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Design of Active Vibration Absorbers Using On-Line Estimation of Parameters and Signals

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1. Introduction

Many engineering systems undergo undesirable vibrations. Vibration control in mechanical systems is an important problem by means of which vibrations are suppressed or at least attenuated. In this direction, the dynamic vibration absorbers have been widely applied in many practical situations because of their low cost/maintenance, efficiency, accuracy and easy installation (Braun et al., 2001; Preumont, 1993). Some of their applications can be found in buildings, bridges, civil structures, aircrafts, machine tools and many other engineering systems (Caetano et al., 2010; Korenev & Reznikov, 1993; Sun et al., 1995; Taniguchi et al., 2008; Weber & Feltrin, 2010; Yang, 2010).

There are three fundamental control design methodologies for vibration absorbers described as passive, semi-active and active vibration control. Passive vibration control relies on the addition of stiffness and damping to the primary system in order to reduce its dynamic response, and serves for specific excitation frequencies and stable operating conditions, but is not recommended for variable excitation frequencies and/or parametric uncertainty. Semiactive vibration control deals with adaptive spring or damper characteristics, which are tuned according to the operating conditions. Active vibration control achieves better dynamic performance by adding degrees of freedom to the system and/or controlling actuator forces depending on feedback and feedforward real-time information of the system, obtained from sensors. For more details about passive, semiactive and active vibration control we refer to the books (Braun et al., 2001; Den Hartog, 1934; Fuller et al, 1997; Preumont, 1993).

On the other hand, many dynamical systems exhibit a structural property called differential flatness. This property is equivalent to the existence of a set of independent outputs, called flat outputs and equal in number to the control inputs, which completely parameterizes every state variable and control input (Fliess et al., 1993; Sira-Ramirez & Agrawal, 2004). By means of differential flatness techniques the analysis and design of a controller is greatly simplified. In particular, the combination of differential flatness with the control approach called Generalized Proportional Integral (GPI) control, based on output measurements and integral reconstructions of the state variables (Fliess et al., 2002), qualifies as an adequate control scheme to achieve the robust asymptotic output tracking and, simultaneously, the cancellation/attenuation of harmonic vibrations. GPI controllers for design of active vibration absorbers have been previously addressed in (Beltran et al., 2003). Combinations of GPI control, sliding modes and on-line algebraic identification of harmonic vibrations for design of adaptive-like active vibration control schemes have been also proposed in (Beltran et al., 2010). A GPI control strategy implemented as a classical compensation network for robust perturbation rejection in mechanical systems has been presented in (Sira-Ramirez et al., 2008). In this chapter a design approach for active vibration absorption schemes in linear mass-spring-damper mechanical systems subject to exogenous harmonic vibrations is presented, which are based on differential flatness and GPI control, but taking the advantage of the interesting energy dissipation properties of passive vibration absorbers. Our design approach considers a mass-spring active vibration absorber as a dynamic controller, which can simultaneously be used for vibration attenuation and desired reference trajectory tracking tasks. The proposed approach allows extending the vibrating energy dissipation property of a passive vibration absorber for harmonic vibrations of any excitation frequency, by applying suitable control forces to the vibration absorber. Two different active vibration control schemes are synthesized, one employing only displacement measurements of the primary system and other using measurements of the displacement of the primary system as well as information of the excitation frequency. The algebraic parametric identification methodology reported by (Fliess & Sira-Ramirez, 2003), which employs differential algebra, module theory and operational calculus, is applied for the on-line estimation of the parameters associated to the external harmonic vibrations, using only displacement measurements of the primary system. Some experimental results on the application of on-line algebraic identification of parameters and excitation forces in vibrating mechanical systems were presented in (Beltran et al., 2004), which show their success in practical implementations.

The real-time algebraic identification of the excitation frequency is combined with a certainty equivalence controller to cancel undesirable harmonic vibrations affecting the primary mechanical system as well as to track asymptotically and robustly a specified output reference trajectory. The adaptive-like control scheme results quite fast and robust against parameter uncertainty and frequency variations.

The main virtue of the proposed identification and adaptive-like control scheme for vibrating systems is that only measurements of the transient input/output behavior are used during the identification process, in contrast to the well-known persisting excitation condition and complex algorithms required by most of the traditional identification methods (Isermann & Munchhof, 2011; Ljung, 1987; Soderstrom, 1989). It is important to emphasize that the proposed results are now possible thanks to the existence of high speed DSP boards with high computational performance operating at high sampling rates.

Finally, some simulation results are provided to show the robust and efficient performance of the proposed active vibration control schemes as well as of the proposed identifiers for on-line estimation of the unknown frequency and amplitude of resonant harmonic vibrations.

2. Vibrating mechanical system

2.1 Mathematical model

Consider the vibrating mechanical system shown in Fig. 1, which consists of an active undamped dynamic vibration absorber (secondary system) coupled to the perturbed mechanical system (primary system). The generalized coordinates are the displacements of both masses, x_1 and x_2, respectively. In addition, u represents the force control input and $f(t)$ some harmonic perturbation, possibly unknown. Here m_1, k_1 and c_1 denote mass, linear stiffness and linear viscous damping on the primary system, respectively. Similarly, m_2, k_2 and c_2 denote mass, stiffness and viscous damping of the dynamic vibration absorber. Note also that, when $u \equiv 0$ the active vibration absorber becomes only a passive vibration absorber.

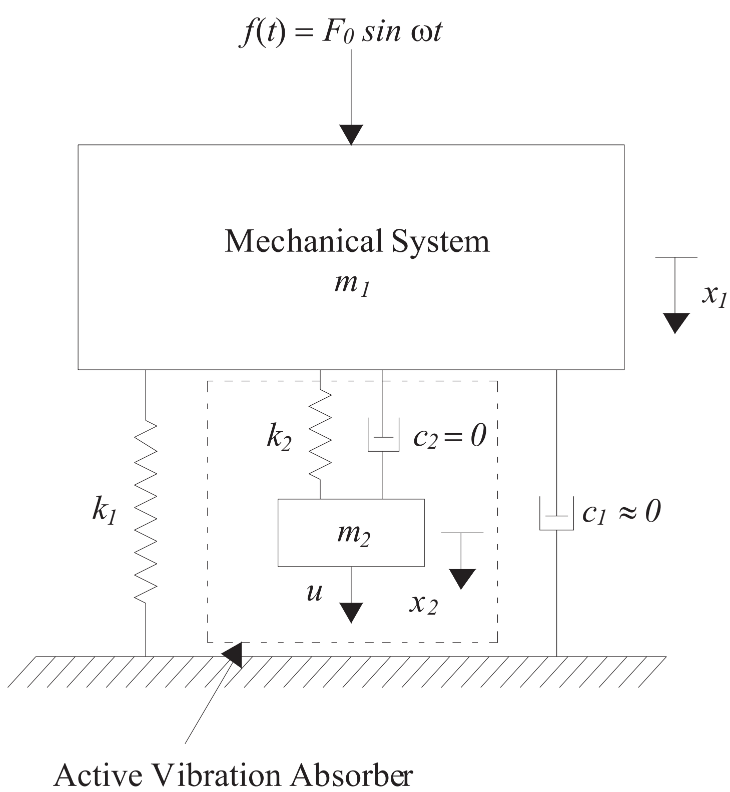

Fig. 1. Schematic diagram of the vibrating mechanical system with active vibration absorber.

The mathematical model of this two degrees-of-freedom system is described by the following two coupled ordinary differential equations

$$\begin{aligned} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + k_2(x_1 - x_2) &= f(t) \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) &= u(t) \end{aligned} \tag{1}$$

where $f(t) = F_0 \sin \omega t$, with F_0 and ω denoting the amplitude and frequency of the excitation force, respecively. In order to simplify the analysis we have assumed that $c_1 \approx 0$.

Defining the state variables as $z_1 = x_1, z_2 = \dot{x}_1, z_3 = x_2$ and $z_4 = \dot{x}_2$, one obtains the following state-space description

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= -\tfrac{k_1+k_2}{m_1} z_1 - \tfrac{c_1}{m_1} z_2 + \tfrac{k_2}{m_1} z_3 + \tfrac{1}{m_1} f(t) \\
\dot{z}_3 &= z_4 \\
\dot{z}_4 &= \tfrac{k_2}{m_2} z_1 - \tfrac{k_2}{m_2} z_3 + \tfrac{1}{m_2} u(t) \\
y &= z_1
\end{aligned} \tag{2}$$

It is easy to verify that the system (2) is completely controllable and observable as well as marginally stable in case of $c_1 = 0$, $f \equiv 0$ and $u \equiv 0$ (asymptotically stable when $c_1 > 0$). Note that, an immediate consequence is that, the output $y = z_1$ has relative degree 4 with respect to u and relative degree 2 with respect to f and, therefore, the so-called disturbance decoupling problem of the perturbation $f(t)$ from the output $y = z_1$, using state feedback, is not solvable (Isidori, 1995).

