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Robust *H*∞ Reliable Control of Uncertain Switched Nonlinear Systems with Time-varying Delay

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1. Introduction

Switched systems are a class of hybrid system consisting of subsystems and a switching law, which define a specific subsystem being activated during a certain interval of time. Many real-world processes and systems can be modeled as switched systems, such as the automobile direction-reverse systems, computer disk systems, multiple work points control systems of airplane and so on. Therefore, the switched systems have the wide project background and can be widely applied in many domains (Wang, W. & Brockett, R. W., 1997; Tomlin, C. et al., 1998; Varaiya, P., 1993). Besides switching properties, when modeling a engineering system, system uncertainties that occur as a result of using approximate system model for simplicity, data errors for evaluation, changes in environment conditions, etc, also exit naturally in control systems. Therefore, both of switching and uncertainties should be integrated into system model. Recently, study of switched systems mainly focuses on stability and stabilization (Sun, Z. D. & Ge, S. S., 2005; Song, Y. et al., 2008; Zhang, Y. et al., 2007). Based on linear matrix inequality technology, the problem of robust control for the system is investigated in the literature (Pettersson, S. & Lennartson, B., 2002). In order to guarantee *H*∞ performance of the system, the robust *H*∞ control is studied using linear matrix inequality method in the literature (Sun, W. A. & Zhao, J., 2005).

In many engineering systems, the actuators may be subjected to faults in special environment due to the decline in the component quality or the breakage of working condition which always leads to undesirable performance, even makes system out of control. Therefore, it is of interest to design a control system which can tolerate faults of actuators. In addition, many engineering systems always involve time delay phenomenon, for instance, long-distance transportation systems, hydraulic pressure systems, network control systems and so on. Time delay is frequently a source of instability of feedback systems. Owing to all of these, we shouldn't neglect the influence of time delay and probable actuators faults when designing a practical control system. Up to now, research activities of this field for switched system have been of great interest. Stability analysis of a class of linear switching systems with time delay is presented in the literature (Kim, S. et al., 2006). Robust *H*∞ control for discrete switched systems with time-varying delay is discussed

in the literature (Song, Z. Y. et al., 2007). Reliable guaranteed-cost control for a class of uncertain switched linear systems with time delay is investigated in the literature (Wang, R. et al., 2006). Considering that the nonlinear disturbance could not be avoided in several applications, robust reliable control for uncertain switched nonlinear systems with time delay is studied in the literature (Xiang, Z. R. & Wang, R. H., 2008). Furthermore, Xiang and Wang (Xiang, Z. R. & Wang, R. H., 2009) investigated robust L_∞ reliable control for uncertain nonlinear switched systems with time delay.

Above the problems of robust reliable control for uncertain nonlinear switched time delay systems, the time delay is treated as a constant. However, in actual operation, the time delay is usually variable as time. Obviously, the system model couldn't be described appropriately using constant time delay in this case. So the paper focuses on the system with time-varying delay. Besides, it is considered that H_∞ performance is always an important index in control system. Therefore, in order to overcome the passive effect of time-varying delay for switched systems and make systems be anti-jamming and fault-tolerant, this paper addresses the robust H_∞ reliable control for nonlinear switched time-varying delay systems subjected to uncertainties. The multiple Lyapunov-Krasovskii functional method is used to design the control law. Compared with the multiple Lyapunov function adopted in the literature (Xiang, Z. R. & Wang, R. H., 2008; Xiang, Z. R. & Wang, R. H., 2009), the multiple Lyapunov-Krasovskii functional method has less conservation because the more system state information is contained in the functional. Moreover, the controller parameters can be easily obtained using the constructed functional.

The organization of this paper is as follows. At first, the concept of robust reliable controller, γ-suboptimal robust H_∞ reliable controller and γ-optimal robust H_∞ reliable controller are presented. Secondly, fault model of actuator for system is put forward. Multiple Lyapunov-Krasovskii functional method and linear matrix inequality technique are adopted to design robust H_∞ reliable controller. Meanwhile, the corresponding switching law is proposed to guarantee the stability of the system. By using the key technical lemma, the design problems of γ-suboptimal robust H_∞ reliable controller and γ-optimal robust H_∞ reliable controller can be transformed to the problem of solving a set of the matrix inequalities. It is worth to point that the matrix inequalities in the γ-optimal problem are not linear, then we make use of variable substitute method to acquire the controller gain matrices and γ-optimal problem can be transferred to solve the minimal upper bound of the scalar γ. Furthermore, the iteration solving process of optimal disturbance attenuation performance γ is presented. Finally, a numerical example shows the effectiveness of the proposed method. The result illustrates that the designed controller can stabilize the original system and make it be of H_∞ disturbance attenuation performance when the system has uncertain parameters and actuator faults.

Notation Throughout this paper, A^T denotes transpose of matrix A, $L_2[0,\infty)$ denotes space of square integrable functions on $[0,\infty)$. $\|x(t)\|$ denotes the Euclidean norm. I is an identity matrix with appropriate dimension. $diag\{a_i\}$ denotes diagonal matrix with the diagonal elements a_i, $i=1,2,\cdots,q$. $S<0$ (or $S>0$) denotes S is a negative (or positive) definite symmetric matrix. The set of positive integers is represented by Z^+. $A \le B$ (or $A \ge B$) denotes $A-B$ is a negative (or positive) semi-definite symmetric matrix. $*$ in $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ represents the symmetric form of matrix, i.e. $* = B^T$.

2. Problem formulation and preliminaries

Consider the following uncertain switched nonlinear system with time-varying delay

$$\dot{x}(t) = \hat{A}_{\sigma(t)}x(t) + \hat{A}_{d\sigma(t)}x(t-d(t)) + \hat{B}_{\sigma(t)}u^f(t) + D_{\sigma(t)}w(t) + f_{\sigma(t)}(x(t),t) \tag{1}$$

$$z(t) = C_{\sigma(t)}x(t) + G_{\sigma(t)}u^f(t) + N_{\sigma(t)}w(t) \tag{2}$$

$$x(t) = \phi(t), t \in [-\rho,0] \tag{3}$$

where $x(t) \in R^m$ is the state vector, $w(t) \in R^q$ is the measurement noise, which belongs to $L_2[0,\infty)$, $z(t) \in R^p$ is the output to be regulated, $u^f(t) \in R^l$ is the control input of actuator fault. The function $\sigma(t):[0,\infty) \to \underline{N} = \{1,2,\cdots,N\}$ is switching signal which is deterministic, piecewise constant, and right continuous, i.e. $\sigma(t):\{(0,\sigma(0)),(t_1,\sigma(t_1)),\cdots,(t_k,\sigma(t_k))\}, k \in Z^+$, where t_k denotes the k th switching instant. Moreover, $\sigma(t) = i$ means that the i th subsystem is activated, N is the number of subsystems. $\phi(t)$ is a continuous vector-valued initial function. The function $d(t)$ denotes the time-varying state delay satisfying $0 \leq d(t) \leq \rho < \infty, \dot{d}(t) \leq \mu < 1$ for constants ρ, μ, and $f_i(\bullet,\bullet): R^m \times R \to R^m$ for $i \in \underline{N}$ are unknown nonlinear functions satisfying

$$\|f_i(x(t),t)\| \leq \|U_i x(t)\| \tag{4}$$

where U_i are known real constant matrices.
The matrices $\hat{A}_i$, $\hat{A}_{di}$ and $\hat{B}_i$ for $i \in \underline{N}$ are uncertain real-valued matrices of appropriate dimensions. The matrices $\hat{A}_i$, $\hat{A}_{di}$ and $\hat{B}_i$ can be assumed to have the form

