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The Eigen Theory of Electromagnetic Waves in Complex Media

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1. Introduction

Since J. C. Maxwell presented the electromagnetic field equations in 1873, the existence of electromagnetic waves has been verified in various medium (Kong, 1986; Monk, 2003). But except for Helmholtz's equation of electromagnetic waves in isotropic media, the laws of propagation of electromagnetic waves in anisotropic media are not clear to us yet. For example, how many electromagnetic waves are there in anisotropic media? How fast can these electromagnetic waves propagate? Where are propagation direction and polarization direction of the electromagnetic waves? What are the space patterns of these waves? Although many research works were made in trying to deduce the equations of electromagnetic waves in anisotropic media based on the Maxwell's equation (Yakhno, 2005, 2006; Cohen, 2002; Haba, 2004), the explicit equations of electromagnetic waves in anisotropic media could not be obtained because the dielectric permittivity matrix and magnetic permeability matrix were all included in these equations, so that only local behaviour of electromagnetic waves, for example, in a certain plane or along a certain direction, can be studied.

On the other hand, it is a natural fact that electric and magnetic fields interact with each other in classical electromagnetics. Therefore, even if most of material studies deal with the properties due to dielectric polarisation, magnetic materials are also capable of producing quite interesting electro-magnetic effects (Lindell et al., 1994). From the bi-anisotropic point of view, magnetic materials can be treated as a subclass of magnetoelectric materials. The linear constitutive relations linking the electric and magnetic fields to the electric and magnetic displacements contain four dyadics, three of which have direct magnetic contents. The magnetoelectric coupling has both theoretical and practical significance in solid state physics and materials science. Though first predicted by Pierre Curie, magnetoelectric coupling was originally thought to be forbidden because it violates time-reversal symmetry, until Landau and Lifshitz (Landau & Lifshitz, 1960) pointed out that time reversal is not a symmetry operation in some magnetic crystal. Based on this argument, Dzyaloshinskii (Dzyaloshinskii, 1960) predicted that magnetoelectric effect should occur in antiferromagnetic crystal Cr_2O_3 , which was verified experimentally by Astrov (Astrov, 1960). Since then the magnetoelectric coupling has been observed in single-phase materials where simultaneous electric and magnetic ordering coexists, and in two-phase composites where the participating phase are piezoelectric and piezomagnetic (Bracke & Van Vliet, 1981; Van Run et al., 1974). Agyei and Birman (Agyei & Birman, 1990) carried out a detailed

analysis of the linear magnetoelectric effect, which showed that the effect should occur not only in some magnetic but also in some electric crystals. Pradhan (Pradhan, 1993) showed that an electric charge placed in a magnetoelectric medium becomes a source of induced magnetic field with non-zero divergence of volume integral. Magnetoelectric effect in two-phase composites has been analyzed by Harshe et al. (Harshe et al., 1993), Nan (Nan, 1994) and Benveniste (Benveniste, 1995). Broadband transducers based on magnetoelectric effect have also been developed (Bracke & Van Vliet, 1981). Although the development mentioned above, no great progress in the theories of electromagnetic waves in bi-anisotropic media because of the difficulties in deal with the bi-coupling in electric field and magnetic one of the Maxwell's equation and the bi-anisotropic constitutive equation by classical electromagnetic theory.

Recently there is a growing interest modeling and analysis of Maxwell's equations (Lee & Madsen, 1990; Monk, 1992; Jin et al., 1999). However, most work is restricted to simple medium such as air in the free space. On the other hand, we notice that lossy and dispersive media are ubiquitous, for example human tissue, water, soil, snow, ice, plasma, optical fibers and radar-absorbing materials. Hence the study of how electromagnetic wave interacts with dispersive media becomes very important. Some concrete applications include geophysical probing and subsurface studied of the moon and other planets (Bui et al., 1991), High power and ultra-wide-band radar systems, in which it is necessary to model ultra-wide-band electromagnetic pulse propagation through plasmas (Dvorak & Dudley, 1995), ground penetrating radar detection of buried objects in soil media (liu & Fan, 1999). The Debye medium plays an important role in electromagnetic wave interactions with biological and water-based substances (Gandhi & Furse, 1997). Until 1990, some paper on modeling of wave propagation in dispersive media started making their appearance in computational electromagnetics community. However, the published papers on modeling of dispersive media are exclusively restricted to the finite-difference time-domain methods and the finite element methods (Li & Chen, 2006; Lu et al., 2004). To our best knowledge, there exist only few works in the literature, which studied the theoretical model for the Maxwell's equation in the complex anisotropic dispersive media, and no explicit equations of electromagnetic waves in anisotropic dispersive media can be obtained due to the limitations of classical electromagnetic theory.

Chiral materials have been recently an interesting subject. In a chiral medium, an electric or magnetic excitation will produce simultaneously both electric and magnetic polarizations. On the other hand, the chiral medium is an object that cannot be brought into congruence with its mirror image by translation and rotation. Chirality is common in a variety of naturally occurring and man-made objects. From an operation point of view, chirality is introduced into the classical Maxwell equations by the Drude-Born-Fedorov relative constitutive relations in which the electric and magnetic fields are coupled via a new materials parameter (Lakhtakia, 1994; Lindell et al., 1994), the chirality parameter. These constitutive relations are chosen because they are symmetric under time reversality and duality transformations. In a homogeneous isotropic chiral medium the electromagnetic fields are composed of left-circularly polarized (LCP) and right-circularly polarized (RCP) components (Jaggard et al., 1979; Athanasiadis & Giotopoulos, 2003), which have different wave numbers and independent directions of propagation. Whenever an electromagnetic wave (LCP, RCP or a linear combination of them) is incident upon a chiral scatterer, then the scattered field is composed of both LCP and RCP components and therefore both LCP and RCP far-field patterns are derived. Hence, in the vector problem we need to specify two

directions of propagation and two polarizations. In recent years, chiral materials have been increasingly studied and there is a growing literature covering both their applications and the theoretical investigation of their properties. It will be noticed that the works dealing with wave phenomena in chiral materials have been mainly concerned with the study of time-harmonic waves which lead to frequency domain studies (Lakhtakia et al., 1989; Athanasiadis et al., 2003).

In this chapter, the idea of standard spaces is used to deal with the Maxwell's electromagnetic equation (Guo, 2009, 2009, 2010, 2010, 2010). By this method, the classical Maxwell's equation under the geometric presentation can be transformed into the eigen Maxwell's equation under the physical presentation. The former is in the form of vector and the latter is in the form of scalar. Through inducing the modal constitutive equations of complex media, such as anisotropic media, bi-anisotropic media, lossy media, dissipative media, and chiral media, a set of modal equations of electromagnetic waves for all of those media are obtained, each of which shows the existence of electromagnetic sub-waves, meanwhile its propagation velocity, propagation direction, polarization direction and space pattern can be completely determined by the modal equations. This chapter will make introductions of the eigen theory to reader in details. Several novel theoretical results were discussed in the different parts of this chapter.

