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Supply Chain Management Systems Advanced Control: MPC on SCM

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1. Introduction

A supply chain is a network of facilities and distribution entities (suppliers, manufacturers, distributors, retailers) that performs the functions of procurement of raw materials, transformation of raw materials into intermediate and finished products and distribution of finished products to customers. Between interconnected entities, there are two types of process flows: information flows, e.g., an order requesting goods, and material flows, i.e., the actual shipment of goods (Figure 1). Key elements to an efficient supply chain are accurate pinpointing of process flows and timing of supply needs at each entity, both of which enable entities to request items as they are needed, thereby reducing safety stock levels to free space and capital. The operational planning and direct control of the network can in principle be addressed by a variety of methods, including deterministic analytical models, stochastic analytical models, and simulation models, coupled with the desired optimization objectives and network performance measures (Beamon, 1998).

The merit of model predictive control (MPC) is its applications in multivariable control in the presence of constraints. The success of MPC is due to the fact that it is perhaps the most general way of posing the control problem in the time domain. The use a finite horizon strategy allows the explicit handling of process and operational constraints by the MPC (Igor, 2008). In a recent paper (Perea et al., 2003), a MPC strategy was employed for the optimization of production/ distribution systems, including a simplified scheduling model for the manufacturing function. The suggested control strategy considers only deterministic type of demand, which reduces the need for an inventory control mechanism (Seferlis et al., 2004:Kapsiotis et al., 1992).

For the purposes of our study and the time scales of interest, a discrete time difference model is developed(Tzafestas, 1997). The model is applicable to multi echelon supply chain networks of arbitrary structure. To treat process uncertainty within the deterministic supply chain network model, a MPC approach is suggested (Wang et al., 2005:Chopra et al., 2004).

Typically, MPC is implemented in a centralized fashion (Wang et al., 2005). The complete system is modeled, and all the control inputs are computed in one optimization problem. In large scale applications, such as power systems, water distribution systems, traffic systems, manufacturing systems, and economic systems, such a centralized control scheme may not suitable or even possible for technical or commercial reasons (Sarimveis et al., 2008), it is useful to have distributed or decentralized control schemes, where local control inputs are computed using local measurements and reduced order models of the local dynamics. The

algorithm uses a receding horizon, to allow the incorporation of past and present control actions to future predictions (Camacho et al., 2004; Findeisen et al., 2007). As well as, further decentralized MPC advantages are less computational complication and lower error risk (Agachi, 2009:Towill, 2008).
So As supply chains can be operated sequentially, local Consecutive model predictive controllers applied to a supply chain management system consist of one plant, two warehouses, four distribution centers and four retailers. Also a move suppression term add to cost function, that increase system robustness toward changes on demands. Through illustrative simulations, it is demonstrated that the model can accommodate supply chain networks of realistic size under disturbances.

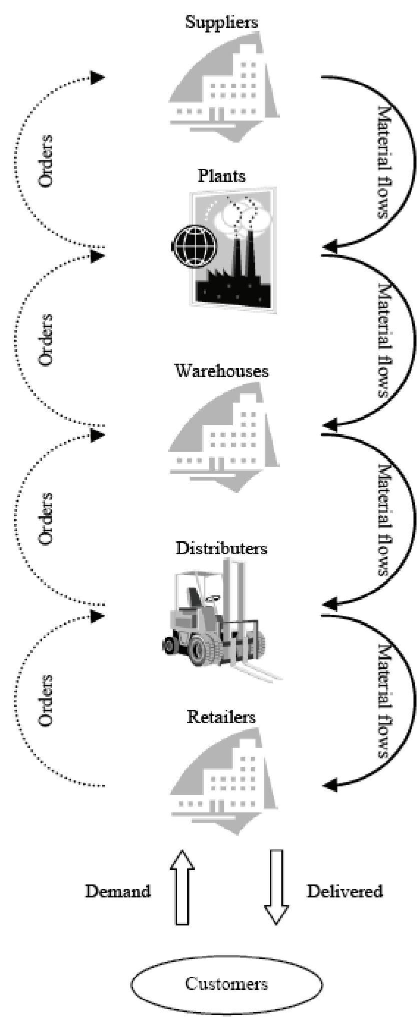

Fig. 1. Schematic of a multi echelon/multi product (A, B, C) supply chain network with process flows

2. Advanced control methods of supply chain management systems-literature review

The utilization of classical control techniques in the supply chain management problem can be traced back to the early 1950s when Simon applied servomechanism continuous-time theory to manipulate the production rate in a simple system involving just a single product. The idea was extended to discrete-time models by Vassian who proposed an inventory control framework based on the *z*-transform methodology. A breakthrough, however, was experienced in the late 1950s by the so-called "industrial dynamics" methodology, which was introduced by the pioneering work of Forrester. The methodology, later referred to as "system dynamics" used a feedback perspective to model, analyze and improve dynamic systems, including the production-inventory system. The scope of the methodology was later broadened to cover complex systems from various disciplines such as social systems, corporate planning and policy design, public management and policy, micro- and macro-economic dynamics, educational problems, biological and medical modeling, energy and the environment, theory development in the natural and social sciences, dynamic decision-making research, strategic planning and more. The book written recently by Sterman is an excellent source of information on the "system dynamics" philosophy and its various applications and includes special chapters on the supply chain management problem.

Forrester's work was appreciated for providing powerful tools to model and simulate complex dynamical phenomena including nonlinear control laws. However, the "industrial dynamics" methodology was criticized for not containing sufficient analytical support and for not providing guidelines to the systems engineers on how to improve performance. Motivated by the need to develop a new framework that could be used as a base for seeking new novel control laws and/or new feedback paths in production/inventory systems, Towill presented the inventory and order based production control system (IOBPCS) in a block diagram form, extending the work of Coyle. It was considered that the system deals with aggregate product levels or alternatively it reflects a single product. The system was subject to many modifications and improvements in subsequent years including extensions to discrete-time systems, thus leading to the IOBPCS family.

The designer has to decide on how the target stock will be set (fixed value or multiple of average sales) and select the three policies (demand policy, inventory policy and pipeline policy), in order to optimize the system with respect to the following performance objectives (Sarimveis et al., 2008):

a. Inventory level recovery.
b. Attenuation of demand rate fluctuations on the ordering rate.

The second objective aims at the reduction of the "bullwhip" effect. The term "bullwhip" was only recently introduced as mentioned in the introduction, but the phenomenon where a small random variation in sales at the marketplace is amplified at each level in the supply chain was already identified by the pioneering work of Forester in industrial dynamics. This was later postulated by Burbidge under the "Law of Industrial Dynamics". The utilization of control engineering principles in tackling the problem by providing supply chain dynamic modeling and re-engineering methodologies was soon recognized as reported by Towill.

The two performance objectives are conflicting. Thus, for each particular supply chain, the control system designer seeks for the best inventory level and ordering rate trade-off. A qualitative look at the two extremes scenarios (perfect satisfaction of each one of the two objectives) clearly shows that a compromise is needed to arrive at a well designed control system. If a fixed ordering rate is used then large inventory deviations are observed, since inventory levels follow any demand variation. This policy (known as Lean Production in manufacturing cites) obviously results in large inventory costs. On the other hand a fixed inventory level (known as Agile Production in manufacturing cites) results in highly variable production schedules and hence, large production costs.