$$[\hat{A}_i, \hat{A}_{di}, \hat{B}_i] = [A_i, A_{di}, B_i] + H_i F_i(t)[E_{1i}, E_{di}, E_{2i}] \tag{5}$$

where $A_i, A_{di}, B_i, H_i, E_{1i}, E_{di}$ and E_{2i} for $i \in \underline{N}$ are known real constant matrices with proper dimensions, H_i, E_{1i}, E_{di} and E_{2i} denote the structure of the uncertainties, and $F_i(t)$ are unknown time-varying matrices that satisfy

$$F_i^T(t)F_i(t) \leq I \tag{6}$$

The parameter uncertainty structure in equation (5) has been widely used and can represent parameter uncertainty in many physical cases (Xiang, Z. R. & Wang, R. H., 2009; Cao, Y. et al., 1998).
In actual control system, there inevitably occurs fault in the operation process of actuators. Therefore, the input control signal of actuator fault is abnormal. We use $u(t)$ and $u^f(t)$ to represent the normal control input and the abnormal control input, respectively. Thus, the control input of actuator fault can be described as

$$u^f(t) = M_i u(t) \tag{7}$$

where M_i is the actuator fault matrix of the form

$$M_i = diag\{m_{i1}, m_{i2}, \cdots, m_{il}\}, \ 0 \leq \underline{m}_{ik} \leq m_{ik} \leq \bar{m}_{ik}, \ \bar{m}_{ik} \geq 1, \ k = 1,2,\cdots,l \tag{8}$$

For simplicity, we introduce the following notation

$$M_{i0} = diag\{\tilde{m}_{i1}, \tilde{m}_{i2}, \cdots, \tilde{m}_{il}\} \tag{9}$$

$$J_i = diag\{j_{i1}, j_{i2}, \cdots, j_{il}\} \tag{10}$$

$$L_i = diag\{l_{i1}, l_{i2}, \cdots, l_{il}\} \tag{11}$$

where $\tilde{m}_{ik} = \frac{1}{2}(\bar{m}_{ik} + \underline{m}_{ik})$, $j_{ik} = \frac{\bar{m}_{ik} - \underline{m}_{ik}}{\bar{m}_{ik} + \underline{m}_{ik}}$, $l_{ik} = \frac{m_{ik} - \tilde{m}_{ik}}{\tilde{m}_{ik}}$

By equation (9)-(11), we have

$$M_i = M_{i0}(I + L_i), \quad |L_i| \le J_i \le I \tag{12}$$

where $|L_i|$ represents the absolute value of diagonal elements in matrix L_i, i.e.

$$|L_i| = diag\{|l_{i1}|, |l_{i2}|, \cdots, |l_{il}|\}$$

Remark 1 $m_{ik} = 1$ means normal operation of the k th actuator control signal of the i th subsystem. When $m_{ik} = 0$, it covers the case of the complete fault of the k th actuator control signal of the i th subsystem. When $\underline{m}_{ik} > 0$ and $m_{ik} \neq 1$, it corresponds to the case of partial fault of the k th actuator control signal of the i th subsystem.

Now, we give the definition of robust H_∞ reliable controller for the uncertain switched nonlinear systems with time-varying delay.

Definition 1 Consider system (1) with $w(t) \equiv 0$. If there exists the state feedback controller $u(t) = K_{\sigma(t)}x(t)$ such that the closed loop system is asymptotically stable for admissible parameter uncertainties and actuator fault under the switching law $\sigma(t)$, $u(t) = K_{\sigma(t)}x(t)$ is said to be a robust reliable controller.

Definition 2 Consider system (1)-(3). Let $\gamma > 0$ be a positive constant, if there exists the state feedback controller $u(t) = K_{\sigma(t)}x(t)$ and the switching law $\sigma(t)$ such that

i.   With $w(t) \equiv 0$, the closed system is asymptotically stable.

ii.   Under zero initial conditions, i.e. $x(t) = 0$ $(t \in [-\rho, 0])$, the following inequality holds

$$\|z(t)\|_2 \le \gamma \|w(t)\|_2, \quad \forall w(t) \in L_2[0, \infty), \quad w(t) \neq 0 \tag{13}$$

$u(t) = K_{\sigma(t)}x(t)$ is said to be γ-suboptimal robust H_∞ reliable controller with disturbance attenuation performance γ. If there exists a minimal value of disturbance attenuation performance γ, $u(t) = K_{\sigma(t)}x(t)$ is said to be γ-optimal robust H_∞ reliable controller.

The following lemmas will be used to design robust H_∞ reliable controller for the uncertain switched nonlinear system with time-varying delay.

Lemma 1 (Boyd, S. P. et al., 1994; Schur complement) For a given matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

with $S_{11} = S_{11}^T$, $S_{22} = S_{22}^T$, $S_{12} = S_{21}^T$, then the following conditions are equivalent

i $S < 0$

ii $S_{11} < 0$, $S_{22} - S_{21}S_{11}^{-1}S_{12} < 0$

iii $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{21} < 0$

Lemma 2 (Cong, S. et al., 2007) For matrices X and Y of appropriate dimension and $Q > 0$, we have

$$X^T Y + Y^T X \le X^T Q X + Y^T Q^{-1} Y$$

Lemma 3 (Lien, C.H., 2007) Let Y, D, E and F be real matrices of appropriate dimensions with F satisfying $F^T = F$, then for all $F^T F \le I$

$$Y + DFE + E^T F^T D^T < 0$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$Y + \varepsilon D D^T + \varepsilon^{-1} E^T E < 0$$

Lemma 4 (Xiang, Z. R. & Wang, R. H., 2008) For matrices R_1, R_2, the following inequality holds

$$R_1 \Sigma(t) R_2 + R_2^T \Sigma^T(t) R_1^T \le \beta R_1 U R_1^T + \beta^{-1} R_2^T U R_2$$

where $\beta > 0$, $\Sigma(t)$ is time-varying diagonal matrix, U is known real constant matrix satisfying $|\Sigma(t)| \le U$, $|\Sigma(t)|$ represents the absolute value of diagonal elements in matrix $\Sigma(t)$.

Lemma 5 (Peleties, P. & DeCarlo, R. A., 1991) Consider the following system

$$\dot{x}(t) = f_{\sigma(t)}(x(t)) \tag{14}$$

where $\sigma(t) : [0, \infty) \to \underline{N} = \{1, 2, \cdots, N\}$. If there exist a set of functions $V_i : R^m \to R$, $i \in \underline{N}$ such that

(i) V_i is a positive definite function, decreasing and radially unbounded;

(ii) $dV_i(x(t))/dt = (\partial V_i / \partial x) f_i(x) \le 0$ is negative definite along the solution of (14);

(iii) $V_j(x(t_k)) \le V_i(x(t_k))$ when the i th subsystem is switched to the j th subsystem $i, j \in N$, $i \ne j$ at the switching instant t_k, $k = Z^+$, then system (14) is asymptotically stable.

3. Main results

3.1 Condition of stability

Consider the following unperturbed switched nonlinear system with time-varying delay

$$\dot{x}(t) = A_{\sigma(t)} x(t) + A_{d\sigma(t)} x(t - d(t)) + f_{\sigma(t)}(x(t), t) \tag{15}$$

$$x(t) = \phi(t), t \in [-\rho, 0] \tag{16}$$

The following theorem presents a sufficient condition of stability for system (15)-(16).