2. Standard spaces of electromagnetic media

In anisotropic electromagnetic media, the dielectric permittivity and magnetic permeability are tensors instead of scalars. The constitutive relations are expressed as follows

$$\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E}, \quad \mathbf{B} = \boldsymbol{\mu} \cdot \mathbf{H} \quad (1)$$

Rewriting Eq.(1) in form of scalar, we have

$$D_i = \varepsilon_{ij} E_j, \quad B_i = \mu_{ij} H_j \quad (2)$$

where the dielectric permittivity matrix $\boldsymbol{\varepsilon}$ and the magnetic permeability matrix $\boldsymbol{\mu}$ are usually symmetric ones, and the elements of the matrixes have a close relationship with the selection of reference coordinate. Suppose that if the reference coordinates is selected along principal axis of electrically or magnetically anisotropic media, the elements at non-diagonal of these matrixes turn to be zero. Therefore, equations (1) and (2) are called the constitutive equations of electromagnetic media under the geometric presentation. Now we intend to get rid of effects of geometric coordinate on the constitutive equations, and establish a set of coordinate-independent constitutive equations of electromagnetic media under physical presentation. For this purpose, we solve the following problems of eigen-value of matrixes.

$$(\boldsymbol{\varepsilon} - \lambda \mathbf{I})\boldsymbol{\phi} = \mathbf{0}, \quad (\boldsymbol{\mu} - \gamma \mathbf{I})\boldsymbol{\varphi} = \mathbf{0} \quad (3)$$

where $\lambda_i (i=1,2,3)$ and $\gamma_i (i=1,2,3)$ are respectively eigen dielectric permittivity and eigen magnetic permeability, which are constants of coordinate-independent. $\boldsymbol{\phi} (i=1,2,3)$ and $\boldsymbol{\varphi} (i=1,2,3)$ are respectively eigen electric vector and eigen magnetic vector, which show the electrically principal direction and magnetically principal direction of anisotropic media, and are all coordinate-dependent. We call these vectors as standard spaces. Thus, the matrix

of dielectric permittivity and magnetic permeability can be spectrally decomposed as follows

$$\boldsymbol{\varepsilon} = \boldsymbol{\Phi} \boldsymbol{\Lambda} \boldsymbol{\Phi}^T, \quad \boldsymbol{\mu} = \boldsymbol{\Psi} \boldsymbol{\Pi} \boldsymbol{\Psi}^T \quad (4)$$

where $\boldsymbol{\Lambda} = \text{diag}[\lambda_1, \lambda_2, \lambda_3]$ and $\boldsymbol{\Pi} = \text{diag}[\gamma_1, \gamma_2, \gamma_3]$ are the matrix of eigen dielectric permittivity and eigen magnetic permeability, respectively. $\boldsymbol{\Phi} = \{\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \boldsymbol{\phi}_3\}$ and $\boldsymbol{\Psi} = \{\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \boldsymbol{\psi}_3\}$ are respectively the modal matrix of electric media and magnetic media, which are both orthogonal and positive definite matrixes, and satisfy $\boldsymbol{\Phi}^T \boldsymbol{\Phi} = \boldsymbol{I}$, $\boldsymbol{\Psi}^T \boldsymbol{\Psi} = \boldsymbol{I}$.

Projecting the electromagnetic physical qualities of the geometric presentation, such as the electric field intensity vector \boldsymbol{E} , magnetic field intensity vector \boldsymbol{H} , magnetic flux density vector \boldsymbol{B} and electric displacement vector \boldsymbol{D} into the standard spaces of the physical presentation, we get

$$\boldsymbol{D}^* = \boldsymbol{\Phi}^T \cdot \boldsymbol{D}, \quad \boldsymbol{E}^* = \boldsymbol{\Phi}^T \cdot \boldsymbol{E} \quad (5)$$

$$\boldsymbol{B}^* = \boldsymbol{\Psi}^T \cdot \boldsymbol{B}, \quad \boldsymbol{H}^* = \boldsymbol{\Psi}^T \cdot \boldsymbol{H} \quad (6)$$

Rewriting Eqs.(5) and (6) in the form of scalar, we have

$$D_i^* = \boldsymbol{\phi}_i^T \cdot \boldsymbol{D} \quad i = 1, 2, 3, \quad E_i^* = \boldsymbol{\phi}_i^T \cdot \boldsymbol{E} \quad i = 1, 2, 3 \quad (7)$$

$$B_i^* = \boldsymbol{\psi}_i^T \cdot \boldsymbol{B} \quad i = 1, 2, 3, \quad H_i^* = \boldsymbol{\psi}_i^T \cdot \boldsymbol{H} \quad i = 1, 2, 3 \quad (8)$$

These are the electromagnetic physical qualities under the physical presentation. Substituting Eq. (4) into Eq. (1) respectively, and using Eqs.(5) and (6) yield

$$D_i^* = \lambda_i E_i^* \quad i = 1, 2, 3 \quad (9)$$

$$B_i^* = \gamma_i H_i^* \quad i = 1, 2, 3 \quad (10)$$

The above equations are just the modal constitutive equations in the form of scalar.

3. Eigen expression of Maxwell's equation

The classical Maxwell's equations in passive region can be written as

$$\boldsymbol{\nabla} \times \boldsymbol{H} = \nabla_t \boldsymbol{D}, \quad \boldsymbol{\nabla} \times \boldsymbol{E} = -\nabla_t \boldsymbol{B} \quad (11)$$

Now we rewrite the equations in the form of matrix as follows

$$\begin{bmatrix} 0 & -\partial_z & \partial_y \\ \partial_z & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{bmatrix} \begin{Bmatrix} H_1 \\ H_2 \\ H_3 \end{Bmatrix} = \nabla_t \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} \quad (12)$$

or

$$[\Delta] \{H\} = \nabla_t \{D\} \quad (13)$$

$$\begin{bmatrix} 0 & -\partial_z & \partial_y \\ \partial_z & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = -\nabla_t \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix} \quad (14)$$

or

$$[\Delta]\{E\} = -\nabla_t\{B\} \quad (15)$$

where $[\Delta]$ is defined as the matrix of electric and magnetic operators. Substituting Eq. (1) into Eqs. (13) and (15) respectively, we have

$$[\Delta]\{H\} = \nabla_t[\varepsilon]\{E\} \quad (16)$$

$$[\Delta]\{E\} = -\nabla_t[\mu]\{H\} \quad (17)$$

Substituting Eq. (16) into (17) or Eq. (17) into (16), yield

$$[\square]\{H\} = -\nabla_t^2[\mu][\varepsilon]\{H\} \quad (18)$$

$$[\square]\{E\} = -\nabla_t^2[\mu][\varepsilon]\{E\} \quad (19)$$

where $[\square] = [\Delta][\Delta]$ is defined as the matrix of electromagnetic operators as follows

$$[\square] = \begin{bmatrix} -(\partial_z^2 + \partial_y^2) & \partial_{xy}^2 & \partial_{xz}^2 \\ \partial_{yx}^2 & -(\partial_x^2 + \partial_z^2) & \partial_{yz}^2 \\ \partial_{zx}^2 & \partial_{zy}^2 & -(\partial_x^2 + \partial_y^2) \end{bmatrix} \quad (20)$$

In another way, substituting Eqs. (5) and (6) into Eqs. (13) and (15), respectively, we have

$$[\Delta][\Phi]\{E^*\} = -\nabla_t[\Psi]\{B^*\} \quad (21)$$

$$[\Delta][\Psi]\{H^*\} = \nabla_t[\Phi]\{D^*\} \quad (22)$$

Rewriting the above in indicial notation, we get

$$\{\Delta_i^*\}E_i^* = -\nabla_t\{\phi_i\}B_i^* \quad i = 1, 2, 3 \quad (23)$$

$$\{\Delta_i^*\}H_i^* = \nabla_t\{\phi_i\}D_i^* \quad i = 1, 2, 3 \quad (24)$$

where, Δ_i^* is the electromagnetic intensity operator, and i th row of $[\Delta^*] = [\Delta][\Phi]$.

4. Electromagnetic waves in anisotropic media

4.1 Electrically anisotropic media

In anisotropic dielectrics, the dielectric permittivity is a tensor, while the magnetic permeability is a scalar. So Eqs. (18) and (19) can be written as follows

$$[\square]\{H\} = -\nabla_t^2 \mu_0 [\varepsilon]\{H\} \quad (25)$$

$$[\square]\{E\} = -\nabla_t^2 \mu_0 [\varepsilon]\{E\} \quad (26)$$

Substituting Eqs. (4) - (6) into Eqs. (25) and (26), we have

$$[\square^*]\{H^*\} = -\nabla_t^2 \mu_0 [\Lambda]\{H^*\} \quad (27)$$

$$[\square^*]\{E^*\} = -\nabla_t^2 \mu_0 [\Lambda]\{E^*\} \quad (28)$$

where $[\square^*] = [\Phi]^T [\square] [\Phi]$ is defined as the eigen matrix of electromagnetic operators under the standard spaces. We can note from Appendix A that it is a diagonal matrix. Thus Eqs. (27) and (28) can be uncoupled in the form of scalar

$$\square_i^* H_i^* + \mu_0 \lambda_i \nabla_t^2 H_i^* = 0 \quad i = 1, 2, 3 \quad (29)$$

$$\square_i^* E_i^* + \mu_0 \lambda_i \nabla_t^2 E_i^* = 0 \quad i = 1, 2, 3 \quad (30)$$

Eqs.(29) and (30) are the modal equations of electromagnetic waves in anisotropic dielectrics.

