Gage Repeatability and Reproducibility Methodologies Suitable for Complex Test Systems in Semi-Conductor Manufacturing

Sandra Healy and Michael Wallace
Analog Devices and University of Limerick
Ireland

1. Introduction

Six sigma is a highly disciplined process that focuses on developing and delivering near-perfect products and services consistently. Six sigma is also a management stragety to use statistical tools and project work to achieve breakthrough profitability and quantum gains in quality. The steps in the six sigma process are Define, Measure, Analyse, Improve, Control or DMAIC for short (Kubiak T.M, Benhow D.W, 2009). The actions that take place in each of these steps are described in brief in table 1 below.

STEP	DISCREPTION
Define	Select the appropriate critical to quality characteristic.
Measure	Gather data to measure the critical to quality characteristic.
Analyse	Identify root causes of deviations from specification.
Improve	Reduce variability or eliminate cause of deviation.
Control	Monitor the process to sustain the improvement.

Table 1. Description of the steps in the DMAIC process.

During the define stage of the DMAIC process, the critical to quality characteristics of the product are clearly identified. Once these are understood, methods of measuring these are defined and described in more detail within the measurement stage. Once the measurement system and test method are identified, a comprehensive measurement system analysis (MSA) is then required. The objective of this MSA is to evaluate the suitability of the measurement method for its intended function within the DMAIC cycle.

The most commonly used methodologies used for MSA are defined in measurement systems analysis reference manual (Measurement Systems Analysis Workgroup, Automotive Industry Action Group, 1998). In this there are three widely used methods to quantify the measurement error. These are in increasing order of complexity: the range method, the average and range method, and ANOVA. These generally use a small sample of parts, measured by a number of different appraisers to generate estimates of the components of measurement error.

With increasing complexity in semiconductor product test, the measurement equipment is generally automated, and test boards are employed that are capable of testing multiple parts in parallel. These introduce additional measurement error components not accounted for in these traditional methodologies. Updated methodologies capable of accounting for this situation are required. The purpose of this chapter is to describe appropriate experimental designs capable for use in MSA in this situation. The experimental designs used are extensively taken from Montgomery (Montgomery D.C., 1996; Montgomery D.C.,Runger G.C., 1993a, 1993b).

2. Review components of MSA

The quality of measurement data is defined by the statistical properties of multiple measurements obtained from a measurement system operating under stable conditions.
The statistical properties most commonly used to characterize the quality of data are the bias and the variance of the measurement system. Bias refers to the location of the average of the data relative to a known reference and is a systematic error component of the measurement system. Variance refers to the spread of the data. These are shown schematically in figure 1.

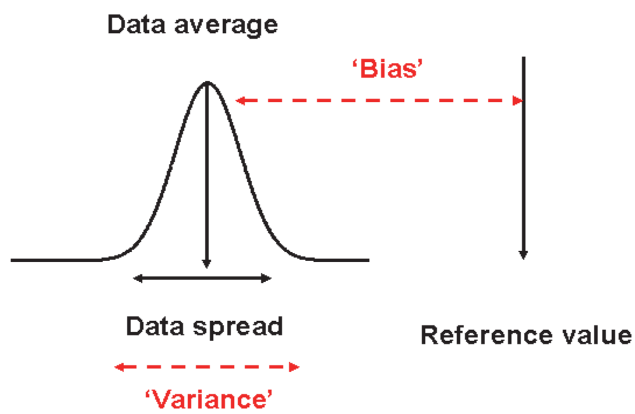

Fig. 1. Schematic of data Bias and Variance

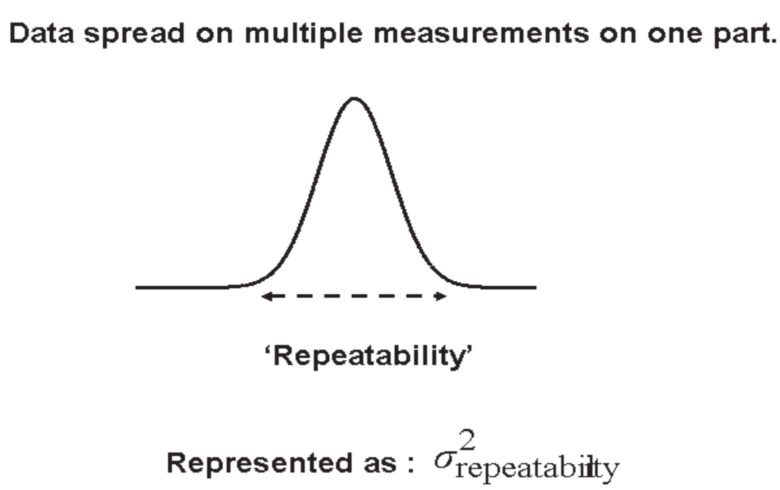

Fig. 2. Schematic test repeatability.

In practice the measurement system or gage is chosen to have a known and acceptable bias, and MSA uses statistical techniques to obtain estimates of the variance.
There are two components of variance for a measurement system. The first is the *repeatability* or *precision* which is the variance within repeated measurements of a given setup by a single appraiser. The second is the *reproducibility* which is the variation in the average measurement made by different appraisers. Repeatability and reproducibility are shown schematically in figure 2 and figure 3.

