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Recent Developments in Seasonal Volatility Models

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1. Introduction

It is well-known that many financial time series such as stock returns exhibit leptokurtosis and time-varying volatility (Bollerslev, 1986; Engle, 1982; Nicholls & Quinn, 1982). The generalized autoregressive conditional heteroscedasticity (GARCH) and the random coefficient autoregressive (RCA) models have been extensively used to capture the time-varying behavior of the volatility. Studies using GARCH models commonly assume that the time series is conditionally normally distributed; however, the kurtosis implied by the normal GARCH tends to be lower than the sample kurtosis observed in many time series (Bollerslev, 1986). Thavaneswaran et al. (2005a) use an ARMA representation to derive the kurtosis of various classes of GARCH models such as power GARCH, non-Gaussian GARCH, non-stationary and random coefficient GARCH. Recently, Thavaneswaran et al. (2009) have extended the results to stationary RCA processes with GARCH errors and Paseka et al. (2010) further extended the results to RCA processes with stochastic volatility (SV) errors.

Seasonal behavior is commonly observed in financial time series, as well as in currency and commodity markets. The opening and closure of the markets, time-of-the-day and day-of-the-week effects, weekends and vacation periods cause changes in the trading volume that translates into regular changes in price variability. Financial, currency, and commodity data also respond to new information entering into the market, which usually follow seasonal patterns (Frank & Garcia, 2009). Recently there has been growing interest in using seasonal volatility models, for example Bollerslev (1996), Baillie & Bollerslev (1990) and Franses & Paap (2000). Doshi et al. (2011) discuss the kurtosis and volatility forecasts for seasonal GARCH models. Ghysels & Osborn (2001) review studies performed on seasonal volatility behavior in several markets. Most of the studies use GARCH models with dummy variables in the volatility equation, and a few of them have been extended to a more flexible form such as the periodic GARCH. However, even though much research has been performed on volatility models applied to market data such as stock returns, more general specifications accounting for seasonal volatility have been little explored.

First, we derive the kurtosis of a simple time series model with seasonal behavior in the mean. Then we introduce various classes of seasonal volatility models and study the moments, forecast error variance, and discuss applications in option pricing. We extend the results for non-seasonal volatility models to seasonal volatility models. For the seasonal GARCH model, we follow the results obtained by Doshi et al. (2011) and extend it to the RCA-seasonal GARCH model. The multiplicative seasonal GARCH model is appropriate for time series where significant autocorrelation exists at seasonal and at adjacent non-seasonal lags. We also propose and derive the expressions for the kurtosis of seasonal SV models and other models such as the RCA with seasonal SV errors.

We also derive the closed-form expression for the variance of the l-steps ahead forecast error in terms of (ψ, Ψ) weights, model parameters and the kurtosis of the error distribution. We show that the kurtosis for the non-seasonal model turns out to be a special case. Option pricing with seasonal GARCH volatility is also discussed in some detail. The moments derived for the seasonal volatility models and the l-steps ahead forecast error variance provide more accurate estimates of market data behavior and help investors, decision makers, and other market participants develop improved trading strategies.

2. Seasonal AR models with GARCH errors

We first start with a seasonal AR(1) model with simple GARCH errors of the form,

$$y_t - \mu = \beta(y_{t-s} - \mu) + \epsilon_{t-1}^2 \epsilon_t \tag{1}$$

where s represents the seasonal period and ϵ_t is a sequence of independent random variables. The following lemma, given in Ghahramani & Thavaneswaran (2007), can be used to derive the second and fourth moments of the process in (1).

Lemma 2.1. For a stationary process and finite eighth moment, the expected value and kurtosis $K^{(y)}$ of the process (1) is given by:

(a)

$$E(y_t - \mu)^2 = \frac{E(\epsilon_{t-1}^4)E(\epsilon_t^2)}{1-\beta^2},$$

(b)

$$K^{(y)} = \frac{E[(y_t-\mu)^4]}{Var(y_t)^2} = \frac{6\beta^2[E(\epsilon_{t-1}^4)E(\epsilon_t^2)]^2 + E(\epsilon_{t-1}^8)E(\epsilon_t^4)(1-\beta^2)}{(1+\beta^2)(E(\epsilon_{t-1}^4)E(\epsilon_t^2))^2},$$

(c) if ϵ_t are assumed to be i.i.d. N(0,σ_ϵ^2), then E[ϵ_t^{2n}] $= ((2n)!/2^n(n!))\sigma_\epsilon^{2n}$ and hence

$$K^{(y)} = \left[\frac{35 - 29\beta^2}{(1+\beta^2)}\right].$$

3. AR Models with seasonal GARCH errors

AR models are the most common representation used in time series analysis. Multiplicative seasonal GARCH errors of the form GARCH(p,q)x$(P,Q)_s$ have been suggested by Doshi et al. (2011). Consider the following model,

$$y_t = \beta y_{t-1} + \epsilon_t \tag{2}$$

$$\epsilon_t = \sqrt{h_t} Z_t \tag{3}$$

$$\theta(B)\Theta(L)h_t = \omega + \alpha(B)\epsilon_t^2 \tag{4}$$

where $\{Z_t\}$ is a sequence of independent and identically distributed (i.i.d.) random variables with zero mean and unit variance, $\alpha(B) = \theta(B)\Theta(L) - \phi(B)\Phi(L)$, $\phi(B) = 1 - \sum_{i=1}^{p} \phi_i B^i$,

$\theta(B) = 1 - \sum_{i=1}^{q} \theta_i B^i$, $\Phi(L) = 1 - \sum_{i=1}^{P} \Phi_i L^i$, $\Theta(L) = 1 - \sum_{i=1}^{Q} \Theta_i L^i$, $L = B^s$, and all coefficients are assumed to be positive.

Letting $u_t = \epsilon_t^2 - h_t$ and $\sigma_u^2 = \text{var}(u_t)$, (4) may be written as,

$$\phi_p(B)\Phi_P(L)\epsilon_t^2 = \omega + \theta_q(B)\Theta_Q(L)u_t, \tag{5}$$

which has a seasonal ARMA(p,q)x$(P,Q)_s$ representation for ϵ_t^2. Note that when $P = Q = 0$, (5) simplifies to an ARMA(max$\{p,q\}, q$) representation for ϵ_t^2, corresponding to the general GARCH(p,q) model.

