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A List Based Detection Technique for MIMO Systems

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1. Introduction

It is well known that multiple transmit and receive antennas can significantly increase the data carrying capacity of wireless systems (1) - (3). Usually a multiple-input multiple-output (MIMO) system requires a channel code to approach capacity (4). However, for such MIMO systems, the optimal joint maximum-likelihood (ML) detection using exhaustive search is too complex and is almost impossible. Motivated by turbo decoding (5), iterative detectors and decoders, treating the channel code as the outer code and the space time mapper as the inner code, have been commonly employed in the literature. Although the iterative decoder structures are extensively investigated in the code theory literature, the design and role of the detector structure still need investigation. In this context, the authors in (6), (9) studied the iterative algorithm for MIMO systems using optimum soft-input/soft-output ML MIMO detector. Unfortunately the optimal MIMO detector complexity increases exponentially with the number of transmit antennas or/and the number of bits per constellation point.

In this chapter, we propose a novel threshold list subset detector (T-LSS) that extends the List subset detector for iterative turbo MIMO systems (20). List based techniques have also been considered by other researchers in various contexts. In (15), a list-sequential detector, based on a modified stack algorithm, is discussed for MIMO systems. Detection over multiple input multiple output channels for uncoded system using Chase (25) type of algorithm has been proposed in (17), and for multiuser detection in spread spectrum systems in (28). The T-LSS detector generates lists of candidate codewords based on the subset sum algorithm (22). The T-LSS detector do not require any search radius as in LSD rather depends on the threshold value. Second, the lists of codewords are generated at each iteration of the receiver by taking into account the *a priori* information fed back from the channel decoder. Third, the candidate codeword selection in T-LSS does not directly involve channel estimation as in LSD. Our results show that for small average list sizes, the performance of T-LSS is almost same as full *A Posteriori Probability* (APP) detection. Moreover, the average complexity of T-LSS is relatively small. Using Extrinsic Information Transfer (EXIT) chart analysis, we can design the optimal list size per iteration. Our results indicate that we can reduce the complexity significantly for an iterative MIMO system by using T-LSS in the first iteration and using very low list sizes optimized using EXIT chart.

The rest of the chapter is organized as follows. Section II describes the system model. Section III describes the T-LSS detector structure in the iterative framework. Performance analysis of T-LSS is given in Section IV and Section V discusses the implementation complexity. Selection

a) Transmitter structure

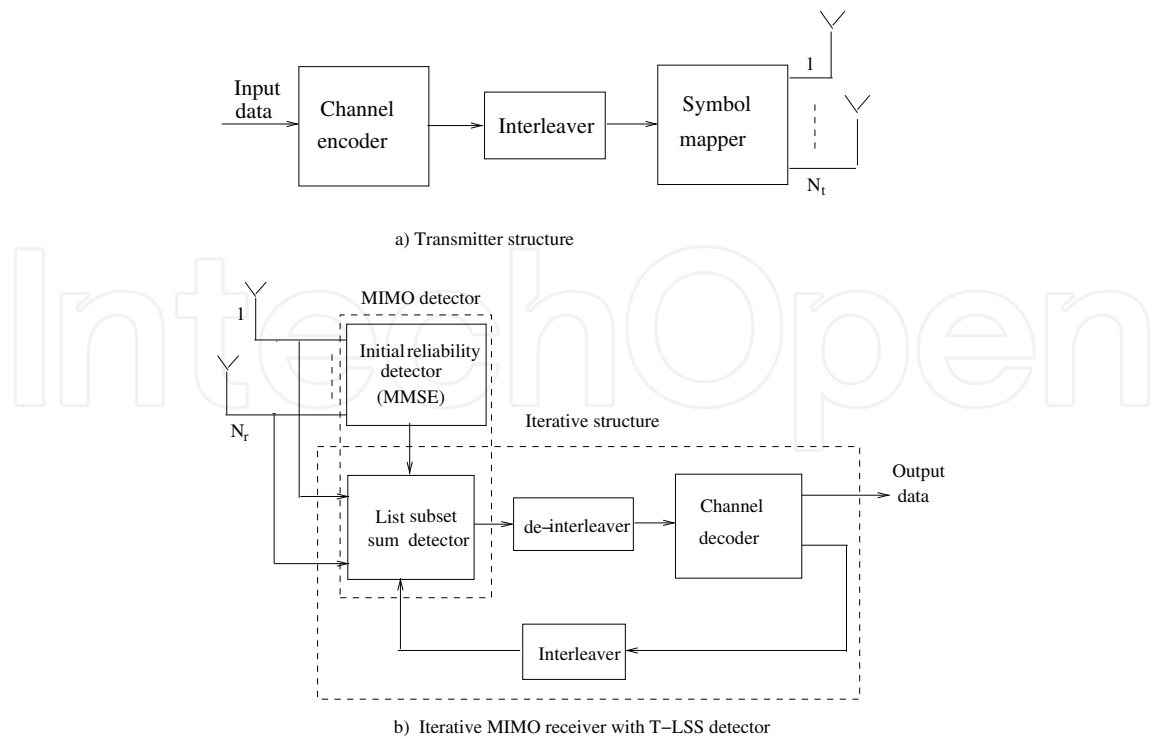b) Iterative MIMO receiver with T–LSS detector

