Passive Robust Control for Internet-Based Time-Delay Switching Systems

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1. Introduction

The Internet is playing an important role in information retrieval, exchange, and applications. Internet-based control, a new type of control systems, is characterized as globally remote monitoring and adjustment of plants over the Internet. In recent years, Internet-based control systems have gained considerable attention in science and engineering [1-6], since they provide a new and convenient unified framework for system control and practical applications. Examples include intelligent home environments, windmill and solar power stations, small-scale hydroelectric power stations, and other highly geographically distributed devices, as well as tele-manufacturing, tele-surgery, and tele-control of spacecrafts.

Internet-based control is an interesting and challenging topic. One of the major challenges in Internet-based control systems is how to deal with the Internet transmission delay. The existing approaches of overcoming network transmission delay mainly focus on designing a model based time-delay compensator or a state observer to reduce the effect of the transmission delay. Being distinct from the existing approaches, literatures (7–9) have been investigating the overcoming of the Internet time-delay from the control system architecture angle, including introducing a tolerant time to the fixed sampling interval to potentially maximize the possibility of succeeding the transmission on time. Most recently, a dual-rate control scheme for Internet-based control systems has been proposed in literature (10). A two-level hierarchy was used in the dual-rate control scheme. At the lower level a local controller which is implemented to control the plant at a higher frequency to stabilize the plant and guarantee the plant being under control even the network communication is lost for a long time. At the higher level a remote controller is employed to remotely regulate the desirable reference at a lower frequency to reduce the communication load and increase the possibility of receiving data over the Internet on time. The local and the remote controller are composed of some modes, which mode is enabled due to the time and state of the network. The mode may changes at instant time $k, k \in \{N_+\}$ and at each instant time only one mode of the controller is enabled. A typical dual-rate control scheme is demonstrated in a process control rig (7; 8) and has shown a great potential to over Internet time-delay and bring this new generation of control systems into industries. However, since the time-delay is variable and the uncertainty of the process parameters is unavoidable, a dual-rate Internet-based control system may be unstable for certain control intervals. The interest in the stability of networked control systems have grown in recent years due to its theoretical and practical significance [11-21], but to our knowledge there are very few reports dealing with the robust passive control for such kind of Internet-based control systems. The robust passive control problem for time-delay systems was dealt with in (24; 25). This motivates the present passivity investigation of multi-rate Internet-based switching control systems with time-delay and uncertainties.

In this paper, we study the modelling and robust passive control for Internet-based switching control systems with multi-rate scheme, time-delay, and uncertainties. The controller is switching between some modes due to the time and state of the network, either different time or the state changing may cause the controller changes its mode and the mode may changes at each instant time. Based on remote control and local control strategy, a new class of multi-rate switching control model with time-delay is formulated. Some new robust passive properties of such systems under arbitrary switching are investigated. An example is given to illustrate the effectiveness of the theoretical results.

Notation: Through the paper I denotes identity matrix of appropriate order, and $*$ represents the elements below the main diagonal of a symmetric block matrix. The superscript $\top$ represents the transpose. $L_2[0,\infty)$ refers to the space of square summable infinite vector sequences. The notation $X > 0(\geq, <, \leq 0)$ denotes a symmetric positive definite (positive semi-definite, negative, negative semi-definite) matrix X. Matrices, if not explicitly stated, are assumed to have compatible dimensions. Let $N = \{1, 2, \cdots\}$ and $N_+ = \{0, 1, 2, \cdots\}$ denote the sets of positive integer and nonnegative integer, respectively.

2. Problem formulation

A typical multi-rate control structure with remote controller and local controller can be shown as Fig. 1. The control architrave gives a discrete dynamical system, where plant is in circle with broken line, $x(k) \in R^n$ is the system state, $z(k) \in R^q$ is the output, and $\omega(k) \in R^p$ is the exogenous input, which is assumed to belong to $L_2[0,\infty)$, $r(k)$ is the input and for the passivity analysis one can let $r(k) = 0$, $u_1(k)$ and $u_2(k)$ are the output of remote control and local control, respectively. A_1, B_1, B_2 and C are parameter matrices of the model with appropriate dimensions, K_{2i} and K_{1j} are control gain switching matrices where the switching rules are given by $i(k) = s(x(k), k)$ and $j(k) = \sigma(x(k), k)$, and $i \in \{1, 2, \cdots, N_1\}, j \in \{1, 2, \cdots, N_2\}, N_1, N_2 \in N$, which imply that the switching controllers have N_1 and N_2 modes, respectively. τ_1 and τ_2 are time-delays caused by communication delay in systems.

