28

Control of a Simple Constrained MIMO System with Steady-state Optimization

František Dušek and Daniel Honc
University of Pardubice
Czech Republic

1. Introduction

This chapter covers two issues (along many others) relating to complex systems control. The main theme is connected with control of Multi-Input Multi-Output (MIMO) systems. If the controlled system has more inputs than outputs (further labelled as MI+MO) there exist many combinations of the inputs for one combination of the outputs. We are able to reach desired system outputs (the main control aim) with many input combinations. This situation is very interesting from practical point of view. Usually optimal inputs combination exists – from some point of view. This combination leads for example to minimum energy consumption, maximum production efficiency or minimum machinery load etc. In practice the set of possible inputs combination is reduced because of constrains and the best feasible combination lies very often on constrain. It would be suitable to extend the controller design to include supplementary demand simultaneously with the fulfilment of the main control aim – to ensure best feasible input combination, too. The common advanced controller like LQ controller has no problem with MIMO system which has different number of inputs and outputs in contrary to standard controllers designed as decentralized control. However the constrains respecting within the standard LQ controller design is not possible. Another advanced controller – Model Predictive Controller (MPC) allows constrains handling (Camacho & Bordons, 1999), (Maciejowski, 2002), (Rossiter, 2003) but the standard controller design doesn't solve which combination of inputs will occur in the steady-state in case of system with more inputs than outputs.
One possibility how to achieve optimal inputs combination is to formulate one term of the cost function connected with inputs penalization as a deviation from ideal inputs combination. This approach is used e.g. in (Novák, 2009) but according to the opinion of authors this approach isn't as universal as following proposal. We have suggested adding another term into the cost function of predictive controller – terminal state in a form of the deviation from desired terminal state. The desired terminal state is chosen that it corresponds to feasible optimal inputs combination and a value of the set-point at the end of the control horizon. Authors call this technique *Steady-state optimization* because the influence of the terminal state deviation comes to light namely in steady-state when the main control aim (desired output combination) is or has been already fulfilled - see also (Dušek & Honc, 2009). The controller ensures both main and supplementary control aims – achievement of desired outputs and inputs moving to an optimal combination. An incorporation of the terminal state into the cost function has also another advantage. The

addition of the terminal state into the cost function is one of the possibilities how to ensure closed-loop stability (Mayne et al., 2000). In (Bitmead et al., 1990) it is proposed using the quadratic form $\mathbf{x}_t^T\mathbf{P}\mathbf{x}_t$, as a terminal cost function where vector $\mathbf{x}_t$ is state at the end of control horizon (terminal state) and the matrix $\mathbf{P}$ is terminal value of the Riccati difference equation.
We propose incorporate the terminal cost function in the form of desired ($\mathbf{x}_w$) and terminated state ($\mathbf{x}_t$) deviation – $(\mathbf{x}_w - \mathbf{x}_t)^T\mathbf{Q}_x(\mathbf{x}_w - \mathbf{x}_t)$ (see Chapter 4.2). The determination of a desired state based on the controlled system steady-state gain matrix is shown for the case of general MIMO system in the article (Dušek & Honc, 2008b) and in detail in (Dušek & Honc, 2008a in Czech). The computation of desired state for the case of MI+MO systems is described in Chapter 4.3.
Suggested technique is applied on the thermostatic bath control. The idealized thermostatic bath (see sketch in Fig. 1) is an example of one of the simplest real constrained system with more inputs than outputs. On this example it is possible to demonstrate another problem we can meet by the control of complex systems – in some cases an advanced controller improves control quality only slightly in comparison with very simple controller. The problem usually arises when one property of the controlled system is dominant. In that case a simple controller respecting the dominant feature can provide satisfactory control. But situation can change dramatically if some specific (or additional) information about the system is available or additional control demands are formulated. Manipulated variables asymmetric constraints are dominant feature of controlled system in our case. Very simple on-off controller based on knowledge of constrains provides similar control quality (from performance measures and control costs points of view) as the sophisticated MPC controller even based on full knowledge of MIMO system dynamic. This holds for the case that we do not know nor do not use information about future reference signal course in MPC controller design. Simple controller do not allow to use such information on the contrary to advanced controller – predictive controller respecting constrains and using future reference course knowledge.

2. Problem formulation, solution fundamentals

In the following text we will show two different control designs for an ideal thermostatic bath. It is possible to describe controlled system behaviour by continues dynamical fourth order mathematical model with four inputs (three are manipulated and constrained) and one controlled output. The model derivation is based on first principle approach (energy conservation law) and a few simplified assumptions. Model parameters are chosen so that the model behaviour is realistic for needs of simulated control experiments. The continuous-time model is numerically transformed into discrete-time state space form for chosen sample time.
The aim of the control is to follow as good as possible a reference signal with respecting the manipulated variable constrains with minimum control cost – energy consumption. Two very different controllers have been designed. The first one is a couple of very simple discrete-time on-off controllers based on system specific feature – two asymmetrically constrained manipulated inputs. The second one is an advanced discrete-time predictive controller with quadratic cost function, finite horizon and banded constrains based on a discrete-time linear state space MI+MO model (TISO – system with two manipulated inputs and one controlled output). The controller cost function is supplemented by a quadratic terminal cost function of the desired and actual terminal state deviation – ensuring steady-

state optimization. An addition of a desired terminal state into controller cost function allows including the demand on minimal energy cost. The minimisation of cost function is made by quadratic programming. The behaviour of both controllers is demonstrated on simulated discrete-time control experiments with continuous-time model of ideal thermostatic bath. Results of simulated controls by on-off controller and proposed predictive controller are discussed. Control responses of the predictive controller without knowledge of future course of the reference signal (only an actual set-point is known) and when the future course is known are compared too. All the computations, results evaluation and visualisation have been made in MATLAB environment.

3. Controlled system

The controlled system is the ideal thermostatic bath which principal sketch is drawn in Fig. 1. Similar real devices are used for controlled heating or cooling of some element. This device is one of the simplest real systems with more inputs then outputs. It is represented by a partially isolated vessel filled with water (denoted C) and placed element D – its temperature T_D is controlled. It is possible to increase the water temperature T_C with electric heating (denoted A). Heating power E is controlled continuously. Cooling helix (denoted B) is used to decrease water temperature - water flows with flow-rate Q through a pipe. Inlet temperature T_{B0} must be lower than a placed element desired temperature T_D. Temperature T_C is affected also by ambient temperature T_o (heating exchange with surroundings because of imperfect isolation). Ambient temperature T_o can cool down the bath if $T_o < T_C$ or heat it if $T_o > T_C$.