Theorem 1 For system (15)-(16), if there exists symmetric positive definite matrices P_i, Q, and the positive scalar δ such that

$$P_i < \delta I \tag{17}$$

$$\begin{bmatrix} A_i^T P_i + P_i A_i + P_j + Q + \delta U_i^T U_i & P_i A_{di} \\ * & -(1-\mu)Q \end{bmatrix} < 0 \qquad (18)$$

where $i \neq j$, $i, j \in \underline{N}$, then systems (15)-(16) is asymptotically stable under the switching law

$$\sigma(t) = \arg \min_{i \in \underline{N}} \{x^T(t) P_i x(t)\} \cdot$$

Proof For the i th subsystem, we define Lyapunov-Krasovskii functional

$$V_i(x(t)) = x^T(t) P_i x(t) + \int_{t-d(t)}^{t} x^T(\tau) Q x(\tau) d\tau$$

where P_i, Q are symmetric positive definite matrices. Along the trajectories of system (15), the time derivative of $V_i(t)$ is given by

$$\begin{aligned} \dot{V}_i(x(t)) &= \dot{x}^T(t) P_i x(t) + x^T(t) P_i \dot{x}(t) + x^T(t) Q x(t) - (1-\dot{d}(t)) x^T(t-d(t)) Q x(t-d(t)) \\ &\leq \dot{x}^T(t) P_i x(t) + x^T(t) P_i \dot{x}(t) + x^T(t) Q x(t) - (1-\mu) x^T(t-d(t)) Q x(t-d(t)) \\ &= 2x^T(t) P_i [A_i x(t) + A_{di} x(t-d(t)) + f_i(x(t), t)] + x^T(t) Q x(t) \\ &\quad - (1-\mu) x^T(t-d(t)) Q x(t-d(t)) \\ &= x^T(t)(A_i^T P_i + P_i A_i + Q) x(t) + 2x^T(t) P_i A_{di} x(t-d(t)) + 2x^T(t) P_i f_i(x(t), t) \\ &\quad - (1-\mu) x^T(t-d(t)) Q x(t-d(t)) \end{aligned}$$

From Lemma 2, it is established that

$$2x^T(t) P_i f_i(x(t), t) \leq x^T(t) P_i x(t) + f_i^T(x(t), t) P_i f_i(x(t), t)$$

From expressions (4) and (17), it follows that

$$2x^T(t) P_i f_i(x(t), t) \leq x^T(t) P_i x(t) + \delta f_i^T(x(t), t) f_i(x(t), t) \leq x^T(t)(P_i + \delta U_i^T U_i) x(t)$$

Therefore, we can obtain that

$$\begin{aligned} \dot{V}_i(x(t)) &\leq x^T(t)(A_i^T P_i + P_i A_i + Q + P_i + \delta U_i^T U_i) x(t) + 2x^T(t) P_i A_{di} x(t-d(t)) \\ &\quad - (1-\mu) x^T(t-d(t)) Q x(t-d(t)) \\ &= \eta^T \Theta_i \eta \end{aligned}$$

where $\eta = \begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}$, $\Theta_i = \begin{bmatrix} A_i^T P_i + P_i A_i + Q + P_i + \delta U_i^T U_i & P_i A_{di} \\ * & -(1-\mu)Q \end{bmatrix}$

From (18), we have

$$\Theta_i + diag\{P_j - P_i, 0\} < 0 \qquad (19)$$

Using η^T and η to pre- and post- multiply the left-hand term of expression (19) yields

$$\dot{V}_i(x(t)) < x^T(t)(P_i - P_j) x(t) \qquad (20)$$

The switching law $\sigma(t) = \arg\min_{i \in \underline{N}}\{x^T(t)P_i x(t)\}$ expresses that for $i,j \in \underline{N}, i \neq j$, there holds the inequality

$$x^T(t)P_i x(t) \leq x^T(t)P_j x(t) \tag{21}$$

(20) and (21) lead to

$$\dot{V}_i(x(t)) < 0 \tag{22}$$

Obviously, the switching law $\sigma(t) = \arg\min_{i \in \underline{N}}\{x^T(t)P_i x(t)\}$ also guarantees that Lyapunov-Krasovskii functional value of the activated subsystem is minimum at the switching instant. From Lemma 5, we can obtain that system (15)-(16) is asymptotically stable. The proof is completed.

∎

Remark 2 It is worth to point that the condition (21) doesn't imply $P_i \leq P_j$, for the state $x(t)$ doesn't represent all the state in domain R^m but only the state of the i th activated subsystem.

3.2 Design of robust reliable controller
Consider system (1) with $w(t) \equiv 0$

$$\dot{x}(t) = \hat{A}_{\sigma(t)}x(t) + \hat{A}_{d\sigma(t)}x(t-d(t)) + \hat{B}_{\sigma(t)}u^f(t) + f_{\sigma(t)}(x(t),t) \tag{23}$$

$$x(t) = \phi(t), t \in [-\rho, 0] \tag{24}$$

By (7), for the i th subsystem the feedback control law can be designed as

$$u^f(t) = M_i K_i x(t) \tag{25}$$

Substituting (25) to (23), the corresponding closed-loop system can be written as

$$\dot{x}(t) = \tilde{A}_i x(t) + \hat{A}_{di}x(t-d(t)) + f_i(x(t),t) \tag{26}$$

where $\tilde{A}_i = \hat{A}_i + \hat{B}_i M_i K_i$, $i \in \underline{N}$.

The following theorem presents a sufficient existing condition of the robust reliable controller for system (23)-(24).

Theorem 2 For system (23)-(24), if there exists symmetric positive definite matrices X_i, S, matrix Y_i and the positive scalar λ such that

$$X_i > \lambda I \tag{27}$$

$$\begin{bmatrix} \Psi_i & A_{di}S & H_i & \Phi_i^T & X_i & X_i & X_i U_i^T \\ * & -(1-\mu)S & 0 & SE_{di}^T & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -S & 0 & 0 \\ * & * & * & * & * & -X_j & 0 \\ * & * & * & * & * & * & -\lambda I \end{bmatrix} < 0 \tag{28}$$

where $i \neq j, i, j \in \underline{N}$, $\Psi_i = A_i X_i + B_i M_i Y_i + (A_i X_i + B_i M_i Y_i)^T$, $\Phi_i = E_{1i} X_i + E_{2i} M_i Y_i$, then there exists the robust reliable state feedback controller

$$u(t) = K_{\sigma(t)} x(t), \ K_i = Y_i X_i^{-1} \tag{29}$$

and the switching law is designed as $\sigma(t) = \arg\min_{i \in \underline{N}}\{x^T(t) X_i^{-1} x(t)\}$, the closed-loop system is asymptotically stable.

Proof From (5) and Theorem 1, we can obtain the sufficient condition of asymptotically stability for system (26)

$$P_i < \delta I \tag{30}$$

$$\begin{bmatrix} \Lambda_{ij} & P_i(A_{di} + H_i F_i(t) E_{di}) \\ * & -(1-\mu)Q \end{bmatrix} < 0 \tag{31}$$

and the switching law is designed as $\sigma(t) = \arg\min_{i \in \underline{N}}\{x^T(t) P_i x(t)\}$, where

$$\Lambda_{ij} = P_i[A_i + B_i M_i K_i + H_i F_i(t)(E_{1i} + E_{2i} M_i K_i)] + [A_i + B_i M_i K_i + H_i F_i(t)(E_{1i} + E_{2i} M_i K_i)]^T P_i$$
$$+ P_j + Q + \delta U_i^T U_i$$

Denote

$$Y_{ij} = \begin{bmatrix} P_i(A_i + B_i M_i K_i) + (A_i + B_i M_i K_i)^T P_i + P_j + Q + \delta U_i^T U_i & P_i A_{di} \\ * & -(1-\mu)Q \end{bmatrix} \tag{32}$$

