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Adaptive Step-size Order Statistic LMS-based Time-domain Equalisation in Discrete Multitone Systems

Suchada Sitjongsataporn and Peerapol Yuvapoositanon
Centre of Electronic Systems Design and Signal Processing (CESdSP)
Mahanakorn University of Technology
Thailand

1. Introduction

Discrete multitone (DMT) is a digital implementation of the multicarrier transmission technique for digital subscriber line (DSL) standard (Golden et al., 2006; Starr et al., 1999). An all-digital implementation of multicarrier modulation called *DMT modulation* has been standardised for asymmetric digital subscriber line (ADSL), ADSL2, ADSL2+ and very high bit rate DSL (VDSL) (ITU, 2001; 2002; 2003). ADSL modems rely on DMT modulation, which divides a broadband channel into many narrowband subchannels and modulated encoded signals onto the narrowband subchannels. The major impairments such as the intersymbol interference (ISI), the intercarrier interference (ICI), the channel distortion, echo, radio-frequency interference (RFI) and crosstalk from DSL systems are induced as a result of large bandwidth utilisation over the telephone line. However, the improvement can be achieved by the equalisation concepts. A time-domain equaliser (TEQ) has been suggested for equalisation in DMT-based systems (Bladel & Moenclaey, 1995; Baldemair & Frenger, 2001; Wang & Adali, 2000) and multicarrier systems (Lopez-Valcarce, 2004).

The so-called shortened impulse response (SIR) which is basically the convolutional result of TEQ and channel impulse response (CIR) is preferably shortened as most as possible. By employing a TEQ, the performance of a DMT system is less sensitive to the choice of length of cyclic prefix. It is inserted between DMT symbols to provide subchannel independency to eliminate intersymbol interference (ISI) and intercarrier interference (ICI). TEQs have been introduced in DMT systems to alleviate the effect of ISI and ICI in case that the length of SIR or shorter than the length of cyclic prefix (F-Boroujeny & Ding, 2001). The target impulse response (TIR) is a design parameter characterising the derivation of the TEQ. By employing a TEQ, the performance of a DMT system is less sensitive to the choice of length of the cyclic prefix. In addition to TEQ, a frequency-domain equaliser (FEQ) is provided for each tone separately to compensate for the amplitude and phase of distortion. An ultimate objective of most TEQ designs is to minimise the mean square error (MSE) between output of TEQ and TIR which implies that TEQ and TIR are optimised in the MSE sense (F-Boroujeny & Ding, 2001).

Existing TEQ algorithms are based upon mainly in the MMSE-based approach (Al-Dhahir & Cioffi, 1996; Lee et al., 1995; Yap & McCanny, 2002; Ysebaert et al., 2003). These include

the MMSE-TEQ design algorithm with the unit tap constraint (UTC) in (Lee et al., 1995) and the unit energy constraint (UEC) in (Ysebaert et al., 2003). Only a few adaptive algorithms for TEQ are proposed in the literature. In (Yap & McCanny, 2002), a combined structure using the order statistic normalised averaged least mean fourth (OS-NALMF) algorithm for TEQ and order statistic normalised averaged least mean square (OS-NALMS) for TIR is presented. The advantage of a class of order statistic least mean square algorithms has been presented in (Haweel & Clarkson, 1992) which are similar to the usual gradient-based least mean square (LMS) algorithm with robust order statistic filtering operations applied to the gradient estimate sequence.

The purpose of this chapter is therefore finding the adaptive low-complexity time-domain equalisation algorithm for DMT-based systems which more robust as compared to existing algorithms. The chapter is organised as follows. In Section 2 , we describe the overview of system and data model. In Section 3 , the MMSE-based time-domain equalisation is reviewed. In Section 4 , the derivation of normalised least mean square (NLMS) algorithm with the constrained optimisation for TEQ and TIR are introduced. We derive firstly the stochastic gradient-based TEQ and TIR design criteria based upon the well known low-complexity NLMS algorithm with the method of Lagrange multiplier. It is simple and robust for ISI and ICI. This leads into Section 5 , where the order statistic normalised averaged least mean square (OS-NALMS) TEQ and TIR are presented. Consequently, the adaptive step-size order statistic normalised averaged least mean square (AS-OSNALMS) algorithms for TEQ and TIR can be introduced as the solution of MSE sense. This allows to track changing channel conditions and be quite suitable and flexible for DMT-based systems. In Section 6 , the analysis of stability of proposed algorithm for TEQ and TIR is shown. In Section 7 and Section 8 , the simulation results and conclusion are presented.

2. System and data model

The basic structure of the DMT transceiver is illustrated in Fig. 1. The incoming bit stream is likewise reshaped to a complex-valued transmitted symbol for mapping in quadrature amplitude modulation (QAM). Then, the output of QAM bit stream is split into N parallel bit streams that are instantaneously fed to the modulating inverse fast Fourier transform (IFFT). After that, IFFT outputs are transformed into the serial symbols including the cyclic prefix (CP) between symbols in order to prevent intersymbol interference (ISI) (Henkel et al., 2002) and then fed to the channel. The transmission channel will be used throughout the chapter is based on parameters in (ITU, 2001). The transmitted signal sent over the channel with impulse response is generally corrupted by the additive white Gaussian noise (AWGN).

The received signal is also equalised by TEQ. The number of coefficients of TEQ is particularly used to make the shortened-channel impulse response (SIR) length, which is the desired length of the channel after equalisation. The frequency-domain equaliser (FEQ) is essentially a one-tap equaliser that is the fast Fourier transform (FFT) of the composite channel of the convolution between the coefficients of the channel (**h**) and the tap-weight vector (**w**) of TEQ. The parallel of received symbols are eventually converted into serial bits in the frequency-domain.

The data model is based on a finite impulse response (FIR) model of transmission channel and will be used for equaliser in DMT-based systems. The basic data model is assumed that the transmission channel, including the transmitter and receiver filter front end. This can be represented with an FIR model **h**. The k-th received sample vector which is used for the detection of the k-th transmitted symbol vector $\mathbf{x}_{k,N}$, is given by
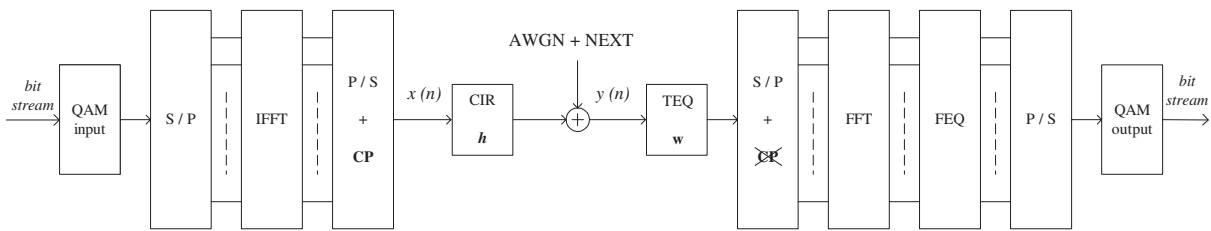
Fig. 1. Block diagram for time-domain equalisation.

$$
\underbrace{\begin{bmatrix} y_{k,l+\Delta} \\ \vdots \\ y_{k,N-l+\Delta} \end{bmatrix}}_{\mathbf{y}_{k,l+\Delta:N-1+\Delta}} = \underbrace{\begin{bmatrix} \mathbf{0}_{(1)} & \overbrace{\begin{bmatrix} [\,\bar{\mathbf{h}}^T\,] & 0 & \cdots \\ & \ddots & \ddots \\ \cdots & 0 & [\,\bar{\mathbf{h}}^T\,] \end{bmatrix}}^{\mathbf{H}_T} & \mathbf{0}_{(2)} \end{bmatrix}}_{\mathbf{H}} \cdot (\mathbf{I} \otimes \mathbf{P}_\nu \mathcal{F}_N^H) \cdot \underbrace{\begin{bmatrix} \mathbf{x}_{k-1,N} \\ \mathbf{x}_{k,N} \\ \mathbf{x}_{k+1,N} \end{bmatrix}}_{\mathbf{x}_{k-1:k+1,N}} + \underbrace{\begin{bmatrix} \eta_{k,l+\Delta} \\ \vdots \\ \eta_{k,N-l+\Delta} \end{bmatrix}}_{\boldsymbol{\eta}_{k,l+\Delta:N-1+\Delta}}, \tag{1}
$$

where

- The notation for the received sample vectors $\mathbf{y}_{k,l+\Delta:N-l+\Delta}$ and the received samples $y_{k,l+\Delta}$ are introduced by

$$
\mathbf{y}_{k,l+\Delta:N-l+\Delta} = \begin{bmatrix} y_{k,l+\Delta} & \cdots & y_{k,N-l+\Delta} \end{bmatrix}^T, \tag{2}
$$

where l determines the first considered sample of the k-th received DMT-symbol and depends on the number of equaliser taps L. The parameter Δ is a synchronisation delay.

- $\bar{\mathbf{h}}$ is the CIR vector $\mathbf{h}$ with coefficients in reverse order.

- $\mathbf{I}$ is an $n \times n$ identity matrix and $\otimes$ denotes the Kronecker product. The $(N+\nu) \times N$ matrix $\mathcal{P}_\nu$, which adds the cyclic prefix of length ν, is introduced by

$$
\mathbf{x}_{k,-\nu:N-1} = \underbrace{\begin{bmatrix} \mathbf{0}_{\nu \times (N-\nu)} & |\mathbf{I}_\nu \\ \hline \mathbf{I}_N \end{bmatrix}}_{\mathcal{P}_\nu} \mathbf{x}_{k,0:N-1}, \tag{3}
$$

where the sample vector $\mathbf{x}_{k,-\nu:-1}$ is called a cyclic prefix (CP).

- $\mathcal{F}_N^H = \mathcal{F}_N^*$ is the $N \times N$ IDFT matrix.

