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A Generalized Algebraic Model for Optimizing Inventory Decisions in a Centralized or Decentralized Three-Stage Multi-Firm Supply Chain with Complete Backorders for Some Retailers

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1. Introduction

Supply chain management has enabled numerous firms to enjoy great advantages by integrating all activities associated with the flow of material, information and capital between suppliers of raw materials and the ultimate customers. The benefits of a property managed supply chain include reduced costs, faster product delivery, greater efficiency, and lower costs for both the business and its customers. These competitive advantages are achieved through improved supply chain relationships and tightened links between chain partners such as suppliers, manufacturing facilities, distribution centers, wholesalers, and end users (Berger *et al.* 2004). Besides integrating all members in a supply chain, to improve the traditional method of solving inventory problems is also necessary. Without using derivatives, Grubbström (1995) first derived the optimal expressions for the classical economic order quantity (EOQ) model using the unity decomposition method, which is an algebraic approach. Adopting this method, Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) respectively derived the optimal expressions for an EOQ and economic production quantity (EPQ) model with complete backorders. In this chapter, a generalized model for a three-stage multi-firm production-inventory integrated system is solved using the methods of complete squares and perfect squares adopted in Leung (2008a,b, 2009a,b, 2010a,b), which are also algebraic approaches; whereby optimal expressions of decision variables and the objective function are derived.

Assume that there is an uninterrupted production run. In the case of lot streaming in each of the upstream stages, shipments can be made from a production batch even before the whole batch is finished. However, some or all suppliers/manufacturers/assemblers cannot accommodate lot streaming because of regulations, material handling equipment, or production restrictions (Silver *et al.* 1998, p. 657). Without lot streaming, no shipments can be made from a production batch until the whole batch is finished. Sucky (2005) discussed the integrated single-vendor single-buyer system, with and without lot streaming, in detail.

In the inventory/production literature, all researchers have constructed their models under the assumption of either allowing lot streaming for all firms involving production (Khouia 2003) or not (Ben-Daya and Al-Nassar 2008), or both extremes (Sucky 2005, and Leung 2010a). The main purpose of the chapter is twofold: First, we build a generalized model incorporating a mixture of the two extremes and allowing compete backorders penalized by linear (i.e. time-dependent) shortage costs, and solve it algebraically. As a result, we can deduce and solve such special models as Khouja (2003), Cárdenas-Barrón (2007), Ben-Daya and Al-Nassar's (2008), Seliaman and Ahmad (2009), and Leung (2009a, 2010a,b). In addition, with appropriate assignments as in Section 5 of Leung (2010a), we can also deduce and solve other special models: Yang and Wee (2002), Wu and Ouyang (2003) or Wee and Chung (2007), and Chung and Wee (2007). Second, we derive expressions for sharing the coordination benefits based on Goyal's (1976) scheme, and introduce a further sharing scheme.

Some good review articles exist that provide an extensive overview of the topic under study and can be helpful as guidance through the literature. We mention surveys by Goyal and Gupta (1989), Goyal and Deshmukh (1992), Bhatnagar *et al.* (1993), Maloni and Benton (1997), Sarmah *et al.* (2006), and Ben-Daya *et al.* (2008). The well-known models of Goyal (1976), Banerjee (1986), Lu (1995), and Hill (1997) are extended by Ben-Daya, *et al.* (2008) as well. Other recently related articles include Chan and Kingsman (2007), Chiou *et al.* (2007), Cha *et al.* (2008), Leng and Parlat (2009 a,b), and Leng and Zhu (2009).

2. Assumptions, symbols and designations

The integrated production-inventory model is developed under the following assumptions:
1. A single item is considered.
2. There are two or more stages.
3. Production and demand rates (with the former greater than the latter) are independent of production or order quantity, and are constant.
4. Unit cost is independent of quantity purchased, and an order quantity will not vary from one cycle to another.
5. Neither a wait-in-process unit, nor a defective-in-transit unit, is considered.
6. Each upstream firm implements perfect inspection to guarantee that defective units are not delivered to any retailer. Three types of inspection suggested in Wee and Chung (2007) are executed .
7. Each type of inspection costs is different for all firms in each stage involving production.
8. Setup or ordering costs are different for all firms in the chain.
9. Holding costs of raw materials are different from those of finished products.
10. Holding costs of raw materials are different for all firms in the chain.
11. Holding costs of finished goods are different for all firms in the chain.
12. Lot streaming is allowed for some firms but no lot streaming is allowed for the rest in each stage involving production.
13. Shortages are allowed for some/all retailers and are completely backordered, and all backorders are made up at the beginning of the next order cycle.
14. All firms have complete information of each other.
15. The number of shipments of each supplier, manufacturer, assembler or retailer is a positive integer.
16. The planning horizon is infinite.

The following symbols (some as defined in Leung 2010b) are used in the expression of the joint total relevant cost per year.

D_{ij} = demand rate of firm $j\ (=1,\cdots,J_i)$ in stage $i\ (=1,\cdots,n)$ [units per year]

P_{ij} = production rate of firm $j\ (=1,\cdots,J_i)$ in stage $i\ (=1,\cdots,n-1)$ [units per year]

$b_{nj} \equiv b_j$ = linear backordering cost of finished goods of firm $j\ (=1,\cdots,J_n)$ in stage n, where $0 < b_j \le \infty$ (Note that $b_j = 0$ means that no costs of operating an inventory system are incurred; this is not realistic and thus excluded, and $b_j = \infty$ means that the penalty of incurring a backorder is too large; this is pragmatic and thus included.) [$ per unit per year]

g_{ij} = holding cost of incoming raw material of firm $j\ (=1,\cdots,J_i)$ in stage $i\ (=1,\cdots,n-1)$ [$ per unit per year]

h_{ij} = holding cost of finished goods of firm $j\ (=1,\cdots,J_i)$ in stage $i(=1,...n)$ [$ per unit per year]

S_{ij} = setup or ordering cost of firm $j\ (=1,\cdots,J_i)$ in stage $i\ (=1,\cdots,n)$ [$ per cycle]

A_{ij} = inspection cost per cycle of firm $j\ (=1,\cdots,J_i)$ in stage $i\ (=1,\cdots,n-1)$ [$ per cycle]

B_{ij} = inspection cost per delivery of firm $j\ (=1,\cdots,J_i)$ in stage $i\ (=1,\cdots,n-1)$ [$ per delivery]

C_{ij} = inspection cost per unit of firm $j\ (=1,\cdots,J_i)$ in stage $i\ (=1,\cdots,n-1)$ [$ per unit]

For a centralized supply chain (or the integrated approach), we have

$t_{nj} \equiv t_j$ = backordering time of firm $j\ (=1,\cdots,J_n)$ in stage n; hereafter called retailer $j\ (=1,\cdots,J_n)$

(t_j are decision variables, each with *non-negative* real values) [a fraction of a year]

$T_{nj}^{(b)} = T_n$ = basic cycle time of retailer $j\ (=1,\cdots,J_n)$

(T_n is a decision variable with *non-negative* real values) [a fraction of a year]

$T_{ij} = T_n \prod_{k=i}^{n-1} K_k = T_n \prod_{k=i}^{n} K_k$ with $K_n \equiv 1$

= integer multiplier cycle time of firm $j\ (=1,\cdots,J_i)$ in stage $i\ (=1,\cdots,n-1)$

($K_1,\cdots,K_{n-1}$ are decision variables, each with *positive* integral values) [a fraction of a year]

TC_{ij} = total relevant cost of firm $j\ (=1,\cdots,J_i)$ in stage $i\ (=1,\cdots,n)$ [$ per year]

$JTC(K_1,\cdots,K_{n-1},T_n,t_j) = \sum_{i=1}^{n} \sum_{j=1}^{J_i} TC_{ij}$ = joint total relevant cost as a function of $K_1,\cdots,K_{n-1}$, T_n and t_j