Then (31) can be written as

$$Y_{ij} + \begin{bmatrix} P_i H_i \\ 0 \end{bmatrix} F_i(t)[E_{1i} + E_{2i} M_i K_i \quad E_{di}] + [E_{1i} + E_{2i} M_i K_i \quad E_{di}]^T F_i^T(t) \begin{bmatrix} P_i H_i \\ 0 \end{bmatrix}^T < 0 \tag{33}$$

By Lemma 3, if there exists a scalar $\varepsilon > 0$ such that

$$Y_{ij} + \varepsilon \begin{bmatrix} P_i H_i \\ 0 \end{bmatrix} \begin{bmatrix} P_i H_i \\ 0 \end{bmatrix}^T + \varepsilon^{-1} [E_{1i} + E_{2i} M_i K_i \quad E_{di}]^T [E_{1i} + E_{2i} M_i K_i \quad E_{di}] < 0 \tag{34}$$

then (31) holds.
(34) can also be written as

$$\begin{bmatrix} \Pi_{ij} & P_i A_{di} + \varepsilon^{-1}(E_{1i} + E_{2i} M_i K_i)^T E_{di} \\ * & -(1-\mu)Q + \varepsilon^{-1} E_{di}^T E_{di} \end{bmatrix} < 0 \tag{35}$$

where

$$\Pi_{ij} = (A_i + B_i M_i K_i)^T P_i + P_i(A_i + B_i M_i K_i) + \varepsilon P_i H_i H_i^T P_i + \varepsilon^{-1}(E_{1i} + E_{2i} M_i K_i)^T (E_{1i} + E_{2i} M_i K_i)$$
$$+ P_j + Q + \delta U_i^T U_i$$

Using $diag\{\varepsilon^{1/2}, \varepsilon^{1/2}\}$ to pre- and post- multiply the left-hand term of expression (35) and denoting $\bar{P}_i = \varepsilon P_i, \bar{Q} = \varepsilon Q$, we have

$$\begin{bmatrix} \bar{\Pi}_{ij} & \bar{P}_i A_{di} + (E_{1i} + E_{2i} M_i K_i)^T E_{di} \\ * & -(1-\mu)\bar{Q} + E_{di}^T E_{di} \end{bmatrix} < 0 \qquad (36)$$

where

$$\begin{aligned} \bar{\Pi}_{ij} &= (A_i + B_i M_i K_i)^T \bar{P}_i + \bar{P}_i (A_i + B_i M_i K_i) + \bar{P}_i H_i H_i^T \bar{P}_i + (E_{1i} + E_{2i} M_i K_i)^T (E_{1i} + E_{2i} M_i K_i) \\ &\quad + \bar{P}_j + \bar{Q} + \varepsilon \delta U_i^T U_i \end{aligned}$$

By Lemma 1, (36) is equivalent to

$$\begin{bmatrix} \bar{\Pi}'_{ij} & \bar{P}_i A_{di} & \bar{P}_i H_i & (E_{1i} + E_{2i} M_i K_i)^T \\ * & -(1-\mu)\bar{Q} & 0 & E_{di}^T \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \qquad (37)$$

where $\bar{\Pi}'_{ij} = (A_i + B_i M_i K_i)^T \bar{P}_i + \bar{P}_i (A_i + B_i M_i K_i) + \bar{P}_j + \bar{Q} + \varepsilon \delta U_i^T U_i$

Using $diag\{\bar{P}_i^{-1}, \bar{Q}^{-1}, I, I\}$ to pre- and post- multiply the left-hand term of expression (37) and denoting $X_i = \bar{P}_i^{-1}, Y_i = K_i \bar{P}_i^{-1}, S = \bar{Q}^{-1}, \lambda = (\varepsilon\delta)^{-1}$, (37) can be written as

$$\begin{bmatrix} \bar{\Pi}''_{ij} & A_{di}S & H_i & (E_{1i}X_i + E_{2i}M_iY_i)^T \\ * & -(1-\mu)S & 0 & SE_{di}^T \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \qquad (38)$$

where $\bar{\Pi}''_{ij} = (A_i X_i + B_i M_i Y_i)^T + (A_i + B_i M_i Y_i) + X_i (X_j^{-1} + S^{-1} + \lambda^{-1} U_i^T U_i) X_i$

Using Lemma 1 again, (38) is equivalent to (28). Meanwhile, substituting $X_i = \bar{P}_i^{-1}, \bar{P}_i = \varepsilon P_i$ and $\lambda = (\varepsilon\delta)^{-1}$ to (30) yields (27). Then the switching law becomes

$$\sigma(t) = \arg\min_{i \in \underline{N}}\{x^T(t) X_i^{-1} x(t)\} \qquad (39)$$

Based on the above proof line, we know that if (27) and (28) holds, and the switching law is designed as (39), the state feedback controller $u(t) = K_{\sigma(t)} x(t)$, $K_i = Y_i X_i^{-1}$ can guarantee system (23)-(24) is asymptotically stable. The proof is completed.   ■

3.3 Design of robust *H*∞ reliable controller

Consider system (1)-(3). By (7), for the *i* th subsystem the feedback control law can be designed as

$$u^f(t) = M_i K_i x(t) \tag{40}$$

Substituting (40) to (1) and (2), the corresponding closed-loop system can be written as

$$\dot{x}(t) = \tilde{A}_i x(t) + \hat{A}_{di} x(t - d(t)) + D_i w(t) + f_i(x(t), t) \tag{41}$$

$$z(t) = \tilde{C}_i x(t) + N_i w(t) \tag{42}$$

where $\tilde{A}_i = \hat{A}_i + \hat{B}_i M_i K_i$, $\tilde{C}_i = C_i + G_i M_i K_i$, $i \in \underline{N}$.

The following theorem presents a sufficient existing condition of the robust H_∞ reliable controller for system (1)-(3).

Theorem 3 For system (1)-(3), if there exists symmetric positive definite matrices X_i, S, matrix Y_i and the positive scalar λ, ε such that

$$X_i > \lambda I \tag{43}$$

$$\begin{bmatrix} \Psi_i & D_i & (C_i X_i + G_i M_i Y_i)^T & A_{di}S & H_i & \Phi_i^T & X_i & X_i & X_i U_i^T \\ * & -\gamma\varepsilon I & N_i^T & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\dfrac{\gamma}{\varepsilon}I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -(1-\mu)S & 0 & SE_{di}^T & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -S & 0 & 0 \\ * & * & * & * & * & * & * & -X_j & 0 \\ * & * & * & * & * & * & * & * & -\lambda I \end{bmatrix} < 0 \tag{44}$$

where $i \neq j, i, j \in \underline{N}$, $\Psi_i = A_i X_i + B_i M_i Y_i + (A_i X_i + B_i M_i Y_i)^T$, $\Phi_i = E_{1i}X_i + E_{2i}M_iY_i$, then there exists the robust H_∞ reliable state feedback controller

$$u(t) = K_{\sigma(t)} x(t) \text{,} \quad K_i = Y_i X_i^{-1} \tag{45}$$

and the switching law is designed as $\sigma(t) = \arg\min_{i \in \underline{N}}\{x^T(t)X_i^{-1}x(t)\}$, the closed-loop system is asymptotically stable with disturbance attenuation performance γ for all admissible uncertainties as well as all actuator faults.

