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# Investigation of Different Control Strategies for the Waste Water Treatment Plant

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## 1. Introduction

Wastewater treatment is just one component in the urban water cycle; however, it is an important component since it ensures that the environmental impact of human usage of water is significantly reduced. It consists of several processes: biological, chemical and physical processes. Wastewater treatment aims to reduce: nitrogen, phosphorous, organic matter and suspended solids. To reduce the amount of these substances, wastewater treatment plants (WWTP) consisting of (in general) four treatment steps, have been designed. The steps are: a primarily mechanical pre-treatment step, a biological treatment step, a chemical treatment step and a sludge treatment step. See Figure 1.

The quality of water is proportional to the quality of life and therefore in modern world the sustainable development concept is to save water. The goal of a wastewater treatment plant is to eliminate pollutant agents from the wastewater by means of physical and (bio) chemical processes. Modern wastewater treatment plants use biological nitrogen removal, which relies on nitrifying and denitrifying bacteria in order to remove the nitrogen from the wastewater.

Biological wastewater treatment plants are considered complex nonlinear systems due to large variations in their flow rates and feed concentrations. In addition, the microorganisms that are involved in the process and their adaptive behaviour coupled with nonlinear dynamics of the system make the WWTP to be really challenging from the modelling and control point of view [Clarke D.W ], [Dutka.A& Ordys], [Grimblea & M. J], [H.Elbahja & P.Vega],[ H.Elbahja & O.Bakka] and [O.Bakka & H.Elbahja].

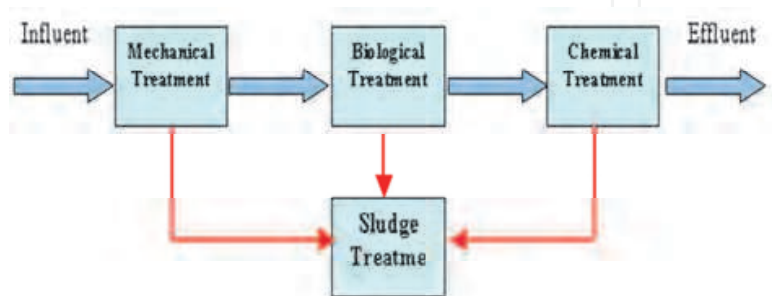


Fig. 1. Layout of a typical wastewater treatment plant

The paper is organized as follows. The modelling of the continuous wastewater treatment is detailed in Section 2. Section 3 is dedicated to the non linear predictive control technique. Observer based Regulator Problem for a WWTP with Constraints on the Control in Section 4. In Section 5 the efficiency of the two controls schemes are illustrated via simulation studies. Finally Section 6 ends the paper.

## 2. Process modelling

A typical, conventional activated sludge plant for the removal of carbonaceous and nitrogen materials consists of an anoxic basin followed by an aerated one, and a settler (figure 2). In the presence of dissolved oxygen, wastewater that is mixed with the returned activated sludge is biodegraded in the reactor. Treated effluent is separated from the sludge is wasted while a large fraction is returned to anoxic reactor to maintain the appropriate substrate-to-biomass ratio. In this study we consider six basic components present in the wastewater: autotrophic bacteria  $X_A$ , heterotrophic bacteria  $X_H$ , readily biodegradable carbonaceous substrates  $S_S$ , nitrogen substrates  $S_{NH}$ ,  $S_{NO}$  and dissolved oxygen  $S_O$ .

In the formulation of the model the following assumptions are considered: the physical properties of fluid are constant; there is no concentration gradient across the vessel; substrates and dissolved oxygen are considered as a rate-limiting with a bi-substrate Monod-type Kinetic; no bio-reaction takes place in the settler and the settler is perfect. Based on the above description and assumptions, we can formulate the full set of ordinary differential equations (mass balance equations), making up the IAWQ AS Model NO.1 [Henze].

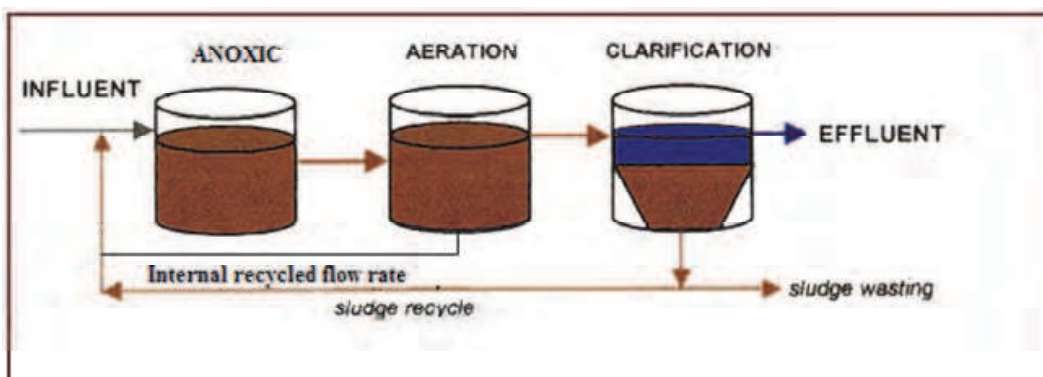


Fig. 2. Pre-denitrification plant design

### 2.1 Modeling of the aerated basin

$$\dot{X}_{A,nit}(t) = (1 + r_1 + r_2) \cdot D_{nit}(X_{A,denit} - X_{A,nit}) + (\mu_{A,nit} - b_A)X_{A,nit} \quad (1)$$