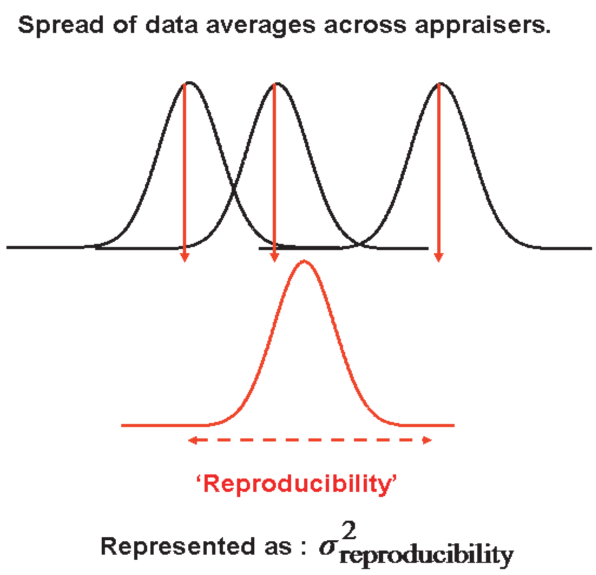

Fig. 3. Schematic of test reproducibility.

The Gage repeatability and reproducibility (Gage R&R) is the combined estimate of the measurement system repeatability and reproducibility variance components. This is given by equation 1.

$$\text{Gage R\&R} = \sqrt{\sigma^2_{repeatability} + \sigma^2_{reproducability}} \quad (1)$$

Within the manufacturing enviornment, this Gage R&R error gets added into the product distribution as a pure error term (Wheeler D, Lyday R, 1989). This has the effect of widening the true product distribution by this amount. Representing the true product distribution as $\sigma_{product}$, the resulting total variation (TV) of the manufacturing distribution is given by equation 2.

$$TV = \sqrt{\sigma^2_{product} + \sigma^2_{R\&R}} \quad (2)$$

This total variation is shown schematically in figure 4. Here the true product distribution is represented by the green curve, while the TV distribution seen in manufacturing is represented by the black curve. This black curve is estimated using equation 2 above.
With a knowledge of the components of total variation, some useful performance metrics for the measurement system can be generated. The most commonly used are (a) the percentage of total variation and (b) the percentage contribution to total variance. These are calculated using equations 3 and 4 respectively.

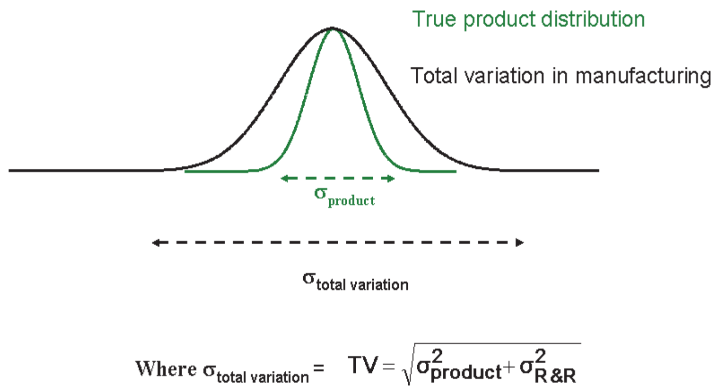

Fig. 4. Schematic total variation in manufacturing

%of total variation:

$$\%GR\,\&\,R = \left(\frac{GageR\,\&\,R}{TV}\right)\times 100 = \left(\frac{\sigma_{R\&R}}{\sqrt{\sigma^2_{product} + \sigma^2_{R\&R}}}\right)\times 100 \tag{3}$$

% contribution to total variance:

$$\%Contribution(GR\,\&\,R) = \left(\frac{GageR\,\&\,R^2}{TV^2}\right)\times 100 = \left(\frac{\sigma^2_{R\&R}}{\sigma^2_{Product} + \sigma^2_{R\&R}}\right)\times 100 \tag{4}$$

These metrics give an indication of how capable the gage is for measuring the critical to quality characteristic. Acceptable regions of gage R&R as defined by the Automotive Industry Action Group (Measurement Systems Analysis Workgroup, Automotive Inductry Action Group, 1998) are as indicated in table 2.

GAGE R&R RANGE	ACTION REQUIRED
<10%	Gage acceptable
10% < Gage R&R < 30%	Action required to understand variance
30% < Gage R&R	Gage unacceptable for use and requires improvement

Table 2. Acceptable regions of Gage R&R.

Note that similar equations can be written for the individual components of variance and also for the product contribution by replacing $\sigma_{R\&R}$ with $\sigma_{repeatability}$, $\sigma_{reproducibility}$ and $\sigma_{product}$ respectively.

Once the MSA indicates that the measurement method is both sufficiently accurate and capable, it can be integrated into the remaining steps of the DMAIC process to analyse, improve and control the characteristic.

3. Review of existing methodologies employed for MSA

Historically gages within the manufacturing enviornment have been manual devices capable of measuring one single critical to quality characteristic. Here the components of variance are (a) the repeatability on a given part, and (b) the reproducibility across operators or appraiser effect. To estimate the components of variance in this instance, a small sample of readings is required by independent appraisers. Typical data collection operations comprised of 5 parts measured by each of 3 appraisers.There are three widely used methods in use to analyse the collected data. These are the range method, the average and range method, and the analysis of variance (ANOVA) method (Measurement Systems Analysis Workgroup, Automotive Inductry Action Group, 1998).

The range method utilises the range of the data collected to generate an estimate of the overall variance. It does not provide estimates of the variance components. The average and range method is more comprehensive in that it utilises the average and range of the data collected to provide estimates of the overall variance and the components of variance i.e. the repeatability and reproducibility. The ANOVA method is the most comprehensive in that it not only provides estimates of the overall variance and the components of variance, it also provides estimates of the interaction between these components. In addition, it enables the use of statistical hypothesis testing on the results to identify statistically significant effects. ANOVA methods capable of replacing the range / average and range methods have previously been described (Measurement Systems Analysis Workgroup, Automotive Inductry Action Group, 1998). A relative comparison of these three methods are summarised in table 3 below.

