \in \mathbb{R}^4, A \in \mathbb{R}^{4 \times 4}, B \in \mathbb{R}^{4 \times 1}, E \in \mathbb{R}^{4 \times 1}, \quad (7)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & 0 & \frac{k_s}{m_s} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & 0 & -\frac{k_s + k_t}{m_u} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_u} \end{bmatrix} z_r \quad (8)$$

The force provided by the electromagnetic actuator as the control input is $u = F_A$.

The system is controllable with controllability matrix,

$$C_k = \begin{bmatrix} 0 & \frac{1}{m_s} & 0 & -\left(\frac{k_s}{m_s^2} + \frac{k_s}{m_s m_u}\right) \\ \frac{1}{m_s} & 0 & -\left(\frac{k_s}{m_s^2} + \frac{k_s}{m_s m_u}\right) & 0 \\ 0 & -\frac{1}{m_u} & 0 & \left(\frac{k_s}{m_s m_u} + \frac{k_s + k_t}{m_u^2}\right) \\ -\frac{1}{m_u} & 0 & \left(\frac{k_s}{m_s m_u} + \frac{k_s + k_t}{m_u^2}\right) & 0 \end{bmatrix}, \quad (9)$$

and flat (Fliess et al., 1993; Sira-Ramírez & Agrawal, 2004), with the flat output given by the following expression relating the displacements of both masses (Chávez et al., 2009):

$$F = m_s x_1 + m_u x_3$$

For simplicity, in the analysis of the differential flatness for the suspension system we have assumed that $k_t z_r = 0$. In order to show the differential parameterization of all the state variables and control input, we first formulate the time derivatives up to fourth order for F , resulting,

$$\begin{aligned} F &= m_s x_1 + m_u x_3 \\ \dot{F} &= m_s x_2 + m_u x_4 \\ \ddot{F} &= -k_t x_3 \\ F^{(3)} &= -k_t x_4 \\ F^{(4)} &= \frac{k_t}{m_u} u - \frac{k_s k_t}{m_u} (x_1 - x_3) + \frac{k_t^2}{m_u} x_3 \end{aligned}$$

Then, the state variables and control input are parameterized in terms of the flat output as follows

$$\begin{aligned}
 x_1 &= \frac{1}{m_s} \left(F + \frac{m_u}{k_t} \ddot{F} \right) \\
 x_2 &= \frac{1}{m_s} \left(\dot{F} + \frac{m_u}{k_t} F^{(3)} \right) \\
 x_3 &= -\frac{1}{k_t} \ddot{F} \\
 x_4 &= -\frac{1}{k_t} F^{(3)} \\
 u &= \frac{m_u}{k_t} F^{(4)} + \left(\frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1 \right) \ddot{F} + \frac{k_s}{m_s} F
 \end{aligned}$$

3.1 Integral reconstructors

The control input u in terms of the flat output and its time derivatives is given by

$$u = \frac{m_u}{k_t} F^{(4)} + \left(\frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1 \right) \ddot{F} + \frac{k_s}{m_s} F \quad (10)$$

where $F^{(4)} = v$ defines an auxiliary control input variable. The expression (10) can be rewritten of the following form:

$$u = d_1 F^{(4)} + d_2 \ddot{F} + d_3 F \quad (11)$$

where

$$\begin{aligned}
 d_1 &= \frac{m_u}{k_t} \\
 d_2 &= \frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1 \\
 d_3 &= \frac{k_s}{m_s}
 \end{aligned}$$

An integral input-output parameterization of the state variables is obtained from equation (11), and given by

$$\begin{aligned}
 \hat{F}^{(3)} &= \frac{1}{d_1} \int u - \frac{d_2}{d_1} \hat{F} - \frac{d_3}{d_1} \int F \\
 \hat{F} &= \frac{1}{d_1} \int^{(2)} u - \frac{d_2}{d_1} F - \frac{d_3}{d_1} \int^{(2)} F \\
 \hat{F} &= \frac{1}{d_1} \int^{(3)} u - \frac{d_2}{d_1} \int F - \frac{d_3}{d_1} \int^{(3)} F
 \end{aligned}$$

For simplicity, we will denote the integral $\int_0^t \phi(\tau) d\tau$ by $\int \phi$ and $\int_0^t \int_0^{\sigma_1} \dots \int_0^{\sigma_{n-1}} \phi(\sigma_n) d\sigma_n \dots d\sigma_1$ by $\int^{(n)} \phi$ with n a positive integer.

The relations between the state variables and the integrally reconstructed states are given by

$$\begin{aligned} F^{(3)} &= \widehat{F}^{(3)} + \frac{1}{2}F^{(3)}(0)t^2 + \ddot{F}(0)t + F^{(3)}(0) + \dot{F}(0) \\ \ddot{F} &= \widehat{\ddot{F}} + F^{(3)}(0)t + \ddot{F}(0) \\ \dot{F} &= \widehat{\dot{F}} + \frac{1}{2}F^{(3)}(0)t^2 + \ddot{F}(0)t + \dot{F}(0) \end{aligned}$$

where $F^{(3)}(0)$, $\ddot{F}(0)$ and $\dot{F}(0)$ are all real constants depending on the unknown initial conditions.

3.2 Sliding mode and GPI control

GPI control is based on the use of integral reconstructors of the unmeasured state variables and the output error is integrally compensated. The sliding surface inspired on the GPI control technique can be proposed as

$$\widehat{\sigma} = \widehat{F}^{(3)} + \alpha_5 \widehat{\ddot{F}} + \alpha_4 \widehat{\dot{F}} + \alpha_3 F + \alpha_2 \int F + \alpha_1 \int^{(2)} F + \alpha_0 \int^{(3)} F \quad (12)$$

The last integral term yields error compensation, eliminating destabilizing effects, those of the structural estimation errors. The ideal sliding condition $\widehat{\sigma} = 0$ results in a sixth order dynamics,

$$F^{(6)} + \alpha_5 F^{(5)} + \alpha_4 F^{(4)} + \alpha_3 F^{(3)} + \alpha_2 \ddot{F} + \alpha_1 \dot{F} + \alpha_0 F = 0 \quad (13)$$

The gains of the controller $\alpha_5, \dots, \alpha_0$ are selected so that the associated characteristic polynomial $s^6 + \alpha_5 s^5 + \alpha_4 s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0$ is Hurwitz. As a consequence, the error dynamics on the switching surface $\widehat{\sigma} = 0$ is globally asymptotically stable.