$$\dot{X}_{H,nit}(t) = (1 + r_1 + r_2) \cdot D_{nit}(X_{H,denit} - X_{H,nit}) + (\mu_{H,nit} - b_H)X_{H,nit} \quad (2)$$

$$\dot{S}_{S,nit}(t) = (1 + r_1 + r_2) \cdot D_{nit}(S_{S,denit} - S_{S,nit}) + (\mu_{H,nit} - \mu_{Ha,nit}) \frac{X_{H,nit}}{Y_H} \quad (3)$$

$$\begin{aligned} \dot{S}_{NH,nit}(t) = & (1 + r_1 + r_2)D_{denit}(S_{NH,denit} - S_{NH,nit})(i_{xb} 1/Y_A)\mu_{A,nit}X_{A,nit} \\ & - (\mu_{H,nit} - \mu_{Ha,nit})i_{xb}X_{H,nit} \end{aligned} \quad (4)$$

$$\dot{S}_{NO,nit}(t) = (1 + r_1 + r_2)D_{nit}(S_{NO,denit}S_{NO,nit}) + \frac{\mu_{A,nit}X_{A,nit}}{Y_A} - \frac{1-Y_H}{2.86Y_H}\mu_{Ha,nit}X_{H,nit} \quad (5)$$

$$\begin{aligned} \dot{S}_{O,nit}(t) = & (1 + r_1 + r_2)D_{nit}(S_{O,denit}S_{O,nit}) + a_0Q_{air}(C_S - S_{O,nit}) \\ & - \frac{(4.57-Y_A)\mu_{A,nit}X_{A,nit}}{Y_A} - \frac{1-Y_H}{Y_H}\mu_{Ha,nit}X_{H,nit} \end{aligned} \quad (6)$$

Where:

$$\begin{aligned} \mu_{A,nit} &= \mu_{max,A} \cdot \frac{S_{NH,nit}}{(K_{NH,A} + S_{NH,nit})} \cdot \frac{S_{O,nit}}{(K_{O,A} + S_{O,nit})} \\ \mu_{H,nit} &= \mu_{max,H} \cdot \frac{S_{S,nit}}{(K_S + S_{S,nit})} \cdot \frac{S_{NH,nit}}{(K_{NH,H} + S_{NH,nit})} \cdot \frac{S_{O,nit}}{(K_{O,H} + S_{O,nit})} \\ \mu_{Ha,nit} &= \mu_{max,H} \cdot \frac{S_{S,nit}}{(K_S + S_{S,nit})} \cdot \frac{S_{NH,nit}}{(K_{NH,H} + S_{NH,nit})} \cdot \frac{K_{O,H}}{(K_{O,H} + S_{O,nit})} \cdot \frac{S_{NO,nit}}{(K_{NO} + S_{NO,nit})} \cdot \eta_{NO} \end{aligned}$$

$\mu_{A,nit}$  and  $\mu_{H,nit}$  are the growth rates of autotrophy and heterotrophy in aerobic conditions and  $\mu_{Ha,nit}$  is the growth rate of heterotrophy in anoxic conditions.

## 2.2 Modeling of the anoxic basin

$$\begin{aligned} \dot{X}_{A,denit}(t) = & D_{denit}(X_{A,in} + r_1X_{A,nit}) - (1 + r_1 + r_2) \cdot D_{denit}X_{A,denit} \\ & + \alpha \cdot r_2 D_{denit}X_{rec} + (\mu_{A,denit} - b_A)X_{A,denit} \end{aligned} \quad (7)$$

$$\begin{aligned} \dot{X}_{H,denit}(t) = & D_{denit}(X_{H,in} + r_1X_{H,nit}) - (1 + r_1 + r_2) \cdot D_{denit}X_{H,denit} \\ & + (1 - \alpha)r_2 D_{denit}X_{rec} + (\mu_{H,denit} - b_H)X_{H,denit} \end{aligned} \quad (8)$$

$$\begin{aligned} \dot{S}_{S,denit}(t) = & -(\mu_{H,denit} - \mu_{Ha,denit}) \frac{X_{H,denit}}{Y_H} - (1 + r_1 + r_2) \cdot D_{denit}S_{S,denit} \\ & + D_{denit}(S_{S,in} - r_1S_{S,nit}) \end{aligned} \quad (9)$$

$$\begin{aligned} \dot{S}_{NH,denit} = & D_{denit}(S_{NH,in} - r_1S_{NH,nit}) - (1 + r_1 + r_2) \cdot D_{denit}S_{NH,denit} \\ & - (i_{xb} + 1/Y_A)\mu_{A,denit}X_{A,denit} - (\mu_{H,denit} - \mu_{Ha,denit})i_{xb}X_{H,denit} \end{aligned} \quad (10)$$

$$\begin{aligned} \dot{S}_{NO,denit}(t) = & D_{denit}(S_{NO,in} - r_1S_{NO,nit}) - (1 + r_1 + r_2) \cdot D_{denit}S_{NO,denit} \\ & + \frac{\mu_{A,denit}X_{A,denit}}{Y_A} - \frac{1-Y_H}{2.86Y_H} \end{aligned} \quad (11)$$

Where:

$$\mu_{A,denit} = \mu_{max,A} \cdot \frac{S_{NH,denit}}{(K_{NH,A} + S_{NH,denit})}$$

$$\mu_{H,denit} = \mu_{max,H} \cdot \frac{S_{S,denit}}{(K_S + S_{S,denit})} \cdot \frac{S_{NH,denit}}{(K_{NH,H} + S_{NH,denit})}$$

$$\mu_{Ha,denit} = \mu_{max,H} \cdot \frac{S_{S,denit}}{(K_S + S_{S,denit})} \cdot \frac{S_{NH,denit}}{(K_{NH,H} + S_{NH,denit})} \cdot \frac{S_{NO,denit}}{(K_{NO} + S_{NO,denit})} \cdot \eta_{NO}$$