For the system given by Fig. 1, it is assumed that, the sampling interval of remote controller is the m multiple of local controller with m being positive integer, and the switching device SW1 closes only at the instant time $k = nm, n \in N_+$, and otherwise, it switches off. Correspondingly, remote controller $u_1(k)$ updates its state at $k = nm, n \in N_+$ only, and otherwise, it keeps invariable. Also, it is assumed that the benchmark of discrete systems is the same as local controller. In this case, the system can be described by the following discrete system with time-delay

$$\begin{cases} x(k+1) = A_1\, x(k) + B_2\, u_2(k) + E\omega(k), \\ u_2(k) \quad = B_1\, u_1(k-\tau_2) - K_{2i}\, x(k), \\ z(k) \quad = C\, x(k) + D\omega(k), \end{cases} \tag{1}$$

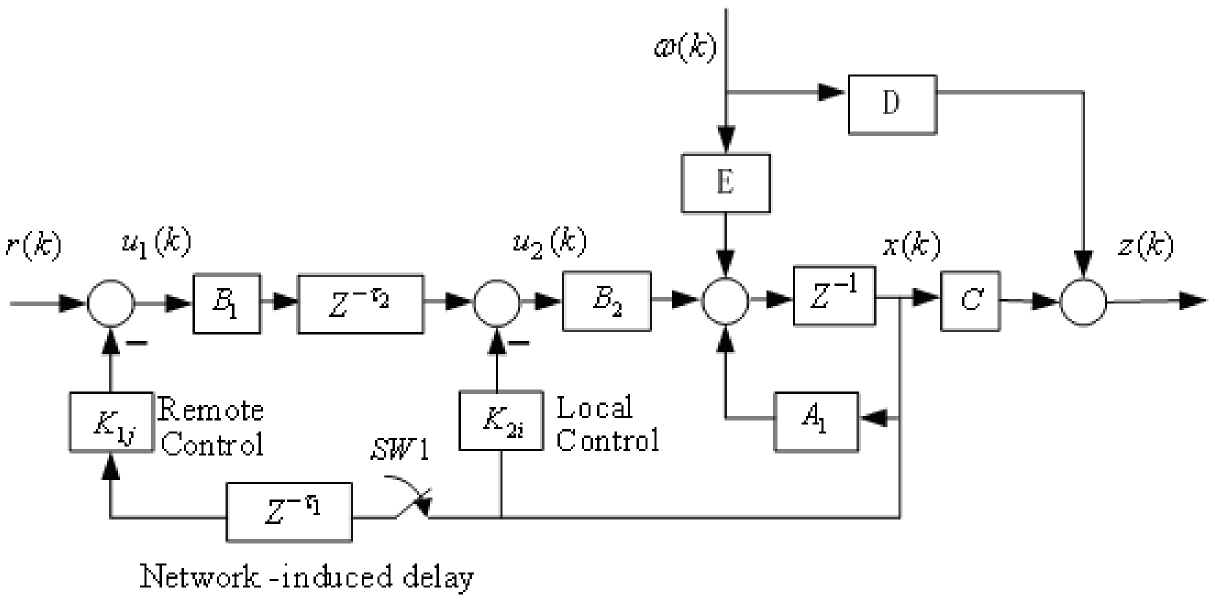

Fig. 1. Multi-rate network control loop with time-delays

where remote controller $u_1(k-\tau_2)$ is given by

$$\begin{cases} u_1(k-\tau_2)=r(k-\tau_2)-K_{1j}\,x(k-\tau_1-\tau_2), k=nm, \\ u_1(k-\tau_2)=r(nm-\tau_2)-K_{1j}\,x(nm-\tau_1-\tau_2), k\in\{nm+1,\cdots,nm+m-1\}, \end{cases} \tag{2}$$

