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# Numerical Approach to Performance Analysis of Multi-Parametric CAC in Multi-Service Wireless Networks

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## 1. Introduction

Cellular wireless network (CWN) consists of radio access points, called base stations (BS), each covering certain geographic area. With the distance power of radio signals fade away (fading or attenuation of signal occurs) which makes possible to use same frequencies over several cells, but in order to avoid interference, this process must be carefully planned. For better use of frequency recourse, existing carrier frequencies are grouped, and number of cells, in which this group of frequencies is used, defines so called frequency reuse factor. Therefore, in densely populated areas with large number of mobile subscribers (MS) small dimensioned cells (micro-cells and pico-cells) are to be used, because of limitations of volumes and frequency reuse factor.

In connection with limitation of transmission spectrum in CWN, problems of allocation of common spectrum among cells are very important. Unit of wireless spectrum, necessary for serving single user is called channel (for instance, time slots in TDMA are considered as channels). There are three solutions for channels allocation problem: Fixed Channel Allocation (FCA), Dynamic Channel Allocation (DCA) and Hybrid Channel Allocation (HCA). Advantages and disadvantage of each of these are well known. At the same time, owing to realization simplicity, FCA scheme is widely used in existing cellular networks. In this paper models with FCA schemes are considered.

Quality of service (QoS) in the certain cell with FCA scheme could be improved by using the effective call admission control (CAC) strategies for the heterogeneous traffics, e.g. see [1]-[3]. Use of such access strategy doesn't require much resource, therefore this method could be considered operative and more defensible for solution of resource shortage problem.

Apart from original (or new) calls (o-calls) flows additional classes of calls that require special approach also exist in wireless cellular networks. These are so-called handover calls (h-calls). This is specific only for wireless cellular networks. The essence of this phenomenon is that moving MS, that already established connection with network, passes boundaries between cells and gets served by new cell. From a new cell's point of view this is h-call, and since the connection with MS has already established, MS handling transfer to new cell must be transparent for user. In other words, in wireless networks the call may occupy channels

from different cells several times during call duration, which means that channel occupation period is not the same as call duration.

Mathematical models of call handling processes in multi-service CWN can be developed adequately enough based on theory of networks of queue with different type of calls and random topology. Such models are researched poorly in literature, e.g. see [4]-[6]. This is explained by the fact, that despite elegance of those models, in practice they are useful only for small dimensional networks and with some limiting simplifying assumptions that are contrary to fact in real functioning wireless networks. In connection with that, in majority of research works models of an isolated cell are analyzed.

In the overwhelming majority of available works one-dimensional (1-D) queuing models of call handling processes in an isolated cell of mono-service CWN are proposed. However these models can not describe studying processes in multi-service CWN since in such networks calls of heterogeneous traffics are quite differ with respect to their bandwidth requirement and arrival rate and channel occupancy time. In connection with that in the given paper two-dimensional (2-D) queuing models of multi-service networks are developed.

In order to be specific we consider integrated voice/data CWN. In such networks real time voice calls (v-calls) are more susceptible to possible losses and delays than non-real time data (original or handover) call (d-calls). That is why a number of different CAC strategies for prioritization of v-calls are suggested in various works, mostly implying use of guard channels (or cutoff strategy) for high priority calls [7], [8] and/or threshold strategies [9] which restrict the number of low priority calls in channels.

In this paper we introduce a unified approach to approximate performance analysis of two multi-parametric CAC in a single cell of un-buffered integrated voice/data CWN which differs from known works in this area. Our approach is based on the principles of theory of phase merging of stochastic systems [10].

The proposed approach allows overcoming an assumption made in almost all of the known papers about equality of handling intensities of heterogeneous calls. Due to this assumption the functioning of the CWN is described with one-dimensional Markov chain (1-D MC) and authors managed simple formulas for calculating the QoS metrics of the system. However as it was mentioned in [11] (pages 267-268) and [12] the assumption of the same mean channel occupancy time even for both original and handover calls of the same class traffic is unrealistic. The presented models are more general in terms of handling intensities and the equality is no longer required.

This paper is organized as follows. In Section 2, we provide a simple algorithm to calculate approximate values of desired QoS metrics of the model of integrated voice/data networks under CAC based on guard channels strategy. Similar algorithm is suggested in Section 3 for the same model under CAC based on threshold strategy. In Section 4, we give results of numerical experiments which indicate high accuracy of proposed approximate algorithms as well as comparison of QoS metrics in different CAC strategies. In Section 5 we provide some conclusion remarks.

## **2. The CAC based on guard channels strategy**

It is undisguised that in an integrating voice/data CWN voice calls of any type (original or handover) have high priority over data calls and within of each flow handover calls have high priority over original calls.

As a means of assigning priorities to handover v-calls (hv-call) in such networks a back-up scheme that involves reserving a particular number of guard channels of a cell expressly for calls of this type are often utilized. According to this scheme any hv-call is accepted if there exists at least one free channel, while calls of remain kind are accepted only when the number of busy channels does not exceed some class-dependent threshold value.

We consider a model of an isolated cell in an integrated voice/data CWN without queues. This cell contains  $N$  channels,  $1 < N < \infty$ . These channels are used by Poisson flows of hv-calls, original v-calls (ov-calls), handover d-calls (hd-calls) and original d-calls (od-calls). Intensity of  $x$ -calls is  $\lambda_x$ ,  $x \in \{ov, hv, od, hd\}$ . As in almost all cited works the values of handover intensities are considered known hereinafter. Although it is apparent that definition of their values depending on intensity of original calls, shape of a cell, mobility of an MS and etc. is rather challenging and complex. However, if we consider the case of a uniform traffic distribution and at most one handover per call, the average handover intensity can be given by the ratio of the average call holding time to the average cell sojourn time [13].

For handle of any narrow-band v-call (either original or handover) one free channel is required only while one wide-band d-call (either original or handover) require simultaneously  $b \geq 1$  channels. Here it is assumed that wide-band d-calls are inelastic, i.e. all  $b$  channels are occupied and released simultaneously (though can be investigated and models with elastic d-calls).

Note that the channels occupancy time considers the both components of occupancy time: the time of calls duration and their mobility. Distribution functions of channel occupancy time of heterogeneous calls are assumed be independent and exponential, but their parameters are different, namely intensity of handling of voice (data) calls equals  $\mu_v$  ( $\mu_d$ ), and generally speaking  $\mu_v \neq \mu_d$ . If during call handling handover procedure is initiated, the remaining handling time of this call in a new cell (yet as an h-call) is also exponentially distributed with the same mean due to memoryless property of exponential distribution.

In a given CAC the procedure by which the channels are engaged by calls of different types is realized in the following way. As it was mentioned before, if upon arrival of an hv-call, there is at least one free channel, this call seizes one of free channels; otherwise this call is blocked. With the purpose of definition of proposed CAC for calls of other types three parameters  $N_1$ ,  $N_2$  and  $N_3$  where  $1 \leq N_1 \leq N_2 \leq N_3 \leq N$  are introduced. It is assumed that  $N_1$  and  $N_2$  are multiples of  $b$ .

Arrived ov-call is accepted if the number of busy channels is less than  $N_3$ , otherwise it is blocked. Arrived od-call (respectively, hd-call) is accepted only in the case at most  $N_1 - b$  (respectively,  $N_2 - b$ ) busy channels, otherwise it is blocked.

Consider the problem of finding the major QoS metrics of the given multi-parametric CAC strategy - blocking (loss) probabilities of calls of each type and overall channels utilization. For simplicity of intermediate mathematical transformations first we shall assume that  $b = 1$ . The case  $b > 1$  is straightforward (see below).

By adopting an assumption for the type of distribution laws governing the incoming traffics and their holding times it becomes possible to describe the operation of an isolated cell by means of a two-dimensional Markov chain (2-D MC), i.e. in a stationary regime the state of the cell at an arbitrary moment of time is described by a 2-D vector  $\mathbf{n} = (n_d, n_v)$ , where  $n_d$  (respectively,  $n_v$ ) is the number of data (respectively, voice) calls in the channels. Then the state space of the corresponding Markov chain describing this call handling scheme is defined thus:

$$S := \left\{ \mathbf{n} : n_d = \overline{0, N_2}, n_v = \overline{0, N}, n_d + n_v \leq N \right\}. \quad (2.1)$$

Elements of generating matrix of this MC  $q(\mathbf{n}, \mathbf{n}'), \mathbf{n}, \mathbf{n}' \in S$  are determined from the following relations:

$$q(\mathbf{n}, \mathbf{n}') = \begin{cases} \lambda_d & \text{if } n_d + n_v \leq N_1 - 1, \mathbf{n}' = \mathbf{n} + \mathbf{e}_1, \\ \lambda_{hd} & \text{if } N_1 \leq n_d + n_v \leq N_2 - 1, \mathbf{n}' = \mathbf{n} + \mathbf{e}_1, \\ \lambda_v & \text{if } n_d + n_v \leq N_3 - 1, \mathbf{n}' = \mathbf{n} + \mathbf{e}_2, \\ \lambda_{hv} & \text{if } N_3 \leq n_d + n_v \leq N - 1, \mathbf{n}' = \mathbf{n} + \mathbf{e}_2, \\ n_d \mu_d & \text{if } \mathbf{n}' = \mathbf{n} - \mathbf{e}_1, \\ n_v \mu_v & \text{if } \mathbf{n}' = \mathbf{n} - \mathbf{e}_2, \\ 0 & \text{in other cases,} \end{cases} \quad (2.2)$$

where  $\lambda_d := \lambda_{od} + \lambda_{hd}$ ,  $\lambda_v := \lambda_{ov} + \lambda_{hv}$ ,  $\mathbf{e}_1 = (1, 0)$ ,  $\mathbf{e}_2 = (0, 1)$ .

State diagram of the model and the system of global balance equations (SGBE) for the steady state probabilities  $p(\mathbf{n})$ ,  $\mathbf{n} \in S$  are shown in [14]. Existence of stationary regime is proved by the fact that all states of finite-dimensional state space  $S$  are communicating.