Proof By (44), we can obtain that

$$\begin{bmatrix} \Psi_i & A_{di}S & H_i & \Phi_i^T & X_i & X_i & X_i U_i^T \\ * & -(1-\mu)S & 0 & SE_{di}^T & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -S & 0 & 0 \\ * & * & * & * & * & -X_j & 0 \\ * & * & * & * & * & * & -\lambda I \end{bmatrix} < 0 \tag{46}$$

From Theorem 2, we know that closed-loop system (41) is asymptotically stable.
Define the following piecewise Lyapunov-Krasovskii functional candidate

$$V(x(t)) = V_i(x(t)) = x^T(t)P_i x(t) + \int_{t-d(t)}^{t} x^T(\tau)Qx(\tau)d\tau, \ t \in [t_n, \ t_{n+1}), \ n = 0,1,\cdots \quad (47)$$

where P_i, Q are symmetric positive definite matrices, and $t_0 = 0$. Along the trajectories of system (41), the time derivative of $V_i(x(t))$ is given by

$$\dot{V}_i(x(t)) \le \xi^T \begin{bmatrix} \Lambda_i & P_i D_i & P_i(A_{di} + H_i F_i(t)E_{di}) \\ * & 0 & 0 \\ * & * & -(1-\mu)Q \end{bmatrix} \xi \quad (48)$$

where

$$\xi = \begin{bmatrix} x^T(t) & w^T(t) & x^T(t-d(t)) \end{bmatrix}^T,$$
$$\Lambda_i = P_i[A_i + B_i M_i K_i + H_i F_i(t)(E_{1i} + E_{2i} M_i K_i)] + [A_i + B_i M_i K_i + H_i F_i(t)(E_{1i} + E_{2i} M_i K_i)]^T P_i$$
$$+ P_i + Q + \delta U_i^T U_i$$

By simple computing, we can obtain that

$$\gamma^{-1} z^T(t)z(t) - \gamma w^T(t)w(t)$$
$$= \xi^T \begin{bmatrix} \gamma^{-1}(C_i + G_i M_i K_i)^T(C_i + G_i M_i K_i) & \gamma^{-1}(C_i + G_i M_i K_i)^T N_i & 0 \\ * & \gamma^{-1} N_i^T N_i - \gamma I & 0 \\ * & * & 0 \end{bmatrix} \xi \quad (49)$$

Denote $X_i = \bar{P}_i^{-1}, Y_i = K_i \bar{P}_i^{-1}, S = \bar{Q}^{-1}, \bar{P}_i = \varepsilon P_i, \bar{Q} = \varepsilon Q$. Substituting them to (44), and using Lemma 1 and Lemma 3, through equivalent transform we have

$$\begin{bmatrix} \gamma^{-1}(C_i + G_i M_i K_i)^T(C_i + G_i M_i K_i) + \Lambda_{ij} & \gamma^{-1}(C_i + G_i M_i K_i)^T N_i + P_i D_i & P_i(A_{di} + H_i F_i(t)E_{di}) \\ * & \gamma^{-1} N_i^T N_i - \gamma I & 0 \\ * & * & -(1-\mu)Q \end{bmatrix} < 0 \quad (50)$$

where

$$\Lambda_{ij} = P_i[A_i + B_i M_i K_i + H_i F_i(t)(E_{1i} + E_{2i} M_i K_i)] + [A_i + B_i M_i K_i + H_i F_i(t)(E_{1i} + E_{2i} M_i K_i)]^T P_i$$
$$+ P_j + Q + \delta U_i^T U_i$$

Obviously, under the switching law $\sigma(t) = \arg \min_{i \in \underline{N}} \{x^T(t)P_i x(t)\}$ there is

$$\gamma^{-1} z^T(t)z(t) - \gamma w^T(t)w(t) + \dot{V}_i(x(t)) < 0 \quad (51)$$

Define

$$J = \int_0^{\infty} (\gamma^{-1} z^T(t)z(t) - \gamma w^T(t)w(t))dt \quad (52)$$

Consider switching signal

$$\sigma(t):\{(0,i^{(0)}),(t_1,i^{(1)}),(t_2,i^{(2)}),\cdots,(t_k,i^{(k)})\}$$

which means the $i^{(k)}$ th subsystem is activated at t_k.

Combining (47), (51) and (52), for zero initial conditions, we have

$$J \leq \int_0^{t_1} (\gamma^{-1}z^T(t)z(t) - \gamma w^T(t)w(t) + \dot{V}_{i^{(0)}}(t))dt + \int_{t_1}^{t_2} (\gamma^{-1}z^T(t)z(t) - \gamma w^T(t)w(t) + \dot{V}_{i^{(1)}}(t))dt + \cdots$$
$$< 0$$

Therefore, we can obtain $\|z(t)\|_2 < \gamma\|w(t)\|_2$. The proof is completed. ∎

When the actuator fault is taken into account in the controller design, we have the following theorem.

Theorem 4 For system (1)-(3), γ is a given positive scalar, if there exists symmetric positive definite matrices X_i, S, matrix Y_i and the positive scalar $\alpha, \varepsilon, \lambda$ such that

$$X_i > \lambda I \tag{53}$$

$$\begin{bmatrix}
\Sigma_i & D_i & \Sigma_{1i}^T & A_{di}S & H_i & \Sigma_{2i}^T & X_i & X_i & X_iU_i^T & Y_i^T M_{i0}J_i^{1/2} \\
* & -\gamma\varepsilon I & N_i^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Sigma_{3i} & 0 & 0 & \alpha G_i J_i E_{2i}^T & 0 & 0 & 0 & 0 \\
* & * & * & -(1-\mu)S & 0 & SE_{di}^T & 0 & 0 & 0 & 0 \\
* & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Sigma_{4i} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -S & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -X_j & 0 & 0 \\
* & * & * & * & * & * & * & * & -\lambda I & 0 \\
* & * & * & * & * & * & * & * & * & -\alpha I
\end{bmatrix} < 0 \tag{54}$$

where $i \neq j, i,j \in \underline{N}$

$$\Sigma_i = A_i X_i + B_i M_{i0} Y_i + (A_i X_i + B_i M_{i0} Y_i)^T + \alpha B_i J_i B_i^T$$
$$\Sigma_{1i} = C_i X_i + G_i M_{i0} Y_i + \alpha G_i J_i B_i^T$$
$$\Sigma_{2i} = E_{1i} X_i + E_{2i} M_{i0} Y_i + \alpha E_{2i} J_i B_i^T$$
$$\Sigma_{3i} = -\frac{\gamma}{\varepsilon}I + \alpha G_i J_i G_i^T, \quad \Sigma_{4i} = -I + \alpha E_{2i} J_i E_{2i}^T$$

then there exists the γ-suboptimal robust H_∞ reliable controller

$$u(t) = K_{\sigma(t)}x(t), \quad K_i = Y_i X_i^{-1} \tag{55}$$

and the switching law is designed as $\sigma(t) = \arg\min_{i \in \underline{N}}\{x^T(t)X_i^{-1}x(t)\}$, the closed-loop system is asymptotically stable.