We assume that $|\beta| < 1$; thus, y_t as given in (2) is stationary. The moving average representation is $y_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$ where $\{\psi_j\}$ is a sequence of constants and $\sum_{j=0}^{\infty} \psi_j^2 < \infty$. The ψ_j's are obtained from $(1-\beta B)\psi(B) = 1$ where $\psi(B) = 1 + \sum_{j=1}^{\infty} \psi_j B^j$.

We also assume that all the zeros of the polynomial $\phi(B)\Phi(L)$ lie outside the unit circle; thus, ϵ_t^2 as given in (5) is stationary. The moving average representation is $\epsilon_t^2 = \mu + \sum_{j=0}^{\infty} \Psi_j u_{t-j}$ where $\{\Psi_j\}$ is a sequence of constants and $\sum_{j=0}^{\infty} \Psi_j^2 < \infty$. The Ψ_j's are obtained from $\Psi(B)\phi(B)\Phi(L) = \theta(B)\Theta(L)$ where $\Psi(B) = 1 + \sum_{j=1}^{\infty} \Psi_j B^j$.

Next, we provide the kurtosis, the forecast, and the forecast error variance for an AR(1)-seasonal GARCH(p,q)x$(P,Q)_s$.

Lemma 3.1. For the stationary AR(1) process y_t with multiplicative seasonal GARCH innovations as in (2)– (4) we have the following relationships:

$$\text{(i)} \quad E(y_t^2) = \frac{E(\epsilon_t^2)}{1-\beta^2}, \tag{6}$$

$$\text{(ii)} \quad E(y_t^4) = \frac{6\beta^2[E(\epsilon_t^2)]^2 + (1-\beta^2)E(\epsilon_t^4)}{(1-\beta^2)(1-\beta^4)}, \tag{7}$$

$$\text{(iii)} \quad K^{(y)} = \frac{E(y_t^4)}{[E(y_t^2)]^2} = \frac{6\beta^2(1-\beta^2)}{1-\beta^4} + \frac{(1-\beta^2)^2}{1-\beta^4}K^{(\epsilon)}. \tag{8}$$

The kurtosis for $\epsilon_t, K^{(\epsilon)}$, is given below.

Lemma 3.2. For the stationary process (3) with finite fourth moment, the kurtosis $K^{(\epsilon)}$ is given by:

(a) $K^{(\epsilon)} = \dfrac{E(Z_t^4)}{E(Z_t^4) - [E(Z_t^4) - 1]\sum_{j=0}^{\infty} \Psi_j^2}$.

(b) The variance of the ϵ_t^2 process is given by $\gamma_0^{\epsilon^2} = \sum_{j=0}^{\infty} \Psi_j^2 \sigma_u^2$

where $\sigma_u^2 = \dfrac{\mu^2(K^{(\epsilon)} - 1)}{\sum_{j=0}^{\infty} \Psi_j^2}$ and $\mu = E(\epsilon_t^2) = \dfrac{\omega}{\left(1 - \sum_{i=1}^{p} \phi_i\right)\left(1 - \sum_{i=1}^{P} \Phi_i\right)}$.

Part (a) is derived in Thavaneswaran et al. (2005a) where examples are given with Ψ-weights derived for non-seasonal GARCH models. The Ψ-weights for examples of seasonal GARCH models, and the proof of part (b), are given in Doshi et al. (2011).

Extending Doshi et al. (2011), we derive the $K^{(y)}$ for AR(1)-seasonal GARCH(p,q)x$(P,Q)_s$ models as follows.

Example 3.1. For a stationary autoregressive process of order one, AR(1), with multiplicative seasonal GARCH $(0,1)$x$(0,1)_s$ errors of the form:

$$\begin{aligned} y_t &= \beta y_{t-1} + \epsilon_t \\ \epsilon_t &= \sqrt{h_t} Z_t \\ \epsilon_t^2 &= \omega + (1-\theta B)(1-\Theta L) u_t \end{aligned}$$

where $u_t = \epsilon_t^2 - h_t$, θ is the moving average parameter and Θ is the seasonal moving average parameter. The Ψ-weights are given in Doshi et al. (2011) as $\Psi_1 = -\theta_1$, $\Psi_s = -\Theta$, $\Psi_{s+1} = \theta\Theta$, and $\Psi_j = 0$ otherwise. It can be shown that $\sum_{j=0}^{\infty} \Psi_j^2 = (1+\theta^2)(1+\Theta^2)$. Then, the kurtosis of y_t is:

$$K^{(y)} = \frac{6\beta^2(1-\beta^2)}{(1-\beta^4)} + \frac{(1-\beta^2)^2}{(1-\beta^4)} \frac{E(Z_t^4)}{E(Z_t^4) - [E(Z_t^4)-1](1+\theta^2)(1+\Theta^2)}, \tag{9}$$

which for a conditionally normally distributed Z_t reduces to:

$$K^{(y)} = \frac{6\beta^2(1-\beta^2)}{(1-\beta^4)} + \frac{(1-\beta^2)^2}{(1-\beta^4)} \frac{3}{[3-2(1+\theta^2)(1+\Theta^2)]}.$$