4.2 Magnetically anisotropic media

In anisotropic magnetics, the magnetic permeability is a tensor, while the dielectric permittivity is a scalar. So Eqs. (18) and (19) can be written as follows

$$[\square]\{H\} = -\nabla_t^2 \varepsilon_0 [\mu]\{H\} \quad (31)$$

$$[\square]\{E\} = -\nabla_t^2 \varepsilon_0 [\mu]\{E\} \quad (32)$$

Substituting Eqs. (4) - (6) into Eqs. (31) and (32), we have

$$[\square^*]\{H^*\} = -\nabla_t^2 \varepsilon_0 [\Pi]\{H^*\} \quad (33)$$

$$[\square^*]\{E^*\} = -\nabla_t^2 \varepsilon_0 [\Pi]\{E^*\} \quad (34)$$

where $[\square^*] = [\Psi]^T [\square] [\Psi]$ is defined as the eigen matrix of electromagnetic operators under the standard spaces. We can also note from Appendix A that it is a diagonal matrix. Thus Eqs. (33) and (34) can be uncoupled in the form of scalar

$$\square_i^* H_i^* + \varepsilon_0 \gamma_i \nabla_t^2 H_i^* = 0 \quad i = 1, 2, 3 \quad (35)$$

$$\square_i^* E_i^* + \varepsilon_0 \gamma_i \nabla_t^2 E_i^* = 0 \quad i = 1, 2, 3 \quad (36)$$

Eqs.(35) and (36) are the modal equations of electromagnetic waves in anisotropic magnetics.

5. Electromagnetic waves in bi-anisotropic media

5.1 Bi-anisotropic constitutive equations

The constitutive equations of bi-anisotropic media are the following (Lindell & Sihvola, 1994; Landau & Lifshitz, 1960)

$$\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E} + \boldsymbol{\xi} \cdot \mathbf{H} \quad (37)$$

$$\mathbf{B} = \boldsymbol{\xi} \cdot \mathbf{E} + \boldsymbol{\mu} \cdot \mathbf{H} \quad (38)$$

where $\boldsymbol{\xi}$ is the matrix of magneto-electric parameter, and a symmetric one.

Substituting Eqs. (5) and (6) into Eqs. (37) and (38), respectively, and multiplying them with the transpose of modal matrix in the left, we have

$$\boldsymbol{\Phi}^T \mathbf{D} = \boldsymbol{\Phi}^T \boldsymbol{\varepsilon} \boldsymbol{\Phi} \mathbf{E}^* + \boldsymbol{\Phi}^T \boldsymbol{\xi} \boldsymbol{\Psi} \mathbf{H}^* \quad (39)$$

$$\boldsymbol{\Psi}^T \mathbf{B} = \boldsymbol{\Psi}^T \boldsymbol{\xi} \boldsymbol{\Phi} \mathbf{E}^* + \boldsymbol{\Psi}^T \boldsymbol{\mu} \boldsymbol{\Psi} \mathbf{H}^* \quad (40)$$

Let $\mathbf{G} = \boldsymbol{\Phi}^T \boldsymbol{\xi} \boldsymbol{\Psi} = \boldsymbol{\Psi}^T \boldsymbol{\xi} \boldsymbol{\Phi}$, that is a coupled magneto-electric matrix, and using Eq. (4), we have

$$\mathbf{D}^* = \boldsymbol{\Lambda} \mathbf{E}^* + \mathbf{G} \mathbf{H}^* \quad (41)$$

$$\mathbf{B}^* = \mathbf{G} \mathbf{E}^* + \boldsymbol{\Pi} \mathbf{H}^* \quad (42)$$

Rewriting the above in indicial notation, we get

$$D_i^* = \lambda_i E_i^* + g_{ij} H_j^* \quad i = 1, 2, 3 \quad j = 1, 2, 3 \quad (43)$$

$$B_i^* = \gamma_i H_i^* + g_{ij} E_j^* \quad i = 1, 2, 3 \quad j = 1, 2, 3 \quad (44)$$

Eqs. (43) and (44) are just the modal constitutive equations for bi-anisotropic media.

5.2 Eigen equations of electromagnetic waves in bi-anisotropic media

Substituting Eqs. (43) and (44) into Eqs. (23) and (24), respectively, we have

$$\{\Delta_i^*\} E_i^* = -\nabla_t \{\phi_i\} (\gamma_i H_i^* + g_{ij} E_j^*) \quad (45)$$

$$\{\Delta_i^*\} H_i^* = \nabla_t \{\phi_i\} (\lambda_i E_i^* + g_{ij} H_j^*) \quad (46)$$

From them, we can get

$$\left(\{\Delta_i^*\} - \nabla_t \{\phi_i\} g_{ij} \delta_{ij} \right)^T \left(\{\Delta_i^*\} + \nabla_t \{\phi_i\} g_{ij} \delta_{ij} \right) E_i^* = -\nabla_t^2 \{\phi_i\} \{\phi_i\}^T \lambda_i \gamma_i E_i^* \quad (47)$$

$$\left(\{\Delta_i^*\} + \nabla_t \{\phi_i\} g_{ij} \delta_{ij} \right)^T \left(\{\Delta_i^*\} - \nabla_t \{\phi_i\} g_{ij} \delta_{ij} \right) H_i^* = -\nabla_t^2 \{\phi_i\} \{\phi_i\}^T \gamma_i \lambda_i H_i^* \quad (48)$$

The above can also be written as the standard form of waves

$$\square_i^* E_i^* + \nabla_i \{\Delta_i^*\}^T \cdot (\{\varphi\} - \{\phi\}) g_{ii} E_i^* + \nabla_i^2 \{\phi\}^T \cdot \{\varphi\} (\lambda_i \gamma_i - g_{ii}^2) E_i^* = 0 \quad i = 1, 2, 3 \quad (49)$$

$$\square_i^* H_i^* + \nabla_i \{\Delta_i^*\}^T \cdot (\{\varphi\} - \{\phi\}) g_{ii} H_i^* + \nabla_i^2 \{\phi\}^T \cdot \{\varphi\} (\lambda_i \gamma_i - g_{ii}^2) H_i^* = 0 \quad i = 1, 2, 3 \quad (50)$$

where, $\square_i^* = \{\Delta_i^*\}^T \cdot \{\Delta_i^*\}$ is the electromagnetic operator. Eqs.(49) and (50) are just equations of electric field and magnetic field for bi-anisotropic media.