Proof By Theorem 3, substituting (12) to (44) yields

$$
T_{ij} + \begin{bmatrix}
B_iM_{i0}L_iY_i + (B_iM_{i0}L_iY_i)^T & 0 & (G_iM_{i0}L_iY_i)^T & 0 & 0 & (E_{2i}M_{i0}L_iY_i)^T & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & 0 & 0 \\
* & * & * & * & * & * & * & * & * & 0
\end{bmatrix} < 0 \quad (56)
$$

where

$$
T_{ij} = \begin{bmatrix}
\Psi_{i0} & D_i & (C_iX_i + G_iM_{i0}Y_i)^T & A_{di}S & H_i & \Phi_{i0}^T & X_i & X_i & X_iU_i^T \\
* & -\gamma\varepsilon I & N_i^T & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -\dfrac{\gamma}{\varepsilon}I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -(1-\mu)S & 0 & SE_{di}^T & 0 & 0 & 0 \\
* & * & * & * & -I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -I & 0 & 0 & 0 \\
* & * & * & * & * & * & -S & 0 & 0 \\
* & * & * & * & * & * & * & -X_j & 0 \\
* & * & * & * & * & * & * & * & -\lambda I
\end{bmatrix}
$$

$\Psi_{i0} = A_iX_i + B_iM_{i0}Y_i + (A_iX_i + B_iM_{i0}Y_i)^T$

$\Phi_{i0} = E_{1i}X_i + E_{2i}M_{i0}Y_i$

Denote

$$
\Xi_i = \begin{bmatrix}
B_iM_{i0}L_iY_i + (B_iM_{i0}L_iY_i)^T & 0 & (G_iM_{i0}L_iY_i)^T & 0 & 0 & (E_{2i}M_{i0}L_iY_i)^T & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & 0 & 0 \\
* & * & * & * & * & * & * & * & * & 0
\end{bmatrix} \quad (57)
$$

Notice that M_{i0} and L_i are both diagonal matrices, then we have

$$
\Xi_i = \begin{bmatrix} B_i \\ 0 \\ G_i \\ 0 \\ 0 \\ E_{2i} \\ 0 \\ 0 \\ 0 \end{bmatrix} L_i \begin{bmatrix} M_{i0}Y_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \left(\begin{bmatrix} B_i \\ 0 \\ G_i \\ 0 \\ 0 \\ E_{2i} \\ 0 \\ 0 \\ 0 \end{bmatrix} L_i \begin{bmatrix} M_{i0}Y_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right)^T
$$

From Lemma 4 and (12), we can obtain that

$$
\Xi_i \le \alpha \begin{bmatrix} B_i \\ 0 \\ G_i \\ 0 \\ 0 \\ E_{2i} \\ 0 \\ 0 \\ 0 \end{bmatrix} J_i \begin{bmatrix} B_i \\ 0 \\ G_i \\ 0 \\ 0 \\ E_{2i} \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \alpha^{-1} \begin{bmatrix} Y_i^T M_{i0} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} J_i \begin{bmatrix} Y_i^T M_{i0} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T
\tag{58}
$$

Then the following inequality

$$
T_{ij} + \alpha \begin{bmatrix} B_i \\ 0 \\ G_i \\ 0 \\ 0 \\ E_{2i} \\ 0 \\ 0 \\ 0 \end{bmatrix} J_i \begin{bmatrix} B_i \\ 0 \\ G_i \\ 0 \\ 0 \\ E_{2i} \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \alpha^{-1} \begin{bmatrix} Y_i^T M_{i0} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} J_i \begin{bmatrix} Y_i^T M_{i0} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T < 0
\tag{59}
$$

can guarantee (56) holds.

By Lemma 1, we know that (59) is equivalent to (54). The proof is completed. ■

Remark 3 (54) is not linear, because there exist unknown variables ε, ε^{-1}. Therefore, we consider utilizing variable substitute method to solve matrix inequality (54). Using $diag\{I, \varepsilon^{-1}, I, I, I, I, I, I, I\}$ to pre- and post- multiply the left-hand term of expression (54), and denoting $\eta = \varepsilon^{-1}$, (54) can be transformed as the following linear matrix inequality

$$
\begin{bmatrix}
\Sigma_i & \eta D_i & \Sigma_{1i}^T & A_{di}S & H_i & \Sigma_{2i}^T & X_i & X_i & X_iU_i^T & Y_i^TM_{i0}J_i^{1/2} \\
* & -\eta\gamma I & \eta N_i^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Sigma_{3i} & 0 & 0 & \alpha G_iJ_iE_{2i}^T & 0 & 0 & 0 & 0 \\
* & * & * & -(1-\mu)S & 0 & SE_{di}^T & 0 & 0 & 0 & 0 \\
* & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Sigma_{4i} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -S & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -X_j & 0 & 0 \\
* & * & * & * & * & * & * & * & -\lambda I & 0 \\
* & * & * & * & * & * & * & * & * & -\alpha I
\end{bmatrix} < 0 \tag{60}
$$

where $\Sigma_{3i} = -\eta\gamma I + \alpha G_iJ_iG_i^T$

Corollary 1 For system (1)-(3), if the following optimal problem

$$
\min_{X_i>0, S>0, \alpha>0, \varepsilon>0, \lambda>0, Y_i} \gamma \tag{61}
$$

s.t. (53) and (54)

has feasible solution $X_i > 0, S > 0, \alpha > 0, \varepsilon > 0, \lambda > 0, Y_i, i \in \underline{N}$, then there exists the γ-optimal robust H_∞ reliable controller

$$
u(t) = K_{\sigma(t)}x(t) \text{，} K_i = Y_iX_i^{-1} \tag{62}
$$

and the switching law is designed as $\sigma(t) = \arg\min_{i\in\underline{N}}\{x^T(t)X_i^{-1}x(t)\}$, the closed-loop system is asymptotically stable.

Remark 4 There exist unknown variables $\gamma\varepsilon$, $\gamma\varepsilon^{-1}$ in (54), so it is difficult to solve the above optimal problem. We denote $\theta = \gamma\varepsilon$, $\chi = \gamma\varepsilon^{-1}$, and substitute them to (54), then (54) becomes a set of linear matrix inequalities. Notice that $\gamma \leq \dfrac{\theta + \chi}{2}$, we can solve the following the optimal problem to obtain the minimal upper bound of γ

$$
\min_{X_i>0, S>0, \alpha>0, \theta>0, \chi>0, \lambda>0, Y_i} \frac{\theta + \chi}{2} \tag{63}
$$

s.t. $\begin{bmatrix}
\Sigma_i & D_i & \Sigma_{1i}^T & A_{di}S & H_i & \Sigma_{2i}^T & X_i & X_i & X_iU_i^T & Y_i^TM_{i0}J_i^{1/2} \\
* & -\theta I & N_i^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Sigma_{3i} & 0 & 0 & \alpha G_iJ_iE_{2i}^T & 0 & 0 & 0 & 0 \\
* & * & * & -(1-\mu)S & 0 & SE_{di}^T & 0 & 0 & 0 & 0 \\
* & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Sigma_{4i} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -S & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -X_j & 0 & 0 \\
* & * & * & * & * & * & * & * & -\lambda I & 0 \\
* & * & * & * & * & * & * & * & * & -\alpha I
\end{bmatrix} < 0 \tag{64}$

$$X_i > \lambda I \qquad (65)$$

where $\varSigma_{3i} = -\chi I + \alpha G_i J_i G_i^T$, then the minimal value of γ can be acquired based on the following steps

Step 1. From (63)-(65), we solve the minimal value $\gamma^{(0)}$ of $\dfrac{\theta + \chi}{2}$, where $\gamma^{(0)}$ is the first

iterative value;

Step 2. Choosing an appropriate step size $\delta = \delta_0$, and let $\gamma^{(1)} = \gamma^{(0)} - \delta_0$, then we substitute

Step 3. $\gamma^{(1)}$ to (60) to solve LMIs. If there is not feasible solution, stop iterating and $\gamma^{(0)}$ is just the optimal performance index; Otherwise, continue iterating until $\gamma^{(k)}$ is feasible solution but $\gamma^{(k+1)}$ is not, then $\gamma = \gamma^{(0)} - k\delta_0$ is the optimal performance index.