The sliding surface $\widehat{\sigma} = 0$ is made globally attractive with the continuous approximation to the discontinuous sliding mode controller as given in (Sira-Ramírez, 1993), i.e., by forcing to satisfy the dynamics,

$$\dot{\widehat{\sigma}} = -\mu[\widehat{\sigma} + \gamma \text{sign}(\widehat{\sigma})] \quad (14)$$

where μ and γ denote real positive constants and “sign” is the standard signum function.

The sliding surface is globally attractive, $\widehat{\sigma} \dot{\widehat{\sigma}} < 0$ for $\widehat{\sigma} \neq 0$, which is a very well known condition for the existence of sliding mode presented in (Utkin, 1978). Then the following sliding-mode controller is obtained

$$u = d_1 v + d_2 \widehat{\ddot{F}} + d_3 F \quad (15)$$

with

$$v = -\alpha_5 \widehat{F}^{(3)} - \alpha_4 \widehat{\ddot{F}} - \alpha_3 \widehat{\dot{F}} - \alpha_2 F - \alpha_1 \int F - \alpha_0 \int^{(2)} F - \mu[\widehat{\sigma} + \gamma \text{sign}(\widehat{\sigma})]$$

This controller requires only the measurement of the variables of state z_s and z_u corresponding to the vertical displacements of the body of the car and the wheel, respectively.

4. Control of hydraulic suspension system

The mathematical model of active suspension system shown in Fig. 1(c) is given by the equations (5) and (6). Using the same state variables definition than the control of electromagnetic suspension system, the representation in the state space form is as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & \frac{k_s}{m_s} & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\frac{k_s+k_t}{m_u} & -\frac{c_s}{m_u} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_u} \end{bmatrix} z_r \quad (16)$$

The net force provided by the hydraulic actuator as control input $u = F_A - F_f$, is the difference between the hydraulic force F_A and the frictional force F_f .

The system is controllable and flat (Fließ et al., 1993; Sira-Ramírez & Agrawal, 2004), with positions of the body of the car and wheel as output $F = m_s x_1 + m_u x_3$, (Chávez et al., 2009). The controllability matrix and coefficients are:

$$C_k = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \quad (17)$$

$$c_{11} = 0, \quad c_{12} = c_{21} = \frac{1}{m_s}, \quad c_{13} = c_{22} = -\left(\frac{c_s}{m_s^2} + \frac{c_s}{m_s m_u}\right),$$

$$c_{14} = c_{23} = \left(\frac{c_s^2}{m_s^2} + \frac{c_s^2}{m_s m_u} - \frac{k_s}{m_s}\right) \frac{1}{m_s} - \left(-\frac{c_s^2}{m_s^2} - \frac{c_s^2}{m_s m_u} + \frac{k_s}{m_s}\right) \frac{1}{m_u},$$

$$c_{24} = \left[\frac{c_s k_s}{m_s^2} - \frac{c_s}{m_s} \left(\frac{c_s^2}{m_s^2} + \frac{c_s^2}{m_s m_u} - \frac{k_s}{m_s}\right) + \frac{c_s k_s}{m_s m_u} + \frac{c_s}{m_s} \left(-\frac{c_s^2}{m_s^2} - \frac{c_s^2}{m_s m_u} + \frac{k_s}{m_s}\right)\right] \frac{1}{m_s} - \left[\frac{c_s k_s}{m_s^2} - \frac{c_s}{m_s} \left(-\frac{c_s^2}{m_s^2} - \frac{c_s^2}{m_s m_u} + \frac{k_s}{m_s}\right) - \frac{c_s k_s}{m_s m_u} + \frac{c_s}{m_s} \left(\frac{c_s^2}{m_s^2} + \frac{c_s^2}{m_s m_u} - \frac{k_s+k_t}{m_s}\right)\right] \frac{1}{m_u},$$

$$c_{31} = 0, \quad c_{32} = c_{41} = -\frac{1}{m_u}, \quad c_{33} = c_{42} = \left(\frac{c_s}{m_u^2} + \frac{c_s}{m_s m_u}\right),$$

$$c_{34} = c_{43} = \left(-\frac{c_s^2}{m_s^2} - \frac{c_s^2}{m_s m_u} + \frac{k_s}{m_u}\right) \frac{1}{m_s} - \left(\frac{c_s^2}{m_s^2} + \frac{c_s^2}{m_s m_u} - \frac{k_s+k_t}{m_u}\right) \frac{1}{m_u},$$

$$c_{44} = \left[-\frac{c_s k_s}{m_s m_u} + \frac{c_s}{m_u} \left(\frac{c_s^2}{m_s^2} + \frac{c_s^2}{m_s m_u} - \frac{k_s}{m_s}\right) - \frac{c_s}{m_u^2} (k_s + k_t) - \frac{c_s}{m_u} \left(-\frac{c_s^2}{m_s^2} - \frac{c_s^2}{m_s m_u} + \frac{k_s}{m_s}\right)\right] \frac{1}{m_s} - \left[\frac{c_s k_s}{m_s m_u} + \frac{c_s}{m_u} \left(-\frac{c_s^2}{m_s^2} - \frac{c_s^2}{m_s m_u} + \frac{k_s}{m_s}\right) + \frac{c_s}{m_u^2} (k_s + k_t) - \frac{c_s}{m_u} \left(\frac{c_s^2}{m_s^2} + \frac{c_s^2}{m_s m_u} - \frac{k_s+k_t}{m_u}\right)\right] \frac{1}{m_u}$$

It is assumed that $k_{z_r} = 0$ in the analysis of the differential flatness for the suspension system. To show the parameterization of the state variables and control input, we first formulate the time derivatives for $F = m_s x_1 + m_u x_3$ up to fourth order, resulting,

$$\begin{aligned} F &= m_s x_1 + m_u x_3 \\ \dot{F} &= m_s x_2 + m_u x_4 \\ \ddot{F} &= -k_t x_3 \\ F^{(3)} &= -k_t x_4 \\ F^{(4)} &= \frac{k_t}{m_u} u - \frac{c_s k_t}{m_u} (x_2 - x_4) - \frac{k_s k_t}{m_u} (x_1 - x_3) + \frac{k_t^2}{m_u} x_3 \end{aligned}$$

Then, the state variables and control input are parameterized in terms of the flat output as follows

$$\begin{aligned} x_1 &= \frac{1}{m_s} \left(F + \frac{m_u}{k_t} \ddot{F} \right), & x_2 &= \frac{1}{m_s} \left(\dot{F} + \left(\frac{m_u}{k_t} \right) F^{(3)} \right) \\ x_3 &= -\frac{1}{k_t} \ddot{F}, & x_4 &= -\frac{1}{k_t} F^{(3)} \\ u &= \frac{m_u}{k_t} F^{(4)} + \left(\frac{c_s m_u}{k_t m_s} + \frac{c_s}{k_t} \right) F^{(3)} + \left(\frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1 \right) \ddot{F} + \frac{c_s}{m_s} \dot{F} + \frac{k_s}{m_s} F \end{aligned}$$