Fig. 1. Block diagram of turbo MIMO system.

of list sizes using EXIT charts is described in Section VI. Section VII discusses numerical results of T-LSS. Finally, conclusions are presented in Section VIII.

Notation: Bold upper case (lower case) letters denote matrices (vectors). The notations $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$, $E[\cdot]$, Re$\{\cdot\}$ and Im$\{\cdot\}$ denote transpose, Hermitian, conjugation, expectation, real and imaginary parts respectively. A^c is the complement of A, $||\cdot||$ is Euclidian norm, and $\binom{m}{k}$ denotes $m!/(m-k)!k!$.

2. System description

Consider a MIMO system with N_t transmit antennas and N_r receive antennas. At the transmitter side, the input data bits are encoded by a channel code, randomly interleaved and then mapped to modulation symbols before being transmitted by the antennas. Denoting a block of information bits by the vector $\mathbf{d}$ and the transfer function of the channel code by $\mathbf{G}$, the output codeword can be written as $\tilde{\mathbf{c}} = \mathbf{G}\mathbf{d}$, and $\mathbf{c} = \Pi(\tilde{\mathbf{c}})$ represents the interleaved sequence of coded bits. The symbol mapper converts this bit sequence into an M-ary symbol sequence as follows. Assume that $\mathbf{c}$ has LN_tM_c elements, where M_c is the number of bits per constellation point and L is a non-negative integer. The symbol mapper first partitions $\mathbf{c}$ into L subvectors, each of length N_tM_c, to form an $N_tM_c \times L$ matrix, $\mathbf{C} = [\mathbf{c}^1, \mathbf{c}^2 ... , \mathbf{c}^L]$, where $\mathbf{c}^l = [c_{(l-1)N_tM_c+1}, c_{(l-1)N_tM_c+2}, ..., c_{lN_tM_c}]^T$, $1 \leq l \leq L$, and c_i is the i-th element of $\mathbf{c}$. Each subvector $\mathbf{c}^l$ is then mapped to the symbol vector $\mathbf{x}^l = [x_1^l, x_2^l, ..., x_{N_t}^l]^T$ through a unique predetermined bit mapping scheme $x_i^l = F[c_{(l-1)N_tM_c+(i-1)M_c+1}, ..., c_{(l-1)N_tM_c+iM_c}]$, $1 \leq i \leq N_t$. We call each sub vector $\mathbf{c}^l$ a code bit vector. The elements of the symbol vector $\mathbf{x}^l$ are then transmitted by the N_t transmit antennas.

Ignoring the superscript (l) for simplicity, the received $N_r \times 1$ vector $\mathbf{r}$ due to the transmission of the $N_t \times 1$ symbol vector $\mathbf{x}^l$ can be written as

$$\mathbf{r} = \mathbf{Hx} + \mathbf{n} \tag{1}$$

where $\mathbf{H}$ is a $N_r \times N_t$ complex MIMO channel matrix, and $\mathbf{n}$ is complex additive noise vector whose elements are independent, complex-valued Gaussian variables with zero mean and variance σ^2 per each dimension.