- The $N \times 1$ transmitted symbol vector $\mathbf{x}_{k,N}$ is introduced by

$$
\mathbf{x}_{k,N} = \begin{bmatrix} x_{k,0} & \cdots & x_{k,N-1} \end{bmatrix}^T = \begin{bmatrix} x_{k,N-1}^* & \cdots & x_{k,\frac{N}{2}+1}^* \end{bmatrix}^T, \tag{4}
$$

- The vector $\boldsymbol{\eta}_{k,l+\Delta:N-1+\Delta}$ is a sample vector with additive channel noise, and its autocorrelation matrix is denoted as $\Sigma_{\boldsymbol{\eta}}^2 = E\{\boldsymbol{\eta}_k \boldsymbol{\eta}_k^T\}$.

- The matrices $\mathbf{0}_{(1)}$ and $\mathbf{0}_{(2)}$ in Eq.(1) are the zero matrices of size $(N-l) \times (N-L+2\nu + \Delta + l)$ and $(N-l) \times (N+\nu-\Delta)$, respectively.

- The transmitted symbol vector is denoted as $\mathbf{x}_{k-1:k+1,N}$, where $\mathbf{x}_{k-1,N}$ and $\mathbf{x}_{k+1,N}$ introduce ISI. The $\mathbf{x}_{k,N}$ is the symbol vector of interest.
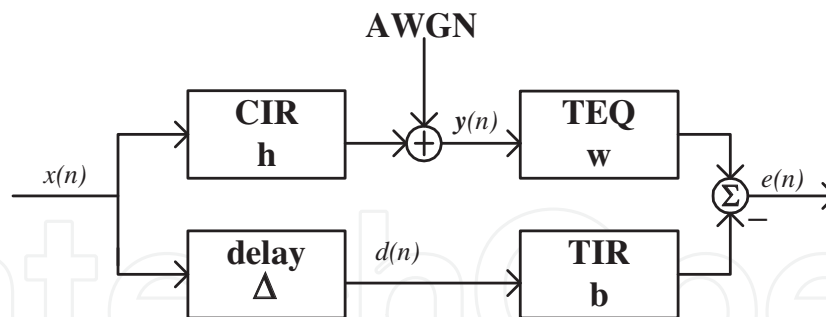
Fig. 2. Block diagram of MMSE-TEQ.

Some notation will be used throughout this chapter as follows: $E\{\cdot\}, (\cdot)^T, (\cdot)^H$ denote as the expectation, transpose and Hermitian operators, respectively. The vectors are in bold lowercase and matrices are in bold uppercase.

3. Minimum mean square error-based time-domain equalisation

The design of minimum mean square error time-domain equalisation (MMSE-TEQ) is based on the block diagram in Figure 2. The transmitted symbol $\mathbf{x}$ is sent over the channel with the impulse response $\mathbf{h}$ and corrupted by AWGN $\boldsymbol{\eta}$. The convolution of the L-tap TEQ filter $\mathbf{w}$ and the CIR $\mathbf{h}$ of $N_h + 1$ samples are sufficiently shortened so that overall of impulse response has length $\nu + 1$ that should make TEQ as a channel shortener $\mathbf{c} = \mathbf{h} * \mathbf{w}$, called the shorten impulse response (SIR). Then the orthogonality between the tones are restored and ISI vanishes (Melsa et al., 1996).

The result of time-domain error e between the TEQ output and the TIR output is then minimised in the mean-square sense as

$$\min_{\mathbf{w},\mathbf{b}} E\{|e|^2\} = \min_{\mathbf{w},\mathbf{b}} E\{|\mathbf{y}^T\mathbf{w} - \mathbf{x}_\Delta^T\mathbf{b}|^2\} \tag{5}$$

$$= \min_{\mathbf{w},\mathbf{b}} \mathbf{w}^T\Sigma_{\mathbf{y}}^2\mathbf{w} + \mathbf{b}^T\Sigma_{\mathbf{x}}^2\mathbf{b} - 2\mathbf{b}^T\Sigma_{\mathbf{xy}}(\Delta)\mathbf{w}, \tag{6}$$

where $\Sigma_{\mathbf{y}}^2 = E\{\mathbf{yy}^T\}$ and $\Sigma_{\mathbf{x}}^2 = E\{\mathbf{xx}^T\}$ are autocorrelation matrices, and where $\Sigma_{\mathbf{xy}}(\Delta) = E\{\mathbf{x}_\Delta\mathbf{y}^T\}$ is a cross-correlation matrix.

To avoid the trivial all-zero solution $\mathbf{w} = \mathbf{0}, \mathbf{b} = \mathbf{0}$, a constraint on the TEQ or TIR is therefore imposed.

Some constraints that are added on the TEQ and TIR (Ysebaert et al., 2003) as follows.

1. The unit-norm constraint (UNC) on the TIR
   By solving Eq.(6) subject to

$$\mathbf{b}^T\mathbf{b} = 1. \tag{7}$$

   The solution of $\mathbf{b}$ is the eigen-vector and $\mathbf{w}$ can be given as

$$\mathbf{w} = (\Sigma_{\mathbf{y}}^2)^{-1}\Sigma_{\mathbf{xy}}^T\mathbf{b}. \tag{8}$$

2. The unit-tap constraint (UTC) on the TEQ
   A UTC on $\mathbf{w}$ can be calculated with the method of the linear equation

$$\mathbf{e}_j^T\mathbf{w} = 1 \quad \text{or} \quad \mathbf{e}_j^T\mathbf{w} = -1, \tag{9}$$

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where $\mathbf{e}_j$ is the canonical vector with element one in the j-th position. By determining the dominant generalised eigen-vector, the vector $\mathbf{w}$ can be obtained as the closed-form solution

$$\mathbf{w} = \frac{\mathbf{A}^{-1}\,\mathbf{e}_j}{\mathbf{e}_j^T\,\mathbf{A}^{-1}\,\mathbf{e}_j}, \tag{10}$$

where $\mathbf{A} = \Sigma_{\mathbf{y}}^2 - \Sigma_{\mathbf{xy}}^T\,(\Sigma_{\mathbf{x}}^2)^{-1}\,\Sigma_{\mathbf{xy}}$.

3. The unit-tap constraint (UTC) on the TIR
Similarly, a UTC on $\mathbf{b}$ can be described as

$$\mathbf{e}_j^T\,\mathbf{b} = 1 \quad \text{or} \quad \mathbf{e}_j^T\,\mathbf{b} = -1\,, \tag{11}$$

After computing the solution for $\mathbf{b}$ as

$$\mathbf{b} = \frac{\mathbf{A}^{-1}\,\mathbf{e}_j}{\mathbf{e}_j^T\,\mathbf{A}^{-1}\,\mathbf{e}_j}\,. \tag{12}$$

The coefficients of TEQ $\mathbf{w}$ can be computed by Eq.(8).

4. The unit-energy constraint (UEC) on TEQ and TIR
Three UECs can be considered as

$$\mathbf{w}^T\Sigma_{\mathbf{y}}^2\mathbf{w} = 1 \quad \text{or} \quad \mathbf{b}^T\Sigma_{\mathbf{x}}^2\mathbf{b} = 1 \quad \text{or} \quad \mathbf{w}^T\Sigma_{\mathbf{y}}^2\mathbf{w} = 1 \ \& \ \mathbf{b}^T\Sigma_{\mathbf{x}}^2\mathbf{b} = 1. \tag{13}$$

It has been shown that each of all constraints results in Eq.(13), which can be incorporated into the one-tap FEQs in frequency domain (Ysebaert et al., 2003).

Most TEQ designs are based on the block-based computation to find TIR (Al-Dhahir & Cioffi, 1996; F-Boroujeny & Ding, 2001; Lee et al., 1995), it will make high computational complexity for implementation. However, this algorithm has much better performance and is used for the reference for on-line technique.

4. The proposed normalised least mean square algorithm for TEQ and TIR

We study the use of the LMS algorithm by means of the simplicity of implementation and robust performance. But the main limitation of the LMS algorithm is slow rate of convergence (Diniz, 2008; Haykin, 2002). Most importantly, the normalised least mean square (NLMS) algorithm exhibits a rate of convergence that is potentially faster than that of the standard LMS algorithm. Following (Haykin, 2002), we derive the normalised LMS algorithm for TEQ and TIR as follows.

Given the channel-filtered input vector $\mathbf{y}(n)$ and the delay input vector $\mathbf{d}(n)$, to determine the tap-weight vector of TEQ $\mathbf{w}(n+1)$ and the tap-weight vector of TIR $\mathbf{b}(n+1)$. So, the change $\delta\mathbf{w}(n+1)$ and $\delta\mathbf{b}(n+1)$ are defined as

$$\delta\mathbf{w}(n+1) = \mathbf{w}(n+1) - \mathbf{w}(n)\,, \tag{14}$$
$$\delta\mathbf{b}(n+1) = \mathbf{b}(n+1) - \mathbf{b}(n)\,, \tag{15}$$

and subject to the constraints

$$\mathbf{w}^H(n+1)\,\mathbf{y}(n) = g_1(n)\,, \tag{16}$$
$$\mathbf{b}^H(n+1)\,\mathbf{d}(n) = g_2(n)\,, \tag{17}$$

where $e(n)$ is the estimation error

$$e(n) = \mathbf{w}^H(n+1)\,\mathbf{y}(n) - \mathbf{b}^H(n+1)\,\mathbf{d}(n)\,. \tag{18}$$

The squared Euclidean norm of the change $\delta\mathbf{w}(n+1)$ and $\delta\mathbf{b}(n+1)$ may be expressed as

$$\|\,\delta\mathbf{w}(n+1)\|^2 = \sum_{k=0}^{M-1} |\,\mathbf{w}_k(n+1) - \mathbf{w}_k(n)|^2\,, \tag{19}$$

$$\|\,\delta\mathbf{b}(n+1)\|^2 = \sum_{k=0}^{M-1} |\,\mathbf{b}_k(n+1) - \mathbf{b}_k(n)|^2\,. \tag{20}$$