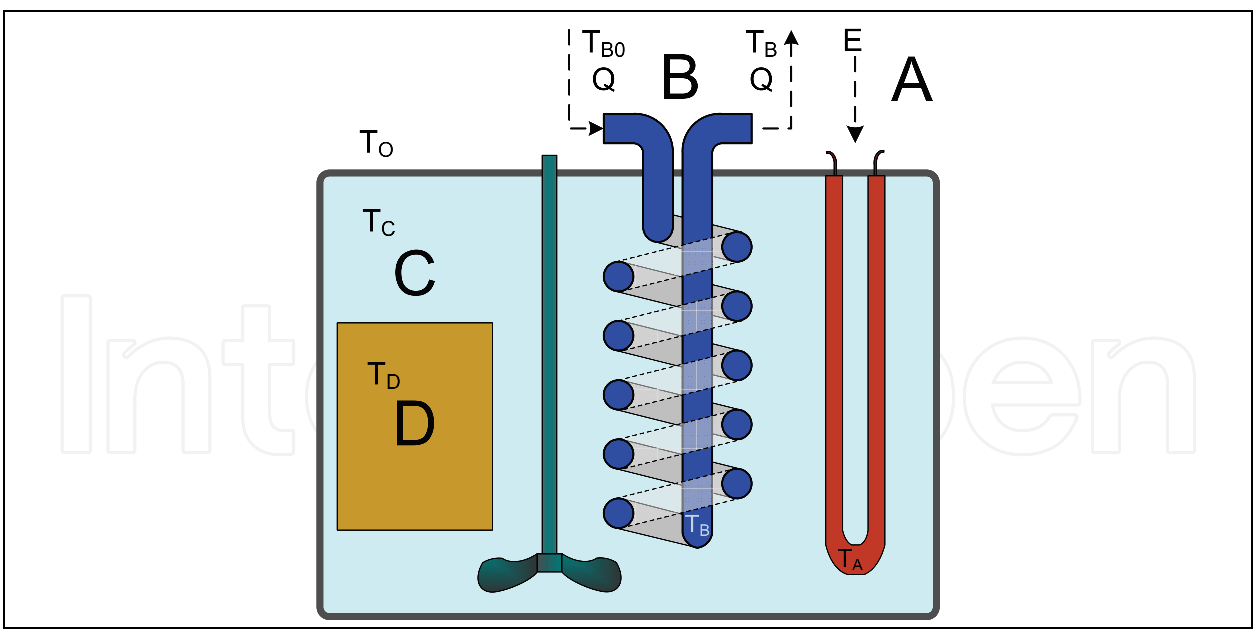

Fig. 1. Thermostatic bath scheme

An ambient temperature T_o is supposed to be constant. A cooling water flow-rate must be within the range $0 \leq Q \leq Q_{max}$, cooling water input temperature $T_{B0min} \leq T_{B0} \leq T_{B0max}$ and heating power $0 \leq E \leq E_{max}$. These asymmetrical constrain lead to special actuating of inputs – it is possible to increase or decrease the state variables only with the particular input.

3.1 Derivation of mathematical model of the plant

The thermostatic bath can be divided into four parts (thermal capacities) according to the scheme in Fig. 1. The state of every part is approximated by "characteristic or average" temperature. The introduction of characteristic temperatures leads to the essential simplification of a process description and hence partial differential equations using is not necessary. Based on the energy balance of every part the whole system can be described under another simplified assumptions (ideal mixing, constant heat transfer coefficients etc.) with a four ordinary differential equations – mathematical model of the plant. The model has eight time depending variables – four input variables (cooling water flow-rate Q with input temperature T_{B0}, heating power E, ambient temperature T_0) and four state variables (characteristic temperature of the heating element T_A, cooling water characteristic temperature T_B, water characteristic temperature T_C and placed element characteristic temperature T_D).
If we put together thermal balances mentioned above and introduce simplified assumptions we get relatively simple dynamic mathematical model of the thermostatic bath as a set of four ordinary differential equations written as

$$E = \alpha_A S_{AC}(T_A - T_C) + m_A c_A \mathrm{d}T_A/\mathrm{d}t \tag{1a}$$

$$Q c_B T_{B0} + \alpha_B S_{BC}(T_C - T_B) = Q c_B T_B + m_B c_B \mathrm{d}T_B/\mathrm{d}t \tag{1b}$$

$$\alpha_A S_{AC}(T_A - T_C) = \alpha_B S_{BC}(T_C - T_B) + \alpha_C S_{C0}(T_C - T_o) + \alpha_D S_{DC}(T_C - T_D) + m_C c_C \mathrm{d}T_C/\mathrm{d}t \tag{1c}$$

$$\alpha_D S_{DC}(T_C - T_D) = m_D c_D \mathrm{d}T_D/\mathrm{d}t \tag{1d}$$

where
T_o is ambient temperature,
$E(t)$ is heating power in the range $0 \le E \le E_{max}$ (increases temperature of A),
$Q(t)$ is flow-rate of the cooling water in the range $0 \le Q \le Q_{max}$ (decreases temperature of B),
$T_{B0}(t)$ is input temperature of the cooling water in the range $T_{B0min} \le T_{B0} \le T_{B0max}$ (decreases temperature of B),
$T_x(t)$ is characteristic temperature (state variables $T_A \dots T_D$),
m_x is mass of individual part,
c_x is specific heat capacity of individual part,
S_{xy} is heat transfer area between two adjacent parts and
α_x is heat transfer coefficient.
Parameters given in Table 1 are used in following simulation experiments.

	units	heating A	cooling B	water C	element D
m_x	kg	0.3	0.1567	4.0	8.93
c_x	$\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$	452	4180	4180	383
S_{xy}	$\mathrm{m^2}$	0.0095	0.065	0.24	0.06
α_x	$\mathrm{J \cdot m^{-2} \cdot s^{-1} \cdot K^{-1}}$	750	500	5	500

Table 1. Process model parameters

The graphs in Fig. 2 demonstrate the basic dynamic behaviour of the system with parameters according to Table 1. In this figure it is depicted the temperature response of placed element T_D (upper graph) to 10 minutes wide pulse of maximal heating power E (middle graph) and 10 minutes wide pulse of minimal cooling water temperature T_{B0} (lower graph). The experiment starts from a system steady-state when all temperatures are the same and equal to the ambient temperature T_0. This steady-state corresponds to heating power equal zero and cooling water temperature equal to ambient temperature. The constant cooling water flow-rate is 0.5 litres per minute. From graphs it is evident that maximal heating is more powerful than maximal cooling.