3. Soft MIMO detectors

3.1 MIMO detector background

In the literature, different optimal and suboptimal structures have been studied. The suboptimal detectors can be classified into linear and nonlinear detectors. Linear detectors include zero-forcing (ZF) and minimum mean-square error (MMSE) detectors, and the nonlinear receivers include decision feedback, nulling-cancelling and variants relying on successive interference cancellation. These suboptimal detectors are easy to implement but their bit error rate (BER) performance is significantly inferior to that of the optimum MIMO detector (10). The authors in (12), (13) studied nulling-cancelling detector with iterative processing.

There has also been considerable progress in using low complexity (near) ML detection based on lattice decoding. In (19), a near optimal detection method is proposed using sphere-decoder (SD) concept, which has low complexity at high SNR. The work in (4) extends the sphere detector algorithm of (19) to coded MIMO systems with complex constellations. It is shown that for good choices of the initial radius and for sufficient list sizes, list sphere decoder (LSD) in a concatenated system can achieve a bit error rate (BER) performance close to capacity limits. However, the worst case complexity of LSD can be very high and the expected complexity is polynomial with the number of transmit antennas (4). Besides, the LSD does not exploit channel decoder information in generating the list of candidate codewords. Based on semi-definite relaxation (SDR) of the ML problem, the authors in (14) proposed a soft Quasi ML detection for MIMO systems.

3.2 Conventional soft MIMO detector

A typical iterative receiver structure consists of an inner MIMO detector and an outer channel decoder. The MIMO detector calculates the log likelihood ratio (LLR) $L_D(c_i|\mathbf{r}) = \log(P(c_i = 1|\mathbf{r})/P(c_i = 0|\mathbf{r}))$, for each bit c_i, $1 \le i \le N_t M_c$, and sends the extrinsic LLR values $L_{\text{ext}}(c_i|\mathbf{r}) = L_D(c_i|\mathbf{r}) - L_A(c_i)$ to the channel decoder, where $L_A(c_i) = \log(P(c_i = 1)/P(c_i = 0))$ is the *a priori* information. A direct computation of detector extrinsic LLR is performed as (9)

$$L_{\text{ext}}(c_i|\mathbf{r}) = \log \frac{\displaystyle\sum_{\mathbf{c}^l \in \text{Ł} \cap c_{i,1}} p\,(\mathbf{r}|\mathbf{c}^l).\exp\left(\frac{1}{2}\mathbf{c}^l{}_{[i]}^T \mathbf{L}_{A,[i]}\right)}{\displaystyle\sum_{\mathbf{c}^l \in \text{Ł} \cap c_{i,0}} p\,(\mathbf{r}|\mathbf{c}^l).\exp\left(\frac{1}{2}\mathbf{c}^l{}_{[i]}^T \mathbf{L}_{A,[i]}\right)} \tag{2}$$

where Ł is a set containing all possible code bit vectors and is called the list. The set $\text{Ł} \cap c_{i,1}$ contains code bit vectors in the list with $c_i = 1$, $(l-1)N_t M_c + 1 \le i \le l N_t M_c$, $\mathbf{L}_{A,[i]}$ denotes the vector of all *a priori* values by omitting the bit i, $\mathbf{c}^l{}_{[i]}^T$ denotes the coded bit vector $\mathbf{c}^l$ by removing the i-th bit, and $p\,(\mathbf{r}|\mathbf{c}^l)$ is the conditional probability density function (pdf).

From (2), it is clear that an exhaustive search over all possible bit vectors requires computation of (2) over a list Ł containing $2^{N_t M_c}$ candidate code bit vectors. This provides MAP detection but its complexity is too high for large $N_t M_c$. In order to reduce complexity, the authors in (4) extend the SD concept for coded turbo MIMO systems by generating a list of code bit vectors from the detector to the decoder. Although the complexity is reduced, the list generation process in (4) does not utilize the information fed back from the channel decoder. Accordingly, the work in (34) modifies the SD algorithm with a Finkle-Pohst MAP search, which is repeated for each iteration. In general, the SD based approach relies on a search radius that sometimes involves trial and error (4). Furthermore, many floating point operations, including QR decompositions or Cholesky decompositions, are required in generating the list of candidate code bit vectors. Therefore, we propose a different soft MIMO detector structure that generates lists requiring only a few floating point operations and mostly comparisons. Besides, instead of using a search radius, we use alternative parameters which are easier to characterize.