Given the tap-weight of TEQ $\mathbf{w}_k(n)$ and TIR $\mathbf{b}_k(n)$ for $k = 0, 1, \ldots, M-1$ in terms of their real and imaginary parts by

$$\mathbf{w}_k(n) = a_k(n) + j\,b_k(n)\,, \tag{21}$$

$$\mathbf{b}_k(n) = u_k(n) + j\,v_k(n)\,. \tag{22}$$

The tap-input vectors $\mathbf{y}(n)$ and $\mathbf{d}(n)$ are defined in term of real and imaginary parts as

$$\mathbf{y}(n) = y_1(n) + j\,y_2(n)\,, \tag{23}$$

$$\mathbf{d}(n) = d_1(n) + j\,d_2(n)\,. \tag{24}$$

Let the constraints $g_1(n)$ and $g_2(n)$ be expressed in terms of their real and imaginary parts as

$$g_1(n) = g_{1a}(n) + j\,g_{1b}(n)\,, \tag{25}$$

$$g_2(n) = g_{2a}(n) + j\,g_{2b}(n)\,. \tag{26}$$

To rewrite the complex constraint of Eq.(16) as the pair of real constraints

$$
\begin{aligned}
g_1(n) &= \sum_{k=0}^{M-1} [\mathbf{w}_k(n+1)]^H\,\mathbf{y}(n) \\
&= \sum_{k=0}^{M-1} \left\{ [a_k(n+1) + j\,b_k(n+1)]^* \, [y_1(n-k) + j\,y_2(n-k)] \right\} \\
&= \sum_{k=0}^{M-1} \left\{ [a_k(n+1)y_1(n-k) + b_k(n+1)y_2(n-k)] \right. \\
&\qquad\qquad \left. + j\,[a_k(n+1)y_2(n-k) - b_k(n+1)y_1(n-k)] \right\} \\
&= g_{1a}(n) + j\,g_{1b}(n)\,.
\end{aligned}
\tag{27}
$$

Therefore,

$$g_{1a}(n) = \sum_{k=0}^{M-1} [a_k(n+1)y_1(n-k) + b_k(n+1)y_2(n-k)]\,, \tag{28}$$

$$g_{1b}(n) = \sum_{k=0}^{M-1} [a_k(n+1)y_2(n-k) - b_k(n+1)y_1(n-k)]\,. \tag{29}$$

To formulate the complex constraint of Eq.(17) as the pair of real constraints.

$$
\begin{aligned}
g_2(n) &= \sum_{k=0}^{M-1} [\mathbf{b}_k(n+1)]^H \mathbf{d}(n) \\
&= \sum_{k=0}^{M-1} \left\{ [u_k(n+1) + j\,v_k(n+1)]^* [d_1(n-k) + j\,d_2(n-k)] \right\} \\
&= \sum_{k=0}^{M-1} \left\{ [u_k(n+1)d_1(n-k) + v_k(n+1)d_2(n-k)] \right. \\
&\qquad\qquad \left. + j\,[u_k(n+1)d_2(n-k) - v_k(n+1)d_1(n-k)] \right\} \\
&= g_{2a}(n) + j\,g_{2b}(n)\,.
\end{aligned}
\tag{30}
$$

Therefore,

$$
g_{2a}(n) = \sum_{k=0}^{M-1} [u_k(n+1)d_1(n-k) + v_k(n+1)d_2(n-k)]\,,
\tag{31}
$$

$$
g_{2b}(n) = \sum_{k=0}^{M-1} [u_k(n+1)d_2(n-k) - v_k(n+1)d_1(n-k)]\,.
\tag{32}
$$

4.1 The proposed normalised least mean square time-domain equalisation (NLMS-TEQ)

We define the real-valued cost function $J_1(n)$ for the constrained optimisation using *Lagrange multiplier*.[1] (Haykin, 2002)

$$
\begin{aligned}
J_1(n) &= \| \delta\mathbf{w}(n+1) \|^2 + \lambda_1 \left\{ [a_k(n+1)y_1(n-k) + b_k(n+1)y_2(n-k)] - g_{1a}(n) \right\} \\
&\quad + \lambda_2 \left\{ [a_k(n+1)y_2(n-k) - b_k(n+1)y_1(n-k)] - g_{1b}(n) \right\} \\
&= \sum_{k=0}^{M-1} \left\{ [a_k(n+1) - a_k(n)]^2 + [b_k(n+1) - b_k(n)]^2 \right\} \\
&\quad + \lambda_1 \left\{ \sum_{k=0}^{M-1} [a_k(n+1)y_1(n-k) + b_k(n+1)y_2(n-k)] - g_{1a}(n) \right\} \\
&\quad + \lambda_2 \left\{ \sum_{k=0}^{M-1} [a_k(n+1)y_2(n-k) - b_k(n+1)y_1(n-k)] - g_{1b}(n) \right\}\,,
\end{aligned}
\tag{33}
$$

where λ_1 and λ_2 are *Lagrange multipliers*. We find the optimum values of $a_k(n+1)$ and $b_k(n+1)$ by differentiating the cost function $J_1(n)$ with respect to these parameters and set the both results equal to zero. Hence,

$$
\frac{\partial J_1(n)}{\partial a_k(n+1)} = 0\,,
$$

---

[1] The method of Lagrange multiplier is defined as a new real-valued Lagrange function $h(w)$

$$
h(w) = f(w) + \lambda_1 Re\,[C(w)] + \lambda_2 Im\,[C(w)]
$$

where $f(w)$ is the real function and $C(w)$ is the complex constraint function. The parameters λ_1 and λ_2 are the Lagrange multipliers, where $\lambda = \lambda_1 + j\,\lambda_2$

and

$$\frac{\partial J_1(n)}{\partial b_k(n+1)} = 0 \, .$$

The results are given by

$$2\left[a_k(n+1) - a_k(n)\right] + \lambda_1 y_1(n-k) + \lambda_2 y_2(n-k) = 0 \, , \tag{34}$$

$$2\left[b_k(n+1) - b_k(n)\right] + \lambda_1 y_2(n-k) - \lambda_2 y_1(n-k) = 0 \, . \tag{35}$$

From Eq.(21) and Eq.(23), we combine these two real results into a single complex one as

$$\frac{\partial J_1(n)}{\partial \mathbf{w}_k(n+1)} = \frac{\partial J_1(n)}{\partial a_k(n+1)} + j \frac{\partial J_1(n)}{\partial b_k(n+1)} = 0 \, . \tag{36}$$

Therefore,

$$
\begin{aligned}
\frac{\partial J_1(n)}{\partial \mathbf{w}_k(n+1)} &= \{2\left[a_k(n+1) - a_k(n)\right] + \lambda_1 y_1(n-k) + \lambda_2 y_2(n-k)\} + \\
&\quad j\{2\left[b_k(n+1) - b_k(n)\right] + \lambda_1 y_2(n-k) - \lambda_2 y_1(n-k)\} \\
&= 2\left[a_k(n+1) + j\, b_k(n+1)\right] - 2\left[a_k(n) + j\, b_k(n)\right] + \\
&\quad \lambda_1\left[y_1(n-k) + j\, y_2(n-k)\right] - j\, \lambda_2\left[y_1(n-k) + j\, y_2(n-k)\right] \\
&= 2\left[a_k(n+1) + j\, b_k(n+1)\right] - 2\left[a_k(n) + j\, b_k(n)\right] + \\
&\quad (\lambda_1 - j\, \lambda_2)\left[y_1(n-k) + j\, y_2(n-k)\right] \\
&= 0 \, .
\end{aligned}
\tag{37}
$$

Thus, we get

$$2\left[\mathbf{w}_k(n+1) - \mathbf{w}_k(n)\right] + \lambda_w^* \mathbf{y}(n-k) = 0, \ for \ k = 0, 1, \ldots, M-1 \tag{38}$$

where λ_w is a complex Lagrange multiplier for TEQ as

$$\lambda_w = \lambda_1 + j\, \lambda_2 \, . \tag{39}$$

In order to find the unknown λ_w^*, we multiply both sides of Eq.(38) by $\mathbf{y}^*(n-k)$ and then sum over all integer values of k for 0 to $M-1$. Thus, we have

$$2\left[\mathbf{w}_k(n+1) - \mathbf{w}_k(n)\right]\mathbf{y}^*(n-k) = -\lambda_w^* \mathbf{y}(n-k)\,\mathbf{y}^*(n-k)$$

$$2\sum_{k=0}^{M-1}\left[\mathbf{w}_k(n+1)\mathbf{y}^*(n-k) - \mathbf{w}_k(n)\mathbf{y}^*(n-k)\right] = -\lambda_w^* \sum_{k=0}^{M-1}|\mathbf{y}(n-k)|^2$$

$$2\left[\mathbf{w}^T(n+1)\,\mathbf{y}^*(n) - \mathbf{w}^T(n)\,\mathbf{y}^*(n)\right] = -\lambda_w^*\|\mathbf{y}(n)\|^2$$

Therefore, the complex conjugate Lagrange multiplier λ_w^* can be formulated as

$$\lambda_w^* = \frac{-2}{\|\mathbf{y}(n)\|^2}\left[\mathbf{w}^T(n+1)\,\mathbf{y}^*(n) - \mathbf{w}^T(n)\,\mathbf{y}^*(n)\right] , \tag{40}$$

where $\|\mathbf{y}(n)\|^2$ is the Euclidean norm of the tap-input vector $\mathbf{y}(n)$.