4.1 Integral reconstructors

The control input u in terms of the flat output and its time derivatives is given by

$$u = \frac{m_u}{k_t} v + \left(\frac{c_s m_u}{k_t m_s} + \frac{c_s}{k_t} \right) F^{(3)} + \left(\frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1 \right) \ddot{F} + \frac{c_s}{m_s} \dot{F} + \frac{k_s}{m_s} F \quad (18)$$

where $F^{(4)} = v$, defines the auxiliary control input. Expression (19) can be rewritten in the following form:

$$u = \eta_1 v + \eta_2 F^{(3)} + \eta_3 \ddot{F} + \eta_4 \dot{F} + \eta_5 F \quad (19)$$

where

$$\begin{aligned} \eta_1 &= \frac{m_u}{k_t} \\ \eta_2 &= \frac{c_s m_u}{k_t m_s} + \frac{c_s}{k_t} \\ \eta_3 &= \frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1 \\ \eta_4 &= \frac{c_s}{m_s}, \quad \eta_5 = \frac{k_s}{m_s} \end{aligned}$$

An integral input-output parameterization of the state variables is obtained from equation (20), and given by

$$\begin{aligned}\widehat{F}^{(3)} &= \frac{1}{\eta_1} \int u - \frac{\eta_2}{\eta_1} \widehat{F} - \frac{\eta_3}{\eta_1} \widehat{F} - \frac{\eta_4}{\eta_1} F - \frac{\eta_5}{\eta_1} \int F \\ \widehat{F} &= \frac{1}{\eta_1} \int^{(2)} u - \frac{\eta_2}{\eta_1} \widehat{F} - \frac{\eta_3}{\eta_1} F - \frac{\eta_4}{\eta_1} \int F - \frac{\eta_5}{\eta_1} \int^{(2)} F \\ \widehat{F} &= \frac{1}{\eta_1} \int^{(3)} u - \frac{\eta_2}{\eta_1} F - \frac{\eta_3}{\eta_1} \int F - \frac{\eta_4}{\eta_1} \int^{(2)} F - \frac{\eta_5}{\eta_1} \int^{(3)} F\end{aligned}$$

For simplicity, we have denoted the integral $\int_0^t \phi(\tau) d\tau$ by $\int \phi$ and $\int_0^t \int_0^{\sigma_1} \dots \int_0^{\sigma_{n-1}} \phi(\sigma_n) d\sigma_n \dots d\sigma_1$ by $\int^{(n)} \phi$ with n as a positive integer.

The relationship between the state variables and the integrally reconstructed state variables is given by

$$\begin{aligned}F^{(3)} &= \widehat{F}^{(3)} + F^{(3)}(0)t^2 + F^{(3)}(0)t + 2\ddot{F}(0)t + \ddot{F}(0) + 2\dot{F}(0) \\ \ddot{F} &= \widehat{F} + \frac{1}{2} F^{(3)}(0)t^2 + F^{(3)}(0)t + \ddot{F}(0)t + \ddot{F}(0) + \dot{F}(0) \\ \dot{F} &= \widehat{F} + \frac{1}{2} F^{(3)}(0)t^2 + \ddot{F}(0)t + \dot{F}(0)\end{aligned}$$

where $F^{(3)}(0)$, $\ddot{F}(0)$ and $\dot{F}(0)$ are all real constants depending on the unknown initial conditions.

4.2 Sliding mode and GPI control

The sliding surface inspired on the GPI control technique is proposed according to equations (12), (13), and (14). This sliding surface is globally attractive (Utkin, 1978). Then the following sliding-mode controller is obtained:

$$u = \eta_1 v + \eta_2 \widehat{F}^{(3)} + \eta_3 \widehat{F} + \eta_4 \widehat{F} + \eta_5 F \quad (20)$$

With

$$v = -\alpha_5 \widehat{F}^{(3)} - \alpha_4 \widehat{F} - \alpha_3 \widehat{F} - \alpha_2 F - \alpha_1 \int F - \alpha_0 \int^{(2)} F - \mu[\widehat{\sigma} + \gamma \text{sign}(\widehat{\sigma})]$$

This controller requires only the measurement of the variables of state z_s and z_u corresponding to the vertical positions of the body of the car and the wheel, respectively.

5. Instrumentation of active suspension system

5.1 Measurements required

The only variables required for implementation of the proposed controllers are the vertical displacement of the body of the car z_s , and the vertical displacement of the wheel z_u . These variables are needed to be measured by sensors.

5.2 Using sensors

In (Chamseddine et al., 2006), the use of sensors in experimental vehicle platforms, as well as in commercial vehicles is presented. The most common sensors, used for measuring the vertical displacement of the body of the car and the wheels, are laser sensors. This type of sensor could be used to measure the variables z_s and z_u needed for implementation of the controllers. Accelerometers or other types of sensors are not needed for measuring the variables \dot{z}_s and \dot{z}_u ; these variables are estimated with the use of integral reconstruction from knowledge of the control input, the flat output and the differentially flat system model.

The schematic diagram of the instrumentation of the active suspension system is illustrated in Fig. 2.