Desired QoS metrics are determined via stationary distribution of initial model. Let  $P_x$  denote the blocking probability of the  $x$ -calls,  $x \in \{hv, ov, hd, od\}$ . Then by using PASTA theorem [15] we obtain:

$$P_{hv} := \sum_{\mathbf{n} \in S} p(\mathbf{n}) \delta(n_d + n_v, N), \quad (2.3)$$

$$P_{ov} := \sum_{\mathbf{n} \in S} p(\mathbf{n}) I(n_d + n_v \geq N_3), \quad (2.4)$$

$$P_{hd} := \sum_{\mathbf{n} \in S} p(\mathbf{n}) I(n_d + n_v \geq N_2), \quad (2.5)$$

$$P_{od} := \sum_{\mathbf{n} \in S} p(\mathbf{n}) I(n_d + n_v \geq N_1), \quad (2.6)$$

where  $I(A)$  denoted the indicator function of event  $A$  and  $\delta(i, j)$  represents Kronecker's symbols.

The mean number of busy channels  $\tilde{N}$  is also calculated via stationary distribution as follows:

$$\tilde{N} := \sum_{k=1}^N kp(k), \quad (2.7)$$

where  $p(k) = \sum_{\mathbf{n} \in S} p(\mathbf{n}) \delta(n_d + n_v, k)$ ,  $k = \overline{1, N}$ , are marginal probability mass functions.

Stationary distribution is determined as a result of solution of an appropriate SGBE of the given 2-D MC. However, to solve the last problem one requires laborious computation efforts for large values of  $N$  since the corresponding SGBE has no explicit solution. Very

often the solution of such problems is evident if the corresponding 2-D MC has reversibility property [16] and hence for it there exists stationary distribution of multiplicative form. Given SGBE has a multiplicative solution only in a special case when  $N_1=N_2=N_3=N$  (even in this case there are known computational difficulties). However, by applying Kolmogorov criteria [16] it is easily verified that the given 2-D MC is not reversible. Indeed, according to mentioned criteria the necessary reversibility condition of 2-D MC consists in the fact that if there exists the transition from state  $(i,j)$  into state  $(i',j')$ , then there must also be the reverse transition from the state  $(i',j')$  to the state  $(i,j)$ . However, for MC considered this condition is not fulfilled. So by the relations (2.2) in the given MC there exists transition  $(n_d, n_v) \rightarrow (n_d-1, n_v)$  with intensity  $n_d \mu_d$  where  $n_d+n_v \geq N_2$ , but the inverse transition not existing.

In [14] a recursive technique has been proposed for solution of mentioned above SGBE. It requires multiple inversion calculation of certain matrices of sufficiently large dimensions that in itself is complex calculating procedure. To overcome the mentioned difficulties, new efficient and refined approximate method for calculation of stationary distribution of the given model is suggested below. The proposed method, due to right selection of state space splitting of corresponding 2-D MC allows one to reduce the solution of the problem considered to calculation by explicit formulae which contain the known (even tabulated) stationary distributions of classical queuing models.

For correct application of phase merging algorithms (PMA) it is assumed below that  $\lambda_v \gg \lambda_d$  and  $\mu_v \gg \mu_d$ . This assumption is not extraordinary for an integrating voice/data CWN, since this is a regime that commonly occurs in multimedia networks, in which wideband d-calls have both longer holding times and significantly smaller arrival rates than narrowband v-calls, e.g. see [17, 18]. Moreover, it is more important to note, that shown below final results are independent of traffic parameters, and are determined from their ratio, i.e. the developed approach can provide a refined approximation even when parameters of heterogeneous traffics are only moderately distinctive.

The following splitting of state space (2.1) is examined:

$$S = \bigcup_{k=0}^{N_2} S_k, \quad S_k \cap S_{k'} = \emptyset, \quad k \neq k', \quad (2.8)$$

where  $S_k := \{\mathbf{n} \in S : n_d = k\}$ .

*Note 1.* The assumption above meets the major requirement for correct use of PMA [10]: state space of the initial model must split into classes, such that transition probabilities within classes are essentially higher than those between states of different classes. Indeed, it is seen from (2.2) that the above mentioned requirement is fulfilled when using splitting (2.8).

Further state classes  $S_k$  combine into separate merged states  $\langle k \rangle$  and the following merging function in state space  $S$  is introduced:

$$U(\mathbf{n}) = \langle k \rangle \text{ if } \mathbf{n} \in S_k, k = \overline{0, N_2}. \quad (2.9)$$

Function (2.9) determines merged model which is one-dimensional Markov chain (1-D MC) with the state space  $\tilde{S} := \{\langle k \rangle : k = \overline{0, N_2}\}$ . Then, according to PMA, stationary distribution of the initial model approximately equals:

$$p(k, i) \approx \rho_k(i) \pi(\langle k \rangle), \quad (k, i) \in S_k, \quad k = \overline{0, N_2}, \quad (2.10)$$

where  $\{\rho_k(i) : (k, i) \in S_k\}$  is stationary distribution of a split model with state space  $S_k$  and  $\{\pi(\langle k \rangle) : \langle k \rangle \in \tilde{S}\}$  is stationary distribution of a merged model, respectively.

State diagram of split model with state space  $S_k$  is shown in fig.1, a. By using (2.2) we conclude that the elements of generating matrix of this 1-D birth-death processes (BDP)  $q_k(i, j)$  are obtained as follows:

$$q_k(i, j) = \begin{cases} \lambda_v & \text{if } i \leq N_3 - k - 1, j = i + 1, \\ \lambda_{hv} & \text{if } N_3 - k \leq i < N, j = i + 1, \\ i\mu_v & \text{if } j = i - 1, \\ 0 & \text{in other cases.} \end{cases}$$

So, stationary distribution within class  $S_k$  is same as that  $M|M|N-k|N-k$  queuing system where service rate of each channel is constant,  $\mu_v$  and arrival rates are variable quantities

$$\begin{cases} \lambda_v & \text{if } i < N_3 - k, \\ \lambda_{hv} & \text{if } j \geq N_3 - k. \end{cases}$$

Hence desired stationary distribution is

$$\rho_k(i) = \begin{cases} \frac{v_v^i}{i!} \rho_k(0) & \text{if } 1 \leq i \leq N_3 - k, \\ \left(\frac{v_v}{v_{hv}}\right)^{N_3-k} \frac{v_{hv}^i}{i!} \rho_k(0) & \text{if } N_3 - k + 1 \leq i \leq N - k, \end{cases} \quad (2.11)$$

where

$$\rho_k(0) = \left( \sum_{i=0}^{N_3-k} \frac{v_v^i}{i!} + \left(\frac{v_v}{v_{hv}}\right)^{N_3-k} \sum_{i=N_3-k+1}^{N-k} \frac{v_{hv}^i}{i!} \right)^{-1}, \quad v_v := \lambda_v / \mu_v, v_{hv} := \lambda_{hv} / \mu_v.$$

Then, from (2.2) and (2.11) by means of PMA elements of generating matrix of a merged model  $q(\langle k \rangle, \langle k' \rangle), \langle k \rangle, \langle k' \rangle \in \tilde{S}$  are found:

$$q(\langle k \rangle, \langle k' \rangle) = \begin{cases} \lambda_d \sum_{i=0}^{N_1-k-1} \rho_k(i) + \lambda_{hd} \sum_{i=N_1-1}^{N_2-k-1} \rho_k(i) & \text{if } 0 \leq k \leq N_1 - 1, k' = k + 1, \\ \lambda_{hd} \sum_{i=0}^{N_2-k-1} \rho_k(i) & \text{if } N_1 \leq k \leq N_2 - 1, k' = k + 1, \\ k\mu_d & \text{if } k' = k - 1, \\ 0 & \text{in other cases.} \end{cases} \quad (2.12)$$

The latter formula allows determining stationary distribution of a merged model. It coincides with an appropriate distribution of state probabilities of a 1-D BDP, for which transition intensities are determined in accordance with (2.12). Consequently, stationary distribution of a merged model is determined as (see fig.1, b):

$$\pi(\langle k \rangle) = \frac{\pi(\langle 0 \rangle)}{k! \mu_d^k} \prod_{i=1}^k q(\langle k-1 \rangle, \langle k \rangle), \quad k = \overline{1, N_2}, \quad (2.13)$$

where  $\pi(\langle 0 \rangle) = \left( 1 + \sum_{k=1}^{N_2} \frac{1}{k! \mu_d^k} \prod_{i=1}^k q(\langle k-1 \rangle, \langle k \rangle) \right)^{-1}$ .

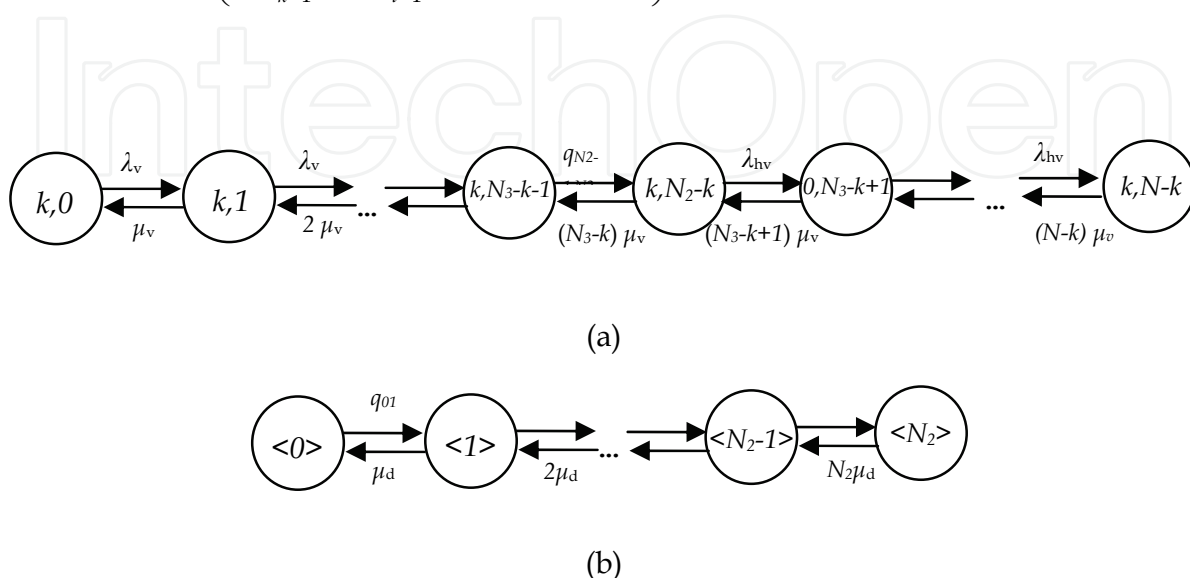


Fig. 1. State diagram of split model with state space  $S_k, k=0,1,\dots,N_2$  (a) and merged model (b).