From the definition of the estimation error $e(n)$ in Eq.(18), the conjugate of $e(n)$ is written as

$$e^*(n) = \mathbf{w}^T(n+1)\,\mathbf{y}^*(n) - \mathbf{b}^T(n+1)\,\mathbf{d}^*(n)\,. \tag{41}$$

The mean-square error $|e(n)|^2$ is minimised by the derivative of $|e(n)|^2$ with respect to $\mathbf{w}(n+1)$ be equal to zero.

$$\frac{\partial |e(n)|^2}{\partial \mathbf{w}(n+1)} = \left[\mathbf{w}^H(n+1)\,\mathbf{y}(n) - \mathbf{b}^H(n+1)\,\mathbf{d}(n)\right]\mathbf{y}^*(n) = 0\,. \tag{42}$$

Hence, we have

$$\mathbf{w}^H(n+1)\,\mathbf{y}(n) = \mathbf{b}^H(n+1)\,\mathbf{d}(n)\,, \tag{43}$$

and the conjugate of Eq.(43) may expressed as

$$\mathbf{w}^T(n+1)\,\mathbf{y}^*(n) = \mathbf{b}^T(n+1)\,\mathbf{d}^*(n)\,. \tag{44}$$

To substitute Eq.(44) and Eq.(41) into Eq.(40) and then formulate λ_w^* as

$$\lambda_w^* = \frac{2}{\|\mathbf{y}(n)\|^2}\,e^*(n)\,. \tag{45}$$

We rewrite Eq.(38) using Eq.(14) by writing,

$$2\,\delta\mathbf{w}(n+1) = -\lambda_w^*\,\mathbf{y}(n) \tag{46}$$

The change $\delta\mathbf{w}(n+1)$ is redefined by substituting Eq.(45) in Eq.(46). We thus have

$$\delta\mathbf{w}(n+1) = \frac{-1}{\|\mathbf{y}(n)\|^2}\,\mathbf{y}(n)\,e^*(n)\,. \tag{47}$$

To introduce a step-size for TEQ denoted by μ_w and then we may express the change $\delta\mathbf{w}(n+1)$ as

$$\delta\mathbf{w}(n+1) = \frac{-\mu_w}{\|\mathbf{y}(n)\|^2}\,\mathbf{y}(n)\,e^*(n)\,. \tag{48}$$

We rewrite the tap-weight vector of TEQ $\mathbf{w}(n+1)$ as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \delta\mathbf{w}(n+1)\,. \tag{49}$$

Finally, we may obtain the tap-weight vector of TEQ $\mathbf{w}(n+1)$ in the well-known NLMS algorithm.

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{\mu_w}{\|\mathbf{y}(n)\|^2}\,\mathbf{y}(n)\,e^*(n)\,. \tag{50}$$

where $e^*(n)$ is described in Eq.(41).

4.2 The proposed normalised least mean square-target impulse response (NLMS-TIR)

We formulate the real-valued cost function $J_2(n)$ for the constrained optimisation problem using *Lagrange multiplier*.

$$
\begin{aligned}
J_2(n) = {} & \| \delta \mathbf{b}(n+1) \|^2 + \lambda_3 \left\{ [u_k(n+1)d_1(n-k) + v_k(n+1)d_2(n-k)] - g_{2a}(n) \right\} \\
& + \lambda_4 \left\{ [u_k(n+1)d_2(n-k) - v_k(n+1)d_1(n-k)] - g_{2b}(n) \right\} \\
= {} & \sum_{k=0}^{M-1} \left\{ [u_k(n+1) - u_k(n)]^2 + [v_k(n+1) - v_k(n)]^2 \right\} \\
& + \lambda_3 \left\{ \sum_{k=0}^{M-1} [u_k(n+1)d_1(n-k) + v_k(n+1)d_2(n-k)] - g_{2a}(n) \right\} \\
& + \lambda_4 \left\{ \sum_{k=0}^{M-1} [u_k(n+1)d_2(n-k) - v_k(n+1)d_1(n-k)] - g_{2b}(n) \right\},
\end{aligned}
\tag{51}
$$

where λ_3 and λ_4 are *Lagrange multipliers*. We find the optimum values of $u_k(n+1)$ and $v_k(n+1)$ by differentiating the cost function $J_2(n)$ with respect to these parameters and then set the results equal to zero. Hence,

$$
\frac{\partial J_2(n)}{\partial u_k(n+1)} = 0 \,,
$$

and

$$
\frac{\partial J_2(n)}{\partial v_k(n+1)} = 0 \,.
$$

The results are

$$
2\left[u_k(n+1) - u_k(n)\right] + \lambda_3 d_1(n-k) + \lambda_4 d_2(n-k) = 0 \,, \tag{52}
$$
$$
2\left[v_k(n+1) - v_k(n)\right] + \lambda_3 d_2(n-k) - \lambda_4 d_1(n-k) = 0 \,. \tag{53}
$$

From Eq.(22) and Eq.(24), we combine these two real results into a single complex one as

$$
\frac{\partial J_2(n)}{\partial \mathbf{b}_k(n+1)} = \frac{\partial J_2(n)}{\partial u_k(n+1)} + j \frac{\partial J_2(n)}{\partial v_k(n+1)} = 0 \,. \tag{54}
$$

Therefore,

$$
\begin{aligned}
\frac{\partial J_2(n)}{\partial \mathbf{b}_k(n+1)} = {} & \left\{ 2\left[u_k(n+1) - u_k(n)\right] + \lambda_3 d_1(n-k) + \lambda_4 d_2(n-k) \right\} + \\
& j\left\{ 2\left[v_k(n+1) - v_k(n)\right] + \lambda_3 d_2(n-k) - \lambda_4 d_1(n-k) \right\} \\
= {} & 2\left[u_k(n+1) + j v_k(n+1)\right] - 2\left[u_k(n) + j v_k(n)\right] + \\
& \lambda_3\left[d_1(n-k) + j d_2(n-k)\right] - j\lambda_4\left[d_1(n-k) + j d_2(n-k)\right] \\
= {} & 2\left[u_k(n+1) + j v_k(n+1)\right] - 2\left[u_k(n) + j v_k(n)\right] + \\
& (\lambda_3 - j\lambda_4)\left[d_1(n-k) + j d_2(n-k)\right] \\
= {} & 0 \,.
\end{aligned}
\tag{55}
$$

Thus, we have

$$2 \left[\mathbf{b}_k(n+1) - \mathbf{b}_k(n) \right] + \lambda_b^* \mathbf{d}(n-k) = 0, \; for \; k = 0, 1, \ldots, M-1 \tag{56}$$

where λ_b is a complex Lagrange multiplier for TIR

$$\lambda_b = \lambda_3 + j \, \lambda_4 \tag{57}$$

To multiply both side of Eq.(56) by $\mathbf{d}^*(n-k)$ to find the unknown λ_b^* and then sum over all possible integer values of k for 0 to $M-1$. Thus, we get

$$2 \left[\mathbf{b}_k(n+1) - \mathbf{b}_k(n) \right] \mathbf{d}^*(n-k) = -\lambda_b^* \, \mathbf{d}(n-k) \, \mathbf{d}^*(n-k)$$

$$2 \sum_{k=0}^{M-1} \left[\mathbf{b}_k(n+1) \mathbf{d}^*(n-k) - \mathbf{b}_k(n) \mathbf{d}^*(n-k) \right] = -\lambda_b^* \sum_{k=0}^{M-1} |\mathbf{d}(n-k)|^2$$

$$2 \left[\mathbf{b}^T(n+1) \, \mathbf{d}^*(n) - \mathbf{b}^T(n) \, \mathbf{d}^*(n) \right] = -\lambda_b^* \|\mathbf{d}(n)\|^2$$

Therefore,

$$\lambda_b^* = \frac{-2}{\|\mathbf{d}(n)\|^2} \left[\mathbf{b}^T(n+1) \, \mathbf{d}^*(n) - \mathbf{b}^T(n) \, \mathbf{d}^*(n) \right] . \tag{58}$$

where $\|\mathbf{d}(n)\|^2$ is the Euclidean norm of the tap-input vector $\mathbf{d}(n)$.
To substitute Eq.(41) and Eq.(44) into Eq.(58) and then formulate λ_b^* as

$$\lambda_b^* = \frac{2}{\|\mathbf{d}(n)\|^2} \, e^*(n) . \tag{59}$$

We rewrite Eq.(56) using Eq.(15) by

$$2 \, \delta\mathbf{b}(n+1) = \lambda_b^* \, \mathbf{d}(n) \tag{60}$$

To redefine the change $\delta\mathbf{b}(n+1)$ by substituting Eq.(59) in Eq.(60). We thus get,

$$\delta\mathbf{b}(n+1) = \frac{1}{\|\mathbf{d}(n)\|^2} \, \mathbf{d}(n) \, e^*(n) . \tag{61}$$

To introduce a step-size for TIR μ_b and then we redefine the change $\delta\mathbf{b}(n+1)$ simply as

$$\delta\mathbf{b}(n+1) = \frac{\mu_b}{\|\mathbf{d}(n)\|^2} \, \mathbf{d}(n) \, e^*(n) , \tag{62}$$

where μ_b is the step-size for the NLMS-TIR.
We rewrite the tap-weight vector of TIR $\mathbf{b}(n+1)$ as

$$\mathbf{b}(n+1) = \mathbf{b}(n) + \delta\mathbf{b}(n+1) . \tag{63}$$

Finally, we may formulate the tap-weight vector of TIR $\mathbf{b}(n+1)$ in the normalised LMS algorithm.

$$\mathbf{b}(n+1) = \mathbf{b}(n) + \frac{\mu_b}{\|\mathbf{d}(n)\|^2} \, \mathbf{d}(n) \, e^*(n) , \tag{64}$$

where $e^*(n)$ is given in Eq.(41).
To comply with the Euclidean norm constraint, the tap-weight vector of TIR $\mathbf{b}(n+1)$ is normalised as

$$\mathbf{b}(n+1) = \frac{\mathbf{b}(n+1)}{\|\mathbf{b}(n+1)\|} . \tag{65}$$

5. Adaptive step-size order statistic-normalised averaged least mean square-based time-domain equalisation

Based on least mean square (LMS) algorithm, a class of adaptive algorihtms employing order statistic filtering of the sampled gradient estimates has been presented in (Haweel & Clarkson, 1992), which can provide with the development of simple and robust adaptive filter across a wide range of input environments. This section is therefore concerned with the development of simple and robust adaptive time-domain equalisation by defining normalised least mean square (NLMS) algorithm.