Example 3.2. For a stationary autoregressive process of order one, AR(1), with multiplicative seasonal GARCH $(0,1)$x$(1,0)_s$ errors of the form,

$$\begin{aligned} y_t &= \beta y_{t-1} + \epsilon_t \\ \epsilon_t &= \sqrt{h_t} Z_t \\ (1-\Phi L)\epsilon_t^2 &= \omega + (1-\theta B) u_t \end{aligned}$$

where Φ is the seasonal autoregressive parameter and θ is the moving average parameter. The Ψ-weights given in Doshi et al. (2011) are as follows: $\Psi_1 = -\theta$, $\Psi_s = -\Phi$, $\Psi_{s+1} = -\theta\Phi$, $\Psi_{2s} = \Phi^2, \ldots, \Psi_{ks} = \Phi^k, \Psi_{ks+1} = -\theta\Phi^k$, where $k = 1, 2, \ldots$ It can be shown that $\sum_{j=0}^{\infty} \Psi_j^2 = (1+\theta^2)/(1-\Theta^2)$. Then, the kurtosis of y_t is:

$$K^{(y)} = \frac{6\beta^2(1-\beta^2)}{(1-\beta^4)} + \frac{(1-\beta^2)^2}{(1-\beta^4)} \frac{E(Z_t^4)}{E(Z_t^4) - [E(Z_t^4)-1]\left(\dfrac{1+\theta^2}{1-\Phi^2}\right)},$$

which for a conditionally normally distributed Z_t reduces to:

$$K^{(y)} = \frac{6\beta^2(1-\beta^2)}{(1-\beta^4)} + \frac{(1-\beta^2)^2}{(1-\beta^4)} \frac{3(1-\Phi^2)}{(1-3\Phi^2-2\theta^2)}.$$

Example 3.3. For a stationary autoregressive process of order one, AR(1), with multiplicative seasonal GARCH $(1,0)\text{x}(1,0)_s$ errors of the form,

$$\begin{aligned} y_t &= \beta y_{t-1} + \epsilon_t \\ \epsilon_t &= \sqrt{h_t} Z_t \\ (1-\phi B)(1-\Phi L)\epsilon_t^2 &= \omega + u_t \end{aligned}$$

where ϕ is the autoregressive parameter and Φ is the seasonal autoregressive parameter. The Ψ-weights given in Doshi et al. (2011) are as follows: $\Psi_1 = \phi, \Psi_2 = \phi^2, \ldots, \Psi_{s-1} = \phi^{s-1}, \Psi_s = \phi^2 + \Phi, \ldots, \Psi_j = \phi\Psi_{j-1} + \Phi\Psi_{j-s} - \phi\Phi\Psi_{j-s}$. It can be shown that $\sum_{j=0}^{\infty} \Psi_j^2 = \dfrac{1 + 2\phi^s\Phi^2 + \Phi^2}{1-\phi^2}$.
Then, the kurtosis of y_t is:

$$K^{(y)} = \frac{6\beta^2(1-\beta^2)}{(1-\beta^4)} + \frac{(1-\beta^2)^2}{(1-\beta^4)} \frac{E(Z_t^4)}{E(Z_t^4) - [E(Z_t^4) - 1]\left(\dfrac{1+2\phi^s\Phi + \Phi^2}{1-\phi^2}\right)},$$

which for a conditionally normally distributed Z_t reduces to:

$$K^{(y)} = \frac{6\beta^2(1-\beta^2)}{(1-\beta^4)} + \frac{(1-\beta^2)^2}{(1-\beta^4)} \frac{3(1-\phi^2)}{(1-3\phi^2-4\phi^s\Phi-2\Phi^2)}.$$

Forecast error variance

Thavaneswaran et al. (2005a) derive the expression for the forecast error variance of various classes of zero mean GARCH(p,q) processes, in terms of the kurtosis and Ψ-weights. Thavaneswaran & Ghahramani (2008) extend the results for ARMA (p,q) processes with GARCH (P,Q) errors. In this section we extend the results to AR models with multiplicative seasonal GARCH$(p,q)\text{x}(P,Q)_s$ errors.

Theorem 3.1. Let $y_n(l)$ be the l-steps-ahead minimum mean square forecast of y_{n+l} and let $e_n^{(y)}(l) = y_{n+l} - y_n(l)$ be the corresponding forecast error. The variance of the l-steps-ahead forecast error of y_{n+l} for the AR(1) model with seasonal GARCH errors as given in (2)- (4) is:

$$\text{Var}[e_n^{(y)}(l)] = \frac{\omega}{\left(1-\sum_{i=1}^{p}\phi_i\right)\left(1-\sum_{i=1}^{P}\Phi_i\right)} \sum_{j=0}^{l-1} \psi_j^2. \tag{10}$$

Proof. The theorem follows from the fact that for a stationary process with uncorrelated error noise ϵ_t the variance of the l-steps ahead forecast error is $\sigma_\epsilon^2 \sum_{j=0}^{l-1} \psi_j^2$ and from part (b) of Lemma 3.2.

We now have expressions for the variance of the l-steps-ahead forecast error of y_{n+l} for the previously discussed AR(1)-GARCH$(p,q)\text{x}(P,Q)_s$ models:

AR(1)-GARCH$(0,1)\text{x}(0,1)_s$ $\qquad$ $\text{Var}[e_n^{(y)}(l)] = \omega \sum_{j=0}^{l-1} \beta^{2j}$

AR(1)-GARCH$(0,1)$x$(1,0)_s$ $\quad$ $\mathrm{Var}[e_n^{(y)}(l)] = \dfrac{\omega}{1-\Phi}\sum_{j=0}^{l-1}\beta^{2j}$

AR(1)-GARCH$(1,0)$x$(1,0)_s$ $\quad$ $\mathrm{Var}[e_n^{(y)}(l)] = \dfrac{\omega}{(1-\phi)(1-\Phi)}\sum_{j=0}^{l-1}\beta^{2j}.$

In the literature on time series analysis, the error variance is estimated by the residual sum of squares. If we denote the squared residual as $Y_t = (y_t - \hat{\beta}y_{t-1})^2$, then we can forecast the conditional variance, $\mathrm{var}(y_t|y_{t-1},\ldots) = h_t$, by using $Y_1,\ldots,Y_{t-1}$.

Theorem 3.2. Let $Y_n(l)$ be the l-steps-ahead minimum mean square forecast of Y_{n+l} and let $e_n^{(Y)}(l) = Y_{n+l} - Y_n(l)$ be the corresponding forecast error. The variance of the l-steps-ahead forecast error of Y_{n+l} is given by:

$$\mathrm{Var}[e_n^{(Y)}(l)] = \sigma_u^2\sum_{j=0}^{l-1}\Psi_j^2 = \frac{\omega^2}{\left[\sum_{j=0}^{\infty}\Psi_j^2\right]\left[1-\sum_{i=1}^{p}\phi_i\right]^2\left[1-\sum_{i=1}^{P}\Phi_i\right]^2}[K^{(\epsilon)}-1]\left[\sum_{j=0}^{l-1}\Psi_j^2\right] \tag{11}$$

where, from (8), $K^{(\epsilon)} = \dfrac{1-\beta^4}{(1-\beta^2)^2}K^{(y)} - \dfrac{6\beta^2}{1-\beta^2}$.