(the objective function) [$ per year]

For a decentralized supply chain (or the independent approach), we have

$\mu_{nj} \equiv \mu_j$ = backordering time of retailer $j\ (=1,\cdots,J_n)$

(μ_j are decision variables, each with *non-negative* real values) [a fraction of a year]

$\tau_{nj}^{(b)} \equiv \tau_n$ = basic cycle time of retailer $j\ (=1,\cdots,J_n)$

(τ_n is a decision variable with *non-negative* real values) [a fraction of a year]

$\tau_{ij} = \tau_n \prod_{k=i}^{n-1} \lambda_k = \tau_n \prod_{k=i}^{n} \lambda_k$ with $\lambda_n \equiv 1$

= integer multiplier cycle time of firm j $(=1,\cdots,J_i)$ in stage i $(=n-1,\cdots,1)$
($\lambda_{n-1},\cdots,\lambda_1$ are decision variables, each with *positive* integral values) [a fraction of a year]

$TC(\tau_n,\mu_j)$ = total relevant cost of all retailers [$ per year]

$TC(\lambda_i)$ = total relevant cost of all firms in stage i $(=n-1,\cdots,1)$ [$ per year]

To simplify the presentation of the subsequent mathematical expressions, we designate

$$\chi_{ij} = \begin{cases} 0 & \text{without lot streaming} \\ 1 & \text{with lot streaming} \end{cases} \text{ and } \bar{\chi}_{ij} = 1 - \chi_{ij} \text{ for } i=1,\cdots,n-1; j=1,\cdots,J_i, \tag{1}$$

$$\varphi_{ij} = \frac{D_{ij}}{P_{ij}} \text{ and } \bar{\varphi}_{ij} = 1 - \varphi_{ij} \text{ for } i=1,\cdots,n-1; j=1,\cdots,J_i, \tag{2}$$

where the former represents the proportion of production that goes to meet demand and the latter reflects the proportion of production allocated to inventory,

$$G_0 \equiv 0 \text{ and } G_i = \sum_{j=1}^{J_i} D_{ij}h_{ij}[\chi_{ij}(\varphi_{ij} - \bar{\varphi}_{ij}) - \bar{\chi}_{ij}] \text{ for } i=1,\cdots,n-1, \tag{3}$$

$$H_i = \sum_{j=1}^{J_i} D_{ij}[\varphi_{ij}g_{ij} + \chi_{ij}\bar{\varphi}_{ij}h_{ij} + \bar{\chi}_{ij}h_{ij}(1+\varphi_{ij})] + G_{i-1} \text{ for } i=1,\cdots,n-1, \tag{4}$$

$$H_n^{(b)} = \sum_{j=1}^{J_n} \frac{D_{nj}b_jh_{nj}}{b_j + h_{nj}} + G_{n-1}, \tag{5}$$

$$S_{iJ} = \sum_{j=1}^{J_i} S_{ij} \text{ for } i=1,\cdots,n, \tag{6}$$

$$A_{iJ} = \sum_{j=1}^{J_i} A_{ij} \text{ for } i=1,\cdots,n-1, \tag{7}$$

$$B_{iJ} = \sum_{j=1}^{J_i} B_{ij} \text{ for } i=1,\cdots,n-1, \tag{8}$$

$$C_{iJ} = \sum_{j=1}^{J_i} C_{ij}D_{ij} \text{ for } i=1,\cdots,n-1, \tag{9}$$

$$\alpha_1 = S_{1J} + A_{1J}, \tag{10}$$

$$\alpha_i = S_{iJ} + A_{iJ} + B_{i-1,J} \text{ for } i=2,\cdots,n-1, \tag{11}$$

$$\alpha_n = S_{nJ} + B_{n-1,J}, \tag{12}$$

and

$$\beta_n = \sum_{i=1}^{n-1} C_{iJ}. \tag{13}$$

Assume that there is an uninterrupted production run. In the case of lot streaming in stage $i \ (= 1, \cdots, n-1)$, shipments can be made from a production batch even before the whole batch is finished. According to Joglekar (1988, pp. 1397-8), the average inventory with lot streaming, for example, in stage 2 of a 3-stage supply chain, is $\frac{T_3 D_{2j}}{2}[\varphi_{2j} + (K_2 - 1)\overline{\varphi}_{2j}]$ units, which is the same as equation (7) of Ben-Daya and Al-Nassar (2008).

Without lot streaming, no shipments can be made from a production batch until the whole batch is finished. The opportunity of lot streaming affects supplier's average inventory. According to Goyal (1988, p. 237), the average inventory without lot streaming, for example, in stage 2 of a 3-stage supply chain, is $\frac{T_3 D_{2j}}{2}(\varphi_{2j} K_2 + K_2 - 1)$ units, which is the same as term 2 in equation (5) of Khouja (2003).

The total relevant cost per year of firm $j \ (= 1, \cdots, J_i)$ in stage $i \ (= 1, \cdots, n-1)$ is given by

$$
\begin{aligned}
TC_{ij} ={}& \frac{\prod_{k=i}^{n} K_k \cdot T_n D_{ij}^2}{2P_{ij}} \cdot (g_{ij} + \overline{\chi}_{ij} h_{ij}) + \frac{(\prod_{k=i}^{n} K_k - \prod_{k=i+1}^{n} K_k) T_n D_{ij}}{2} \cdot h_{ij} \overline{\chi}_{ij} \\
&+ \frac{\prod_{k=i+1}^{n} K_k \cdot T_n D_{ij}^2}{2P_{ij}} \cdot \chi_{ij} h_{ij} + \frac{(\prod_{k=i}^{n} K_k - \prod_{k=i+1}^{n} K_k) T_n D_{ij}(1 - \frac{D_{ij}}{P_{ij}})}{2} \cdot h_{ij} \chi_{ij} \\
&+ \frac{S_{ij}}{\prod_{k=i}^{n} K_k \cdot T_n} + \frac{A_{ij}}{\prod_{k=i}^{n} K_k \cdot T_n} + \frac{B_{ij}}{\prod_{k=i+1}^{n} K_k \cdot T_n} + C_{ij} D_{ij},
\end{aligned} \tag{14}
$$

where without lot streaming, term 1 represents the sum of holding cost of raw material while they are being converted into finished goods and the cost of holding finished goods during the production process, and term 2 represents the holding cost of finished goods after production; but with lot streaming, term 1 represents the sum of holding cost of raw material while they are being converted into finished goods, and terms 3 and 4 represent the holding cost of finished goods during a production cycle; term 5 represents the setup cost, and the last three terms represent the sum of inspection costs.