3.3 Proposed soft MIMO detector

In our novel receiver structure, we reduce the list size significantly so that the computation of (2) is performed only for a few candidate bit words. Toward this end, we propose to optimize the list from iteration to iteration. Thus, for the k-th iteration between the detector and the decoder, $L_{\text{ext}}^{(k)}(c_i|\mathbf{r})$ is computed as

$$L_{\text{ext}}^{(k)}(c_i|\mathbf{r}) = \log \frac{\displaystyle\sum_{\mathbf{c}^l \in Ł^{(k)} \bigcap c_{i,1}} p\,(\mathbf{r}|\mathbf{c}^l).\exp\left(\frac{1}{2}\mathbf{c}^{l^T}_{[i]}\mathbf{L}_{A,[i]}\right)}{\displaystyle\sum_{\mathbf{c}^l \in Ł^{(k)} \bigcap c_{i,0}} p\,(\mathbf{r}|\mathbf{c}^l).\exp\left(\frac{1}{2}\mathbf{c}^{l^T}_{[i]}\mathbf{L}_{A,[i]}\right)} \tag{3}$$

where $Ł^{(k)}$ is the list of code bit vectors in the k-th iteration. Note that unlike 2, $Ł^{(k)}$ does not usually contain all possible code bit vectors. The detector constructs the list based on the LLR information $\{\alpha_i\}$ for each bit. In the first iteration, we use an initial reliability detector to generate $\{\alpha_i\}$. For subsequent iterations, $\{\alpha_i\}$ are obtained from the channel decoder. Thus, our receiver consists of three components : (1) an initial reliability detector, (2) a list subset sum detector and (3) a channel decoder as shown in Fig. 1. The initial reliability detector and the list subset detector form the proposed MIMO detector.

3.3.1 Initial reliability detector

The initial reliability detector is used only during the first iteration to assist in the generation of the list $Ł^{(1)}$. It consists of a linear minimum mean squared error (MMSE) detector. Such detectors are known to have information lossless property (24). Other detectors namely zero forcing (ZF), zero forcing with successive interference cancellation (ZF-SC), minimum mean squared error with successive interference cancellation (MMSE-SC) can also be used to obtain the initial reliability of bits. Let $\mathbf{y} = \mathbf{P}^H\mathbf{r}$ be the $N_t \times 1$ vector output of MMSE detector, where $\mathbf{P} = (\mathbf{H}\mathbf{R}_x\mathbf{H}^H + \sigma^2\mathbf{I})^{-1}\mathbf{H}$ is the MMSE detector coefficients (10), $\mathbf{R}_x = E[\mathbf{x}\mathbf{x}^H]$, and $\mathbf{I}$ is the identity matrix of size $N_r \times N_r$. Assuming MMSE output as Gaussian (21), the MMSE output is used to obtain the initial reliability of the $N_t M_c$ bits. The reliability of the coded bit c_i is measured by the magnitude of the log-likelihood ratio, $\log(P(c_i = 1|y_t)/P(c_i = 0|y_t))$, where $t = \lceil i/M_c \rceil$. As an example, for quadrature phase-shift keying (QPSK) with constellation

points $(1 + j)/\sqrt{2}$, $(-1 + j)/\sqrt{2}$, $(-1 - j)/\sqrt{2}$, $(1 - j)/\sqrt{2}$ representing bit pairs (0,0), (0,1), (1,1), and (1,0) respectively, $\text{Im}\{y_t\}$ and $\text{Re}\{y_t\}$ give the reliabilities of the first and the second bit respectively (32). Let us denote the reliability of the coded bits by $\{\alpha_i\}$ and let α_i^h be the hard decision of α_i.