with $i\in\{1,2,\cdots,N_1\}, j\in\{1,2,\cdots,N_2\}, k,n\in N_+$ and $N_1,N_2\in N$. Moreover, it follows from (1) and (2) that, for $k=nm$,

$$\begin{cases} x(k+1)=(A_1-B_2K_{2i})\,x(k)-B_2\,B_1\,K_{1j}\,x(k-\tau_1-\tau_2)+B_2\,B_1\,r(k-\tau_2)+E\omega(k), \\ \quad z(k) \ \ =C\,x(k)+D\omega(k), \end{cases} \tag{3}$$

and for $k\in\{nm+1,\cdots,nm+m-1\}$,

$$\begin{cases} x(k+1)=(A_1-B_2K_{2i})\,x(k)-B_2\,B_1\,K_{1j}\,x(nm-\tau_1-\tau_2)+B_2\,B_1\,r(nm-\tau_2)+E\omega(k), \\ \quad z(k) \ \ =C\,x(k)+D\omega(k). \end{cases} \tag{4}$$

For the passivity analysis, one can let $r(k)=0$, and then the system (3) and (4) become

$$\begin{cases} x(k+1)=(A_1-B_2K_{2i})\,x(k)-B_2\,B_1\,K_{1j}\,x(k-\tau)+E\omega(k), k=nm, \\ x(k+1)=(A_1-B_2K_{2i})\,x(k)-B_2\,B_1\,K_{1j}\,x(nm-\tau)+E\omega(k), k\in\{nm+1,\cdots,nm+m-1\}, \\ \quad z(k) \ \ =C\,x(k)+D\omega(k), \end{cases} \tag{5}$$

where $\tau=\tau_1+\tau_2>0, k\in N_+, n\in N_+, m>0$ is a positive integer.
Obviously, if define $A_i=A_1-B_2K_{2i}$, $B_j=-B_2B_1K_{1j}$, then the controlled system (5) becomes

$$\begin{cases} x(k+1) &= A_ix(k)+B_jx(k-\tau)+E\omega(k), k=nm, \\ x(k+1) &= A_ix(t)+B_jx(nm-\tau)+E\omega(k), k\in\{nm+1,\cdots,nm+m-1\}, \\ z(k) &= C\,x(k)+D\omega(k), \end{cases} \tag{6}$$

where A_1, B_1, B_2, C, D, E are matrices with appropriate dimensions, K_{1j} and K_{2i} are mode gain matrices of the remote controller and local controller. At each instant time k, there is only one mode of each controller is enabled. $\tau = \tau_1 + \tau_2 > 0$ and $m > 0$ are integers, $k \in N_+$, $n = 0, 1, 2, \cdots$.
Furthermore, note that, as $k = nm + s$ with $s = 0, 1, \cdots, m-1$, and $nm - \tau = k - (\tau + s)$ then (6) can be rewritten as

$$\begin{cases} x(k+1) = A_i\, x(k) + B_j\, x(k-h) + E\omega(k), \\ z(k) \quad\;\; = C\, x(k) + D\omega(k), \end{cases} \tag{7}$$

with $0 \leq \tau \leq h \leq \tau + m - 1$. Accordingly, for the case of time-varying structured uncertainties (7) becomes

$$\begin{cases} x(k+1) = (A_i + \Delta A(k))\, x(k) + (B_j + \Delta B(k))\, x(k-h) + (E + \Delta E)\omega(k), \\ z(k) \quad\;\; = C\, x(k) + D\omega(k), \end{cases} \tag{8}$$

with $0 \leq \tau \leq h \leq \tau + m - 1$, and $\Delta A(k)$, $\Delta B(k)$ and ΔE being structured uncertainties, and are assumed to have the form of

$$\Delta A(k) = D_1 F(k) E_a, \quad \Delta B(k) = D_1 F(k) E_b, \quad \Delta E(k) = D_1 F(k) E_e, \tag{9}$$

where D_1, E_a, E_b and E_e are known constant real matrices with appropriate dimensions. It is assumed that

$$F^\top(k) F(k) \leq I, \quad \forall k. \tag{10}$$

In what follows, the the passive control for the hybrid model (7) and (8) are first studied, and then, an example of systems (8) is investigated.