Due to their dynamic and uncertain nature, production/inventory problems can be naturally formulated as dynamic programs. Dynamic programming is the standard procedure to obtain an optimal state feedback control law for stochastic optimal control problems.
MPC has now become a standard control methodology for industrial and process systems. Its wide adoption from the industry is largely based on the inherent ability of the method to handle efficiently constraints and nonlinearities of multi-variable dynamical systems. MPC is based on the following simple idea: at each discrete time instance the control action is obtained by solving *on-line* a finite-horizon *open-loop* optimal control problem, using the current state of the system as the initial state. A finite-optimal control sequence is obtained, from which only the first element is kept and applied to the system. The procedure is repeated after each state transition. Its main difference from stochastic dynamic programming and optimal control is that the control input is not computed a priori as an explicit function of the state vector. Thus, MPC is prevalent in the control of complex systems where the solution of the dynamic programming equations is computationally intractable due to the curse of dimensionality. However, when the optimal control problem is stochastic in nature, one can only obtain suboptimal solutions, due to the open-loop nature of the methodology (Sarimveis et al., 2008).
The significance of the basic idea implicit in the MPC has been recognized a long-time ago in the operations management literature as a tractable scheme for solving stochastic multi-period optimization problems, such as production planning and supply chain management, under the term rolling horizon. For a review of rolling horizons in operation management problems and interesting trade-offs between horizon lengths and costs of forecasts, we refer the reader to Sethi and Sorger and Chand et al. Kapsiotis and Tzafestas were the first to apply MPC to an inventory management problem, for a single inventory site. They included a penalty term for deviations from an inventory reference trajectory in order to compensate for production lead times. Tzafestas et al., considered a generalized production planning problem that includes both production/inventory and marketing decisions. They employed a linear econometric model concerning sales as a function of advertisement effort so as to approximate a nonlinear Vidale–Wolfe process. The dynamics of sales are coupled with an inventory balance equation. The optimal control problem is formulated as an MPC, where the control variables are the advertisement effort and the production levels. The objective function penalizes deviations from desired sales and inventory levels. Perea-Lopez et al. employed MPC to manage a multi-product, multi-echelon production and distribution network with lead times, allowing no backorders. They formulated the optimal control problem as a large scale mixed integer linear programming (MILP) problem, due to discontinuous decisions allowed in their model. In their formulation the demand is considered to be deterministic. They tested their formulation in a quite complex supply chain producing three products and consisting of three factories, three warehouses, four distribution centers and 10 retailers servicing 20 customers. They compared their centralized approach against two decentralized approaches. The first decentralized approach optimizes distribution only and uses heuristic rules for production/inventory planning. The second approach optimizes manufacturing while allowing the distribution network to follow heuristic rules. Through simulations, they inferred that the centralized approach exhibits superior performance (Sarimveis et al., 2008).
Seferlis and Gianellos developed a two-layered hierarchical control scheme, where a decentralized inventory control policy is embedded within an MPC framework. Inventory levels at the storage nodes and backorders at the order receiving nodes are the state

variables for the linear state space model. The control variables are the product quantities transferred through the network permissible routes and the amounts delivered to the customers. Backorders are considered as output variables. Deterministic transportation delays are also included in the model. The cost function of the MPC consists of four terms, the first two being inventory and transportation costs, the third being a quadratic function that penalizes backorders at retailers and the last term being a quadratic move suppression term that penalizes deviations of decision variables between consecutive time periods. In order to account for demand uncertainty, they employed an autoregressive integrated moving average (ARIMA) forecasting model for the prediction of future product demand variation. Based on historical demand they performed identification of the order and parameters of the ARIMA model (Sarimveis et al., 2008).

PID controllers were embedded for each inventory node and each product. These local controllers are responsible for maintaining the inventory levels close to the pre-specified target levels. Hence, the incoming flows to the inventory nodes are selected as the manipulated variables for the PID controllers. This way a decoupling between inventory level maintenance and satisfaction of primary control objectives (e.g. customer satisfaction) is achieved, permitting the MPC configuration to react faster to disturbances in demand variability and transportation delays. However, tuning of the localized PID controllers requires a time consuming trial-and-error procedure based on simulations. In their experiments, assuming that demand is deterministic and performing a step change, they observed an amplification of set point deviations for upstream nodes (bullwhip). For stochastic demand variation, they noted that the centralized approach requires a much larger control horizon to achieve a comparable performance with their two-layered strategy.

Braun et al., developed a linear MPC framework for large scale supply chain problems resulting from the semiconductor industry. Through experiments, they showed that MPC can handle adequately uncertainty resulting from model mismatch (lead times) and demand forecasting errors. Due to the complexity of large scale supply chains, they proposed a decentralized scheme where a model predictive controller is created for each node, i.e production facility, warehouse and retailers. Inventory levels are treated as state variables for each node, the manipulated variables are orders and production rates, and demands are treated as disturbances. The goal of the MPC controller is to keep the inventory levels as close as possible to the target values while satisfying constraints with respect to production and transportation capacities. Their simulations showed that using move suppression (i.e. the term in the objective function that penalizes large deviations on control variables between two consecutive time instants), backorders can be eliminated. It is well known in the MPC community that the move suppression term has the effect of making the controller less sensitive to prediction inaccuracies, although usually at the price of degrading set point tracking performance. Through simulations, Braun et al. and Wang et al. justified further the significance of move suppression penalties as a means for increased robustness against model mismatch and hedging against inaccurate demand forecasts.

Wang et al. treated demand as a load disturbance and they considered it as a stochastic signal driven by integrated white-noise (the discrete-time analog of Brownian motion). They applied a state estimation-based MPC in order to increase the system performance and robustness with respect to demand variability and erroneous forecasts.

Assuming no information on disturbances, they employed a Kalman filter to estimate the state variables, where the filter gain is a tuning parameter based on the signal-to-noise ratio. Through simulations they concluded that when there is a large error between the average of actual demands and the forecast, a larger filter gain can make the controller compensate for the error sufficiently fast (Sarimveis et al., 2008).