5.3 Applications

5.3.1 Bi-isotropic media

The constitutive equations of bi-isotropic media are the following

$$\mathbf{D} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix} \cdot \mathbf{E} + \begin{bmatrix} \xi & 0 & 0 \\ 0 & \xi & 0 \\ 0 & 0 & \xi \end{bmatrix} \cdot \mathbf{H} \quad (51)$$

$$\mathbf{B} = \begin{bmatrix} \xi & 0 & 0 \\ 0 & \xi & 0 \\ 0 & 0 & \xi \end{bmatrix} \cdot \mathbf{E} + \begin{bmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{bmatrix} \cdot \mathbf{H} \quad (52)$$

The eigen values and eigen vectors of those matrix are the following

$$\mathbf{A} = \text{diag}[\varepsilon, \varepsilon, \varepsilon], \quad \mathbf{H} = \text{diag}[\mu, \mu, \mu] \quad (53)$$

$$\Phi = \Psi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (54)$$

We can see from the above equations that there is only one eigen-space in isotropic medium, which is a triple-degenerate one, and the space structure is the following

$$\mathbf{W} = W_1^{(3)}[\phi_1, \phi_2, \phi_3] \quad (55)$$

$$\phi_1^* = \phi_1^* = \frac{\sqrt{3}}{3}[1, 1, 1]^T \quad (56)$$

Then the eigen-qualities and eigen-operators of bi-isotropic medium are respectively shown as follows

$$E_1^* = \phi_1^{*T} \cdot \mathbf{E} = \frac{\sqrt{3}}{3}(E_1 + E_2 + E_3) \quad (57)$$

$$\square_1^* = -(\partial_x^2 + \partial_y^2 + \partial_z^2), \quad g_{11} = \xi \quad (58)$$

So, the equation of electromagnetic wave in bi-isotropic medium becomes

$$(\partial_x^2 + \partial_y^2 + \partial_z^2)E_1^* = (\mu\varepsilon - \xi^2)\partial_t^2 E_1^* \quad (59)$$

the velocity of electromagnetic wave is

$$c^{(1)} = \frac{1}{\sqrt{\mu\varepsilon - \xi^2}} \quad (60)$$

5.3.2 Dzyaloshinskii's bi-anisotropic media

Dzyaloshinskii's constitutive equations of bi-anisotropic media are the following

$$\mathbf{D} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \cdot \mathbf{E} + \begin{bmatrix} \xi & 0 & 0 \\ 0 & \xi & 0 \\ 0 & 0 & \xi_z \end{bmatrix} \cdot \mathbf{H} \quad (61)$$

$$\mathbf{B} = \begin{bmatrix} \xi & 0 & 0 \\ 0 & \xi & 0 \\ 0 & 0 & \xi_z \end{bmatrix} \cdot \mathbf{E} + \begin{bmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \cdot \mathbf{H} \quad (62)$$

The eigen values and eigen vectors of those matrix are the following

$$\mathbf{\Lambda} = \text{diag}[\varepsilon, \varepsilon, \varepsilon_z], \quad \mathbf{\Pi} = \text{diag}[\mu, \mu, \mu_z] \quad (63)$$

$$\mathbf{\Phi} = \mathbf{\Psi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (64)$$

We can see from the above equations that there are two eigen-spaces in Dzyaloshinskii's bi-anisotropic medium, in which one is a binary-degenerate one, the space structure is the following

$$\mathbf{W} = W_1^{(2)}[\phi_1, \phi_2] \oplus W_2^1[\phi_3] \quad (65)$$

Then the eigen-qualities and eigen-operators of Dzyaloshinskii's bi-anisotropic medium are respectively shown as belows

$$E_2^* = \phi_2^T \cdot \mathbf{E} = E_3, \quad |E_1^*| = \sqrt{(\mathbf{E} - \phi_2^T E_2^*)^T (\mathbf{E} - \phi_2^T E_2^*)} = \sqrt{E_1^2 + E_2^2} \quad (66)$$

$$\alpha_1^* = -(\partial_x^2 + \partial_y^2 + 2\partial_z^2 - 2\partial_{xy}^2), \quad \alpha_2^* = -(\partial_x^2 + \partial_y^2), \quad g_{11} = \xi, \quad g_{22} = \xi_z \quad (67)$$

So, the equations of electromagnetic wave in Dzyaloshinskii's bi-anisotropic medium become

$$(\partial_x^2 + \partial_y^2 + 2\partial_z^2 - 2\partial_{xy}^2)\sqrt{E_1^2 + E_2^2} = (\mu\varepsilon - \xi^2)\partial_t^2 \sqrt{E_1^2 + E_2^2} \quad (68)$$

$$(\partial_x^2 + \partial_y^2)E_3 = (\mu_z \varepsilon_z - \xi_z^2)\partial_t^2 E_3 \quad (69)$$

the velocities of electromagnetic wave are

$$c^{(1)} = \frac{1}{\sqrt{\mu\epsilon - \xi^2}} \quad (70)$$

$$c^{(2)} = \frac{1}{\sqrt{\mu_z \epsilon_z - \xi_z^2}} \quad (71)$$

It is seen both from bi-isotropic media and Dzyaloshinskii's bi-anisotropic medium that the electromagnetic waves in bi-anisotropic medium will go faster due to the bi-coupling between electric field and magnetic one.

6. Electromagnetic waves in lossy media

6.1 The constitutive equation of lossy media

The constitutive equation of lossy media is the following

$$\mathbf{D} = \boldsymbol{\epsilon} \cdot \mathbf{E} + \int_i \boldsymbol{\sigma} \cdot \frac{d\mathbf{E}}{d\tau} d\tau \quad (72)$$

It is equivalent to the following differential constitutive equation

$$\dot{\mathbf{D}} = \boldsymbol{\epsilon} \cdot \dot{\mathbf{E}} + \boldsymbol{\sigma} \cdot \mathbf{E} \quad (73)$$

Let

$$\mathbf{D}^c = \boldsymbol{\epsilon} \cdot \mathbf{E} \quad , \quad \dot{\mathbf{D}}^d = \boldsymbol{\sigma} \cdot \mathbf{E} \quad (74)$$

Eq.(73) can be written as

$$\dot{\mathbf{D}} = \dot{\mathbf{D}}^c + \dot{\mathbf{D}}^d \quad (75)$$

or

$$\nabla_t \{D\} = ([\boldsymbol{\epsilon}] \nabla_t + [\boldsymbol{\sigma}]) \{E\} \quad (76)$$

Using Eq.(5), the above becomes

$$\nabla_t \{D^*\} = ([\boldsymbol{\Phi}]^T [\boldsymbol{\epsilon}] [\boldsymbol{\Phi}] \nabla_t + [\boldsymbol{\Phi}]^T [\boldsymbol{\sigma}] [\boldsymbol{\Phi}]) \{E^*\} \quad (77)$$

According to Appendix B and Eq.(77), we have

$$\nabla_t \{D^*\} = ([A] \nabla_t + [\Gamma]) \{E^*\} \quad (78)$$

Rewriting the above in indicial notation, we get

$$\nabla_t D_i^* = (\lambda_i \nabla_t + \eta_i) E_i^* \quad (79)$$

Eq.(79) is just the modal constitutive equations for lossy media.

6.2 Eigen equations of electromagnetic waves in lossy media

Substituting Eqs. (10) and (79) into Eqs. (23) and (24), respectively, we have

$$\{\Delta_i^*\} E_i^* = -\nabla_t \{\phi_i\} \gamma_i H_i^* \quad i = 1, 2, 3 \quad (80)$$

$$\{\Delta_i^*\} H_i^* = \{\phi_i\} (\lambda_i \nabla_t + \eta_i) E_i^* \quad i = 1, 2, 3 \quad (81)$$

From them, we can get

$$\square_i^* E_i^* + \nabla_{tt} \xi_i \gamma_i \lambda_i E_i^* + \nabla_t \xi_i \gamma_i \eta_i E_i^* = 0 \quad i = 1, 2, 3 \quad (82)$$

$$\square_i^* H_i^* + \nabla_{tt} \xi_i \gamma_i \lambda_i H_i^* + \nabla_t \xi_i \gamma_i \eta_i H_i^* = 0 \quad i = 1, 2, 3 \quad (83)$$

where $\xi_i = \{\phi_i^*\}^T \cdot \{\phi_i^*\}$. Eqs.(82) and (83) are just equations of electric field and magnetic field for bi-anisotropic media.