Then by using (2.11) and (2.13) from (2.10) stationary distribution of the initial 2-D MC can be found. So, summarizing above given and omitting the complex algebraic transformations the following approximate formulae for calculation of QoS metrics (2.3)-(2.7) can be suggested:

$$P_{hv} \approx \sum_{k=0}^{N_2} \pi(\langle k \rangle) \rho_k(N-k); \quad (2.14)$$

$$P_{ov} \approx \sum_{k=0}^{N_2} \pi(\langle k \rangle) \sum_{i=N_3-k}^{N-k} \rho_k(i); \quad (2.15)$$

$$P_{hd} \approx \sum_{k=0}^{N_2} \pi(\langle k \rangle) \sum_{i=N_2-k}^{N-k} \rho_k(i); \quad (2.16)$$

$$P_{od} \approx \sum_{k=0}^{N_1-1} \pi(\langle k \rangle) \sum_{i=N_1-k}^{N-k} \rho_k(i) + \sum_{k=N_1}^{N_2} \pi(\langle k \rangle); \quad (2.17)$$

$$\tilde{N} \approx \sum_{i=1}^N i \sum_{k=0}^{f_{N_2}(i)} \pi(\langle k \rangle) \rho_k(i-k). \quad (2.18)$$



Hereinafter  $f_k(x) = \begin{cases} x & \text{if } 1 \leq x \leq k, \\ k & \text{if } k \leq x \leq N. \end{cases}$

Now we can develop the following algorithm to calculate the QoS metrics of investigated multi-parametric CAC for the similar model with wide-band d-calls, i.e. when  $b > 1$ .

Step 1. For  $k = 0, 1, \dots, [N_2/b]$  calculate the following quantities

$$\rho_k(i) = \begin{cases} \frac{v_v^i}{i!} \rho_k(0) & \text{if } 1 \leq i \leq N_3 - kb, \\ \left(\frac{v_v}{v_{hv}}\right)^{N_3 - kb} \frac{v_{hv}^i}{i!} \rho_k(0) & \text{if } N_3 - kb + 1 \leq i \leq N - kb, \end{cases}$$

$$\text{where } \rho_k(0) = \left( \sum_{i=0}^{N_3 - kb} \frac{v_v^i}{i!} + \left(\frac{v_v}{v_{hv}}\right)^{N_3 - kb} \sum_{i=N_3 - kb + 1}^{N - kb} \frac{v_{hv}^i}{i!} \right)^{-1};$$

$$\pi(< k >) = \frac{\pi(< 0 >)}{k! \mu_d^k} \prod_{i=1}^k q(< k-1 >, < k >),$$

$$\text{where } \pi(< 0 >) = \left( 1 + \sum_{k=1}^{[N_2/b]} \frac{1}{k! \mu_d^k} \prod_{i=1}^k q(< k-1 >, < k >) \right)^{-1},$$

$$q(< k >, < k' >) = \begin{cases} \lambda_d \sum_{i=0}^{N_1 - kb - 1} \rho_k(i) + \lambda_{hd} \sum_{i=N_1 - kb}^{N_2 - kb - 1} \rho_k(i) & \text{if } 0 \leq k \leq [N_1/b] - 1, k' = k + 1, \\ \lambda_{hd} \sum_{i=0}^{N_2 - kb - 1} \rho_k(i) & \text{if } [N_1/b] \leq k \leq [N_2/b] - 1, k' = k + 1, \\ k \mu_d & \text{if } k' = k - 1, \\ 0 & \text{in other cases.} \end{cases}$$

Step 2. Calculate the approximate values of QoS metrics:

$$P_{hv} \approx \sum_{k=0}^{[N_2/b]} \pi(< k >) \rho_k(N - kb);$$

$$P_{ov} \approx \sum_{k=0}^{[N_2/b]} \pi(< k >) \sum_{i=N_3 - kb}^{N - kb} \rho_k(i);$$

$$P_{hd} \approx \sum_{k=0}^{[N_2/b]} \pi(< k >) \sum_{i=N_2 - kb}^{N - kb} \rho_k(i);$$

$$P_{od} \approx \sum_{k=0}^{[N_1/b] - 1} \pi(< k >) \sum_{i=N_1 - kb}^{N - kb} \rho_k(i) + \sum_{k=[N_1/b]}^{[N_2/b]} \pi(< k >);$$

$$\tilde{N} \approx \sum_{i=1}^N i \sum_{k=0}^{f_{\lfloor N_2/b \rfloor}(i)} \pi(< k >) \rho_k(i-k).$$

Henceforth  $[x]$  denote the integer part of  $x$ .

Now consider some important special cases of the investigated multi-parametric CAC (for the sake of simplicity consider case  $b=1$ ).

1. CAC based on Complete Sharing (CS). Under given CAC strategy, no distinction is made between v-calls and d-calls for channel access, i.e. it is assumed that  $N_1=N_2=N_3=N$ . In other words, we have 2-D Erlang's loss model. It is obvious, that in this case blocking probabilities of calls from heterogeneous traffics are equal each other, i.e. this probability according to PASTA theorem coincides with probability of that the arrived call of any type finds all channels of a cell occupied. Then from (2.11)-(2.18) particularly we get the following convolution algorithms for calculation of QoS metrics in the given model:

$$P_{hw} = P_{ov} = P_{hd} = P_{od} \approx \sum_{k=0}^N E_B(v_v, N-k) \pi(< k >), \quad (2.19)$$

$$\tilde{N} \approx \sum_{i=1}^N i \sum_{k=0}^i \theta_{i-k}(v_v, N-k) \pi(< k >) \quad (2.20)$$

Here

$$\pi(< k >) = \frac{v_d^k}{k!} \prod_{i=0}^{k-1} (1 - E_B(v_v, N-i)) \pi(< 0 >), \quad k = \overline{1, N}, \quad (2.21)$$

where  $\pi(< 0 >) = \left( 1 + \sum_{k=1}^N \frac{v_d^k}{k!} \prod_{i=0}^{k-1} (1 - E_B(v_v, N-i)) \right)^{-1}$ .

Henceforth  $E_B(v, m)$  denote the Erlang's B-formula for the model  $M/M/m/m$  with load  $v$  erl, and  $\theta_i(v, m), i=0,1,\dots,m$ , denote the steady state probabilities in the same model, i.e.

$$\theta_i(v, m) = \left( \frac{v^i}{i!} \right) \left( \sum_{j=0}^m \frac{v^j}{j!} \right)^{-1}, \quad i = \overline{0, m}; \quad E_B(v, m) := \theta_m(v, m). \quad (2.22)$$

Note that developed above analytic results for the CS-strategy is similar in spirit to proposed in [18] algorithm for nearly decomposable 2-D MC.

2. CAC with Single Parameter. Given strategy tell the difference between v-calls and d-calls but do not take into account distinctions between original and handover calls within each traffic, i.e. it is assumed that  $N_1=N_2$  and  $N_3=N$  where  $N_2 < N_3$ . For this case from (2.11)-(2.18) we get the following approximate formulae for calculating the blocking probabilities of v-calls ( $P_v$ ) and d-calls ( $P_d$ ) and mean number of busy channels:

$$P_v = P_{hw} = P_{ov} \approx \sum_{k=0}^{N_2} E_B(v_v, N-k) \pi(< k >), \quad (2.23)$$

$$P_d = P_{hd} = P_{od} \approx \sum_{k=0}^{N_2} \pi(< k >) \sum_{i=N_2-k}^{N-k} \theta_i(v_v, N-k), \quad (2.24)$$

$$\tilde{N} \approx \sum_{i=1}^N i \sum_{k=0}^{f_{N_2}(i)} \theta_{i-k}(v_v, N-k) \pi(< k >). \quad (2.25)$$

Here

$$\pi(< k >) = \frac{v_d^k}{k!} \prod_{i=1}^k \Lambda(i) \pi(< 0 >), \quad k = \overline{1, N_2}, \quad (2.26)$$

$$\text{where } \pi(< 0 >) = \left( 1 + \sum_{j=1}^{N_2} \frac{v_d^j}{j!} \prod_{i=1}^j \Lambda(i) \right)^{-1}, \quad \Lambda(i) := \theta_0(v_v, N-i+1) \sum_{j=0}^{N_2-i} \frac{v_v^j}{j!}.$$

3. Mono-service CWN with guard channels. Last results can be interpreted for the model of isolated cell in mono-service CWN with guard channels for h-calls, i.e. for the model in which distinctions between original and handover calls of single traffic is taken into account. Brief description of the model is following. The network supports only the original and handover calls of single traffic that arrive according Poisson processes with rates  $\lambda_o$  and  $\lambda_{hr}$ , respectively. Assume that the o-call (h-call) holding times have an exponential distribution with mean  $\mu_o$  ( $\mu_{hr}$ ) but their parameters are different, i.e. generally speaking  $\mu_o \neq \mu_{hr}$ , see [11] and [12].

In a cell mentioned one-parametric CAC strategy based on guard channels scheme is realized in the following way [19]. If upon arrival of an h-call, there is at least one free channel, this call seizes one of free channels; otherwise h-call is dropped. Arrived o-call is accepted only in the case at least  $g+1$  free channels (i.e. at most  $N-g-1$  busy channels), otherwise o-call is blocked. Here  $g \geq 0$  denotes the number of guard channels that are reserved only for h-calls.