Following (Haweel & Clarkson, 1992), we present the NLMS algorithm which replaces linear smoothing of gradient estimates by order statistic averaged LMS filter. A class of order statistic normalised averaged LMS algorithm with the adaptive step-size scheme for the proposed NLMS algorithm in Eq.(50) and Eq.(64) that are shown as (Sitjongsataporn & Yuvapoositanon, 2007).

$$\widehat{\mathbf{w}}(n+1) = \widehat{\mathbf{w}}(n) - \frac{\mu_w(n)}{\|\mathbf{y}(n)\|^2} \, \mathcal{M}_w \, \boldsymbol{a}_w \,, \tag{66}$$

$$\widehat{\mathbf{b}}(n+1) = \widehat{\mathbf{b}}(n) + \frac{\mu_b(n)}{\|\mathbf{d}(n)\|^2} \, \mathcal{M}_b \, \boldsymbol{a}_b \,, \tag{67}$$

with

$$\mathcal{M}_w = \tilde{T}\{ \, \tilde{e}^*(n)\mathbf{y}(n), \, \tilde{e}^*(n-1)\mathbf{y}(n-1), \dots, \, \tilde{e}^*(n-N_w+1)\mathbf{y}(n-N_w+1)\} \,, \tag{68}$$

$$\mathcal{M}_b = \tilde{T}\{ \, \tilde{e}^*(n)\mathbf{d}(n), \, \tilde{e}^*(n-1)\mathbf{d}(n-1), \dots, \, \tilde{e}^*(n-N_b+1)\mathbf{d}(n-N_b+1)\} \,, \tag{69}$$

$$\tilde{e}(n) = \widehat{\mathbf{w}}^H(n)\mathbf{y}(n) - \widehat{\mathbf{b}}^H(n)\mathbf{d}(n) \,, \tag{70}$$

and

$$\boldsymbol{a}_w = [a_w(1), a_w(2), \dots, a_w(N_w)] \,, \qquad a_w(i) = 1/N_w \,; \qquad i = 1, 2, \dots, N_w. \tag{71}$$

$$\boldsymbol{a}_b = [a_b(1), a_b(2), \dots, a_b(N_b)] \,, \qquad a_b(j) = 1/N_b \,; \qquad j = 1, 2, \dots, N_b. \tag{72}$$

where $\tilde{e}(n)$ is a priori estimation error and $\tilde{T}\{\cdot\}$ operation denotes as the algebraic ordering transformation. The parameters $\boldsymbol{a}_w$ and $\boldsymbol{a}_b$ are the average of the gradient estimates of weighting coefficients as described in (Chambers, 1993). The parameters $\mu_w(n)$ and $\mu_b(n)$ are the step-size of $\widehat{\mathbf{w}}(n)$ and $\widehat{\mathbf{b}}(n)$. The parameters N_w and N_b are the number of tap-weight vectors for TEQ and TIR, respectively.

Following (Benveniste et al., 1990), we demonstate the derivation of adaptive step-size algorithms of $\mu_w(n)$ and $\mu_b(n)$ based on the proposed NLMS algorithm in Eq.(50) and Eq.(64). The cost function $J_{min}(n)$ may be expressed as

$$J_{min}(n) = \min_{\mathbf{w},\mathbf{b}} E\{|e(n)|^2\} \,, \tag{73}$$

$$e(n) = \mathbf{w}^H(n+1) \, \mathbf{y}(n) - \mathbf{b}^H(n+1) \, \mathbf{d}(n) \,. \tag{74}$$

We then form the stochastic approximation equations for $\mu_w(n+1)$ and $\mu_b(n+1)$ as (Kushner & Yang, 1995)

$$\mu_w(n+1) = \mu_w(n) + \alpha_w\{-\nabla J_{min}(\mu_w)\} \,, \tag{75}$$

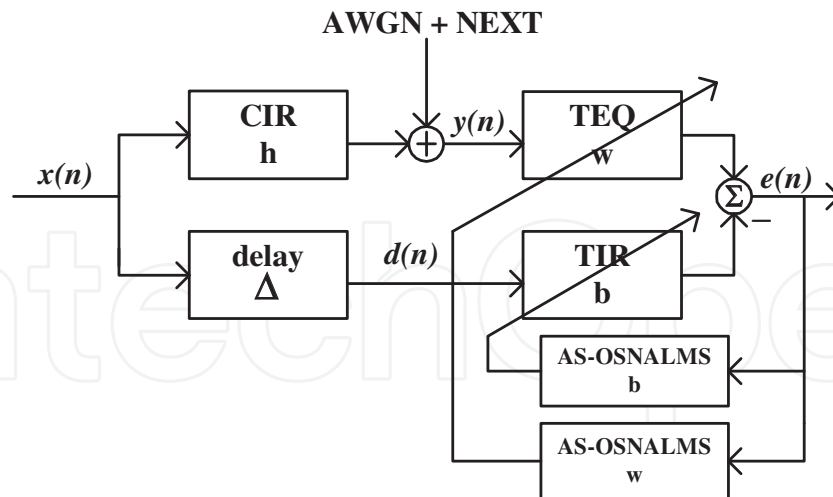$$\mu_b(n+1) = \mu_b(n) + \alpha_b\{-\nabla J_{min}(\mu_b)\} \,, \tag{76}$$

Fig. 3. Block diagram of adaptive step-size order statistic normalised averaged least mean square (AS-OSNALMS) TEQ and TIR.

where $\nabla J_{min}(\mu_w)$ and $\nabla J_{min}(\mu_b)$ denote as the value of the gradient vectors. The parameters α_w and α_b are the adaptation constant of μ_w and μ_b, respectively.

By differentiating the cost function in Eq.(73) with respect to μ_w and μ_b, we get

$$\frac{\partial J_{min}}{\partial \mu_w} = \nabla J_{min}(\mu_w) = e(n)\, \mathbf{y}^T(n)\Psi_w \,, \tag{77}$$

$$\frac{\partial J_{min}}{\partial \mu_b} = \nabla J_{min}(\mu_b) = -e(n)\, \mathbf{d}^T(n)\Psi_b \,, \tag{78}$$

where $\Psi_w = \frac{\partial \mathbf{w}(n)}{\partial \mu_w}$ and $\Psi_b = \frac{\partial \mathbf{b}(n)}{\partial \mu_b}$ are the derivative of $\mathbf{w}(n+1)$ in Eq.(50) with respect to $\mu_w(n)$ and of $\mathbf{b}(n+1)$ in Eq.(64) with respect to $\mu_b(n)$ (Moon & Stirling, 2000).

By substituting Eq.(77) and Eq.(78) in Eq.(75) and Eq.(76), we get the adaptive step-size $\mu_w(n)$ and $\mu_b(n)$ as

$$\mu_w(n+1) = \mu_w(n) - \alpha_w \{ e(n)\, \mathbf{y}^T(n)\Psi_w \} \,, \tag{79}$$

$$\mu_b(n+1) = \mu_b(n) + \alpha_b \{ e(n)\, \mathbf{d}^T(n)\Psi_b \} \,, \tag{80}$$

where

$$\Psi_w(n+1) = \left[\mathbf{I} - \frac{\mathbf{y}(n)}{\|\mathbf{y}(n)\|^2} \mu_w(n)\, \mathbf{y}^T(n) \right] \Psi_w(n) - \frac{\mathbf{y}(n)}{\|\mathbf{y}(n)\|^2} e^*(n) \,, \tag{81}$$

$$\Psi_b(n+1) = \left[\mathbf{I} - \frac{\mathbf{d}(n)}{\|\mathbf{d}(n)\|^2} \mu_b(n)\, \mathbf{d}^T(n) \right] \Psi_b(n) + \frac{\mathbf{d}(n)}{\|\mathbf{d}(n)\|^2} e^*(n) \,. \tag{82}$$

Then, we apply the order statistic scheme in Eq.(81) and Eq.(82) as

$$\widetilde{\Psi}_w(n+1) = \left[\mathbf{I} - \frac{\mathbf{y}(n)}{\|\mathbf{y}(n)\|^2} \mu_w(n)\, \mathbf{y}^T(n) \right] \widetilde{\Psi}_w(n) - \frac{\mathcal{M}_w\, a_w}{\|\mathbf{y}(n)\|^2} \,, \tag{83}$$

$$\widetilde{\Psi}_b(n+1) = \left[\mathbf{I} - \frac{\mathbf{d}(n)}{\|\mathbf{d}(n)\|^2} \mu_b(n)\, \mathbf{d}^T(n) \right] \widetilde{\Psi}_b(n) + \frac{\mathcal{M}_b\, a_b}{\|\mathbf{d}(n)\|^2} \,, \tag{84}$$

where $\mathcal{M}_w$, $\mathcal{M}_b$, $\boldsymbol{a}_w$, $\boldsymbol{a}_b$ and $\tilde{e}(n)$ are given in Eq.(68)-Eq.(72).

6. Stability analysis of the proposed AS-OSNALMS TEQ and TIR

In this section, the stability of the proposed AS-OSNALMS algorithm for TEQ and TIR are based upon the NLMS algorithm as given in (Haykin, 2002). This also provides for the optimal step-size parameters for TEQ and TIR.

According to the tap-weight estimate vector $\widehat{\mathbf{w}}(n)$ and $\widehat{\mathbf{b}}(n)$ computed in Eq.(66) and Eq.(67), the difference between the optimum tap-weight vector $\mathbf{w}^{opt}$ and $\widehat{\mathbf{w}}(n)$ is calculated by the weight-error vector of TEQ as

$$\Delta \mathbf{w}(n) = \mathbf{w}^{opt} - \widehat{\mathbf{w}}(n), \tag{85}$$

and, in the similar fashion, the weight-error vector of TIR is given by

$$\Delta \mathbf{b}(n) = \mathbf{b}^{opt} - \widehat{\mathbf{b}}(n), \tag{86}$$

By substituting Eq.(66) and Eq.(67) from $\mathbf{w}^{opt}$ and $\mathbf{b}^{opt}$, we have

$$\Delta \mathbf{w}(n+1) = \Delta \mathbf{w}(n) + \frac{\mu_w(n)}{\|\mathbf{y}(n)\|^2} \mathcal{M}_w \, \boldsymbol{a}_w, \tag{87}$$

where $\mathcal{M}_w$ and $\boldsymbol{a}_w$ are defined in Eq.(68) and Eq.(71).

$$\Delta \mathbf{b}(n+1) = \Delta \mathbf{b}(n) - \frac{\mu_b(n)}{\|\mathbf{d}(n)\|^2} \mathcal{M}_b \, \boldsymbol{a}_b, \tag{88}$$

where $\mathcal{M}_b$ and $\boldsymbol{a}_b$ are given in Eq.(69) and Eq.(72).