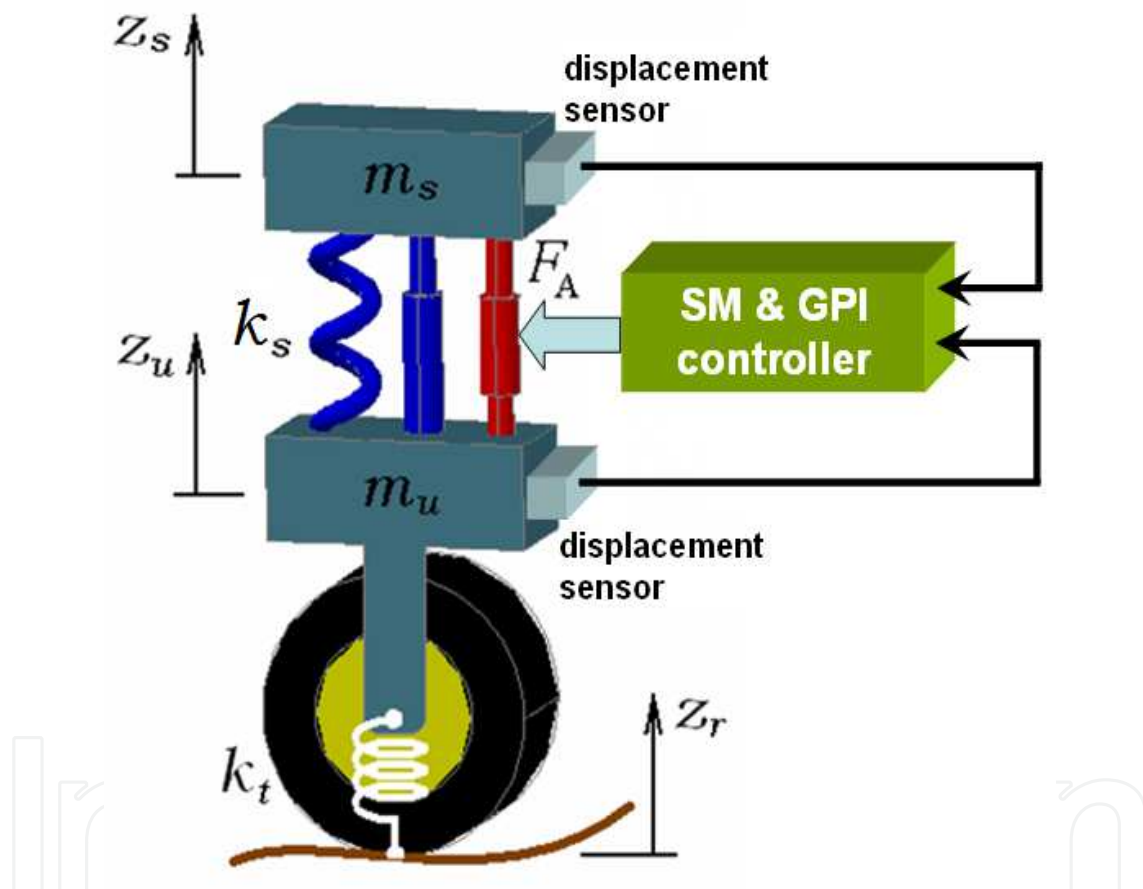


Fig. 2. Schematic diagram of the instrumentation of the active suspension system.

6. Simulation results with MATLAB/Simulink

The simulation results were obtained by means of MATLAB/Simulink[®], with the Runge-Kutta numerical method and a fixed integration step of 1 ms .

6.1 Parameters and type of road disturbance

The numerical values of the quarter-car suspension model parameters (Sam & Hudha, 2006) chosen for the simulations are shown in Table 1.

Parameter	Value
Sprung mass, m_s	282 [kg]
Unsprung mass, m_u	45 [kg]
Spring stiffness, k_s	17900 [$\frac{N}{m}$]
Damping constant, c_s	1000 [$\frac{N \cdot s}{m}$]
Tire stiffness, k_t	165790 [$\frac{N}{m}$]

Table 1. Vehicle suspension system parameters for a quarter-car model.

In this simulation study, the road disturbance is shown in Fig. 3 and set in the form of (Sam & Hudha, 2006):

$$z_r = a \frac{1 - \cos(8\pi t)}{2}$$

with $a = 0.11$ [m] for $0.5 \leq t \leq 0.75$, $a = 0.55$ [m] for $3.0 \leq t \leq 3.25$ and 0 otherwise.

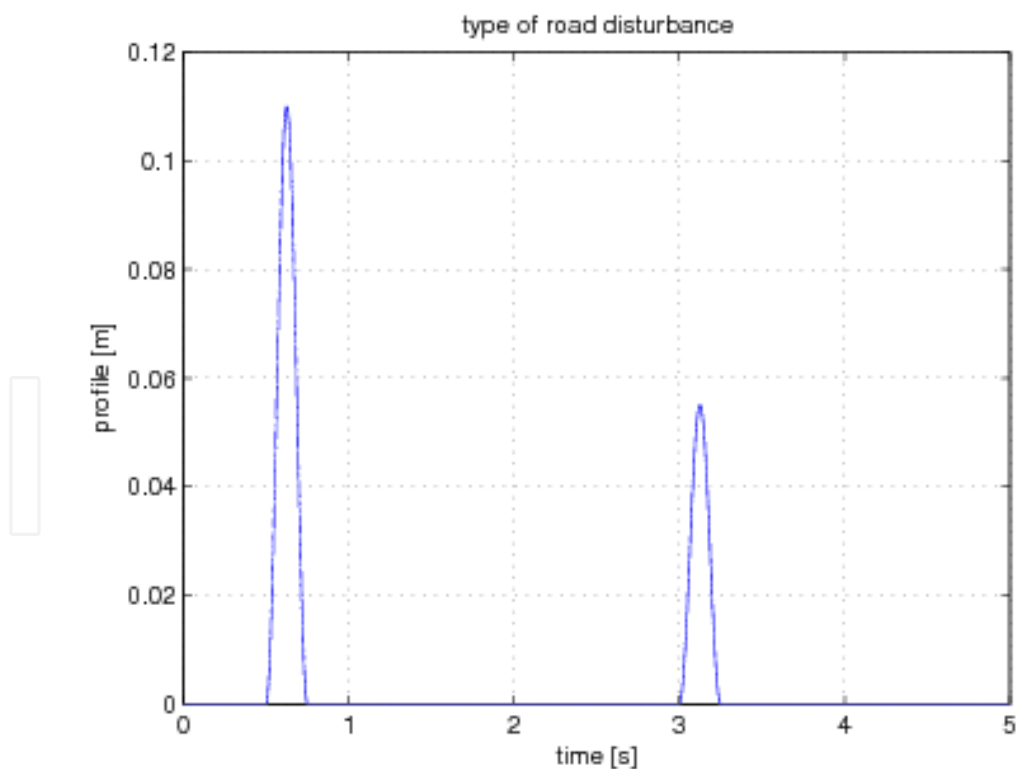


Fig. 3. Type of road disturbance.

The road disturbance was programmed into Simulink blocks, as shown in Fig. 4. Here, the block called “conditions” was developed as a Simulink subsystem block Fig. 5.