3. Passivity analysis

On the basis of models (7) and (8), consider the following discrete-time nominal switching system with time-delay:

$$\begin{cases} x(k+1) = A_i x(k) + B_j x(k-h) + E\omega(k), \\ z(k) = Cx(k) + D\omega(k), \\ x(k) = \phi(k),\, k \in [-h, 0], \\ i(k) = s(x(k), k), \\ j(k) = \sigma(x(k), k), \end{cases} \tag{11}$$

where s and σ are switching rules, $i \in \{1, \cdots, N_1\}$, $j \in \{1, \cdots, N_2\}$, $N_1, N_2 \in N$, $A_i, B_j \in R^{n\times n}$ are ith and jth switching matrices of system (11), $h \in N$ is the time delay, and $\phi(\cdot)$ is the initial condition.
For the case of structured uncertainties, it can be described by

$$\begin{cases} x(k+1) = A_i(k)x(k) + B_j(k)x(k-h) + E(k)\omega(k), \\ z(k) = Cx(k) + D\omega(k), \\ x(k) = \phi(k),\, k \in [-h, 0], \\ i(k) = s(x(k), k), \\ j(k) = \sigma(x(k), k), \end{cases} \tag{12}$$

where $A_i(k) = A_i + \Delta A(k), B_j(k) = B_j + \Delta B(k)$, $E(k) = E + \Delta E(k)$, and it is assumed that (9) and (10) are satisfied. Our problem is to test whether system (11) and (12) are passive with the switching controllers. To this end, we introduce the following fact and related definition of passivity.

Lemma 1 (22). The following inequality holds for any $a \in R^{n_a}, b \in R^{n_b}, N \in R^{n_a \times n_b}, X \in R^{n_a \times n_a}, Y \in R^{n_a \times n_b}$, and $Z \in R^{n_b \times n_b}$:

$$-2a^\top Nb \leq \begin{bmatrix} a \\ b \end{bmatrix}^\top \begin{bmatrix} X & Y-N \\ * & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}, \tag{13}$$

where $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geq 0$.

Lemma 2 (23). Given matrices $Q = Q^\top, H, E$ and $R = R^\top > 0$ of appropriate dimensions,

$$Q + HFE + E^\top F^\top H^\top < 0 \tag{14}$$

holds for all F satisfying $F^\top F \leq R$, if and only if there exists some $\lambda > 0$ such that

$$Q + \lambda HH^\top + \lambda^{-1} E^\top RE < 0. \tag{15}$$

Definition 1 (26) The dynamical system (11) is called passive if there exists a scalar β such that

$$\sum_{k=0}^{k_f} \omega^\top(k) z(k) \geq \beta, \quad \forall \omega \in L_2[0,\infty), \quad \forall k_f \in N,$$

where β is some constant which depends on the initial condition of system.

In the sequel, we provide condition under which a class of discrete-time switching dynamical systems with time-delay and uncertainties can be guaranteed to be passive.

System (11) can be recast as

$$\begin{cases} y(k) = x(k+1) - x(k), \\ 0 = (A_i + B_j - I)x(k) - y(k) - B_j \sum_{l=k-h}^{k-1} y(l) + E\omega(k), \\ z(k) = Cx(k) + D\omega(k), \\ x(k) = \phi(k),\ k \in [-h, 0] \\ i(k) = s(x(k), k), \\ j(k) = \sigma(x(k), k). \end{cases} \tag{16}$$

It is noted that (11) is completely equivalent to (16).