The stability analysis of the proposed AS-OSNALMS TEQ and TIR are based on the mean square deviation (MSD) as

$$\mathcal{D}_w(n) = E\{\|\Delta\mathbf{w}(n)\|^2\}, \tag{89}$$

$$\mathcal{D}_b(n) = E\{\|\Delta\mathbf{b}(n)\|^2\}, \tag{90}$$

where $\mathcal{D}_w(n)$ and $\mathcal{D}_b(n)$ denote as the MSD on TEQ and TIR.

By taking the squared Euclidean norms of both sides of Eq.(87) and Eq.(88), we get

$$\|\Delta\mathbf{w}(n+1)\|^2 = \|\Delta\mathbf{w}(n)\|^2 + 2\frac{\mu_w(n)}{\|\mathbf{y}(n)\|^2} \Delta\mathbf{w}^H(n) \cdot (\mathcal{M}_w \, \boldsymbol{a}_w)$$
$$+ \frac{\mu_w^2(n)}{\|\mathbf{y}(n)\|^2} \frac{(\mathcal{M}_w \, \boldsymbol{a}_w)^H (\mathcal{M}_w \, \boldsymbol{a}_w)}{\|\mathbf{y}(n)\|^2}, \tag{91}$$

$$\|\Delta\mathbf{b}(n+1)\|^2 = \|\Delta\mathbf{b}(n)\|^2 - 2\frac{\mu_b(n)}{\|\mathbf{d}(n)\|^2} \Delta\mathbf{b}^H(n) \cdot (\mathcal{M}_b \, \boldsymbol{a}_b)$$
$$+ \frac{\mu_b^2(n)}{\|\mathbf{d}(n)\|^2} \frac{(\mathcal{M}_b \, \boldsymbol{a}_b)^H (\mathcal{M}_b \, \boldsymbol{a}_b)}{\|\mathbf{d}(n)\|^2}. \tag{92}$$

Then taking expectations and rearranging terms with Eq.(89) and Eq.(90), the MSD of $\widehat{\mathbf{w}}(n)$ is defined by

$$\mathcal{D}_w(n+1) = \mathcal{D}_w(n) + 2\,\mu_w(n)\,E\{\Re(\Delta\mathbf{w}^H(n)\,\boldsymbol{\xi}_w(n))\}$$
$$+ \mu_w^2(n)\,E\{\Re(\boldsymbol{\xi}_w^H(n)\,\boldsymbol{\xi}_w(n))\}, \tag{93}$$

where $\boldsymbol{\xi}_w(n)$ is given by

$$\boldsymbol{\xi}_w(n) = E\left\{\frac{\mathcal{M}_w \, \boldsymbol{a}_w}{\|\mathbf{y}(n)\|^2}\right\}, \tag{94}$$

and $\Re(\cdot)$ denote as the real operator.

Thus, the MSD of $\widehat{\mathbf{b}}(n)$ can be computed as

$$\mathcal{D}_b(n+1) = \mathcal{D}_b(n) - 2\,\mu_b(n)\,E\left\{\Re\left(\Delta \mathbf{b}^H(n)\,\boldsymbol{\xi}_b(n)\right)\right\}$$
$$+ \mu_b^2(n)\,E\left\{\Re\left(\boldsymbol{\xi}_b^H(n)\,\boldsymbol{\xi}_b(n)\right)\right\}, \tag{95}$$

where $\boldsymbol{\xi}_b(n)$ is calculated by

$$\boldsymbol{\xi}_b(n) = E\left\{\frac{\mathcal{M}_b \, \boldsymbol{a}_b}{\|\mathbf{d}(n)\|^2}\right\}. \tag{96}$$

Following these approximations

$$\lim_{n\to\infty} \mathcal{D}_w(n+1) = \lim_{n\to\infty} \mathcal{D}_w(n), \tag{97}$$

$$\lim_{n\to\infty} \mathcal{D}_b(n+1) = \lim_{n\to\infty} \mathcal{D}_b(n), \tag{98}$$

are taken into Eq.(93) and Eq.(95). The normalised step-size parameters $\mu_w(n)$ and $\mu_b(n)$ are bounded as

$$0 < \mu_w(n) < 2\left|\Re\left(\frac{\Delta \mathbf{w}^H(n)\,\boldsymbol{\xi}_w(n)}{\boldsymbol{\xi}_w^H(n)\,\boldsymbol{\xi}_w(n)}\right)\right|, \tag{99}$$

$$0 < \mu_b(n) < 2\,\Re\left(\frac{\Delta \mathbf{b}^H(n)\,\boldsymbol{\xi}_b(n)}{\boldsymbol{\xi}_b^H(n)\,\boldsymbol{\xi}_b(n)}\right). \tag{100}$$

Therefore, the optimal step-size parameters μ_w^{opt} and μ_b^{opt} can be formulated by

$$\mu_w^{opt} = \left|\Re\left(\frac{\Delta \mathbf{w}^H(n)\,\boldsymbol{\xi}_w(n)}{\boldsymbol{\xi}_w^H(n)\,\boldsymbol{\xi}_w(n)}\right)\right|, \tag{101}$$

$$\mu_b^{opt} = \Re\left(\frac{\Delta \mathbf{b}^H(n)\,\boldsymbol{\xi}_b(n)}{\boldsymbol{\xi}_b^H(n)\,\boldsymbol{\xi}_b(n)}\right). \tag{102}$$

7. Simulation results

We implemented the ADSL transmission channel based on parameters as follows: the sampling rate f_s = 2.208 MHz, the size of FFT N = 512, and the input signal power of -40dBm/Hz. The standard ADSL system parameters were shown in Table 1. The ADSL downstream starting at active tones 38 up to tone 255 that comprises 512 coefficients of channel impulse response. The signal to noise ratio gap of 9.8dB, the coding gain of 4.2dB and the noise margin of 6dB were chosen for all active tones. The additive white Gaussian noise (AWGN) with a power of -140dBm/Hz and near-end cross talk (NEXT) from 24

Asymmetric Digital Subscriber Line (ADSL) Specifications			
Taps of $\widehat{\mathbf{w}}$ (N_w)	32	FFT size (N)	512
Taps of $\widehat{\mathbf{b}}$ (N_b)	32	Cyclic prefix (ν)	32
Sampling rate (f_s)	2.208 MHz	Signal to noise ratio gap	9.8 dB
Tone spacing	4.3125 KHz	Noise margin	6 dB
TX-DMT block (M)	400	Coding gain	4.2 dB
TX sequence	$M \times N$	Input power	-40dBm/Hz
Input impedance	100 Ω	AWGN power	-140dBm/Hz

Table 1. The standard ADSL system for simulation.

ADSL disturbers were included over the entire test channel. The optimal synchronisation delay (Δ) can be obtained from the proposed algorithm that was equal to 45. The ADSL downstream simulations with the carrier serving area (CSA) loop no. 1 was the representative of simulations with all 8 CSA loops as detailed in (Al-Dhahir & Cioffi, 1996). The CSA#1 loop is a 7700 ft, 26 gauge loop with 26 gauge bridged tap of length of 600 ft at 5900 ft.

The initial parameters of the proposed AS-OSNALMS algorithm were $\widehat{\mathbf{w}}(0) = \widehat{\mathbf{b}}(0) = \widetilde{\Psi}_w(0) = \widetilde{\Psi}_b(0) = [0.001 \ 0 \ \cdots \ 0]^T$ and of NLMS algorithm were $\mu_w = 0.15, \mu_b = 0.075$. The NLMS algorithm was calculated with the fixed step-size for TEQ and TIR with the method as described in Section 4. Fig. 4 depicts the original simulated channel, SIR and TIR of the proposed AS-OSNALMS algorithm which compared with SIR of MMSE-UEC. It is noted that the comparable lengths of SIR and TIR of proposed algorithm are shorter than the original channel. This explains the channel-shortening capability of the proposed algorithm. Fig. 5 illustrates the MSE curves of proposed AS-OSNALMS and NLMS algorithms. The MSE curve of proposed algorithm is shown to converge to the MMSE. Fig. 6 and Fig. 7 show the mean square deviation (MSD) on TEQ and TIR of proposed AS-OSNALMS and NLMS algorithms. The trajectories of $\mu_w(n)$ and $\mu_b(n)$ at the different of initial step-size μ_{w_0} and μ_{b_0} are presented with the fixed at the adaptation parameters α_w and α_b in Fig. 8 and Fig. 9 and with the different α_w and α_b in Fig. 10 and Fig. 11. Comparing the proposed AS-OSNALMS algorithm with the fixed at the adaptation parameters, it has been shown that the proposed algorithms have faster initial convergence rate with the different setting of initial step-size and adaptation parameters. Their are shown to converge to their own equilibria.