Proof. The proof follows from part (b) of Lemma 3.2.

We now have expressions for the variance of the l-steps-ahead forecast error of Y_{n+l} for the previously discussed AR(1)-GARCH(p,q)x$(P,Q)_s$ models:

AR(1)-GARCH$(0,1)$x$(0,1)_s$ $\quad$ $\mathrm{Var}[e_n^{(Y)}(l)] = \dfrac{(K^{(\epsilon)}-1)\mu^2}{(1+\theta^2)(1+\Theta^2)}\sum_{j=0}^{l-1}\Psi_j^2$

AR(1)-GARCH$(0,1)$x$(1,0)_s$ $\quad$ $\mathrm{Var}[e_n^{(Y)}(l)] = \dfrac{(K^{(\epsilon)}-1)\mu^2(1-\Phi^2)}{1+\theta^2}\sum_{j=0}^{l-1}\Psi_j^2$

AR(1)-GARCH$(1,0)$x$(1,0)_s$ $\quad$ $\mathrm{Var}[e_n^{(Y)}(l)] = \dfrac{(K^{(\epsilon)}-1)\mu^2(1-\phi^2)}{1+2\phi^s\Phi+\Phi^2}\sum_{j=0}^{l-1}\Psi_j^2$

which are similar to the expressions given in Doshi et al. (2011). Here, $K^{(\epsilon)}$ is given in Theorem 3.2. and expressions for $K^{(y)}$ are given in Examples 3.1, 3.2, and 3.3.

4. RCA models with seasonal GARCH errors

The random coefficient autoregressive (RCA) model as proposed by Nicholls & Quinn (1982) has the form,

$$y_t = (\beta + b_t)y_{t-1} + \epsilon_t \tag{12}$$

where $\begin{pmatrix} b_t \\ \epsilon_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_\epsilon^2 \end{pmatrix}\right)$ and $\beta^2 + \sigma_b^2 < 1$.

Thavaneswaran et al. (2009) derive the moments for the RCA model with GARCH(p, q) errors. Here we propose the RCA model with seasonal GARCH innovations of the following form,

$$y_t = (\beta + b_t)y_{t-1} + \epsilon_t \tag{13}$$

$$\epsilon_t = \sqrt{h_t} Z_t \tag{14}$$

$$\theta(B)\Theta(L)h_t = \omega + \alpha(B)\epsilon_t^2 \tag{15}$$

where $Z_t, \theta(B), \Theta(L), \alpha(B)$ were defined in Section 2.
The general expression for the kurtosis $K^{(y)}$ parallels the one in Thavaneswaran et al. (2009) for non-seasonal GARCH innovations and can be written as follows.
Lemma 4.1. For the stationary RCA process y_t with GARCH innovations as in (13)– (15) we have the following relationships:

$$\text{(i)}\quad E(y_t^2) = \frac{E(\epsilon_t^2)}{1-(\beta^2+\sigma_b^2)}, \tag{16}$$

$$\text{(ii)}\quad E(y_t^4) = \frac{6(\beta^2+\sigma_b^2)[E(\epsilon_t^2)]^2 + [1-(\beta^2+\sigma_b^2)]E(\epsilon_t^4)}{1-(3\sigma_b^4+\beta^4+6\beta^2\sigma_b^2)[1-(\beta^2+\sigma_b^2)]}, \tag{17}$$

$$\text{(iii)}\quad K^{(y)} = \frac{6(\beta^2+\sigma_b^2)[1-(\beta^2+\sigma_b^2)]}{1-(3\sigma_b^4+\beta^4+6\beta^2\sigma_b^2)} + \frac{[1-(\beta^2+\sigma_b^2)]^2}{1-(3\sigma_b^4+\beta^4+6\beta^2\sigma_b^2)}K^{(\epsilon)}. \tag{18}$$

If Z_t is normally distributed, then the above equations can be written as:

$$\text{(i)}\quad E(y_t^2) = \frac{E(h_t)}{1-(\beta^2+\sigma_b^2)}, \tag{19}$$

$$\text{(ii)}\quad E(y_t^4) = \frac{6(\beta^2+\sigma_b^2)}{[1-(\beta^2+\sigma_b^2)](1-6\beta^2\sigma_b^2-\beta^4-3\sigma_b^4)}[E(h_t)]^2 + \frac{3E(h_t^2)}{1-6\beta^2\sigma_b^2-\beta^4-3\sigma_b^4}, \tag{20}$$

$$\text{(iii)}\quad K^{(y)} = \frac{6(\beta^2+\sigma_b^2)[1-(\beta^2+\sigma_b^2)]}{1-6\beta^2\sigma_b^2-\beta^4-3\sigma_b^4} + \frac{3(1-\beta^2-\sigma_b^2)}{1-6\beta^2\sigma_b^2-\beta^4-3\sigma_b^4}\frac{E(h_t^2)}{[E(h_t)]^2}. \tag{21}$$

Thavaneswaran et al. (2005a) show that:

$$\frac{E(h_t^2)}{[E(h_t)]^2} = \frac{1}{E(Z_t^4) - [E(Z_t^4)-1]\sum_{j=0}^{\infty}\Psi_j^2},$$

which for a conditionally normally distributed ϵ_t reduces to $\dfrac{1}{3-2\sum_{j=0}^{\infty}\Psi_j^2}$.
Example 4.1. RCA(1) with multiplicative seasonal GARCH (0,1)x(0,1) process

$$y_t = (\beta + b_t)y_{t-1} + \epsilon_t$$
$$\epsilon_t = \sqrt{h_t} Z_t$$
$$\epsilon_t^2 = \omega + (1-\theta B)(1-\Theta L)u_t$$

where $u_t = \epsilon_t^2 - h_t$.The Ψ-weights are given in example 3.1. Then, the kurtosis of y_t for a conditionally normally distributed Z_t is:

$$K^{(y)} = \frac{6(\sigma_b^2+\beta^2)(1-\beta^2-\sigma_b^2)}{1-6\beta^2\sigma_b^2-\beta^4-3\sigma_b^4} + \frac{3(1-\beta^2-\sigma_b^2)}{(1-6\beta^2\sigma_b^2-\beta^4-3\sigma_b^4)[3-2(1+\theta^2)(1+\Theta^2)]}.$$