Dunbar and Desa applied a recently developed distributed/decentralized implementation of nonlinear MPC to the problem of dynamic supply chain management problem, reminiscent of the classic MIT "Beer Game". By this implementation, each subsystem is optimized for its own policy, and communicates the most recent policy to those subsystems to which it is coupled. The supply network consists of three nodes, a retailer, a manufacturer and a supplier. Information flows (i.e., flows moving upstream) are assumed to have no time delays (lead times). On the other hand, material flows (i.e., flows moving downstream) are assumed to have transportation delays. The proposed continuous-time dynamic model is characterized by three state variables, namely, inventory level, unfulfilled orders and backlog for each node. The control inputs are the order rates for each node. Demand rates and acquisition rates (i.e., number of items per day acquired from the upstream node) are considered as disturbances. The control objective is to minimize the total cost, which includes avoiding backorders and keeping unfulfilled orders and inventory levels low. Their model demonstrates bidirectional coupling between nodes, meaning that differential equation models of each stage depend upon the state and input of other nodes. Hence, cycles of information dependence are present in the chain. These cycles complicate decentralized/distributed MPC implementations since at each time period coupled stages must estimate states and inputs of one another. To address this issue, the authors assumed that coupled nodes receive the previously computed predictions from neighboring nodes prior to each update, and rely on the remainder of these predictions as the assumed prediction at each update. To bound the discrepancy between actual and assumed predictions, a move suppression term is included in the objective function. Thus, with the decentralized scheme, an MPC controller is designed for each node, which updates its policy in parallel with the other nodes based on estimates regarding information for interconnected variables. Through simulations, they concluded that the decentralized MPC scheme performs better than a nominal feedback control derived in Sterman, especially when accurate forecasts regarding customer demand exist. However, both approaches exhibit non-zero steady-state error with respect to unfulfilled demands when a step increase is applied to the customer demand rate. Furthermore, the bullwhip effect is observed in their simulations (Sarimveis et al., 2008).

Based on the model of Lin et al., Lin et al. presented a minimum variance control (MVC) system, where two separate set points are posed. An ARIMA model is used as a mechanism to forecast customer demands. The system proved superior to other approaches such as the order-up-to-level policy, PI control in maintaining proper inventory levels without causing the "bullwhip" effect, whether the customer demand trend is stationary or not (Sarimveis et al., 2008).

Yildirim et al. studied a dynamic planning and sourcing problem with service level constraints. Specifically, the manufacturer must decide how much to produce, where to produce, when to produce, how much inventory to carry, etc., in order to fulfill random customer demands in each period. They formulated the problem as a multi-period stochastic programming problem, where service level constraints appear in the form of chance constraints. In order to obtain the optimal feedback control one should be able to solve the resulting stochastic dynamic program. However, due to the curse of dimensionality the problem is computationally intractable. Thus, in order to obtain a sub-optimal solution they formulated the problem as a static deterministic optimization problem. They approximated the service level chance constraints with deterministic equivalent constraints by specifying certain minimum cumulative production quantities that depend on the service level requirements. The rolling horizon procedure is applied on-line following the MPC philosophy, i.e. by solving the resulting mathematical programming problem at each discrete-time instance, applying only the first decision and moving to a new state, where the procedure is repeated. The authors compared their approach to certain threshold subcontracting policies yielding similar results.

Describing uncertainties in a stochastic framework is the standard practice used by the operations research community. For example, in the majority of papers reviewed so far, uncertainties concerning customer demands, machine failures and lead times were mostly described by probability distributions and stochastic processes. However, in many practical situations one may not be able to identify the underlying probability distributions or such a stochastic description may simply not exist. On the other hand, based on historical data or experience one can easily infer bounds on the magnitude of the uncertain parameters (Sarimveis et al., 2008).

Having realized this fact a long-time ago, the control engineering community has developed the necessary theoretical and algorithmic machinery for this type of problems, the so-called robust control theory. In this framework, uncertainties are unknown-but-bounded quantities and constraints dictated by performance specifications and physical limitations are usually hard, meaning that they must be satisfied for all realizations of the uncertain quantities. In the robust control framework, models can be usually "infected" with two types of uncertainty; exogenous disturbances (e.g. customer demands) and plant-model mismatch, that is, uncertainties due to modeling errors (Sarimveis et al., 2008).

The aim of this review paper was to present alternative control philosophies that have been applied to the dynamic supply chain management problem. Representative references were provided that can guide the reader to explore in depth the methodologies of his/her choice. The efforts started in the early 1950s by applying classical control techniques where the analysis was performed in the frequency domain. More recently, highly sophisticated optimal control methods have been proposed mainly based on the time domain. However, many recent reports state that the majority of companies worldwide still suffer from poor supply chain management. Moreover, undesired phenomena, such as the "bullwhip" effect have not yet been remedied. The applicability of control methodologies in real life supply chain problems is thus, naturally questioned.

It is true that in many methodologies that have been presented in this paper, the assumptions on which they are based are often not valid in reality. For example, lead times are not fixed and are not known with accuracy, as many models assume. Inventory levels should be bounded below by zero and above due to warehouse capacities, but these bounds are not always taken into account. The same happens with the production rates which are limited by the machinery capacities. Another limitation is that single stage systems are usually studied, assuming production of a single product or aggregated production. In real life systems, various products are produced with different production rates and different lead times, which, however, share common machinery and storage facilities. Horizontal integration is often represented by considering the supply chain stages in a raw, while interconnections between different level and same level stages are ignored. Finally, raw material costs which may be variable, labor costs and inventory costs are rarely taken explicitly into account. From the above discussion, it is evident that despite the considerable advances that have occurred throughout the years in controlling supply chain systems, there is still plenty of room for further improvements. Elimination of the above limitations will lead to new methodologies of more applicability. Therefore, dynamic control of supply chain systems remains an open and active research area. Among the alternative methodologies that have been presented in this review paper, we would like to draw the attention of the reader to the MPC framework which has become extremely popular in the engineering community, as it proved successful in facing problems similar to the ones mentioned above. Among other advantages, the MPC framework can easily incorporate bounds on the manipulated and controlled variables and leads to the formulation of computationally tractable optimization problems (Sarimveis et al., 2008).

3. MPC for multi echelon supply chain management system

Supply chains are complicated dynamical systems triggered by customer demands. Over the past decade, supply chain management and control has become a strategic focus of leading manufacturing companies. This has been caused by rapid changes in environments in which the companies operate, characterized by high globalization of markets and ever increasing customer demands for higher levels of service and quality. Proper selection of equipment, machinery, buildings and transportation fleets is a key component for the success of such systems. However, efficiency of supply chains mostly depends on management decisions, which are often based on intuition and experience. Due to the increasing complexity of supply chain systems (which is the result of changes in customer preferences, the globalization of the economy and the stringy competition among companies), these decisions are often far from optimum. Another factor that causes difficulties in decision making is that different stages in supply chains are often supervised by different groups of people with different managing philosophies. From the early 1950s it became evident that a rigorous framework for analyzing the dynamics of supply chains and taking proper decisions could improve substantially the performance of the systems. Due to the resemblance of supply chains to engineering dynamical systems, control theory has provided a solid background for building such a framework. During the last half century many mathematical tools emerging from the control literature have been applied to the supply chain management problem. These tools vary from classical transfer function analysis to highly sophisticated control methodologies, such as MPC and neuro dynamic programming.

In this work, a discrete time difference model is developed. The model is applicable to multi echelon supply chain networks of arbitrary structure, that DP denote the set of desired products in the supply Chain and these can be manufactured at plants, P, by utilizing various resources, RS. The manufacturing function considers independent production lines for the distributed products. The products are subsequently transported to and stored at warehouses, W. Products from warehouses are transported upon customer demand, either to distribution centers, D, or directly to retailers, R. Retailers receive time varying orders from different customers for different products. Satisfaction of customer demand is the primary target in the supply chain management mechanism. Unsatisfied demand is recorded as backorders for the next time period. A discrete time difference model is used for description of the supply chain network dynamics. It is assumed that decisions are taken within equally spaced time periods (e.g. hours, days, or weeks). The duration of the base time period depends on the dynamic characteristics of the network. As a result, dynamics of higher frequency than that of the selected time scale are considered negligible and completely attenuated by the network (Perea, 2007).