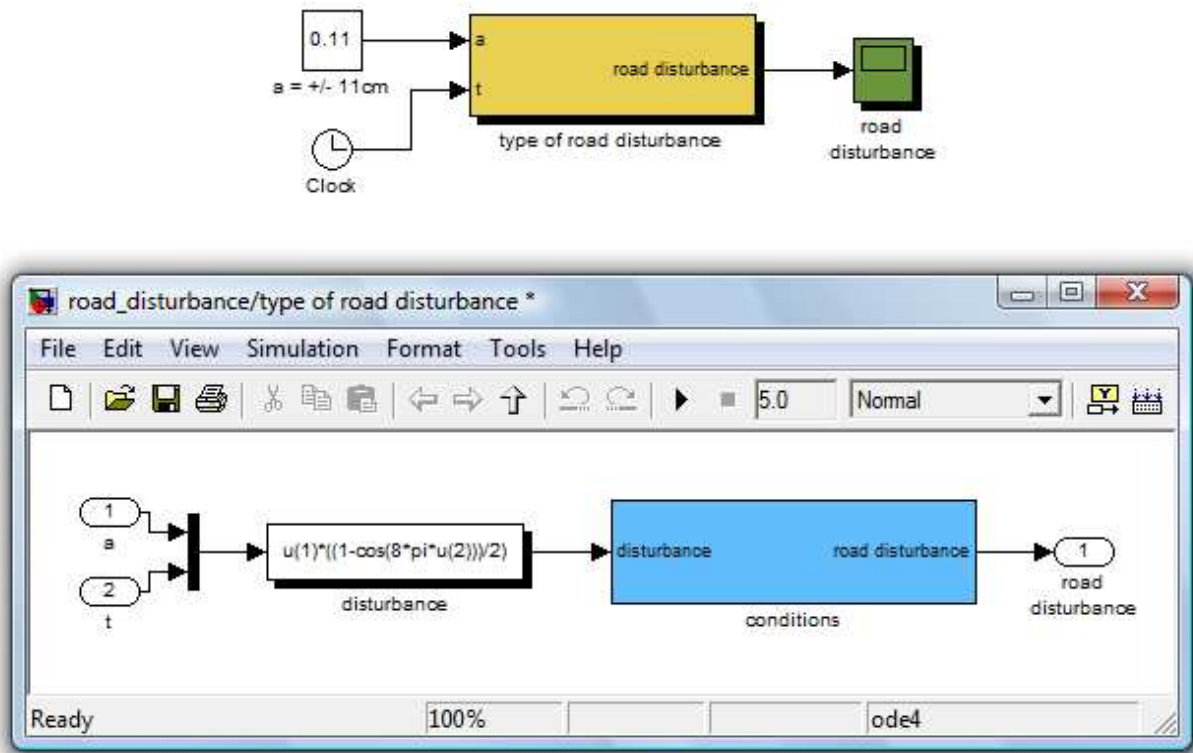


Fig. 4. Type of road disturbance in Simulink.

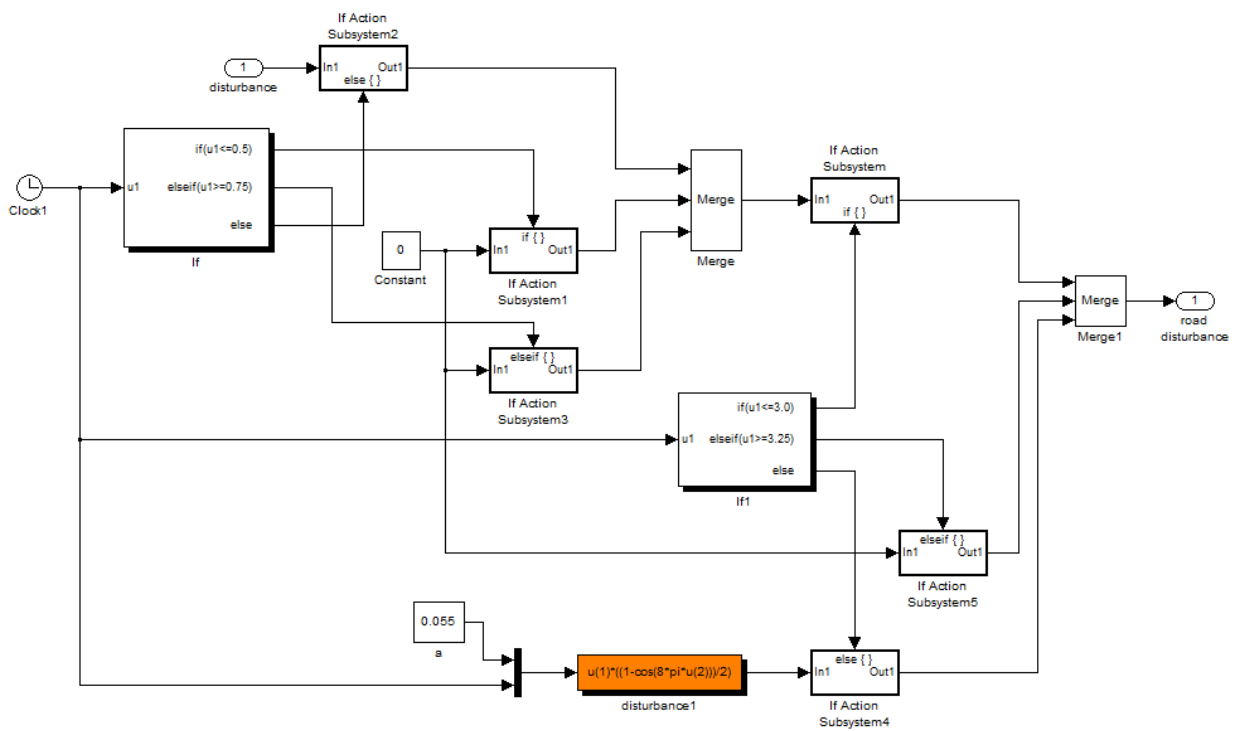


Fig. 5. Conditions of road disturbance in Simulink.

6.2 Passive vehicle suspension system

Some simulation results of the passive suspension system performance are shown in Fig. 6. The Simulink model of the passive suspension system used for the simulations is shown in Fig. 7.