### 2.3 Modeling of the settler

$$\dot{X}_{rec} = (1 + r_2)D_{dec}(X_{A,nit} + X_{H,nit}) - (r_2 + \omega)D_{dec}X_{rec} \quad (12)$$

$r_1, r_2$  and  $\omega$  represent respectively, the ratio of the internal recycled flow  $Q_{r1}$  to influent flow  $Q_{in}$ , the ratio of the recycled flow  $Q_{r2}$  to the influent flow,  $C_S$  is the maximum dissolved oxygen concentration.  $D_{nit}$ ,  $D_{denit}$  and  $D_{dec}$  are the dilution rates in respectively, nitrification, denitrification basins and settler tank;  $X_{rec}$  is the concentration of the recycled biomass. The other variables and parameters of the system equations (1)-(13) are also defined.

## 3. Control of global nitrogen and dissolved oxygen concentrations

The implementation of efficient modern control strategies in bioprocesses [Hajji, S., Farza, Hammouri, H., & Farza, Shim, H.], highly depends on the availability of on-line information about the key biological process components like biomass and substrate. But due to lack or prohibitive cost, in many instances, of on-line sensors for these components and due to expense and duration (several days or hours) of laboratory analyses, there is a need to develop and implement algorithms which are capable of reconstructing the time evolution of the unmeasured state variables on the base of the available on-line data. However, because of the nonlinear feature of the biological processes dynamics and the usually large uncertainty of some process parameters, mainly the process kinetics, the implementation of extended versions of classical observers proves to be difficult in practical applications, and the design of new methods is undoubtedly an important research matter nowadays. In that context, Extended Kalman Filter (EKF) is presented in this work.

### 3.1 Method presentation of the Extended Kalman Filter

The aim of the estimation procedure is to compute estimated values of the unavailable state variables of the process [ $X_{A,nit}(t)$ ,  $X_{A,denit}(t)$ ,  $X_{H,nit}(t)$ ,  $X_{H,denit}(t)$ ,  $S_{S,nit}(t)$ ,  $S_{S,denit}(t)$ ,  $X_{rec}(t)$ ] and the specific growth rate  $\mu(t)$  using the concentrations [ $S_{NH,nit}(t)$ ,  $S_{NH,denit}(t)$ ,  $S_{NO,nit}(t)$ ,  $S_{NO,denit}(t)$ ,  $S_{O,nit}(t)$ ] as measurable variables. The EKF estimator uses a non-linear mathematical model of the process and a number of measures for estimating the states and parameters not measurable. The estimation is realised in three stages: prediction, observation and registration.

The EKF estimator uses a non-linear mathematical model of the process and a number of measures for estimating the states and parameters not measurable. The estimation is realised in three stages: prediction, observation and registration.

Let a dynamic non-linear system be characterised by a model in the state space form as following:

$$\frac{dX(t)}{dt} = f(X(t), u(t), t) + v(t) \quad (13)$$

Where:

$X(t)$ : Represents the state vector of dimension  $n$ .

$f(\cdot)$ : Non-linear function of  $X(t)$  and  $u(t)$ .

$u(t)$ : Represents the input vector of dimension  $m$ .

$v(t)$ : Vector of noise on the state equation of dimension  $n$ , assumed Gaussian white noise, medium null and covariance matrix known  $q(t) = \text{Var}(v(t))$ .

The state of the system is observed by  $m$  discrete measures related to the state  $X(t)$  by the following equation of observation:

$$Z(t_k) = h(X(t_k), t_k) + \omega(t_k) \quad (14)$$

Where:

$Z(t_k)$ : Represents the observation vector of dimension  $n$ .

$h(\cdot)$ : Observation matrix of dimension  $m \times n$ .

$t_k$ : Observation instant.

$\omega(t_k)$ : Vector of noise on the measure, of dimension  $m$ , independent of  $v(t)$  assumed Gaussian white noise, medium null and covariance matrix known  $r(t) = \text{Var}(\omega(t))$ .

- The EKF algorithm corresponding to the continuous process in discrete observation, where the measurements are acquired at regular intervals, is given by [17]:
- Initialisation filter  $t = t_0$ :

$$X(t_0) = E(X(t_0)) \quad (15)$$

$$L(t_0) = \text{Var}(X(t_0)) \quad (16)$$

- Between two instant of observation:
- The estimated state  $\hat{X}(t)$  and its associated covariance matrix  $L(t)$  are integrated by the equations:

$$\frac{d\hat{X}(t)}{dt} = f(\hat{X}(t), u(t), t) \quad (17)$$

$$\frac{dL(t)}{dt} = F(t)L(t) + L(t)F^T(t) + q(t) \quad (18)$$

$$F(t) = \frac{\partial f(\hat{X}(t), u(t), t)}{\partial \hat{X}} \quad (19)$$

Then we have, before the observation at  $t = t_{k-}$ , an estimated of  $\hat{X}(t_{k-})$  and its covariance matrix  $L(t_{k-})$ .