To cancel the exogenous harmonic vibrations on the primary system, the dynamic vibration absorber should apply an equivalent force to the primary system, with the same amplitude but in opposite phase (sign). This means that the vibration energy injected to the primary system, by the exogenous vibration $f(t)$, is transferred to the vibration absorber through the coupling elements (i.e., spring k_2). Of course, this vibration control method is possible under the assumption of perfect knowledge of the exogenous vibrations and stable operating conditions (Preumont, 1993).

In this work we will apply the algebraic identification method to estimate the parameters associated to the harmonic force $f(t)$ and then, propose the design of an active vibration controller based on state feedback and feedforward information of $f(t)$.

2.2 Passive vibration absorber

It is well known that a passive vibration absorber can only cancel the vibration $f(t)$ affecting the primary system if and only if the excitation frequency ω coincides with the uncoupled natural frequency of the absorber (Den Hartog, 1934), that is,

$$\omega_2 = \sqrt{\frac{k_2}{m_2}} = \omega \tag{3}$$

See Fig. 2, where X_1 denotes the steady-state maximum amplitude of $x_1(t)$ and δ_{st} the static deflection of the primary system under the constant force F_0. Note, however, that the interconnection of the passive vibration absorber to the primary system slightly changes the natural frequencies in both uncoupled subsystems and, hence, when $\omega \neq \omega_2$ and close to those resonant frequencies the amplitudes might be large or theoretically infinite. This situation clearly leads to large displacements and could damage of any physical system.

In what follows we shall use an active vibration absorber based on Generalized PI control (GPI) to provide some robustness with respect to variations on the excitation frequency ω, uncertain system parameters and initial conditions.

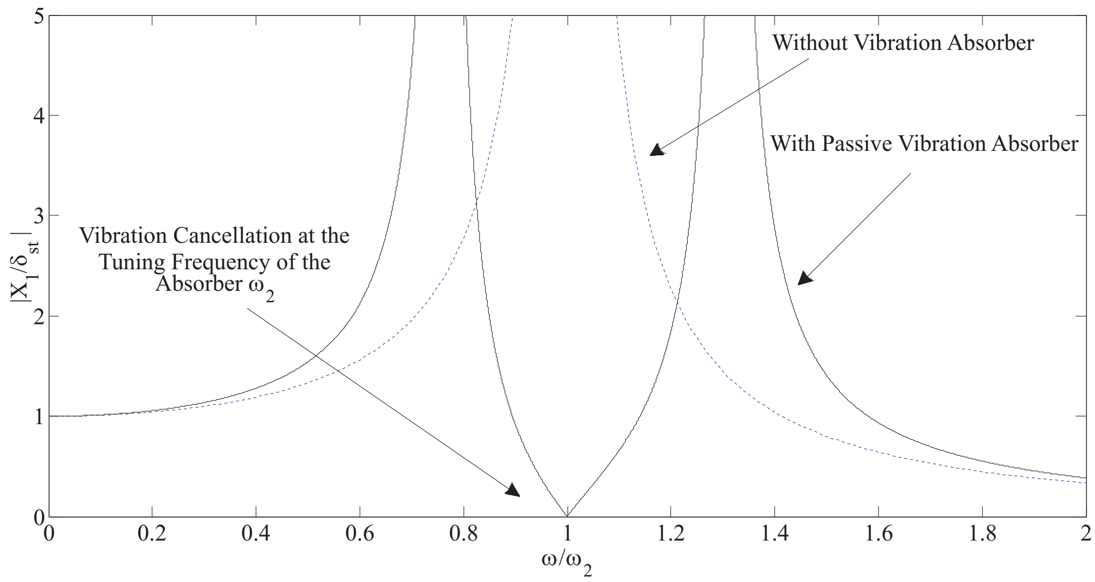

Fig. 2. Frequency response of the vibrating mechanical system with passive vibration absorber.

2.3 Differential flatness

Because the system (2) is completely controllable from u then, it is differentially flat, with flat output given by $y = z_1$. Then, all the state variables and the control input can be differentially parameterized in terms of the flat output y and a finite number of its time derivatives (Fliess et al., 1993; Sira-Ramirez & Agrawal, 2004).
In fact, from y and its time derivatives up to fourth order one can obtain that

$$\begin{aligned} y &= z_1 \\ \dot{y} &= z_2 \\ \ddot{y} &= -\tfrac{k_1+k_2}{m_1} z_1 + \tfrac{k_2}{m_1} z_3 \\ y^{(3)} &= -\tfrac{k_1+k_2}{m_1} z_2 + \tfrac{k_2}{m_1} z_4 \\ y^{(4)} &= \left[\tfrac{(k_1+k_2)^2}{m_1^2} + \tfrac{k_2^2}{m_1 m_2}\right] z_1 - \left[\tfrac{k_2(k_2+k_1)}{m_1^2} + \tfrac{k_2^2}{m_1 m_2}\right] z_3 + \tfrac{k_2}{m_1 m_2} u \end{aligned} \tag{4}$$

where $c_1 = 0$ and $f \equiv 0$. Therefore, the differential parameterization results as follows

$$\begin{aligned} z_1 &= y \\ z_2 &= \dot{y} \\ z_3 &= \tfrac{k_1+k_2}{k_2} y + \tfrac{m_1}{k_2} \ddot{y} \\ z_4 &= \tfrac{k_1+k_2}{k_2} \dot{y} + \tfrac{m_1}{k_2} y^{(3)} \\ u &= k_1 y + \left(m_1 + m_2 + \tfrac{k_1}{k_2} m_2\right) \ddot{y} + \tfrac{m_1 m_2}{k_2} y^{(4)} \end{aligned} \tag{5}$$

Then, the flat output y satisfies the following input-output differential equation

$$y^{(4)} = a_0 y + a_2 \ddot{y} + bu \tag{6}$$

where

$$a_0 = -\frac{k_1 k_2}{m_1 m_2}$$
$$a_2 = -\left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2}\right)$$
$$b = \frac{k_2}{m_1 m_2}$$

From (6) one obtains the following differential flatness-based controller to asymptotically track some desired reference trajectory $y^*(t)$:

$$u = b^{-1}\left(v - a_0 y - a_2 \ddot{y}\right) \tag{7}$$

with

$$v = (y^*)^{(4)}(t) - \beta_6\left[y^{(3)} - (y^*)^{(3)}(t)\right] - \beta_5\left[\ddot{y} - \ddot{y}^*(t)\right] - \beta_4\left[\dot{y} - \dot{y}^*(t)\right] - \beta_3\left[y - y^*(t)\right]$$

The use of this controller yields the following closed-loop dynamics for the trajectory tracking error $e = y - y^*(t)$:

$$e^{(4)} + \beta_6 e^{(3)} + \beta_5 \ddot{e} + \beta_4 \dot{e} + \beta_3 e = 0 \tag{8}$$

Therefore, selecting the design parameters β_i, $i = 3, ..., 6$, such that the associated characteristic polynomial for (8) be *Hurwitz*, i.e., all its roots lying in the open left half complex plane, one can guarantee that the error dynamics be globally asymptotically stable.
Nevertheless, this controller is not robust with respect to exogenous signals or parameter uncertainties in the model. In case of $f(t) \neq 0$, the parameterization should explicitly include the effect of f and its time derivatives up to second order. In addition, the implementation of this controller requires measurements of the time derivatives of the flat output up to third order and vibration signal and its time derivatives up to second order.
Remark. In spite of the linear models under study, it results important to emphasize the great potential of the differential flatness approach for nonlinear flat systems, which can be analyzed using similar arguments (Fliess & Sira-Ramirez, 2003). In fact, the proposed results can be generalized to some classes of nonlinear mechanical systems.
Next, we will synthesize two controllers based on the Generalized PI (GPI) control approach combined with differential flatness and passive absorption, in order to get robust controllers against external vibrations.