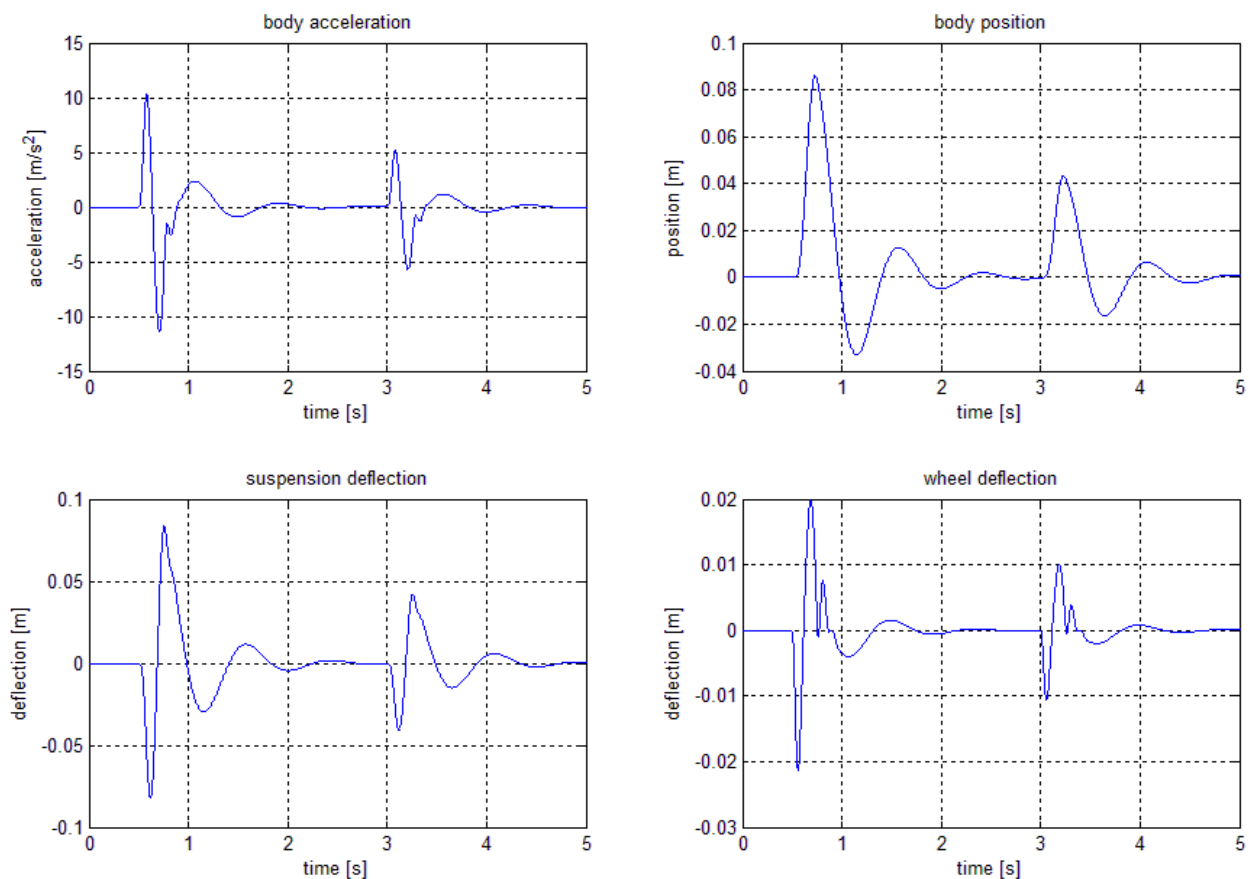


Fig. 6. Simulation results of passive suspension system, where the suspension deflection is given by $(z_s - z_u)$ and the tire deflection by $(z_u - z_r)$.

6.3 Control of electromagnetic suspension system

It is desired to stabilize the system at the positions $z_s = 0$ and $z_u = 0$. The controller gains were obtained by forcing the closed loop characteristic polynomial to be given by the following Hurwitz polynomial:

$$p_{d1}(s) = (s + p_1)(s + p_2)(s^2 + 2\zeta_1\omega_{n1}s + \omega_{n1}^2)^2$$

with $p_1 = 90$, $p_2 = 90$, $\zeta_1 = 0.7071$, $\omega_{n1} = 80$, $\mu = 95$ y $\gamma = 95$.

The Simulink model of the sliding mode based GPI controller of the active suspension system is shown in Fig. 8. The simulation results are illustrated in Fig. 9. It can be seen the high vibration attenuation level of the active vehicle suspension system compared with the passive counterpart.

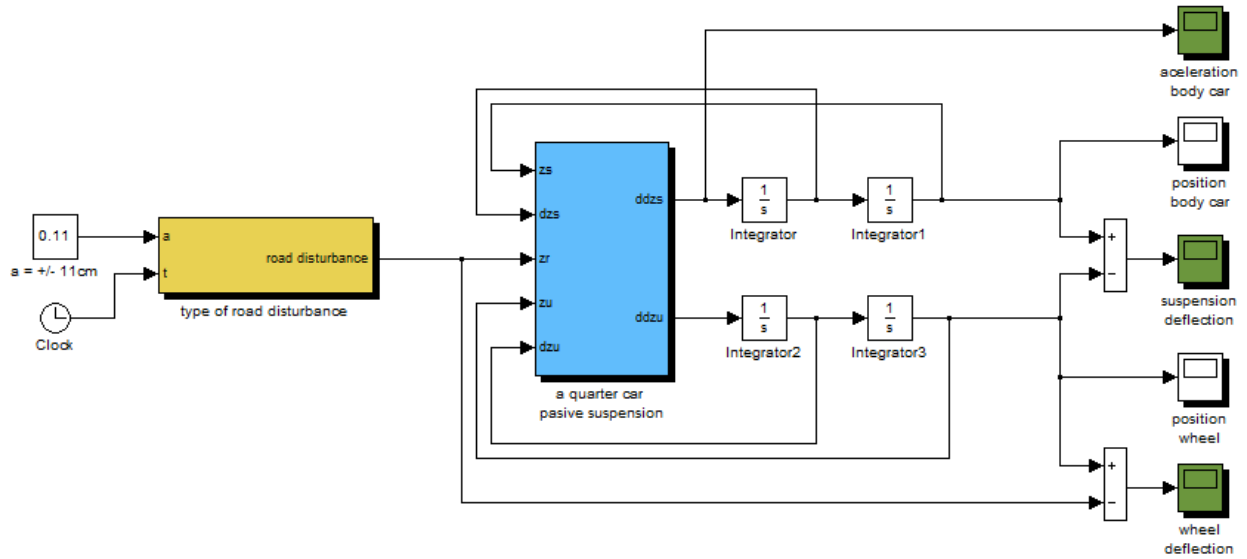


Fig. 7. Simulink model of the passive suspension system.