Incorporating designation (2) in equation (14) yields

$$
\begin{aligned}
TC_{ij} ={}& \frac{D_{ij}[\varphi_{ij} g_{ij} + \chi_{ij} \overline{\varphi}_{ij} h_{ij} + \overline{\chi}_{ij} h_{ij}(1 + \phi_{ij})] T_n \prod_{k=i}^{n} K_k}{2} + \frac{D_{ij} h_{ij}[\chi_{ij}(\varphi_{ij} - \overline{\varphi}_{ij}) - \overline{\chi}_{ij}] T_n \prod_{k=i+1}^{n} K_k}{2} \\
&+ \frac{S_{ij} + A_{ij}}{T_n \prod_{k=i}^{n} K_k} + \frac{B_{ij}}{T_n \prod_{k=i+1}^{n} K_k} + C_{ij} D_{ij} \quad \text{for } i = 1, \cdots, n-1; \ j = 1, \cdots, J_i.
\end{aligned} \tag{15}
$$

The total relevant cost per year of retailer $j \ (= 1, \cdots, J_n)$, each associated with complete backorders and each backorder penalized by a linear cost, is given by

$$TC_{nj} = \frac{D_{nj}h_{nj}(T_n - t_j)^2}{2T_n} + \frac{D_{nj}b_j t_j^2}{2T_n} + \frac{S_{nj}}{T_n} \text{ for } j = 1, \cdots, J_n, \quad (16)$$

where term 1 represents the holding cost of finished goods, term 2 represents the backordering cost of finished goods, and term 3 represents the ordering cost.
Expanding equation (16) and grouping like terms yield

$$TC_{nj} = \frac{D_{nj}(b_j + h_{nj})}{2T_n}\left[t_j^2 - \frac{2h_{nj}T_n t_j}{b_j + h_{nj}} \right] + \frac{D_{nj}h_{nj}T_n}{2} + \frac{S_{nj}}{T_n}.$$

Using the complete squares method (by taking half the coefficient of v_j) advocated in Leung (2008a,b, 2010a), we have

$$TC_{nj} = \frac{D_{nj}(b_j + h_{nj})}{2T_n}\left(t_j - \frac{h_{nj}T_n}{b_j + h_{nj}} \right)^2 - \frac{D_{nj}h_{nj}^2 T_n}{2(b_j + h_{nj})} + \frac{D_{nj}h_{nj}T_n}{2} + \frac{S_{nj}}{T_n}$$

$$= \frac{D_{nj}(b_j + h_{nj})}{2T_n}\left(t_j - \frac{h_{nj}T_n}{b_j + h_{nj}} \right)^2 + \frac{D_{nj}b_j h_{nj}T_n}{2(b_j + h_{nj})} + \frac{S_{nj}}{T_n}. \quad (17)$$

3. An algebraic solution to an integrated model of a three-stage multi-firm supply chain

Incorporating designations (3) to (9) with $n = 3$ in equations (15) and (17) yield the total relevant cost per year in stage i ($= 1, 2, 3$) given by

$$\sum_{j=1}^{J_1} TC_{1j} = \frac{H_1 K_1 K_2 T_3}{2} + \frac{G_1 K_2 T_3}{2} + \frac{S_{1J} + A_{1J}}{K_1 K_2 T_3} + \frac{B_{1J}}{K_2 T_3} + C_{1J}, \quad (18)$$

$$\sum_{j=1}^{J_2} TC_{2j} = \frac{(H_2 - G_1)K_2 T_3}{2} + \frac{G_2 T_3}{2} + \frac{S_{2J} + A_{2J}}{K_2 T_3} + \frac{B_{2J}}{T_3} + C_{2J}, \quad (19)$$

and

$$\sum_{j=1}^{J_3} TC_{3j} = \frac{1}{2T_3}\sum_{j=1}^{J_3} D_{3j}(b_j + h_{3j})\left(t_j - \frac{h_{3j}T_3}{b_j + h_{3j}} \right)^2 + \frac{T_3}{2}\sum_{j=1}^{J_3}\frac{D_{3j}b_j h_{3j}}{(b_j + h_{3j})} + \frac{S_{3J}}{T_3}$$

$$= \frac{(H_3^{(b)} - G_2)T_3}{2} + \frac{S_{3J}}{T_3} + \frac{1}{2T_3}\sum_{j=1}^{J_3} D_{3j}(b_j + h_{3j})\left(t_j - \frac{h_{3j}T_3}{b_j + h_{3j}} \right)^2. \quad (20)$$

The joint total relevant cost per year for the supply chain integrating multiple suppliers $(i = 1; j = 1, \cdots, J_1)$, multiple manufacturers $(i = 2; j = 1, \cdots, J_2)$ and multiple retailers $(i = 3; j = 1, \cdots, J_3)$ is given by

$$JTC(K_1, K_2, T_3, t_j) = \sum_{j=1}^{J_1} TC_{1j} + \sum_{j=1}^{J_2} TC_{2j} + \sum_{j=1}^{J_3} TC_{3j} . \tag{21}$$

Substituting equations (18) to (20) in (21) and incorporating designations (10) to (13) with $n = 3$ yield

$$JTC(K_1, K_2, T_3, t_j) = \frac{1}{T_3}\left(\frac{\alpha_1}{K_1 K_2} + \frac{\alpha_2}{K_2} + \alpha_3\right) + T_3\left(\frac{H_1 K_1 K_2 + H_2 K_2 + H_3^{(b)}}{2}\right)$$
$$+ \frac{1}{2T_3}\sum_{j=1}^{J_3} D_{3j}(b_j + h_{3j})\left(t_j - \frac{h_{3j} T_3}{b_j + h_{3j}}\right)^2 + \beta_3. \tag{22}$$

Adopting the perfect squares method advocated in Leung (2008a, p. 279) to terms 1 and 2 of equation (22), we have

$$JTC(K_1, K_2, T_3, t_j) = \left[\sqrt{\frac{1}{T_3}\left(\frac{\alpha_1}{K_1 K_2} + \frac{\alpha_2}{K_2} + \alpha_3\right)} - \sqrt{T_3\left(\frac{H_1 K_1 K_2 + H_2 K_2 + H_3^{(b)}}{2}\right)}\right]^2$$
$$+ \sqrt{2\left(\frac{\alpha_1}{K_1 K_2} + \frac{\alpha_2}{K_2} + \alpha_3\right)(H_1 K_1 K_2 + H_2 K_2 + H_3^{(b)})}$$
$$+ \frac{1}{2T_3}\sum_{j=1}^{J_3} D_{3j}(b_j + h_{3j})\left(t_j - \frac{h_{3j} T_3}{b_j + h_{3j}}\right)^2 + \beta_3. \tag{23}$$

For two fixed positive integral values of the decision variables K_1 and K_2, equation (23) has a unique minimum value when the two quadratic non-negative terms, depending on T_3 and t_j, are made equal to zero. Therefore, the optimal value of the decision variables and the resulting minimum cost are denoted and determined by

$$T^\circ(K_1, K_2) = \sqrt{2\left(\frac{\alpha_1}{K_1 K_2} + \frac{\alpha_2}{K_2} + \alpha_3\right)\left(\frac{1}{H_1 K_1 K_2 + H_2 K_2 + H_3^{(b)}}\right)}, \tag{24}$$

$$t_j^\circ(K_1, K_2) = \frac{h_{3j} T^\circ(K_1, K_2)}{b_j + h_{3j}} \quad \text{for } j = 1, \cdots, J_3, \tag{25}$$

and

$$JTC^\circ(K_1, K_2) \equiv JTC[K_1, K_2, T^\circ(K_1, K_2), t_j^\circ(K_1, K_2)]$$

$$= \sqrt{2\left(\frac{\alpha_1}{K_1 K_2} + \frac{\alpha_2}{K_2} + \alpha_3\right)(H_1 K_1 K_2 + H_2 K_2 + H_3^{(b)})} + \beta_3 . \tag{26}$$

Multiplying out the two factors inside the square root in equation (26) yields

$$JTC^{\circ}(K_1,K_2) = \sqrt{2} \cdot \sqrt{\frac{\alpha_1 H_2}{K_1} + \alpha_2 H_1 K_1 + \frac{\alpha_2 H_3^{(b)}}{K_2} + \alpha_3 H_2 K_2 + \frac{\alpha_1 H_3^{(b)}}{K_1 K_2} + \alpha_3 H_1 K_1 K_2 + \alpha_1 H_1 + \alpha_2 H_2 + \alpha_3 H_3^{(b)}} + \beta_3.$$

Clearly, to minimize $JTC^{\circ}(K_1,K_2)$ is equivalent to minimize

$$\zeta(K_1,K_2) = \frac{\alpha_1 H_2}{K_1} + \alpha_2 H_1 K_1 + \frac{\alpha_2 H_3^{(b)}}{K_2} + \alpha_3 H_2 K_2 + \frac{\alpha_1 H_3^{(b)}}{K_1 K_2} + \alpha_3 H_1 K_1 K_2. \tag{27}$$

We observe from equation (27) that there are two options to determine the optimal integral values of K_1 and K_2 as shown below.