4. Numerical example

In this section, an example is given to illustrate the effectiveness of the proposed method. Consider system (1)-(3) with parameter as follows (the number of subsystems $N = 2$)

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}, \ A_2 = \begin{bmatrix} 1 & -3 \\ 0 & -2 \end{bmatrix}, \ A_{d1} = \begin{bmatrix} -5 & 0 \\ 0 & -4 \end{bmatrix}, \ A_{d2} = \begin{bmatrix} -3 & -1 \\ 0 & -6 \end{bmatrix}, \ B_1 = \begin{bmatrix} -5 & 7 \\ -9 & 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} -8 & 2 \\ 2 & 6 \end{bmatrix}$$

$$E_{11} = \begin{bmatrix} 2 & 5 \\ 0 & 0 \end{bmatrix}, \ E_{12} = \begin{bmatrix} 1 & 2 \\ 4 & 0 \end{bmatrix}, \ E_{21} = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}, \ E_{22} = \begin{bmatrix} 2 & 0 \\ 0.2 & 0 \end{bmatrix}, \ E_{d1} = \begin{bmatrix} -1 & 0 \\ 1 & 0.1 \end{bmatrix}, \ E_{d2} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -0.8 & 0.5 \end{bmatrix}, \ C_2 = \begin{bmatrix} 0.3 & -0.8 \end{bmatrix}, \ G_1 = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \ G_2 = \begin{bmatrix} -0.1 & 0 \end{bmatrix}, \ D_1 = \begin{bmatrix} 2 & -1 \\ -4 & 0 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 3 & -6 \\ -5 & 12 \end{bmatrix}, \ H_1 = H_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0 \end{bmatrix}, \ N_1 = N_2 = \begin{bmatrix} 0.01 & 0 \end{bmatrix}$$

The time-varying delay $d(t) = 0.5e^{-t}$, the initial condition $x(t) = \begin{bmatrix} 2 & -1 \end{bmatrix}^T$, $t \in [-0.5, 0]$,

uncertain parameter matrices $F_1(t) = F_2(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \sin t \end{bmatrix}$, $\gamma = 0.8$ and nonlinear functions is

selected as

$$f_1(x(t), t) = \begin{bmatrix} x_1(t)\cos(t) \\ 0 \end{bmatrix}, \ f_2(x(t), t) = \begin{bmatrix} 0 \\ x_2(t)\cos(t) \end{bmatrix}$$

then $U_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $U_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

When $M_1 = M_2 = I$, from Theorem 3 and using LMI toolbox in Matlab, we have

$$X_1 = \begin{bmatrix} 0.6208 & 0.0909 \\ 0.0909 & 0.1061 \end{bmatrix}, \ X_2 = \begin{bmatrix} 0.2504 & 0.1142 \\ 0.1142 & 0.9561 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} 0.6863 & 0.5839 \\ -3.2062 & -0.3088 \end{bmatrix}, \ Y_2 = \begin{bmatrix} 0.8584 & -1.3442 \\ -0.5699 & -5.5768 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.1123 & 0.0370 \\ 0.0370 & 0.1072 \end{bmatrix}, \ \varepsilon = 19.5408, \ \lambda = 0.0719$$

Then robust H_∞ controller can be designed as

$$K_1 = \begin{bmatrix} 0.3426 & 5.2108 \\ -5.4176 & 1.7297 \end{bmatrix}, \ K_2 = \begin{bmatrix} 4.3032 & -1.9197 \\ 0.4056 & -5.8812 \end{bmatrix}$$

Choosing the switching law $\sigma(t) = \arg\min_{i \in \underline{N}} \{x^T(t) X_i^{-1} x(t)\}$, the switching domain is

$$\Omega_1 = \{x(t) \in R^2 \,|\, x^T(t) X_1^{-1} x(t) \le x^T(t) X_2^{-1} x(t)\}$$

$$\Omega_2 = \{x(t) \in R^2 \,|\, x^T(t) X_1^{-1} x(t) > x^T(t) X_2^{-1} x(t)\}$$

i.e.

$$\Omega_1 = \{x(t) \in R^2 \,|\, x^T(t) \begin{bmatrix} -2.3815 & -1.0734 \\ -1.0734 & 9.6717 \end{bmatrix} x(t) \le 0\}$$

$$\Omega_2 = \{x(t) \in R^2 \,|\, x^T(t) \begin{bmatrix} -2.3815 & -1.0734 \\ -1.0734 & 9.6717 \end{bmatrix} x(t) > 0\}$$

The switching law is

$$\sigma(t) = \begin{cases} 1 & x(t) \in \Omega_1 \\ 2 & x(t) \in \Omega_2 \end{cases}$$

The state responses of the closed-loop system are shown in Fig. 1.
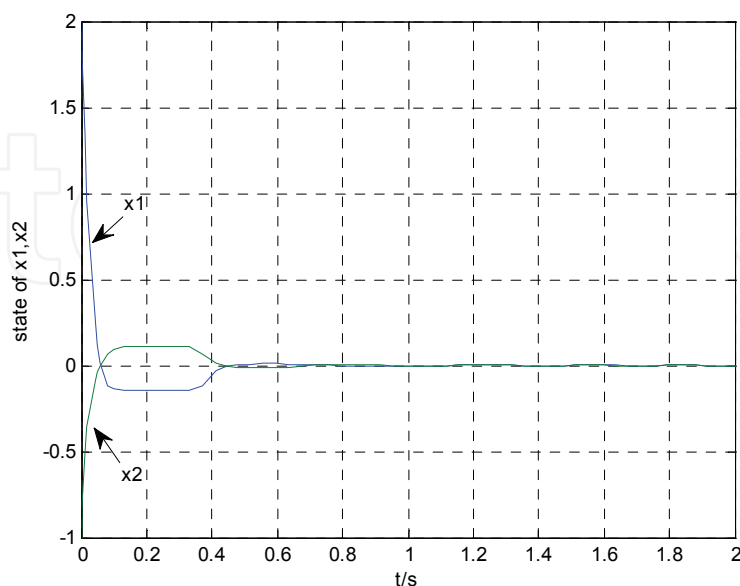
Fig. 1. State responses of the closed-loop system with the normal switched controller when the actuator is normal

The Fig. 1 illustrates that the designed normal switched controller can guarantee system is asymptotically stable when the actuator is normal.

However, in fact, the actuator fault can not be avoided. Here, we assume that the actuator fault model with parameters as follows

For subsystem 1

$$0.04 \leq m_{11} \leq 1 , \ 0.1 \leq m_{12} \leq 1.2$$

For subsystem 2

$$0.1 \leq m_{21} \leq 1 , \ 0.04 \leq m_{22} \leq 1$$

Then we have

$$M_{10} = \begin{bmatrix} 0.52 & 0 \\ 0 & 0.65 \end{bmatrix}, \ J_1 = \begin{bmatrix} 0.92 & 0 \\ 0 & 0.85 \end{bmatrix}$$

$$M_{20} = \begin{bmatrix} 0.55 & 0 \\ 0 & 0.52 \end{bmatrix}, \ J_2 = \begin{bmatrix} 0.82 & 0 \\ 0 & 0.92 \end{bmatrix}$$

Choosing the fault matrices of subsystem 1 and subsystem 2 are

$$M_1 = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.1 \end{bmatrix}, \ M_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.04 \end{bmatrix}$$

Then the above switched controller still be used to stabilize the system, the simulation result of the state responses of closed-loop switched system is shown in Fig. 2.
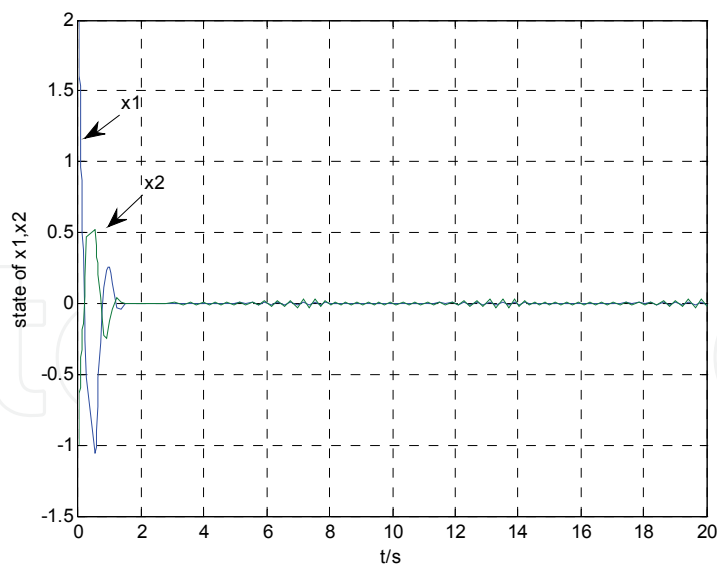
Fig. 2. State responses of the closed-loop system with the normal switched controller when the actuator is failed

Obviously, it can be seen that system state occurs vibration and the system can not be stabilized effectively.