6.3 Applications

In this section, we discuss the propagation laws of electromagnetic waves in an isotropic lossy medium. The material tensors in Eqs.(1) and (72) are represented by the following matrices

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{11} \end{bmatrix}, \quad \boldsymbol{\mu} = \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{11} & 0 \\ 0 & 0 & \mu_{11} \end{bmatrix}, \quad \boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{11} & 0 \\ 0 & 0 & \sigma_{11} \end{bmatrix} \quad (84)$$

The eigen values and eigen vectors of those matrix are the following

$$\boldsymbol{\Lambda} = \text{diag}[\varepsilon_{11}, \varepsilon_{11}, \varepsilon_{11}], \quad \boldsymbol{\Pi} = \text{diag}[\mu_{11}, \mu_{11}, \mu_{11}], \quad \boldsymbol{\Gamma} = \text{diag}[\sigma_{11}, \sigma_{11}, \sigma_{11}] \quad (85)$$

$$\boldsymbol{\Phi} = \boldsymbol{\Psi} = \boldsymbol{\Theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (86)$$

We can see from the above equations that there is only one eigen-space in an isotropic lossy medium, which is a triple-degenerate one, and the space structure is the following.

$$\boldsymbol{W}_{mag} = W_1^{(3)}[\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \boldsymbol{\phi}_3], \quad \boldsymbol{W}_{ele} = W_1^{(3)}[\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3] \quad (87)$$

where, $\boldsymbol{\phi}_1^* = \frac{\sqrt{3}}{3}\{1, 1, 1\}^T$, $\boldsymbol{\varphi}_1^* = \frac{\sqrt{3}}{3}\{1, 1, 1\}^T$, $\xi_1 = 1$.

Then the eigen-qualities and eigen-operators of an isotropic lossy media are respectively shown as follows

$$E_1^* = \frac{\sqrt{3}}{3}(E_1 + E_2 + E_3) \quad (88)$$

$$H_1^* = \frac{\sqrt{3}}{3}(H_1 + H_2 + H_3) \quad (89)$$

$$\epsilon_1^* = \frac{1}{3}[-(\partial_x^2 + \partial_y^2 + \partial_z^2)] \quad (90)$$

So, the equation of electromagnetic wave in lossy media becomes

$$(\partial_x^2 + \partial_y^2 + \partial_z^2)E_1^* = \frac{1}{c^2}\partial_t^2 E_1^* + \frac{1}{\tau^2}\partial_t E_1^* \quad (91)$$

Rewriting it in the component form, we have

$$(\partial_x^2 + \partial_y^2 + \partial_z^2)E_1 = \frac{1}{c^2}\partial_t^2 E_1 + \frac{1}{\tau^2}\partial_t E_1 \quad (92)$$

$$(\partial_x^2 + \partial_y^2 + \partial_z^2)E_2 = \frac{1}{c^2}\partial_t^2 E_2 + \frac{1}{\tau^2}\partial_t E_2 \quad (93)$$

$$(\partial_x^2 + \partial_y^2 + \partial_z^2)E_3 = \frac{1}{c^2}\partial_t^2 E_3 + \frac{1}{\tau^2}\partial_t E_3 \quad (94)$$

where, c is the velocity of electromagnetic wave, τ is the lossy coefficient of electromagnetic wave

$$c = \frac{1}{\sqrt{\mu_{11}\epsilon_{11}}}, \quad \tau = \frac{1}{\sqrt{\mu_{11}\sigma_{11}}} \quad (95)$$

Now, we discuss the the propagation laws of a plane electromagnetic wave in x-axis. In this time, Eq. (92) becomes

$$\frac{\partial}{\partial x^2} E_1 = \frac{1}{c^2}\partial_t^2 E_1 + \frac{1}{\tau^2}\partial_t E_1 \quad (96)$$

Let the solution of Eq. (96) is as follows

$$E_1 = A \exp[i(kx - \omega t)] \quad (97)$$

Substituting the above into Eq. (96), we have

$$k^2 = \frac{\omega^2}{c^2} + i \frac{\omega}{\tau^2} \quad (98)$$

From Eq.(96), we can get

$$\bar{k} = k_1 + ik_2 \quad (99)$$

where

$$k_1 = \frac{\omega}{c} \left[\frac{1 + \left(1 + \frac{c^4}{\omega^2 \tau^4}\right)^{\frac{1}{2}}}{2} \right]^{\frac{1}{2}}, \quad k_2 = \frac{\omega}{c} \left[\frac{-1 + \left(1 + \frac{c^4}{\omega^2 \tau^4}\right)^{\frac{1}{2}}}{2} \right]^{\frac{1}{2}}.$$

Then, the solutions of electromagnetic waves are the following

$$E_1 = A e^{-k_2 x} \cdot e^{i(k_1 x - \omega t)} = \bar{A} \cdot e^{i(k_1 x - \omega t)} \quad (100)$$

It is an attenuated sub-waves.

7. Electromagnetic waves in dispersive media

7.1 The constitutive equation of dispersive media

The general constitutive equations of dispersive media are the following

$$\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E} + \boldsymbol{\varepsilon}_1 \cdot \dot{\mathbf{E}} + \boldsymbol{\varepsilon}_2 \cdot \ddot{\mathbf{E}} + \dots \quad (101)$$

$$\mathbf{B} = \boldsymbol{\mu} \cdot \mathbf{H} + \boldsymbol{\mu}_1 \cdot \dot{\mathbf{H}} + \boldsymbol{\mu}_2 \cdot \ddot{\mathbf{H}} + \dots \quad (102)$$

where $\boldsymbol{\varepsilon}_i, (i=1,2,\dots)$ and $\boldsymbol{\mu}_i, (i=1,2,\dots)$ are the higher order dielectric permittivity matrix and the magnetic permeability matrix respectively, and all symmetric ones.

Substituting Eqs. (5) and (6) into Eqs. (101) and (102), respectively, and multiplying them with the transpose of modal matrix in the left, we have

$$\mathbf{D}^* = \boldsymbol{\Phi}^T \boldsymbol{\varepsilon} \boldsymbol{\Phi} \mathbf{E}^* + \boldsymbol{\Phi}^T \boldsymbol{\varepsilon}_1 \boldsymbol{\Phi} \dot{\mathbf{E}}^* + \boldsymbol{\Phi}^T \boldsymbol{\varepsilon}_2 \boldsymbol{\Phi} \ddot{\mathbf{E}}^* + \dots \quad (103)$$

$$\mathbf{B}^* = \boldsymbol{\Psi}^T \boldsymbol{\mu} \boldsymbol{\Psi} \mathbf{H} + \boldsymbol{\Psi}^T \boldsymbol{\mu}_1 \boldsymbol{\Psi} \dot{\mathbf{H}} + \boldsymbol{\Psi}^T \boldsymbol{\mu}_2 \boldsymbol{\Psi} \ddot{\mathbf{H}} + \dots \quad (104)$$

It can be proved that there exist same standard spaces for various order electric and magnetic fields in the condition close to the thermodynamic equilibrium. Then, we have

$$D_i^* = \lambda_i E_i^* + \lambda_i^{(1)} \dot{E}_i^* + \lambda_i^{(2)} \ddot{E}_i^* + \dots \quad (105)$$

$$B_i^* = \gamma_i H_i^* + \gamma_i^{(1)} \dot{H}_i^* + \gamma_i^{(2)} \ddot{H}_i^* + \dots \quad (106)$$

Eqs. (105) and (106) are just the modal constitutive equations for the general dispersive media.