3. Generalized PI control

3.1 Control scheme using displacement measurement on the primary system

Since the system (2) is observable for the flat output y then, all the time derivatives of the flat output can be reconstructed by means of integrators, that is, they can be expressed in terms of the flat output y, the input u and iterated integrals of the input and the output variables (Fliess et al., 2002).
For simplicity, we will denote the integral $\int_0^t \varphi(\tau)\, d\tau$ by $\int \varphi$ and $\int_0^t \int_0^{\sigma_1} \cdots \int_0^{\sigma_{n-1}} \varphi(\sigma_n)\, d\sigma_n \cdots d\sigma_1$ by $\int^{(n)} \varphi$ with n a positive integer. The integral input-output

parameterization of the time derivatives of the flat output is given, modulo initial conditions, by

$$\begin{aligned} \widehat{\dot{y}} &= a_0 \int^{(3)} y + a_2 \int y + b \int^{(3)} u \\ \widehat{\ddot{y}} &= a_0 \int^{(2)} y + a_2 y + b \int^{(2)} u \\ \widehat{y^{(3)}} &= a_0 \int y + a_2 \widehat{\dot{y}} + b \int u \end{aligned} \tag{9}$$

These expressions were obtained by successive integrations of the last equation in (6). For non-zero initial conditions, the relations linking the actual values of the time derivatives of the flat output to the structural estimates in (9) are given as follows

$$\begin{aligned} \dot{y} &= \widehat{\dot{y}} + e_{12} t^2 + e_{11} t + e_{11} \\ \ddot{y} &= \widehat{\ddot{y}} + g_{11} t + g_{10} \\ y^{(3)} &= \widehat{y^{(3)}} + h_{12} t^2 + h_{11} t + h_{10} \end{aligned} \tag{10}$$

where $e_{1i}, g_j, h_i, i = 0, \ldots, 2, j = 0, \ldots, 1$, are real constants depending on the unknown initial conditions.
For the design of the GPI controller, the time derivatives of the flat output are replaced for their structural estimates (9) into (7). This, however, implies that the closed-loop system would be actually excited by constant values, ramps and quadratic functions. To eliminate these destabilizing effects of such structural estimation errors, one can use the following controller with iterated integral error compensation:

$$\begin{aligned} u &= b^{-1} \left(v - a_0 y - a_2 \widehat{\ddot{y}} \right) \\ v &= (y^*)^{(4)}(t) - \beta_6 \left[\widehat{y^{(3)}} - (y^*)^{(3)}(t) \right] - \beta_5 \left[\widehat{\ddot{y}} - \ddot{y}^*(t) \right] - \beta_4 \left[\widehat{\dot{y}} - \dot{y}^*(t) \right] \\ & -\beta_3 \left[y - y^*(t) \right] - \beta_2 \xi_1 - \beta_1 \xi_2 - \beta_0 \xi_3 \\ \dot{\xi}_1 &= y - y^*(t), \quad \xi_1(0) = 0 \\ \dot{\xi}_2 &= \xi_1, \qquad \xi_2(0) = 0 \\ \dot{\xi}_3 &= \xi_2, \qquad \xi_3(0) = 0 \end{aligned} \tag{11}$$

The use of this controller yields the following closed-loop system dynamics for the tracking error, $e = y - y^*(t)$, described by

$$e^{(7)} + \beta_6 e^{(6)} + \beta_5 e^{(5)} + \beta_4 e^{(4)} + \beta_3 e^{(3)} + \beta_2 \ddot{e} + \beta_1 \dot{e} + \beta_0 e = 0 \tag{12}$$

The coefficients β_i, $i = 0, ..., 6$, have to be selected in such way that the characteristic polynomial of (12) be *Hurwitz*. Thus, one can conclude that $\lim_{t\to\infty} e(t) = 0$, i.e., the asymptotic output tracking of the reference trajectory $\lim_{t\to\infty} y(t) = y^*(t)$.

3.1.1 Robustness analysis with respect to external vibrations

Now, consider that the passive vibration absorber is tuned at the uncoupled natural frequency of the primary system, that is, $\omega_2 = \omega_1$. The transfer function of the closed-loop system from the perturbation $f(t)$ to the output $y = z_1$ is then given by

$$
\begin{aligned}
G(s) =& \frac{x_1(s)}{f(s)} \\
=& \frac{\mu\left(m_2 s^2 + k_2\right)\left(m_2 s^3 + \beta_6 m_2 s^2 + \beta_5 m_2 s - \beta_6 k_2 \mu - 2\beta_6 k_2 + \beta_4 m_2 - 2k_2 s - k_2 s \mu\right)}{m_2^3\left(s^7 + \beta_6 s^6 + \beta_5 s^5 + \beta_4 s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s + \beta_0\right)}
\end{aligned}
\tag{13}
$$

where $\mu = m_2/m_1$ is the mass ratio.
Then, for the harmonic perturbation $f(t) = F_0 \sin\omega t$, the steady-state magnitude of the primary system is computed as

$$|X_1| = \frac{\mu}{m_2^3} F_0 \sqrt{\frac{A(\omega)}{B(\omega)}} \tag{14}$$

where

$$
\begin{aligned}
A(\omega) =& \left(k_2 - m_2\omega^2\right)^2 \Big[\left(-\beta_6 m_2 \omega^2 - \beta_6 k_2 \mu - 2\beta_6 k_2 + \beta_4 m_2\right)^2 \\
& + \left(-m_2\omega^3 + \beta_5 m_2 \omega - 2k_2\omega - k_2\omega\mu\right)^2\Big] \\
B(\omega) =& \left(-\beta_6\omega^6 + \beta_4\omega^4 - \beta_2\omega^2 + \beta_0\right)^2 + \left(-\omega^7 + \beta_5\omega^5 - \beta_3\omega^3 + \beta_1\omega\right)^2
\end{aligned}
$$

Note that $X_1 \equiv 0$ exactly when $\omega = \omega_2 = \sqrt{\frac{k_2}{m_2}}$, independently of the selected gains of the control law in (11), corresponding to the dynamic performance of the passive vibration control scheme. This clearly corresponds to a finite zero in the above transfer function $G(s)$, situation where the passive vibration absorber is well tuned.
Thus, the control objective for (11) is to add some robustness when $\omega \neq \omega_2$ and improve the performance of the closed-loop system using small control efforts and taking advantage of the passive vibration absorber (when $\omega = \omega_2$ the system can operate with $u \equiv 0$).
In Fig. 3 we can observe that, the active vibration absorber can attenuate vibrations for any excitation frequency, including vibrations with multiple harmonic signals. In fact, it is still possible to minimize the attenuation level by adding a proper viscous damping to the absorber (Korenev & Reznikov, 1993; Rao, 1995).

3.2 Control scheme using displacement measurement on the primary system and excitation frequency

Consider the perturbed system (2). The state variables and the control input u can be expressed in terms of the flat output y, the perturbation f and their time derivatives:

$$
\begin{aligned}
z_1 &= y \\
z_2 &= \dot{y} \\
z_3 &= \tfrac{k_1+k_2}{k_2} y + \tfrac{m_1}{k_2}\ddot{y} - \tfrac{1}{k_2} f(t) \\
z_4 &= \tfrac{k_1+k_2}{k_2} \dot{y} + \tfrac{m_1}{k_2} y^{(3)} - \tfrac{1}{k_2} \dot{f}(t) \\
u &= \tfrac{m_1 m_2}{k_2} y^{(4)} + k_1 y + \left(m_1 + m_2 + \tfrac{k_1}{k_2} m_2\right)\ddot{y} - f(t) - \tfrac{m_2}{k_2}\ddot{f}(t)
\end{aligned}
\tag{15}
$$