By using the described above approach and omitting the known intermediate transformations we conclude that QoS metrics of the given model are calculated as follows:

$$P_o \approx \sum_{k=0}^{N-g} \pi(< k >) \sum_{i=N-g-k}^{N-k} \theta_i(v_h, N-k), \quad (2.27)$$

$$P_h \approx \sum_{k=0}^{N-g} E_B(v_h, N-k) \pi(< k >), \quad (2.28)$$

$$\tilde{N} \approx \sum_{i=1}^N i \sum_{k=0}^{f_{N-g}(i)} \theta_{i-k}(v_h, N-k) \pi(< k >). \quad (2.29)$$

Here

$$\pi(< k >) = \frac{v_o^k}{k!} \prod_{i=1}^k \Lambda(i) \pi(< 0 >), \quad k = \overline{1, N-g}, \quad (2.30)$$

where  $\nu_o = \lambda_o / \mu_o$ ,  $\nu_h = \lambda_h / \mu_h$ ;

$$\pi(< 0 >) = \left( 1 + \sum_{j=1}^{N-g} \frac{\nu_o^j}{j!} \prod_{i=1}^j \Lambda(i) \right)^{-1}, \quad \Lambda(i) = \theta_0(\nu_h, N-i+1) \sum_{j=0}^{N-g-i} \frac{\nu_h^j}{j!}.$$

Formulas (2.27)-(2.30) coincided with ones for CAC with single parameter in integrated voice/data networks if we set  $g:=N-N_2$ ,  $\nu_o:=\nu_d$ ,  $\nu_h:=\nu_v$ . And in case  $g=0$  we get the results for CAC based on CS-strategy, see (2.19)-(2.21). Also from (2.28) we get the following unimprovable limits for  $P_h$

$$E_B(\nu_h, N) \leq P_h \leq E_B(\nu_h, g).$$

In the proposed algorithms the computational procedures contains the well-known Erlang's B-formula as well as expressions within that formula which has even been tabulated [20]. Thus, complexity of the proposed algorithms to calculate QoS metrics of investigated multi-parametric CAC based on guard channels are almost congruous to that of Erlang's B-formula. Direct calculations by Erlang's B-formula bring known difficulties at large values of  $N$  because of large factorials and exponents. To overcome these difficulties the known effective recurrent formulae can be used, e.g. see [8].

### 3. The CAC based on threshold strategy

Now consider an alternative CAC in integrated voice/data networks which based on threshold strategy. More detailed description of the given CAC is follows. As in CAC based on guard channels, we assume that arrived an hv-call is accepted as long as at least one free channel is available; otherwise it is blocked. With the purpose of definition of CAC based on threshold strategy for calls of other types three parameters  $R_1$ ,  $R_2$  and  $R_3$  where  $1 \leq R_1 \leq R_2 \leq R_3 \leq N$  are introduced. Then proposed CAC defines the following rules for admission of heterogeneous calls: an od-call (respectively, hd-call and ov-call) is accepted only if the number of calls of the given type in progress is less than  $R_1$  (respectively,  $R_2$  and  $R_3$ ) and a free channel is available; otherwise it is blocked.

For the sake of simplicity we shall assume that  $b=1$ . The case  $b>1$  is straightforward (see section 2). The state of the system under given CAC at any time also is described by 2-D vector  $\mathbf{n}=(n_d, n_v)$ , where  $n_d$  (respectively,  $n_v$ ) is the number of data (respectively, voice) calls in the channels. Then state space of appropriate 2-D MC is given by:

$$S := \left\{ \mathbf{n} : n_d = \overline{0, R_2}, n_v = \overline{0, N}; n_d + n_v \leq N \right\} \quad (3.1)$$

*Note 2.* Hereinafter, for simplicity, we use same notations for state spaces, stationary distribution and etc. in different CAC strategy. This should not cause misunderstanding, as it will be clear what model is considered from the context.

The elements of generating matrix of the appropriate 2-D MC in this case is determined as follows:

$$q(\mathbf{n}, \mathbf{n}') = \begin{cases} \lambda_d & \text{if } n_d \leq R_1 - 1, \mathbf{n}' = \mathbf{n} + \mathbf{e}_1, \\ \lambda_{hd} & \text{if } R_1 \leq n_d \leq R_2 - 1, \mathbf{n}' = \mathbf{n} + \mathbf{e}_1, \\ \lambda_v & \text{if } n_v \leq R_3 - 1, \mathbf{n}' = \mathbf{n} + \mathbf{e}_2, \\ \lambda_{hv} & \text{if } R_3 \leq n_v \leq N - 1, \mathbf{n}' = \mathbf{n} + \mathbf{e}_2, \\ n_d \mu_d & \text{if } \mathbf{n}' = \mathbf{n} - \mathbf{e}_1, \\ n_v \mu_v & \text{if } \mathbf{n}' = \mathbf{n} - \mathbf{e}_2, \\ 0 & \text{in other cases.} \end{cases} \quad (3.2)$$

Blocking probability of hv-calls and mean number of busy channels are defined similarly to (2.3) and (2.7), respectively. The other QoS metrics are defined as following marginal distributions of initial chain:

$$P_{ov} := \sum_{\mathbf{n} \in S} p(\mathbf{n}) I(n_v \geq R_3), \quad (3.3)$$

$$P_{hd} := \sum_{\mathbf{n} \in S} p(\mathbf{n}) \delta(n_d, R_2) + \sum_{\mathbf{n} \in S} p(\mathbf{n}) \delta(n_d + n_v, N) I(n_d < R_2), \quad (3.4)$$

$$P_{od} := \sum_{\mathbf{n} \in S} p(\mathbf{n}) I(n_d \geq R_1) + \sum_{\mathbf{n} \in S} p(\mathbf{n}) \delta(n_d + n_v, N) I(n_d < R_1). \quad (3.5)$$

Unlike CAC based on guard channel strategy, it is easily to show that under this one there is no circulation flow in the state diagram of the underlying 2-D MC, i.e. it is reversible [16]. In other words, there is general solution of the system of local balance equations (SLBE) in this chain. Therefore, we can express any state probability  $p(n_d, n_v)$  by state probability  $p(0,0)$  by choosing any path between these states in the state diagram. So, in case  $R_2 + R_3 \leq N$  we get following multiplicative solution for stationary distribution of the underlying 2-D MC:

$$p(n_d, n_v) = \begin{cases} \frac{v_d^{n_d}}{n_d!} \cdot \frac{v_v^{n_v}}{n_v!} \cdot p(0,0), & \text{if } n_d \leq R_1, n_v \leq R_3, \\ \frac{v_d^{n_d}}{n_d!} \cdot \frac{v_{hv}^{n_v}}{n_v!} \left( \frac{v_v}{v_{hv}} \right)^{R_3} \cdot p(0,0), & \text{if } n_d \leq R_1, R_3 < n_v \leq N, \\ \frac{v_{hd}^{n_d}}{n_d!} \cdot \frac{v_v^{n_v}}{n_v!} \left( \frac{v_d}{v_{hd}} \right)^{R_1} \cdot p(0,0), & \text{if } R_1 < n_d \leq R_2, n_v \leq R_3, \\ \frac{v_{hd}^{n_d}}{n_d!} \cdot \frac{v_{hv}^{n_v}}{n_v!} \left( \frac{v_d}{v_{hd}} \right)^{R_1} \cdot \left( \frac{v_v}{v_{hv}} \right)^{R_3} \cdot p(0,0), & \text{if } R_1 < n_d \leq R_2, R_3 < n_v \leq N, \end{cases} \quad (3.6)$$

where  $p(0,0)$  is determined from normalizing condition:

$$p(0,0) = \left( \sum_{n \in S_1} \frac{v_d^{n_d}}{n_d!} \cdot \frac{v_v^{n_v}}{n_v!} + \left( \frac{v_v}{v_{hv}} \right)^{R_3} \sum_{n \in S_2} \frac{v_d^{n_d}}{n_d!} \cdot \frac{v_{hv}^{n_v}}{n_v!} + \left( \frac{v_d}{v_{hd}} \right)^{R_1} \sum_{n \in S_3} \frac{v_{hd}^{n_d}}{n_d!} \cdot \frac{v_v^{n_v}}{n_v!} + \left( \frac{v_d}{v_{hd}} \right)^{R_1} \left( \frac{v_v}{v_{hv}} \right)^{R_3} \sum_{n \in S_4} \frac{v_{hd}^{n_d}}{n_d!} \cdot \frac{v_{hv}^{n_v}}{n_v!} \right)^{-1}$$

Here we use the following notations:  $v_d := \lambda_d / \mu_d$ ,  $v_{hd} := \lambda_{hd} / \mu_d$ ;

$$S_1 := \{n \in S : n_d \leq R_1, n_v \leq R_3\}, S_2 := \{n \in S : n_d \leq R_1, R_3 + 1 \leq n_v \leq N\},$$

$$S_3 := \{n \in S : R_1 + 1 \leq n_d \leq R_2, n_v \leq R_3\}, S_4 := \{n \in S : R_1 + 1 \leq n_d \leq R_2, R_3 + 1 \leq n_v \leq N\}.$$

In the case  $R_2+R_3>N$  stationary distribution has the following form:

$$p(n_d, n_v) = \begin{cases} \frac{v_d^{n_d}}{n_d!} \cdot \frac{v_v^{n_v}}{n_v!} \cdot p(0,0), & \text{if } 0 \leq n_d \leq R_1, 0 \leq n_v \leq R_3, \\ \frac{v_{hd}^{n_d}}{n_d!} \cdot \frac{v_v^{n_v}}{n_v!} \cdot \left(\frac{v_d}{v_{hd}}\right)^{R_2} \cdot p(0,0), & \text{if } R_1 + 1 \leq n_d \leq R_2, 0 \leq n_v \leq N - n_d, \\ \frac{v_d^{n_d}}{n_d!} \cdot \frac{v_{hv}^{n_v}}{n_v!} \cdot \left(\frac{v_v}{v_{hv}}\right)^{R_3} \cdot p(0,0), & \text{if } 0 \leq n_d \leq N - R_3 - 1, R_3 + 1 \leq n_v \leq N, \end{cases} \quad (3.7)$$

where  $p(0,0) = \left( \sum_{n \in T_1} \frac{v_d^{n_d}}{n_d!} \cdot \frac{v_v^{n_v}}{n_v!} + \left(\frac{v_d}{v_{hd}}\right)^{R_1} \sum_{n \in T_2} \frac{v_{hd}^{n_d}}{n_d!} \cdot \frac{v_v^{n_v}}{n_v!} + \left(\frac{v_v}{v_{hv}}\right)^{R_3} \sum_{n \in T_3} \frac{v_d^{n_d}}{n_d!} \cdot \frac{v_{hv}^{n_v}}{n_v!} \right)^{-1}$  ;

$$T_1 := \{n \in S : 0 \leq n_d \leq R_1, 0 \leq n_v \leq R_3\}, T_2 := \{n \in S : R_1 + 1 \leq n_d \leq R_2, 0 \leq n_v \leq N - n_d\},$$

$$T_3 := \{n \in S : 0 \leq n_d \leq N - R_3 - 1, R_3 + 1 \leq n_v \leq N\}.$$

The exact method to determine the steady state probabilities in terms of a multiplicative representation (3.6) (or (3.7)) for large values of  $N$  encounters numerical problems such as imprecision and overflow. These are related to the fact that with such a method the entire state space has to be generated, and large factorials and powers close to the zero of the quantities (for low loads) or large values (for high loads) have to be calculated, i.e. there arises the problem of exponent overflow or underflow. Hence we can use developed approximate method to determine the QoS metrics of the model under using the proposed CAC based on threshold strategy even when state space (3.1) is large.