Example 4.2. RCA(1) with multiplicative seasonal GARCH (0,1)x(1,0) process

$$\begin{aligned} y_t &= (\beta+b_t)y_{t-1}+\epsilon_t \\ \epsilon_t &= \sqrt{h_t}Z_t \\ (1-\Phi L)\epsilon_t^2 &= \omega+(1-\theta B)u_t \end{aligned}$$

The Ψ-weights are given in example 3.2. Then, the kurtosis of y_t for a conditionally normally distributed Z_t is:

$$K^{(y)} = \frac{6(\sigma_b^2+\beta^2)(1-\beta^2-\sigma_b^2)}{1-6\beta^2\sigma_b^2-\beta^4-3\sigma_b^4} + \frac{3(1-\beta^2-\sigma_b^2)}{(1-6\beta^2\sigma_b^2-\beta^4-3\sigma_b^4)\left[3-2\left(\frac{1+\theta^2}{1-\Phi^2}\right)\right]}.$$

Example 4.3. RCA(1) with multiplicative seasonal GARCH (1,0)x(1,0) process

$$\begin{aligned} y_t &= (\beta+b_t)y_{t-1}+\epsilon_t \\ \epsilon_t &= \sqrt{h_t}Z_t \\ (1-\phi B)(1-\Phi L)\epsilon_t^2 &= \omega+u_t \end{aligned}$$

The Ψ-weights are given in example 3.3. Then, the kurtosis of y_t for a conditionally normally distributed Z_t is:

$$K^{(y)} = \frac{6(\sigma_b^2+\beta^2)(1-\beta^2-\sigma_b^2)}{1-6\beta^2\sigma_b^2-\beta^4-3\sigma_b^4} + \frac{3(1-\beta^2-\sigma_b^2)}{(1-6\beta^2\sigma_b^2-\beta^4-3\sigma_b^4)\left[3-2\left(\frac{1+2\phi^s\Phi+\Phi^2}{1-\Phi^2}\right)\right]}.$$

Forecast error variance

Thavaneswaran & Ghahramani (2008) derive the expression for the variance of the forecast error for a RCA(1) process with non-seasonal GARCH (1,1) errors. In this section we expand the results for the more general RCA(1) process with seasonal GARCH(p,q)x$(P,Q)_s$ errors.

Theorem 4.1. Let $y_n(l)$ be the l-steps-ahead minimum mean square forecast of y_{n+l} and let $e_n^{(y)}(l) = y_{n+l} - y_n(l)$ be the corresponding forecast error. The variance of the l-steps-ahead forecast error of y_{n+l} for the RCA(1) model with seasonal GARCH errors as given in (13)- (15) is:

$$\mathrm{Var}[e_n^{(y)}(l)] = \frac{\omega(1-\beta^2)}{\left(1-\sum_{i=1}^{p}\phi_i\right)\left(1-\sum_{i=1}^{P}\Phi_i\right)(1-\beta^2-\sigma_b^2)}\sum_{j=0}^{l-1}\beta^{2j}. \tag{22}$$

Proof. The y_t process is second order stationary with autocorrelation $\rho_k = \beta^k$ and variance $\sigma_\epsilon^2/(1-\beta^2-\sigma_b^2)$. Hence, y_t has a valid moving average representation of the form $y_t^* =$

$\sum_{j=0}^{\infty} \beta^j a_{t-j}$, where a_t is an uncorrelated sequence with variance σ_a^2. By equating the variance of y_t^* to the variance of y_t we have $\sigma_\epsilon^2/(1-\beta^2-\sigma_b^2) = \sigma_a^2/(1-\beta^2)$, and $\sigma_a^2 = \sigma_\epsilon^2(1-\beta^2)/(1-\beta^2-\sigma_b^2)$.

Note: When $\sigma_b^2 = 0$, $\text{var}[e_n^{(y)}(l)]$ in Theorem 4.1 reduces to $\text{var}[e_n^{(y)}(l)]$ in Theorem 3.1 for the AR model with seasonal GARCH errors.

We now have expressions for the variance of the l-steps-ahead forecast error of y_{n+l} for the previously discussed RCA(1)-GARCH(p,q)x$(P,Q)_s$ models:

$$\text{RCA(1)-GARCH}(0,1)\text{x}(0,1)_s \qquad \text{Var}[e_n^{(y)}(l)] = \frac{\omega(1-\beta^2)}{(1-\beta^2-\sigma_b^2)} \sum_{j=0}^{l-1} \beta^{2j}$$

$$\text{RCA(1)-GARCH}(0,1)\text{x}(1,0)_s \qquad \text{Var}[e_n^{(y)}(l)] = \frac{\omega(1-\beta^2)}{(1-\Phi)(1-\beta^2-\sigma_b^2)} \sum_{j=0}^{l-1} \beta^{2j},$$

$$\text{RCA(1)-GARCH}(1,0)\text{x}(1,0)_s \qquad \text{Var}[e_n^{(y)}(l)] = \frac{\omega(1-\beta^2)}{(1-\phi)(1-\Phi)(1-\beta^2-\sigma_b^2)} \sum_{j=0}^{l-1} \beta^{2j}.$$

