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Robust Modeling and Control Issues of Parallel Manipulators with Actuation Redundancy

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1. Introduction

Parallel manipulators, often called parallel kinematics machines (PKM), are controlled non-linear dynamical systems. From a mechanical point of view PKM are (holonomically) constrained mechanical systems characterized by a power transmission between input and output. A so-called end-effector (EE), representing the mechanical output, is connected to a fixed platform by several (often identical) serial linkages, and the constraints reflect the existence of closed loops formed by these chains. Each chain is equipped with one or more actuators, representing the mechanical inputs. The modeling, identification, and control of PKM have advanced in the last decades culminating in successful industrial implementations. Yet the acceptance of PKM is far beyond that of the well-established serial manipulators. This is mainly due to the limited workspace, drastically varying static and dynamic properties, the abundance of singularities within the workspace, and the seemingly complex control.

Traditionally the number of mechanical inputs of a PKM equals its mechanical degree-of-freedom (DOF) so that the PKM is non-redundantly actuated. A means to overcome the aforementioned mechanical limitations is the inclusion of additional actuators, commonly by adding further limbs to the moving platform without increasing the DOF of the PKM. As a simple example consider the PKM in figure 1. The EE can be positioned in the plane thus possesses 2 degrees of freedom. Also the PKM as a whole has the DOF 2 so that two actuators would be sufficient for controlling this PKM. Yet the PKM is actuated by 3 actuators, which gives rise to actuation redundancy in the sense that the actuator forces are not independent. Such actuation redundancy has the potential to increase the EE-acceleration, to homogenize stiffness and manipulability, and to eliminate input singularities (where the motion of the moving platform is not controllable by the actuators), and thus to increase the usable workspace. The design of RA-PKM, and the possible dexterity improvement were addressed in several publications as for instance Garg et al. (2009); Gogu (2007); Krut et al. (2004); Kurtz & Hayward (1992); Lee et al. (1998); Nahon & Angeles (1989); O'Brien & Wen (1999); Shin et al. (2011); Wu et al. (2009).

The existence of redundant actuators allows for control forces that have no effect on the PKM motion but rather lead to mechanical prestress within the PKM. This effect can be exploited for different second-level control tasks such as backlash avoidance and stiffness control. In

*All control concepts proposed in this chapter were implemented by Timo Hufnagel at the Heilbronn University.

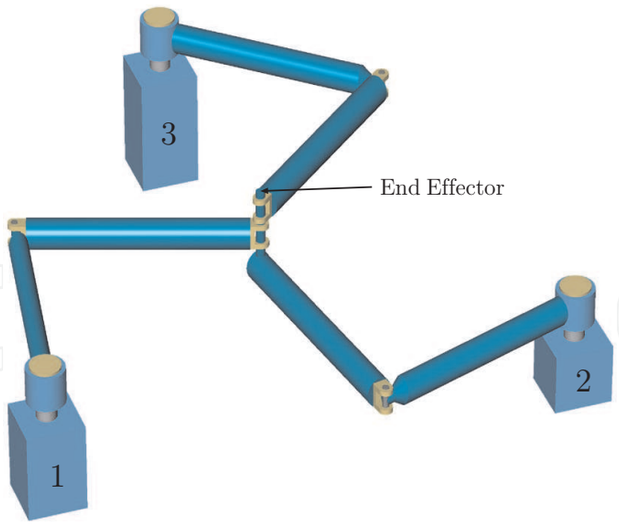

Fig. 1. Multibody model of a 2 DOF planar RA-PKM.

a power port representation of the PKM control system this means that control forces are possible without a (mechanical) power inflow into the system. This also means that in the non-linear control system there are more control vector fields than the dimension of the state space manifold, and that the control forces are not unique. Several strategies for redundancy resolution were proposed exploiting redundancy for second-level control tasks, see Lee et al. (2005); Müller (2005; 2006).

While the described advantages make the redundant actuation scheme attractive its implementation raises several challenges, however. These challenges are due to model uncertainties, synchronization errors in decentralized control schemes, as well to the lack of a globally valid dynamics model. The effect of uncertainties has been analyzed in Müller (2008) and Müller (2010) and it was show that geometric imperfections cause a qualitative change of the way in which actuation forces act upon the PKM. This is in sharp contrast to non-redundantly actuated manipulators where geometric uncertainties simply cause quantitative control errors and so impair the control performance. Moreover, geometric uncertainties of redundantly actuated PKM (RA-PKM) lead to antagonistic control forces proportional to the linear feedback gains. It turns out that this can severely deteriorate the integrity of the controlled RA-PKM.

Although model-based control concepts have been proposed for a long time and implemented recently, robotic manipulators are dominantly controlled by means of decentralized control schemes in practice. Now the indiscriminate application of decentralized control methods to RA-PKM leads again to the problem of antagonistic control forces. In contrast to intentionally generated counteracting control forces, generating desired prestresses, the latter are uncontrolled parasitic control forces. This is an inherent problem of the decentralized control of RA-PKM that can be observed even for a perfect matching of model and plant. To eliminate such antagonistic control forces a so-called antagonism filter (AF) was proposed in Müller & Hufnagel (2011).

The motion equations governing the PKM dynamics form the basis for any model-based control. Aiming on an efficient formulation, the motion equations are usually derived in terms of a minimal number of generalized coordinates that constitute a local parameterization of the configuration space. A well-known problem of this formulation is that such minimal

coordinates are not globally valid on the entire configuration space. Configurations where these coordinates become invalid are called parameterization singularities. That is, it is not possible to uniquely determine any PKM configuration by one specific set of minimal coordinates. The most natural and practicable choice of minimal coordinates is to use the actuator (input) coordinates as they can be measured. Then, the parameterization singularities are also input singularities. An ad hoc method to cope with this phenomenon is to switch between different minimal coordinates as proposed in Hufnagel & Müller (2011). This is a computationally expensive approach since it requires monitoring the numerical conditioning of the constraint equations, and the entire set of motion equation must be changed accordingly. To avoid such switching a novel formulation of motion equations that does use minimal coordinates was proposed in Müller (2011). This formulation is robust with respect to input respectively parameterization singularities and hence does not require switching between different parameterizations. Exponentially stable trajectory tracking can be shown when this formulation is employed in a computed torque and augmented PD control scheme.
These observations call for robust modeling and control concepts. This chapter reports some recent developments in modeling and control of RA-PKM.

2. The PKM control problem

2.1 Manipulator dynamics

Aiming for efficient real-time implementations model-based control schemes for PKM are based on a dynamic model in terms of minimal coordinates as pursued in Cheng et al. (2003); Müller (2005); Nakamura & Ghodoussi (1989); Thanh et al. (2009). A PKM is a force-controlled mechanism with kinematic loops. Following the standard approach in multibody dynamics the Lagrangian motion equations of first kind are first derived for the unconstrained system (opening kinematic loops), and the Lagrange multipliers are eliminated by projecting these equations to the configuration space defined by the (holonomic) geometric constraints. This approach is know as the coordinate partitioning method, see Wehage & Haug (1982).
Denote with $\mathbf{q} \in \mathbb{V}^n$ the vector of joint variables $q^a, a = 1, \ldots, n$ of the unconstrained system, obtained by opening the kinematic loops, where in each kinematic loop one joint is removed. The loop closure is enforced by the corresponding loop constraints giving rise to a set of r geometric and kinematic loop constraints

$$\begin{aligned} \mathbf{0} &= \mathbf{h}(\mathbf{q}), \ \mathbf{h}(\mathbf{q}) \in \mathbb{R}^r \\ \mathbf{0} &= \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}, \ \mathbf{J}(\mathbf{q}) \in \mathbb{R}^{r,n}. \end{aligned} \tag{1}$$

The PKM in figure 1, for instance, comprises two independent kinematic loops that can be opened by removing the two joints at the EE connecting the kinematic chain formed by joints 2 and 5 with that formed by joints 1 and 4, and joints 3 and 6, respectively. The resulting unconstrained tree-system in figure 2 has $n = 6$ generalized coordinates $q^a, a = 1, \ldots, 6$.
With the constraints (1) the Lagrangian motion equations of the PKM are

$$\mathbf{G}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}, t) + \mathbf{J}^T(\mathbf{q})\boldsymbol{\lambda} = \mathbf{u}. \tag{2}$$

$\mathbf{G}$ is the generalized mass matrix of the tree-system, $\mathbf{C}\dot{\mathbf{q}}$ represents generalized Coriolis and centrifugal forces, $\mathbf{Q}$ represents all remaining forces, including EE loads, and $\mathbf{u}$ are the generalized control forces. The Lagrange multipliers $\boldsymbol{\lambda}$ can be identified with the