As in section 2, we assume that  $\lambda_v \gg \lambda_d$  and  $\mu_v \gg \mu_d$  and examine the following splitting of the state space (3.1):

$$S = \bigcup_{k=0}^{R_2} S_k, \quad S_k \cap S_{k'} = \emptyset, \quad k \neq k',$$

where  $S_k := \{n \in S : n_d = k\}$ .

Next classes of states  $S_k$  are combined into individual merged states  $\langle k \rangle$  and in (3.1) the merged function with range  $\tilde{S} := \{ \langle k \rangle : k = 0, 1, \dots, R_2 \}$  which is similar to (2.9) is introduced. As in exact algorithm in order to find stationary distribution within splitting classes  $S_k$  we will distinguish two cases: 1)  $R_2+R_3 \leq N$  and 2)  $R_2+R_3 > N$ . In first case the elements of generating matrix of appropriate 1-D BDP are same for all splitting models, i.e.

$$q_k(i, j) = \begin{cases} \lambda_v & \text{if } i \leq R_3 - 1, j = i + 1, \\ \lambda_{hv} & \text{if } R_3 \leq i \leq N - 1, j = i + 1, \\ i\mu_v & \text{if } j = i - 1, \\ 0 & \text{in other cases.} \end{cases}$$

From last formula we conclude that stationary distribution within class  $S_k$  is same as that  $M|M|N-k|N-k$  queuing system with state-dependent arrival rates and constant service rate of each channel, i.e.

$$\rho_k(i) = \begin{cases} \frac{v_v^i}{i!} \rho_k(0) & \text{if } 1 \leq i \leq R_3, \\ \left(\frac{v_v}{v_{hv}}\right)^{R_3} \frac{v_{hv}^i}{i!} \rho_k(0) & \text{if } R_3 + 1 \leq i \leq N - k, \end{cases} \quad (3.8)$$

where  $\rho_k(0) = \left( \sum_{i=0}^{R_3} \frac{v_v^i}{i!} + \left(\frac{v_v}{v_{hv}}\right)^{R_3} \sum_{i=R_3+1}^{N-k} \frac{v_{hv}^i}{i!} \right)^{-1}$ .

So, from (3.2) and (3.8) we conclude that elements of generating matrix of the merged model are

$$q(\langle k \rangle, \langle k' \rangle) = \begin{cases} \lambda_d(1 - \rho_k(N - k)) & \text{if } 0 \leq k \leq R_1 - 1, k' = k + 1, \\ \lambda_{hd}(1 - \rho_k(N - k)) & \text{if } R_1 \leq k \leq R_2 - 1, k' = k + 1, \\ k\mu_d & \text{if } k' = k - 1, \\ 0 & \text{in other cases.} \end{cases} \quad (3.9)$$

Distribution of merged model is calculated by using (3.9) and has the following form:

$$\pi(\langle k \rangle) = \frac{\pi(\langle 0 \rangle)}{k! \mu_d^k} \prod_{i=1}^k q(\langle k - 1 \rangle, \langle k \rangle), \quad k = \overline{1, R_2}, \quad (3.10)$$

where  $\pi(\langle 0 \rangle) = \left( 1 + \sum_{k=1}^{R_2} \frac{1}{k! \mu_d^k} \prod_{i=1}^k q(\langle k - 1 \rangle, \langle k \rangle) \right)^{-1}$ .

Finally the following approximate formulae to calculate the desired QoS metrics under using the proposed CAC based on threshold strategy are obtained:

$$P_{hw} \approx \sum_{k=0}^{R_2} \pi(\langle k \rangle) \rho_k(N - k); \quad (3.11)$$

$$P_{ov} \approx \sum_{k=0}^{R_2} \pi(\langle k \rangle) \sum_{i=R_3}^{N-k} \rho_k(i); \quad (3.12)$$

$$P_{hd} \approx \pi(\langle R_2 \rangle) + \sum_{k=0}^{R_2-1} \pi(\langle k \rangle) \rho_k(N - k); \quad (3.13)$$

$$P_{od} \approx \sum_{k=R_1}^{R_2} \pi(\langle k \rangle) + \sum_{k=0}^{R_1-1} \pi(\langle k \rangle) \rho_k(N - k); \quad (3.14)$$

$$N_{ov} \approx \sum_{k=1}^N k \sum_{i=0}^{f_{R_2}(k)} \pi(< i >) \rho_i(k-i). \quad (3.15)$$

In second case (i.e. when  $R_2+R_3>N$ ) distributions for splitting models with state space  $S_k$  for  $k=0,1,\dots,N-R_3-1$  are calculated by using relations (3.8) while distributions for splitting models with state space  $S_k$  for  $k=N-R_3,\dots,R_2$  coincides with distributions of model  $M/M/N-k/N-k$  with load  $v_v$  erl, see (2.22). And all stage of developed procedure to calculate the QoS metrics are same with first case except the calculating of  $P_{ov}$ . Last QoS metric in this case is calculated as follows:

$$P_{ov} \approx \sum_{k=0}^{N-R_3} \pi(< k >) \sum_{i=R_3}^{N-k} \rho_k(i) + \sum_{k=N-R_3+1}^{R_2} \pi(< k >) \rho_k(N-k). \quad (3.16)$$

Now consider some special cases. First of all note that CAC based on CS-strategy is a special case of proposed one when  $R_1=R_2=R_3=N$ . It is important to note that if we set in developed approximate algorithm the indicated value of parameters we obtain exactly the results which were established in section 2, see (2.19)-(2.21).

1. CAC with Single Parameter. As in section 2, let us examine subclass of investigated CAC in which distinction is made only between voice and data traffics, i.e. it is assumed that  $R_1=R_2$  and  $R_3=N$  where  $R_2<R_3$ . For this case from (3.8)-(3.15) we get the following simple approximate formulae for calculating the blocking probabilities of v-calls ( $P_v$ ) and d-calls ( $P_d$ ):

$$P_v = P_{hv} = P_{ov} \approx \sum_{k=0}^{R_2} \pi(< k >) E_B(v_v, N-k), \quad (3.17)$$

$$P_d = P_{hd} = P_{od} \approx \sum_{k=0}^{R_2-1} \pi(< k >) E_B(v_v, N-k) + \pi(< R_2 >). \quad (3.18)$$

Here

$$\pi(< k >) = \frac{v_d^k}{k!} \prod_{i=0}^{k-1} (1 - E_B(v_d, N-i)), k = \overline{1, R_2}, \quad (3.19)$$

where  $\pi(< 0 >) = \left( 1 + \sum_{k=1}^{R_2} \frac{v_d^k}{k!} \prod_{i=0}^{k-1} (1 - E_B(v_d, N-i)) \right)^{-1}$ .

Mean number of busy channels is calculated as follows:

$$\tilde{N} \approx \sum_{i=1}^N i \sum_{k=0}^{f_{R_2}(i)} \theta_{i-k}(v_v, N-k) \pi(< k >). \quad (3.20)$$

Note that if in formulas (3.17)-(3.20) set  $R_2=N$  then obtains the results for CAC based on CS-strategy, see (2.19)-(2.21).

2. Mono-service CWN with individual pools for heterogeneous calls. In a given CAC the entire pool of  $N$  channels is divided into three pools, an individual pool consisting of  $r_o$



channels (for the o-calls alone),  $r_h$  channels (for the o-calls alone) and a common pool consisting of  $N-r_o-r_h$  channels (for the o- and the h-calls). Assume that  $N > r_o+r_h$ , since in case  $N=r_o+r_h$  there is trivial CAC based on Complete Partitioning (CP) strategy, i.e. initial system is divided into two separate subsystems where one of them contains  $r_h$  channels for handling only h-calls whereas second one with  $r_o$  channels handle only o-calls.

If there is at least one free channel (either in the appropriate individual or common pool) at the moment an o-call (h-call) arrives, it is accepted for servicing; otherwise, the call is lost. Note that the process by which the channels are engaged by heterogeneous calls is realized in the following way. If there is one free channel in own pool at the moment an o-call (h-call) arrives, it engages a channel from the own individual pool, while if there is no free channel in the own individual pool, the o-call (h-calls) utilize channels from the common pool. Upon completion of servicing of an o-call (h-call) in the individual pool, the relinquished channel is transferred to the common pool if there is an o-call (h-call) present there, while the channel in the common pool that has finished servicing the o-call (h-call) is switched to the appropriate individual pool. This procedure is called channel reallocation method [21].