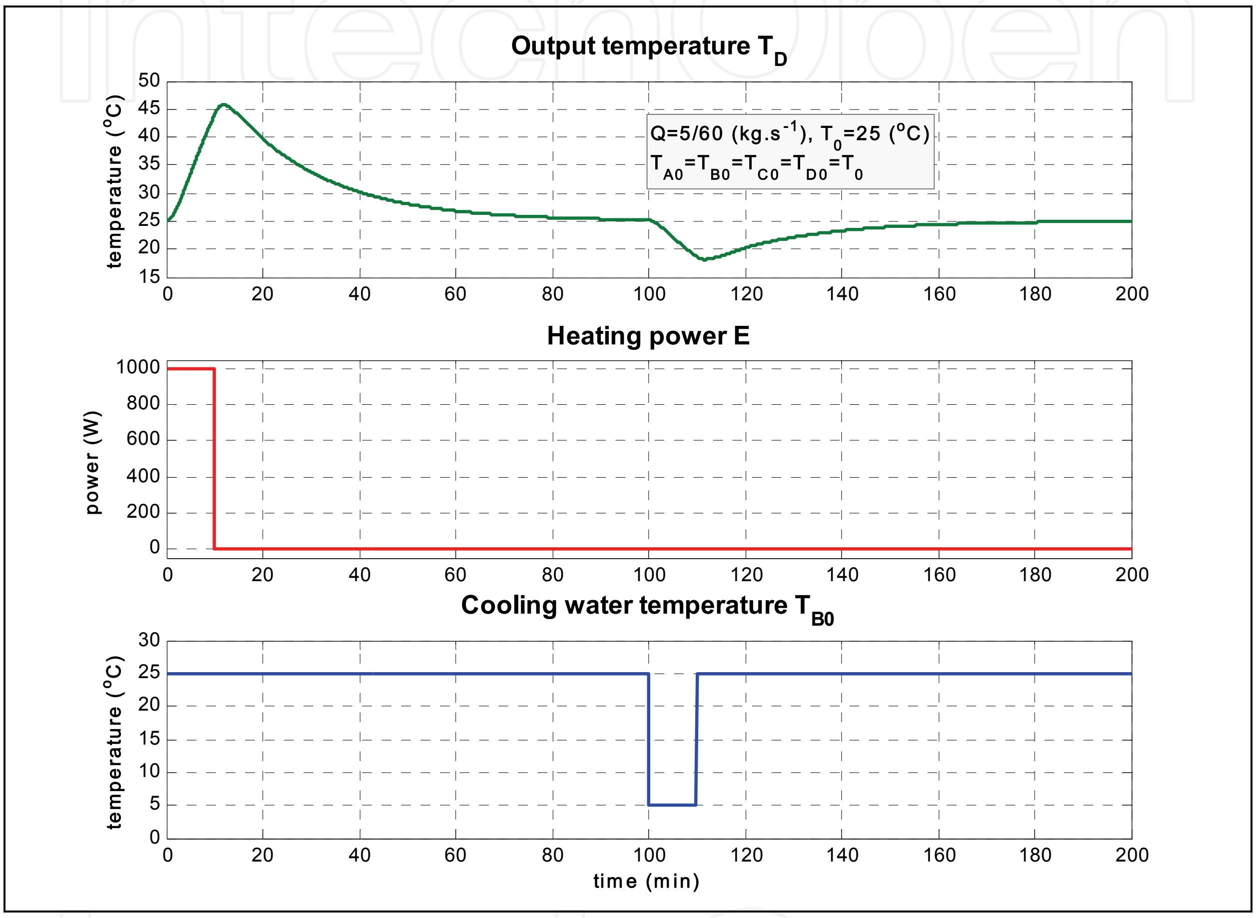

Fig. 2. Output variable response to inputs changes

3.2 Continuous-time mathematical model in a standard form

From the control point of view the system has three manipulated variables (Q, T_{B0}, E), one measured disturbance (T_0), four state variables (T_A, T_B, T_C and T_D) and one controlled variable (T_D). To get linear system suitable for the control design we choose only the input temperature T_{B0} and heating power E as manipulated variables. The dynamic of input temperature T_{B0} refrigerating is neglected to simplify the thermostatic bath description. The cooling water flow-rate Q is supposed to be constant. This "non practical" choice is made due to simplification of predictive controller design – to avoid problem with nonlinear system control design. For needs of this text it isn't important whether the manipulated variable is cooling water flow-rate or temperature.

The equations (1a) – (1d) can be rewritten in a matrix form of standard continuous-time state space model as

$$dx/dt = A_c x + B_c u \tag{2a}$$

$$y = C_c x \tag{2b}$$

Integral part of the process description is information about the manipulated variables constrains.

$$0 \le E \le E_{max} \tag{2c}$$

$$T_{B0min} \le T_{B0} \le T_{B0max} \tag{2d}$$

where

x(t) is vector of state variables T_A, T_B, T_C and T_D,

$$\mathbf{x}(t) = [T_A(t), T_B(t), T_C(t), T_D(t)]^T$$

u(t) is vector of inputs E, T_{B0} and T_0,

$$\mathbf{u}(t) = [E(t), T_{B0}(t), T_0(t)]^T$$

$y(t)$ is output variable T_D and

$\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$ are matrices of continuous-time state space model parameters (see Eq. 2e)

$$\mathbf{A}_c = \begin{bmatrix} -\frac{\alpha_A S_{AC}}{m_A c_A} & 0 & \frac{\alpha_A S_{AC}}{m_A c_A} & 0 \\ 0 & -\frac{\alpha_B S_{BC}}{m_B c_B} - \frac{Q}{m_B} & \frac{\alpha_B S_{BC}}{m_B c_B} & 0 \\ \frac{\alpha_A S_{AC}}{m_C c_C} & \frac{\alpha_B S_{BC}}{m_C c_C} & -\frac{\alpha_A S_{AC} + \alpha_B S_{BC} + \alpha_C S_{C0} + \alpha_D S_{DC}}{m_C c_C} & \frac{\alpha_D S_{DC}}{m_C c_C} \\ 0 & 0 & \frac{\alpha_D S_{DC}}{m_D c_D} & -\frac{\alpha_D S_{DC}}{m_D c_D} \end{bmatrix}$$

$$\mathbf{B}_c = \begin{bmatrix} \frac{1}{m_A c_A} & 0 & 0 \\ 0 & \frac{Q}{m_B} & 0 \\ 0 & 0 & \frac{\alpha_C S_{C0}}{m_C c_C} \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{C}_c = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \tag{2e}$$

The continuous-time mathematical model (2) with parameters given by Table 1 was used in simulation control experiments as a plant (process) model.

3.3 Discrete-time mathematical model for MPC control design

A standard predictive controller design is based on a discrete-time linear time invariant (LTI) process model. If we suppose constant cooling water flow-rate Q than the matrices $\mathbf{A}_c$ and $\mathbf{B}_c$ in (2e) are constant (time invariant) for given values of thermostatic bath parameters. Now we can transform the linear continues-time model (2) into equivalent linear discrete-time state space model (3) or an input-output model under the "zero order hold" assumption - that the value of inputs between two equidistant sample times are constant. We get the values of discrete-time state space model matrices **A**, **B** and **C** for given sample time T numerically (in MATLAB with function c2d)

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \tag{3a}$$

$$y(k) = \mathbf{C}\mathbf{x}(k) \tag{3b}$$

$$\mathbf{u}_{\min} \le \mathbf{u}(\mathrm{k}) \le \mathbf{u}_{max} \tag{3c}$$

where

$\mathbf{x}(k)$ is vector of sampled state variables T_A, T_B, T_C and T_D,

$$\mathbf{x}(k) = [T_A(k), T_B(k), T_C(k), T_D(k)]^T$$

$\mathbf{u}(k)$ is vector of discrete-time inputs E, T_{B0} and T_0,

$$\mathbf{u}(k) = [E(k), T_{B0}(k), T_0(k)]^T$$

$$\mathbf{u}_{\min} = [0, T_{B0\min}, T_0]^T$$

$$\mathbf{u}_{\max} = [E_{\max}, T_{B0\max}, T_0]^T$$

$y(k)$ is sampled output variable T_D and

A, **B**, **C** are discrete-time model parameters (matrices).