METHOD	ADVANTAGE	DISADVANTAGE
Range method.	Simple calculation method.	Estimates overall variance only - excludes estimate of the components of R&R.
Average and range method.	Simple calculation method. Enables estimate of overall variance and component variance.	Estimates overall variance and components but excludes estimate of interaction effects.
ANOVA method.	Enables estimates of overall variance and all components including interaction terms. More accuracy in the calculated estimates. Enables statistical hypothesis testing.	Detailed calculations - require automation.

Table 3. Compare and contrast historical methods for Gage R&R

The metrics generated from these gage R&R studies are typically the percentage total variance and the percentage contribution to total variance of the repeatability, the reproducibility or appraiser effect, and the product effect. A typical gage R&R results table is shown in table 4.

With increasing complexity in semiconductor test manufacturing, automated test equipment is used to generate measurement data for many critical to quality characteristic on any given product. Additional sources of test variance can be recognised within this complex test system. More advanced ANOVA methods are required to enable MSA in this situation.

Note that for cycle time and cost reasons, the data collection steps have an additional constraint in that the number of experimental runs must be minimised. Design of experiments is used to achieve this optimization.

Estimate of Variance component	**Standard Deviation**	**% of Total Variation**	**% Contribution.**
Equipment Variation or Repeatability.	Equipment Variaiton (EV) = $\sigma_{repetability}$	$\left(\frac{\sigma_{repeatability}}{\sqrt{\sigma^2_{product}+\sigma^2_{R\&R}}}\right)\times 100$	$\left(\frac{\sigma^2_{repeatability}}{\sigma^2_{product}+\sigma^2_{R\&R}}\right)\times 100$
Appraiser or Operator Variation.	Appraiser Variation (AV) = $\sigma_{reproducibility}$	$\left(\frac{\sigma_{reproducibility}}{\sqrt{\sigma^2_{product}+\sigma^2_{R\&R}}}\right)\times 100$	$\left(\frac{\sigma^2_{reproducibility}}{\sigma^2_{product}+\sigma^2_{R\&R}}\right)\times 100$
Interaction variation.	Appraiser by product interaction = $\sigma_{interaction}$	$\left(\frac{\sigma_{Interaction}}{\sqrt{\sigma^2_{product}+\sigma^2_{R\&R}}}\right)\times 100$	$\left(\frac{\sigma^2_{Interaction}}{\sigma^2_{product}+\sigma^2_{R\&R}}\right)\times 100$
System or Gage Variation.	Gage R&R = $\sigma_{R\&R}$	$\left(\frac{\sigma_{R\&R}}{\sqrt{\sigma^2_{product}+\sigma^2_{R\&R}}}\right)\times 100$	$\left(\frac{\sigma^2_{R\&R}}{\sigma^2_{product}+\sigma^2_{R\&R}}\right)\times 100$
Product Variation.	Product variation (PV) = $\sigma_{product}$	$\left(\frac{\sigma_{product}}{\sqrt{\sigma^2_{product}+\sigma^2_{R\&R}}}\right)\times 100$	$\left(\frac{\sigma^2_{product}}{\sigma^2_{product}+\sigma^2_{R\&R}}\right)\times 100$

Table 4. Measurement systems analysis metrics evaluating Gage R&R.

4. MSA for complex test systems

With increased complexity and cost pressure within the semiconductor manufacture environment, the test equipment used is automated and often tests multiple devices in parallel. This introduces additional components of variance of test error. These are illustrated in figure 5. The components of variance in this instance can be identified as follows.

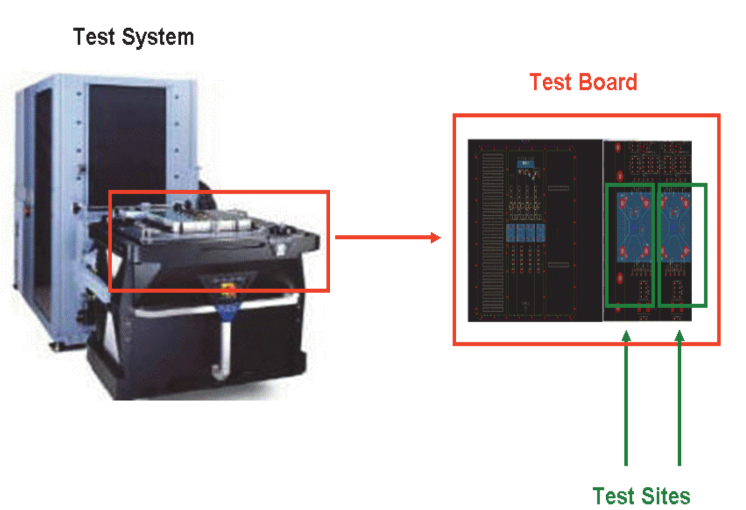

Fig. 5. Components of test variance in manufacturing-System, Boards, Sites

The test repeatability or replicate error is the variance seen on one unit on one test set-up. Because test repeatability may vary across the expected device performance window i.e. a range effect, multiple devices from across the expected range are used in the investigation of test repeatability error.

As the test operation is fully automated, the traditional appraiser affect is replaced by the test setup reproducibility. The test reproducibility therefore comes from the physical components of the test system setup. These are identified as the testers and the test boards used on the systems. In addition, when multi-site testing is employed allowing testing of multiple devices in parallel across multiple sites on a given test board, the test sites themselves contribute to test reproducibility.