Option (1): Equation (27) can be written as

$$\zeta^{(1)}(K_1,K_2) = \frac{\alpha_1 H_2}{K_1} + \alpha_2 H_1 K_1 + \frac{H_3^{(b)}(\frac{\alpha_1}{K_1}+\alpha_2)}{K_2} + \alpha_3(H_1 K_1 + H_2)K_2.$$

To minimize $\zeta^{(1)}(K_1,K_2)$ is equivalent to separately minimize

$$\phi_2^{(1)}(K_1,K_2) \equiv \frac{H_3^{(b)}(\frac{\alpha_1}{K_1}+\alpha_2)}{K_2} + \alpha_3(H_1 K_1 + H_2)K_2, \tag{28}$$

and

$$\phi_1^{(1)}(K_1) \equiv \frac{\alpha_1 H_2}{K_1} + \alpha_2 H_1 K_1. \tag{29}$$

The validity of the equivalence is based on the following two-step minimization procedure.

Step (1): Because $\zeta^{(1)}(K_1,K_2) = \phi_1^{(1)}(K_1) + \phi_2^{(1)}(K_1,K_2)$, it is partially minimized by minimizing $\phi_1^{(1)}(K_1)$. As a result, the optimal integral value of K_1, denoted by $K_1^{(1)*}$ and given by expression (32) is obtained.

Step (2): Because $K_1^{(1)*}$ is fixed, to minimize $\zeta^{(1)}(K_1^{(1)*},K_2)$ is equivalent to minimize $\phi_2^{(1)}(K_1^{(1)*},K_2)$. As a result, a local optimal integral value of K_2, denoted by $K_2^{(1)*}$ and given by expression (33), and a local minimum, namely $\zeta^{(1)}(K_1^{(1)*},K_2^{(1)*})$ are obtained.

Hence, the joint total relevant cost per year can be minimized by first choosing $K_1 = K_1^{(1)*}$ and next $K_2 = K_2^{(1)*} \equiv K_2(K_1^{(1)*})$ such that

$$\phi_1^{(1)}(K_1) < \phi_1^{(1)}(K_1-1) \quad \text{and} \quad \phi_1^{(1)}(K_1) \leq \phi_1^{(1)}(K_1+1), \tag{30}$$

and

$$\phi_2^{(1)}(K_1^{(1)*},K_2) < \phi_2^{(1)}(K_1^{(1)*},K_2-1) \quad \text{and} \quad \phi_2^{(1)}(K_1^{(1)*},K_2) \leq \phi_2^{(1)}(K_1^{(1)*},K_2+1). \tag{31}$$

Two closed-form expressions, derived in the Appendix, for determining the optimal integral values of K_1 and K_2 are denoted and given by

$$K_1^{(1)*} = \left\lfloor \sqrt{\frac{\alpha_1 H_2}{\alpha_2 H_1} + 0.25} + 0.5 \right\rfloor, \tag{32}$$

and

$$K_2^{(1)*} = \left\lfloor \sqrt{\frac{H_3^{(b)}(\frac{\alpha_1}{K_1^{(1)*}} + \alpha_2)}{\alpha_3(H_1 K_1^{(1)*} + H_2)} + 0.25} + 0.5 \right\rfloor, \tag{33}$$

where $\lfloor x \rfloor$ is the largest integer $\leq x$.

Option (2): Equation (27) can also be written as

$$\zeta^{(2)}(K_1, K_2) = \frac{\alpha_2 H_3^{(b)}}{K_2} + \alpha_3 H_2 K_2 + \frac{\alpha_1(H_2 + \frac{H_3^{(b)}}{K_2})}{K_1} + H_1(\alpha_2 + \alpha_3 K_2)K_1.$$

To minimize $\zeta^{(2)}(K_1, K_2)$ is equivalent to separately minimize

$$\phi_2^{(2)}(K_1, K_2) \equiv \frac{\alpha_1(H_2 + \frac{H_3^{(b)}}{K_2})}{K_1} + H_1(\alpha_2 + \alpha_3 K_2)K_1,$$

and

$$\phi_1^{(2)}(K_2) \equiv \frac{\alpha_2 H_3^{(b)}}{K_2} + \alpha_3 H_2 K_2.$$

Similarly, the joint total relevant cost per year can be minimized by first choosing $K_2 = K_2^{(2)*}$ and next $K_1 = K_1^{(2)*} \equiv K_1(K_2^{(2)*})$ determined by

$$K_2^{(2)*} = \left\lfloor \sqrt{\frac{\alpha_2 H_3^{(b)}}{\alpha_3 H_2} + 0.25} + 0.5 \right\rfloor, \tag{34}$$

and

$$K_1^{(2)*} = \left\lfloor \sqrt{\frac{\alpha_1(H_2 + \frac{H_3^{(b)}}{K_2^{(2)*}})}{H_1(\alpha_2 + \alpha_3 K_2^{(2)*})} + 0.25} + 0.5 \right\rfloor. \tag{35}$$

Both options must be evaluated for a problem (see the numerical example in Section 6). However, Option (1), evaluating in the order of K_1 and K_2, might dominate Option (2), evaluating in the order of K_2 and K_1, when the holding costs decrease from upstream to downstream firms. A formal analysis is required to confirm this conjecture.

3.1 Deduction of Leung's (2010a) model without inspection

Suppose that for $i = 1$, 2 and all j; $\chi_{ij} = 1$ and $A_{ij} = B_{ij} = C_{ij} = 0$. Then we obtain the results shown in Subsection 3.1 of Leung (2010a).
Suppose that for $i = 1$, 2 and all j; $\chi_{ij} = 0$ and $A_{ij} = B_{ij} = C_{ij} = 0$. Then we obtain the results shown in Subsection 3.2 of Leung (2010a).

3.2 Deduction of Leung's (2010b) model without shortages

Suppose that for all j, $b_j = \infty$. Then $H_3^{(b)}$ becomes $H_3 \equiv \sum_{j=1}^{J_3} D_{3j} h_{3j} + G_2$. Then, we obtain the results shown in Section 3 of Leung (2010b).

4. The global minimum solution

It is apparent from the term in equation (26), namely $H_3^{(b)} = \sum_{j=1}^{J_3} \frac{D_{3j} b_j h_{3j}}{b_j + h_{3j}} - G_2$ that it will be optimal to incur some backorders towards the end of an order cycle if neither $h_{3j} = \infty$ nor $b_j = \infty$ occurs.

This brief checking is also valid for any n-stage $(n = 2, 3, \cdots)$ single/multi-firm supply chain with/without lot streaming and with complete backorders. However, when both a linear and fixed backorder costs are considered, the checking of global minimum is not so obvious, see Sphicas (2006).