3.4 Prediction equations in matrix form

If we use cost function in a general matrix form then it is suitable to formulate future process output directly in a matrix form and not in the original iterative form (3a). Because we will also need a state prediction at the end of the prediction horizon we will formulate the state prediction equation for N sample step ahead, too. Based on knowledge of the actual state $\mathbf{x}(k)$ and a vector of future inputs $\mathbf{u}_N$ we can write these two prediction matrix equations as

$$\mathbf{y}_N = \mathbf{S}_{yx}\mathbf{x}(k) + \mathbf{S}_{yu}\mathbf{u}_N \tag{4a}$$

$$\mathbf{x}(k+N) = \mathbf{S}_{xx}\mathbf{x}(k) + \mathbf{S}_{xu}\mathbf{u}_N \tag{4b}$$

where

$\mathbf{y}_N$ is vector of future output T_D at time k, k+1, …, k+N-1,

$$\mathbf{y}_N = [T_D(k+1), T_D(k+2), \ldots, T_D(k+N)]^T$$

$\mathbf{x}(\mathrm{k})$ is vector of state variables T_A, T_B, T_C and T_D at time k,

$\mathbf{u}_N$ is vector of future inputs E, T_{B0} and T_0 at time k, k+1, …, k+N-1,

$$\mathbf{u}_N = [\mathbf{u}^T(k), \mathbf{u}^T(k+1), \ldots, \mathbf{u}^T(k+N-1)]^T \text{ and}$$

$\mathbf{S}_{xx}$, $\mathbf{S}_{xu}$, $\mathbf{S}_{yx}$, $\mathbf{S}_{xu}$ are constant matrices depending on the process matrices **A**, **B** and **C**

$$\mathbf{S}_{xx} = \mathbf{A}^N \qquad \mathbf{S}_{xu} = \begin{bmatrix} \mathbf{A}^{N-1}\mathbf{B} & \mathbf{A}^{N-2}\mathbf{B} & \ldots & \mathbf{AB} & \mathbf{B} \end{bmatrix}$$
$$\mathbf{S}_{yx} = \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{N-2} \\ \mathbf{CA}^{N-1} \end{bmatrix} \qquad \mathbf{S}_{xu} = \begin{bmatrix} \mathbf{CB} & 0 & \ldots & 0 & 0 \\ \mathbf{CAB} & \mathbf{CB} & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{CA}^{N-3}\mathbf{B} & \mathbf{CA}^{N-4}\mathbf{B} & \ldots & \mathbf{CB} & 0 \\ \mathbf{CA}^{N-2}\mathbf{B} & \mathbf{CA}^{N-3}\mathbf{B} & \ldots & \mathbf{CAB} & \mathbf{CB} \end{bmatrix} \tag{4c}$$

4. Control design

The main control objectives are to follow the reference signal, to respect manipulated variables constrains and at the same time to minimize energy costs for heating and refrigerating of the cooling water (we do not consider dynamics of the cooling water input temperature). It is evident that the set-point of steady-state output temperature is reachable with many combinations of heating power and cooling water temperature. The system has different overall energy consumption for each combination. From energy consumption point of view the ideal combination in steady-state is when heating power equals to zero and temperature of cooling water equals to surrounding temperature. This ideal combination with zero energy cost is feasible only in the situation that desired temperature is equal to surrounding temperature. Because of the imperfect thermostat insulation it is necessary either permanently to heat or to cool in all other cases. Hence if it is necessary to heat then refrigeration must be off and vice versa. This idea is the principle of the simplest on-off controller without any tuneable parameters described in Chapter 4.1. Based on this idea it is also possible to design many other simple controllers with some solution for the system with more inputs than outputs. Well known is for example the technique called split range in which the output of a controller is split into two or more manipulated variables.
But these solutions are made ad hoc. More systematic and general way is to use MIMO controller. Such a controller based on principles of model predictive control is described in Chapter 4.2.

4.1 On-off controller

It is possible to control the thermostatic bath with objectives and conditions mentioned above by an on-off controller (to switch between minimal and maximal cooling water input temperature and heating power according to the sign of the control error). This approach uses only one dominant characteristic – asymmetrical manipulated variable actuating. The control error performance measure is comparable with a sophisticated predictive controller without using any information about the future set-point. This strategy uses the only information about manipulated variables constraints ($E_{\max}$, $T_{B0\min}$, $T_{B0,\max}$), actual value of output variable T_D and actual value of set point w at discrete time k and there are no tuneable controller parameters. The resulting very simple thermostatic bath on-of controller is given by (5)

$$e = w(k) - T_D(k) \tag{5a}$$

$$\text{if } e<0 \text{ then } \{E=E_{\max}\ T_{B0}=T_{B0,\max}\} \text{ else } \{E=0\ T_{B0}=T_{B0,\min}\} \tag{5b}$$

where

$w(k)$ is an actual set-point – desired value of output T_D at time k,
e is an actual control error,
E is actual heating power and
T_{B0} is actual cooling water input temperature.

Such a very primitive strategy has interesting features. It is a feedback control with a huge feedback gain and closed loop stability is ensured by respecting the constraints. It is simple variant of adaptive control approach called in literature as Self-Oscillating Adaptive Systems (Åström & Wittenmark, 1995). Big feedback gain causes controller insensitivity to changing of process properties (Wellstead & Zarrop, 1991) and we can suppose operation without problems even in case when the cooling water flow-rate is used as a manipulated variable – in case of nonlinear system. The disadvantage is permanent control variables switching between minimal and maximal values and thereby permanently alternating of the controlled variable in the steady-state. The control quality and control costs are worse than in case of controller with continuous output – see Fig. 3.

4.2 Predictive controller

Predictive controller design is open methodology and it allows incorporating many of control demands and other information. The control objective is formulated as a minimization of a discrete-time cost function that is constrained. It means that the dependencies given by the process model have to be respected. From math point of view it is a task of finding constrained extreme. If the cost function is quadratic with finite horizons, process model is linear and variables are unconstrained then the analytic solution of the cost function minimization exists in a form of matrix equations. If inputs, outputs or states are linearly constrained then it is possible to solve arising task numerically with quadratic programming techniques.
We formulate the discrete-time quadratic cost function on finite horizon of length N steps (both predictive and control horizon) in matrix form (6a) and inputs constrains (6b) as

$$\begin{aligned} J(\Delta\mathbf{u}_N,\mathbf{w}_N,\mathbf{x}_w;N) &= \mathbf{e}_N^T\mathbf{Q}\mathbf{e}_N + \Delta\mathbf{u}_N^T\mathbf{R}\Delta\mathbf{u}_N + \Delta\mathbf{x}^T(N)\mathbf{Q}_x\Delta\mathbf{x}(N) \\ \mathbf{e}_N = \mathbf{w}_N - \mathbf{y}_N \qquad \mathbf{u}_N &= \mathbf{u}_{N,0} + \Delta\mathbf{u}_N \qquad \Delta\mathbf{x}(N) = \mathbf{x}_w - \mathbf{x}(k+N) \end{aligned} \tag{6a}$$