3.3.2 List subset sum detector

The list subset sum detector takes reliability values $\{\alpha_i\}$ as input soft information to produce a list $Ł^{(k)}$ of most likely code bit vectors. This list is used to produce extrinsic LLR $L_{\text{ext}}(c_i|\mathbf{r})$ for each bit c_i. It consists of two parts : a list generator and an extrinsic LLR calculator.
List generator: The motivation of list based algorithms is that flipping of α_i^h at a few bit positions may produce the MAP code bit vector in $Ł^{(k)}$ of 3 with high probability. We use the MAP metric of the transmitted vector $\mathbf{c}$, denoted by $J(\mathbf{c})$ as (27)

$$J(\mathbf{c}) = \sum_{i \in S_c} |\alpha_i| - \sum_{i \in S_c^n} |\alpha_i| = \sum_{i \in U} |\alpha_i| - 2 \sum_{i \in S_c^n} |\alpha_i| \tag{4}$$

where $U = \{1, 2, ..., N_t M_c\}$ is the set of bit positions, $S_c = \{i|c_i = \alpha_i^h, i \in U\}$ and $S_c^n = \{i|c_i \neq \alpha_i^h, i \in U\}$. Since the first term in (4) is independent of the value of $\mathbf{c}$, $J(\mathbf{c})$ depends only on the second term $\sum_{i \in S_c^n} |\alpha_i|$. Hence we need to find set of vectors which minimizes the second term. i.e. the perturbation vectors should be designed in ascending order of the sum $S = \sum_{i \in S_c^n} |\alpha_i|$. That is the problem is equivalent to the subset-sum problem (22), where the objective is to find all subsets of a set of numbers that have sum less than or equal to a given number V_{th}. Let $Ł^{(k)}$ be the set of all perturbation vectors whose reliability sum S is below a certain threshold V_{th}, and $\gamma^{(k)}$ be the cardinality of $Ł^{(k)}$. The choice of V_{th} will be discussed in the next section. Let us define a vector $\mathbf{p}_i$ as a zero element vector with a 1 at the i-th bit position. The weight of the vector $\{\alpha_i\}$ is defined as $w = \sum_{i=1}^{N_t M_c} |\alpha_i|$. The list is generated as follows :

Step 1: Sort $|\alpha_j|$, $j = 1, ..., N_t M_c$ in the ascending order. Set the size of the list $\Lambda = 1$, bit position $i = 1$, first term in the list $\mathbf{v}_{\text{old}}^1 = [0\ 0\ 0...0]$ and the weight $w_{\text{old}}^1 = 0$.
Step 2: For each vector in the list $m = 1$ to Λ, create $\mathbf{v}_{\text{new}}^m = \mathbf{v}_{\text{old}}^m + \mathbf{p}_i$, and modify the weight of each vector as $w_{\text{new}}^m = w_{\text{old}}^m + |\alpha_i|$.
Step 3: Merge lists $\mathbf{v}_{\text{new}}$ and $\mathbf{v}_{\text{old}}$ to form a new list $\mathbf{v}_{\text{new}}$.
Step 4: If $k = 1$ i.e., in the first iteration between the detector and the decoder, keep only the vectors in $\mathbf{v}_{\text{new}}$ that have a sum less than or equal to V_{th}.
Step 5: If $k > 1$, then sort vectors in $\mathbf{v}_{\text{new}}$ in ascending order according to their weights w_{new}. Truncate the size of $\mathbf{v}_{\text{new}}$ to $\Lambda^{(k)}$, if the size is greater than $\Lambda^{(k)}$, where $\Lambda^{(k)}$ is the maximum size of the list, and set the value of $\Lambda^{(k)}$ to the size of $\mathbf{v}_{\text{new}}$.
Step 6: Set $\mathbf{v}_{\text{old}} \leftarrow \mathbf{v}_{\text{new}}$.
Step 7: Set $i \leftarrow i + 1$. Repeat Step 2 to 7 if $i \leq N_t M_c$.
Step 8: Re-arrange the positions of the elements in each test vector in $\mathbf{v}_{\text{new}}$ back to their original positions before Step 1, and then add each vector to $\boldsymbol{\alpha}^h$ to generate the test list $Ł^{(k)}$.
Extrinsic LLR calculator : Once the list $Ł^{(k)}$ is available, the extrinsic LLR values $L_{\text{ext}}^{(k)}(c_i|\mathbf{r})$ are computed for each coded bit c_i. For higher number of transmit antennas, computation of (3) is very cumbersome, and max-log approximation is used as (4)