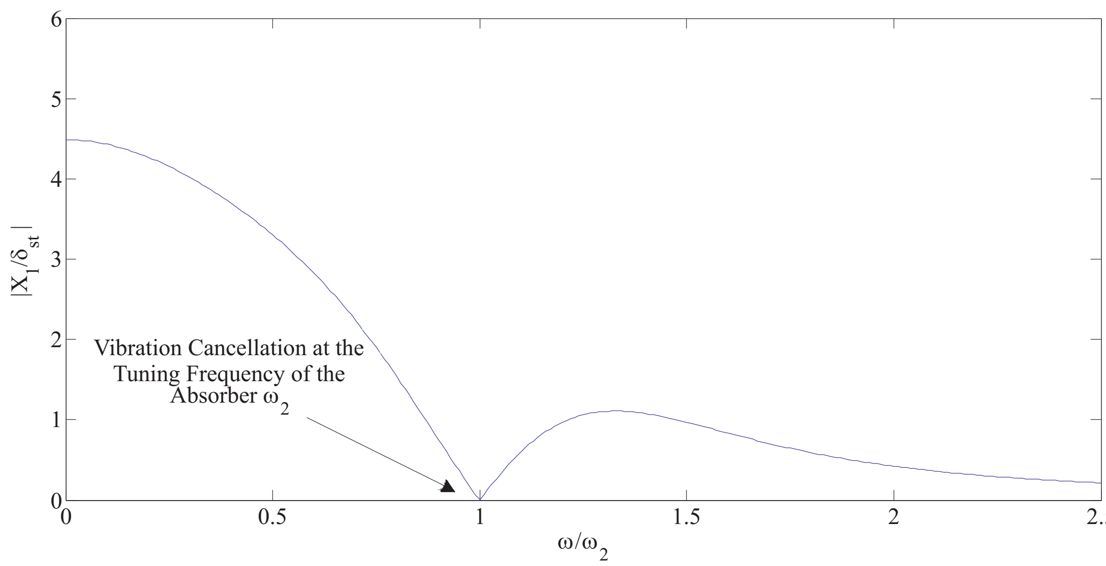

Fig. 3. Frequency response of the vibrating mechanical system using an active vibration absorber with controller (11).

Furthermore, when $f(t) = F_0 \sin \omega t$ the flat output y satisfies the following input-output differential equation:

$$y^{(4)} = -\frac{k_1 k_2}{m_1 m_2} y - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2} \right) \ddot{y} + \left(\frac{k_2}{m_1 m_2} - \frac{\omega^2}{m_1} \right) F_0 \sin \omega t + \frac{k_2}{m_1 m_2} u \qquad (16)$$

Taking two additional time derivatives of (16) results in

$$y^{(6)} = -\frac{k_1 k_2}{m_1 m_2} \ddot{y} - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2} \right) y^{(4)} + \frac{k_2}{m_1 m_2} \ddot{u} - \left(\frac{k_2}{m_1 m_2} - \frac{\omega^2}{m_1} \right) \omega^2 F_0 \sin \omega t \qquad (17)$$

Multiplication of (16) by ω^2 and adding it to (17) leads to

$$y^{(6)} + d_1 y^{(4)} + d_2 \ddot{y} + d_3 y = d_4 \left(\ddot{u} + \omega^2 u \right) \qquad (18)$$

where

$$\begin{aligned} d_1 &= \tfrac{k_1 + k_2}{m_1} + \tfrac{k_2}{m_2} + \omega^2 \\ d_2 &= \left(\tfrac{k_1 + k_2}{m_1} + \tfrac{k_2}{m_2} \right) \omega^2 + \tfrac{k_1 k_2}{m_1 m_2} \\ d_3 &= \tfrac{k_1 k_2}{m_1 m_2} \omega^2 \\ d_4 &= \tfrac{k_2}{m_1 m_2} \end{aligned}$$

A differential flatness-based dynamic controller, using feedback measurements of the flat output y and its time derivatives up to fifth order as well as feedforward measurements of the excitation frequency ω, is proposed by the following dynamic compensator:

$$\begin{aligned} &\ddot{u} + \omega^2 u = d_4^{-1} v + d_4^{-1} \left(d_1 y^{(4)} + d_2 \ddot{y} + \tfrac{k_1 k_2}{m_1 m_2} \omega^2 y \right) \\ &v = y^{*(6)} - \alpha_{10} \left[y^{(5)} - y^{*(5)} \right] - \alpha_9 \left[y^{(4)} - y^{*(4)} \right] - \alpha_8 \left[y^{(3)} - y^{*(3)} \right] \\ &\quad - \alpha_7 \left[\ddot{y} - \ddot{y}^* \right] - \alpha_6 \left[\dot{y} - \dot{y}^* \right] - \alpha_5 \left[y - y^* \right] \end{aligned} \qquad (19)$$

with zero initial conditions (i.e., $u(0) = \dot{u}(0) = 0$). It is important to remark that, the above differential equation resembles an exosystem (linear oscillator) tuned at the known excitation frequency ω (feedforward action) and injected by feedback terms involving the flat output y and its desired reference trajectory y^*.
On the other hand, one can note that the time derivatives of the flat output admit an integral input-output parameterization, obtained after some algebraic manipulations, given by

$$\begin{aligned}
\widehat{\dot{y}} &= -d_1 \int y - d_2 \int^{(3)} y - d_3 \int^{(5)} y + d_4 \int^{(3)} u \\
\widehat{\ddot{y}} &= -d_1 y - d_2 \int^{(2)} y - d_3 \int^{(4)} y + d_4 \int^{(2)} u + d_4\omega^2 \int^{(4)} u \\
\widehat{y}^{(3)} &= -d_1 \widehat{\dot{y}} - d_2 \int y - d_3 \int^{(3)} y + d_4 \int u + d_4\omega^2 \int^{(3)} u \\
\widehat{y}^{(4)} &= -d_1 \widehat{\ddot{y}} - d_2 y - d_3 \int^{(2)} y + d_4 u + d_4\omega^2 \int^{(2)} u \\
\widehat{y}^{(5)} &= -d_1 \widehat{y}^{(3)} - d_2 \widehat{\dot{y}} - d_3 \int y + d_4 \dot{u} + d_4\omega^2 \int u
\end{aligned} \tag{20}$$

The differences in the structural estimates of the time derivatives of the flat output with respect to the actual time derivatives are given by

$$\begin{aligned}
\dot{y} &= \widehat{\dot{y}} + p_4 t^4 + p_3 t^3 + p_2 t^2 + p_1 t + p_0 \\
\ddot{y} &= \widehat{\ddot{y}} + p_4 t^3 + p_3 t^2 + p_2 t + p_1 \\
y^{(3)} &= \widehat{y}^{(3)} + q_4 t^4 + q_3 t^3 + q_2 t^2 + q_1 t + q_0 \\
y^{(4)} &= \widehat{y}^{(4)} + r_3 t^3 + r_2 t^2 + r_1 t + r_0 \\
y^{(5)} &= \widehat{y}^{(5)} + s_4 t^4 + s_3 t^3 + s_2 t^2 + s_1 t + s_0
\end{aligned}$$

where p_i, q_i, r_j, s_i, $i = 0,...,4$, $j = 0, ..., 3$, are real constants depending on the unknown initial conditions.
Finally, the differential flatness based GPI controller is obtained by replacing the actual time derivatives of the flat output in (19) by their structural estimates in (20) but using additional iterated integral error compensations as follows

$$\begin{aligned}
&\ddot{u} + \omega^2 u = d_4^{-1} v + d_4^{-1}\left(d_1 \widehat{y}^{(4)} + d_2 \widehat{\ddot{y}} + d_3 y\right) \\
&v = y^{*(6)} - \alpha_{10}\left[\widehat{y^{(5)}} - y^{*(5)}\right] - \alpha_9\left[\widehat{y^{(4)}} - y^{*(4)}\right] - \alpha_8\left[\widehat{y^{(3)}} - y^{*(3)}\right] \\
&\quad - \alpha_7\left[\widehat{\ddot{y}} - \ddot{y}^*\right] - \alpha_6\left[\widehat{\dot{y}} - \dot{y}^*\right] - \alpha_5\left[y - y^*\right] \\
&\quad - \alpha_4\xi_1 - \alpha_3\xi_2 - \alpha_2\xi_3 - \alpha_1\xi_4 - \alpha_0\xi_5 \\
&\dot{\xi}_1 = y - y^*, \quad \xi_1(0) = 0 \\
&\dot{\xi}_2 = \xi_1, \qquad \xi_2(0) = 0 \\
&\dot{\xi}_3 = \xi_2, \qquad \xi_3(0) = 0 \\
&\dot{\xi}_4 = \xi_3, \qquad \xi_4(0) = 0 \\
&\dot{\xi}_5 = \xi_4, \qquad \xi_5(0) = 0
\end{aligned} \tag{21}$$

This feedback and feedforward active vibration controller depends on the measurements of the flat output y and the excitation frequency ω, therefore, this dynamic controller can compensate simultaneously two harmonic components, corresponding to the tuned (passive) vibration absorber (ω_2) and the actual excitation frequency (ω).