7.2 Eigen equations of electromagnetic waves in dispersive media

Substituting Eqs. (105) and (106) into Eqs. (23) and (24), respectively, we have

$$\{\Delta_i^*\} E_i^* = -\nabla_i \{\varphi_i\} (\gamma_i H_i^* + \gamma_i^{(1)} \dot{H}_i^* + \gamma_i^{(2)} \ddot{H}_i^* + \dots) \quad (107)$$

$$\{\Delta_i^*\} H_i^* = \nabla_t \{\phi_i\} (\lambda_i E_i^* + \lambda_i^{(1)} \dot{E}_i^* + \lambda_i^{(2)} \ddot{E}_i^* + \dots) \quad (108)$$

From them, we can get

$$\frac{1}{\xi_i} \square_i^* E_i^* + \gamma_i \lambda_i \nabla_{tt} E_i^* + (\gamma_i \lambda_i^{(1)} + \gamma_i^{(1)} \lambda_i) \nabla_{ttt} E_i^* + (\gamma_i \lambda_i^{(2)} + \gamma_i^{(1)} \lambda_i^{(1)} + \gamma_i^{(2)} \lambda_i) \nabla_{tttt} E_i^* + \dots = 0 \quad (109)$$

$$\frac{1}{\xi_i} \square_i^* H_i^* + \gamma_i \lambda_i \nabla_{tt} H_i^* + (\gamma_i \lambda_i^{(1)} + \gamma_i^{(1)} \lambda_i) \nabla_{ttt} H_i^* + (\gamma_i \lambda_i^{(2)} + \gamma_i^{(1)} \lambda_i^{(1)} + \gamma_i^{(2)} \lambda_i) \nabla_{tttt} H_i^* + \dots = 0 \quad (110)$$

Eqs.(109) and (110) are just equations of electric field and magnetic field for general dispersive media.

7.3 Applications

In this section, we discuss the propagation laws of electromagnetic waves in an one-order dispersive medium. The material tensors in Eqs.(101) and (102) are represented by the following matrices

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{11} \end{bmatrix}, \quad \boldsymbol{\mu} = \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{11} & 0 \\ 0 & 0 & \mu_{11} \end{bmatrix}, \quad \boldsymbol{\varepsilon}_1 = \begin{bmatrix} \varepsilon'_{11} & 0 & 0 \\ 0 & \varepsilon'_{11} & 0 \\ 0 & 0 & \varepsilon'_{11} \end{bmatrix}, \quad \boldsymbol{\mu}_1 = \begin{bmatrix} \mu'_{11} & 0 & 0 \\ 0 & \mu'_{11} & 0 \\ 0 & 0 & \mu'_{11} \end{bmatrix} \quad (111)$$

The eigen values and eigen vectors of those matrix are the following

$$\boldsymbol{\Lambda} = \text{diag}[\varepsilon_{11}, \varepsilon_{11}, \varepsilon_{11}], \quad \boldsymbol{\Lambda}_1 = \text{diag}[\varepsilon'_{11}, \varepsilon'_{11}, \varepsilon'_{11}] \quad (112)$$

$$\boldsymbol{\Pi} = \text{diag}[\mu_{11}, \mu_{11}, \mu_{11}], \quad \boldsymbol{\Pi}_1 = \text{diag}[\mu'_{11}, \mu'_{11}, \mu'_{11}] \quad (113)$$

$$\boldsymbol{\Phi} = \boldsymbol{\Psi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (114)$$

We can see from the above equations that there is only one eigen-space in isotropic one-order dispersive medium, which is a triple-degenerate one, and the space structure is the following

$$\boldsymbol{W}_{mag} = W_1^{(3)} [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \boldsymbol{\phi}_3], \quad \boldsymbol{W}_{ele} = W_1^{(3)} [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3] \quad (115)$$

where, $\boldsymbol{\phi}_1^* = \frac{\sqrt{3}}{3} \{1, 1, 1\}^T$, $\boldsymbol{\varphi}_1^* = \frac{\sqrt{3}}{3} \{1, 1, 1\}^T$, $\xi_1 = 1$. Thus the eigen-qualities and eigen-operators of isotropic one-order dispersive medium are known as same as Eqs. (88) - (90). The equations of electromagnetic wave in one-order dispersive medium become

$$(\partial_x^2 + \partial_y^2 + \partial_z^2) E_1^* = \frac{1}{c^2} \nabla_{tt} E_1^* + (\mu_{11} \varepsilon_{11}^{(1)} + \mu_{11}^{(1)} \varepsilon_{11}) \nabla_{ttt} E_1^* \quad (116)$$

$$(\partial_x^2 + \partial_y^2 + \partial_z^2)H_1^* = \frac{1}{c^2} \nabla_{tt} H_1^* + (\mu_{11} \epsilon_{11}^{(1)} + \mu_{11}^{(1)} \epsilon_{11}) \nabla_{ttt} H_1^* \quad (117)$$

in which c is the velocity of electromagnetic wave

$$c = \frac{1}{\sqrt{\mu_{11} \epsilon_{11}}} \quad (118)$$

Now, we discuss the propagation laws of a plane electromagnetic wave in x-axis. In this time, Eq.(116) becomes

$$\frac{\partial}{\partial x^2} E_1 = \frac{1}{c^2} \nabla_{tt} E_1 + (\mu_{11} \epsilon_{11}^{(1)} + \mu_{11}^{(1)} \epsilon_{11}) \nabla_{ttt} E_1 \quad (119)$$

Let

$$E_1 = A \exp[i(kx - \omega t)] \quad (120)$$

Substituting the above into Eq.(119), we have

$$k^2 = \frac{\omega^2}{c^2} - i\omega^3 (\mu_{11} \epsilon_{11}^{(1)} + \mu_{11}^{(1)} \epsilon_{11}) \quad (121)$$

From the above, we can get

$$\bar{k} = k_1 + ik_2 \quad (122)$$

where

$$k_1 = \frac{\omega}{c} \left[\frac{1 + \left(1 + c^4 (\mu_{11} \epsilon_{11}^{(1)} + \mu_{11}^{(1)} \epsilon_{11})^2 \omega^2\right)^{\frac{1}{2}}}{2} \right]^{\frac{1}{2}}, \quad k_2 = \frac{\omega}{c} \left[\frac{-1 + \left(1 + c^4 (\mu_{11} \epsilon_{11}^{(1)} + \mu_{11}^{(1)} \epsilon_{11})^2 \omega^2\right)^{\frac{1}{2}}}{2} \right]^{\frac{1}{2}}.$$

Then, the solutions of electromagnetic waves are

$$E_1 = A e^{-k_2 x} \cdot e^{i(k_1 x - \omega t)} = \bar{A} \cdot e^{i(k_1 x - \omega t)} \quad (123)$$

It is an attenuated sub-waves.

8. Electromagnetic waves in chiral media

8.1 The constitutive equation of chiral media

The constitutive equations of chiral media are the following

$$\mathbf{D} = \boldsymbol{\epsilon} \cdot \mathbf{E} - \boldsymbol{\chi} \cdot \nabla_t \mathbf{H} \quad (124)$$

$$\mathbf{B} = \boldsymbol{\chi} \cdot \nabla_t \mathbf{E} + \boldsymbol{\mu} \cdot \mathbf{H} \quad (125)$$

where χ is the matrix of chirality parameter, and a symmetric one.

Substituting Eqs. (5) and (6) into Eqs. (124) and (125), respectively, and multiplying them with the transpose of modal matrix in the left, we have

$$\mathbf{D}^* = \Phi^T \boldsymbol{\varepsilon} \Phi \mathbf{E}^* - \Phi^T \chi \Psi \nabla_t \mathbf{H}^* \quad (126)$$

$$\mathbf{B}^* = \Psi^T \chi \Phi \nabla_t \mathbf{E}^* + \Psi^T \boldsymbol{\mu} \Psi \mathbf{H}^* \quad (127)$$

Let $\Gamma = \Psi^T \chi \Phi$, that is a coupled chiral matrix, and using Eq. (4), we have

$$\mathbf{D}^* = \Lambda \mathbf{E}^* - \Gamma^T \nabla_t \mathbf{H}^* \quad (128)$$

$$\mathbf{B}^* = \Gamma^T \nabla_t \mathbf{E}^* + \Pi \mathbf{H}^* \quad (129)$$

For most chiral, $\Gamma = \text{diag}[\zeta_1, \zeta_2, \zeta_3]$. Then we have

$$D_i^* = \lambda_i E_i^* - \zeta_i \nabla_t H_i^* \quad (130)$$

$$B_i^* = \zeta_i \nabla_t E_i^* + \gamma_i H_i^* \quad (131)$$

Eqs.(130) and (131) are just the modal constitutive equations for anisotropic chiral media.