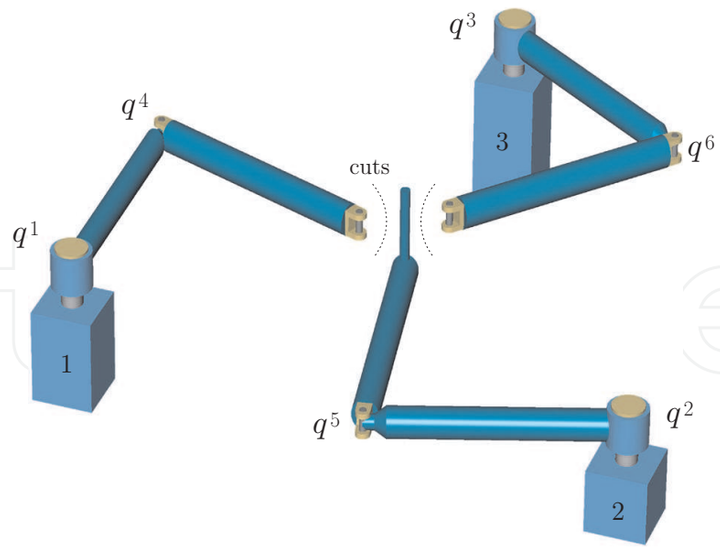

Fig. 2. Definition of loop constraints for the planar 2 DOF RA-PKM.

constraint reactions in cut-joints. The vector $\mathbf{q} \in \mathbb{V}^n$ represents the PKM configuration. The configuration space (c-space) of the PKM model is defined by the geometric constraints:

$$V := \{\mathbf{q} \in \mathbb{V}^n | \mathbf{h}(\mathbf{q}) = \mathbf{0}\}. \tag{3}$$

If $\mathbf{J}$ has locally full rank r, one can select $\delta := n - r$ joint variables, called independent coordinates, such that the admissible configurations $\mathbf{q} \in V$ are functions of these independent coordinates. This induces a coordinate partitioning. If the rank of $\mathbf{J}$ is constant, the c-space is smooth δ-dimensional manifold and δ is the DOF of the PKM, see Müller (2009). A configuration $\mathbf{q}$ where the rank of $\mathbf{J}$ changes is called a c-space singularity since then V is not a smooth manifold in $\mathbf{q}$.
Denote with $\mathbf{q}_1$ and $\mathbf{q}_2$ respectively the vector of dependent and independent coordinates, the velocity constraints can be written as

$$\mathbf{J}_1\dot{\mathbf{q}}_1 + \mathbf{J}_2\dot{\mathbf{q}}_2 = \mathbf{0}, \tag{4}$$

where $\mathbf{J} = (\mathbf{J}_1, \mathbf{J}_2)$, with $\mathbf{J}_1(\mathbf{q}) \in \mathbb{R}^{r,r}$, $\mathbf{J}_2(\mathbf{q}) \in \mathbb{R}^{r,\delta}$. By definition of independent coordinates $\mathbf{J}_1$ has full rank, and the generalized velocities can be expressed as

$$\dot{\mathbf{q}} = \mathbf{F}\dot{\mathbf{q}}_2, \text{ where } \mathbf{F} := \begin{pmatrix} -\mathbf{J}_1^{-1}\mathbf{J}_2 \\ \mathbf{I}_\delta \end{pmatrix}. \tag{5}$$

where the matrix $\mathbf{F}$ is an orthogonal complement of $\mathbf{J}$ because $\mathbf{JF} \equiv \mathbf{0}$. The time derivative of (5) yields the accelerations $\ddot{\mathbf{q}} = \mathbf{F}\ddot{\mathbf{q}}_2 + \dot{\mathbf{F}}\dot{\mathbf{q}}_2$.
Due to the existence of kinematic loops, PKM comprise passive joints and only the m control forces corresponding to the active joints are present in $\mathbf{u}$. Denote with $\mathbf{c} \equiv (c_1, \ldots, c_m)$ the vector of generalized control forces in the actuated joints, and let $\mathbf{A}$ be that part of $\mathbf{F}$ so that $\mathbf{F}^T\mathbf{u} = \mathbf{A}^T\mathbf{c}$. This means that if $\mathbf{q}_a$ denote the vector of m actuator coordinates, then $\dot{\mathbf{q}}_a = \mathbf{A}\dot{\mathbf{q}}_2$.

Projecting the motion equations (2) of the tree-system to the configuration space V, with the help of the orthogonal complement $\mathbf{F}$, and with (5), yields

$$\overline{\mathbf{G}}(\mathbf{q})\,\ddot{\mathbf{q}}_2 + \overline{\mathbf{C}}\,(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}}_2 + \overline{\mathbf{Q}}(\mathbf{q},\dot{\mathbf{q}},t) = \mathbf{A}^T(\mathbf{q})\,\mathbf{c} \tag{6}$$

where

$$\overline{\mathbf{G}} := \mathbf{F}^T\mathbf{G}\mathbf{F},\ \overline{\mathbf{C}} := \mathbf{F}^T(\mathbf{C}\mathbf{F} + \mathbf{G}\dot{\mathbf{F}}),\ \overline{\mathbf{Q}} := \mathbf{F}^T\mathbf{Q}. \tag{7}$$

The $\delta = n - m$ equations (6) together with the r dynamic constraints $\mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}} = \mathbf{0}$ yield a system of n ODE's in the n generalized coordinates $\mathbf{q}$ that govern the PKM dynamics when controlled via the generalized control forces $\mathbf{c}$. The equations (6) have been first proposed by Voronets (1901) and are a special kind of Maggi's equations, Maggi (1901). They have been proposed for use in multibody dynamics in Angles & Lee (1988); Wehage & Haug (1982), and were put forward for PKM modeling in Cheng et al. (2003); Müller (2005); Thanh et al. (2009). The PKM control problem can now be represented as the control-affine control system

$$\dot{\mathbf{x}} = \mathbf{f}\,(\mathbf{x}) + \sum_{i=1}^{m} \mathbf{g}^i\,(\mathbf{x})\,c_i \tag{8}$$

with state vector $\mathbf{x} := (\mathbf{q}_2, \dot{\mathbf{q}}_2)$, where

$$\mathbf{f} := \begin{pmatrix} \dot{\mathbf{q}}_2 \\ -\overline{\mathbf{G}}^{-1}(\overline{\mathbf{C}}\dot{\mathbf{q}}_2 + \overline{\mathbf{Q}}) \end{pmatrix}, \quad \mathbf{g} := \begin{pmatrix} \mathbf{0} \\ \overline{\mathbf{G}}^{-1}\mathbf{A}^T \end{pmatrix}. \tag{9}$$

$\mathbf{f}$ is the drift vector field, and the columns $\mathbf{g}^i, i = 1,\ldots,m$ are the control vector fields that determine how the control forces affect the system's state.

2.2 Actuation concepts

Based on the control system (8) different actuation schemes can be distinguished. Actuation refers to the immediate effect of control forces in a given state of the PKM. Apparently the degree of actuation has to do with the number of independent control vector fields. The degree of actuation (DOA) can be defined as the number of independent input vector fields $\mathbf{g}^i$ in the control system (8). With regular $\overline{G}$ the DOA is

$$\alpha\,(q) := \operatorname{rank} \overline{M}\,(q)\,. \tag{10}$$

If $\alpha\,(q) < \delta$, the system is called *underactuated*, and if $\alpha\,(q) = \delta$, it is called *full-actuated* at q. The system is called *redundantly actuated* at q, if $m - \alpha\,(q) > 0$ and *non-redundantly actuated* at q, if $m = \alpha\,(q)$. Apparently a system can be redundantly underactuated. Configurations q where the DOA changes, i.e. when α is not constant in a neighborhood of q, are called *input singularities*, see Müller (2009); Zlatanov & Fenton (1998).