The closed-loop system dynamics, expressed in terms of the tracking error $e = y - y^*(t)$, is described by

$$\begin{aligned} &e^{(11)} + \alpha_{10}e^{(10)} + \alpha_9 e^{(9)} + \alpha_8 e^{(8)} + \alpha_7 e^{(7)} + \alpha_6 e^{(6)} + \alpha_5 e^{(5)} + \alpha_4 e^{(4)} \\ &+\alpha_3 e^{(3)} + \alpha_2 \ddot{e} + \alpha_1 \dot{e} + \alpha_0 e = 0 \end{aligned} \tag{22}$$

Therefore, the design parameters α_i, $i = 0, ..., 10$, have to be selected such that the associated characteristic polynomial for (22) be *Hurwitz*, thus guaranteeing the desired asymptotic output tracking when one can measure the excitation frequency ω.

3.2.1 Robustness with respect to external vibrations

Fig. 4 shows the frequency response of the closed-loop system, using an active vibration absorber based on differential flatness and measurements of y and ω. Note that this active

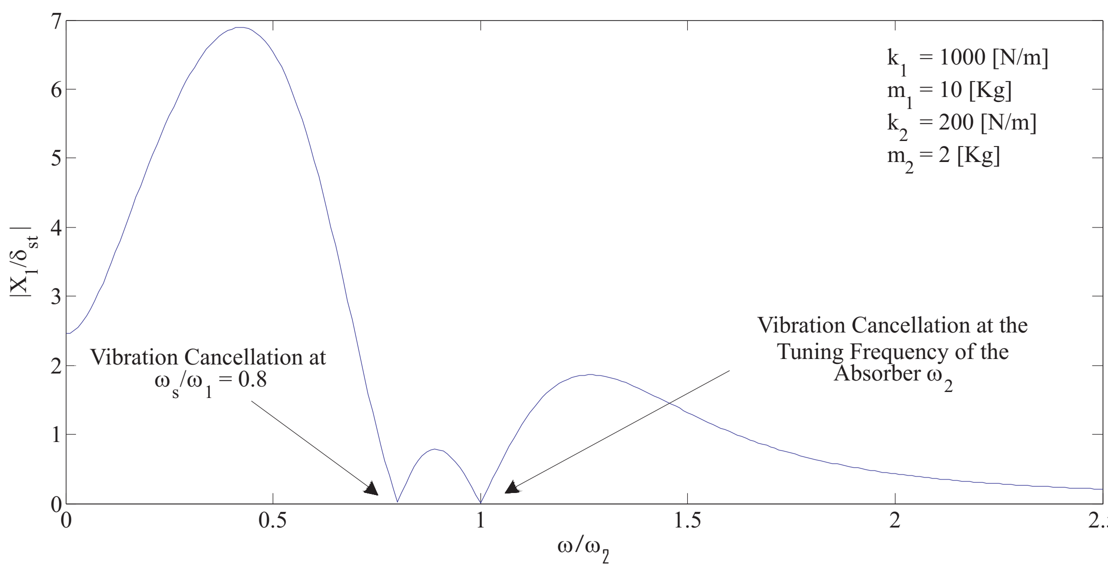

Fig. 4. Frequency response of the vibrating mechanical system using the active vibration absorber with controller (21).

vibration absorber employs the measurement of the excitation frequency ω and, therefore, such harmonic vibrations can always be cancelled (i.e., $X_1 = 0$). Moreover, this absorber is also useful to eliminate vibrations of the form $f(t) = F_0 [\sin(\omega_s t) + \sin(\omega_2 t)]$, where ω_s is the measured frequency (affecting the feedforward control action) and ω_2 is the design frequency of the passive absorber.

3.3 Simulation results

Some numerical simulations were performed on a vibrating mechanical platform from *Educational Control Products (ECP), model 210/210a Rectilinear Control System*, characterized by the set of system parameters given in Table 1.

The controllers (11) and (21) were specified in such a way that one could prove how the active vibration absorber cancels the two harmonic vibrations affecting the primary system and the asymptotic output tracking of the desired reference trajectory.

$m_1 = 10\text{kg}$	$m_2 = 2\text{kg}$
$k_1 = 1000\frac{\text{N}}{\text{m}}$	$k_2 = 200\frac{\text{N}}{\text{m}}$
$c_1 \approx 0\frac{\text{N}}{\text{m/s}}$	$c_2 \approx 0\frac{\text{N}}{\text{m/s}}$

Table 1. System parameters for the primary and secondary systems.

The planned trajectory for the flat output $y = z_1$ is given by

$$y^*(t) = \begin{cases} 0 & \text{for } 0 \leq t < T_1 \\ \psi(t, T_1, T_2)\,\bar{y} & \text{for } T_1 \leq t \leq T_2 \\ \bar{y} & \text{for } t > T_2 \end{cases}$$

where $\bar{y} = 0.01\text{m}$, $T_1 = 5\text{s}$, $T_2 = 10\text{s}$ and $\psi(t, T_1, T_2)$ is a Bézier polynomial, with $\psi(T_1, T_1, T_2) = 0$ and $\psi(T_2, T_1, T_2) = 1$, described by

$$\psi(t) = \left(\frac{t - T_1}{T_2 - T_1}\right)^5 \left[r_1 - r_2\left(\frac{t - T_1}{T_2 - T_1}\right) + r_3\left(\frac{t - T_1}{T_2 - T_1}\right)^2 - ... - r_6\left(\frac{t - T_1}{T_2 - T_1}\right)^5\right]$$

with $r_1 = 252$, $r_2 = 1050$, $r_3 = 1800$, $r_4 = 1575$, $r_5 = 700$, $r_6 = 126$.
Fig. 5 shows the dynamic behavior of the closed-loop system with the controller (11). One can observe the vibration cancellation on the primary system and the output tracking of the pre-specified reference trajectory. The controller gains were chosen so that the characteristic polynomial of the closed-loop tracking error dynamics (12) is a *Hurwitz* polynomial given by

$$p_{d1}(s) = (s + p_1)\left(s^2 + 2\zeta_1\omega_{n1}s + \omega_{n1}^2\right)^3$$

with $\zeta_1 = 0.5$, $\omega_{n1} = 12\text{rad/s}$, $p_1 = 6$.

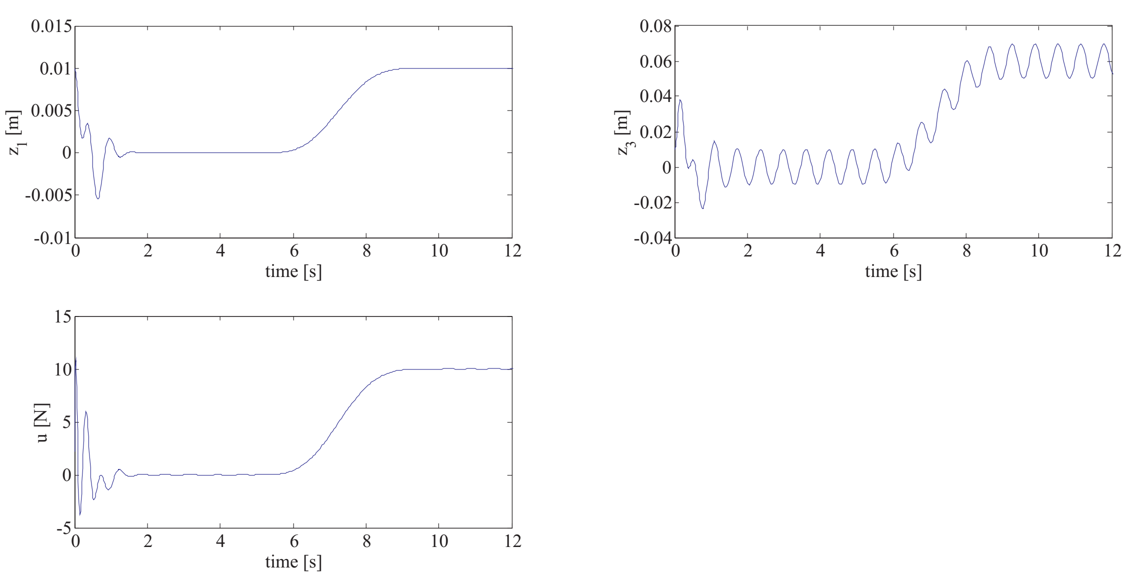

Fig. 5. Active vibration absorber using measurements of y and ω for harmonic vibration $f = 2\sin(10t)$ N.