Theorem 4.2. Let $Y_t = [y_t - (\hat{\beta} + b_t)y_{t-1}]^2$. Also, let $Y_n(l)$ be the l-steps-ahead minimum mean square forecast of Y_{n+l} and let $e_n^{(Y)}(l) = Y_{n+l} - Y_n(l)$ be the corresponding forecast error. The variance of the l-steps-ahead forecast error of Y_{n+l} for the RCA(1) model with seasonal GARCH errors as given in (13)- (15) is:

$$\text{Var}[e_n^{(Y)}(l)] = \sigma_u^2 \sum_{j=0}^{l-1} \Psi_j^2 = \frac{\omega^2}{\left[\sum_{j=0}^{\infty} \Psi_j^2\right] \left[1 - \sum_{i=1}^{p} \phi_i\right]^2 \left[1 - \sum_{i=1}^{P} \Phi_i\right]^2} [K^{(\epsilon)} - 1] \left[\sum_{j=0}^{l-1} \Psi_j^2\right] \tag{23}$$

where, from (18), $K^{(\epsilon)} = \dfrac{1-(3\sigma_b^4 + \beta^4 + 6\beta^2\sigma_b^2)}{[1-(\beta^2+\sigma_b^2)]^2} K^{(y)} - \dfrac{6(\beta^2+\sigma_b^2)}{1-(\beta^2+\sigma_b^2)}$.

Proof. The proof follows from part (b) of Lemma 3.2.

Note: When $\sigma_b^2 = 0$, $K^{(\epsilon)}$ in Theorem 4.2 reduces to $K^{(\epsilon)}$ in Theorem 3.2 for the AR model with seasonal GARCH errors.

We now have expressions for the variance of the l-steps-ahead forecast error for the previously discussed RCA(1)-GARCH(p,q)x$(P,Q)_s$ models:

$$\text{RCA(1)-GARCH}(0,1)\text{x}(0,1)_s \qquad \text{Var}[e_n^{(Y)}(l)] = \frac{(K^{(\epsilon)}-1)\mu^2}{(1+\theta^2)(1+\Theta^2)} \sum_{j=0}^{l-1} \Psi_j^2$$

$$\text{RCA(1)-GARCH}(0,1)\text{x}(1,0)_s \qquad \text{Var}[e_n^{(Y)}(l)] = \frac{(K^{(\epsilon)}-1)\mu^2(1-\Phi^2)}{1+\theta^2} \sum_{j=0}^{l-1} \Psi_j^2$$

$$\text{RCA(1)-GARCH}(1,0)\text{x}(1,0)_s \qquad \text{Var}[e_n^{(Y)}(l)] = \frac{(K^{(\epsilon)}-1)\mu^2(1-\phi^2)}{1+2\phi^s\Phi+\Phi^2} \sum_{j=0}^{l-1} \Psi_j^2$$

which are similar to the expressions given in Doshi et al. (2011). Here, $K^{(\epsilon)}$ is given in Theorem 4.2. and expressions for $K^{(y)}$ for a conditionally normally distributed ϵ_t are given in Examples 4.1, 4.2, and 4.3.

5. RCA models with seasonal SV errors

We start with Taylor's (2005) stochastic volatility (SV) model and propose its seasonal form,

$$y_t = (\beta + b_t)y_{t-1} + \epsilon_t \tag{24}$$

$$\epsilon_t = Z_t e^{\frac{1}{2}h_t} \qquad Z_t \sim N(0,1) \tag{25}$$

$$\phi(B)\Phi(L)h_t = \omega + v_t \qquad v_t \sim N(0,\sigma_v^2) \tag{26}$$

where ϵ_t and h_t are innovations of the observed time series y_t and the unobserved stochastic volatility, respectively. Also, $\phi(B) = 1 - \sum_{i=1}^{q} \phi_i B^i$, $\Phi(L) = 1 - \sum_{i=1}^{Q} \Phi_i L^i$, and $L = B^s$, where s is the seasonal period. We assume that all the zeros of the polynomial $\phi(B)\Phi(L)$ lie outside the unit circle; thus, h_t as given in (26) is stationary. The moving average representation is $h_t = \omega + \sum_{j=0}^{\infty} \Psi_j v_{t-j}$ where $\{\Psi_j\}$ is a sequence of constants and $\sum_{j=0}^{\infty} \Psi_j^2 < \infty$. The Ψ_j's are obtained from $\phi(B)\Phi(L)\Psi(B) = 1$ where $\Psi(B) = 1 + \sum_{j=1}^{\infty} \Psi_j B^j$.

RCA models with SV innovations have been studied in Paseka et al. (2010). Here we consider the seasonal version of the SV process and we study the moment properties of RCA models with seasonal SV innovations.

Theorem 5.1. Suppose y_t is an RCA model with seasonal SV innovations as in (24)– (26). Then, we have the following relationship:

(i) $E(y_t^2) = \dfrac{E(\epsilon_t^2)}{1-(\beta^2+\sigma_b^2)}$,

(ii) $E(y_t^4) = \dfrac{6(\sigma_b^2+\beta^2)[E(\epsilon_t^2)]^2 + [1-(\beta^2+\sigma_b^2)]E(\epsilon_t^4)}{1-(3\sigma_b^4+\beta^4+6\beta^2\sigma_b^2)[1-(\beta^2+\sigma_b^2)]}$,

(iii) $K^{(y)} = \dfrac{6(\sigma_b^2+\beta^2)[1-(\beta^2+\sigma_b^2)]}{1-(3\sigma_b^4+\beta^4+6\beta^2\sigma_b^2)} + \dfrac{[1-(\beta^2+\sigma_b^2)]^2}{1-(3\sigma_b^4+\beta^4+6\beta^2\sigma_b^2)}K^{(\epsilon)}$,

(iv) $K^{(\epsilon)} = 3e^{\sigma_v^2 \sum_{j=0}^{\infty} \Psi_j^2}$

where $E(\epsilon_t^2) = \exp\left\{\mu_{h_t} + \frac{1}{2}\sigma_{h_t}^2\right\}$, $E(\epsilon_t^4) = 3\exp\left\{2\mu_{h_t} + 2\sigma_{h_t}^2\right\}$, the mean of the h_t process is $\mu_{h_t} = \dfrac{\omega}{(1-\sum_{i=1}^{q}\phi_i)(1-\sum_{i=1}^{Q}\Phi_i)}$ and the variance of h_t is $\sigma_{h_t}^2 = \sigma_v^2 \sum_{j=0}^{\infty} \Psi_j^2$.