$$L_{\text{ext}}^{(k)}(c_i|\mathbf{r}) \approx \frac{1}{2} \max_{\mathbf{c}^l \in \text{Ł}^{(k)} \cap c_{i,+1}} \left\{ -\frac{1}{\sigma^2} ||\mathbf{r} - \mathbf{Hx}||^2 + \mathbf{c}_{[i]}^{lT} \mathbf{L}_{A,[i]} \right\}$$
$$- \frac{1}{2} \max_{\mathbf{c}^l \in \text{Ł}^{(k)} \cap c_{i,0}} \left\{ -\frac{1}{\sigma^2} ||\mathbf{r} - \mathbf{Hx}||^2 + \mathbf{c}_{[i]}^{lT} \mathbf{L}_{A,[i]} \right\} \tag{5}$$

After all the code bit vectors $\mathbf{c}^l$, $1 \leq l \leq L$, have been processed by the MIMO detector, the extrinsic LLR values associated with $\mathbf{c}$ are made available to the channel decoder as *a priori* information after de-interleaving:

$$L^A(\tilde{\mathbf{c}}) = \Pi^{-1}\{L_{\text{ext}}(\mathbf{c})\} \tag{6}$$

3.4 Channel decoder

The decoder considered in this chapter is a soft-input soft-output (SISO) convolutional decoder based on the BCJR algorithm (30). This decoder is based on trellis structure, where a branch metric computation uses the soft information $L^A(\tilde{\mathbf{c}})$ coming from the detector. Based on forward and reverse recursions of BCJR, the LLR for each coded and data bit is obtained. The *a priori* information $L^A(\tilde{c}_i)$ is removed from the LLR of each coded bit to produce extrinsic information $L^{ext}(\tilde{c}_i)$. Finally, the extrinsic information is fed back to the detector as *a priori* information after interleaving as

$$L_A(\mathbf{c}) = \Pi\{L^{\text{ext}}(\tilde{\mathbf{c}})\} \tag{7}$$

The *a priori* information $L_A c_i$ for each bit c_i is used by the MIMO detector as described in Section III A. The iteration between the detector and the decoder are repeated to decrease the BER. After the last iteration, the LLR of data bits are subjected to hard decision to produce final data decisions.

4. BER analysis of MIMO detector

In this section, we consider BER analysis of the MIMO detector. It is useful for two reasons. First, it directly provides analytical performance results of the proposed detector when used in an uncoded MIMO system. Second, we use this analysis to show selection of V_{th} used in the list generation algorithm. We follow a procedure similar to (32). However, our final results are different from (32).
We consider QPSK modulation. The probability of an error e in bit c_i can be written as

$$P_{b,i} = \sum_{u=0}^{1} P(e|c_i = u)P(c_i = u) \tag{8}$$

Assume bits 0 and 1 to be equally likely and $P(e|c_i = u)$ to be same for $u = 0$ and 1. We can then write $P_{b,i} = P(e|c_i = 0)$. When the MIMO detector uses V_{th} to control the list, we can write

$$P_{b,i} = P(e|c_i = 0, \alpha_i \leq V_{\text{th}})P(\alpha_i \leq V_{\text{th}}|c_i = 0)$$
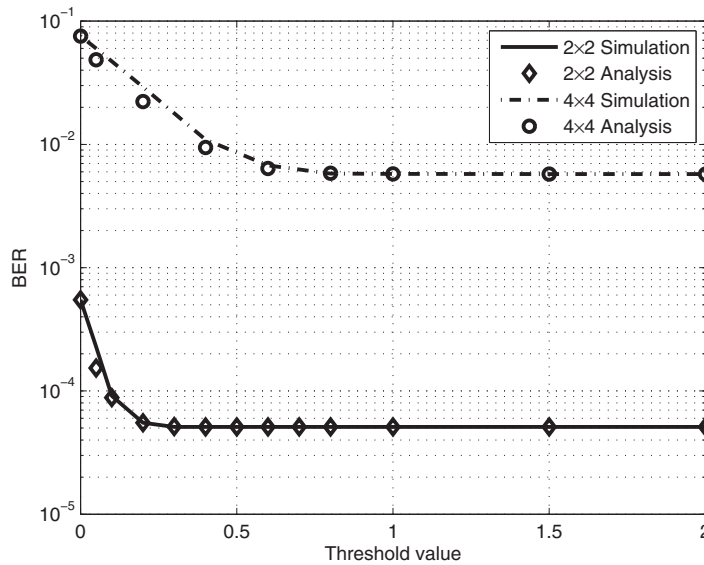$$+ P(e|c_i = 0, \alpha_i > V_{\text{th}})P(\alpha_i > V_{\text{th}}|c_i = 0) \tag{9}$$