Fig. 6 shows the robust performance of closed-loop system employing the controller (21). One can see that the active vibration absorber eliminates vibrations containing two different harmonics. The design parameters were selected to have a sixth order closed-loop characteristic polynomial

$$p_{d2}(s) = (s + p_2)\left(s^2 + 2\zeta_2\omega_{n2}s + \omega_{n2}^2\right)^5$$

with

$$\zeta_2 = 0.5, \quad \omega_{n2} = 10\text{rad/s}, \quad \mathrm{p}_2 = 5.$$

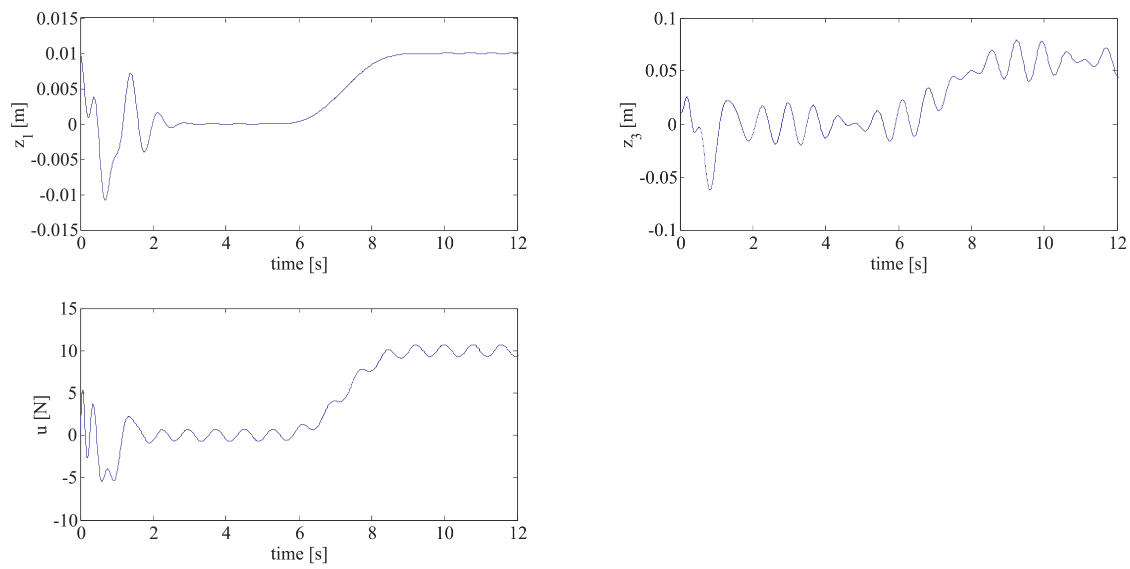

Fig. 6. Active vibration absorber using measurements of y and ω, for external vibration $f = 2\left[\sin(8.0109t) + \sin(10t)\right]$ N.

4. Algebraic identification of harmonic vibrations

The algebraic identification methodology (Fliess & Sira-Ramirez, 2003) can be applied to estimate the parameters associated to exogenous harmonic vibrations affecting a mechanical vibrating system (Beltran et al., 2004).
To do that, consider the input-output differential equation (16), where only displacement measurements of the primary system $y = z_1$ and the control input u are available for the identification process of the parameters associated with the harmonic signal $f(t) = F_0 \sin \omega t$, that is,

$$y^{(4)} = -\frac{k_1 k_2}{m_1 m_2} y - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2}\right)\ddot{y} + \left(\frac{k_2}{m_1 m_2} - \frac{\omega^2}{m_1}\right) F_0 \sin \omega t + \frac{k_2}{m_1 m_2} u \tag{23}$$

Next we will proceed to synthesize two algebraic identifiers for the excitation frequency ω and amplitude F_0.

4.1 Identification of the excitation frequency ω

The differential equation (23) is expressed in notation of operational calculus as

$$m_1 s^4 Y(s) + \left(k_1 + k_2 + \frac{m_1 k_2}{m_2}\right) s^2 Y(s) + \frac{k_1 k_2}{m_2} Y(s)$$
$$= \frac{k_2}{m_2} U(s) + \left(\frac{k_2}{m_2} - \omega^2\right) F_0 \frac{\omega}{s^2+\omega^2} + a_3 s^3 + a_2 s^2 + a_1 s + a_0 \tag{24}$$

where a_i, $i = 0,...,3$, denote unknown real constants depending on the system initial conditions. Now, equation (24) is multiplied by $\left(s^2+\omega^2\right)$, leading to

$$\left(s^2+\omega^2\right)\left[\left(s^4 Y + \frac{k_2}{m_2} s^2 Y\right) m_1 + \left(s^2 Y + \frac{k_2}{m_2} Y\right) k_1 + k_2 s^2 Y\right]$$
$$= \frac{k_2}{m_2}\left(s^2+\omega^2\right) u + \left(\frac{k_2}{m_2} - \omega^2\right) F_0 \omega + \left(s^2+\omega^2\right)\left(a_3 s^3 + a_2 s^2 + a_1 s + a_0\right) \tag{25}$$

This equation is then differentiated six times with respect to s, in order to eliminate the constants a_i and the unknown amplitude F_0. The resulting equation is then multiplied by s^{-6} to avoid differentiations with respect to time in time domain, and next transformed into the time domain, to get

$$\left[a_{11}(t) + \omega^2 a_{12}(t)\right] m_1 + \left[a_{12}(t) + \omega^2 b_{12}(t)\right] k_1 = c_1(t) + \omega^2 d_1(t) \tag{26}$$

where $\Delta t = t - t_0$ and

$$\begin{aligned}
a_{11}(t) &= m_2 g_{11}(t) + k_2 g_{12}(t) \\
a_{12}(t) &= m_2 g_{12}(t) + k_2 g_{13}(t) \\
b_{12}(t) &= m_2 g_{13}(t) + k_2 \int_{t_0}^{(6)} (\Delta t)^6 z_1 \\
c_1(t) &= k_2 g_{14}(t) - k_2 m_2 g_{12}(t) \\
d_1(t) &= k_2 \int_{t_0}^{(6)} (\Delta t)^6 u - k_2 m_2 g_{13}(t)
\end{aligned}$$

with

$$\begin{aligned}
g_{11}(t) = {} & 720 \int_{t_0}^{(6)} y - 4320 \int_{t_0}^{(5)} (\Delta t) y + 5400 \int_{t_0}^{(4)} (\Delta t)^2 y - 2400 \int_{t_0}^{(3)} (\Delta t)^3 y \\
& + 450 \int_{t_0}^{(2)} (\Delta t)^4 y - 36 \int_{t_0} (\Delta t)^5 y + (\Delta t)^6 y \\
g_{12}(t) = {} & 360 \int_{t_0}^{(6)} (\Delta t)^2 y - 480 \int_{t_0}^{(5)} (\Delta t)^3 y + 180 \int_{t_0}^{(4)} (\Delta t)^4 y \\
& - 24 \int_{t_0}^{(3)} (\Delta t)^5 y + \int_{t_0}^{(2)} (\Delta t)^6 y
\end{aligned}$$

$$g_{13}(t) = 30\int_{t_0}^{(6)} (\Delta t)^4 y - 12\int_{t_0}^{(5)} (\Delta t)^5 y + \int_{t_0}^{(4)} (\Delta t)^6 y$$
$$g_{14}(t) = 30\int_{t_0}^{(6)} (\Delta t)^4 u - 12\int_{t_0}^{(5)} (\Delta t)^5 u + \int_{t_0}^{(4)} (\Delta t)^6 u$$

Finally, solving for the excitation frequency ω in (26) leads to the following on-line algebraic identifier:

$$\omega_e^2 = \frac{N_1(t)}{D_1(t)} = \frac{c_1(t) - a_{11}(t)\, m_1 - a_{12}(t)\, k_1}{a_{12}(t)\, m_1 + b_{12}(t)\, k_1 - d_1(t)} \tag{27}$$

This estimation is valid if and only if the condition $D_1(t) \neq 0$ holds in a sufficiently small time interval $(t_0, t_0 + \delta_0]$ with $\delta_0 > 0$. This nonsingularity condition is somewhat similar to the well-known persistent excitation property needed by most of the asymptotic identification methods (Isermann & Munchhof, 2011; Ljung, 1987; Soderstrom, 1989). In particular, this obstacle can be overcome by using numerical resetting algorithms or further integrations on $N_1(t)$ and $D_1(t)$ (Sira-Ramirez et al., 2008).