8. Conclusion

In this chapter, we present the proposed adaptive step-size order statistic LMS-based TEQ and TIR for DMT-based systems. We introduce how to derive the updated tap-weight vector $\widehat{\mathbf{w}}(n)$ and $\widehat{\mathbf{b}}(n)$ as the solution of constrained optimisation to obtain a well-known NLMS algorithm, which an averaged order statistic scheme is replaced linear smoothing of the gradient estimation. We demonstrate the derivation of adaptive step-size mechanism for the proposed order statistic normalised averaged least mean square algorithm. The proposed algorithms for TEQ and TIR can adapt automatically the step-size parameters. The adaptation of MSE, MSD of TEQ and MSD of TIR curves of the proposed algorithms are shown to converge to the MMSE in the simulated channel. According to the simulation results, the proposed algorithms provide a good approach and are appeared to be robust in AWGN and NEXT channel as compared to the existing algorithm.
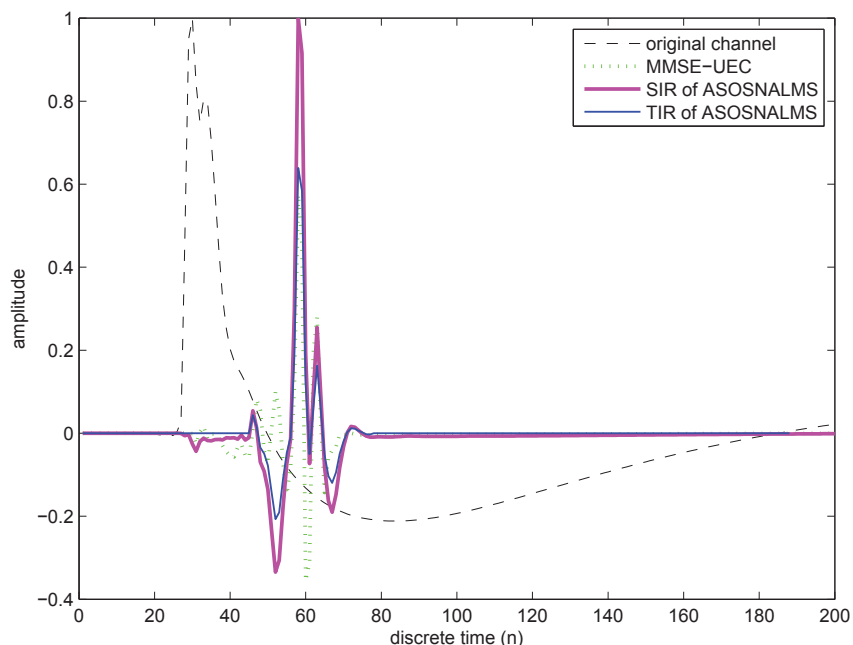
Fig. 4. Original channel, SIR of proposed ASOS-NALMS and TIR of AS-OSNALMS which compared with SIR of MMSE-UEC, when the samples of CSA loop are loop #1. Other parameters are $\mu_{w_0} = 0.415, \mu_{b_0} = 0.095, \alpha_w = 1.25 \times 10^{-6}$ and $\alpha_b = 1.5 \times 10^{-6}$.
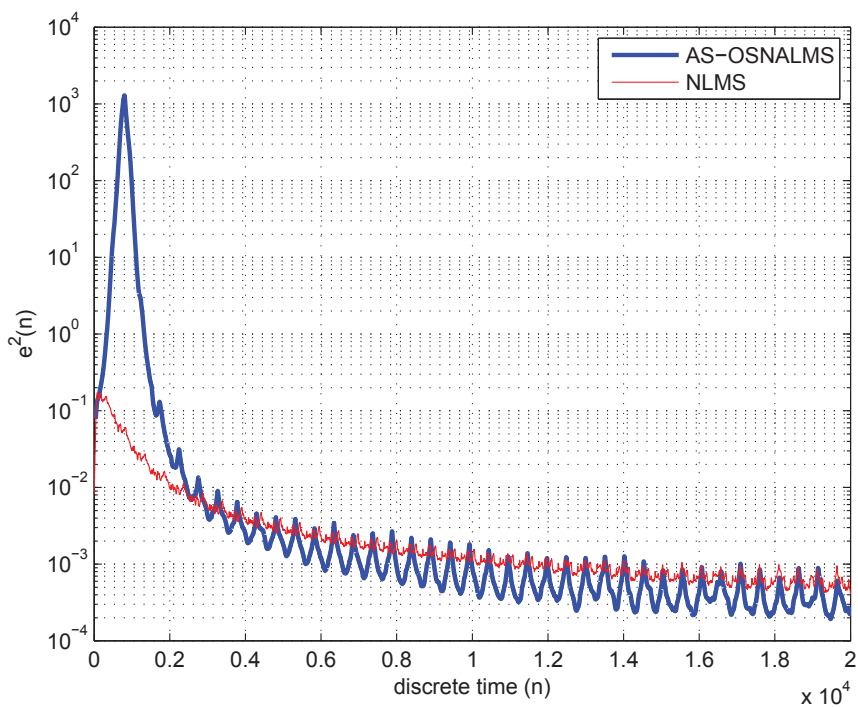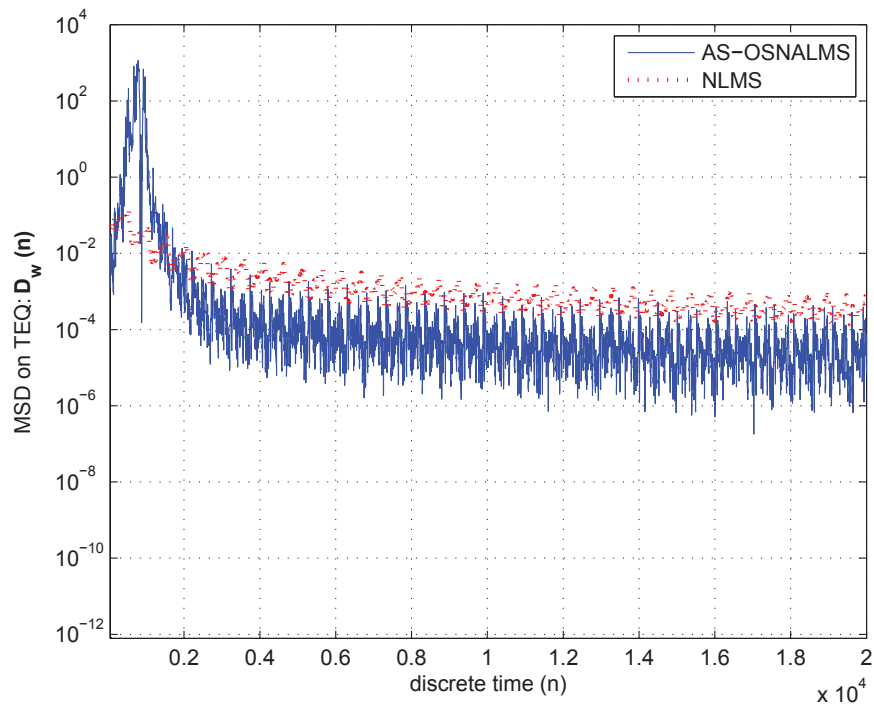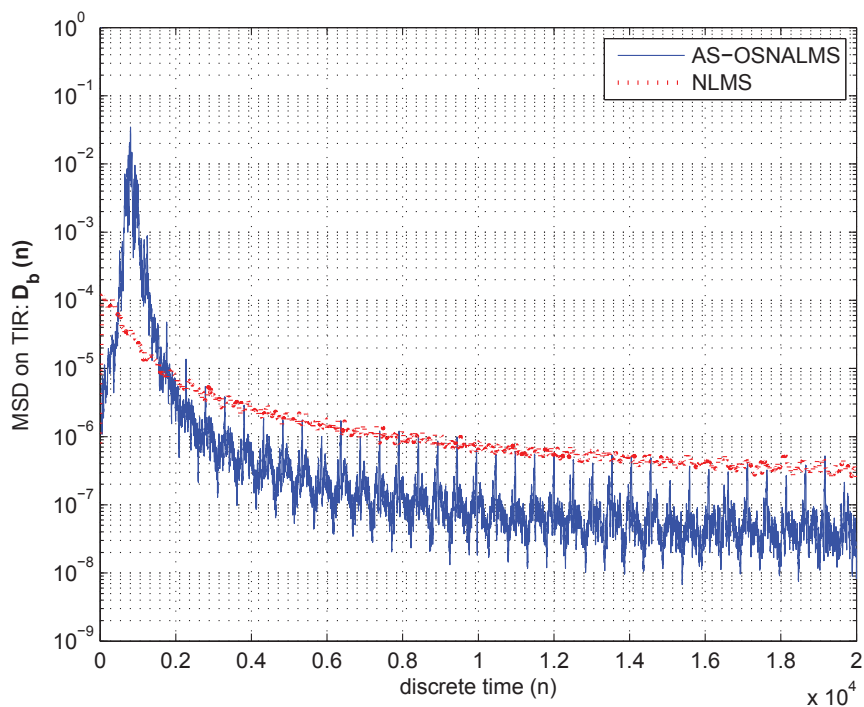

Fig. 5. Learning Curves of MSE of proposed AS-OSNALMS and NLMS algorithms for TEQ and TIR, when the samples of CSA loop are loop #1. Other parameters of AS-OSNALMS algorithm are $\mu_{w_0} = 0.415, \mu_{b_0} = 0.095, \alpha_w = 1.25 \times 10^{-6}, \alpha_b = 1.5 \times 10^{-6}$ and of NLMS alorithm are $\mu_w = 0.15, \mu_b = 0.075$.

Fig. 6. Learning Curves of MSD $\mathcal{D}_w(n)$ of proposed AS-OSNALMS and NLMS algorithms for TEQ, when the samples of CSA loop are loop #1. Other parameters of AS-OSNALMS algorithm are $\mu_{w_0} = 0.415$, $\mu_{b_0} = 0.095$, $\alpha_w = 1.25 \times 10^{-6}$, $\alpha_b = 1.5 \times 10^{-6}$ and of NLMS algorithm are $\mu_w = 0.15$, $\mu_b = 0.075$.