In investigating tester to tester and board to board effects a fixed number of specific testers and boards will be chosen from the finite population of testers and boards. Because these are being specifically chosen, a suitable experimental design in this case is a Fixed Effects Model in which the fixed factors are the testers and the boards.

In investigating multisite site-to-site effects, the variation across the devices used within the sites is confounded with the site-to-site variation. The devices used within the sites are effectively a nuisance effect and need to be blocked from the site to site effects. In this instance a suitable experimental design is a blocked design.

5. Fixed effects experimental design for test board and tester effects

In this instance there are two experimental factors – the test boards and the test systems. The MSA therefore requires a two factor experimental design. For the example of two factors at two levels, the data collection runs are represented by an array shown in table 5. To ensure an appropriate number of data points are collected in each run, 30 repeats or replicates are performed.

Run number	Tester level	Board level
1	1	1
2	1	2
3	2	1
4	2	2

Table 5. Experimental Array - 2 Factors at 2 Levels.

An example dataset is shown in figure 6. This shows data from a measurement on a temperature sensor product. Data were collected from devices across two test boards and two test systems. Both the tester to tester and board to board variations are seen in the plot.

5.1 Fixed effects statistical model

Because the testers and boards are chosen from a finite population of testers and boards, in this instance a suitable statistical model is given by equation 5 (Montgomery D.C, 1996):

$$Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + e_{ijk} \qquad \begin{aligned} i &= 1 \text{ to } t \\ j &= 1 \text{ to } b \\ k &= 1 \text{ to } r \end{aligned} \tag{5}$$

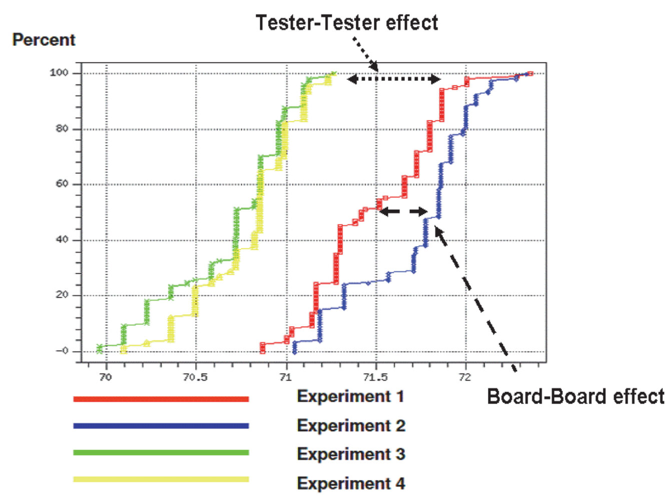

Fig. 6. Example data Fixed Effects Model- Across Boards and Testers.

Where Y_{ijk} are the experimentally measured data points.
μ is the overall experimental mean.
τ_i is the effect of tester 'i'.
β_j is the effect of board 'j'.
$(\tau\beta)_{ij}$ is the interaction effect between testers and boards.
k is the replicate of each experiment.
e_{ijk} is the random error term for each experimental measurement.

Here it is assumed that τ_i , β_j , $(\tau\beta)_{ij}$ and e_{ijk} are random independent variables, where $\{\tau_i\} \sim N(0, \sigma^2_T)$, $\{\beta_j\} \sim N(0, \sigma^2_B)$ and $\{e_{ijk}\} \sim N(0, \sigma^2_R)$.

The analysis of the model is carried out in two stages. The first partitions the total sum of squares (SS) into its constituent parts. The second stage uses the model defined in equation 5 and derives expressions for the expected mean squares (EMS). By equating the SS to the EMS the model estimates are calculated. Both the SS and the EMS are summarised in an ANOVA table.

5.2 Derivation of expression for SS

The results of this data collection are represented by the generalized experimental result Y_{hk}, where h= 1 … s is the total number of set-ups or experimental runs, and k= 1 … r is the number of replicates performed on each experimental run. Using the dot notation, the following terms are defined:

Set-up Total: $Y_{h.} = \sum_{k=1}^{r} Y_{hk}$ denotes the sum of all replicates for a given set-up.

Overall Total: $Y_{..} = \sum_{h=1}^{s}\sum_{k=1}^{r} Y_{hk}$ denotes the sum of all data points.

Overall Mean: $\overline{Y}_{..} = \left(\sum_{h=1}^{s}\sum_{k=1}^{r} Y_{hk}\right) / (sr)$ denotes the average of all data points.

The effect of each factor is analysed using 'contrasts'. The contrast of a factor is a measure of the change in the *total* of the results produced by a change in the level of the factor. Here a simplified "-" and "+" notation is used to denote the two levels. The contrast of a factor is the difference between the sum of the set-up totals at the "+" level of the factor and the sum of the set-up totals at the "-" level of the factor. The array is rewritten to indicate the contrast effects of each factor as shown in table 6.