4.2 Identification of the amplitude F_0

To synthesize an algebraic identifier for the amplitude F_0 of the harmonic vibrations acting on the mechanical system, the input-output differential equation (23) is expressed in notation of operational calculus as follows

$$\begin{aligned} m_1 s^4 Y(s) + \left(k_1 + k_2 + \frac{m_1 k_2}{m_2}\right) s^2 Y(s) + \frac{k_1 k_2}{m_2} Y(s) \\ = \frac{k_2}{m_2} U(s) + \left(\frac{k_2}{m_2} - \omega^2\right) F(s) + a_3 s^3 + a_2 s^2 + a_1 s + a_0 \end{aligned} \tag{28}$$

Taking derivatives, four times, with respect to s makes possible to remove the dependence on the unknown constants a_i. The resulting equation is then multiplied by s^{-4}, and next transformed into the time domain, to get

$$\begin{aligned} m_1 P_1(t) + \left(k_1 + k_2 + \frac{m_1 k_2}{m_2}\right) P_2(t) + \frac{k_1 k_2}{m_2} \int_{t_0}^{(4)} (\Delta t)^4 z_1 \\ = \frac{k_2}{m_2} \int_{t_0}^{(4)} (\Delta t)^4 u + F_0 \left(\frac{k_2}{m_2} - \omega^2\right) \int_{t_0}^{(4)} (\Delta t)^4 \sin \omega t \end{aligned} \tag{29}$$

where

$$P_1(t) = 24\int_{t_0}^{(4)} z_1 - 96\int_{t_0}^{(3)} (\Delta t) z_1 + 72\int_{t_0}^{(2)} (\Delta t)^2 z_1 - 16\int_{t_0} (\Delta t)^3 z_1 + (\Delta t)^4 z_1$$
$$P_2(t) = 12\int_{t_0}^{(4)} (\Delta t)^2 z_1 - 8\int_{t_0}^{(3)} (\Delta t)^3 z_1 + \int_{t_0}^{(2)} (\Delta t)^4 z_1$$

It is important to note that equation (29) still depends on the excitation frequency ω, which can be estimated from (27). Therefore, it is required to synchronize both algebraic identifiers for ω and F_0. This procedure is sequentially executed, first by running the identifier for ω and, after some small time interval with the estimation $\omega_e(t_0 + \delta_0)$ is then started the algebraic identifier for F_0, which is obtained by solving

$$N_2(t) - D_2(t)F_0 = 0 \tag{30}$$

where

$$N_2(t) = m_1 P_1(t) + \left(k_1 + k_2 + \frac{m_1 k_2}{m_2}\right) P_2(t) + \frac{k_1 k_2}{m_2}\int_{t_0+\delta_0}^{(4)} (\Delta t)^4 z_1 - \frac{k_2}{m_2}\int_{t_0+\delta_0}^{(4)} (\Delta t)^4 u$$
$$D_2(t) = \left(\frac{k_2}{m_2} - \omega_e^2\right)\int_{t_0+\delta_0}^{(4)} (\Delta t)^4 \sin\left[\omega_e(t_0+\delta_0)t\right]$$

In this case if the condition $D_2(t) \neq 0$ is satisfied for all $t \in (t_0+\delta_0, t_0+\delta_1]$ with $\delta_1 > \delta_0 > 0$, then the solution of (30) yields an algebraic identifier for the excitation amplitude

$$F_{0e} = \frac{N_2(t)}{D_2(t)},\ \forall t \in (t_0+\delta_0, t_0+\delta_1] \tag{31}$$

4.3 Adaptive-like active vibration absorber for unknown harmonic forces

The active vibration control scheme (21), based on the differential flatness property and the GPI controller, can be combined with the on-line algebraic identification of harmonic vibrations (27) and (31), where the estimated harmonic force is computed as

$$f_e(t) = F_{0e}\sin(\omega_e t) \tag{32}$$

resulting some certainty equivalence feedback/feedforward control law. Note that, according to the algebraic identification approach, providing fast identification for the parameters associated to the harmonic vibration (F_0, ω) and, as a consequence, a fast estimation of this perturbation signal, the proposed controller is similar to an adaptive control scheme. From a theoretical point of view, the algebraic identification is instantaneous (Fliess & Sira-Ramirez, 2003). In practice, however, there are modelling errors and many other factors that complicate the real-time algebraic computation. Fortunately, the identification algorithms and closed-loop system are robust against such difficulties.

4.4 Simulation results

Fig. 7 shows the identification process of the excitation frequency of the resonant harmonic perturbation $f(t) = 2\sin(8.0109t)$ N and the robust performance of the adaptive-like control scheme (21) for reference trajectory tracking tasks, which starts using the nominal frequency value $\omega = 10$rad/s, which corresponds to the known design frequency for the passive vibration absorber, and at $t > 0.1$s this controller uses the estimated value of the resonant excitation frequency. Here it is clear how the frequency identification is quickly performed (before $t = 0.1$s and it is almost exact with respect to the actual value.

One can also observe that, the resonant vibrations affecting the primary mechanical system are asymptotically cancelled from the primary system response in a short time interval. It is also evident the presence of some singularities in the algebraic identifier, i.e., when its denominator $D_1(t)$ is zero. The first singularity, however, occurs about $t = 0.727$s, which is too much time (more than 7 times) after the identification has been finished.

Fig. 8 illustrates the fast and effective performance of the on-line algebraic identifier for the amplitude of the harmonic force $f(t) = 2\sin(8.0109t)$ N. First of all, it is started the identifier for ω, which takes about $t < 0.1$s to get a good estimation. After the time interval $(0, 0.1]$s, where $t_0 = 0$s and $\delta_0 = 0.1$s with an estimated value $\omega_e(t_0 + \delta_0) = 8.0108$rad/s, it is activated the identifier for the amplitude F_0.

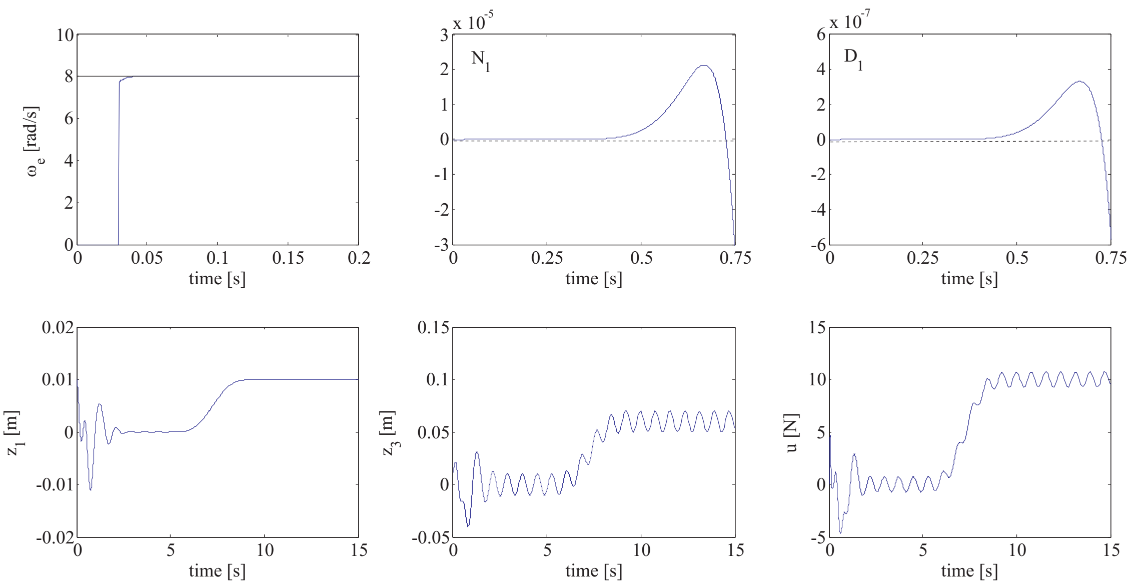

Fig. 7. Controlled system responses and identification of frequency for $f(t) = 2\sin(8.0109t)$ [N].