Plants P, warehouses W, distribution centers D, and retailers R constitute the nodes of the system. For each node, k, there is a set of upstream nodes and a set of downstream nodes, indexed by (k',k''). Upstream nodes can supply node k and downstream nodes can be supplied by k. All valid (k',k) and/or (k,k'') pairs constitute permissible routes within the network. All variables in the supply chain network (e.g. inventory, transportation loads) valid for bulk commodities and products. For unit products, continuous variables can still be utilized, with the addition of a post-processing rounding step to identify neighbouring integer solutions. This approach, though clearly not formally optimal, may be necessary to retain computational tractability in systems of industrial relevance.
A product balance around any network node involves the inventory level in the node at time instances t and $t - 1$, as well as the total inflow of products from upstream nodes and total outflow to downstream nodes. The following balance equation is valid for nodes that are either warehouses or distribution centers:

$$\begin{aligned} & y_{i,k}(t) = y_{i,k}(t-1) + \sum_{k'} x_{i,k',k}(t - L_{k',k}) - \sum_{k''} x_{i,k,k''}(t), \\ & \forall k \in \{W, D\}, \qquad t \in T, \qquad i \in DP \end{aligned} \tag{1}$$

where $y_{i,k}$ is the inventory of product i stored in node k; $x_{i,k',k}$ denotes the amount of the i-th product transported through route (k',k); $L_{k',k}$ denotes the transportation lag (delay time) for route (k',k), i.e. the required time periods for the transfer of material from the supplying node to the current node. The transportation lag is assumed to be an integer multiple of the base time period.
For retailer nodes, the inventory balance is slightly modified to account for the actual delivery of the i-th product attained, denoted by $d_{i,k}(t)$.

$$\begin{aligned} & y_{i,k}(t) = y_{i,k}(t-1) + \sum_{k'} x_{i,k',k}(t - L_{k',k}) - d_{i,k}(t), \\ & \forall k \in \{R\}, \qquad t \in T, \qquad i \in DP. \end{aligned} \tag{2}$$

The amount of unsatisfied demand is recorded as backorders for each product and time period. Hence, the balance equation for back orders takes the following form:

$$\begin{aligned} & BO_{i,k}(t) = BO_{i,k}(t-1) + R_{i,k}(t) - d_{i,k}(t) - LO_{i,k}(t), \\ & \forall k \in \{R\}, \qquad t \in T, \qquad i \in DP. \end{aligned} \tag{3}$$

where $R_{i,k}$ denotes the demand for the i-th product at the k-th retailer node and time period t. $LO_{i,k}$ denotes the amount of cancelled back orders (lost orders) because the network failed to satisfy them within a reasonable time limit. Lost orders are usually expressed as a percentage of unsatisfied demand at time t. Note that the model does not require a separate balance for customer orders at nodes other than the final retailer nodes (Sterman et al, 2002).
MPC is a model based control strategy that calculates at each sampling time via optimization the optimal control action to maintain the output of the plant close to the desired reference. In fact, MPC stands for a family of methods that select control actions based on online optimization of an objective function. MPC has gained wide acceptance in the chemical and other process industries as the basis for advanced multivariable control schemes. In MPC, a system model and current and historical measurements of the process are used to predict the system behavior at future time instants. A control relevant objective function is then

optimized to calculate a sequence of future control moves that must satisfy system constraints. The first predicted control move is implemented and at the next sampling time the calculations are repeated using updated system states (illustrated in Figure 2). MPC represents a general framework for control system implementation that accomplishes both feedback and feed forward control action on a dynamical system. The appeal of MPC over traditional approaches to control design include (1) the ability to handle large multivariable problems, (2) the explicit handling of constraints on system input and output variables, and (3) its relative ease of use. MPC applied to supply chain management relies on dynamical models of material flow to predict inventory changes among the various nodes of the supply chain. These model predictions are used to adjust current and future order quantities requested from upstream nodes such that inventory will reach the targets necessary to satisfy demand in a timely manner (Wang et al, 2007). The control system aims at operating the supply chain at the optimal point despite the influence of demand changes. The control system is required to possess built in capabilities to recognize the optimal operating policy through meaningful and descriptive cost performance indicators and mechanisms to successfully alleviate the detrimental effects of demand uncertainty and variability. The main objectives of the control strategy for the supply chain network can be summarized as follows: (i) maximize customer satisfaction, and (ii) minimize supply chain operating costs.

The first target can be attained by the minimization of back orders (i.e. unsatisfied demand) over a time period because unsatisfied demand would have a strong impact on company reputation and subsequently on future demand and total revenues. The second goal can be achieved by the minimization of the operating costs that include transportation and inventory costs that can be further divided into storage costs and inventory assets in the supply chain network. Based on the fact that past and present control actions affect the future response of the system, a receding time horizon is selected. Over the specified time horizon the future behavior of the supply chain is predicted using the described difference model (Eqs. (1)–(3)). In this model, the state variables are the product inventory levels at the storage nodes, *y*, and the back orders, *BO*, at the order receiving nodes. The manipulated (control or decision) variables are the product quantities transferred through the network's permissible routes, *x*, and the delivered amounts to customers, *d*. Finally, the product back orders, *BO*, are also matched to the output variables. The inventory target levels (e.g. inventory setpoints) are time invariant parameters. The control actions that minimise a performance index associated with the outlined control objectives are then calculated over the receding time horizon. At each time period the first control action in the calculated sequence is implemented. The effect of unmeasured demand disturbances and model mismatch is computed through comparison of the actual current demand value and the prediction from a stochastic disturbance model for the demand variability. The difference that describes the overall demand uncertainty and system variability is fed back into the MPC scheme at each time period facilitating the corrective action that is required.

The centralized mathematical formulation of the performance index considering simultaneously back orders, transportation and inventory costs takes the following form:

$$
\begin{aligned}
J_{total} = & \sum_{t}^{t+P} \sum_{k\in\{W,D,R\}} \sum_{i\in DP} \left\{ w_{y,i,k} (y_{i,k}(t) - y_{s,i,k}(t))^2 \right\} + \sum_{t}^{t+M} \sum_{k\in\{W,D,R\}} \sum_{i\in DP} \left\{ w_{x,i,k',k} (x_{i,k',k}(t))^2 \right\} \\
& + \sum_{t}^{t+P} \sum_{k\in\{R\}} \sum_{i\in DP} \left\{ w_{BO,i,k} (BO_{i,k}(t))^2 \right\} + \sum_{t}^{t+M} \sum_{k\in\{W,D,R\}} \sum_{i\in DP} \left\{ w_{\Delta x,i,k',k} (x_{i,k',k}(t) - x_{i,k',k}(t-1))^2 \right\}.
\end{aligned}
\tag{4}
$$