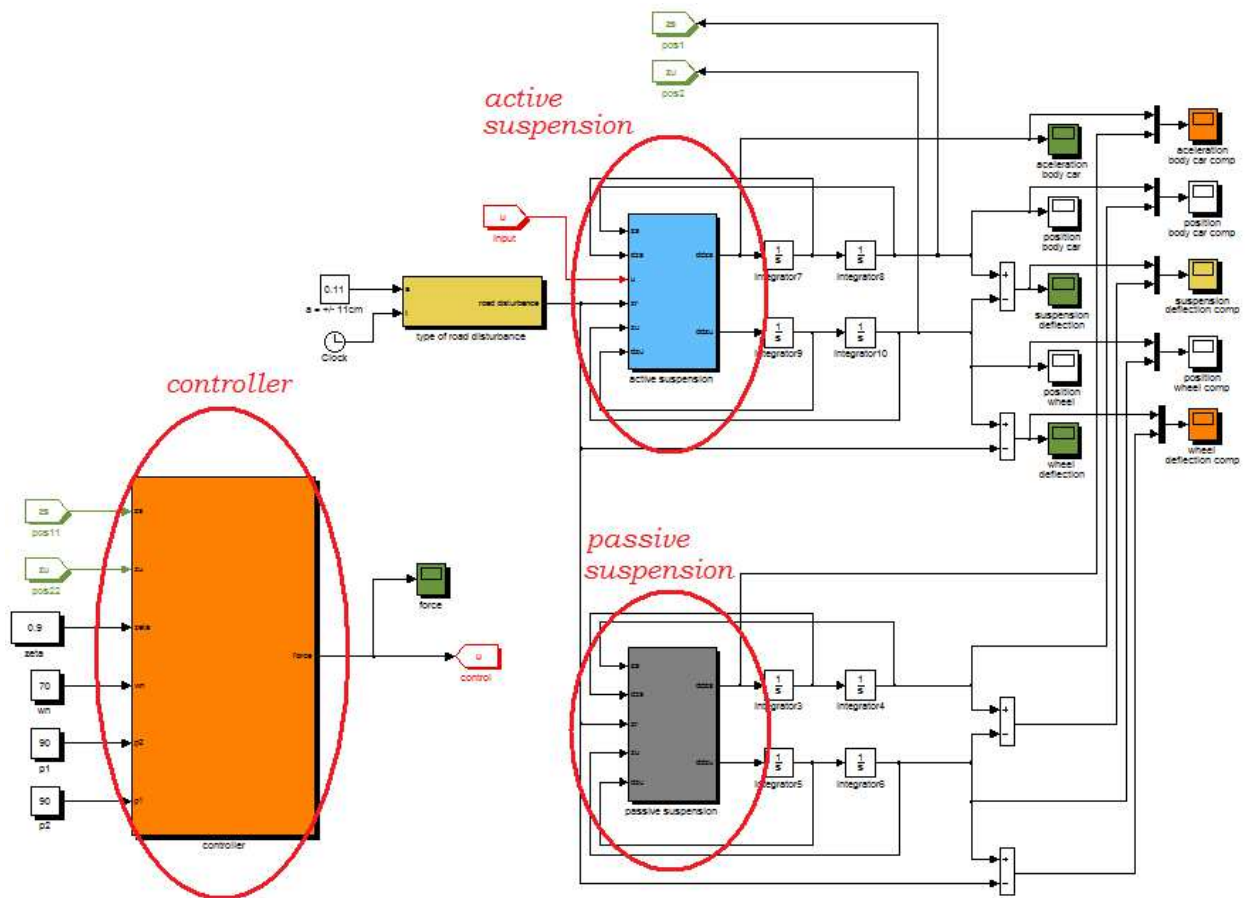


Fig. 8. Simulink model of the sliding mode based GPI controller.

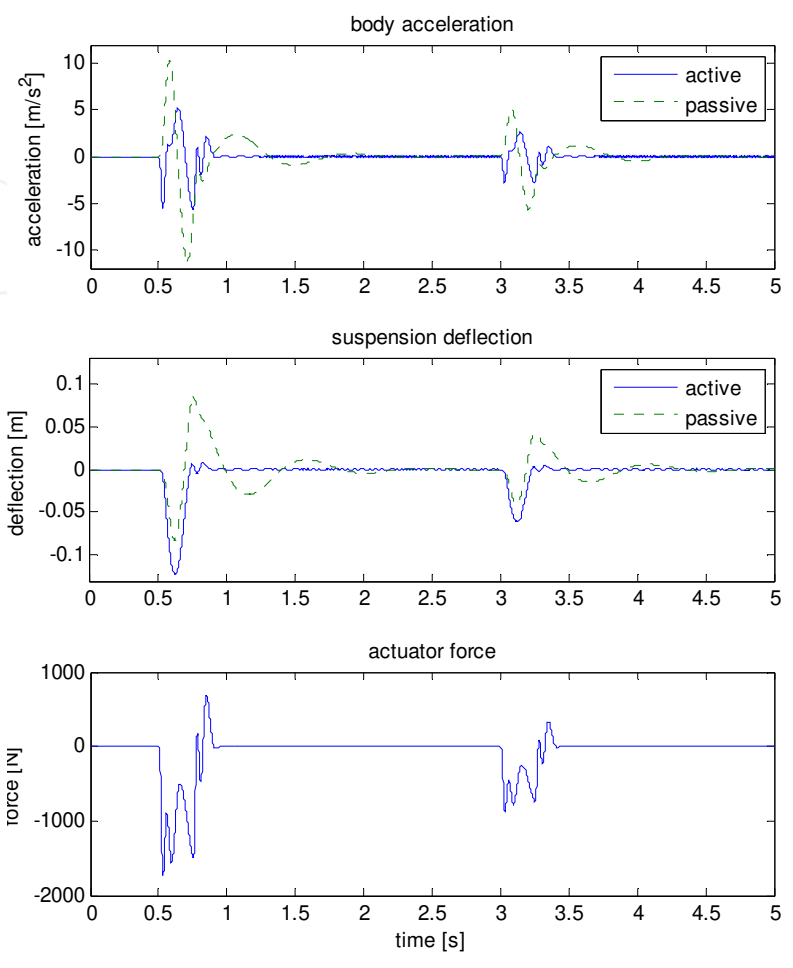
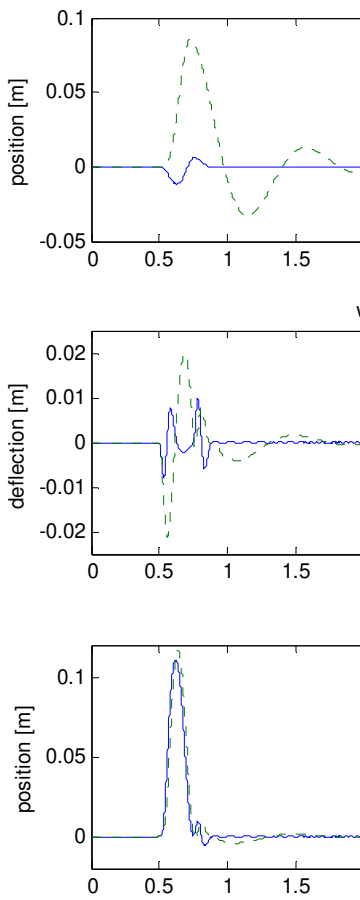


Fig. 9. Simulation results of the sliding mode based GPI controller of the electromagnetic suspension system.