5. Expressions for sharing the coordination benefits

Recall that the basic cycle time and the associated integer multipliers in a decentralized supply chain are denoted by τ_n and $\lambda_{n-1}, \cdots, \lambda_2, \lambda_1$ together with $\lambda_n \equiv 1$, respectively. Then equation (20) can be written as

$$TC(\tau_3, \mu_j) = \frac{(H_3^{(b)} - G_2)\tau_3}{2} + \frac{S_{3J}}{\tau_3} + \frac{1}{2\tau_3} \sum_{j=1}^{J_3} D_{3j}(b_j + h_{3j}) \left(\mu_j - \frac{h_{3j}\tau_3}{b_j + h_{3j}} \right)^2 , \qquad (36)$$

which, on applying the perfect squares method to the first two terms, yields the economic order interval and backordering intervals for each retailer in stage 3 given by

$$\tau_3^* = \sqrt{\frac{2S_{3J}}{H_3^{(b)} - G_2}} , \qquad (37)$$

$$\mu_j^* = \frac{h_{3j}\tau_3^*}{b_j + h_{3j}} , \qquad (38)$$

and the resulting minimum total relevant cost per year given by

$$TC_3^* \equiv TC(\tau_3^*, \mu_j^*) = \sqrt{2S_{3J}(H_3^{(b)} - G_2)} . \qquad (39)$$

Assume that the demand for the item with which each distributor in stage 2 is faced is a stream of $\tau_3^* D_{3j}$ units of demand at fixed intervals of τ_3^* year. Given these streams of demand, Rosenblatt and Lee (1985, p. 389) showed that each distributor's economic production interval should be some integer multiple of τ_3^*. As a result, equation (19) can be written as

$$TC(\lambda_2) = \lambda_2 \left[\frac{(H_2 - G_1)\tau_3^*}{2} \right] + \frac{1}{\lambda_2} \left(\frac{S_{2J} + A_{2J}}{\tau_3^*} \right) + \frac{G_2 \tau_3^*}{2} + \frac{B_{2J}}{\tau_3^*} + C_{2J} \ . \tag{40}$$

Hence, the total relevant cost in stage 2 per year can be minimized by choosing $\lambda_2 = \lambda_2^*$ such that

$$TC(\lambda_2) < TC(\lambda_2 - 1) \quad \text{and} \quad TC(\lambda_2) \leq TC(\lambda_2 + 1) \ ,$$

which, on following the derivation given in the Appendix, yields a closed-form expression for determining the optimal integral value of λ_2 given by

$$\lambda_2^* = \left\lfloor \sqrt{\frac{2(S_{2J} + A_{2J})}{(H_2 - G_1)(\tau_3^*)^2} + 0.25} + 0.5 \right\rfloor . \tag{41}$$

Similarly, equation (18) can be written as

$$TC(\lambda_1) = \lambda_1 \left(\frac{H_1 \lambda_2^* \tau_3^*}{2} \right) + \frac{1}{\lambda_1} \left(\frac{S_{1J} + A_{1J}}{\lambda_2^* \tau_3^*} \right) + \frac{G_1 \lambda_2^* \tau_3^*}{2} + \frac{B_{1J}}{\lambda_2^* \tau_3^*} + C_{1J} \ , \tag{42}$$

which can be minimized by choosing $\lambda_1 = \lambda_1^*$ given by

$$\lambda_1^* = \left\lfloor \sqrt{\frac{2(S_{1J} + A_{1J})}{H_1 (\lambda_2^* \tau_3^*)^2} + 0.25} + 0.5 \right\rfloor . \tag{43}$$

We readily deduce from equations (36) to (43) the expressions for $n\ (= 2,\ 3,\ 4, \cdots)$ stages given by

$$\tau_n^* = \sqrt{\frac{2S_{nJ}}{H_n^{(b)} - G_{n-1}}} \ , \tag{44}$$

$$\mu_j^* = \frac{h_{nj} \tau_n^*}{b_j + h_{nj}} \ , \tag{45}$$

$$\lambda_n^* \equiv 1 \quad \text{and} \quad \lambda_i^* = \left\lfloor \sqrt{\frac{2(S_{iJ} + A_{iJ})}{(H_i - G_{i-1})\left(\tau_n^* \prod\limits_{k=i+1}^{n} \lambda_k^* \right)^2} + 0.25} + 0.5 \right\rfloor \quad \text{for } i = n-1, \cdots, 1 \ , \tag{46}$$

$$TC_n^* \equiv TC(\tau_n^*, \mu_j^*) = \sqrt{2S_{nJ}(H_n^{(b)} - G_{n-1})} \ , \tag{47}$$

and

$$TC_i^* \equiv TC(\lambda_i^*) = \frac{(H_i - G_{i-1})\tau_n^* \prod_{k=i}^n \lambda_k^*}{2} + \frac{G_i \tau_n^* \prod_{k=i+1}^n \lambda_k^*}{2} + \frac{S_{iJ} + A_{iJ}}{\tau_n^* \prod_{k=i}^n \lambda_k^*} + \frac{B_{iJ}}{\tau_n^* \prod_{k=i+1}^n \lambda_k^*} + C_{iJ} \quad \text{for } i = n-1, \cdots, 1. \quad (48)$$

The judicious scheme for allocating the coordination benefits, originated from Goyal (1976), is explicitly expressed as follows:

$$\text{Share}_i = \text{Total saving} \times \frac{TC_i^*}{\sum_{i=1}^n TC_i^*} = \left(\sum_{i=1}^n TC_i^* - JTC_n^*\right) \times \frac{TC_i^*}{\sum_{i=1}^n TC_i^*}, \quad (49)$$

where $JTC_n^* \equiv JTC^\circ(K_1^*, K_2^*, \cdots, K_{n-1}^*)$. Hence, the total relevant cost, after sharing the benefits, in stage i per year is denoted and given by

$$TC_i^\circ = TC_i^* - \text{Share}_i = JTC_n^* \times \frac{TC_i^*}{\sum_{i=1}^n TC_i^*}. \quad (50)$$

In addition, the percentages of cost reduction in each stage and the entire supply chain are the same because $\frac{TC_i^* - TC_i^\circ}{TC_i^*} = \frac{\text{Share}_i}{TC_i^*} = \frac{\text{Total saving}}{\sum_{i=1}^n TC_i^*}$, and total saving and $\sum_{i=1}^n TC_i^*$ are constants.

More benefits have to be allocated to retailers so as to convince them of their coordination when $TC_n^\circ > TC_n^{**}$, where $TC_n^{**} \equiv \sum_{j=1}^{J_n} \sqrt{2 S_{nj} D_{nj} h_{nj} \left(\frac{b_j}{b_j + h_j}\right)}$ = the minimum total relevant cost of all retailers based on the EOQ model with complete backorders penalized by a linear shortage cost (see, e.g. Moore *et al.* 1993, pp. 338-344). Even if $TC_n^\circ \leq TC_n^{**}$, additional benefits should be allocated to the retailers to enhance their interests in coordination. The reason is that if the retailers insist on employing their respective EOQ cycle times, then clearly the corresponding total relevant cost of all firms in stage i ($= 1, \cdots, n-1$) denoted by TC_i^+ is higher than TC_i^* which in turn is higher than TC_i°, i.e. $TC_i^+ \geq TC_i^* \geq TC_i^\circ$ ($i = 1, \cdots, n-1$). As a result, the retailers are crucial to realize the coordination.

Because we consider a non-serial supply chain (where each stage has more than one firm, but a serial supply chain has only one firm), not necessarily tree-like, a reasonable scheme is explicitly proposed as follows:

$$\text{Adjusted Share}_i = \begin{cases} [\text{Share}_i - \chi(TC_n^\circ - TC_n^{**})(\frac{J_i}{\sum_{i=1}^{n-1} J_i})](1 - \frac{J_i}{\sum_{i=1}^{n-1} J_i}) & \text{for } i = 1, \cdots, n-1, \\ \text{Share}_n + \sum_{i=1}^{n-1} \chi(TC_n^\circ - TC_n^{**})(\frac{J_i}{\sum_{i=1}^{n-1} J_i}) + \sum_{i=1}^{n-1} [\text{Share}_i - \chi(TC_n^\circ - TC_n^{**})(\frac{J_i}{\sum_{i=1}^{n-1} J_i})](\frac{J_i}{\sum_{i=1}^{n-1} J_i}) & \text{for } i = n, \end{cases} \quad (51)$$

where $\chi = \begin{cases} 0 & \text{if } TC_n^\circ \leq TC_n^{**}, \\ 1 & \text{if } TC_n^\circ > TC_n^{**}. \end{cases}$ Obviously, if $(TC_n^\circ - TC_n^{**}) > \sum_{i=1}^{n-1} \text{Share}_i$, then no coordination exists.