The performance index, *J*, in compliance with the outlined control objectives consists of four quadratic terms. Two terms account for inventory and transportation costs throughout the supply chain over the specified prediction and control horizons(P , M). A term penalizes back orders for all products at all order receiving nodes (e.g. retailers) over the prediction horizon P. Also a term penalizes deviations for the decision variables (i.e. transported product quantities) from the corresponding value in the previous time period over the control horizon M. The term is equivalent to a penalty on the rate of change in the manipulated variables and can be viewed as a move suppression term for the control system. Such a policy tends to eliminate abrupt and aggressive control actions and subsequently, safeguard the network from saturation and undesired excessive variability induced by sudden demand changes. In addition, transportation activities are usually preferred to resume a somewhat constant level rather than fluctuate from one time period to another.
However, the move suppression term would definitely affect control performance leading to a more sluggish dynamic response. The weighting factors, $w_{y,i,k}$, reflect the inventory storage costs and inventory assets per unit product, $w_{x,i,k',k}$, account for the transportation cost per unit product for route (k',k) .Weights $w_{BO,i,k}$ correspond to the penalty imposed on unsatisfied demand and are estimated based on the impact service level has on the company reputation and future demand. Weights $w_{\Delta x,i,k',k}$, are associated with the penalty on the rate of change for the transferred amount of the *i*-th product through route (k',k) . Even though, factors $w_{y,i,k}$, $w_{x,i,k',k}$ and $w_{BO,i,k}$ are cost related that can be estimated with a relatively good accuracy, factors $w_{\Delta x,i,k',k}$ are judged and selected mainly on grounds of desirable achieved performance.
The weighting factors in cost function also reflect the relative importance between the controlled (back orders and inventories) and manipulated (transported products) variables. Note that the performance index of cost function reflects the implicit assumption of a constant profit margin for each product or product family. As a result, production costs and revenues are not included in the index.
But in this paper, a consecutive decentralized formulation will used, namely centralized cost function divided to decentralized cost functions for each stage(warehouse, distribution center, retailer):

$$\begin{aligned} J_1 = &\sum_{t}^{t+P} \sum_{i \in DP} \left\{ w_{y,i,k} (y_{i,k}(t))^2 \right\} \\ &+ \sum_{t}^{t+M} \sum_{i \in DP} \left\{ w_{x,i,k',k} (x_{i,k',k}(t))^2 \right\} \\ &+ \sum_{t}^{t+M} \sum_{i \in DP} \left\{ w_{\Delta x,i,k',k} (x_{i,k',k}(t) - x_{i,k',k}(t-1))^2 \right\}, \quad k \in W \end{aligned} \tag{5}$$

$$\begin{aligned} J_2 = &\sum_{t}^{t+P} \sum_{i \in DP} \left\{ w_{y,i,k} (y_{i,k}(t))^2 \right\} \\ &+ \sum_{t}^{t+M} \sum_{i \in DP} \left\{ w_{x,i,k',k} (x_{i,k',k}(t))^2 \right\} \\ &+ \sum_{t}^{t+M} \sum_{i \in DP} \left\{ w_{\Delta x,i,k',k} (x_{i,k',k}(t) - x_{i,k',k}(t-1))^2 \right\}, \quad k \in D \end{aligned} \tag{6}$$

$$J_3 = \sum_{t}^{t+P} \sum_{i \in DP} \left\{ w_{y,i,k} (y_{i,k}(t))^2 \right\} + \sum_{t}^{t+M} \sum_{i \in DP} \left\{ w_{x,i,k',k} (x_{i,k',k}(t))^2 \right\} + \sum_{t}^{t+P} \sum_{i \in DP} \left\{ w_{BO,i,k} (BO_{i,k}(t))^2 \right\} + \sum_{t}^{t+M} \sum_{i \in DP} \left\{ w_{\Delta x,i,k',k} (x_{i,k',k}(t) - x_{i,k',k}(t-1))^2 \right\}, \quad k \in R. \tag{7}$$

Therefore by this implementation, As supply chains can be operated sequentially, i.e., stages update their policies in series, synchronously, each node by a decentralized model predictive controller optimizes for its own policy, and communicates the most recent policy to those nodes to which it is coupled. In fact, MPC's of retailers (with Eqs. (1),(5)) only will optimized for its own policy and then will sent its optimal inputs to upstream joint nodes to those nodes which it is coupled (distribution centers), as measurable disturbances. Also model predictive controllers of distribution centers (with Eqs. (1),(6)) only will optimized for its own policy and then will sent its optimal inputs to upstream joint nodes to those nodes which it is coupled (warehouse centers), as measurable disturbances. Finally, model predictive controllers of warehouses (with Eqs. (2),(3),(7)) will optimized for its own optimal inputs.

Two types of sequential decentralized MPC can be used. In first method, each node completely by a decentralized MPC optimizes for its own policy. At each time period, the first decentralized MPC action in the calculated sequence is implemented until MPC process complete.

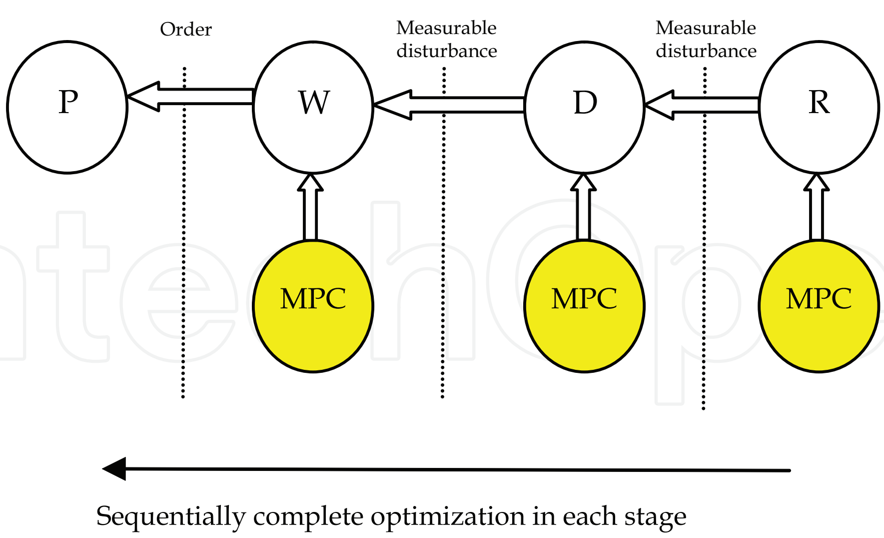

Fig. 2. Procedure of first consecutive decentralized MPC

In fact, local decentralized model predictive controllers corresponding to retailers will done for regulating inventory level in R and then will sent its MPC optimal inputs at long prediction horizon to upstream joint nodes to those nodes which it is coupled (distribution centers), as measurable disturbances. Also model predictive controllers corresponding to distribution centers will optimized and then will sent its optimal inputs to upstream joint nodes to those nodes which it is coupled (warehouse centers), as measurable disturbances. Finally, model predictive controllers corresponding to warehouses will optimized for its own optimal inputs by local method. In fact decentralized model predictive controllers, sequentially operate (Figure 2).