2.3 Inverse dynamics solution

The inverse dynamics problem is to find the control forces $\mathbf{c}$ required for controlling the PKM along a prescribed target trajectory $\mathbf{q}\,(t)$. The system (6) has no unique solution for $\mathbf{c}$. Moreover, it is clear that only those $\mathbf{c}$ in the range of the control matrix $\mathbf{A}^T$ are effective control forces, and that arbitrary forces $\mathbf{c}^0$ (prestress) in the null-space of $\mathbf{A}^T$ can be superposed. If $\mathbf{c}^0$ is the vector of desired prestress, for given $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$, a solution such that

$\left(\mathbf{c}-\mathbf{c}^0\right)^T \mathbf{W}\left(\mathbf{c}-\mathbf{c}^0\right) \rightarrow \min$ is given by

$$\begin{aligned}\mathbf{c} &= \left(\mathbf{A}^T\right)_{\mathbf{W}}^{+} \mathbf{F}^T(\mathbf{G}\left(\mathbf{q}\right)\ddot{\mathbf{q}} + \mathbf{C}\left(\mathbf{q},\dot{\mathbf{q}}\right)\dot{\mathbf{q}} + \mathbf{Q}\left(\mathbf{q},\dot{\mathbf{q}}\right)) + \mathbf{N}_{\mathbf{A}^T,\mathbf{W}}\mathbf{c}^0 \\ &= \left(\mathbf{A}^T\right)_{\mathbf{W}}^{+} (\overline{\mathbf{G}}\left(\mathbf{q}\right)\ddot{\mathbf{q}}_2 + \overline{\mathbf{C}}\left(\mathbf{q},\dot{\mathbf{q}}\right)\dot{\mathbf{q}}_2 + \overline{\mathbf{Q}}\left(\mathbf{q},\dot{\mathbf{q}}\right)) + \mathbf{N}_{\mathbf{A}^T,\mathbf{W}}\mathbf{c}^0 \end{aligned} \tag{11}$$

where $\left(\mathbf{A}^T\right)_{\mathbf{W}}^{+} := \mathbf{W}^{-1}\mathbf{A}\left(\mathbf{A}^T\mathbf{W}^{-1}\mathbf{A}\right)^{-1}$ is the weighted right pseudoinverse, and $\mathbf{N}_{\mathbf{A}^T,\mathbf{W}} := \left(\mathbf{I}_m - \left(\mathbf{A}^T\right)_{\mathbf{W}}^{+}\mathbf{A}^T\right)$ is a projector to the null-space of $\mathbf{A}^T$. $\mathbf{W}$ is a symmetric positive definite weighting matrix for the drive forces in accordance with the drive capabilities. The pseudoinverse solution in (11) delivers the controls that produce the desired motion, where the drive load is balanced between the individual drives according to the weights. The second part of the control vector $\mathbf{c}$ is the null-space component generating prestress that is closest to the desired $\mathbf{c}^0$. The possibility of generating control forces in the null-space has been used for backlash avoidance and stiffness control Müller (2005; 2006); Valasek et al. (2002); Yi et al. (1989).

3. Decentralized control schemes

3.1 Peculiarities of PKM with actuation redundancy

Decentralized control of individual actuators without taking into account the dynamics of the controlled system is still the standard control method in industrial applications. Moreover the majority of contemporary robotic manipulators are controlled by a decentralized PD law in favor of its simple computation and low-cost setup. In contrast to model-based control, the actuators are controlled independently, without reference to the dynamics of the non-linear control system, exclusively upon the individual commands obtained from motion planning and inverse kinematics. In other words, it is assumed that all actuators can be independently controlled without mutual interference. This applies to serial manipulators as well as to PKM without actuation redundancy as summarized in Paccot et al. (2009) and Thanh et al. (2009). However, since in case of RA-PKM more actuators are activated than required, decentralized control of RA-PKM naturally leads to conflicting control forces, reflected by undesired prestresses and an increased power consumption as observed in Saglia et al. (2009),Valasek et al. (2005), Wang et al. (2009). Hence antagonistic control forces cannot be attributed to model uncertainties alone, as analyzed in Müller (2010), which could be minimized using model identification methods. Even more such counteraction is inherent to the decentralized control method. In order to eliminate contradicting control forces the actuation redundancy must be resolved also within decentralized control, which requires a kinematic model. In the following the interplay of measurement errors and decentralized control is discussed.

As above denote with $\mathbf{q}$ and $\mathbf{q}_\mathrm{a}$ the actual generalized coordinates of the plant (which is unknown). Further a measurement error is assumed caused by a constant (calibration) offset $\Delta\mathbf{q}_\mathrm{a}$. The vector of measured actuator coordinates is then introduced as $\widetilde{\mathbf{q}}_\mathrm{a} := \mathbf{q}_\mathrm{a} + \Delta\mathbf{q}_\mathrm{a}$. If $\mathbf{q}_\mathrm{a}^\mathrm{d}\left(t\right)$ denotes the desired actuator motion, $\mathbf{e}_\mathrm{a} := \mathbf{q}_\mathrm{a} - \mathbf{q}_\mathrm{a}^\mathrm{d}$ is the actual tracking error. Due to the calibration offset the measured tracking error is $\widetilde{\mathbf{e}}_\mathrm{a} = \widetilde{\mathbf{q}}_\mathrm{a} - \mathbf{q}_\mathrm{a}^\mathrm{d} = \mathbf{e}_\mathrm{a} + \Delta\mathbf{q}_\mathrm{a}$.

The simple PD control law that independently regulates the m actuator positions upon these measurements is

$$\mathbf{c} = -\mathbf{K}_\mathrm{P}\widetilde{\mathbf{e}}_\mathrm{a} - \mathbf{K}_\mathrm{D}\dot{\widetilde{\mathbf{e}}}_\mathrm{a}. \tag{12}$$

$\mathbf{K}_\mathrm{P}$ and $\mathbf{K}_\mathrm{D}$ are diagonal positive definite gain matrices. The effect of measurement errors can be understood considering the pose $\widetilde{\mathbf{q}}_\mathrm{a}$ that is attained as result of a constant setpoint command $\mathbf{q}_\mathrm{a}^\mathrm{d}$. The stationary control forces, not producing any motion, are those $\mathbf{c}^0$ in the null-space of $\mathbf{A}^T(\mathbf{q})$, i.e.

$$\mathbf{0} = \mathbf{A}^T(\mathbf{q})\,\mathbf{c}^0 = -\mathbf{A}^T(\mathbf{q})\,\mathbf{K}_\mathrm{P}\widetilde{\mathbf{e}}_\mathrm{a} = -\mathbf{A}^T(\mathbf{q})\,\mathbf{K}_\mathrm{P}\left(\mathbf{e}_\mathrm{a} + \Delta\mathbf{q}_\mathrm{a}\right). \tag{13}$$

If there are no measurement errors (and no model uncertainties) it is $\Delta\mathbf{q}_\mathrm{a} = \mathbf{0}$, and the PKM converges to $\mathbf{e}_\mathrm{a} = 0$ since $\mathbf{q}_\mathrm{a}$ and $\mathbf{q}_\mathrm{a}^\mathrm{d}$ satisfy the geometric constraints. Since generally $\Delta\mathbf{q}_\mathrm{a}$ do not comply with the geometric constraints the PKM converges to a $\mathbf{q}_\mathrm{a}$ such that $\mathbf{e}_\mathrm{a} + \Delta\mathbf{q}_\mathrm{a}$ is in the null-space of $\mathbf{A}^T\mathbf{K}_\mathrm{P}$. The exact value of $\mathbf{q}$ depends on external forces, e.g. gravity and process loads. The attained steady state error is

$$\mathbf{e}_\mathrm{a} = -\mathbf{K}_\mathrm{P}^{-1}\left(\mathbf{A}^T\right)^+\mathbf{K}_\mathrm{P}\Delta\mathbf{q}_\mathrm{a} \tag{14}$$

with the right pseudoinverse $\left(\mathbf{A}^T\right)^+ = \mathbf{A}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}$ of $\mathbf{A}^T$. In this equilibrium configuration the PD controllers yield the steady state actuator forces $\mathbf{c}^0 = \mathbf{K}_\mathrm{P}\left(\mathbf{e}_\mathrm{a} + \Delta\mathbf{q}_\mathrm{a}\right)$. Since they are in the null-space of $\mathbf{A}^T$ the RA-PKM stays at rest.

The consideration so far applies to constant calibration offsets. A further source of systematic measurement errors is the encoder resolution within the actuators. Assuming exact calibration, for simplicity, the error measurement, and thus $\Delta\mathbf{q}_\mathrm{a}$, change discontinuously according to the resolution. Then the solution for the steady state error (14) is only piecewise valid and may not be unique. Moreover the interplay of antagonistic forces and the quantization of $\widetilde{\mathbf{q}}_\mathrm{a}$ can cause alternating control forces, and hence excited vibrations. This is clear by noting that $\Delta\mathbf{q}_\mathrm{a}$ changes in discrete steps so that during the settling process the components of $\mathbf{e}_\mathrm{a} + \Delta\mathbf{q}_\mathrm{a}$ in the null-space of $\mathbf{A}^T\mathbf{K}_\mathrm{P}$ are changing discontinuously and thus cause discontinuous control forces.

The crucial point is that the decentralized control scheme (13) is not restricted to the range of $\mathbf{A}^T$, and so yields antagonistic control forces in the null-space of $\mathbf{A}^T$. Such contradictory control forces are on the one hand due to measurement errors but are on the other hand inevitably caused by the uncoordinated control of individual actuators with (12). In the decentralized PD control the individual controller for each kinematic chain, connecting the EE to the base, acts independently without respecting the coordination within the closed kinematic loops of the RA-PKM. Consider this phenomenon for the 2 DOF RA-PKM in figure 1. The manipulator can be viewed as the cooperation of two (virtually independent) non-redundantly actuated 5-bar linkages as shown in figure 3. Consider the situation when the EE is to follow a straight line depicted in figure 3. The three base joints, with coordinates q^1, q^2, and q^3, are actuated. Two of these actuators are sufficient for this 2 DOF system, however. For instance, the PKM can be controlled by joints 1 and 2, or joints 2 and 3. Consequently the motion control of the 5-bar loop consisting of joints 1 and 2, and the other 5-bar loop with joints 2 and 3 must be synchronized. Controlling the loops independently, the control commands for steering the EE along the straight line, determined from (12), drive either 5-bar loop along a straight line in the $q^1 - q^2$ and $q^2 - q^3$ joint subspace, respectively. The latter correspond to EE-curves depicted in figure 3. It is apparent that the control commands of the 5-bar linkages are contradicting due to the missing synchronization of the two loops. This is an inherent problem of the decentralized control, which does exist for the model-based control schemes that a priori respects such interdependencies.