- Updating the gain

$$K(t_k) = L(t_{k-})H^T(\hat{X}(t_{k-}), t_k) \left[ H(\hat{X}(t_{k-}), t_k)L(t_{k-})H^T(\hat{X}(t_{k-}), t_k) + r(t_k) \right]^{-1} \quad (20)$$

- Update of the estimated state

$$\hat{X}(t_k) = \hat{X}(t_{k-}) + K(t_k)[Z(t_k) - h(\hat{X}(t_{k-}), t_k)] \quad (21)$$

- Update of the covariance matrix

$$L(t_{k+}) = L(t_{k-}) - K(t_k)H(\hat{X}(t_{k-}), t_k)L(t_{k-}) \quad (22)$$

$$H(\hat{X}(t_{k-})) = \frac{\partial h(\hat{X}(t_{k-}), t_k)}{\partial \hat{X}(t_{k-})} \quad (23)$$

The estimator EKF is an iterative algorithm. The final results of each step of calculation are used as initial conditions for the next step.

### 3.2 The non linear GPC

The control objective is to make the effluent organics concentration below certain regulatory limits. A multivariable non linear generalized predictive control strategy based on  $NH_4$ ,  $NO_3$  and  $O_2$  measurements is developed, enabling the control of the nitrogen and the dissolved oxygen concentrations, by acting on the internal flow and aeration flow rates,  $Q_{r1}$  and  $Q_{air}$ , at desired levels. The dynamics of the WWTP are represented by the equations below. The system is discretized using Euler integration method and re-arranged into the state dependent coefficient form the state-space model [10, 11].

State and control dependent matrices in general may be formulated in an infinite number of ways. Finally we can write the discrete model in the following matrix form:

$$x_{n+1} = \tilde{A}(x_n)x_n + \tilde{B}(x_n)u_n \quad (24)$$

$$y_n = \tilde{C}(x_n)x_n \quad (25)$$

The state dependant form of the model, in state space format is substituted to the traditional GPC format, allowing for inherent integral action within the model, including the control increment as system input to the state space model.

Thus, an extra system state is included.

$$\chi_{n+1} = A(\chi_n)\chi_n + B(\chi_n)\Delta u_n \quad (26)$$

$$y_n = C(\chi_n)\chi_n \quad (27)$$

Where:

$$A(\chi_n) = \begin{bmatrix} A(\chi_n) & B(\chi_n) \\ 0 & I \end{bmatrix}, B(\chi_n) = \begin{bmatrix} B(\chi_n) \\ I \end{bmatrix},$$

$$C(\chi_n) = [C(\chi_n) \ 0], \chi_n = \begin{bmatrix} x_n \\ u_{n-1} \end{bmatrix}, \Delta u_n = u_n - u_{n-1}$$

To derive the non-linear predictive control algorithm the assumption on the future trajectory of the system must be made. For a moment assume, that the future trajectory for the state of the system is known. State-space model (26), (27) matrices may be re-calculated for the future using the future trajectory. The resulting state-space model may be seen as a time-varying linear model and for this model the controller is designed. Therefore the following notation for state dependent matrices  $A_n = A(\chi_n)$   $B_n = B(\chi_n)$   $C_n = C(\chi_n)$  is used in the remaining part of the paper. Now, the future trajectory for the system has to be



determined. In the classic predictive control strategy the vector of current and future controls is calculated. For the receding horizon control technique only the first control is used for the plant inputs manipulation, remaining part is not.

But this part may be employed in the next iteration of the algorithm to predict the future trajectory.

The cost function of the GPC controller here is defined as:

$$J_n = \sum_{i=1}^{Ne} \{ (sp_{n+i} - y_{n+i})^T \Lambda_E^i (sp_{n+i} - y_{n+i}) \} + \sum_{i=1}^{Nu} \{ \Delta u_{n+i-1}^T \Lambda_u^i \Delta u_{n+i-1} \} \quad (28)$$

Where  $sp_n$  is a vector of size  $n_y$  of set point at time  $n$ ,  $\Lambda_E^i, i = 1 \dots Ne$  and  $\Lambda_u^j, j = 1 \dots Nu$  are weighting matrices (symmetric) and  $Ne, Nu$  are positive integer numbers greater or equal one. Next the following vectors containing current and future values of the control  $u_n$ , and future values of state  $x_n$ , and output  $y_n$  are introduced:

$$\begin{aligned} X_{n+1,Ne} &= [\chi_{n+1}^T, \dots, \chi_{n+Ne}^T]^T, \\ \Delta U_{n,Nu} &= [\Delta u_n^T, \dots, \Delta u_{n+Nu-1}^T]^T \\ Y_{n+1,Ne} &= [y_{n+1}^T, \dots, y_{n+Ne}^T]^T, \\ R_{n+1,Ne} &= [sp_{n+1}^T, \dots, sp_{n+Ne}^T]^T \end{aligned} \quad (29)$$

The cost function (28) with notation (29) may be written in the vector form:

$$J_n = (R_{n+1,Ne} - Y_{n+1,Ne})^T \Lambda_E (R_{n+1,Ne} - Y_{n+1,Ne}) + \Delta U_{n,Nu}^T \Lambda_u \Delta U_{n,Nu} \quad (30)$$

With:

$$\Lambda_E = \text{diag}(\Lambda_E^1, \Lambda_E^2, \dots, \Lambda_E^{Ne}), \Lambda_u = \text{diag}(\Lambda_u^1, \Lambda_u^2, \dots, \Lambda_u^{Nu})$$

It is possible now to determine the future state prediction. For  $j = 1, \dots, Ne$  the future state predictions may be obtained from:

$$\begin{aligned} \chi_{n+j} &= [A_{n+j-1} A_{n+j-2} \dots A_n] \chi_n + [A_{n+j-1} A_{n+j-2} \dots A_{n+1}] \cdot \\ & B_n \Delta u_n + [A_{n+j-1} A_{n+j-2} \dots A_{n+2}] B_{n+1} \Delta u_{n+1} + \dots \\ & + [A_{n+j-1} A_{n+j-2} \dots A_{n+Nu}] B_{n-1+\min(j,Nu)} \Delta u_{n-1+\min(j,Nu)} \end{aligned} \quad (31)$$

Note that to obtain the state prediction at time instance  $n+j$  the knowledge of matrix predictions  $A_n \dots A_{n+i-1}$  and  $B_n \dots B_{n-l+\min(j,Nu)}$  is required. The control increments after the control horizon are assumed to be zero.