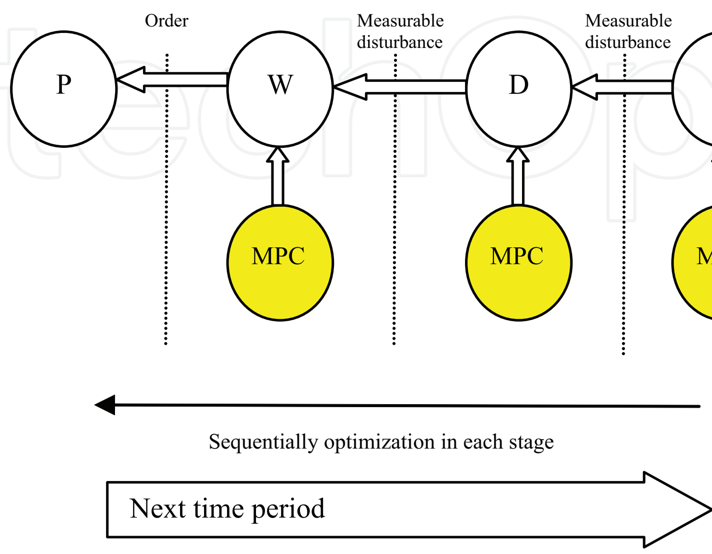

Fig. 3. Procedure of second consecutive decentralized MPC

4. Simulations

A four echelon supply chain system is used in the simulated examples. The supply chain network consists of one production nodes, two warehouse nodes, four distribution centers, and four retailer nodes (Figure 4). All possible connections between immediately successive echelons are permitted. One product family consist of 12 products is being distributed through the network. Inventory setpoints, maximum storage capacities at every node, and transportation cost data for each supplying route are reported in Table 1.

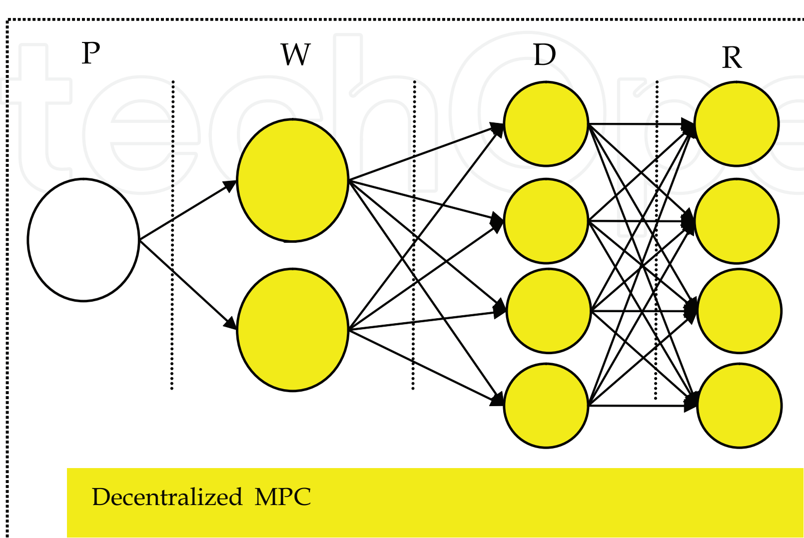

Fig. 4. MPC framework in a multiechelon supply chain management system

Echelon	W	D	R
Max inventory level	1400	500	150
Inventory setpoint	320	220	35
Transportation cost (move suppression cost)	P to W 0.5	W to D 1	D to R 1
Inventory weights	1	1	1
Back order weights	-	-	1
Delays	2	3	2

Table 1. Given values of supply chain

A prediction horizon of 25 time periods and a control horizon of 20 time periods were selected and was considered $LO_{i,k} = 0$ for every times. So each delay was replaced by its 4th order Pade approximation (after system model transform to continues time model and then return to discrete time model). In this part, two method of consecutive decentralize MPC that beforehand was stated, applying to large scale supply chain to constant demands equal 4. The simulated scenarios lasted for 70 time periods. As demand is constant, both method have equal response to constant demand that is presented in figure 5 (average inventory levels in each echelon). The move suppression term would definitely affect control performance leading to a more sluggish dynamic response.

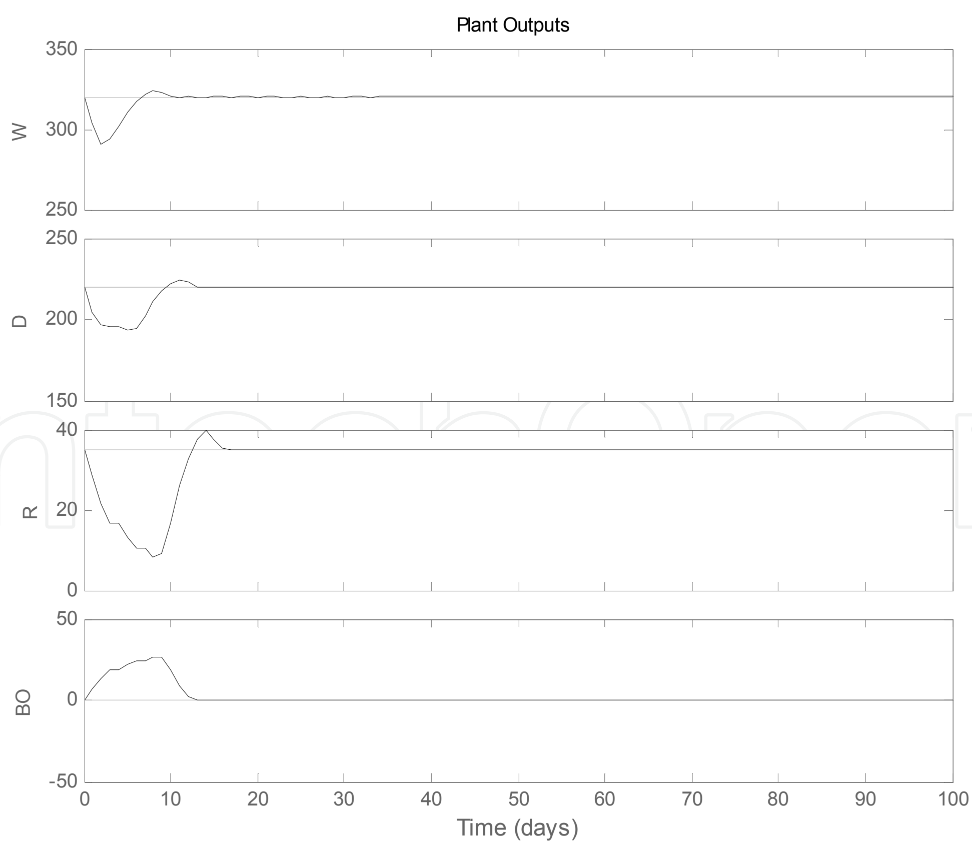

Fig. 5. Discrete time dynamic response to a 4 unit constant demand for networks with different transportation delays L = [2 3 2] (first and second method)