The rationale behind equation (51) is that we compensate, if applicable, the retailers for the increased cost of $(TC_n^\circ - TC_n^{**} > 0)$, and share additional coordination benefits to them, in

proportion to the number of firms in each of the upstream stages. In addition, equation (51) is simplified to

$$\text{Adjusted Share}_i = \begin{cases} \text{Share}_i(1 - \frac{J_i}{\sum_{i=1}^{n-1} J_i}) - \chi(TC_n^\circ - TC_n^{**})(\frac{J_i}{\sum_{i=1}^{n-1} J_i})(1 - \frac{J_i}{\sum_{i=1}^{n-1} J_i}) & \text{for } i = 1, \cdots, n-1, \\ \text{Share}_n + \sum_{i=1}^{n-1} \text{Share}_i(\frac{J_i}{\sum_{i=1}^{n-1} J_i}) + \sum_{i=1}^{n-1} \chi(TC_n^\circ - TC_n^{**})(\frac{J_i}{\sum_{i=1}^{n-1} J_i})(1 - \frac{J_i}{\sum_{i=1}^{n-1} J_i}) & \text{for } i = n. \end{cases} \tag{52}$$

Hence, the total relevant costs, after adjusting the shares of the benefits, in stage i per year are denoted and given by

$$TC_i^{\circ\circ} = TC_i^* - \text{Adjusted Share}_i \quad \text{for } i = 1, \cdots, n, \tag{53}$$

and the adjusted percentages of cost reduction are given by $\frac{TC_i^* - TC_i^{\circ\circ}}{TC_i^*}$ $(i = 1, \cdots, n)$ or $\frac{TC_n^{**} - TC_n^{\circ\circ}}{TC_n^{**}}$ if $\chi = 1$.

6. A numerical example

(A 3-stage multi-firm centralized/decentralized supply chain, with/without lot streaming, with/without linear backorder costs, and with inspections)
Suppose that an item has almost the same characteristics as those on page 905 of Leung (2010b) as follows:

Two suppliers $(i = 1; j = 1, 2)$:

$\chi_{11} = 0$, $D_{11} = 100,000$ units per year, $P_{11} = 300,000$ units per year,

$g_{11} = \$0.08$ per unit per year, $h_{11} = \$0.8$ per unit per year, $S_{11} = \$600$ per setup,

$A_{11} = \$30$ per setup, $B_{11} = \$3$ per delivery, $C_{11} = \$0.0005$ per unit,

$\chi_{12} = 1$, $D_{12} = 80,000$, $P_{12} = 160,000$, $g_{12} = 0.09$, $h_{12} = 0.75$, $S_{12} = 550$, $A_{12} = 50$, $B_{12} = 4$, $C_{12} = 0.0007$.

Four manufacturers $(i = 2; j = 1, \cdots, 4)$:

$\chi_{21} = 1$, $D_{21} = 70,000$, $P_{21} = 140,000$, $g_{21} = 0.83$, $h_{21} = 2$, $S_{21} = 300$, $A_{21} = 50$, $B_{21} = 8$, $C_{21} = 0.001$;

$\chi_{22} = 0$, $D_{22} = 50,000$, $P_{22} = 150,000$, $g_{22} = 0.81$, $h_{22} = 2.1$, $S_{22} = 310$, $A_{22} = 45$, $B_{22} = 7$, $C_{22} = 0.0009$;

$\chi_{23} = 0$, $D_{23} = 40,000$, $P_{23} = 160,000$, $g_{23} = 0.79$, $h_{23} = 1.8$, $S_{23} = 305$, $A_{23} = 48$, $B_{23} = 7.5$, $C_{23} = 0.0012$;

$\chi_{24} = 1$, $D_{24} = 20,000$, $P_{24} = 100,000$, $g_{24} = 0.85$, $h_{24} = 2.2$, $S_{24} = 285$, $A_{24} = 60$, $B_{24} = 9.5$, $C_{24} = 0.0015$.

Six retailers $(i = 3; j = 1, \cdots, 6)$:

$D_{31} = 40,000$, $b_1 = \$3.5$ per unit per year, $h_{31} = 5$, $S_{31} = \$50$ per order; $D_{32} = 30,000$, $b_2 = 5.3$, $h_{32} = 5.1$, $S_{32} = 48$;

$D_{33} = 20,000$, $b_3 = 4.8$, $h_{33} = 4.8$, $S_{33} = 51$; $D_{34} = 35,000$, $b_4 = 5.3$, $h_{34} = 4.9$, $S_{34} = 52$;
$D_{35} = 45,000$, $b_5 = 5.2$, $h_{35} = \infty$, $S_{35} = 50$; $D_{36} = 10,000$, $b_6 = \infty$, $h_{36} = 5$, $S_{36} = 49$.

Table 1 shows the optimal results of the integrated approach, obtained using designations (2) to (13), and equations (18) to (20), (24) to (26) and (32) to (35). Detailed calculations to reach Table 1 are given in the Appendix. Thus, each of the two suppliers fixes a setup every 41.67 days, each of the four manufacturers fixes a setup every 41.67 days and each of the six retailers places an order every 13.89 days, coupled with the respective backordering times: 8.17, 6.81, 6.95, 6.67, 13.89 and 0 days.

Note that the yearly cost saving, compared with no shortages, is 8.20% ($= \frac{69,719.47 - 63,999.43}{69,719.47}$), where the figure \$69,719.47 is obtained from the last column of Table 1 in Leung (2010b). The comparison is feasible because the assignments of $b_5 = 5.2$ and $h_{35} = \infty$ (causing all negative inventory) has the same cost effect as $b_5 = \infty$ and $h_{35} = 5.2$ (all positive inventory) on retailer 5.

Stage	Integer multiplier	Cycle time (year)	Cycle time (days)	Yearly cost ($)
Suppliers	1	0.11415	41.67	13,337.04
Manufacturers	3	0.11415	41.67	31,716.19
Retailers	–	0.03805	13.89	18,946.20
Entire supply chain	–	–	–	63,999.43

Table 1. Results for the centralized model

When the ordering decision is governed by the adjacent downstream stage, Table 2 shows the optimal results of the independent approach, obtained using equations (44), (46) to (48) with $n = 3$. Table 3 shows the results after sharing the coordination benefits, obtained using equations (49) and (50). Detailed calculations to reach Tables 2 and 3 are also given in the Appendix.

Stage	Integer multiplier	Cycle time (year)	Cycle time (days)	TC_i^* ($ per year)
Suppliers	1	0.09636	35.16	14,955.80
Manufacturers	3	0.09636	35.16	31,283.07
Retailers	–	0.03212	11.72	18,677.85
Entire supply chain	–	–	–	64,916.72

Table 2. Results for the decentralized model

Stage	Yearly saving ($) or penalty (–$)	Share ($ per year)	TC_i° ($ per year)	Yearly cost reduction (%)
Suppliers	1618.76	211.33	14,744.47	1.41
Manufacturers	–433.12	442.04	30,841.03	1.41
Retailers	–268.35	263.92	18,413.93	1.41
Entire supply chain	917.29	917.29	63,999.43	1.41

Table 3. Results after sharing the coordination benefits

Table 3 shows that the centralized replenishment policy increases the costs of the four manufacturers and six retailers, while decreases the cost of the two suppliers. According to Goyal's (1976) saving-sharing scheme, the increased costs of the manufacturers and retailers must be covered so as to motivate them to adopt the centralized replenishment policy, and the total yearly saving of \$917.29 is shared to assure equal yearly cost reduction of 1.41% through all three stages or the entire chain.