Next introduce the following notation:

$$\left[ \prod_{k=1}^m A_{n+k} \right] = \begin{cases} A_{n+m} A_{n+m-1} \dots A_{n+1} & \text{if } l \leq m \\ I & \text{if } l > m \end{cases}$$

Then (31) may be represented as:

$$\chi_{n+j} = \left[ \prod_{k=0}^{j-1} A_{n+k} \right] \chi_n + \left[ \prod_{k=1}^{j-1} A_{n+k} \right] B_n \Delta u_n + \left[ \prod_{k=2}^{j-1} A_{n+k} \right] B_{n+1} \Delta u_{n+1}$$



$$+ \dots + \left[ \prod_{k=Nu}^{j-1} A_{n+k} \right] B_{n-1+\min(j,Nu)} \Delta u_{n-1+\min(j,Nu)} \quad (32)$$

From (29) and (32) the following equation for the future state predictions vector  $X_{n+1,N}$  is obtained:

$$X_{n,Ne} = \Omega_{n,Ne} A_n \chi_n + \Psi_{n,Ne} \Delta u_{n,Nu} \quad (33)$$

Where

$$\Omega_{n,Ne} = \left[ \left[ \prod_{k=1}^0 A_{n+k} \right]^T \left[ \prod_{k=1}^1 A_{n+k} \right]^T \dots \left[ \prod_{k=1}^{Ne-1} A_{n+k} \right]^T \right]$$

$$\Psi_{n,Ne} = \begin{bmatrix} \left[ \prod_{k=1}^0 A_{n+k} \right] B_n & 0 \\ \left[ \prod_{k=1}^1 A_{n+k} \right] B_n & \left[ \prod_{k=2}^0 A_{n+k} \right] B_{n+1} \\ \vdots & \vdots \\ \left[ \prod_{k=1}^{Ne-1} A_{n+k} \right] B_n & \left[ \prod_{k=2}^{Ne-1} A_{n+k} \right] B_{n+1} \\ \vdots & 0 \\ \vdots & \vdots \\ \vdots & \left[ \prod_{k=Nu}^{Ne-1} A_{n+k} \right] B_{n+Nu-1} \end{bmatrix}$$

From the output equation (27) it is clear that

$$y_{n+j} = C_{n+j} \chi_{n+j} \quad (34)$$

Combining (29) and (34) the following relationship between vectors  $X_{n+1,N}$  and  $Y_{n+1,N}$  is obtained:

$$Y_{n+1,Ne} = \Theta_{n,Ne} X_{n+1,Ne} \quad (35)$$

Where:

$$\Theta_{n,Ne} = \text{diag}(C_{n+1}, C_{n+2}, \dots, C_{n+Ne})$$

Finally substituting in (35)  $X_{n+1,N}$  by (33) the following equation for output prediction is obtained:

$$Y_{n+1,Ne} = \Phi_{n,Ne} A_n \chi_n + S_{n,Ne} \Delta U_{n,Nu} \quad (36)$$

Where:

$$\Phi_{n,Ne} = \Theta_{n,Ne} \Omega_{n,Ne} \quad S_{n,Ne} = \Theta_{n,Ne} \Psi_{n,Ne}$$

Substituting  $Y_{n+1,N}$  in the cost function (30) by the equation (36) and performing the static optimization the control minimizing the given cost function is finally derived:

$$\Delta U_{n,Nu} = (S_{n,Ne}^T \Lambda_E S_{n,Ne} + \Lambda_u)^{-1} S_{n,Ne} \Lambda_E (R_{n+1,Ne} - \Phi_{n,Ne} A_n \chi_n) \quad (37)$$

## 4. Observer based regulator problem with constraints on the control

### 4.1 Linearization

Through linearization, the model equations are written in the standard form of state equations, as follows:

$$\begin{cases} \dot{x} = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (38)$$

For the model ASM1 simplified through linearization, the state, input and output vectors are given by the equation (14)-(16):

$$x(t) = [X_{A,nit}(t) \ X_{H,nit}(t) \ S_{S,nit}(t) \ S_{NH,nit}(t) \ S_{NO,nit}(t) \ S_{O,nit}(t) \ X_{A,denit}(t) \ X_{H,denit}(t) \ S_{S,denit}(t) \ S_{NH,denit}(t) \ S_{NO,denit}(t) \ X_{rec}(t)]^T \quad (39)$$

$$y(t) = [S_{NH,nit}(t) \ S_{NO,nit}(t) \ S_{O,nit}(t)]^T \quad (40)$$

$$u(t) = [Q_{r1} \ Q_{r2} \ Q_{air}]^T \quad (41)$$

We present the constraint on the control as follows:

$$\begin{cases} -Q_{r1} \leq Q_{r1} \leq 4Q_{r1} \\ -Q_{r2} \leq Q_{r2} \leq Q_{r2} \\ -100 \leq Q_{air} \leq 260 \end{cases}$$

For the steady-state functioning point:

$$x(t) = [69.6 \ 623 \ 13.5 \ 3.2 \ 10.4 \ 2.4 \ 68.9 \ 624.6 \ 20.9 \ 8.9 \ 5.3 \ 1356.8]^T \quad (42)$$

### 4.2 Decomposition

Any representation in the state space can be transformed into the equivalent form by the transformation  $Z = T_0x$  [10]:

$$\begin{cases} \dot{Z} = \bar{A}Z + \bar{B}u \\ y(t) = \bar{C}Z \end{cases} \quad (43)$$

With:

$$\bar{A} = \begin{pmatrix} A_{n0} & A_{12} \\ 0 & A_0 \end{pmatrix}; \bar{B} = \begin{pmatrix} B_{n0} \\ B_0 \end{pmatrix}; \bar{C} = (0 \ C_0); Z = \begin{pmatrix} Z_{n0} \\ Z_0 \end{pmatrix}$$

Where  $(A_0 \ C_0)$  is observable but in our case the pair  $(A_0 \ B_0)$  is controllable.