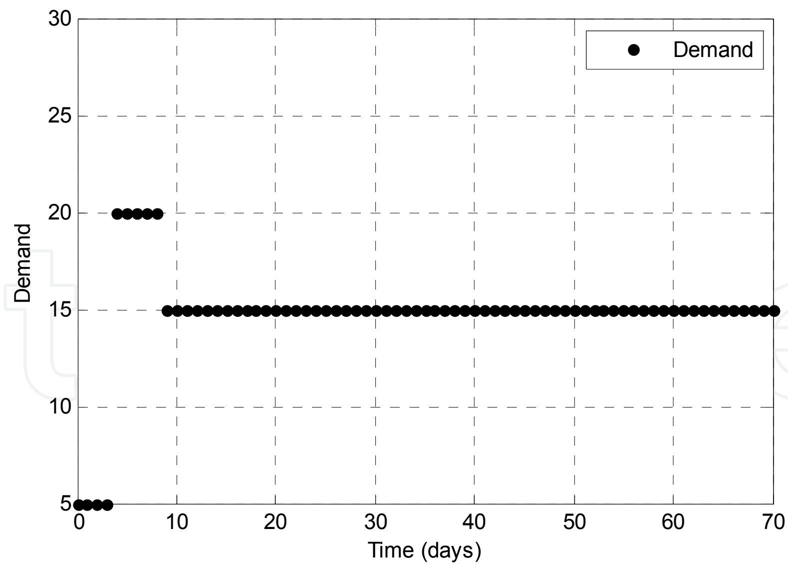

Fig. 6. Discrete pulsatory customer demand

In fact, if suddenly demand changed, the first method can not predict this changes and has not efficiency. Instead second method by online demand prediction in its formulation is efficient. As second method by online demand prediction in its formulation is rather efficient to first method, in this part, second decentralized MPC method applied to the supply chain network with pulsatory variations of customer demand that are seeing in figure 6, once with no move suppression term (Figure 7), and once with move suppression term (Figure 8).

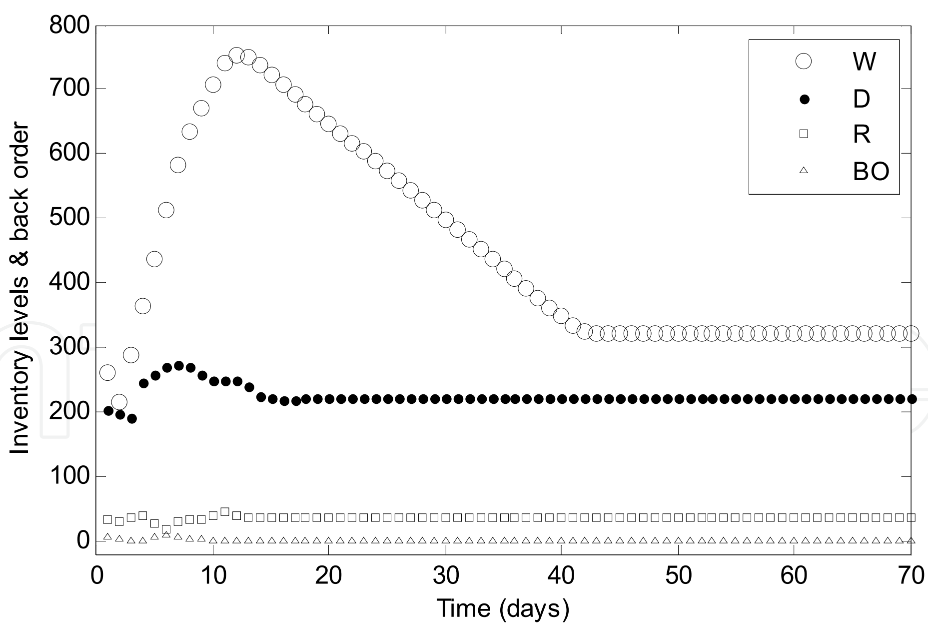

Fig. 7. Inventory levels control by second method of consecutive decentralized MPC toward discrete pulsatory demand with without move suppression effect

Also second decentralized MPC method applied to the supply chain network with pulsatory variations of customer demand that are seeing in figure 9, once with no move suppression term (Figure 10), and once with move suppression term (Figure 11).

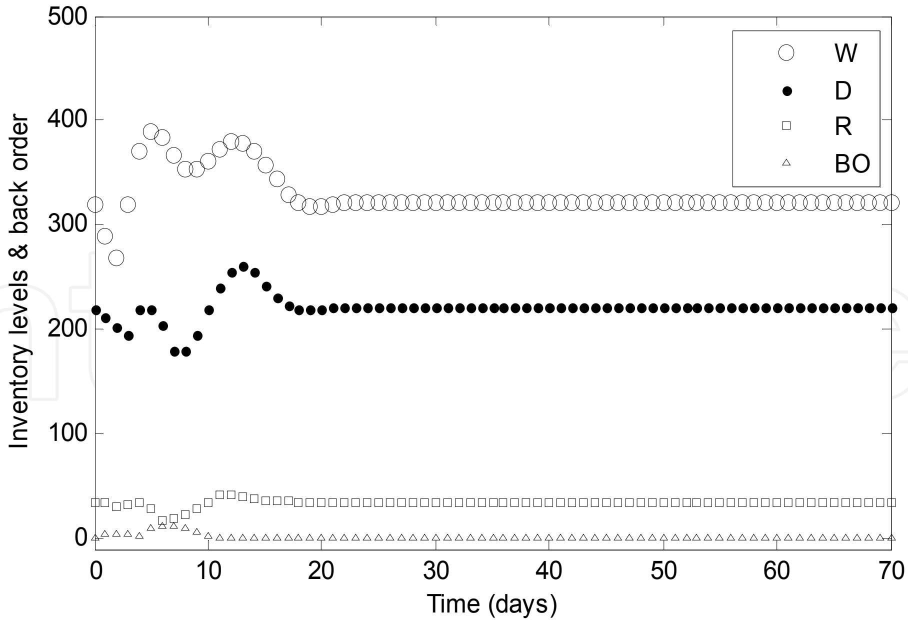

Fig. 8. Inventory levels control by second method of consecutive decentralized MPC toward discrete pulsatory demand with move suppression effect

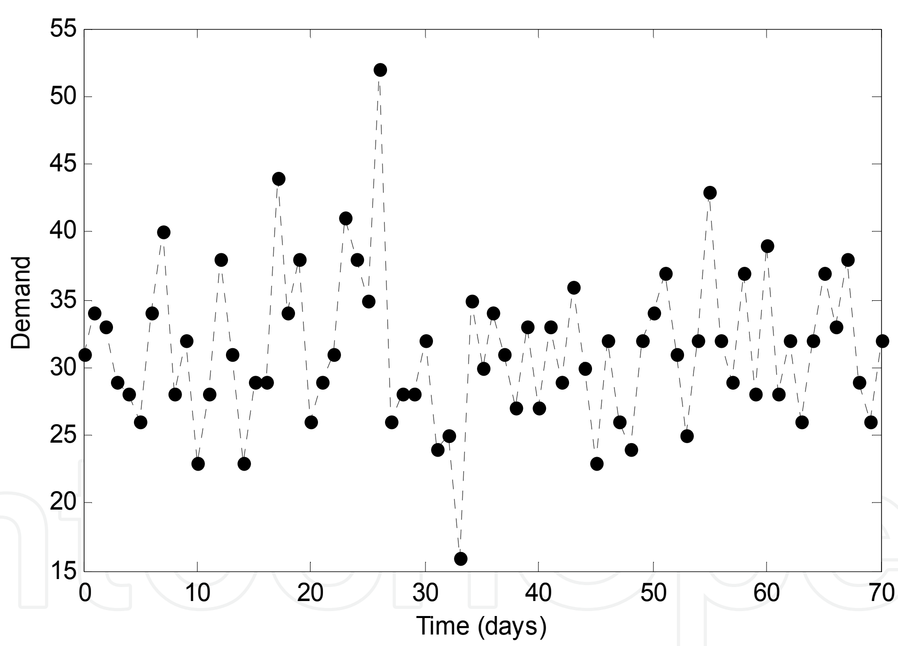

Fig. 9. Discrete stochastic demand of gamma distribution

Therefore by using of move suppression, amplitude of variation of outputs will be decreased. So move suppression term increased system robustness toward changes on demands (Figure 11).