Theorem 1. System (11) is passive under arbitrary switching rules s and σ, if there exist matrices $P_1 > 0, P_2, P_3, W_1, W_2, W_3, M_1, M_2, S_1 > 0, S_2 > 0$ such that the following LMIs hold

$$\Lambda = \begin{bmatrix} Q_1 & Q_2 & P_2^\top B_j - M_1 & P_2^\top E - C \\ * & Q_3 & P_3^\top B_j - M_2 & P_3^\top E \\ * & * & -S_2 & 0 \\ * & * & * & -(D + D^\top) \end{bmatrix} < 0, \tag{17}$$

and

$$\begin{bmatrix} W & M \\ M^\top & S_1 \end{bmatrix} \geq 0, \tag{18}$$

for $i \in \{1, \cdots, N_1\}, j \in \{1, \cdots, N_2\}, N_1, N_2 \in N$, where

$$\begin{aligned}
&Q_1 = P_2^\top (A_i - I) + (A_i - I)^\top P_2 + hW_1 + M_1 + M_1^\top + S_2,\\
&Q_2 = (A_i - I)^\top P_3 + P_1^\top - P_2^\top + hW_2 + M_2^\top,\\
&Q_3 = -P_3 - P_3^\top + hW_3 + P_1 + hS_1,\\
&W = \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}, M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}.
\end{aligned}$$

Proof. Construct Lyapunov function as

$$V(k) = x^\top(k) P_1 x(k) + \sum_{\theta=-h+1}^{0} \sum_{l=k-1+\theta}^{k-1} y^\top(l) S_1 y(l) + \sum_{l=k-h}^{k-1} x^\top(l) S_2 x(l),$$

then

$$\begin{aligned}
\Delta V(k) =& V(k+1) - V(k)\\
=& 2x^\top(k) P_1 y(k) + x^\top(k) S_2 x(k) + y^\top(k)(P_1 + hS_1) y(k)\\
& - x^\top(k-h) S_2 x(k-h) - \sum_{l=k-h}^{k-1} y^\top(l) S_1 y(l),
\end{aligned} \tag{19}$$

where

$$2x^\top(k) P_1 y(k) = 2\eta^\top(k) P^\top \{ \begin{bmatrix} y(k) \\ (A_i + B_j - I)x(k) - y(k) + E\omega(k) \end{bmatrix} - \sum_{l=k-h}^{k-1} \begin{bmatrix} 0 \\ B_j \end{bmatrix} y(l) \}, \tag{20}$$

with $\eta^\top(k) = \begin{bmatrix} x^\top(k) & y^\top(k) \end{bmatrix}, P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$, and

$$\begin{aligned}
& 2\eta^\top(k) P^\top \begin{bmatrix} y(k) \\ (A_i + B_j - I)x(k) - y(k) + E\omega(k) \end{bmatrix}\\
=& 2\eta^\top(k) P^\top \{ \begin{bmatrix} 0 \\ A_i - I \end{bmatrix} x(k) + \begin{bmatrix} I \\ -I \end{bmatrix} y(k) + \begin{bmatrix} 0 \\ B_j \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ E\omega(k) \end{bmatrix} \}.
\end{aligned} \tag{21}$$

According to Lemma 1 we get that

$$\begin{aligned}
& -2 \sum_{l=k-h}^{k-1} \eta^\top(k) P^\top \begin{bmatrix} 0 \\ B_j \end{bmatrix} y(l)\\
\leq & \sum_{l=k-h}^{k-1} \begin{bmatrix} \eta(k) \\ y(l) \end{bmatrix}^\top \begin{bmatrix} W & M - P^\top \begin{bmatrix} 0 \\ B_j \end{bmatrix} \\ * & S_1 \end{bmatrix} \begin{bmatrix} \eta(k) \\ y(l) \end{bmatrix}\\
=& \eta^\top(k) hW \eta(k) + 2\eta^\top(k)(M - P^\top \begin{bmatrix} 0 \\ B_j \end{bmatrix})(x(k) - x(k-h)) + \sum_{l=k-h}^{k-1} y^\top(l) S_1 y(l),
\end{aligned} \tag{22}$$