So we obtain the following system of equations:

$$\begin{cases} \dot{Z}_{n0} = A_{n0}Z_{n0} + A_{12}Z_0 + B_{n0} \\ \dot{Z}_0 = A_0Z_0 + B_0u \\ y = C_0Z_0 \end{cases} \quad (44)$$

### 4.3 Luenberger observer

An observer is a mathematical structure that combines sensor output and plant excitation signals with models of the plant and sensor. An observer provides feedback signals that are superior to the sensor output alone.

When faced with the problem of controlling a system, some scheme must be devised to choose the input vector  $x(t)$  so that the system behaves in an acceptable manner. Since the state vector  $y(t)$  contains all the essential information about the system, it is reasonable to base the choice of  $x(t)$  solely on the values of  $y(t)$  and perhaps also  $t$ . In other words,  $x$  is determined by a relation of the form  $x(t) = F[y(t), t]$ .

This is, in fact, the approach taken in a large portion of present day control system literature. Several new techniques have been developed to find the function  $F$  for special classes of control problems. These techniques include dynamic programming [Labarrere, M., Krief]-[Dutka, A., Ordys, A., Grimble], Pontryagin's maximum principle [K. K. Maitra], and methods based on Lyapunov's theory [J. Oreilly].

In most control situations, however, the state vector is not available for direct measurement. This means that it is not possible to evaluate the function  $F[y(t), t]$ . In these cases either the method must be abandoned or a reasonable substitute for the state vector must be found.

In this chapter it is shown how the available system inputs and outputs may be used to construct an estimate of the system state vector. The device which reconstructs the state vector is called an observer. The observer itself as a time-invariant linear system driven by the inputs and outputs of the system it observes.

To observe the system state, sometimes he can go to estimate the entire state vector then part is available as a linear combination of the output [J. Oreilly]. We suppose that we have  $p$  linear combinations, we will present the case where one has this information and cannot rebuilt that  $(n-p)$  linear combination of system states or

$$z(.) = TZ_0(.) \quad (45)$$

Is a linear combination, with the matrix  $T$  of dimension  $(n-p, n)$ . The estimated state is then obtained by:

$$\hat{Z}_0 = \begin{pmatrix} C_0 \\ T \end{pmatrix}^{-1} \begin{pmatrix} y(.) \\ z(.) \end{pmatrix} = \begin{pmatrix} V & P \end{pmatrix} \begin{pmatrix} y(.) \\ z(.) \end{pmatrix} \quad (46)$$

The matrix  $T$  is chosen in such a way that the matrix  $\begin{pmatrix} C_0 \\ T \end{pmatrix}$  is invertible. Furthermore the amount  $TZ_0(.)$  can be measured which leads us to generate  $z(.)$ , from an auxiliary dynamical system as follows:

$$\delta z(.) = Dz(.) + Ey(.) + Gu(.) \quad (47)$$

Where  $z(.)$  is the state of the observer dynamics. Note here that the matrices  $V, C_0, T, P$ , verify

$$VC_0 + PT = II \quad (48)$$

The control problem with constraint via an observer of minimal order may be solved in the following way:

How to choose the state feedback  $F$ :

$$u(.) = \text{sat} \left( F\hat{Z}_0(.) \right) \quad (49)$$

And matrices  $D, E$  and  $G$  calculated such that the asymptotic stability and the constraints on inputs are guaranteed

The observation error in this case is given by

$$\epsilon(.) = z(.) - TZ_0(.) \quad (50)$$

We recall that the matrices of the observer of minimal order is given by [11]:

$$D = TA_0P, E = TA_0V, G = TB_0 \quad (51)$$

Which is equivalent to write that the check matrices in the following relation

$$TA_0 - EC_0 = DT \quad (52)$$

Where the matrices T and P are chosen to ensure asymptotic stability of the matrix D, in order to see vanish asymptotically non sampling error, indeed:

$$\begin{aligned} \delta\epsilon(.) &= \delta z(.) - T\delta Z_0 \\ &= Dz(.) + Ey(.) + Gu(.) - T(A_0Z_0(.) + B_0u(.)) \\ &= Dz(.) + EC_0Z_0(.) - TA_0Z_0(.) \\ &= Dz(.) - DTZ_0(.) \\ &= D\epsilon(.) \end{aligned}$$

For the observation error, we define the field  $D(II, \epsilon_{max}, \epsilon_{min})$  that give us the limits within which we allow change of error  $\epsilon(.)$ . The reconstruction error is always given by

$$e(.) = \hat{Z}_0(.) - Z_0(.) \quad (53)$$

Is related to the error of observation:

$$\begin{aligned} e(.) &= Vy(.) + Pz(.) - Z_0(.) \\ &= VC_0Z_0(.) + Pz(.) - (VC_0 + PT)Z_0(.) \\ &= P(z(.) - TZ_0(.)) \\ &= P\epsilon(.) \end{aligned}$$