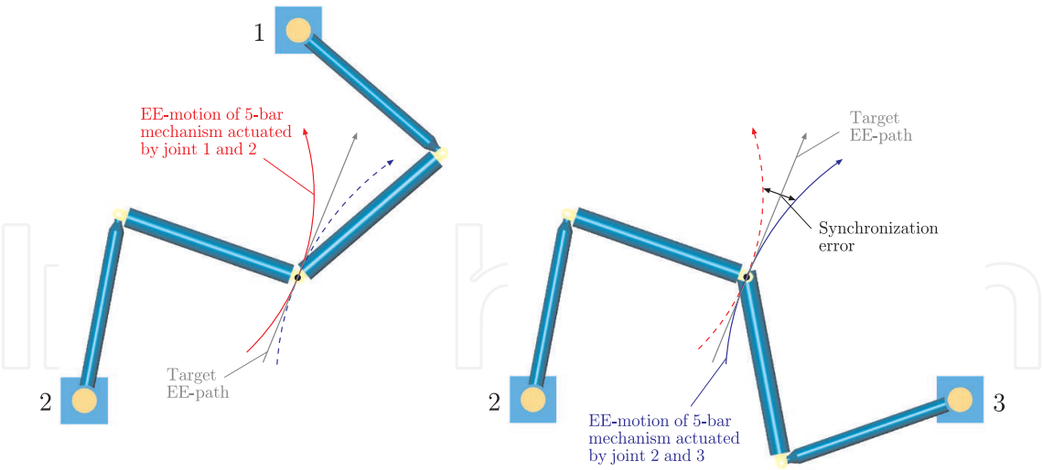

Fig. 3. Explanation of the synchronization error in decentralized control.

3.2 Antagonism filter - a method for reducing counteraction

Apparently the antagonistic actuator forces are those control commands that are (unintentionally) in the null-space of $\mathbf{A}^T$. Elimination of these antagonistic control forces is hence equivalent to the removal of those components of the control forces $\mathbf{c}$ that are in the null-space of $\mathbf{A}^T$. This is readily achieved using the projector

$$\mathbf{R}_{\mathbf{A}^T} := \mathbf{I} - \mathbf{N}_{\mathbf{A}^T} = \left(\mathbf{A}^T\right)^+ \mathbf{A}^T \tag{15}$$

onto the range of $\mathbf{A}^T$, where $\mathbf{N}_{\mathbf{A}^T}$ is the null-space projector. This projector can be applied to any actuator force commands (not necessarily computed from (12)), and return the effective control forces

$$\mathbf{c}_{\text{eff}} = \mathbf{R}_{\mathbf{A}^T}\mathbf{c}. \tag{16}$$

These control forces can be applied to the PKM without changing the drive action since $\mathbf{A}^T\mathbf{R}_{\mathbf{A}^T} = \mathbf{A}^T$. Because $\mathbf{R}_{\mathbf{A}^T}$ eliminates the antagonistic actuator forces it is called the *antagonism filter* (AF) in Müller & Hufnagel (2011).

In practice the individual actuators are position/velocity controlled rather than force controlled. Since this splitting concerns the actuator forces it needs to be transformed to the position and velocity command. Therefore the error vector in (12) is projected to the range of $\mathbf{A}^T\mathbf{K}_\text{P}$ and $\mathbf{A}^T\mathbf{K}_\text{D}$, respectively, so that

$$\mathbf{e}_{\text{eff}} = \mathbf{R}_{\mathbf{A}^T\mathbf{K}_\text{P}}\widetilde{\mathbf{e}}_\text{a}, \quad \dot{\mathbf{e}}_{\text{eff}} = \mathbf{R}_{\mathbf{A}^T\mathbf{K}_\text{D}}\dot{\widetilde{\mathbf{e}}}_\text{a}. \tag{17}$$

Then the corrected command sent to the individual PD controllers is

$$\mathbf{q}^\text{d}_{\text{eff}} = \mathbf{q}_\text{a} - \mathbf{e}_{\text{eff}}, \quad \dot{\mathbf{q}}^\text{d}_{\text{eff}} = \dot{\mathbf{q}}_\text{a} - \dot{\mathbf{e}}_{\text{eff}}. \tag{18}$$

3.3 Case study: Planar 2 DOF RA-PKM

The AF has been applied to decentralized control of the planar 2 DOF redundantly full-actuated PKM in figure 1. The testbed was developed at the Heilbronn University as reported in Hufnagel & Müller (2011) and Müller & Hufnagel (2011), where all experiments were carried out. The prototype consists of arm segments with a length of 200 mm. The three revolute joints are located at an equilateral triangle with lateral length 400 mm. The base

joints are actuated with DC motors (Maxon Re30). The RA-PKM can be controlled either with individual PD controllers or a model-base control scheme (see section 4.5). The PD controllers are designed for a load corresponding to the weight of one arm (134 g). In the following results for the PD controller are reported.

As example the PKM is controlled within 5 s along the triangular EE trajectory in figure 4. The trajectory is planned according to the maximal acceleration of about 0.5 $\frac{m}{s^2}$. The required actuator motions are determined from the inverse kinematics that are the target trajectories of the PD controllers. The manipulator was calibrated manually in order to reduce calibration errors. The required joint torques are shown in figure 5 and the joint errors in figure 6. The

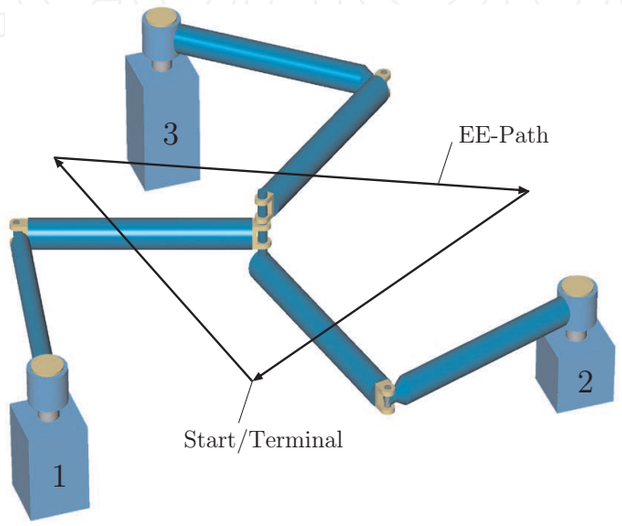

Fig. 4. Triangular EE-path along which the RA-PKM is controlled with the CTC.

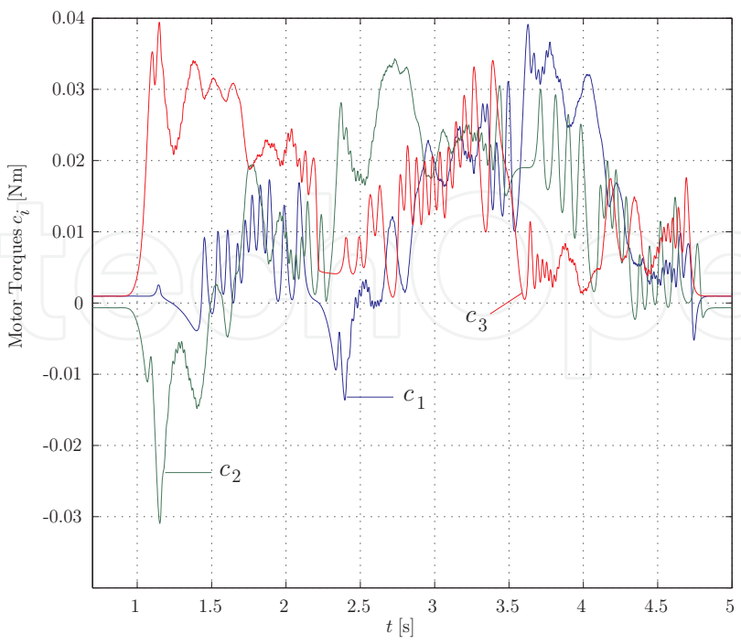

Fig. 5. Joint torques when the RA-PKM is controlled by decentralized PD controllers.