Run number	Tester level	Board level	Tester x Board Interaction	Generalized Experimental Result
1	-	-	+	Y_{hk}, where: h= 1 to s set-ups (= 4) k= 1 to r replicates (= 30)
2	-	+	-	
3	+	-	-	
4	+	+	+	

Table 6. Fixed Effects Array with 2 Level Contrasts

The contrasts are determined for each of the factors as follows:

$$\text{Tester contrast} = -Y_{1.} - Y_{2.} + Y_{3.} + Y_{4.}$$

$$\text{Board contrast} = -Y_{1.} + Y_{2.} - Y_{3.} + Y_{4.}$$

$$\text{Interaction contrast} = +Y_{1.} - Y_{2.} - Y_{3.} + Y_{4.}$$

The SS for each factor are written as:

Tester: $$SS_T = [-Y_{1.} - Y_{2.} + Y_{3.} + Y_{4.}]^2 / (sr) \quad (6)$$

Board: $$SS_B = [-Y_{1.} + Y_{2.} - Y_{3.} + Y_{4.}]^2 / (sr) \quad (7)$$

Interaction (TXS): $$SS_{TxB} = [+ Y_{1.} - Y_{2.} - Y_{3.} + Y_{4.}]^2 / (sr) \quad (8)$$

Total: $$SS_{TOTAL} = \left(\sum_{h=1}^{s}\sum_{k=1}^{r} Y^2_{hk}\right) - Y^2_{..} / (sr) \quad (9)$$

Residual: $$SS_R = SS_{TOTAL} - (SS_T + SS_B + SS_{TxB}) \quad (10)$$

5.3 Derivation of expression for EMS and ANOVA table

Expressions for the EMS of each factor are also needed. This is found by substituting the equation for the linear statistical model into the SS equations and simplifying. In this case the EMS are as follow.

Tester: $$EMS_T = \sigma^2_R + r\sigma^2_{TxB} + br\sigma^2_T \quad (11)$$

Board: $$EMS_B = \sigma^2_R + r\sigma^2_{TxB} + tr\sigma^2_B \quad (12)$$

Interaction : $$EMS_{TXB} = \sigma^2_R + r\sigma^2_{TxB} \quad (13)$$

Residual: $$EMS_R = \sigma^2_R \quad (14)$$

These EMS are equated to the MS from the experimental data and solved to find the variance attributable to each factor in the experimental design.
The results of this analysis is summarised in an ANOVA table. The terms presented in this ANOVA table are as follows. The SS are the calculated sum of squares from the

experimental data for each factor under investigation. The DOF are the degrees of freedom associated with the experimental data for each factor. The MS is the mean square calculated using the SS and DOF. The EMS is estimated mean square for each factor derived from the theoretical model. For the design of experiment presented in this section the ANOVA table is shown in table 7 below.

Source	SS	DOF	MS	EMS
Tester	Eq. (6)	t – 1	$SS_T/(t-1)$	$\sigma^2_R + r\sigma^2_{TxB} + br\sigma^2_T$
Board	Eq. (7)	b – 1	$SS_B/(b-1)$	$\sigma^2_R + r\sigma^2_{TxB} + tr\sigma^2_B$
Interaction	Eq. (8)	(t – 1)(b – 1)	$SS_{TxB}/((t-1)(b-1))$	$\sigma^2_R + r\sigma^2_{TxB}$
Residual	Eq. (10)	tb(r – 1)	$SS_R/(tb(r-1))$	σ^2_R
Total	Eq. (9)	tbr – 1	Sum of above	

Table 7. Fixed Effects ANOVA Table

5.4 Output of ANOVA – complete estimate of robust test statistics

Equating the MS from the experimental data to the EMS from the model analysis, it is possible to solve for the variance estimate due to each source. From the ANOVA table the best estimate for σ^2_T, σ^2_B, σ^2_{TxB} and σ^2_R are derived as S^2_T, S^2_B, S^2_{TxB} and S^2_R respectively. The calculations on the ANOVA outputs to generate these estimates are listed in table 8.

Source	Variance Estimate
Tester	$S^2_T = \dfrac{MS_T - \sigma^2_R - r\sigma^2_{TxB}}{br}$
Board	$s^2_B = \dfrac{MS_B - \sigma^2_R - r\sigma^2_{TxB}}{tr}$
Interaction	$S^2_{TxB} = \dfrac{MS_{TxB} - \sigma^2_R}{r}$
Residual	$S^2_R = MS_R$
Total	Sum of above

Table 8. Fixed Effects Model Results Table

Note that because each setup is measured a number of times on each device, the residual contains the replicate or repeatability effect.

5.5 Example test data – experimental results

For the example dataset, there are two testers and two boards, hence t = b = 2. In addition during data collection there were 30 replicates done on each site, hence r = 30. Using these values and the raw data from the dataset, the ANOVA results are in tables 9 and 10 below.

Here the dominant source of variance is the test system variance, with $S^2_T = 0.403$. This has a P value < 0.01, indicating that this effect is highly significant. The variances from all other sources are negligible in comparison, with S^2_R, S^2_{TXB}, S^2_B variances of 0.015, 0.008, and 0.001 respectively.

Source	SS	DOF	MS	F	P
Tester	24.465	1	24.465	1631	<0.01
Board	0.303	1	0.303	20.2	0.58
Interaction	0.243	1	0.243	15.2	0.62
Residual	1.791	116	0.015		
Total	26.730	119	0.230		

Table 9. Example Data - ANOVA Table Results

Source	Variance Estimate
Tester	$S^2_T = 0.403$
Board	$S^2_B = 0.001$
Interaction	$S^2_{TxR} = 0.008$
Residual	$S^2_R = 0.015$
Total	$S^2_T + S^2_B + S^2_{TxR} + S^2_R = 0.427$

Table 10. Example Data - Calculation of Variances

6. Blocked experimental design for estimating multi-site test boards

For cost reduction, multisite test boards is employed allowing multiple parts to be tested in parallel. In analysing the effect of each test site, the variance of the part is confounded into the variance of the test site. In this instance the variability of the parts becomes a nuisance factor that will affect the response. Because this nuisance factor is known and can be controlled, a blocking technique is used to systematically eliminate the part effect from the site effects.