Fig. 7. Learning Curves of MSD $\mathcal{D}_b(n)$ of proposed AS-OSNALMS and NLMS algorithms for TIR, when the samples of CSA loop are loop #1. Other parameters of AS-OSNALMS algorithm are $\mu_{w_0} = 0.415$, $\mu_{b_0} = 0.095$, $\alpha_w = 1.25 \times 10^{-6}$, $\alpha_b = 1.5 \times 10^{-6}$ and of NLMS algorithm are $\mu_w = 0.15$, $\mu_b = 0.075$.
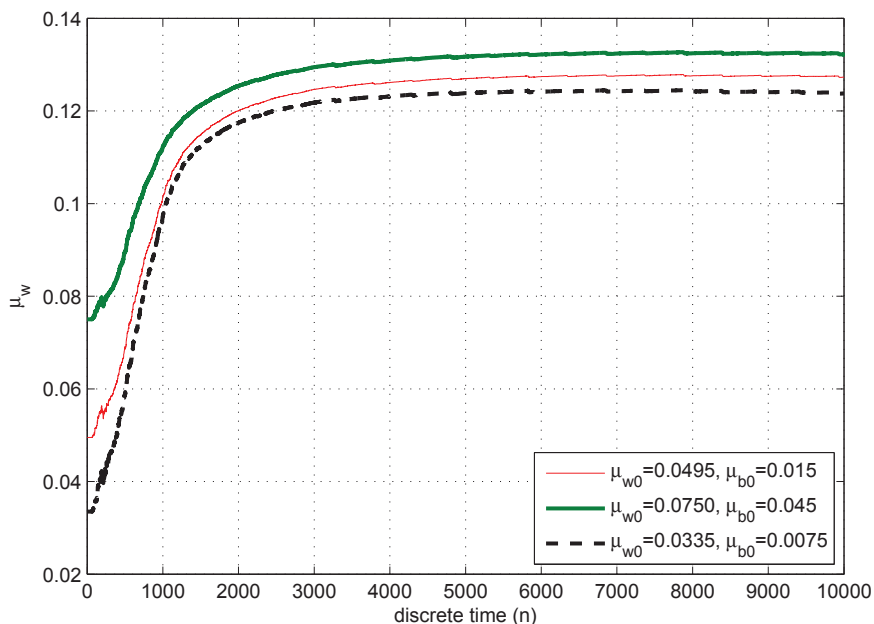
Fig. 8. Trajectories of μ_w of proposed AS-OSNALMS algorithm for TEQ using different setting of μ_{w_0} and μ_{b_0} for TEQ and TIR with fixed at $\alpha_w = 4.45 \times 10^{-4}$ and $\alpha_b = 1.75 \times 10^{-4}$, when the samples of CSA loop are loop #1.
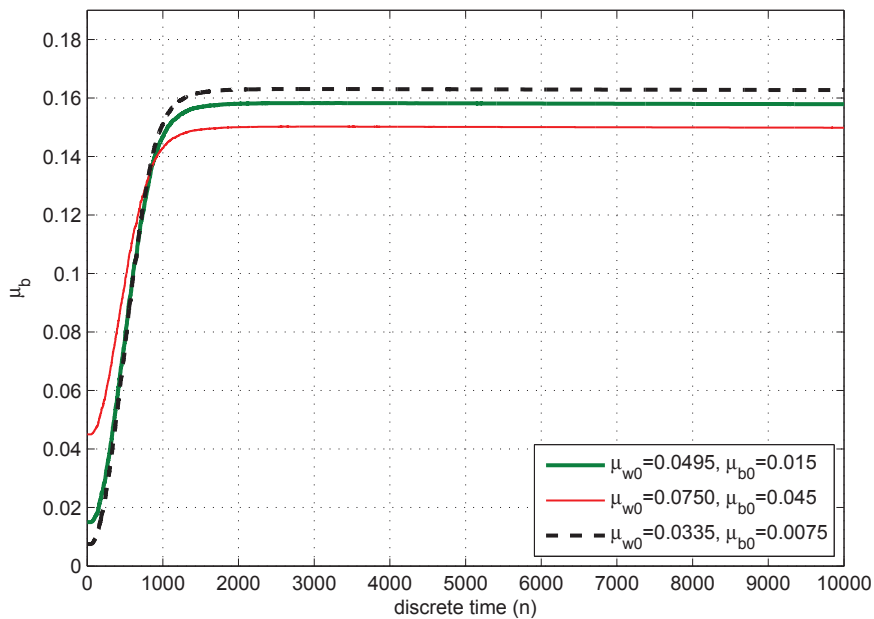

Fig. 9. Trajectories of μ_b of proposed AS-OSNALMS algorithm for TIR using different setting of μ_{w_0} and μ_{b_0} for TEQ and TIR with fixed at $\alpha_w = 4.45 \times 10^{-4}$ and $\alpha_b = 1.75 \times 10^{-4}$, when the samples of CSA loop are loop #1.
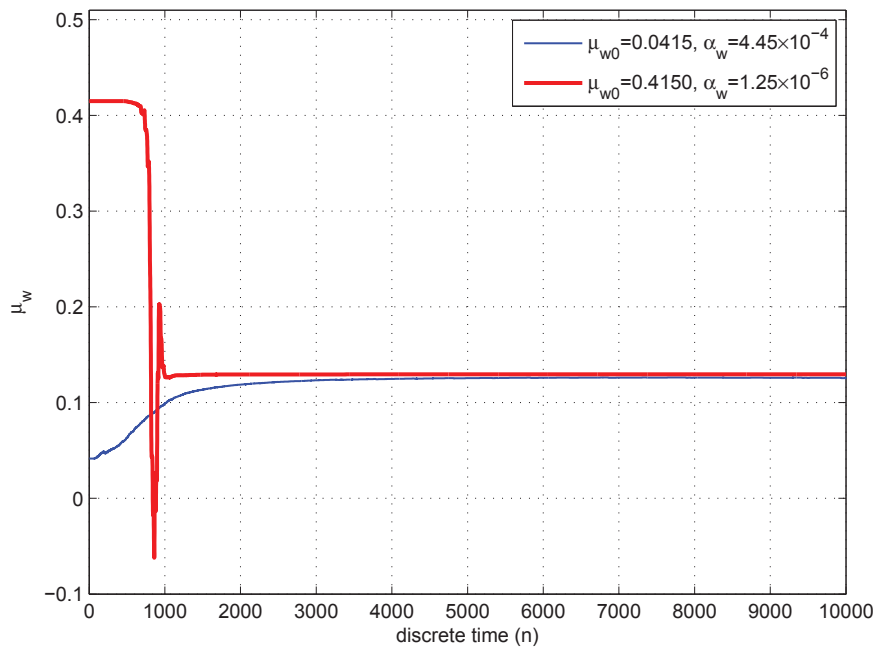
Fig. 10. Trajectories of μ_w of proposed AS-OSNALMS algorithm for TEQ using different setting of μ_{w_0} and μ_{b_0} for TEQ and TIR with different at $\alpha_w = 4.45 \times 10^{-4}$ and $\alpha_w = 1.25 \times 10^{-6}$, when the samples of CSA loop are loop #1.
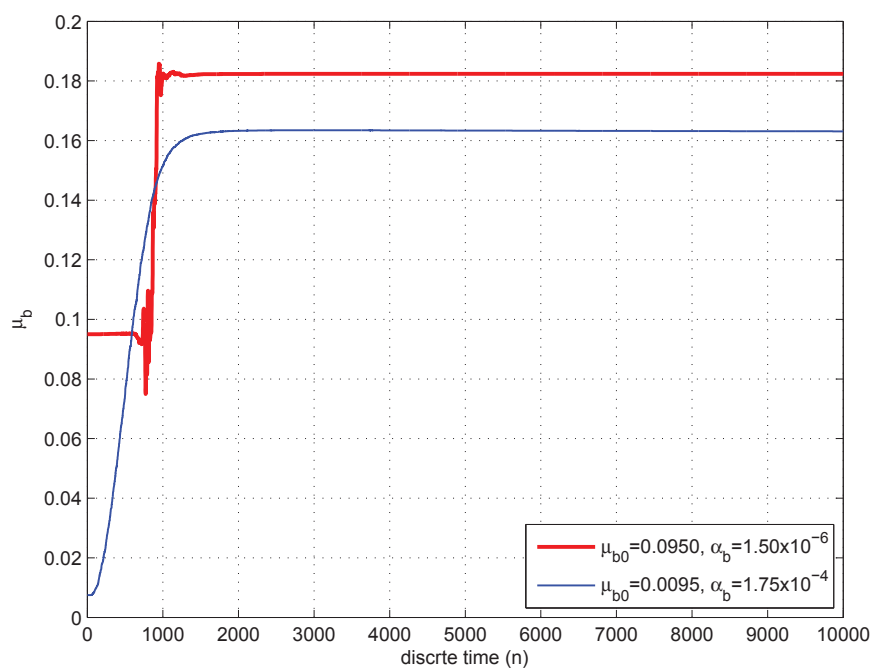
Fig. 11. Trajectories of μ_b of proposed AS-OSNALMS algorithm for TIR using different setting of μ_{w_0} and μ_{b_0} for TEQ and TIR with different at $\alpha_b = 1.75 \times 10^{-4}$ and $\alpha_b = 1.5 \times 10^{-6}$, when the samples of CSA loop are loop #1.

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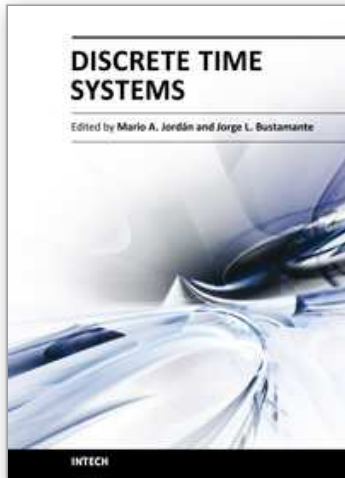
Discrete-Time Systems comprehend an important and broad research field. The consolidation of digital-based computational means in the present, pushes a technological tool into the field with a tremendous impact in areas like Control, Signal Processing, Communications, System Modelling and related Applications. This book attempts to give a scope in the wide area of Discrete-Time Systems. Their contents are grouped conveniently in sections according to significant areas, namely Filtering, Fixed and Adaptive Control Systems, Stability Problems and Miscellaneous Applications. We think that the contribution of the book enlarges the field of the Discrete-Time Systems with signification in the present state-of-the-art. Despite the vertiginous advance in the field, we also believe that the topics described here allow us also to look through some main tendencies in the next years in the research area.

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