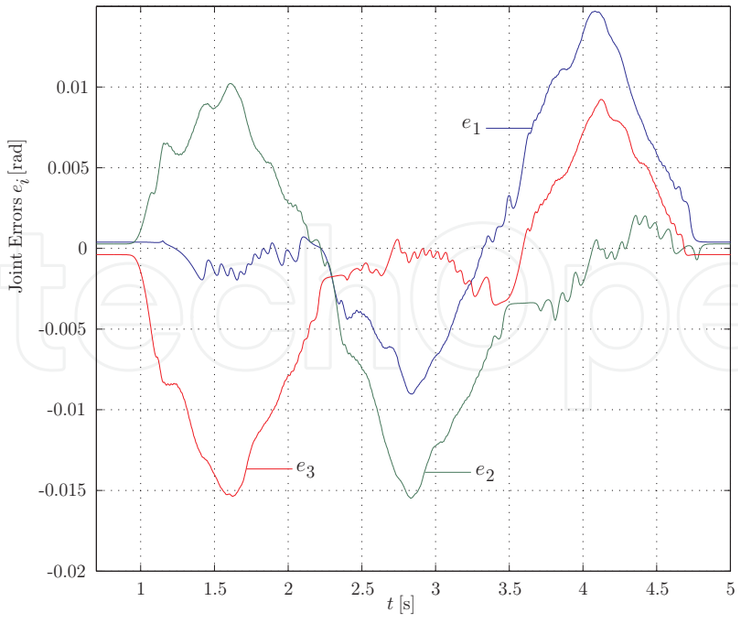

Fig. 6. Actuator tracking errors when the RA-PKM is controlled by decentralized PD controllers.

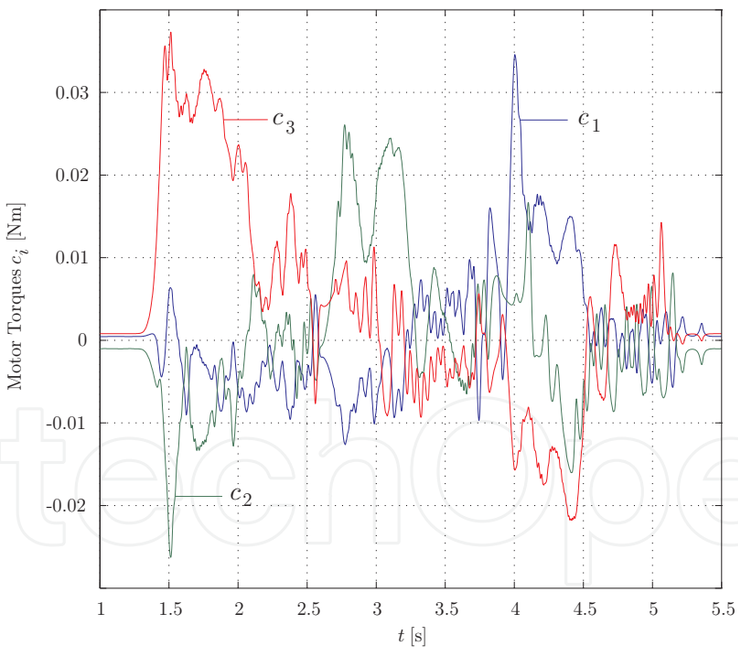

Fig. 7. Joint torques when the AF is applied to decentralized PD control of the RA-PKM.

initial and final torques and errors in the actuator joint angles when the PKM is at rest are due to the remaining calibration uncertainty and encoder quantization. These steady state drive torques are in accordance with (13) and (14). Application of the AF (15) reduces the overall drive torques as shown in figure 7. The antagonistic actuator torques are almost removed by the AF. A small part of the constant initial and final actuator torques remain, however.

Motor i	Electric Energy Consumption E_i^{el}	
	without AF	with AF
1	1.3347 Ws	0.4574 Ws
2	1.5414 Ws	0.7010 Ws
3	1.6844 Ws	1.0801 Ws

Table 1. Energy consumption if the PKM is manually recalibrated.

This can be explained by noting that the control matrix $\mathbf{A}^T$ and the AF are computed upon the measurement $\widetilde{\mathbf{q}}_a$, and that there are always model imperfections. A reduction of electric energy translates directly to the reduction of actuator torques, and the reduced actuator torques are reflected by a reduced electrical power consumption E_i^{el} of motor i. Table 1 shows the energy consumptions when the RA-PKM is controlled with and without application of the AF. A significant reduction of electrical energy consumption and thus of the drive torques is apparent. The performed mechanical work is indeed not altered by AF.

4. Model-based control schemes

4.1 Augmented PD and computed torque controller

Two accepted methods for the model-based control of robotic manipulators are the augmented PD (APD) and computed torque control (CTC) schemes Asada & Slotine (1986); Murray et al. (1993). Both schemes consist of a non-linear feedforward term, that delivers the control forces required for steering the PKM along the desired trajectory, and a linear feedback term to compensate drifts from the desired motion. Now the feedforward term requires the inverse dynamics solution (11). These control methods, originally derived for non-redundantly actuated systems, can be directly adopted for RA-PKM as in Cheng et al. (2003); Müller (2005); Paccot et al. (2009). The APD can be used in the form (omitting null-space components)

$$\mathbf{c} = \left(\mathbf{A}^T(\mathbf{q})\right)^+ \left[\overline{\mathbf{G}}\,(\mathbf{q})\,\ddot{\mathbf{q}}_2^{\mathrm{d}} + \overline{\mathbf{C}}\,(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}}_2^{\mathrm{d}} + \overline{\mathbf{Q}}\,(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{K}_{\mathrm{D}}\dot{\mathbf{e}}_2 - \mathbf{K}_{\mathrm{P}}\mathbf{e}_2\right], \tag{19}$$

wherein $\mathbf{q}^{\mathrm{d}}(t)$ is the desired path, and $\mathbf{e}_2(t) := \mathbf{q}_2(t) - \mathbf{q}_2^{\mathrm{d}}(t)$ is the tracking error of the independent coordinates. The gain matrices $\mathbf{K}_{\mathrm{D}}$ and $\mathbf{K}_{\mathrm{P}}$ are diagonal and positive definite. An adopted form of the CTC law for RA-PKM is

$$\mathbf{c} = \left(\mathbf{A}^T(\mathbf{q})\right)^+ \left[\overline{\mathbf{G}}\,(\mathbf{q})\,\mathbf{v}_2 + \overline{\mathbf{C}}\,(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}}_2 + \overline{\mathbf{Q}}\,(\mathbf{q},\dot{\mathbf{q}})\right], \tag{20}$$

with $\mathbf{v}_2 := \ddot{\mathbf{q}}_2^{\mathrm{d}} - \mathbf{K}_{\mathrm{D}}\dot{\mathbf{e}}_2 - \mathbf{K}_{\mathrm{P}}\mathbf{e}_2$. Perfect matching of model and plant presumed, both control laws applied to (6) result in exponentially stable trajectory tracking for sufficiently large gains $\mathbf{K}_{\mathrm{D}}$ and $\mathbf{K}_{\mathrm{P}}$, provided $\overline{\mathbf{G}}$ is regular. The latter assumption only fails in configuration space singularities of the PKM and in singularities of the parameterization of the model. This is in particular critical for RA-PKM as explained in the next section.

4.2 Parameterization-singularities of the dynamic model

It is well-known that there is generally no choice of minimal coordinates that is valid for the entire motion range of the manipulator. Parameterization-singularities refer to configurations

where a selected set of independent coordinates becomes invalid. This problem is usually solved ad-hoc by switching between different mathematical models as proposed in Hufnagel & Müller (2011). There is, however, no general approach to cope with this problem. While this is essentially a numerical problem of the particular PKM model it does have a great significance for PKM control. In particular the improper selection of generalized coordinates can severely deteriorate the stability of model-based control schemes.

The inherent problem of the minimal coordinate formulation is the need for selecting independent minimal coordinates $\mathbf{q}_2$. Since they are local coordinates on the c-space V the PKM configuration cannot be expressed globally in terms of these minimal coordinates. That is, any such minimal coordinates are only valid in a limited range of motion, and a collection of different sets of minimal coordinates is necessary to cover the entire c-space. The switching method proposed in Hufnagel & Müller (2011) switches between such local coordinates.

From a practical point of view it makes sense to use δ actuator coordinates as independent coordinates in the motion equations (6). That is, $\mathbf{q}_2$ is a subset of $\mathbf{q}_\mathrm{a}$. In other words the PKM is considered as non-redundantly actuated and its motion equations are parameterized in terms of δ actuator coordinates. Consequently, parameterization-singularities are exactly the input-singularities of the non-redundantly actuated PKM. For example, the planar 2 DOF RA-PKM in figure 1 is naturally parameterized in terms of $\delta = 2$ out of the $m = 3$ actuator coordinates. This leads to parameterization-singularities shown in figure 8. Figure 8 a) shows two configurations where the actuator coordinates q^1 and q^2 are not valid as independent coordinates for the PKM model. In these configurations the PKM configuration is not uniquely determined by the motion joints of 1 and 2 so that q^1 and q^2 fail as independent coordinates. Alternatively joints 1 and 3 could be used to control the PKM. That is, q^1 and q^3 would constitute independent coordinates of the minimal coordinate model (6). This parameterization exhibits the singular configurations in figure 8 b), however. Similar singularities exist when q^2 and q^3 are used as independent, and moreover there are parameterization-singularities for any choice of two actuator angles.