**Lemma:** The field  $D(II, u_{max}, u_{min}) \times D(II, \epsilon_{max}, \epsilon_{min})$  is positively invariant with respect to the system trajectory  $\begin{pmatrix} u(.) \\ \epsilon(.) \end{pmatrix}$  only hosts and if so, there exists a matrix  $H \in R^{m \times n}$  Such that:

1.  $HF = FA_0 + FB_0F$  (54)
2.  $\tilde{M}q_\epsilon \leq 0$

Where:

$$M = \begin{pmatrix} H & L_r \\ 0 & D \end{pmatrix} ; \quad q_\epsilon = \begin{pmatrix} u_{max} \\ \epsilon_{max} \\ u_{min} \\ \epsilon_{min} \end{pmatrix} ; \quad L_r = -FVC_0A_0P$$

For every pair  $(u(0), \epsilon(0)) \in D(II, u_{max}, u_{min}) \times D(II, \epsilon_{max}, \epsilon_{min})$

**Proof:** We start by writing the equation for the evolution of the control  $u(t)$  always in the case of a linear behaviour using previous relationship.

$$\begin{aligned} \delta u(.) &= F\delta\hat{Z}_0 \\ &= F\delta(Pz(.) + VC_0Z_0(.)) \\ &= FP\delta z(.) + FVC_0\delta Z_0(.) \end{aligned}$$

$$\begin{aligned}
&= FP(Dz(.) + Ey(.) + Gu(.)) + FVC_0(A_0Z_0(.) + B_0u(.)) \\
&= FP(TA_0Pz(.) + TA_0Vy(.)) + (FPTB_0 + FVC_0B_0)u(.) + FVC_0A_0Z_0(.) \\
&= FPTA_0(Pz(.) + Vy(.)) + F(PT + VC_0)B_0u(.) + FVC_0A_0Z_0(.) \\
&= FPTA_0\hat{Z}_0(.) + FB_0u(.) + FVC_0A_0(\hat{Z}_0(.) - e(.)) \\
&= F(PT + VC_0)A_0\hat{Z}_0(.) + FB_0F\hat{Z}_0(.) - FVC_0A_0e(.) \\
&= (FA_0 + FB_0F)\hat{Z}_0(.) - FVC_0A_0e(.) \\
&= HF\hat{Z}_0(.) - FVC_0A_0P\epsilon(.) \\
&= Hu(.) + L_r\epsilon(.)
\end{aligned}$$

Is then augmented system consisting of control  $u(t)$  and error  $\epsilon(.)$ , we get

$$\begin{pmatrix} \delta u(.) \\ \delta \epsilon(.) \end{pmatrix} = \begin{pmatrix} H & L_r \\ 0 & D \end{pmatrix} \begin{pmatrix} u(.) \\ \epsilon(.) \end{pmatrix}$$

## 5. Simulation results

Simulation experiments for the first strategies of control were carried out by numerical integration of the complete model of the biological process. Numerical values of the parameters appearing in the model equations are given in the table I and table II.

Variable	Value	Description
$V_{nit}$	$1000m^3$	volume of nitrification basin
$V_{denit}$	$250m^3$	volume of denitrification basin
$V_{dec}$	$1250m^3$	volume of settler
$Q_{in}$	$3000m^3/j$	influent flow rate
$Q_{r1}$	$2955m^3/j$	recycled flow rate
$Q_{r2}$	$1500m^3/j$	intern recycled flow rate
$Q_w$	$45m^3/j$	waste flow rate
$X_{A,in}$	$0 mg/l$	autotrophs in the influent
$X_{H,in}$	$30 mg/l$	heterotrophs in the influent
$S_{S,in}$	$200 mg/l$	substrate in the influent
$S_{NH,in}$	$30 mg/l$	ammonium in the influent
$S_{NO,in}$	$2 mg/l$	nitrate in the influent
$S_{O,in}$	$0mg/l$	oxygen in the influent

Table I. Process characteristics.

Simulation results are given in figure 3 for the NLGPC strategies. The perturbations pursued on the control variables are due to measurement noises. The output variables evolution that are the global nitrogen and the dissolved oxygen concentrations, and their corresponding reference trajectories are 7 and 3, respectively. The figure 4 presents the results of simulation for the second controller.

Parameter	Value	Description
$Y_A$	0.24	yield of autotroph mass
$Y_H$	0.67	yield of heterotroph mass
$i_{xb}$	0.086	
$K_S$	20mg/l	affinity constant
$K_{NH,A}$	1mg/l	affinity constant
$K_{NH,H}$	0.05mg/l	affinity constant
$K_{NO}$	0.5mg/l	affinity constant
$K_{O,A}$	0.4mg/l	affinity constant
$K_{O,H}$	0.2mg/l	affinity constant
$\mu_{Amax}$	0.8l/j	maximum specific growth rate
$\mu_{Hmax}$	0.6l/j	maximum specific growth rate
$b_A$	0.2l/j	decay coefficient of autotrophs
$b_H$	0.68l/j	decay coefficient of heterotrophs
$\eta_{NO}$	0.8l/j	correction factor for anoxic growth