8.2 Eigen equations of electromagnetic waves in chiral media

Substituting Eqs. (130) and (131) into Eqs. (23) and (24), respectively, we have

$$\{\Delta_i^*\} E_i^* = -\nabla_t \{\phi_i\} (\zeta_i \nabla_t E_i^* + \gamma_i H_i^*) \quad i = 1, 2, 3 \quad (132)$$

$$\{\Delta_i^*\} H_i^* = \nabla_t \{\phi_i\} (\lambda_i E_i^* - \zeta_i \nabla_t H_i^*) \quad i = 1, 2, 3 \quad (133)$$

From them, we can get

$$\square_i^* E_i^* + \xi_i \zeta_i^2 \nabla_{\text{ttt}} E_i^* + (2\zeta_i \partial_i^* + \xi_i \lambda_i \gamma_i) \nabla_{\text{tt}} E_i^* = 0 \quad i = 1, 2, 3 \quad (134)$$

$$\square_i^* H_i^* + \xi_i \zeta_i^2 \nabla_{\text{ttt}} H_i^* + (2\zeta_i \partial_i^* + \xi_i \lambda_i \gamma_i) \nabla_{\text{tt}} H_i^* = 0 \quad i = 1, 2, 3 \quad (135)$$

where $\xi_i = \{\phi_i\}^T \cdot \{\phi_i\} = 1$, $\partial_i^* = \{\Delta_i^*\}^T \cdot \{\phi_i\}$. Eqs.(134) and (135) are just equations of electric field and magnetic field for chiral media.

8.3 Applications

In this section, we discuss the propagation laws of electromagnetic waves in an isotropic chiral medium. The material tensors in Eqs.(124) and (125) are represented by the following matrices

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{11} \end{bmatrix}, \quad \boldsymbol{\mu} = \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{11} & 0 \\ 0 & 0 & \mu_{11} \end{bmatrix}, \quad \chi = \begin{bmatrix} \chi_{11} & 0 & 0 \\ 0 & \chi_{11} & 0 \\ 0 & 0 & \chi_{11} \end{bmatrix} \quad (136)$$

The eigen values and eigen vectors of those matrix are the following

$$\mathbf{A} = \text{diag}[\varepsilon_{11}, \varepsilon_{11}, \varepsilon_{11}], \quad \mathbf{\Pi} = \text{diag}[\mu_{11}, \mu_{11}, \mu_{11}], \quad \mathbf{\Gamma} = \text{diag}[\chi_{11}, \chi_{11}, \chi_{11}] \quad (137)$$

$$\mathbf{\Phi} = \mathbf{\Psi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (138)$$

We can see from the above equations that there is only one eigen-space in isotropic medium, which is a triple-degenerate one, and the space structure is the following

$$\mathbf{W}_{mag} = W_1^{(3)}[\phi_1, \phi_2, \phi_3], \quad \mathbf{W}_{ele} = W_1^{(3)}[\varphi_1, \varphi_2, \varphi_3] \quad (139)$$

where, $\phi_1^* = \frac{\sqrt{3}}{3}\{1, 1, 1\}^T$, $\varphi_1^* = \frac{\sqrt{3}}{3}\{1, 1, 1\}^T$, $\xi_1 = 1$.

Then the eigen-qualities and eigen-operators of isotropic chiral medium are respectively shown as follows

$$E_1^* = \frac{\sqrt{3}}{3}(E_1 + E_2 + E_3), \quad H_1^* = \frac{\sqrt{3}}{3}(H_1 + H_2 + H_3) \quad (140)$$

$$\square_1^* = \frac{1}{3}[-(\partial_x^2 + \partial_y^2 + \partial_z^2)], \quad \partial_1^* = \frac{\sqrt{3}}{3}(\partial_x + \partial_y + \partial_z) \quad (141)$$

So, the equations of electromagnetic wave in isotropic chiral medium become

$$(\partial_x^2 + \partial_y^2 + \partial_z^2)E_1^* = \chi_{11}^2 \nabla_{\text{int}} E_1^* + \left[2\sqrt{3}\chi_{11}(\partial_x + \partial_y + \partial_z) + \frac{1}{c^2} \right] \nabla_{\text{tt}} E_1^* \quad (142)$$

$$(\partial_x^2 + \partial_y^2 + \partial_z^2)H_1^* = \chi_{11}^2 \nabla_{\text{int}} H_1^* + \left[2\sqrt{3}\chi_{11}(\partial_x + \partial_y + \partial_z) + \frac{1}{c^2} \right] \nabla_{\text{tt}} H_1^* \quad (143)$$

where, c is the velocity of electromagnetic wave

$$c = \frac{1}{\sqrt{\mu_{11}\varepsilon_{11}}} \quad (144)$$

Now, we discuss the propagation laws of a plane electromagnetic wave in x-axis. In this time, Eq.(142) becomes

$$\frac{\partial}{\partial x^2} E_1 = \chi_{11}^2 \nabla_{\text{int}} E_1 + \left[2\sqrt{3}\chi_{11} \frac{\partial}{\partial x} + \frac{1}{c^2} \right] \nabla_{\text{tt}} E_1 \quad (145)$$

Let

$$E_1 = A \exp[i(kx - \omega t)] \quad (146)$$

Substituting the above into Eq.(145), we have

$$k^2 = \left(\frac{1}{c^2} + i2\sqrt{3}\chi_{11}k \right) \omega^2 - \chi_{11}^2 \omega^4 \quad (147)$$

or

$$k^2 - i2\sqrt{3}\chi_{11}\omega^2 k + (\chi_{11}^2 \omega^2 - \mu_{11}\epsilon_{11}) \omega^2 = 0 \quad (148)$$

1. when $\omega^2 - \frac{\mu_{11}\epsilon_{11}}{\chi_{11}^2} < 0$

By Eq.(148), we have

$$k_1 = k'_1 + ik''_1 = i\chi_{11}\omega^2 \left[\sqrt{3} + (x + iy) \right] \quad (149)$$

$$k_2 = k'_2 + ik''_2 = i\chi_{11}\omega^2 \left[\sqrt{3} - (x + iy) \right] \quad (150)$$

From them, we can get

$$k'_1 = -\chi_{11}\omega^2 y, \quad k''_1 = (\sqrt{3} + x)\chi_{11}\omega^2 \quad (151)$$

$$k'_2 = \chi_{11}\omega^2 y, \quad k''_2 = (\sqrt{3} - x)\chi_{11}\omega^2 \quad (152)$$

$$\text{where, } x = \frac{\sqrt{2}}{2} \sqrt{3 + \left[9 + \frac{\left(\frac{\mu_{11}\epsilon_{11}}{\chi_{11}^2} - \omega^2 \right)^2}{\omega^4} \right]^{\frac{1}{2}}}, \quad y = \frac{\sqrt{2}}{2} \sqrt{-3 + \left[9 + \frac{\left(\frac{\mu_{11}\epsilon_{11}}{\chi_{11}^2} - \omega^2 \right)^2}{\omega^4} \right]^{\frac{1}{2}}}.$$

Then, the solution of electromagnetic waves is the following

$$E_1 = A_1 e^{-k''_1 x} \cdot e^{i(k'_1 x - \omega t)} + A_2 e^{-k''_2 x} \cdot e^{i(k'_2 x - \omega t)} \quad (153)$$

It is composed of two attenuated sub-waves.