Now it is important to notice that any switching to different independent coordinates $\mathbf{q}_2$ causes a complete change of the motion equations (6). Such a switching method is thus computationally rather complex and accompanied by a high implementation effort. Its application to general RA-PKM requires monitoring the numerical conditioning of the orthogonal complement $\mathbf{F}$ in (5) in order to detect switching points. Only for simple mechanisms, such as the reported 2 DOF RA-PKM in figure 1, the switching points can be determined explicitly giving rise to a switching map.

4.3 A robust formulation of the dynamic model in redundant coordinates

The minimal coordinate formulation (6) is prone to parameterization-singularities. Coordinate switching methods, introduced to cope with this problem, are computationally rather complex for general PKM. An approach that completely avoids the use of independent coordinates was proposed in Müller (2011) where the motion equations are expressed in terms of n redundant coordinates. The idea is to eliminate the Lagrange multipliers from (2) by means of a projector to the null-space of $\mathbf{J}$. That is, instead of premultiplication with $\mathbf{F}^T$, (2) is premultiplied with a null-space projector determined from the pseudoinverse of $\mathbf{J}$. As long as the PKM does not encounter c-space singularities, where rank $\mathbf{J}$ drops, the constraint Jacobian $\mathbf{J}$ is always full rank r, and its right pseudoinverse is given by $\mathbf{J}^+ = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1}$. The corresponding projector to the null-space of $\mathbf{J}$ is then $\mathbf{N}_\mathbf{J} = \mathbf{I}_n - \mathbf{J}^+\mathbf{J}$. This projector does not

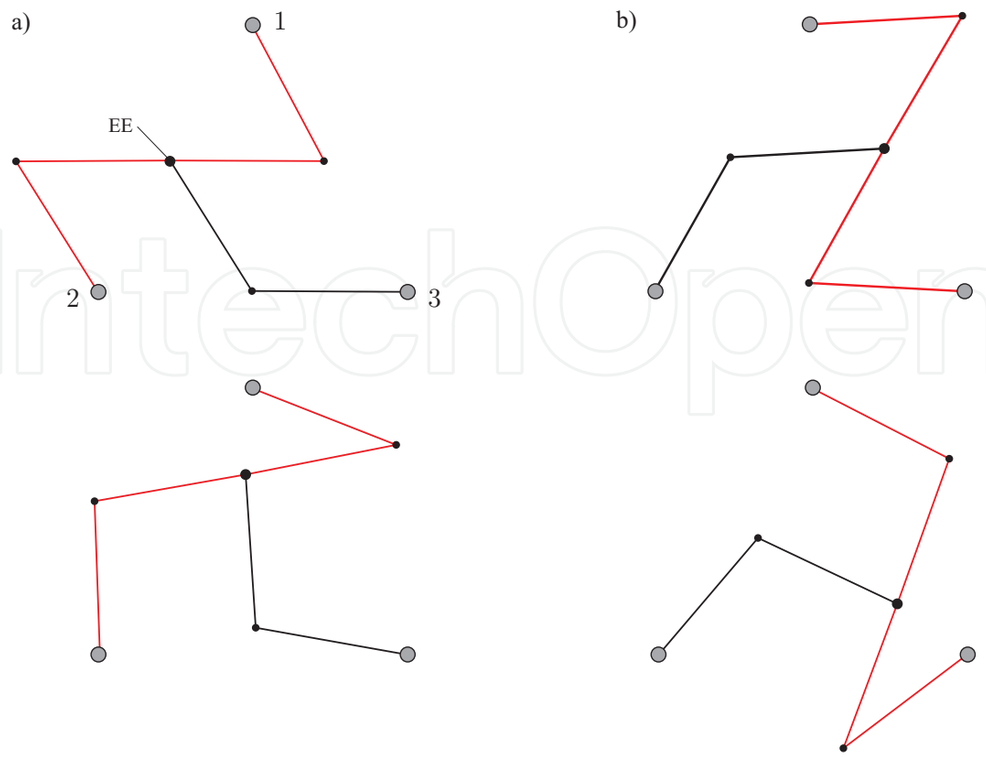

Fig. 8. Different parameterization singularities if the 2 DOF planer PKM is non-redundantly actuated. The mechanism shown in color is the equivalent non-redundantly actuated mechanisms being instantaneously in an input-singularity.

require the selection of minimal coordinates, it is a rank deficient $n \times n$ matrix with constant rank $\delta = n - r$, and $\mathbf{N_J} = \mathbf{N_J}^T = \mathbf{N_J}\mathbf{N_J}$.
As with the orthogonal complement the null-space projector must be partitioned according to the coordinates of the passive and actuated joints. To this end the coordinate vector is rearranged as $\mathbf{q} = (\mathbf{q}_\mathrm{p}, \mathbf{q}_\mathrm{a})$, Denote with $\mathbf{q}_\mathrm{p}$ and $\mathbf{q}_\mathrm{a}$ the vector consisting of the m coordinates of passive joints and δ coordinates of actuator joints, respectively. With the assumption that the PKM configuration is determined by the m actuator coordinates, δ actuator coordinates serve as minimal coordinates, so that $\mathbf{q}_\mathrm{a} = (..., \mathbf{q}_2)$. Then the projector can be partitioned as

$$\mathbf{N_J} = \begin{pmatrix} \widetilde{\mathbf{P}} \\ \widetilde{\mathbf{A}} \end{pmatrix} \begin{matrix} - \ (n-m) \times n \\ - \ m \times n \end{matrix} \tag{21}$$

where $\widetilde{\mathbf{A}}$ corresponds to the actuator coordinates so that $\dot{\mathbf{q}}_\mathrm{a} = \widetilde{\mathbf{A}}\dot{\mathbf{q}}$. Premultiplication of the motion equations (2) with $\mathbf{N_J}^T = \mathbf{N_J}$ and $\mathbf{J}\mathbf{N_J} = \mathbf{0}$ yields a system of n motion equations

$$\mathbf{N_J}(\mathbf{G}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}, t)) = \widetilde{\mathbf{A}}^T\mathbf{c}. \tag{22}$$

Only $\delta = n - r$ of these n equations are independent, however. The $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ in (22) must satisfy the constraints. If this cannot be ensured, in particular if $\mathbf{q}$ are determined from measured values, they must be projected according to $\dot{\mathbf{q}}_\mathrm{proj} = \mathbf{N_J}\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}_\mathrm{proj} = \mathbf{N_J}\ddot{\mathbf{q}} + \dot{\mathbf{N}}_\mathbf{J}\dot{\mathbf{q}}$. This leads

to

$$\widetilde{\mathbf{G}}\left(\mathbf{q}\right)\ddot{\mathbf{q}}+\widetilde{\mathbf{C}}\left(\mathbf{q},\dot{\mathbf{q}}\right)\dot{\mathbf{q}}+\widetilde{\mathbf{Q}}\left(\mathbf{q}\right)=\widetilde{\mathbf{A}}^{T}\mathbf{c} \tag{23}$$

with

$$\widetilde{\mathbf{G}}:=\mathbf{N}_{\mathbf{J}}^{T}\mathbf{G}\mathbf{N}_{\mathbf{J}},\ \widetilde{\mathbf{C}}:=\mathbf{N}_{\mathbf{J}}^{T}\left(\mathbf{C}\mathbf{N}_{\mathbf{J}}+\mathbf{G}\dot{\mathbf{N}}_{\mathbf{J}}\right),\ \widetilde{\mathbf{Q}}:=\mathbf{N}_{\mathbf{J}}^{T}\mathbf{Q}. \tag{24}$$

The time derivative in (21) is readily found to be $\dot{\mathbf{N}}_{\mathbf{J}}=-\left(\mathbf{B}^{+}\dot{\mathbf{B}}\mathbf{N}_{\mathbf{J}}\right)-\left(\mathbf{B}^{+}\dot{\mathbf{B}}\mathbf{N}_{\mathbf{J}}\right)^{T}$.
The dynamics model (22) in redundant coordinates is globally valid in all regular configurations as it does not involve any minimal coordinates.