$$\mathbf{u}_{N,\min} \le \mathbf{u}_N \le \mathbf{u}_{N,\max} \tag{6b}$$

where
$\mathbf{y}_N$ is vector of predicted process outputs (see Eq. 4a),
$\mathbf{w}_N$ is vector of future reference signal,
$\mathbf{u}_N$ is vector of future process inputs,
$\mathbf{u}_{N,0}$ is vector of supposed (known) future process inputs,
$\Delta\mathbf{u}_N$ is vector of computed deviations from supposed process inputs (this vector contains only the manipulated inputs),
$\mathbf{x}_w$ is desired terminal state (see Chapter 4.3),
$\mathbf{x}(k+N)$ is predicted terminal state (see Eq. 4b),
N is length of control and prediction horizon (number of samples),
$\mathbf{Q}$, $\mathbf{Q}_x$, $\mathbf{R}$ are square weighting matrices and
$\mathbf{u}_{N,\min}$, $\mathbf{u}_{N,\max}$ are vectors of input constrains.
The cost function (6a) is composed of three parts. All parts are quadratic function of adequate deviations. The first two parts are functions of the all points over the whole horizon and the last part is a function of the last point of horizon only. The first part is a function of control error (the deviation between output and reference signal). It ensures a satisfaction of the main control aim – following the reference signal as close as possible.
The second part is a function of manipulated variables and ensures that the main control aim isn't fulfilled at any cost – infinite or very large values of manipulated variables. The disadvantage of standard form (without deviations) is arising of a steady-state control error. We use this term in a form of deviations of inputs from supposed future inputs. The deviation decreases a steady-state control error and incorporating of supposed course of

inputs $\mathbf{u}_{N,0}$ allows involving known unmanipulated inputs or disturbances (ambient temperature in our case). Supposed inputs can be also used for optimization of inputs values combination in case of MI+MO systems – similar way as in (Novák, 2009).
The third part is the deviation of a desired and actual state at the end of horizon (terminal state). Adding a terminal cost function ensures closed-loop stability (Mayne et al., 2000) but also leads to arising of a steady-state control error. Proposal of a terminal cost function in the quadratic form was made by (Bitmead et al., 1990). We propose the quadratic terminal cost function of desired and terminal state deviation. The deviation decreases a steady-state control error and desired terminal state allows taking into account additional control requirement. We use the desired state to steady-state optimization in case of MI+MO systems. The desired state computation for MI+MO system is described in following chapter 4.3.
If we use prediction equations (4a), (4b) to eliminate process output $\mathbf{y}_N$ and terminal state $\mathbf{x}(k+N)$ then the cost function (6a) and constraints (6b) can be rewritten into form

$$\begin{aligned} J(\Delta\mathbf{u}_N) &= \Delta\mathbf{u}_N^T\mathbf{M}\Delta\mathbf{u}_N + \Delta\mathbf{u}_N^T\mathbf{m}_k + \mathbf{m}_k^T\Delta\mathbf{u}_N + c = \Delta\mathbf{u}_N^T\mathbf{M}\Delta\mathbf{u}_N + 2\mathbf{m}_k^T\Delta\mathbf{u}_N + c \\ \mathbf{M} &= \mathbf{R} + \mathbf{S}_{yu}^T\mathbf{Q}\mathbf{S}_{yu} + \mathbf{S}_{xu}^T\mathbf{Q}_x\mathbf{S}_{xu} \\ \mathbf{m} &= -\left\{\mathbf{S}_{yu}^T\mathbf{Q}\left[\mathbf{w}_N - \mathbf{S}_{yx}\mathbf{x}(k) - \mathbf{S}_{yu}\mathbf{u}_{N,0}\right] + \mathbf{S}_{xu}^T\mathbf{Q}_x\left[\mathbf{x}_w - \mathbf{S}_{xx}\mathbf{x}(k) - \mathbf{S}_{xu}\mathbf{u}_{N,0}\right]\right\} \end{aligned} \tag{7a}$$

$$\mathbf{u}_{N,\min} - \mathbf{u}_{N,0} \le \Delta\mathbf{u}_N \le \mathbf{u}_{N,\max} - \mathbf{u}_{N,0} \tag{7b}$$

The minimization of (7a) regarding to $\Delta\mathbf{u}_N$ without constrains (7b) is possible in explicit form (7c) on condition that the matrix **M** is positive definite and symmetric.

$$\Delta\mathbf{u}_N = -\mathbf{M}^{-1}\mathbf{m} \tag{7c}$$

The minimization of (7a) with constrains (7b) is a task of quadratic programming. In both cases we get a vector of future manipulated inputs deviation $\Delta\mathbf{u}_N$ that in combination of supposed inputs $\mathbf{u}_{N,0}$ gives vector of optimal process inputs $\mathbf{u}_N$. The calculated value of optimal inputs depends on actual state $\mathbf{x}$(k), future course of reference signal $\mathbf{w}_N$, desired terminal state $\mathbf{x}_w$ and supposed future inputs $\mathbf{u}_{N,0}$. If the actual state isn't measured, then a state estimator based on state space model (3a), (3b) can be used. If the future course of reference signal isn't known then the actual set point is used as a future course of constant reference signal. The calculation of desired terminal state is described in next chapter. The vector of supposed inputs $\mathbf{u}_{N,0}$ can be constructed from actual values of inputs which are supposed to be constant in the future or if we know the future course of some inputs (as known future disturbances) then we can add this information in corresponding part of $\mathbf{u}_{N,0}$.
We apply only the control actions of the first member $\mathbf{u}$(k) from the optimal vector $\mathbf{u}_N$ and the minimization is repeated in the next sample time.

4.3 Calculation of desired state

The calculation of the desired state $\mathbf{x}_w$ is a fundamental part of steady-state optimization. It is based on a non square steady-state gain matrix **Z** of a MI+MO system model (3)

$$y_\infty = \mathbf{Z}\mathbf{u} \tag{8a}$$

$$\mathbf{Z} = \mathbf{C}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} \tag{8b}$$

The calculation is described by equations (9a – 9d) and the solution is valid for steady state. The minimization of quadratic cost function (9a) represents the requirement of a minimal quadratic distance between ideal inputs values $\mathbf{u}_{\text{ideal}}$ and accessible inputs $\tilde{\mathbf{u}}$. At the same time the equation (9b) has to be respected. This equation arising from (3) represents a requirement that accessible inputs $\tilde{\mathbf{u}}$ lead the system (in steady-state) to set-point at the end of horizon $\mathbf{w}(k+N)$. These two equations formulate a standard task of constrained extreme minimization. The solution of this task is a value of $\tilde{\mathbf{u}}$ which can be recalculated to desired state $\mathbf{x}_{\text{w}}$ using the equation (9d).