Proof. Parts (i) to (iii) are similar to Paseka et al. (2010) for an RCA-non seasonal SV process. Part (iv) follows from the above expressions for $E(\epsilon_t^2)$ and $E(\epsilon_t^4)$ as follows:

$$K^{(\epsilon)} = \frac{E(\epsilon_t^4)}{[E(\epsilon_t^2)]^2} = \frac{3e^{2\mu_{h_t}+2\sigma_{h_t}^2}}{(e^{\mu_{h_t}+1/2\sigma_{h_t}^2})^2} = 3e^{\sigma_{h_t}^2} = 3e^{\sigma_v^2\sum_{j=0}^{\infty}\Psi_j^2}.$$

Next, we illustrate applications of Theorem 5.1 with three examples.

Example 5.1. RCA with autoregressive [AR(1)] SV process

$$y_t = (\beta + b_t)y_{t-1} + \epsilon_t$$

$$\epsilon_t = Z_t e^{\frac{1}{2}h_t}$$

$$(1-\phi B)h_t = \omega + v_t$$

The Ψ-weights are $\Psi_j = \phi^j, j \geq 1$. Therefore, $\sum_{j=0}^{\infty} \Psi_j^2 = 1 + \phi^2 + \phi^4 + \ldots = \frac{1}{1-\phi^2}$. Then, the kurtosis of y_t is:

$$K^{(y)} = \frac{6(\sigma_b^2 + \beta^2)[1 - (\beta^2 + \sigma_b^2)]}{1 - (3\sigma_b^4 + \beta^4 + 6\beta^2\sigma_b^2)} + 3\frac{[1 - (\beta^2 + \sigma_b^2)]^2}{1 - (3\sigma_b^4 + \beta^4 + 6\beta^2\sigma_b^2)} \exp\left\{\frac{\sigma_v^2}{1-\phi^2}\right\}.$$

Example 5.2. RCA with pure seasonal autoregressive [AR$(1)_s$] SV process

$$\begin{aligned} y_t &= (\beta + b_t)y_{t-1} + \epsilon_t \\ \epsilon_t &= Z_t e^{\frac{1}{2}h_t} \\ (1 - \Phi B^s)h_t &= \omega + v_t \end{aligned}$$

The Ψ-weights are $\Psi_j = \Phi^j, j \geq 1$. Therefore, $\sum_{j=0}^{\infty} \Psi_j^2 = 1 + \Phi^2 + \Phi^4 + \ldots = \frac{1}{1-\Phi^2}$. Then, the kurtosis of y_t is:

$$K^{(y)} = \frac{6(\sigma_b^2 + \beta^2)[1 - (\beta^2 + \sigma_b^2)]}{1 - (3\sigma_b^4 + \beta^4 + 6\beta^2\sigma_b^2)} + 3\frac{[1 - (\beta^2 + \sigma_b^2)]^2}{1 - (3\sigma_b^4 + \beta^4 + 6\beta^2\sigma_b^2)} \exp\left\{\frac{\sigma_v^2}{1-\Phi^2}\right\}.$$

Example 5.3. RCA with multiplicative seasonal autoregressive [AR(1)x$(1)_s$] SV process

$$\begin{aligned} y_t &= (\beta + b_t)y_{t-1} + \epsilon_t \\ \epsilon_t &= Z_t e^{\frac{1}{2}h_t} \\ (1 - \phi B)(1 - \Phi B^s)h_t &= \omega + v_t \end{aligned}$$

The Ψ-weights are $\Psi_1 = \phi + \Phi$, and $\Psi_j = (\phi + \Phi)\Psi_{j-1} + \phi\Phi\Psi_{j-2}$, $j \geq 2$. Then, the kurtosis of y_t is:

$$K^{(y)} = \frac{6(\sigma_b^2 + \beta^2)[1 - (\beta^2 + \sigma_b^2)]}{1 - (3\sigma_b^4 + \beta^4 + 6\beta^2\sigma_b^2)} + 3\frac{[1 - (\beta^2 + \sigma_b^2)]^2}{1 - (3\sigma_b^4 + \beta^4 + 6\beta^2\sigma_b^2)} e^{\sigma_{h_t}^2}$$

where $\sigma_{h_t}^2 = \frac{(1+\phi^s)\sigma_v^2}{(1-\phi^2)(1-\Phi^2)(1-\Phi\phi^s)}$.

Recently, Gong & Thavaneswaran (2009) discussed the filtering of SV models. The prediction of discrete SV models can be obtained by using the recursive method proposed in Gong & Thavaneswaran (2009).

6. Option pricing with seasonal volatility

Option pricing based on the Black-Scholes model is widely used in the financial community. The Black-Scholes formula is used for the pricing of European-style options. The model has traditionally assumed that the volatility of returns is constant. However, several studies have shown that asset returns exhibit variances that change over time. Duan (1995) proposes an option pricing model for an asset with returns following a GARCH process. Badescu & Kulpeger (2008); Elliot et al. (2006); Heston & Nandi (2000) and others derived closed form option pricing formulas for different models which are assumed to follow a GARCH volatility process. Most recently, Gong et al. (2010) derive an expression for the call price as an expectation with respect to random GARCH volatility. The model is then evaluated in terms of the moments of the volatility process. Their results indicate that the suggested model outperforms the classic Black-Scholes formula. Here we extend Gong et al. (2010) and propose an option pricing model with seasonal GARCH volatility as follows:

$$dS_t = rS_t dt + \sigma_t S_t dW_t \tag{27}$$

$$y_t = \log\left(\frac{S_t}{S_{t-1}}\right) - E\left[\log\left(\frac{S_t}{S_{t-1}}\right)\right] = \sigma_t Z_t \tag{28}$$

$$\theta(B)\Theta(L)\sigma_t^2 = \omega + \alpha(B)y_t^2 \tag{29}$$

where S_t is the price of the stock, r is the risk-free interest rate, $\{W_t\}$ is a standard Brownian motion, σ_t is the time-varying seasonal volatility process, $\{Z_t\}$ is a sequence of i.i.d. random variables with zero mean and unit variance and $\alpha(B)$, $\theta(B)$ and $\Theta(L)$ have been defined in (4).