From described above model we conclude that it correspond to general CAC based on threshold strategy in case  $R_1=R_2=N-r_h$  and  $R_3=N-r_o$ . Therefore, taking into account (3.8)-(3.15) we find the following approximate formulae to calculate the QoS metrics of the given model:

$$P_o \approx E_B(v_h, N-r_o) \sum_{k=0}^{r_o} \pi(<k>) + \sum_{k=r_o+1}^{N-r_h-1} E_B(v_h, N-k) \pi(<k>) + \pi(<N-r_h>), \quad (3.21)$$

$$P_h \approx E_B(v_h, N-r_o) \sum_{k=0}^{r_o} \pi(<k>) + \sum_{k=r_o+1}^{N-r_h} E_B(v_h, N-k) \pi(<k>), \quad (3.22)$$

$$\tilde{N} \approx \sum_{k=1}^{N-r_h} k \sum_{i=0}^k \pi(<i>) \rho_i(k-i) + \sum_{k=N-r_h+1}^N k \sum_{i=r_o-N+k}^{N-r_h} \pi(<i>) \rho_i(k-i), \quad (3.23)$$

where

$$\rho_k(i) = \begin{cases} \theta_i(v_h, N-r_o), & \text{if } 0 \leq k \leq r_o, 0 \leq i \leq N-r_o, \\ \theta_i(v_h, N-k), & \text{if } r_o+1 \leq k \leq N-r_h, 0 \leq i \leq N-k; \end{cases} \quad (3.24)$$

$$\pi(<k>) = \begin{cases} \frac{v_o^k}{k!} \pi(<0>), & \text{if } 1 \leq k \leq r_o, \\ \frac{v_o^k}{k!} \prod_{i=N-k+1}^{N-r_o} (1-E_B(v_h, i)) \pi(<0>), & \text{if } r_o+1 \leq k \leq N-r_h, \end{cases} \quad (3.25)$$

$$\pi(<0>) = \left( \sum_{i=0}^{r_o} \frac{v_o^i}{i!} + \sum_{k=r_o+1}^{N-r_h} \frac{v_o^k}{k!} \prod_{i=N-k+1}^{N-r_h} (1-E_B(v_h, i)) \right)^{-1}.$$

Note that in special case  $r_o=0$  the proposed CAC coincides with the one investigated in [9]. It is evident from derived formulas that in case approximate calculation of QoS metrics we

don't have to generate the entire state space of the initial model and calculate its stationary distribution in order to calculate the QoS metrics of the CAC based on individual pools for heterogeneous calls. These parameters may be found by means of simple computational procedures which contain the Erlang's B-formula and terms within that formula. Note that for  $r_o=r_h=0$  this scheme becomes fully accessible by both types of calls, i.e. CAC based on CS-strategy takes place.

#### 4. Numerical results

For realization of the above derived algorithms a software package was developed to investigate the behavior of the QoS metrics as a function of the variation in the values of cell's load and structure parameters as well as CAC parameters. First briefly consider some results for the CAC based on guard channels strategy in integrated voice/data model with four classes of calls.

The developed approximate formulas allow without essential computing difficulties to carry out the authentic analysis of QoS metrics in any range of change of values of loading parameters of the heterogeneous traffic, satisfying to the assumption concerning their ratio (i.e. when  $\lambda_v \gg \lambda_d$  and  $\mu_v \gg \mu_d$ ) and also at any number of channels of cell. Some results are shown in figures 2-4 where  $N=16$ ,  $N_3=14$ ,  $N_2=10$ ,  $\lambda_{ov}=10$ ,  $\lambda_{hv}=6$ ,  $\lambda_{od}=4$ ,  $\lambda_{hd}=3$ ,  $\mu_v=10$ ,  $\mu_d=2$ . Behavior of the studied curves fully confirms all theoretical expectations.

In the given model at the fixed value of the total number of channels ( $N$ ) it is possible to change values of three threshold parameters ( $N_1$ ,  $N_2$  and  $N_3$ ). In other words, there is three degree of freedom. Let's note, that the increase in value of one of parameters (in admissible area) favorably influences on blocking probability of calls of corresponding type only (see fig.2 and 3). So, in these experiments, the increase in value of parameter  $N_1$  leads to reduction of blocking probability of od-calls but other blocking probabilities (i.e.  $P_{hv}$ ,  $P_{ov}$  and  $P_{hd}$ ) increase. At the same time, the increase in value of any parameter leads to increase in overall channels utilization (see fig.4).

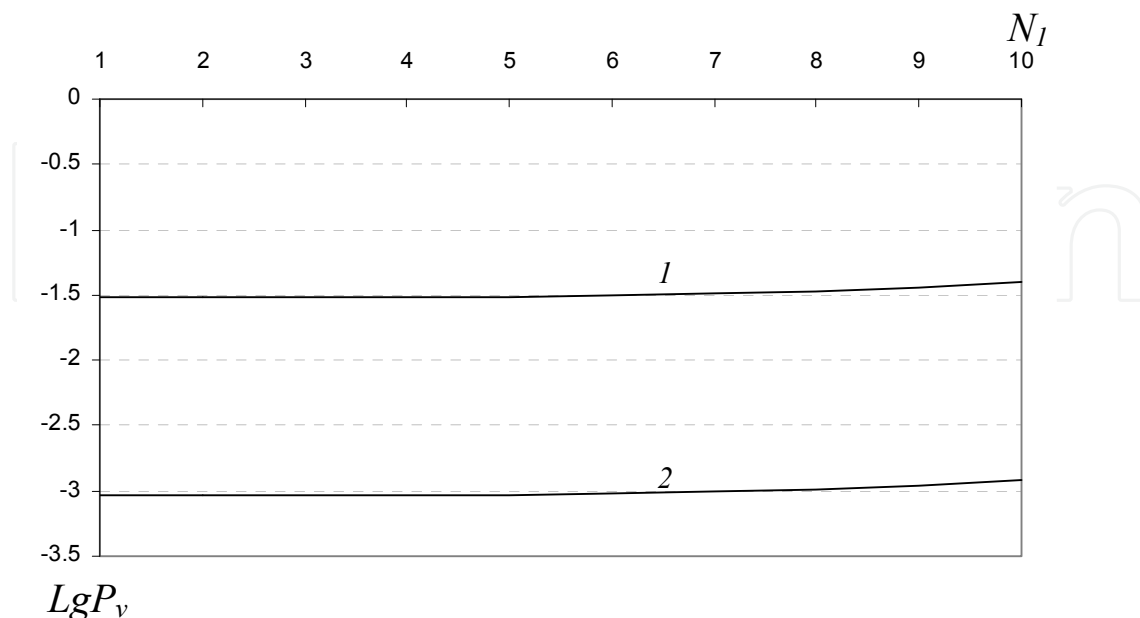


Fig. 2. Blocking probability of v-calls versus  $N_1$ : 1 -  $P_{ov}$ ; 2 -  $P_{hv}$ .

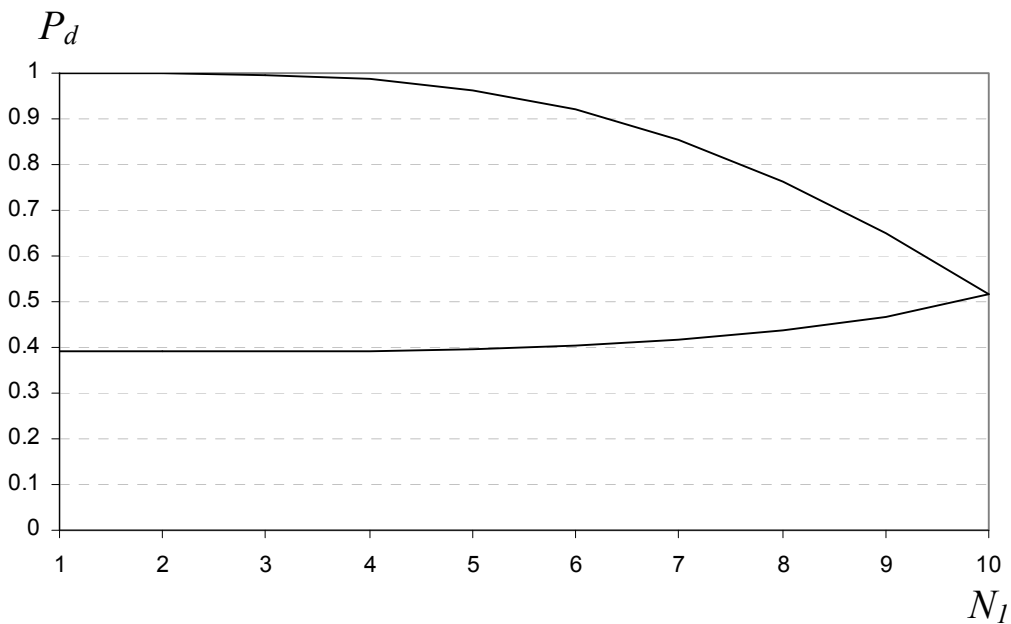


Fig. 3. Blocking probability of d-calls versus  $N_1$ : 1- $P_{od}$ ; 2-  $P_{hd}$ .

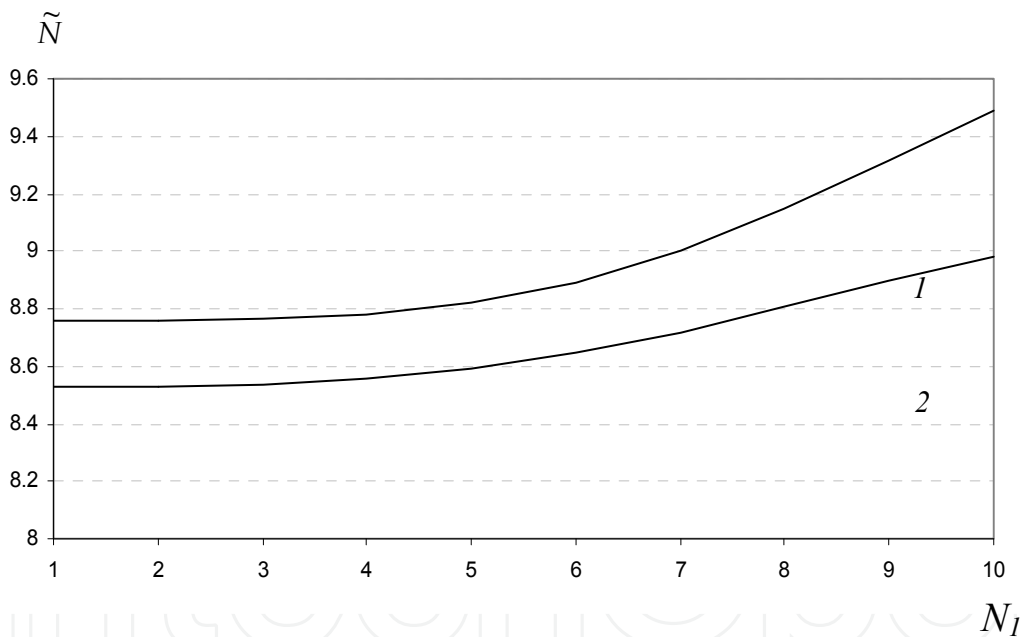


Fig. 4. Average number of busy channels versus  $N_1$ : 1 -  $N_3=15$ ; 2 -  $N_3=11$ .