$$\min_{\tilde{\mathbf{u}}}\left[\left(\mathbf{u}_{ideal}-\tilde{\mathbf{u}}\right)^T \mathbf{I}\left(\mathbf{u}_{ideal}-\tilde{\mathbf{u}}\right)\right] \tag{9a}$$

$$\mathbf{Z}\tilde{\mathbf{u}} = \mathbf{w}(k+N) \tag{9b}$$

$$\mathbf{u}_{\min} \le \tilde{\mathbf{u}} \le \mathbf{u}_{\max} \tag{9c}$$

$$\mathbf{x}_w = \left(\mathbf{I}-\mathbf{A}\right)^{-1}\mathbf{B}\tilde{\mathbf{u}} \tag{9d}$$

We can get the solution of unconstrained extreme task – by considering only equations (9a) and (9b) – in explicit form by Lagrange's multipliers. If we rewrite (9a), (9b) into form of (10a) then we get the searched input $\tilde{\mathbf{u}}$ as part of the solution of matrix expression (10b)

$$\left(\mathbf{u}_{ideal}-\tilde{\mathbf{u}}\right)^T \mathbf{I}\left(\mathbf{u}_{ideal}-\tilde{\mathbf{u}}\right)+\left[\mathbf{Z}\tilde{\mathbf{u}}-\mathbf{w}(k+N)\right]^T \boldsymbol{\lambda} = 0 \tag{10a}$$

$$\begin{bmatrix}\tilde{\mathbf{u}} \\ \boldsymbol{\lambda}\end{bmatrix} = \begin{bmatrix} 2\mathbf{I} & \mathbf{Z}^T \\ \mathbf{Z} & 0 \end{bmatrix}^{-1} \begin{bmatrix}\mathbf{u}_{ideal} \\ \mathbf{w}(k+N)\end{bmatrix} \tag{10b}$$

In case of constrained inputs – by considering also equation (9c) - the problem is formulated as quadratic programming task and the searched input $\tilde{\mathbf{u}}$ has to be computed numerically.

5. Simulated control experiments

The simulated experiments demonstrate the discrete-time control of continuous-time MI+MO system (thermostatic bath) with simple on-off controller (5) and predictive controller without (7) and with steady-state optimization (8). Placed element temperature T_{D} is controlled – responses of simulated reference signal tracking are depicted in Figures 3-5. The control by predictive controller is shown for two situations – without and with future reference signal course knowledge. All experiments are made under identical conditions and the control performance is evaluated by two measures. First measure C_{quality} (quality) represents a value of quadratic control error area and the second measure C_{cost} (cost) is price of total energy consumption for the heating E_{heat} and cooling E_{cool}. The energy consumption for refrigerating E_{cool} (cooling water temperature decreasing at chosen constant flow-rate) is supposed (for needs of following simulations) to be equivalent to energy consumption for cooling water heating about same temperature difference with efficiency e_{f}=0.5 (50%). These two measures can be written as

$$C_{quality} = T\sum_{k=0}^{N_s}\left[w(k) - T_D(k)\right]^2 \tag{11a}$$

$$C_{\cos t} = k_{price}\left(E_{heat} + E_{cool}\right)$$

$$E_{heat} = T\sum_{k=0}^{N_s} E(k) \qquad E_{cool} = T\frac{Qc_B}{e_f}\sum_{k=0}^{N_s-1}\left(T_o - T_{B0}(k)\right) \tag{11b}$$

where

k_{price} is electric energy price per unit,
T is a controller sample time,
e_{f} is efficiency of cooling and
N_{s} is number of samples during experiment.

The ideal thermostatic bath is simulated as a continuous-time system (Eq. 1) with parameters given in Table 1. All control experiments start from steady-state ($\mathbf{x}_0$, $\mathbf{u}_0$) and respect inputs variables ranges. Conditions and constrains are listed in Table 2. The input values $\mathbf{u}_0$ leading to the steady-state $\mathbf{x}_0$ are no optimal from energy consumption point of view. These values were chosen to show the influence of the steady-state optimization for case of predictive controller.

	Input variables ranges and initial inputs $\mathbf{u}_0$				Steady-state $\mathbf{x}_0$, $\mathbf{u}_0$	
	E [W]	Q [kg s^{-1}]	T_{B0} [°C]	T_o [°C]	T_A [°C]	64.63
$\mathbf{u}_{\max}$	1000	0.5/60	25	25	T_B [°C]	22.02
$\mathbf{u}_0$	250	0.5/60	15	25	T_C [°C]	29.54
$\mathbf{u}_{\min}$	0	0.5/60	5	25	T_D [°C] = y_0	29.54

Table 2. Input variables and steady-state

The on-off controller is realized as discrete-time system (with zero-order hold terms on the outputs with sample time T = 10 s). Its response is depicted in Fig. 3. The achieved values of control quality and costs measures in this experiment are used as a standard and marked as 100%. The control quality is apparently bad (the output oscillates) but the computed control quality value is comparable with predictive controller without knowledge of future reference signal course. The on-off controller responds immediately to changes in reference signal with maximal values of heating or cooling and hence the output response is as quick as possible. In spite of the fact that the on-off controller ensures that the heating and cooling doesn't actuate concurrently the energy consumption is high because of heating and cooling switching to their maximal values.

Predictive controller with steady-state optimization and inputs constrains (6) is realized as a discrete-time system (with zero-order hold terms on the outputs with sample time T = 10 s). The control horizon is N = 60 samples (that is N×T = 600 s = 10 min). Control response of the predictive controller without future reference signal knowledge is depicted in Fig. 4. It means that the controller has information about actual value of set-point and the future reference signal is assumed to be constant and equal to set-point at current time. If the actual set-point changes then the constant future reference signal over the whole control horizon changes too. The values of control measures are relative to corresponding values achieved in control with on-off controller and expressed as percentages.