4.4 Model-based control in redundant coordinates

4.4.1 Inverse dynamics

In order to use the dynamics formulation in redundant coordinates within a model-based controller (22), respectively (23), must be solved for $\mathbf{c}$. The general inverse dynamics solution (neglecting prestress forces) is

$$\mathbf{c}=\left(\widetilde{\mathbf{A}}^{T}\right)^{+}\mathbf{N}_{\mathbf{J}}(\mathbf{G}\left(\mathbf{q}\right)\ddot{\mathbf{q}}+\mathbf{C}\left(\mathbf{q},\dot{\mathbf{q}}\right)\dot{\mathbf{q}}+\mathbf{Q}\left(\mathbf{q},\dot{\mathbf{q}},t\right)). \tag{25}$$

The crucial point here is the computation of the pseudoinverse. The $n\times m$ matrix $\widetilde{\mathbf{A}}^{T}$ is not regular since rank $\widetilde{\mathbf{A}}^{T}=\delta<m<n$. Hence the closed form Moore-Penrose pseudoinverse is not applicable. The singular value decomposition (SVD) can always be used to iteratively determine the pseudoinverse. This is generally not applicable for real-time applications due to its numerical complexity. Now if the redundantly actuated PKM does not poses input-singularities, the PKM configuration is uniquely determined by the $m>\delta$ input coordinates. Hence at any time rank $\widetilde{\mathbf{A}}=\delta$, and a full-rank $\delta\times n$ submatrix $\widetilde{\mathbf{A}}_1$ can be separated so that

$$\widetilde{\mathbf{A}}=\begin{pmatrix}\widetilde{\mathbf{A}}_1\\ \widetilde{\mathbf{A}}_2\end{pmatrix} \tag{26}$$

with the remaining $(m-\delta)\times n$ matrix $\widetilde{\mathbf{A}}_2$. Upon this partitioning the following explicit expression for the pseudoinverse was presented in Müller (2011):

$$\left(\widetilde{\mathbf{A}}^{T}\right)^{+}=\begin{pmatrix}\left(\widetilde{\mathbf{A}}_1^{T}\right)^{+}\left(\mathbf{I}_n-\widetilde{\mathbf{A}}_2^{T}\left(\mathbf{I}_{m-\delta}+\mathbf{B}^{T}\mathbf{B}\right)^{-1}\mathbf{B}^{T}(\widetilde{\mathbf{A}}_1^{T})^{+}\right)\\ \left(\mathbf{I}_{m-\delta}+\mathbf{B}^{T}\mathbf{B}\right)^{-1}\mathbf{B}^{T}\left(\widetilde{\mathbf{A}}_1^{T}\right)^{+}\end{pmatrix} \tag{27}$$

with $\mathbf{B}=\left(\widetilde{\mathbf{A}}_1^{T}\right)^{+}\widetilde{\mathbf{A}}_2^{T}$ and the left pseudoinverse $\left(\widetilde{\mathbf{A}}_1^{T}\right)^{+}=\left(\widetilde{\mathbf{A}}_1\widetilde{\mathbf{A}}_1^{T}\right)^{-1}\widetilde{\mathbf{A}}_1$.
The partitioning (26) is equivalent to selecting δ independent coordinates. As already discussed such a selection, and thus the submatrix $\widetilde{\mathbf{A}}_1$, is not unique, which rises again the problem of selecting δ independent coordinates out of the m actuator coordinates. Consequently the full-rank $\delta\times n$ matrix $\widetilde{\mathbf{A}}_1$ must be selected depending on the actual PKM pose. The important difference to the minimal coordinate formulation is that only the submatrix $\widetilde{\mathbf{A}}$ must be selected whereas the motion equations (22) and (23) are globally valid and remain unaltered in the entire motion range. Switching is only performed within the pseudoinverse computation for $\widetilde{\mathbf{A}}^{T}$. The selection of a proper submatrix requires monitoring the rank of the $\delta\times\delta$ matrix $\widetilde{\mathbf{A}}_1\widetilde{\mathbf{A}}_1^{T}$.

4.4.2 Augmented PD control

The control task is to minimize the tracking error of the actuator coordinates given a target trajectory with $\mathbf{q}_{\mathrm{a}}^{\mathrm{d}}(t)$. With the above formulation it is straightforward to introduce the following APD control scheme

$$\begin{aligned}\mathbf{c} &= \left(\widetilde{\mathbf{A}}^T\right)^+ \left(\widetilde{\mathbf{G}}(\mathbf{q})\ddot{\mathbf{q}}^{\mathrm{d}} + \widetilde{\mathbf{C}}(\dot{\mathbf{q}},\mathbf{q})\dot{\mathbf{q}}^{\mathrm{d}} + \widetilde{\mathbf{Q}}(\mathbf{q},\dot{\mathbf{q}},t) - \mathbf{K}_{\mathrm{P}}\mathbf{e} - \mathbf{K}_{\mathrm{D}}\dot{\mathbf{e}}\right) \\ &= \left(\widetilde{\mathbf{A}}^T\right)^+ \mathbf{N}_{\mathbf{J}}(\mathbf{G}(\mathbf{q})\ddot{\mathbf{q}}^{\mathrm{d}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}^{\mathrm{d}} + \mathbf{Q}(\mathbf{q},\dot{\mathbf{q}},t) - \mathbf{K}_{\mathrm{P}}\mathbf{e} - \mathbf{K}_{\mathrm{D}}\dot{\mathbf{e}})\end{aligned} \tag{28}$$

with error vector $\mathbf{e} := \mathbf{q} - \mathbf{q}^{\mathrm{d}}$. Now the gain matrices measure the errors in the m actuator coordinates. That is, assuming the coordinate partitioning $\mathbf{q} \equiv (\mathbf{q}_{\mathrm{p}}, \mathbf{q}_{\mathrm{a}})$, they have the form

$$\mathbf{K} = \mathrm{diag}\,(0,\ldots,0,K_1,\ldots,K_m)\,. \tag{29}$$

The closed loop dynamics, when (28) is applied to the model (23), is governed by

$$\widetilde{\mathbf{G}}\ddot{\mathbf{q}} - \mathbf{D}\widetilde{\mathbf{G}}\ddot{\mathbf{q}}^{\mathrm{d}} + \widetilde{\mathbf{C}}\dot{\mathbf{q}} - \mathbf{D}\widetilde{\mathbf{C}}\dot{\mathbf{q}}^{\mathrm{d}} + \widetilde{\mathbf{Q}} - \mathbf{D}\widetilde{\mathbf{Q}} + \mathbf{D}\mathbf{K}_{\mathrm{D}}\dot{\mathbf{e}} + \mathbf{D}\mathbf{K}_{\mathrm{P}}\mathbf{e} = \mathbf{0} \tag{30}$$

with $\mathbf{D} = \widetilde{\mathbf{A}}^T\left(\widetilde{\mathbf{A}}^T\right)^+ \neq \mathbf{I}_n$. The rank deficiency of $\widetilde{\mathbf{A}}$ implies that $\mathbf{D} \neq \mathbf{I}_n$.
It can be shown that the APD control scheme (28) achieves exponential trajectory tracking on the c-space V using the Lyapunov function

$$\mathcal{V}(\dot{\mathbf{e}},\mathbf{e},t) = \frac{1}{2}\dot{\mathbf{e}}^T\widetilde{\mathbf{G}}(\mathbf{q})\dot{\mathbf{e}} + \frac{1}{2}\mathbf{e}^T\mathbf{K}_{\mathrm{P}}\mathbf{e} + \frac{1}{2}\varepsilon\mathbf{e}^T\widetilde{\mathbf{G}}(\mathbf{q})\dot{\mathbf{e}} \tag{31}$$

with $\varepsilon > 0$. $\mathcal{V}(\dot{\mathbf{e}},\mathbf{e},t)$ is positive definite, and $\dot{\mathcal{V}}$ is negative definite for all trajectories in V. These properties are directly inherited from the minimal coordinate formulation via the projection onto V with the projector $\mathbf{N}_{\mathbf{J}}$.