Other direction of researches consists in an estimation of accuracy of the developed approximate formulas to calculate the QoS metrics. Exact values (EV) of QoS metrics are determined from SGBE. It is important to note, that under fulfilling of the mentioned assumptions related to ratio of loading parameters of heterogeneous traffic the exact and approximate values (AV) almost completely coincide for all QoS metrics. Therefore these comparisons here are not shown. At the same time, it is obvious that finding the exact values of QoS metrics on the basis of the solution of SGBE appears effective only for models with the moderate dimension.

It is important to note sufficiently high accuracy of suggested formulae even for the case when accepted assumption about ratio of traffic loads is not fulfilled. To facilitate the computation efforts, as exact values of QoS metrics we use their values that calculated from explicit formulas, see [22], pages 131-135. In mentioned work appropriate results are obtained for the special case  $b=1$  and  $\mu_v=\mu_d$ . Let's note, that condition  $\mu_v=\mu_d$  contradicts our assumption  $\mu_v \gg \mu_d$ . The comparative analysis of results is easy for executing by means of tables 1-3 where initial data are  $N=16, N_3=14, N_2=10, \lambda_{ov}=10, \lambda_{hw}=6, \lambda_{od}=4, \lambda_{hd}=3, \mu_v=\mu_d=2$ . Apparently from these tables, the highest accuracy of the developed approximate formulas is observed at calculation of QoS metric for v-calls since for them the maximal difference between exact and approximate values does not exceed 0.001 (see tabl.1). Small deviations take place at calculation of QoS metrics for d-calls, but also thus in the worst cases the

$N_1$	$P_{ov}$		$P_{hw}$	
	EV	AV	EV	AV
1	0.03037298	0.03465907	0.00092039	0.00119181
2	0.03037774	0.03469036	0.00092054	0.00119309
3	0.03040249	0.03482703	0.00092129	0.00119878
4	0.03048919	0.03521813	0.00092392	0.00121521
5	0.03072036	0.03604108	0.00093092	0.00125021
6	0.03122494	0.03741132	0.00094621	0.00130942
7	0.03217389	0.03932751	0.00097497	0.00139396
8	0.03377398	0.04168754	0.00102345	0.00150073
9	0.03627108	0.04432985	0.00109912	0.00162373
10	0.03997025	0.04706484	0.00121112	0.00175503

Table 1. Comparison for v-calls in CAC based on guard channels.

$N_1$	$P_{od}$		$P_{hd}$	
	EV	AV	EV	AV
1	0.99992793	0.99985636	0.39177116	0.35866709
2	0.99925564	0.99855199	0.39183255	0.35886135
3	0.99612908	0.99271907	0.39215187	0.35969536
4	0.98645464	0.97565736	0.39327015	0.36203755
5	0.96398536	0.93891584	0.39625194	0.36685275
6	0.92198175	0.87621832	0.40276033	0.37462591
7	0.85564333	0.78660471	0.41500057	0.38506671
8	0.76370389	0.67487475	0.43563961	0.39731190
9	0.64880652	0.55004348	0.46784883	0.41028666
10	0.51556319	0.42295366	0.51556319	0.42295366

Table 2. Comparison for d-calls in CAC based on guard channels.

$N_1$	EV	AV
1	8.75786133	8.52991090
2	8.75908958	8.53136014
3	8.76473770	8.53753920
4	8.78196778	8.55476428
5	8.82125679	8.58985731
6	8.89293266	8.64583980
7	9.00241811	8.71992002
8	9.14705952	8.80533833
9	9.31596095	8.89429324
10	9.49204395	8.97976287

Table 3. Comparison for average number of busy channels in CAC based on guard channels.

absolute error of the proposed formulas does not exceed 0.09, that are quite comprehensible in engineering practice (see tabl.2). Similar results are observed for an average number of occupied channels of cell (see tabl.3). It is important to note, that numerous numerical experiments have shown, that at all admissible loads accuracy of the proposed approximate formulas grows with increase in the value of total number of channels.

It is clear that in terms of simplicity and efficiency, the proposed approach is emphatically superior to the approach based on the use of a balance equations for the calculate QoS metrics of the given CAC in the model with non-identical channel occupancy time.

Let's note, that high accuracy at calculation of QoS metrics for v-calls is observed even at those loadings which do not satisfy any of accepted above assumptions concerning ratio of intensities of heterogeneous traffic. So, for example, at the same values of number of channels and parameters of strategy, at  $\lambda_{ov}=4$ ,  $\lambda_{hw}=3$ ,  $\lambda_{od}=10$ ,  $\lambda_{hd}=6$ ,  $\mu_v=\mu_d=2$  (i.e. when assumptions  $\lambda_v \gg \lambda_d$ ,  $\mu_v \gg \mu_d$  are not fulfilled) the absolute error for mentioned QoS metric does not exceed 0.002. Similar results are observed and for an average number of occupied channels of cell. However, the proposed approximate formulas show low accuracy for d-calls since for them the maximal absolute error exceeds 0.2.

Numerical experiments with the CAC based on threshold strategy are carried out also. Due to limitation of volume of work these results here are not resulted. As in CAC based on guard channels, the increase in value of one of parameters (in admissible area) favorably affect the blocking probability of calls of corresponding type only. So, the increase in value of parameter  $R_1$  leads to reduction of blocking probability of od-calls but other blocking probabilities (i.e.  $P_{hw}$ ,  $P_{ov}$  and  $P_{hd}$ ) increase. At the same time, the increase in value of any parameter leads to increase in overall channels utilization.

The very high precision of the proposed approximate method should also be noted. Thus, in this case comparative analysis of approximate results and the results obtained using a multiplicative solution (for small values of channels) shows that their differences is negligible. Moreover, in some cases these results completely coincide. But in terms of simplicity and efficiency, the proposed approximate approach is emphatically superior to the approach based on the use of a multiplicative solution. For the sake of brief these results are not shown here.

At the end of this section we conducted research on comparative analysis of QoS metrics of two schemes: CAC based on guard channels scheme and CAC based threshold strategy.

Comparison was done in the broad range of number of channels and load parameters. In each access strategy the total number of channels is fixed and controllable parameters are  $N_1, N_2, N_3$  (for CAC based on guard channels scheme) and  $R_1, R_2, R_3$  (for CAC based on threshold strategy). As it mentioned above, behavior of QoS metrics with respect to indicated controllable parameters in different CAC are same.

Some results of comparison are shown in fig.5-9 where label 1 and 2 denotes QoS metrics for CAC based on guard channels and CAC based on threshold strategies, respectively. The input data of model are chosen as follows:  $N=16, R_3=14, R_2=12, \lambda_{ov}=10, \lambda_{hv}=6, \lambda_{od}=4, \lambda_{hd}=3, \mu_v=10, \mu_d=2$ . In graphs the parameter of the CAC based on guard channels (i.e.  $N_1$ ) is specified as X-line and as it has been specified above, it corresponds to parameter  $R_1$  of the CAC based on threshold strategy.

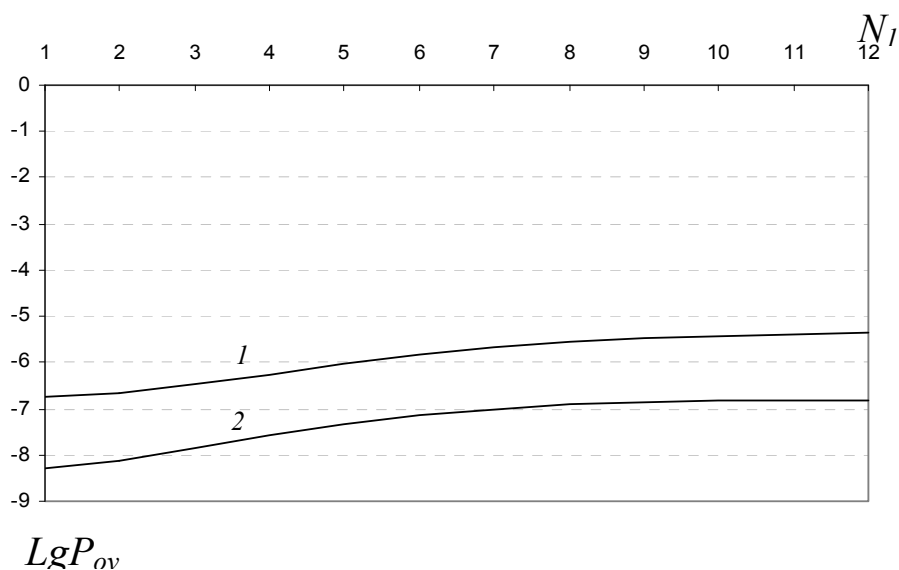


Fig. 5. Comparison for  $P_{ov}$  under different CAC.