Table II. Kinetic parameters and stoichiometric coefficient characteristics

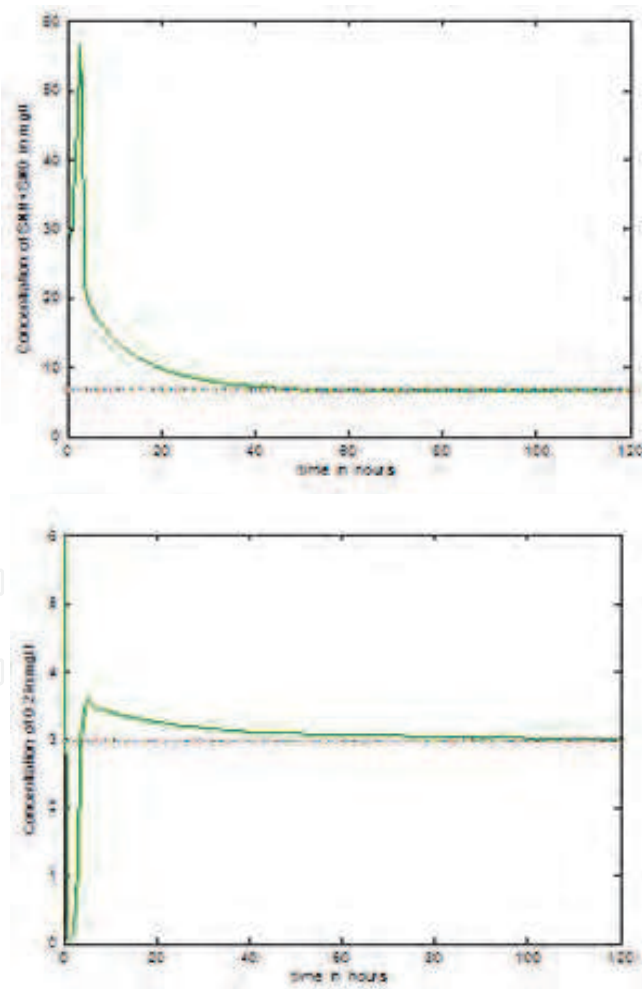


Fig. 3. Evolution of the dissolved oxygen and the global nitrogen concentrations for Non linear system with first controller.

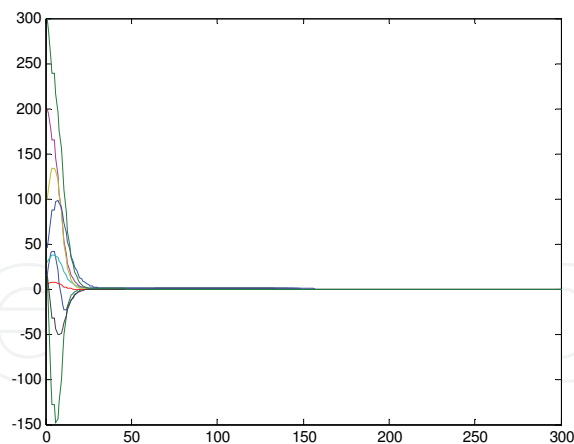


Fig. 4. Evolution of all the states of the linear system with the second controller.

## 6. Conclusion

Controlling the complex behaviour of the Wastewater Treatment Plant is a challenging mission and requires good control strategies. The process has many variables and presents large time constants. In addition, the process is constantly submitted to significant influent disturbances. These facts make mathematical models and computer simulation to be indispensable in developing new and efficient model based control architecture. This paper presents a part from control, the estimation procedure to compute estimated values of the unavailable state variables of the process, in order to have a more realistic simulation.

In one hand this paper, presents estimation and a predictive non linear controller for a biological nutrient removal have been proposed. The observer performs the twin task of states reconstruction and parameters estimation. The control and estimation techniques developed are based on direct exploitation of the full non-linear IAWQ model Simulation studies show either the efficiency of the non-linear controller in regulation or the effectiveness and the robustness of the estimation scheme, in reconstruction of the unmeasured variables and online estimation of the specific growth rates. The application of estimators such as 'intelligent sensors' to identify important biological variables and parameters with physical meaning constitutes an interesting alternative to the lack of sophisticated instrumentation and provides real time information on the process. In the other hand we introduced the observers in the control loop of a linear system with input constraints. This work is an extension of the theory of control systems with constraints by applying the concept of invariance positive. It addresses the problem of applicability such method in case the states of the systems studied are not measurable or not available at the measure. We presented the case of the observer which part of the information output is used to complete part of the state vector to estimate.

## 7. Acknowledgment

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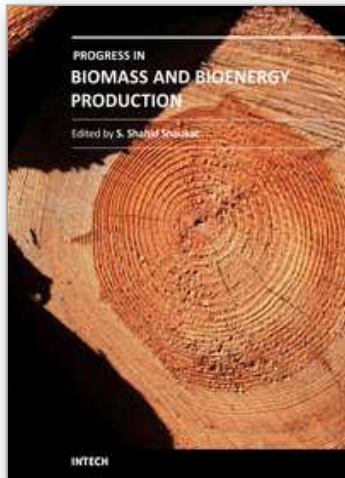


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## **Progress in Biomass and Bioenergy Production**

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Alternative energy sources have become a hot topic in recent years. The supply of fossil fuel, which provides about 95 percent of total energy demand today, will eventually run out in a few decades. By contrast, biomass and biofuel have the potential to become one of the major global primary energy source along with other alternate energy sources in the years to come. A wide variety of biomass conversion options with different performance characteristics exists. The goal of this book is to provide the readers with current state of art about biomass and bioenergy production and some other environmental technologies such as Wastewater treatment, Biosorption and Bio-economics. Organized around providing recent methodology, current state of modelling and techniques of parameter estimation in gasification process are presented at length. As such, this volume can be used by undergraduate and graduate students as a reference book and by the researchers and environmental engineers for reviewing the current state of knowledge on biomass and bioenergy production, biosorption and wastewater treatment.

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