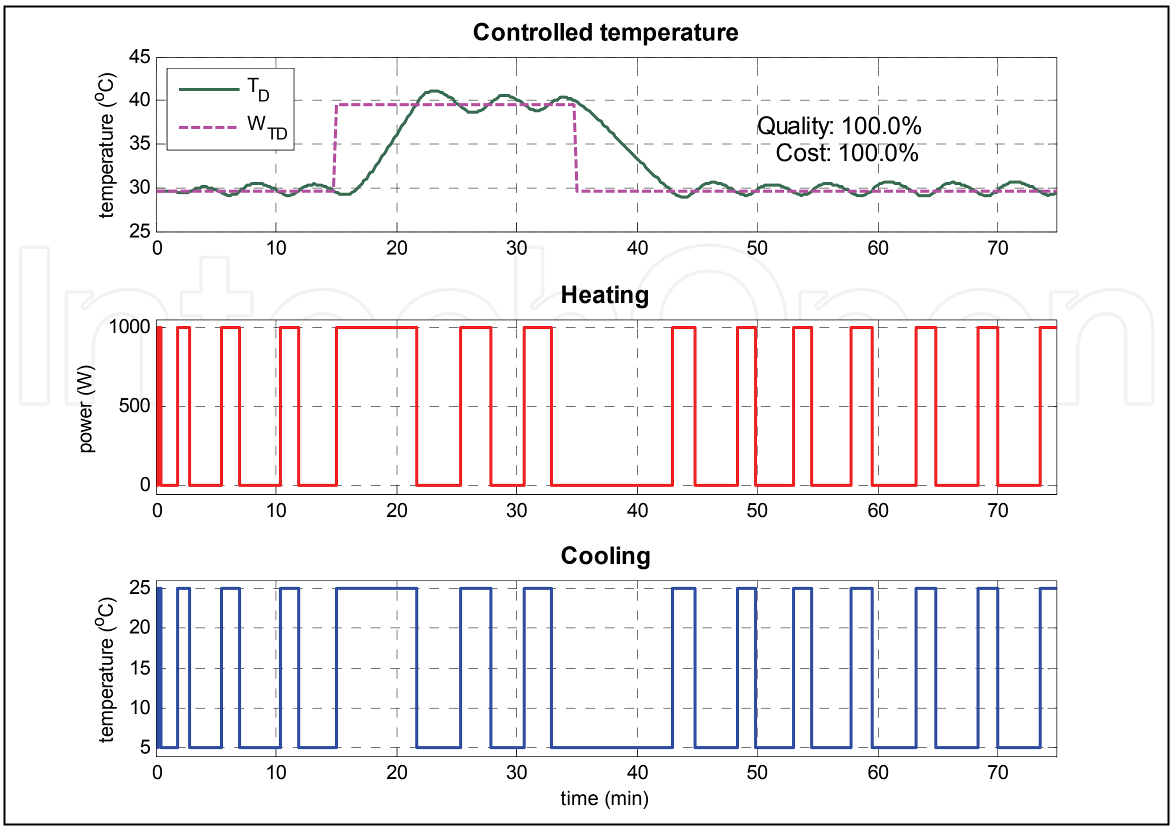

Fig. 3. On-off controller

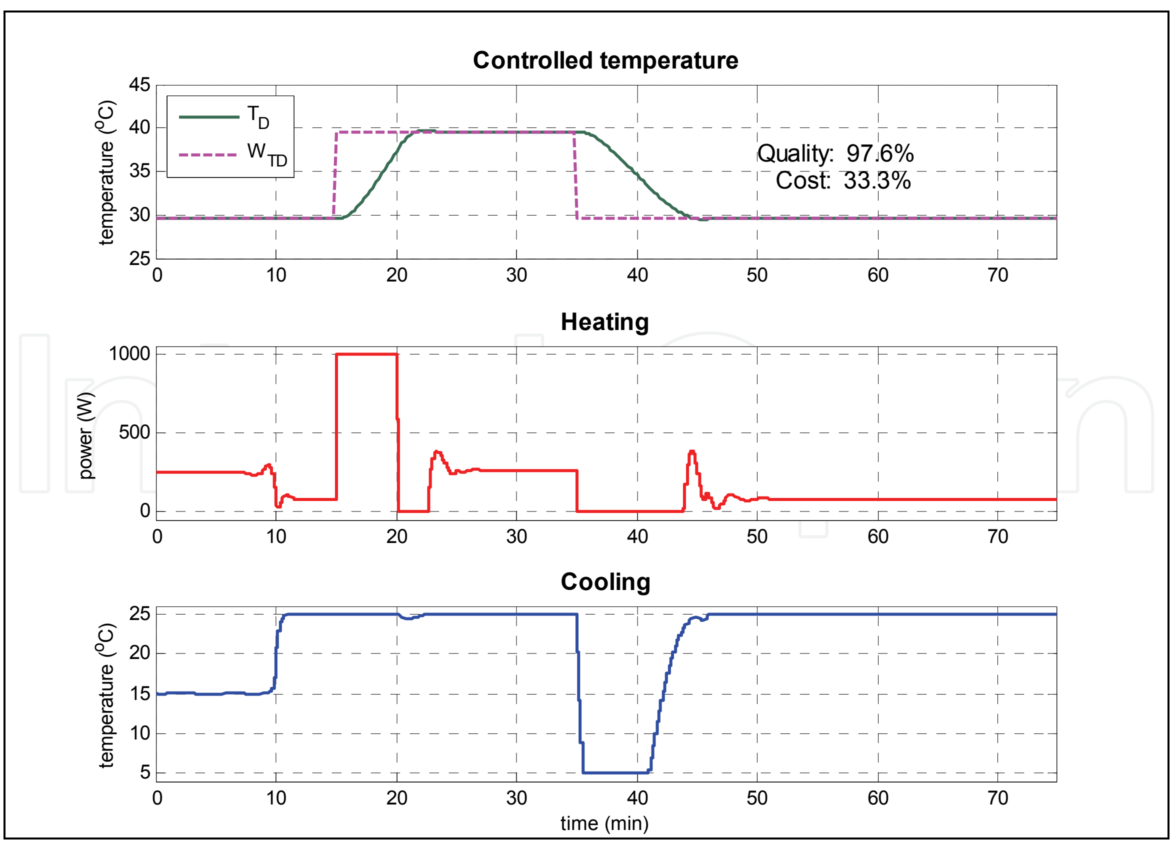

Fig. 4. Predictive controller – without future reference signal knowledge

The first 15 minutes of experiment depicted in the Fig. 4 demonstrates the steady-state optimization - the controller manipulates inputs without output change. The inputs achieve their optimal values after time corresponding to length of horizon (10 minutes). Because of the absent of a future reference signal knowledge the controller react only to actual set-point. The control quality (97.6%) is comparable with on-off controller but the energy consumption is significantly better (33.3%).
A control response of the predictive controller with steady-state optimization and with knowledge of the future reference signal is depicted in the Fig. 5. This experiment demonstrates best control approach from the point of control quality and energy consumption. The controller uses maximum of accessible information. Due to prediction horizon and future reference signal knowledge the controller can act before the actual set-point change. The time of advance controller reaction depends on both system dynamic and constrains. Hence it can be different when the set-point changes up and down. On this experiment we can also see that the control quality is preferred before control cost. There are parts of control where heating and cooling act simultaneously. We can see this in transient state only. This behaviour also depends on the choice of weighting matrices in the cost function (6a).