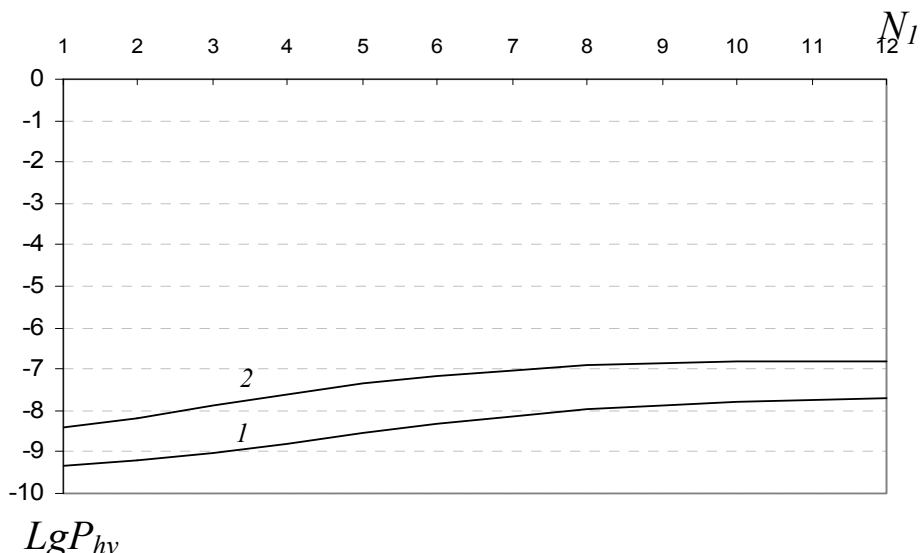


Fig. 6. Comparison for  $P_{hv}$  under different CAC.

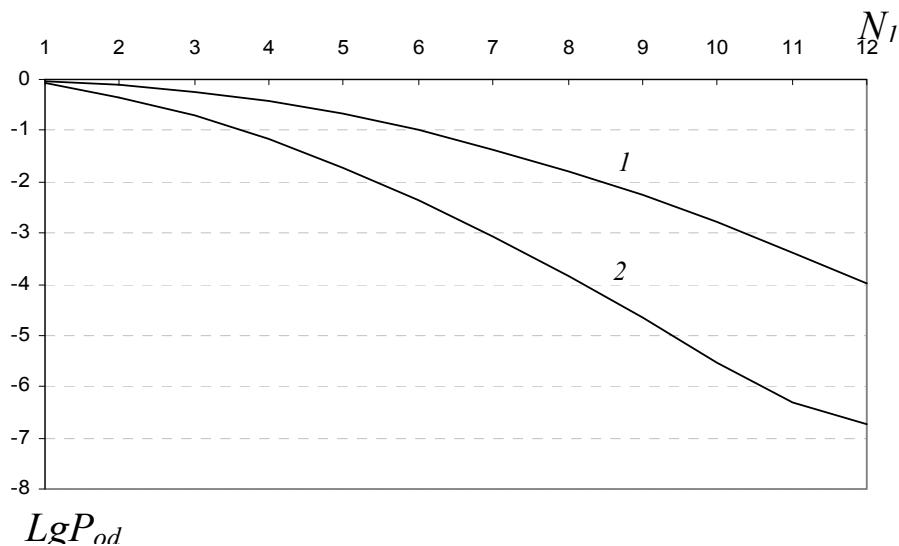


Fig. 7. Comparison for  $P_{od}$  under different CAC.

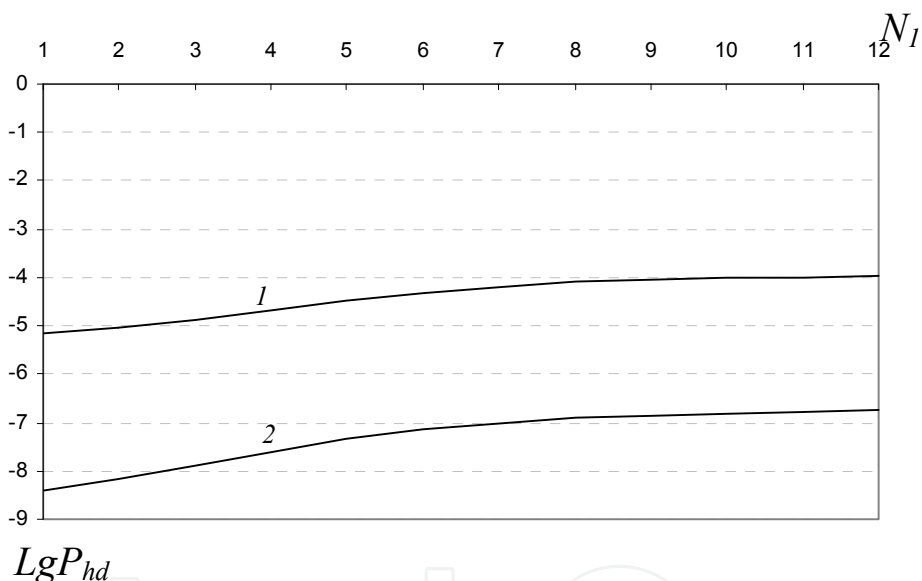


Fig. 8. Comparison for  $P_{hd}$  under different CAC

From these graphs we conclude that, for the chosen initial data three QoS metrics, except for blocking probability of hv-calls, essentially better under using CAC based on threshold strategy. The average number of occupied channels in both strategies is almost same. However, quite probably, that at other values of initial data QoS metrics (either all or some of them) in CAC based on guard channels will be better than the CAC based on threshold strategy.

It is important to note that with the given number of channels, loads and QoS requirements either of CAC strategy may or may not meet the requirements. For instance, in the model of mono-service CWN for the given values of  $N=100$ ,  $\nu_o=50$  erl,  $\nu_h=35$  erl following requirements  $P_o \leq 0.1$ ,  $P_h \leq 0.007$  and  $\tilde{N} \geq 80$  are not met with CAC based on guard channels irrespective of value of parameter  $g$  (number of guard channels), whereas CAC based on individual pool only for h-calls (i.e.  $r_o=0$ ) meets the requirements at  $r_h=40$ . However, for the

same given initial data, requirements  $P_o \leq 0.3$ ,  $P_h \leq 0.0001$  and  $\tilde{N} \geq 60$  are only met by CAC based on guard channel scheme at  $g=20$ , and never met by CAC based on individual pool strategy irrespective of value of its parameter  $r_h$ . Thus it is possible to find optimal (in given context) strategy at the given loads without changing number of channels.

Apparently, both strategies have the same implementation complexity. That is why the selection of either of them at each particular case must be based on the answer to the following question: does it meet the given QoS requirements? These issues are subjects to separate investigation.

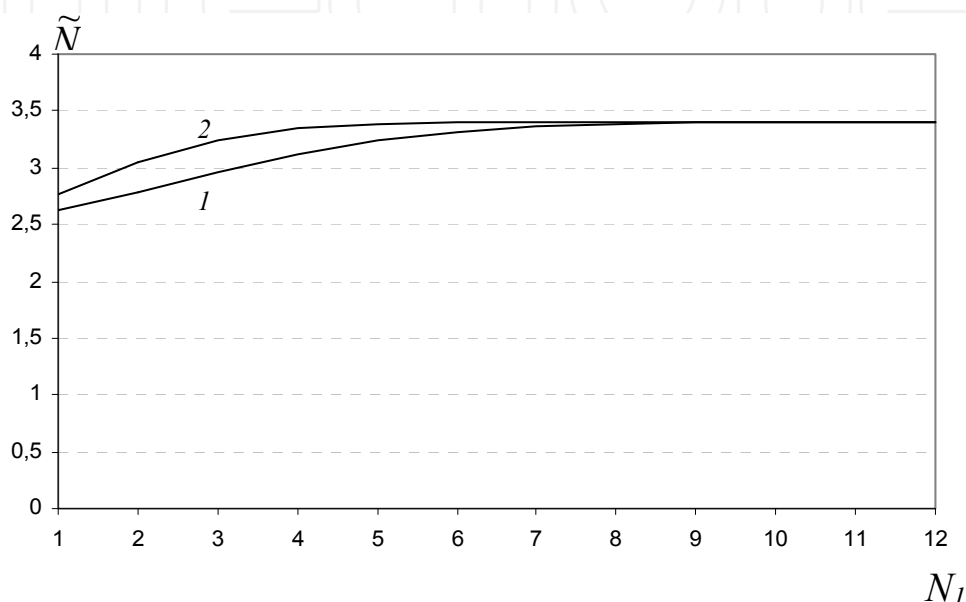


Fig. 9. Comparison for  $\tilde{N}$  under different CAC.

## 5. Conclusion

In this paper effective and refined approximate approach to performance analysis of un-buffered integrated voice/data CWN under different multi-parametric CAC has been proposed. Note that many well-known results related to mono-service CWN are special cases of proposed ones. In the almost all available works devoted mono-service CWN the queuing model is investigated with assumption that both handover and original calls are identical in terms of channel occupancy time. This assumption is rather limiting and unreal. Here models of un-buffered integrated voice/data CWN are explored with more general parameter requirements. Performed numerical results demonstrate high accuracy of the developed approximate method.

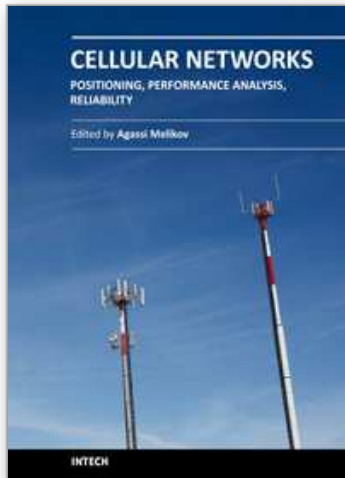
It is important to note that the proposed approach may be facilitate the solution of problems related to selecting the optimal (in given sense) values of parameters of investigated multi-parametric CAC. These problems are subjects to separate investigation.

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## **Cellular Networks - Positioning, Performance Analysis, Reliability**

Edited by Dr. Agassi Melikov

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Wireless cellular networks are an integral part of modern telecommunication systems. Today it is hard to imagine our life without the use of such networks. Nevertheless, the development, implementation and operation of these networks require engineers and scientists to address a number of interrelated problems. Among them are the problem of choosing the proper geometric shape and dimensions of cells based on geographical location, finding the optimal location of cell base station, selection the scheme dividing the total net bandwidth between its cells, organization of the handover of a call between cells, information security and network reliability, and many others. The book focuses on three types of problems from the above list - Positioning, Performance Analysis and Reliability. It contains three sections. The Section 1 is devoted to problems of Positioning and contains five chapters. The Section 2 contains eight Chapters which are devoted to quality of service (QoS) metrics analysis of wireless cellular networks. The Section 3 contains two Chapters and deal with reliability issues of wireless cellular networks. The book will be useful to researches in academia and industry and also to post-graduate students in telecommunication specialitiies.

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