Take the example of a quad site tester in which 4 parts are tested in 4 independent sites in parallel. In this instance the variability of the parts needs to be removed from the overall experimental error. A design that will accomplish this involves testing each of 4 parts inserted in each of the 4 sites. The parts are systematically rotated across the sites during each experimental run. This is in effect a blocked experimental design. The experimental array for this example is shown in table 11, using parts labled A to D.

Run	Site1	Site2	Site3	Site4
1	A	B	C	D
2	B	C	D	A
3	C	D	A	B
4	D	A	B	C

Table 11. Example Array Blocked Experimental Design.

An example dataset from a quad site test board is shown in figure 7. This shows data from a temperature sensor product. Data were collected using 4 parts rotated across the 4 test sites as indicated in the array above.

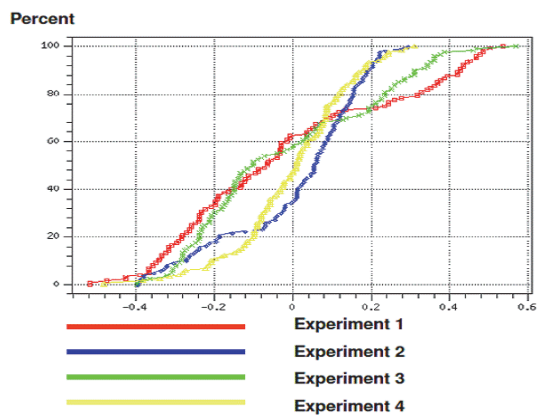

Fig. 7. Example data Blocked Experimental Design – Parts And Sites.

6.1 Blocked design statistical model

In this instance a suitable statistical model is given by equation 15 (Montgomery D.C, 1996):

$$Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + e_{ijk} \qquad \begin{array}{l} i = 1 \text{ to } p \\ j = 1 \text{ to } s \\ k = 1 \text{ to } r \end{array} \tag{15}$$

Where Y_{ijk} are the experimentally measured data points.
μ is the overall experimental mean.
τ_i is the effect of device 'i'.
β_j is the effect of site 'j'.
$(\tau\beta)_{ij}$ is the interaction effect between devices and sites.
k is the replicate of each experiment.
e_{ijk} is the random error term for each experimental measurement.

Here it is assumed that τ_i, β_j, $(\tau\beta)_{ij}$ and e_{ijk} are random independent variables, where $\{\tau_i\} \sim N(0, \sigma^2_P)$, $\{\beta_j\} \sim N(0, \sigma^2_S)$ and $\{e_{ijk}\} \sim N(0, \sigma^2_R)$.

As before, the analysis of the model is carried out in two stages. The first partitions the total SS into its constituent parts. The second uses the model as defined and derives expressions for the EMS. By equating the SS to the EMS the model estimates are calculated. Both the SS and the EMS are summarised in an ANOVA table.

6.2 Derivation of expression for SS

The generalised experimental array is redrawn in the more general form in table 12.

	Site 1	Site 2	Site 3	Site j	Part Total
Part 1	Y_{11k}	Y_{12k}	Y_{13k}	Y_{1jk}	$Y_{1..}$
Part 2	Y_{21k}	Y_{22k}	Y_{23k}	Y_{2jk}	$Y_{2..}$
Part 3	Y_{31k}	Y_{32k}	Y_{33k}	Y_{3jk}	$Y_{3..}$
Part i	Y_{i1k}	Y_{i2k}	Y_{i3k}	Y_{ijk}	$Y_{i..}$
Site Total	$Y_{.1.}$	$Y_{.2.}$	$Y_{.3.}$	$Y_{.j.}$	$Y_{...}$

Table 12. Generalised Array – Blocked Experimental Design.

The results of this data collection are represented by the generalised experimental result Y_{ijk}, where i= 1 to p is the total number of parts, j= 1 to s is the total number of sites, and k= 1 to r is the number of replicates performed on each experimental run.
Using the dot notation, the following terms are written:

Parts total: $Y_{i..} = \sum_{j=1}^{s}\sum_{k=1}^{r} Y_{ijk}$ is the sum of all replicates for each part.

Site total: $Y_{.j.} = \sum_{i=1}^{p}\sum_{k=1}^{r} Y_{ijk}$ is the sum of all replicates on a particular site.

Overall total: $Y_{...} = \sum_{i=1}^{p}\sum_{j=1}^{s}\sum_{k=1}^{r} Y_{ijk}$ is the overall sum of measurements.

The SS for each factor are written as:
Parts:

$$SS_P = \left(\sum_{i=1}^{p} Y_{i..}^2\right) / (sr) - Y_{...}^2 / (psr) \tag{16}$$

Sites:

$$SS_S = \left(\sum_{j=1}^{S} Y_{.j.}^2\right) / (pr) - Y_{...}^2 / (psr) \tag{17}$$

Interaction:

$$SS_{PXS} = \left(\sum_{i=1}^{p}\sum_{j=1}^{s} Y_{ij.}^2\right) / (r) - \left(\sum_{j=1}^{s} Y_{.j.}^2\right) / (pr) - \left(\sum_{i=1}^{p} Y_{i..}^2\right) / (sr) + Y_{...}^2 / (psr) \tag{18}$$

Total:

$$SS_{TOTAL} = \sum_{i=1}^{p}\sum_{j=1}^{s}\sum_{k=1}^{r} Y_{ijk}^2 - \frac{Y^2_{...}}{psr} \tag{19}$$