Because $18,413.93 = TC_3^\circ > TC_3^{**} = 17,913.57$, we have $\chi = 1$. Table 4 shows the adjusted results, obtained using equations (52) and (53), and indicates that the retailers' yearly cost reduction increases from 1.41% to 4.56% (which is rather significant), and the suppliers' and manufacturers' yearly cost reductions are *at least* ($\frac{TC_i^+ - TC_i^{\circ\circ}}{TC_i^+} \geq \frac{TC_i^* - TC_i^{\circ\circ}}{TC_i^*}$ because $TC_i^+ \geq TC_i^* \geq TC_i^\circ$, $i = 1, 2$) 0.20% and 0.12%, respectively. However, if the retailers regard 0.49% as the relevant comparison figure and as insignificant, all the coordination benefit may be allocated to them, and hence this figure becomes 0.85% ($= \frac{17,913.57 - 17,826.42 + 29.70 + 36.16}{17,913.57}$).

If they consider 0.85% insignificant, negotiation between all the upstream stages and the retailers is the last resort.

Stage	Adjusted share ($ per year)	$TC_i^{\circ\circ}$ ($ per year)	Adjusted yearly cost reduction (%)
Suppliers	29.70	14,926.10	0.20
Manufacturers	36.16	31,246.91	0.12
Retailers	851.43	17,826.42	4.56 (or 0.49)
Entire supply chain	917.29	63,999.43	1.41

Table 4. Results after adjusting the shares of the coordination benefits

The final remark for this example is that we need not assume that, for instance, supplier 1 supplies manufacturers 1 and 2, and supplier 2 supplies manufacturers 3 and 4. The mild condition for a non-serial supply chain is to satisfy the equality: $\sum_{j=1}^{J_1} D_{1j} = \sum_{j=1}^{J_2} D_{2j} = \sum_{j=1}^{J_3} D_{3j}$.

7. Conclusions and future research

The main contribution of the chapter to the literature is threefold: First, we establish the *n*-stage ($n = 2, 3, 4, \cdots$) model, which is more pragmatic than that of Leung (2010b), by including Assumption (13). Secondly, we derive expressions for sharing the coordination benefits based on Goyal's (1976) scheme, and on a further sharing scheme. Thirdly, we deduce and solve such special models as Leung (2009a, 2010a,b).

The limitation of our model manifest in the numerical example is that the number of suppliers in Stage 1 is arbitrarily assigned. Concerning the issue of "How many suppliers are best?", we can refer to Berger *et al.* (2004), and Ruiz-Torres and Mahmoodi (2006, 2007) to decide the optimal number of suppliers at the very beginning.

Three ready extensions of our model that warrant future research endeavors in this field are: First, following the evolution of three-stage multi-firm supply chains shown in Section 3, we can readily formulate and algebraically analyze the integrated model of a four- or higher-stage multi-firm supply chain. In addition, a remark relating to determining optimal integral

values of K's is as follows: To be more specific, letting $n = 4$, we have *at most* 6 (= 3×2×1) options to determine the optimal values of K_1, K_2 and K_3 (see Leung 2009a, 2010a,b). However, Option (1), evaluating in the order of K_1, K_2 and K_3, might dominate other options when the holding costs decrease from upstream to downstream firms. Although this conjecture is confirmed by the numerical example in this chapter and those in Leung (2010a,b), a formal analysis is still necessary.

Secondly, using complete and perfect squares, we can solve the integrated model of a *n*-stage multi-firm supply chain either for an equal-cycle-time, or an integer multiplier at each stage, with not only a linear (see Leung 2010a) but also a fixed shortage cost for either the complete, or a fixed ratio partial backordering allowed for some/all downstream firms (i.e. retailers), and with lot streaming allowed for some/all upstream firms (i.e. suppliers, manufacturers and assemblers).

Thirdly, severity of green issues gives rise to consider integrated deteriorating production-inventory models incorporating the factor of environmental consciousness such as Yu *at al.* (2008), Chung and Wee (2008), and Wee and Chung (2009). Rework, a means to reduce waste disposal, is examined in Chiu *et al.* (2006) or Leung (2009b) who derived the optimal expressions for an EPQ model with complete backorders, a random proportion of defectives, and an immediate imperfect rework process while Cárdenas-Barrón (2008) derived those for an EPQ model with no shortages, a fixed proportion of defectives, and an immediate or a *N*-cycle perfect rework process. Reuse, another means to reduce waste disposal, is investigated in El Saadany and Jaber (2008), and Jaber and Rosen (2008). Incorporating rework or reuse in our model will be a challenging piece of future research.

Appendix

A1. Derivation of equations (32) and (33)

Substituting equation (29) in the two conditions of (30) yields the following inequality

$$K_1(K_1 - 1) < \frac{\alpha_1 H_2}{\alpha_2 H_1} < K_1(K_1 + 1) .$$

We can derive a closed-form expression concerning the optimal integer $K_1^{(1)*}$ as follows:

$$K_1(K_1 - 1) + 0.25 < \frac{\alpha_1 H_2}{\alpha_2 H_1} + 0.25 < K_1(K_1 + 1) + 0.25$$

$$\Leftrightarrow (K_1 - 0.5)^2 < \frac{\alpha_1 H_2}{\alpha_2 H_1} + 0.25 < (K_1 + 0.5)^2$$

$$\Leftrightarrow K_1 - 0.5 < \sqrt{\frac{\alpha_1 H_2}{\alpha_2 H_1} + 0.25} < K_1 + 0.5$$

$$\Leftrightarrow \sqrt{\frac{\alpha_1 H_2}{\alpha_2 H_1} + 0.25} - 0.5 \leq K_1 < \sqrt{\frac{\alpha_1 H_2}{\alpha_2 H_1} + 0.25} + 0.5 .$$

From the last inequality, we can deduce that the optimal integer $K_1^{(1)*}$ is represented by expression (32). In an analogous manner, the optimal integer $K_2^{(1)*}$ represented by expression (33) is derived.

A2. Detailed calculations for the numerical example

Designations (2) to (13) give

$$\varphi_{11} = \tfrac{1}{3} \, , \; \varphi_{12} = 0.5 \, , \; \varphi_{21} = 0.5 \, , \; \varphi_{22} = \tfrac{1}{3} \, , \; \varphi_{23} = 0.25 \, , \; \varphi_{24} = 0.2 \, ,$$

$$\alpha_1 = 1150 + 80 = 1230 \, , \; \alpha_2 = 1200 + 203 + 7 = 1410 \, , \; \alpha_3 = 300 + 32 = 332 \, ,$$

$$\beta_3 = 50 + 56 + 70 + 45 + 48 + 30 = 299 \, ,$$

$$G_0 \equiv 0 \, , \; G_1 = -100{,}000(0.8) + 80{,}000(0.75)(\tfrac{1}{3} - \tfrac{2}{3}) = -100{,}000 \, ,$$

$$G_2 = 70{,}000(2)(0.5 - 0.5) - 50{,}000(2.1) - 40{,}000(1.8) + 20{,}000(2.2)(0.2 - 0.8) = -203{,}400 \, ,$$

$$H_1 = 100{,}000(\tfrac{1}{3} \times 0.88 + 0.8) + 80{,}000(0.5 \times 0.09 + 0.5 \times 0.75) = 142{,}933.33 \, ,$$

$$H_2 = 70{,}000(0.5 \times 0.83 + 0.5 \times 2) + 50{,}000(\tfrac{1}{3} \times 2.91 + 2.1) + 40{,}000(0.25 \times 2.59 + 1.8) + 20{,}000(0.2 \times 0.85 + 0.8 \times 2.2)$$
$$- 100{,}000 = 99{,}050 + 153{,}500 + 97{,}900 + 38{,}600 - 100{,}000 = 289{,}050,$$

$$H_3^{(b)} = \frac{40{,}000(3.5)(5)}{3.5+5} + \frac{30{,}000(5.3)(5.1)}{5.3+5.1} + \frac{20{,}000(4.8)(4.8)}{4.8+4.8} + \frac{35{,}000(5.3)(4.9)}{5.3+4.9} + 45{,}000(5.2) + 10{,}000(5) - 203{,}400 = 378{,}036.84 \, .$$