The simulation comparisons of Fig. 1 and Fig. 2 shows that the design method for normal switched controller may lose efficacy when the actuator is failed.

Then for the above fault model, by Theorem 3 and using LMI toolbox in Matlab, we have

$$X_1 = \begin{bmatrix} 0.0180 & 0.0085 \\ 0.0085 & 0.0123 \end{bmatrix}, \; X_2 = \begin{bmatrix} 0.0436 & -0.0007 \\ -0.0007 & 0.0045 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} 0.4784 & 0.6606 \\ -0.5231 & -0.0119 \end{bmatrix}, \; Y_2 = \begin{bmatrix} 0.7036 & -0.1808 \\ -0.1737 & -0.5212 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.0130 & 0.0000 \\ 0.0000 & 0.0012 \end{bmatrix}, \; \alpha = 0.0416, \; \varepsilon = 946.1561, \; \lambda = 0.0036$$

Then robust H_∞ reliable controller can be designed as

$$K_1 = \begin{bmatrix} 1.8533 & 52.3540 \\ -42.5190 & 28.3767 \end{bmatrix}, \; K_2 = \begin{bmatrix} 15.5242 & -37.5339 \\ -5.8387 & -116.0295 \end{bmatrix}$$

Choosing the switching law as

$$\sigma(t) = \begin{cases} 1 & x(t) \in \Omega_1 \\ 2 & x(t) \in \Omega_2 \end{cases}$$

where

$$\Omega_1 = \{x(t) \in R^2 \big| x^T(t) \begin{bmatrix} 59.6095 & -60.5390 \\ -60.5390 & -100.9210 \end{bmatrix} x(t) \le 0\}$$

$$\Omega_2 = \{x(t) \in R^2 \big| x^T(t) \begin{bmatrix} 59.6095 & -60.5390 \\ -60.5390 & -100.9210 \end{bmatrix} x(t) > 0\}$$

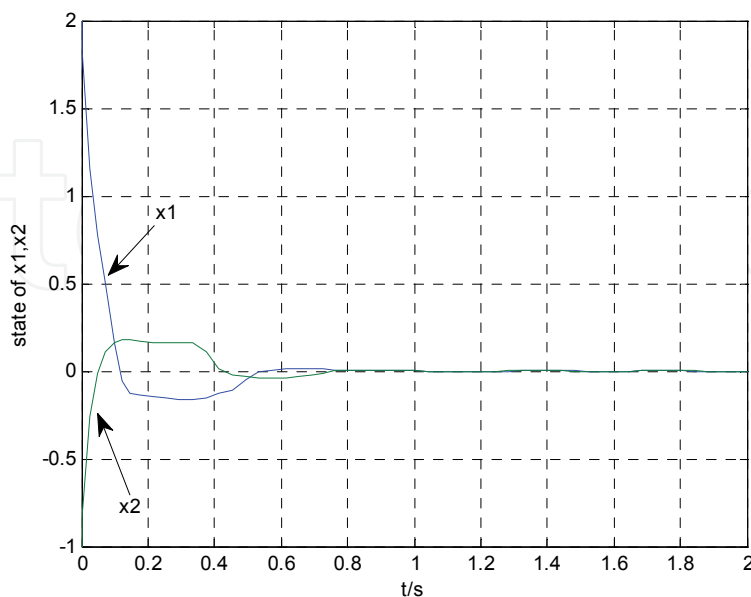The state responses of the closed-loop system are shown in Fig. 3.

Fig. 3. State responses of the closed-loop system with the reliable switched controller when the actuator is failed

It can be seen that the designed robust H_∞ reliable controller makes the closed-loop switched system is asymptotically stable for admissible uncertain parameter and actuator fault.

The simulation of Fig. 3 also shows that the design method of robust H_∞ reliable controller can overcome the effect of time-varying delay for switched system.

Moreover, by Corollary 1, based on the solving process of Remark 4 we can obtain the optimal H_∞ disturbance attenuation performance $\gamma = 0.54$, the optimal robust H_∞ reliable controller can be designed as

$$K_1 = \begin{bmatrix} 9.7714 & 115.4893 \\ -69.8769 & 41.1641 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 9.9212 & -106.5624 \\ -62.1507 & -608.0198 \end{bmatrix}$$

The parameter matrices X_1, X_2 of the switching law are

$$X_1 = \begin{bmatrix} 0.0031 & 0.0011 \\ 0.0011 & 0.0018 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 0.0119 & -0.0011 \\ -0.0011 & 0.0004 \end{bmatrix}$$

5. Conclusion

In order to overcome the passive effect of time-varying delay for switched systems and make systems be anti-jamming and fault-tolerant, robust H_∞ reliable control for a class of uncertain switched systems with actuator faults and time-varying delays is investigated. At first, the concept of robust reliable controller, γ-suboptimal robust H_∞ reliable controller and γ-optimal robust H_∞ reliable controller are presented. Secondly, fault model of actuator for switched systems is put forward. Multiple Lyapunov-Krasovskii functional method and linear matrix inequality technique are adopted to design robust H_∞ reliable controller. The matrix inequalities in the γ-optimal problem are not linear, then we make use of variable substitute method to acquire the controller gain matrices. Furthermore, the iteration solving process of optimal disturbance attenuation performance γ is presented. Finally, a numerical example shows the effectiveness of the proposed method. The result illustrates that the designed controller can stabilize the original system and makes it be of H_∞ disturbance attenuation performance when the system has uncertain parameters and actuator faults. Our future work will focus on constructing the appropriate multiply Lyapunov-Krasovskii functional to obtain the designed method of time delay dependent robust H_∞ reliable controller.

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7. References

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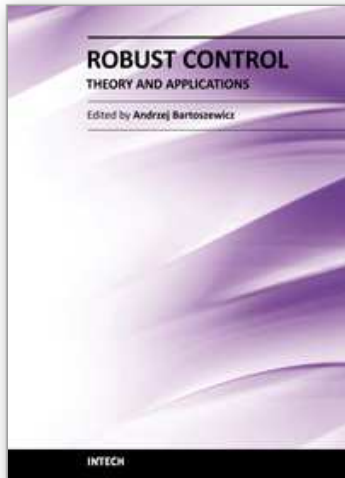
Robust Control, Theory and Applications

Edited by Prof. Andrzej Bartoszewicz

The main objective of this monograph is to present a broad range of well worked out, recent theoretical and application studies in the field of robust control system analysis and design. The contributions presented here include but are not limited to robust PID, H-infinity, sliding mode, fault tolerant, fuzzy and QFT based control systems. They advance the current progress in the field, and motivate and encourage new ideas and solutions in the robust control area.

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