Residual:

$$SS_R = SS_{TOTAL} - (SS_S + SS_P + SS_{PxS}). \tag{20}$$

6.3 Derivation of expression for EMS and ANOVA table

Expressions for the EMS for each factor are also needed. This is found by substituting the equation for the linear statistical model into the SS equations and simplifying. In this case the EMS are as follows.
Parts:

$$EMS_P = \sigma^2_R + r\sigma^2_{PxS} + sr\sigma^2_P \tag{21}$$

Sites:

$$EMS_S = \sigma^2_R + r\sigma^2_{PxS} + pr\sigma^2_S \tag{22}$$

Interaction:

$$EMS_{PXS} = \sigma^2_R + r\sigma^2_{PxS} \quad (23)$$

Residual:

$$EMS_R = \sigma^2_R \quad (24)$$

These are equated to the MS from the experimental data. These results for the blocked experimental design are summarised in the ANOVA table shown in table 13.

Source	SS	DOF	MS	EMS
Parts	Eq. (16)	p – 1	$SS_P/(p-1)$	$\sigma^2_R + r\sigma^2_{PxS} + sr\sigma^2_P$
Sites	Eq. (17)	s – 1	$SS_S/(s-1)$	$\sigma^2_R + r\sigma^2_{PxS} + pr\sigma^2_S$
Interaction	Eq. (18)	(s – 1)(p – 1)	$SS_{PxS}/((s-1)(p-1))$	$\sigma^2_R + r\sigma^2_{PxS}$
Residual	Eq. (20)	sp(r – 1)	$SS_R/(sp(r-1))$	σ^2_R
Total	Eq. (19)	spr – 1		

Table 13. ANOVA Table - Blocked Design.

6.4 Output of ANOVA – complete estimate of robust test statistics

Equating the MS from the experimental data to the EMS from the model analysis, it is possible to solve for the variance due to each source. From the ANOVA table the best estimate for σ^2_P, σ^2_S, σ^2_{PxS} and σ^2_R are derived as S^2_P, S^2_S, S^2_{PxS} and S^2_R respectively. The calculations on the ANOVA outputs to generate these estimates are listed in table 14.

Source	Variance Estimate
Parts	$S_P^2 = \dfrac{MS_P - \sigma_R^2 - r\sigma_{PxS}^2}{sr}$
Sites	$S_S^2 = \dfrac{MS_S - \sigma_R^2 - r\sigma_{PxS}^2}{pr}$
Interaction	$S_{PXS}^2 = \dfrac{MS_{PxS} - \sigma_R^2}{r}$
Residual	$S_R^2 = MS_R$

Table 14. Results Table – Blocked Design.

Note that because each setup is measured a number of times on each part, the residual contains the replicate effect.

6.5 Example test data – experimental results

For the example from a quad site test board, there are 4 sites and 4 parts rotated across these sites, hence s = p = 4. In addition during data collection there were 30 replicates done on each site, hence r = 30. Using these values and the raw data from the dataset, the results of the ANOVA are shown in tables 15 and 16.

Source	SS	DOF	MS	F	p
Parts	0.063	3	0.021	2.6	0.05
Sites	8.800	3	2.933	366.6	<0.01
Interaction	9.414	9	1.04	130.0	<0.01
Residual	4.057	464	0.008		
Total	22.335	479			

Table 15. Example Data - ANOVA Table.

Source	*Variance Estimate*
Parts	$S^2_P = 0$
Sites	$S^2_S = 0.021$,
Interaction	$S^2_{PxS} = 0.035$
Residual	$S^2_R = 0.009$

Table 16. Example Data Calculation of Variance.

Here the dominant sources of variance are the test site variance, with $S^2_S = 0.021$, and the interaction variance estimate $S^2_{PxS} = 0.015$. Both these effects are highly significant with P values < 0.01. The variances estimates from other sources are negligible in comparison, with S^2_R, S^2_P of 0.009, and 0 respectively.
Figure 8 shows a replot of the original data with results grouped by site. It is clearly seen that site 4 has an offset difference of about 0.2 compared to the other sites. It is primarily this offset that is responsible for the site variance reported in the ANOVA.

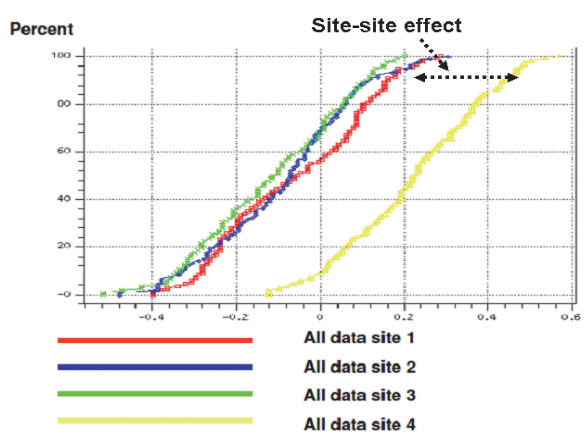

Fig. 8. Temperature Sensor Offset – Replotted by Site.

7. Complete experimental design for MSA on quad site test system

For a complete MSA on a quad site test system both the fixed effects and blocked experimental design are brought together. This enables optimisation within the data collection stage. The complete experimental design is shown in table 17. Here four parts are used – these are labelled A to D. These are rotated across the test sites in runs 1 through to 4. The data from these first 4 rows is analysed as a blocked experimental design to estimate the site-to-site and part-to-part effects. In runs 5 to 7 a second test board and test system are used to test the parts. The data from row 1 and rows 5 through to 7 is analysed as a fixed experimental design to estimate the tester-to-tester and board-to-board effects.