The price of a call option can be calculated using the option pricing formula given in Gong et al. (2010). The call price is derived as a first conditional moment of a truncated lognormal distribution under the martingale measure, and it is based on estimates of the moments of the GARCH process. The call price based on the Black-Scholes model with seasonal GARCH volatility is given by:

$$\begin{aligned} C(S,T) = {} & S\left(f[E(\sigma_t^2)] + \frac{1}{2}f''[E(\sigma_t^2)]\left(\frac{1}{3}\kappa^{(y)} - 1\right)E^2(\sigma_t^2)\right) \\ & - Ke^{-rT}\left(g[E(\sigma_t^2)] + \frac{1}{2}g''[E(\sigma_t^2)]\left(\frac{1}{3}\kappa^{(y)} - 1\right)E^2(\sigma_t^2)\right), \end{aligned} \tag{30}$$

where f and g are twice differentiable functions, S is the initial value of S_t, K is the strike price, T is the expiry date, σ_t is a stationary process with finite fourth moment, and $\kappa^{(y)} = \frac{E(y_t^4)}{[E(y_t^2)]^2}$. Also, $f[E(\sigma_t^2)]$, $g[E(\sigma_t^2)]$, $f''[E(\sigma_t^2)]$, and $g''[E(\sigma_t^2)]$ are given by:

$$f[E(\sigma_t^2)] = \mathrm{N}(d) = \mathrm{N}\left(\frac{\log(S/K) + rT + \frac{1}{2}E(\sigma_t^2)}{\sqrt{E(\sigma_t^2)}}\right),$$

$$g[E(\sigma_t^2)] = \mathrm{N}\left(d - \sqrt{E(\sigma_t^2)}\right) = \mathrm{N}\left(\frac{\log(S/K) + rT - \frac{1}{2}E(\sigma_t^2)}{\sqrt{E(\sigma_t^2)}}\right),$$

$$\begin{aligned} f''[E(\sigma_t^2)] = {} & \frac{1}{\sqrt{2\pi}}\left[-\left(\frac{E(\sigma_t^2) - 2(\log(S/K) + rT)}{4E(\sigma_t^2)\sqrt{E(\sigma_t^2)}}\right)\left(\frac{[E(\sigma_t^2)]^2 - 4(\log(S/K) + rT)^2}{8[E(\sigma_t^2)]^2}\right)\right. \\ & \left. + \left(\frac{6(\log(S/K) + rT) - E(\sigma_t^2)}{8[E(\sigma_t^2)]^2\sqrt{E(\sigma_t^2)}}\right)\right] \times \exp\left\{-\frac{(2(\log(S/K) + rT) + E(\sigma_t^2))^2}{8E(\sigma_t^2)}\right\}, \end{aligned}$$

$$g''[E(\sigma_t^2)] = \frac{1}{\sqrt{2\pi}} \Bigg[\left(\frac{E(\sigma_t^2) + 2(\log(S/K) + rT)}{4E(\sigma_t^2)\sqrt{E(\sigma_t^2)}} \right) \left(\frac{[E(\sigma_t^2)]^2 - 4(\log(S/K) + rT)^2}{[E(\sigma_t^2)]^2} \right)$$

$$+ \left(\frac{6(\log(S/K) + rT) + E(\sigma_t^2)}{8[E(\sigma_t^2)]^2\sqrt{E(\sigma_t^2)}} \right) \Bigg] \exp\left\{ -\frac{(2(\log(S/K) + rT) - E(\sigma_t^2))^2}{8E(\sigma_t^2)} \right\},$$

where N denotes the standard normal CDF, and under the option pricing model with seasonal GARCH volatility,

$$E(\sigma_t^2) = \frac{\omega}{\left(1 - \sum_{i=1}^{p} \phi_i\right)\left(1 - \sum_{i=1}^{P} \Phi_i\right)},$$

$$\kappa^{(y)} = \frac{3}{3 - 2\sum_{j=1}^{\infty} \Psi_j^2}.$$

7. Concluding remarks

In this chapter we propose various classes of seasonal volatility models. We consider time series processes such as AR and RCA with multiplicative seasonal GARCH errors and SV errors. The multiplicative seasonal volatility models are suitable for time series where autocorrelation exists at seasonal and at adjacent non-seasonal lags. The models introduced here extend and complement the existing volatility models in the literature to seasonal volatility models by introducing more general structures.

It is well-known that financial time series exhibit excess kurtosis. In this chapter we derive the kurtosis for different seasonal volatility models in terms of model parameters. We also derive the closed-from expression for the variance of the l-steps ahead forecast error of i) y_{n+l} in terms of ψ-weights and model parameters, and of ii) squared series Y_{n+l} in terms of Ψ-weights, model parameters and the kurtosis of ϵ_t. The results are a generalization of existing results for non-seasonal volatility processes. We provide examples for all the different classes of models considered and discussed them in some detail (i.e. AR(1)-GARCH$(p,q) \times (P,Q)_s$, RCA(1)-GARCH$(p,q) \times (P,Q)_s$ and RCA(1)-seasonal SV).

The results are primarily oriented to financial time series applications. Financial time series often meet the large dataset demands of the volatility models studied here. Also, financial data dynamics in higher order moments are of interest to many market participants. Specifically, we consider the Black-Scholes model with seasonal GARCH volatility and show that the moments of the seasonal volatility process can be used to evaluate the call price for European options.

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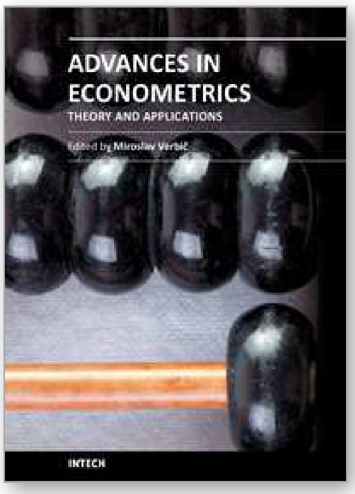

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