where

$$\begin{bmatrix} W & M \\ * & S_1 \end{bmatrix} \geq 0.$$

From (19)-(22) we can get

$$\begin{aligned}\Delta V(k) - 2z^\top(k)\omega(k) = & 2\eta^\top(k)P^\top \begin{bmatrix} 0 & I \\ A_i - I & -I \end{bmatrix}\eta(k) + \eta^\top(k)hW\eta(k) \\ & + 2\eta^\top(k)Mx(k) + 2\eta^\top(k)(P^\top \begin{bmatrix} 0 \\ B_j \end{bmatrix} - M)x(k-h) + x^\top(k)S_2x(k) \\ & + y^\top(k)(P_1 + hS_1)y(k) - x^\top(k-h)S_2x(k-h) + 2\eta^\top(k)P^\top \begin{bmatrix} 0 \\ E\omega(k) \end{bmatrix} \\ & - 2(x^\top(k)C^\top\omega(k) + \omega^\top(k)D^\top\omega(k)).\end{aligned}$$

Let $\xi^\top(k) = [x^\top(k), y^\top(k), x^\top(k-h), \omega^\top(k)]$, then $\Delta V(k) - 2z^\top(k)\omega(k) \le \xi(k)^\top v \xi(k)$, where

$$v = \begin{bmatrix} \phi & P^\top \begin{bmatrix} 0 \\ B_j \end{bmatrix} - M & \begin{bmatrix} P_2^\top E - C^\top \\ P_3^\top E \end{bmatrix} \\ * & -S_2 & 0 \\ * & * & -(D + D^\top) \end{bmatrix}, \tag{23}$$

and

$$\phi = P^\top \begin{bmatrix} 0 & I \\ A_i - I & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A_i - I & -I \end{bmatrix}^\top P + hW + \begin{bmatrix} M & 0 \end{bmatrix} + \begin{bmatrix} M^\top \\ 0 \end{bmatrix} + \begin{bmatrix} S_2 & 0 \\ 0 & P_1 + hS_1 \end{bmatrix}.$$

If $v < 0$, then $\triangle V(k) - 2z^\top(k)\omega(k) < 0$, which gives

$$\sum_{k=0}^{k_f} \omega^\top(k)z(k) > \frac{1}{2}\sum_{k=0}^{k_f} \triangle V(k) = \frac{1}{2}[V(k_f + 1) - V(0)].$$

Furthermore, since $V(k) = V(x(k)) \ge 0$, it follows that

$$\sum_{k=0}^{k_f} \omega^\top(k)z(k) \ge -\frac{1}{2}V(0) \equiv \beta, \quad \forall \omega \in L_2[0,\infty), \quad \forall k_f \in N,$$

which implies from Definition 1 that the system (11) is passive. Using the Schur complement (23) is equivalent to (17). This complete the proof.

Theorem 2. System (12) is passive under arbitrary switching rules s and σ, if there exist matrices $P_1 > 0, P_2, P_3, W_1, W_2, W_3, M_1, M_2, S_1 > 0, S_2 > 0$ such that the following LMIs holds

$$\begin{bmatrix} Q_1 + E_a^\top E_a & Q_2 & P_2^\top B_j - M_1 + E_a^\top E_b & P_2^\top E - C^\top + E_a^\top E_e & P_2^\top D_1 \\ * & Q_3 & P_3^\top B_j - M_2 & P_3^\top E & P_3^\top D_1 \\ * & * & -S_2 + E_b^\top E_b & E_b^\top E_e & 0 \\ * & * & * & -(D + D^\top) + E_e^\top E_e & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{24}$$

and

$$\begin{bmatrix} W & M \\ M^\top & S_1 \end{bmatrix} \ge 0, \tag{25}$$

for

$$i \in \{1, \cdots, N_1\}, j \in \{1, \cdots, N_2\}, N_1, N_2 \in N,$$

where Q_1, Q_2, Q_3, W, M are defined in Theorem 1 and E_a, E_b, E_e are given by (9) and (10).