5. Conclusion

Supply chain management system is a network of facilities and distribution entities: suppliers, manufacturers, distributors, retailers. The control system aims at operating the

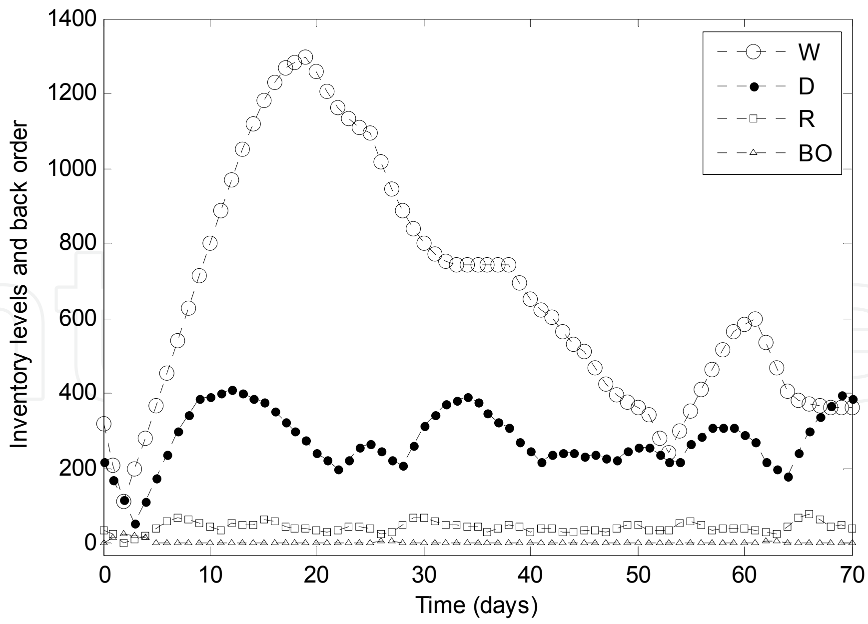

Fig. 10. Inventory levels control by second method of consecutive decentralized MPC toward discrete stochastic demand of gamma distribution without move suppression effect

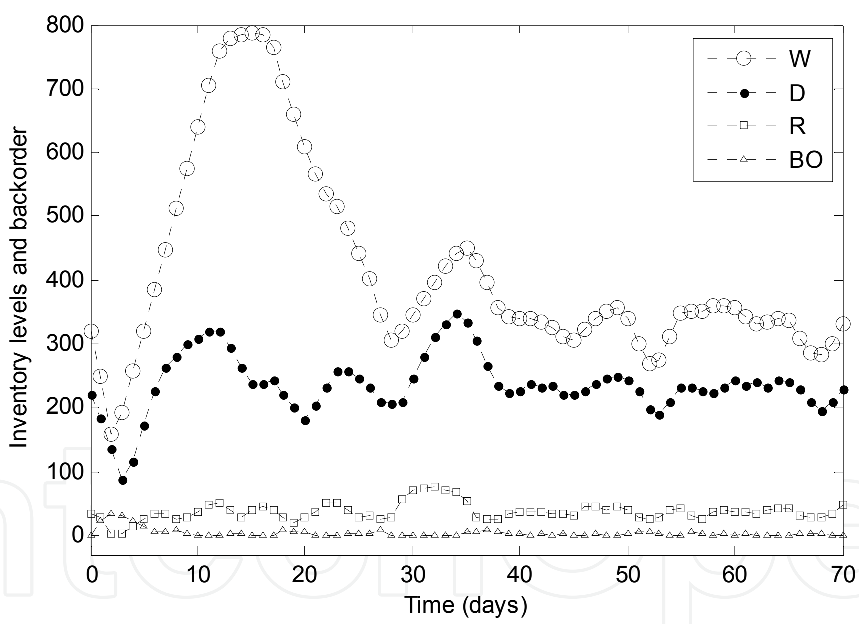

Fig. 11. Inventory levels control by second method of consecutive decentralized MPC toward discrete stochastic demand of gamma distribution with move suppression effect

supply chain at the optimal point despite the influence of demand changes. As supply chains can be operated sequentially, local Consecutive model predictive controllers applying to a supply chain management system consist of four echelon. Two types of sequential decentralized MPC used. In first method, each node completely by a decentralized model predictive controller optimized for its own policy, and in second method, decentralized model predictive controllers in each stage are updated in each time period. As second method by online demand prediction in its formulation is rather efficient to first method, in this part, second decentralized MPC method applied to the supply chain network with pulsatory variations of customer demand. Also second decentralized MPC method applied to the supply chain network with pulsatory variations of customer demand. In fact, if suddenly demand changed, the first method can not predict this changes and has not efficiency. Instead second method by online demand prediction in its formulation is efficient. Also a move suppression term add to cost function, that increase system robustness toward changes on demands.

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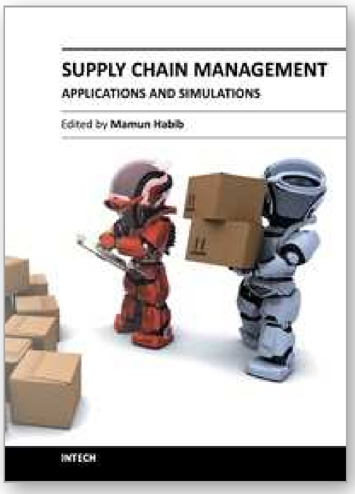

Supply Chain Management - Applications and Simulations

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Supply Chain Management (SCM) has been widely researched in numerous application domains during the last decade. Despite the popularity of SCM research and applications, considerable confusion remains as to its meaning. There are several attempts made by researchers and practitioners to appropriately define SCM. Amidst fierce competition in all industries, SCM has gradually been embraced as a proven managerial approach to achieving sustainable profits and growth. This book "Supply Chain Management - Applications and Simulations" is comprised of twelve chapters and has been divided into four sections. Section I contains the introductory chapter that represents theory and evolution of Supply Chain Management. This chapter highlights chronological prospective of SCM in terms of time frame in different areas of manufacturing and service industries. Section II comprised five chapters those are related to strategic and tactical issues in SCM. Section III encompasses four chapters that are relevant to project and technology issues in Supply Chain. Section IV consists of two chapters which are pertinent to risk managements in supply chain.