Run	Tester	Board	Site 1	Site 2	Site 3	Site 4
1	1	1	A	B	C	D
2	1	1	B	C	D	A
3	1	1	C	D	A	B
4	1	1	D	A	B	C
5	1	2	A	B	C	D
6	2	1	A	B	C	D
7	2	2	A	B	C	D

Table 17. Complete experimental design for quad site example

7.1 Complete experimental design for MSA on quad site test system

Example results obtained using this design of experiment are shown in table 18 and table 19 below. Table 18 presents the blocked design results, while table 19 presents the fixed design results. Note that 30 repeats were done for each experimental run.

Source	SS	DOF	MS	F	P
Tester	0.01199	1	0.01199	1.38	0.24
Board	0.01337	1	0.01337	1.54	0.21
Interaction	2.08E-05	116	1.79E-07	2.07E-05	1
Repeatability	1.031162	119	0.00866		

Table 18. Fixed Factor Design Experimental Results.

Source	SS	DOF	MS	F	P
Parts	4.1325	3	1.3775	152.30	<0.01
Sites	9.0550	3	3.0183	333.72	<0.01
Interaction	0.1653	9	0.0183	2.030	0.04
Repeatability	4.1966	464	0.0090		

Table 19. Blocked Design Experimental Results.

From the ANOVA tables it is seen that both the sites and parts are statistically significant with P values < 0.01, while the tester and board effects are not showing significance. The variance estimates from both the fixed and blocked design are summarised in Table 20. The total variance is obtained by summing the components of variance for both the fixed effects design and the blocked design. The repeatability is taken as the largest value obtained from either designs.

Source	Variance Estimate
Fixed effects model results	
Tester = S^2_T	5.18E-5
Board = S^2_B	7.47E-7
TXB = S^2_{TXB}	0.0000
Repeatability = S^2_R	0.0086
Blocked design results	
Parts = S^2_P	0.0226
Sites = S^2_S	0.0499
PXS = S^2_T	0.0031
Repeatability = S^2_R	0.0090
Test Gage R&R	0.0616
Total Variance (TV) = sum all components	0.0846

Table 20. Calculation of Components of Variances.

Using the equations (3) and (4) from section 2, the overall MSA metrics including gage R&R results from these ANOVA are presented in table 21 .

Component	Variance Estimate	Standard Deviation	% Total Variance	%Contribution to variance
Components R&R :				
Tester	5.18E-05	0.0071	2.4	0.06
Board	7.47E-07	0.0008	0.2	0.00
TesterXboard	0	0	0.0	0.00
Site	0.0499	0.2233	76.8	58.9
SiteXPart		0	0.0	0.00
Repeatability	0.0090	0.0948	32.6	10.6
Overall Gage R&R	0.0616	0.2481	85.3	72.8
Part	0.0226	0.1503	51.6	26.7
Total Variation	0.0846	0.2908	100.0	100

Table 21. Calculation of MSA metrics from experimental dataset.

8. Conclusions

Traditional measurement systems analysis methodologies are aimed at obtaining estimates of test error components. These are identified as equipment repeatability and reproducibility effects arising from independent appraisers. Gage R&R metrics can be generated using the data gathered. The most commonly used metrics are the percentage of total variation, and the percentage contribution to overall variance of each component.

With increasing complexity in semiconductor product test, the measurement equipment is generally automated, and test boards are employed that are capable of testing multiple parts in parallel. This introduces additional variance components not accounted for in these traditional methodologies. These components are identified as the tester, board and test sites effects. Updated ANOVA methodologies capable of accounting for this situation are required to enable MSA.

The purpose of this chapter is to describe the appropriate experimental designs appropriate for use in MSA in this situation. As the testers and boards come from a fixed population, a suitable design of experiments for tester-to-tester and board-to-board effects is a fixed effects experimental model. To evaluate site-to-site effects, the variation of the parts must be blocked from the variation of the sites. A suitable design of experiments for site-to-site and part-to-part effects is a blocked experimental design. Within this the parts are rotated across the test sites to allow the independent variation of both the parts and the sites.
The derivations of the ANOVA tables for both designs are presented. The data collection operation is optimised by merging the two designs. Experimental data gathered on a product within a manufacturing environment is analysed using these designs, and the results discussed. These designs enable the performance of MSA within the semiconductor environment in a streamlined fashion.

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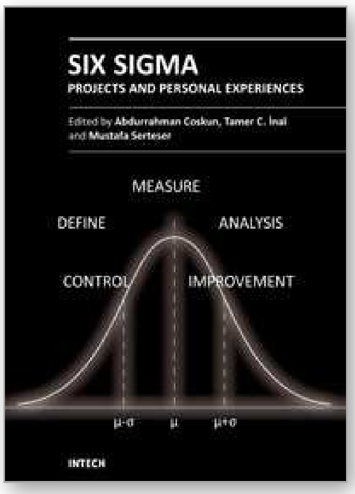

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In the new millennium the increasing expectation of customers and products complexity has forced companies to find new solutions and better alternatives to improve the quality of their products. Lean and Six Sigma methodology provides the best solutions to many problems and can be used as an accelerator in industry, business and even health care sectors. Due to its flexible nature, the Lean and Six Sigma methodology was rapidly adopted by many top and even small companies. This book provides the necessary guidance for selecting, performing and evaluating various procedures of Lean and Six Sigma. In the book you will find personal experiences in the field of Lean and Six Sigma projects in business, industry and health sectors.

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InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821