4.4.3 Computed torque control

The standard CTC scheme is easily adapted to the redundant coordinate formulation as

$$\begin{aligned}\mathbf{c} &= \left(\widetilde{\mathbf{A}}^T\right)^+ \left(\widetilde{\mathbf{G}}(\mathbf{q})\mathbf{v} + \widetilde{\mathbf{C}}(\dot{\mathbf{q}},\mathbf{q})\dot{\mathbf{q}} + \widetilde{\mathbf{Q}}(\mathbf{q},\dot{\mathbf{q}},t)\right) \\ &= \left(\widetilde{\mathbf{A}}^T\right)^+ \mathbf{N}_{\mathbf{J}}\left(\mathbf{G}(\mathbf{q})\mathbf{v} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q},\dot{\mathbf{q}},t)\right)\end{aligned} \tag{32}$$

with $\mathbf{v} = \ddot{\mathbf{q}} - \mathbf{K}_{\mathrm{P}}\mathbf{e} - \mathbf{K}_{\mathrm{D}}\dot{\mathbf{e}}$ and the gain matrices in (29). It is readily shown that the CTC (32) achieves exponential trajectory tracking on V. Choose local coordinates $\mathbf{q}_2$ on V, and let $\mathbf{P}_1$ be a projector to the vector space of dependent velocities $\dot{\mathbf{q}}_1$, and $\mathbf{P}_2$ a projector to the vector space of independent velocities $\dot{\mathbf{q}}_2$. Then the dynamics of the closed loop splits into

$$\mathbf{P}_1\left(\widetilde{\mathbf{G}}\ddot{\mathbf{q}} + \widetilde{\mathbf{C}}\dot{\mathbf{q}} + \widetilde{\mathbf{Q}}\right) = \mathbf{0} \tag{33}$$

$$\mathbf{P}_2\widetilde{\mathbf{G}}\left(\ddot{\mathbf{e}} + \mathbf{K}_{\mathrm{D}}\dot{\mathbf{e}} + \mathbf{K}_{\mathrm{P}}\mathbf{e}\right) = \mathbf{0}. \tag{34}$$

The first $n - \delta$ equations are automatically satisfied for trajectories in V if the second system is satisfied. The second system, consisting of δ equations, governs the error dynamics in term of

independent coordinates. For trajectories in V this is equivalent to the system of δ equations

$$\overline{\mathbf{G}}\left(\ddot{\mathbf{e}}_2 + \mathbf{F}^T\mathbf{K}_\mathrm{D}\mathbf{F}\dot{\mathbf{e}}_2 + \mathbf{F}^T\mathbf{K}_\mathrm{P}\mathbf{F}\mathbf{e}_2\right) = \mathbf{0} \tag{35}$$

with positive definite $\overline{\mathbf{G}} = \mathbf{F}^T\widetilde{\mathbf{G}}\mathbf{F}$ in (7). The stability of the controller is ensured with positive definite gains.

4.5 Case study: Planar 2 DOF RA-PKM

The proposed CTC control scheme in redundant coordinates was implemented in a prototype of the planar 2 DOF PKM in figure 1, which is briefly described in section 3.3. As discussed in section 2.1 the dynamic model is given in terms of $n = 6$ joint angles, giving rise to a system of $n = 6$ motion equations (22). The projected 6×3 control matrix $\widetilde{M}$ has rank 2.
The manipulator is controlled along the EE-path in figure 4. If two actuator joint angles are used as independent coordinates the minimal coordinate model (6) exhibits parameterization singularities as described in section 4.2. Moreover the EE-path passes such singularities several times and the minimal coordinate model is not valid. The redundant coordinate formulation does not suffer from such singularities.

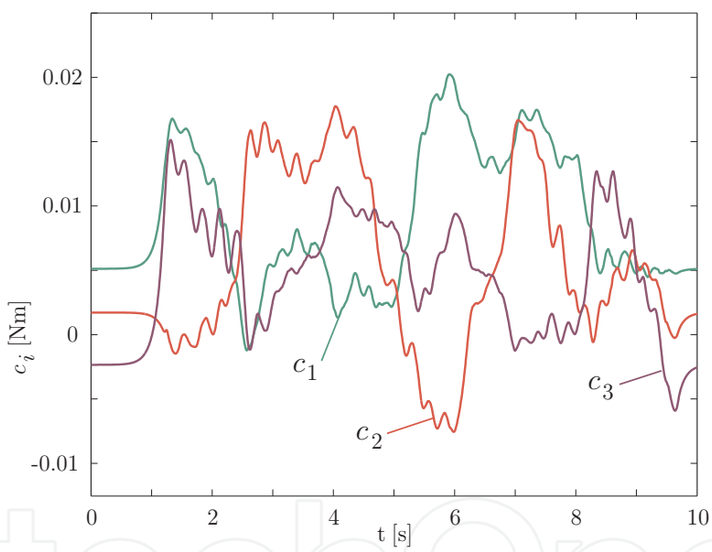

Fig. 9. Actuator torques when the RA-PKM is controlled along the EE-path of figure 4 by a CTC model in terms of redundant coordinates.

Figure 9 shows the actuator torques when the RA-PKM is controlled by the CTC scheme (32). Figure 10 shows the corresponding actuator tracking errors. Apparently the motion and the torque evolution is smooth and unaffected by any singularities thanks to the redundant coordinate formulation. Notice that also for this CTC method there are non-zero drive torques even if the RA-PKM is at rest. This is again due to measurement errors in conjunction with actuation redundancy. The crucial point in the inverse dynamics formulation (25) is the computation of the pseudoinverse (27). This requires identification of a full rank submatrix $\widetilde{\mathbf{A}}_1$, i.e. δ actuator coordinates representing valid local coordinates on V. A straightforward implementation is to select one of the three combinations $\mathbf{q}_2^{(1)} = (q^1, q^2)$, $\mathbf{q}_2^{(2)} = (q^1, q^3)$, and

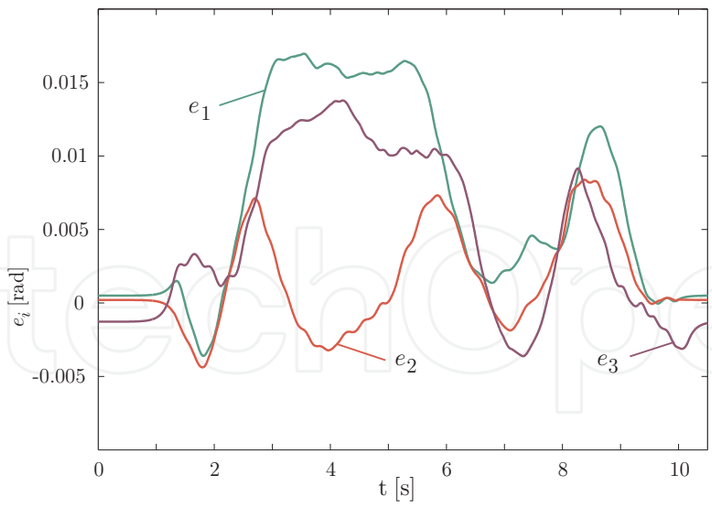

Fig. 10. Joint tracking error when the RA-PKM is controlled by a CTC model in redundant coordinates.

$\mathbf{q}_2^{(3)} = (q^2, q^3)$ based on the infinity norm of the corresponding $\widetilde{\mathbf{A}}_1$. That is, the submatrix $\widetilde{\mathbf{A}}_1$, corresponding to a certain $\mathbf{q}_2^{(i)}$, is selected for which $||\widetilde{\mathbf{A}}_1||_\infty$ is minimal.

5. Summary

Redundant actuation has the potential to improve the kinematic and dynamic performance of PKM. This redundancy is easily taken into account within the design process so to optimize dexterity. It turns out, however, that the control of RA-PKM poses several challenges. A problem that is peculiar to RA-PKM is the existence of antagonistic control forces. Such forces can be employed purposefully to avoid backlash or to modulate the EE stiffness, but impair the performance and stability of decentralized control schemes for RA-PKM. In this chapter the applicability of decentralized control schemes is analyzed, and it is shown that they inherently cause antagonistic control forces. As remedy a so-called antagonism filter is proposed that eliminates antagonistic control forces. Another problem arises in the model-based control of RA-PKM. Since the dynamics model is commonly formulated in terms of a set of independent actuator coordinates the model of the RA-PKM becomes invalid at the input singularities of the non-redundantly actuated PKM. This would limit the controllable motion range to that of the non-redundant PKM. To overcome this problem a formulation in redundant coordinates was presented that does not require the selection of independent actuator coordinates. This formulation is valid in the entire motion range. Thereupon an augmented PD and computed torque controller was proposed. Experimental results are reported for the planar 2 DOF RA-PKM that confirmed the robustness of these control schemes.

6. Acknowledgment

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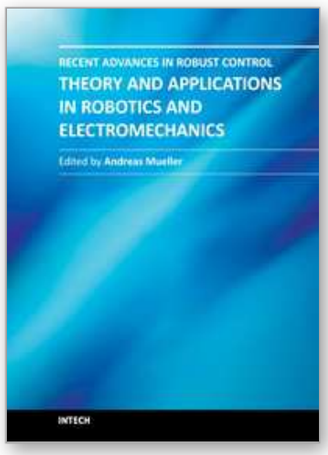

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Robust control has been a topic of active research in the last three decades culminating in H_2/H_\infty and \mu design methods followed by research on parametric robustness, initially motivated by Kharitonov's theorem, the extension to non-linear time delay systems, and other more recent methods. The two volumes of Recent Advances in Robust Control give a selective overview of recent theoretical developments and present selected application examples. The volumes comprise 39 contributions covering various theoretical aspects as well as different application areas. The first volume covers selected problems in the theory of robust control and its application to robotic and electromechanical systems. The second volume is dedicated to special topics in robust control and problem specific solutions. Recent Advances in Robust Control will be a valuable reference for those interested in the recent theoretical advances and for researchers working in the broad field of robotics and mechatronics.

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