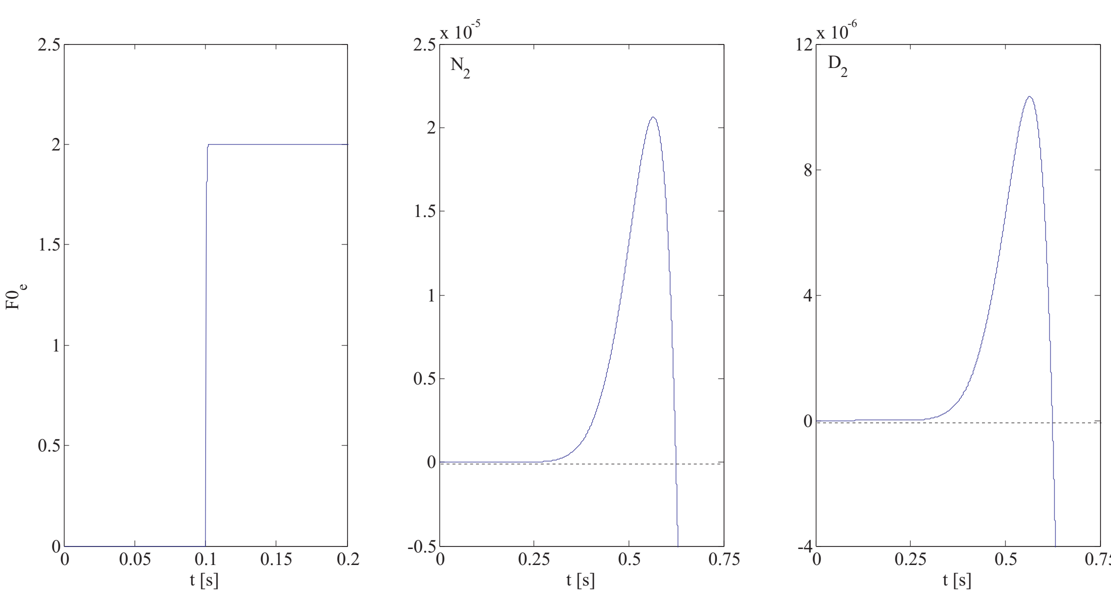

Fig. 8. Identification of amplitude for $f(t) = 2\sin(8.0109t)$ [N].

One can also observe that the first singularity occurs when the numerator $N_2(t)$ and denominator $D_2(t)$ are zero. However the first singularity is presented about $t = 0.702$s, and therefore the identification process is not affected.

Now, Figs. 9 and 10 present the robust performance of the on-line algebraic identifiers for the excitation frequency ω and amplitude F_0. In this case, the primary system was forced by external vibrations containing two harmonics, $f(t) = 2[\sin(8.0109t) + 10\sin(10t)]$ N. Here, the frequency $\omega_2 = 10$rad/s corresponds to the known tuning frequency of the passive vibration absorber, which does not need to be identified. Once again, one can see the fast and effective estimation of the resonant excitation frequency $\omega = 8.0109$rad/s and amplitude $F_0 = 2$N as well as the robust performance of the proposed active vibration control scheme (21) based on differential flatness and GPI control, which only requires displacement measurements of the primary system and information of the estimated excitation frequency.

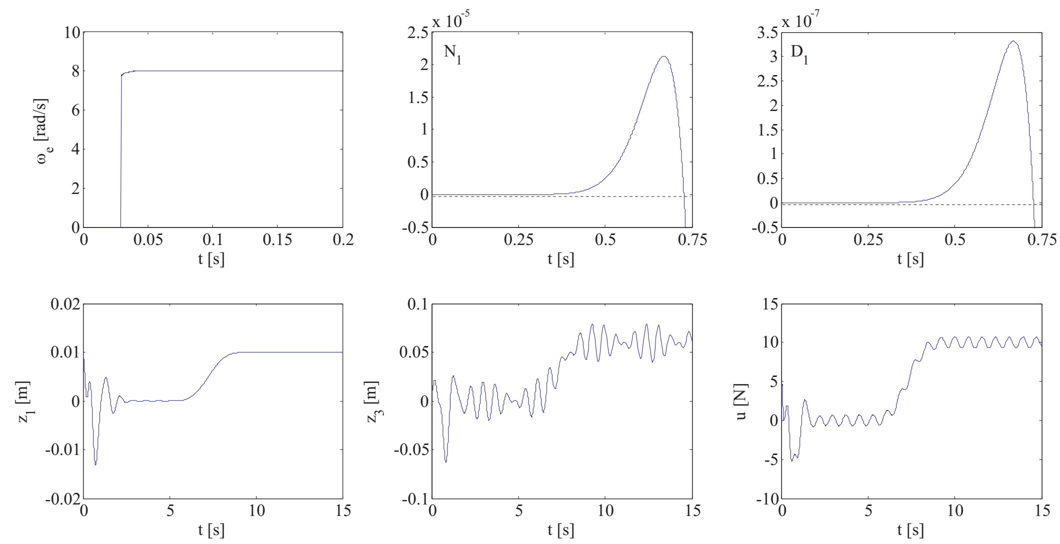

Fig. 9. Controlled system responses and identification of the unknown resonant frequency for .$f(t) = 2[\sin(8.0109t) + 10\sin(10t)]$ [N].

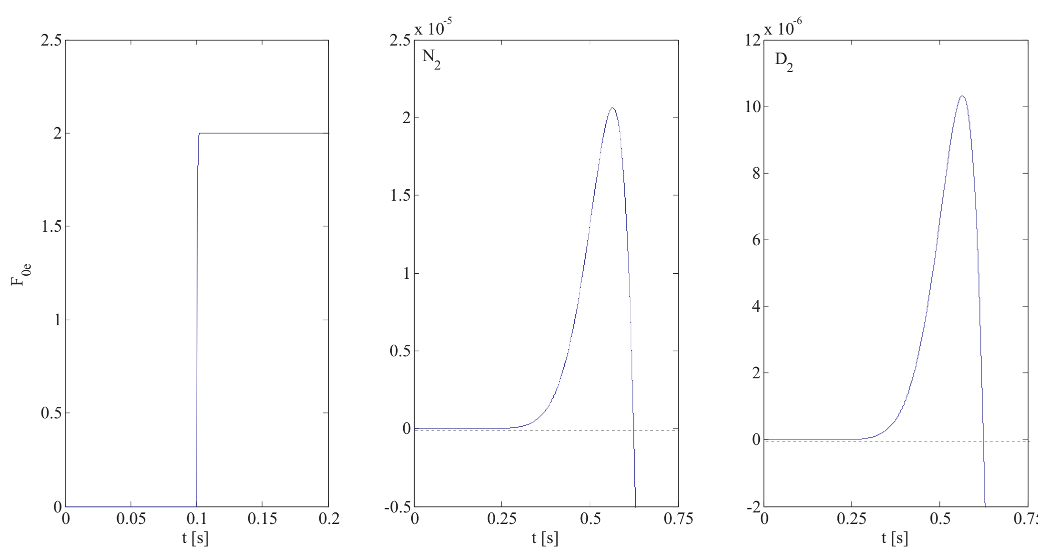

Fig. 10. Identification of amplitude for $f(t) = 2[\sin(8.0109t) + 10\sin(10t)]$ [N].

5. Conclusions

In this chapter we have described the design approach of a robust active vibration absorption scheme for vibrating mechanical systems based on passive vibration absorbers, differential flatness, GPI control and on-line algebraic identification of harmonic forces.

The proposed adaptive-like active controller is useful to completely cancel any harmonic force, with unknown amplitude and excitation frequency, and to improve the robustness of passive/active vibrations absorbers employing only displacement measurements of the primary system and small control efforts. In addition, the controller is also able to asymptotically track some desired reference trajectory for the primary system.

In general, one can conclude that the adaptive-like vibration control scheme results quite fast and robust in presence of parameter uncertainty and variations on the amplitude and excitation frequency of harmonic perturbations.

The methodology can be applied to rotor-bearing systems and some classes of nonlinear mechanical systems.

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This book focuses on the important and diverse field of vibration analysis and control. It is written by experts from the international scientific community and covers a wide range of research topics related to design methodologies of passive, semi-active and active vibration control schemes, vehicle suspension systems, vibration control devices, fault detection, finite element analysis and other recent applications and studies of this fascinating field of vibration analysis and control. The book is addressed to researchers and practitioners of this field, as well as undergraduate and postgraduate students and other experts and newcomers seeking more information about the state of the art, challenging open problems, innovative solution proposals and new trends and developments in this area.

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