Proof. Replacing A_i, B_j and E in (17) with $A_i + D_1F(k)E_a$, $B_j + D_1F(k)E_b$ and $E + D_1F(k)E_e$, respectively, we find that (17) for (12) is equivalent to the following condition

$$\Lambda + \begin{bmatrix} P_2^\top D_1 \\ P_3^\top D_1 \\ 0 \\ 0 \end{bmatrix} F(k) \begin{bmatrix} E_a & 0 & E_b & E_e \end{bmatrix} + \begin{bmatrix} E_a^\top \\ 0 \\ E_b^\top \\ E_e^\top \end{bmatrix} F^\top(k) \begin{bmatrix} D_1^\top P_2 & D_1^\top P_3 & 0 & 0 \end{bmatrix} < 0.$$

By Lemma 2, a sufficient condition guaranteeing (17) for (12) is that there exists a positive number $\lambda > 0$ such that

$$\lambda\Lambda + \lambda^2 \begin{bmatrix} P_2^\top D_1 \\ P_3^\top D_1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} D_1^\top P_2 & D_1^\top P_3 & 0 & 0 \end{bmatrix} + \begin{bmatrix} E_a^\top \\ 0 \\ E_b^\top \\ E_e^\top \end{bmatrix} \begin{bmatrix} E_a & 0 & E_b & E_e \end{bmatrix} < 0. \tag{26}$$

Replacing $\lambda P, \lambda S_1, \lambda S_2, \lambda M$ and λW with P, S_1, S_2, M and W respectively, and applying the Schur complement shows that (26) is equivalent to (24). This completes the proof.

4. A numerical example

In this section, we shall present an example to demonstrate the effectiveness and applicability of the proposed method. Consider system (12) with parameters as follows:

$$A_1 = \begin{bmatrix} -6 & -6 \\ 2 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -4 & -6 \\ 4 & -4 \end{bmatrix}, B_1 = \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} -2 & 0 \\ -3 & -1 \end{bmatrix},$$
$$B_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$C = \begin{bmatrix} 0.1 & -0.2 \end{bmatrix}, E = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, E_a = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{bmatrix}, E_b = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.3 \end{bmatrix}, D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$D = 0.1, h = 5.$$

Applying Theorem 2, with $i \in \{1, 2\}, j \in \{1, 2, 3\}$. It has been found by using software LMIlab that the switching discrete time-delay system (12) is the passive and we obtain the solution as follows:

$$P_1 = 10^{-3} \times \begin{bmatrix} 0.1586 & 0.0154 \\ * & 0.2660 \end{bmatrix}, P_2 = \begin{bmatrix} 0.5577 & 0.3725 \\ -1.6808 & 1.0583 \end{bmatrix}, P_3 = \begin{bmatrix} 0.1689 & -0.0786 \\ -0.0281 & 0.1000 \end{bmatrix},$$
$$S_1 = 10^{-4} \times \begin{bmatrix} 0.4207 & 0.0405 \\ * & 0.6941 \end{bmatrix}, S_2 = \begin{bmatrix} 2.6250 & 0.8397 \\ * & 2.0706 \end{bmatrix}, W_1 = \begin{bmatrix} 0.2173 & -0.0929 \\ * & 0.0988 \end{bmatrix},$$
$$W_2 = \begin{bmatrix} 0.0402 & -0.0173 \\ * & 0.0182 \end{bmatrix}, W_3 = \begin{bmatrix} 0.0075 & -0.0032 \\ * & 0.0034 \end{bmatrix}, M_1 = 10^{-4} \times \begin{bmatrix} -0.0640 & -0.2109 \\ 0.1402 & -0.5777 \end{bmatrix},$$
$$M_2 = 10^{-5} \times \begin{bmatrix} 0.0985 & -0.4304 \\ 0.1231 & -0.9483 \end{bmatrix}.$$

5. Conclusions

In this paper, based on remote control and local control strategy, a class of hybrid multi-rate control models with uncertainties and switching controllers have been formulated and their passive control problems have been investigated. Using the Lyapunov-Krasovskii function approach on an equivalent singular system, some new conditions in form of LMIs have been derived. A numerical example has been shown to verify the effectiveness of the proposed control and passivity methods.

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The main objective of this monograph is to present a broad range of well worked out, recent theoretical and application studies in the field of robust control system analysis and design. The contributions presented here include but are not limited to robust PID, H-infinity, sliding mode, fault tolerant, fuzzy and QFT based control systems. They advance the current progress in the field, and motivate and encourage new ideas and solutions in the robust control area.

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