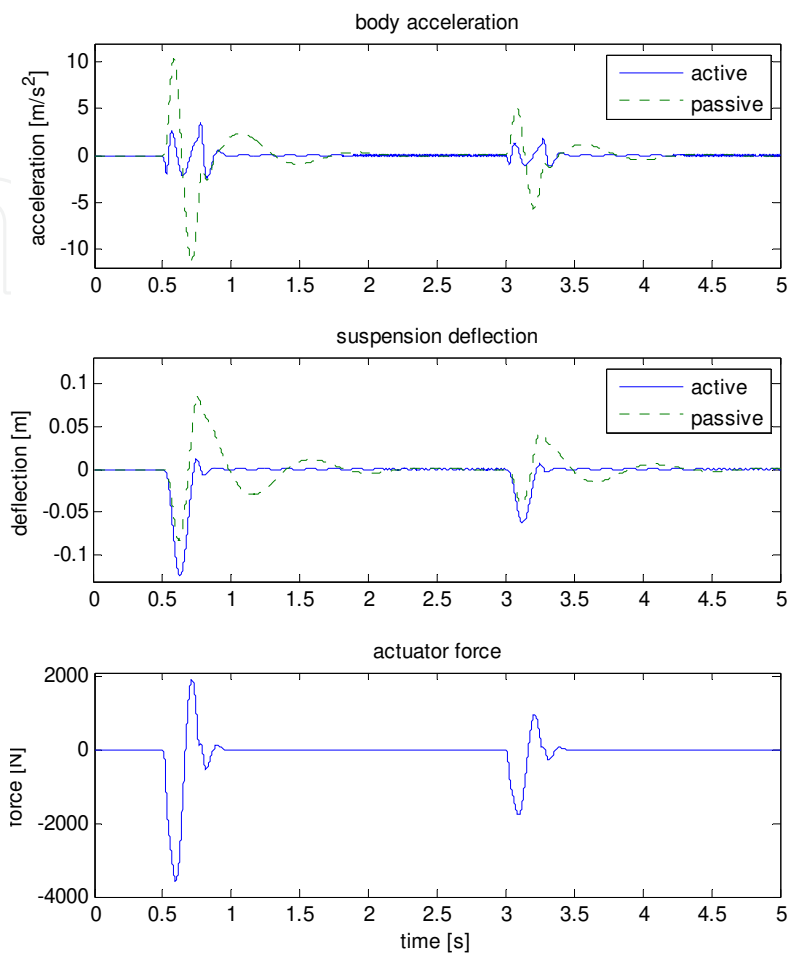
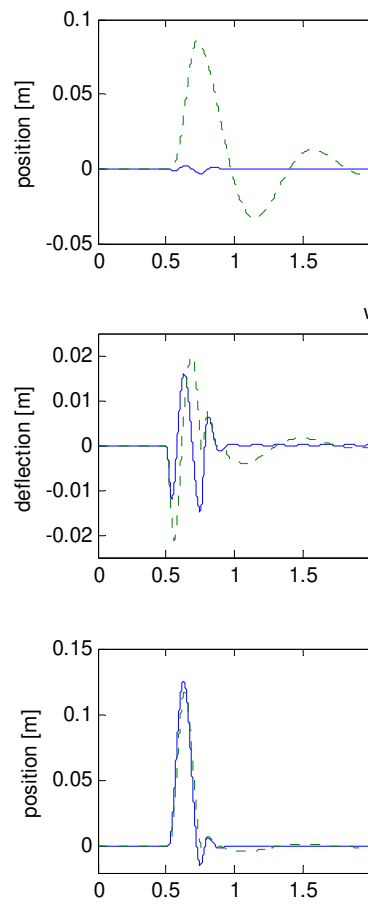


Fig. 10. Simulation results of sliding mode based GPI controller of hydraulic suspension system.

6.3 Control of hydraulic suspension system

It is desired to stabilize the system in the positions $z_s = 0$ and $z_u = 0$. The controller gains were obtained by forcing the closed loop characteristic polynomial to be given by the following Hurwitz polynomial:

$$p_{d2}(s) = (s + p_3)(s + p_4)(s^2 + 2\zeta_2\omega_{n2}s + \omega_{n2}^2)^2$$

with $p_3 = 90$, $p_4 = 90$, $\zeta_2 = 0.9$, $\omega_{n2} = 70$, $\mu = 95$ and $\gamma = 95$. The performance of the sliding mode based GPI controller is depicted in Fig. 10. One can see the high attenuation level of road-induced vibrations with respect to passive suspension system.

The same Matlab/Simulink simulation programs were used to implement the controllers for the electromagnetic and hydraulic active suspension systems. For the electromagnetic active suspension system, it is assumed that $c_z = 0$.

7. Conclusions

In this chapter we have presented an approach of robust active vibration control schemes for electromagnetic and hydraulic vehicle suspension systems based on Generalized Proportional-Integral control, differential flatness and sliding modes. Two controllers have been proposed to attenuate the vibrations induced by unknown exogenous disturbance excitations due to irregular road surfaces. The main advantage of the controllers proposed, is that they require only measurements of the position of the car body and the tire. Integral reconstruction is employed to get structural estimates of the time derivatives of the flat output, needed for the implementation of the controllers proposed. The simulation results show that the stabilization of the vertical position of the quarter of car is obtained within a period of time much shorter than that of the passive suspension system. The fast stabilization with amplitude in acceleration and speed of the body of the car is observed. Finally, the robustness of the controllers to stabilize to the system before the unknown disturbance is verified.

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Applications of MATLAB in Science and Engineering

Edited by Prof. Tadeusz Michalowski

ISBN 978-953-307-708-6

Hard cover, 510 pages

Publisher InTech

Published online 09, September, 2011

Published in print edition September, 2011

The book consists of 24 chapters illustrating a wide range of areas where MATLAB tools are applied. These areas include mathematics, physics, chemistry and chemical engineering, mechanical engineering, biological (molecular biology) and medical sciences, communication and control systems, digital signal, image and video processing, system modeling and simulation. Many interesting problems have been included throughout the book, and its contents will be beneficial for students and professionals in wide areas of interest.

How to reference

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Esteban Chávez Conde, Francisco Beltrán Carbajal Antonio Valderrábano González and Ramón Chávez Bracamontes (2011). Generalized PI Control of Active Vehicle Suspension Systems with MATLAB, Applications of MATLAB in Science and Engineering, Prof. Tadeusz Michalowski (Ed.), ISBN: 978-953-307-708-6, InTech, Available from: <http://www.intechopen.com/books/applications-of-matlab-in-science-and-engineering/generalized-pi-control-of-active-vehicle-suspension-systems-with-matlab>

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University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
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Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
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