Equations (32), (33) and (26) give

$$K_1^{(1)*} = \left\lfloor \sqrt{\frac{1230(289{,}050)}{1410(142{,}933.33)} + 0.25} + 0.5 \right\rfloor = \lfloor 1.92 \rfloor = 1 \, ,$$

$$K_2^{(1)*} = \left\lfloor \sqrt{\frac{378{,}036.84(\frac{1230}{1}+1410)}{332(142{,}933.33 \times 1 + 289{,}050)} + 0.25} + 0.5 \right\rfloor = \lfloor 3.18 \rfloor = 3 \, ,$$

$$JTC^\circ(1, \, 3) = \sqrt{2(\tfrac{1230}{3} + \tfrac{1410}{3} + 332)(142{,}933.33 \times 3 + 289{,}050 \times 3 + 378{,}036.84)} + 299$$

$$= \sqrt{2(1212)(1{,}673{,}986.83)} + 299 = \$63{,}999.42 \;\; \text{per year}.$$

Equations (34), (35) and (26) give

$$K_2^{(2)*} = \left\lfloor \sqrt{\frac{1410(378{,}036.84)}{332(289{,}050)} + 0.25} + 0.5 \right\rfloor = \lfloor 2.91 \rfloor = 2 \, ,$$

$$K_1^{(2)*} = \left\lfloor \sqrt{\frac{1230(289{,}050 + \frac{378{,}036.84}{2})}{142{,}933.33(1410 + 332 \times 2)} + 0.25} + 0.5 \right\rfloor = \lfloor 1.99 \rfloor = 1 \, ,$$

$$JTC^\circ(1, \, 2) = \sqrt{2(\tfrac{1230}{2} + \tfrac{1410}{2} + 332)(142{,}933.33 \times 2 + 289{,}050 \times 2 + 378{,}036.84)} + 299$$

$$= \sqrt{2(1652)(1{,}242{,}003.50)} + 299 = \$64{,}358.19 \;\; \text{per year}.$$

Hence, the optimal integral values of K_1 and K_2 are 1 and 3, and equations (24) and (25) give the optimal basic cycle time and backordering times:

$$T_3^* \equiv T^{\circ}(1,\ 3) = \sqrt{\frac{2(1212)}{1,673,986.83}} = 0.03805 \text{ year} \cong 13.89 \text{ days},$$

$$t_1^* \equiv t_1^{\circ}(1,\ 3) = \frac{5(0.03805)}{3.5+5} = 0.02238 \text{ year} \cong 8.17 \text{ days},$$

$$t_2^* \equiv t_2^{\circ}(1,\ 3) = \frac{5.1(0.03805)}{5.3+5.1} = 0.01866 \text{ year} \cong 6.81 \text{ days},$$

$$t_3^* \equiv t_3^{\circ}(1,\ 3) = \frac{4.8(0.03805)}{4.8+4.8} = 0.01903 \text{ year} \cong 6.95 \text{ days},$$

$$t_4^* \equiv t_4^{\circ}(1,\ 3) = \frac{4.9(0.03805)}{5.3+4.9} = 0.01828 \text{ year} \cong 6.67 \text{ days},$$

$$t_5^* \equiv t_5^{\circ}(1,\ 3) = 0.03805 \text{ year} \cong 13.89 \text{ days} \text{ (all backorders)},$$

$$t_6^* \equiv t_6^{\circ}(1,\ 3) = 0 \text{ (no backorders)}.$$

The three yearly costs are obtained using equations (18) to (20) as follows:

$$\sum_{j=1}^{2} TC_{1j} = \frac{142,933.33(1)(3)(0.03805)}{2} - \frac{100,000(3)(0.03805)}{2} + \frac{1150+80}{1(3)(0.03805)} + \frac{7}{3(0.03805)} + 50 = \$13,337.04 \text{ per year},$$

$$\sum_{j=1}^{4} TC_{2j} = \frac{(289,050+100,000)(3)(0.03805)}{2} - \frac{203,400(0.03805)}{2} + \frac{1200+203}{3(0.03805)} + \frac{32}{0.03805} + 249 = \$31,716.19 \text{ per year},$$

$$\sum_{j=1}^{6} TC_{3j} = \frac{(378,036.84+203,400)(0.03805)}{2} + \frac{300}{0.03805} = \$18,946.20 \text{ per year}.$$

In particular, the optimal solution to the model based on the equal-cycle-time coordination mechanism is as follows:

$$JTC^{\circ}(1,\ 1) = \sqrt{2(1230+1410+332)(142,933.33+289,050+378,036.84)} + 299$$

$$= \sqrt{2(2972)(810,020.17)} + 299 \ = \$69,687.47 \text{ per year},$$

which is 8.89% $(= \frac{69,687.47-63,999.42}{63,999.42})$ higher than $JTC_3^* \equiv JTC^{\circ}(1,\ 3)$,

$$T^{\circ}(1,\ 1) = \sqrt{\frac{2(2972)}{810,020.17}} = 0.08566 \text{ year} \cong 31.27 \text{ days}.$$

When the ordering decision is governed by the adjacent downstream stage, equations (44) and (46) with $n = 3$ give

$$\tau_3^* = \sqrt{\frac{2(300)}{378,036.84+203,400}} = 0.03212 \text{ year} \cong 11.72 \text{ days},$$

$$\lambda_2^* = \left\lfloor \sqrt{\frac{2(1200+203)}{(289,050+100,000)(0.03212)^2} + 0.25} + 0.5 \right\rfloor = \lfloor 3.19 \rfloor = 3,$$

$$\lambda_1^* = \left\lfloor \sqrt{\frac{2(1150+80)}{142,933.33(3 \times 0.03212)^2} + 0.25} + 0.5 \right\rfloor = \lfloor 1.95 \rfloor = 1.$$

The three yearly costs are obtained using equations (47) and (48) with $n = 3$ as follows:

$$TC_3^* = \sqrt{2(300)(378,036.84 + 203,400)} = \$18,677.85 \text{ per year},$$

$$TC_2^* = \frac{(289,050+100,000)(3)(0.03212)}{2} - \frac{203,400(0.03212)}{2} + \frac{1200+203}{3(0.03212)} + \frac{32}{0.03212} + 249 = \$31,283.07 \text{ per year},$$

$$TC_1^* = \frac{142,933.33(1)(3)(0.03212)}{2} - \frac{100,000(3)(0.03212)}{2} + \frac{1150+80}{1(3)(0.03212)} + \frac{7}{3(0.03212)} + 50 = \$14,955.80 \text{ per year}.$$

The results for the decentralized model are summarized in Table 2, and the results after sharing the coordination benefits are summarized in Table 3, in which columns 3 and 4 are obtained using equations (49) and (50), respectively.

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Supply Chain Management

Edited by Dr. pengzhong Li

The purpose of supply chain management is to make production system manage production process, improve customer satisfaction and reduce total work cost. With indubitable significance, supply chain management attracts extensive attention from businesses and academic scholars. Many important research findings and results had been achieved. Research work of supply chain management involves all activities and processes including planning, coordination, operation, control and optimization of the whole supply chain system. This book presents a collection of recent contributions of new methods and innovative ideas from the worldwide researchers. It is aimed at providing a helpful reference of new ideas, original results and practical experiences regarding this highly up-to-date field for researchers, scientists, engineers and students interested in supply chain management.

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