2. when $\omega^2 - \frac{\mu_{11}\epsilon_{11}}{\chi_{11}^2} > 0$

By Eq.(148), we have

$$k_1 = k'_1 + ik''_1 = i\chi_{11}\omega^2 \left[\sqrt{3} + \sqrt{3 + \frac{1}{\omega^2} \left(\omega^2 - \frac{\mu_{11}\epsilon_{11}}{\chi_{11}^2} \right)} \right] \quad (154)$$

$$k_2 = k'_2 + ik''_2 = i\chi_{11}\omega^2 \left[\sqrt{3} - \sqrt{3 + \frac{1}{\omega^2} \left(\omega^2 - \frac{\mu_{11}\epsilon_{11}}{\chi_{11}^2} \right)} \right] \quad (155)$$

where

$$k'_1 = 0, k''_1 = \chi_{11} \omega^2 \left[\sqrt{3} + \sqrt{3 + \frac{1}{\omega^2} \left(\omega^2 - \frac{\mu_{11} \epsilon_{11}}{\chi_{11}^2} \right)} \right] \quad (156)$$

$$k'_2 = -\chi_{11} \omega^2 \left[\sqrt{3 + \frac{1}{\omega^2} \left(\omega^2 - \frac{\mu_{11} \epsilon_{11}}{\chi_{11}^2} \right)} - \sqrt{3} \right], k''_2 = 0 \quad (157)$$

Then, the solution of electromagnetic waves is the following

$$E_1 = A_1 e^{-k'_1 x} e^{-i\omega t} + A_2 e^{i(k'_2 x - \omega t)} \quad (158)$$

It is seen that there only exists an electromagnetic sub-wave in opposite direction.

3. when $\omega^2 - \frac{\mu_{11} \epsilon_{11}}{\chi_{11}^2} = 0$

By Eq.(148), we have

$$k = k' + ik'' = i2\sqrt{3}\chi_{11}\omega^2 \quad (159)$$

where

$$k' = 0, k'' = 2\sqrt{3}\chi_{11}\omega^2 \quad (160)$$

Then, the solution of electromagnetic waves is the following

$$E = A e^{-k'' x} \cdot e^{-i\omega t} \quad (161)$$

No electromagnetic sub-waves exist now.

9. Conclusion

In this chapter, we construct the standard spaces under the physical presentation by solving the eigen-value problem of the matrixes of dielectric permittivity and magnetic permeability, in which we get the eigen dielectric permittivity and eigen magnetic permeability, and the corresponding eigen vectors. The former are coordinate-independent and the latter are coordinate-dependent. Because the eigen vectors show the principal directions of electromagnetic media, they can be used as the standard spaces. Based on the spaces, we get the modal equations of electromagnetic waves for anisotropic media, bi-anisotropic media, dispersive medium and chiral medium, respectively, by converting the classical Maxwell's vector equation to the eigen Maxwell's scalar equation, each of which shows the existence of an electromagnetic sub-wave, and its propagation velocity, propagation direction, polarization direction and space pattern are completely determined in the equations. Several novel results are obtained for anisotropic media. For example, there is only one kind of electromagnetic wave in isotropic crystal, which is identical with the classical result; there are two kinds of electromagnetic waves in uniaxial crystal; three kinds of electromagnetic waves in biaxial crystal and three kinds of distorted

electromagnetic waves in monoclinic crystal. Also for bi-anisotropic media, there exist two electromagnetic waves in Dzyaloshinskii's bi-anisotropic media, and the electromagnetic waves in bi-anisotropic medium will go faster duo to the bi-coupling between electric field and magnetic one. For isotropic dispersive medium, the electromagnetic wave is an attenuated sub-waves. And for chiral medium, there exist different propagating states of electromagnetic waves in different frequency band, for example, in low frequency band, the electromagnetic waves are composed of two attenuated sub-waves, in high frequency band, there only exists an electromagnetic sub-wave in opposite direction, and in the critical point, no electromagnetic can propagate. All of these new theoretical results need to be proved by experiments in the future.

10. Appendix A: Proof of the eigenmode of electromagnetic operator matrix

The Maxwell's equation of anisotropic dielectrics is the following

$$[\square]\{H\} = -\nabla_t^2 \mu_0 [\varepsilon]\{H\} \quad (\text{A1})$$

Using the representation transform relationship Eq. (6), we have

$$[\square][\Phi]\{H^*\} = -\nabla_t^2 \mu_0 [\varepsilon][\Phi]\{H^*\} \quad (\text{A2})$$

Substituting the spectral decomposition matrix of dielectric permittivity Eq. (4) into above, we have

$$[\square][\Phi]\{H^*\} = -\nabla_t^2 \mu_0 [\Phi][\Lambda]\{H^*\} \quad (\text{A3})$$

Comparing the both sides of above equation, we can get

$$[\square][\Phi] = -\nabla_t^2 \mu_0 [\Phi][\Lambda] \quad (\text{A4})$$

Multiplying the both sides of above with the transpose of modal matrix in the left, we have

$$[\Phi]^T [\square][\Phi] = -\nabla_t^2 \mu_0 [\Lambda] \quad (\text{A5})$$

It is seen that the right side above is a diagonal matrix, which shows that the electromagnetic operators matrix can also be spectrally decomposed in standard spaces, then we get

$$[\square^*] = -\nabla_t^2 \mu_0 [\Lambda] \quad (\text{A6})$$

Rewriting above in the form of scalar, we have

$$\square_i^* = -\nabla_t^2 \mu_0 \lambda_i \quad (\text{A7})$$

11. Appendix B: Spectrally decomposition of lossy matrix

The Helmholtz's free energy of electromagnetic system with lossy property is the following

$$\psi = \psi(\mathbf{D}, \mathbf{B}, \mathbf{D}^d) = \frac{1}{2}(\mathbf{D} - \mathbf{D}^d)\boldsymbol{\epsilon}^{-1}(\mathbf{D} - \mathbf{D}^d) + \frac{1}{2}\mathbf{B}\boldsymbol{\mu}^{-1}\mathbf{B} \quad (\text{B1})$$

Differentiating the above with lossy variable, and using Eq.(1), we have

$$\mathbf{R} = -\frac{\partial \psi}{\partial \mathbf{D}^d} = (\mathbf{D} - \mathbf{D}^d)\boldsymbol{\epsilon}^{-1} = \mathbf{E} \quad (\text{B2})$$

According to the Onsager's principle, for the process of closing to equilibrium, the rate of lossy variable is proportion to the driving force, that is

$$\frac{\partial \psi}{\partial D_i^d} + \beta_{ij} \frac{dD_j^d}{dt} = 0 \quad (\text{B3})$$

where, $\beta_{ij} = \beta_{ji}$ is the general friction coefficient. Rewriting the above in matrix form, we have

$$\{R\} = [B]\{\dot{D}^d\} \quad (\text{B4})$$

Projecting the lossy electric displacement vector \mathbf{D} into the standard spaces of the physical presentation, we get

$$\{D^d\} = a\{\varphi\} \quad (\text{B5})$$

Using Eq.(B2), we have

$$\{R\} = \omega \dot{a}\{\varphi\} \quad (\text{B6})$$

Substituting Eqs. (B5) and (B6) into Eq. (B4), the condition of non-zero solution to α is the following

$$([B] - \omega[I])\{\varphi\} = 0 \quad (\text{B7})$$

It is seen that the general friction coefficient matrix can also be spectrally decomposed in standard spaces, so we have

$$[B] = [\Phi][\Omega][\Phi]^T \quad (\text{B8})$$

Comparing Eq. (B4) with Eq. (74), it is known that we can also spectrally decompose the lossy matrix in standard spaces

$$[\sigma] = [\Phi]^T [\Gamma][\Phi] \quad (\text{B9})$$

where, $\boldsymbol{\Gamma} = \boldsymbol{\Omega}^{-1} = \text{diag}[\eta_1, \eta_2, \eta_3]$.

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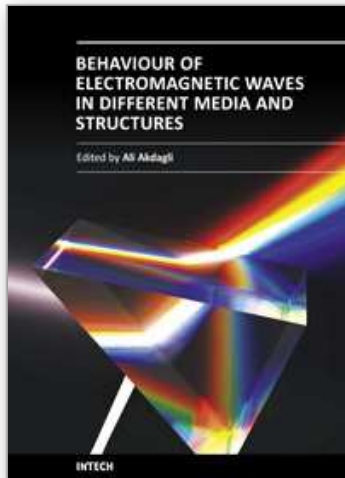
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