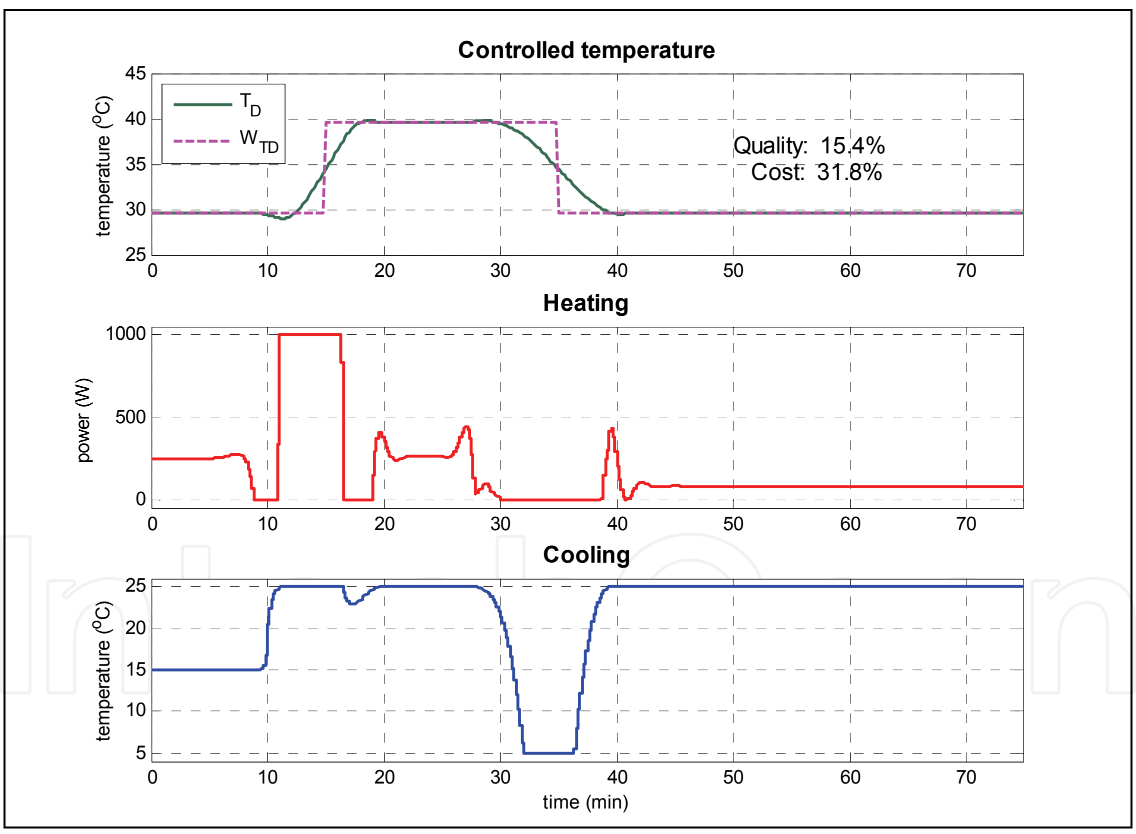

Fig. 5. Predictive controller – with future reference signal knowledge

6. Conclusion

Control design is often "made-to-measure problem" especially if one feature of the controlled process is dominant and therefore affecting control possibilities. Even quite sophisticated generally designed controller does not improve control quality compared to simple solution respecting the dominant properties. This was illustrated on an example of a MI+MO system thermostatic bath – a system with two constrained inputs and one controlled output. If the set-point changes significantly the controller can not do anything else than to set both control variables on their appropriate limits because of constrains on heating and cooling. Control response of quite complicated predictive controller will be improved if additional information and requirements are implemented within the control design. Process dynamics knowledge including cross couplings, was fully used only if we considered known future reference signal.

Another problem connected with systems with more inputs then outputs was illustrated on the mentioned example of MI+MO system. To solve the problem of indeterminate inputs combinations in case of MI+MO processes control we propose to add the "steady-state optimization" to controller design. Under the steady-state optimization we understand that we need to find such an inputs combination that is as close as possible to ideal process inputs and at the same time reaching the set-point in steady-state. We can observe the effect of the steady-state optimization during the first 15 minutes of the control response in Fig 4. The "ideal" desired input variables combination for the steady-state in our case is zero heating power and maximal cooling water input temperature – that is a combination with lowest energy cost. Future control error and terminal state error is minimized in every time instant as a result of the cost function form with respect to manipulated variables constrains. The effect of steady-state optimization is nice to see in steady-state but it takes effect continuously.

To add the steady-state optimization to a predictive controller design we use the terminal cost function. The quadratic terminal cost function was originally introduced to ensure controller stability. We modified the criterion so that the deviation of a desired and the predicted terminal state is used instead the terminal state only. The computation of the desired terminal state is based on a desired input variables combination, value of set-point at the end of control horizon and no square steady-state gain matrix. The solution is formulated as a standard constrained extreme finding task where the inputs constraints can be included, too.

7. Acknowledgements

The research has been supported in the program of Ministry of Education of Czech Republic MSM 0021627505 in part "Řízení, optimalizace a diagnostika složitých systémů". This support is gratefully acknowledged.

8. References

Åström, K.J. & Wittenmark, B. (1995). *Adaptive Control*. 2nd ed. Addison-Wesley, ISBN 0-201-55866-1, Boston, USA

Bitmead, R. R. ; Gevers, M. & Wertz, M. (1990). *Adaptive optimal control – The thinking man's GPC,* Englewood Cliffs, Prentice-Hall, New Jersey USA

Camacho, E. F. & Bordons, C. (1999). *Model Predictive Control*, Springer Verlag, ISBN 3-540-76241-8, London, UK

Dušek, F. & Honc, D. (2008a). Transformace soustav s různým počtem vstupů a výstupů pro decentralizované řízení (in Czech). *Automatizace,* 51, 7-8, 2008, 458-462, ISSN 0005-125X

Dušek, F. & Honc, D. (2008b). Static compensator for non-square MIMO systems. *Proceedings of 8th International Scientific-Technical Conference Process Control 2008*, pp. 13 (full text on CD), ISBN 978-80-7395-077-4, Kouty nad Desnou, June 9 - 12 2008. University of Pardubice, Czech Republic

Dušek, F. & Honc, D. (2009). Terminal State in a Predictive Controller Cost Function. In: *17th International Conference on Process Control '09*, pp. 409-413, ISBN 978-80-227-3081-5, Štrbské pleso, June 9 – 12 2009, Slovak University of Technology in Bratislava, Slovak Republic

Goodwin, G.C. ; Graebe, S.F. & Salgado, M.E. (2001). *Control System Design*, Prentice-Hall, ISBN 0272-1708, New Jersey, USA

Maciejowski, J.M. (2002). *Predictive Control with Constrains*, Pearson Education Ltd., ISBN 0-201-39823-0, Harlow, UK.

Mayne, D. Q. ; Rawlings, J. B. ; Rao, C. V. & Scokaert, P. O. M. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, 36, 6, 789-814, ISSN 0005-1098

Novák, J. ; Chalupa, P. & Bobál V. (2009). Local Model Networks for Modelling and Predictive Control of Nonlinear Systems, *Proceedings of 23rd European Conference on Modelling and Simulation*, pp. 557-562, Universidad Rey Juan Carlos Madrid, June 9-12 2009, Madrid, Spain

Rossiter, J. A. (2003). *Model-Based Predictive Control, a Practical Approach*, CRC Press LLC, ISBN 0-8493-1291-4, Florida, USA

Wellstead, P. E. & Zarrop, M. B. (1991). *Self-tuning Systems: Control and Signal Processing.* John Wiley&Sons, ISBN 0-471-92883-6, Chichester, England

Robust Control, Theory and Applications
Edited by Prof. Andrzej Bartoszewicz

ISBN 978-953-307-229-6
Hard cover, 678 pages
Publisher InTech
Published online 11, April, 2011
Published in print edition April, 2011

The main objective of this monograph is to present a broad range of well worked out, recent theoretical and application studies in the field of robust control system analysis and design. The contributions presented here include but are not limited to robust PID, H-infinity, sliding mode, fault tolerant, fuzzy and QFT based control systems. They advance the current progress in the field, and motivate and encourage new ideas and solutions in the robust control area.