Fig. 2. Effects of V_{th} on the BER of T-LSS based MIMO system.

Let us denote $P(\alpha_i > V_{\text{th}}|c_i = 0)$ as $P_{\delta,i}$. When $\alpha_i > V_{\text{th}}$, the decision on the i-th bit is 1. Hence $P(e|c_i = 0, \alpha_i > V_{\text{th}}) = 1$. We approximate $p(\alpha_i|c_i = 0)$ to be Gaussian with mean μ_i and variance σ_i^2. Thus, we get, $P_{\delta,i} = Q((V_{\text{th}} + |\mu_i|)/\sigma_i^2)$ where $Q(.)$ is the Gaussian tail probability (10). The evaluation of the term $P(e|c_i = 0, \alpha_i \leq V_{\text{th}})$ is difficult. However, it can be approximately evaluated as follows.

Consider asymptotically large V_{th}, i.e., $V_{\text{th}} \to \infty$. In this case, the list contains all possible $2^{N_t M_c}$ members. Obviously, in this case, $P(e|c_i = 0, \alpha_i \leq V_{\text{th}} \to P_{\text{ML}})$, where P_{ML} is the bit error probability for a maximum likelihood detector. Even for a moderately large V_{th}, since we are interested in the near-ML region of operation, we can still write $P(e|c_i = 0, \alpha_i \leq V_{\text{th}}) \approx P_{\text{ML}}$. Therefore, from (9)

$$P_{b,i} \approx (1 - P_{\delta,i})P_{\text{ML}} + P_{\delta,i} \tag{10}$$

The average BER can be written as

$$P_b = \frac{1}{N_t M_c} \sum_{i=1}^{N_t M_c} P_{b,i} = P_{\text{ML}} + \frac{1 - P_{ML}}{N_t M_c} \sum_{i=1}^{N_t M_c} P_{\delta,i} \tag{11}$$

We have found this expression to provide accurate results in our extensive simulations. As an example, Figure 2 shows our results for various values of V_{th}. The channel is randomly generated and kept fixed at an SNR of 10 dB. For the 2×2 system, the rows of the channel matrix is are [1.3-j0.19 0.08-j0.16], and [0.84-j0.09 -0.85-j0.81] respectively. For the 4×4 system, the rows are [0.27-j0.23 0.35-j0.21 0.21+j1.51 0.91-j0.52], [0.82+j0.46 0.40+j0.50 -0.69+j0.024 0.35-j1.202], [1.16+j0.56 0.38-j0.03 0.47-j0.27 1.11+j1.25], and [-0.02-j0.14 -0.15-j0.36 0.69+j0.75 1.16+j0.24] respectively.

The selection of V_{th} is done as follows. Suppose we want P_b to be within a fraction of ϵ of P_{ML}, i.e., $P_b = (1 + \epsilon)P_{\text{ML}}$. From (11), we get

$$\sum_{i=1}^{N_t M_c} P_{\delta,i} = (\frac{\epsilon P_{\text{ML}}}{1 - P_{\text{ML}}})N_t M_c \tag{12}$$

which gives

$$\sum_{i=1}^{N_t M_c} Q\left(\frac{V_{\text{th}} + |\mu_i|}{\sigma_i}\right) = \left(\frac{\epsilon P_{\text{ML}}}{1 - P_{\text{ML}}}\right) N_t M_c \tag{13}$$

Thus, V_{th} can be found by solving (13). The parameter P_{ML} is difficult to obtain (31). We propose to use interference-free error performance P_{IF} in place of P_{ML}. This is given by

$$P_{\text{IF}} = \frac{1}{N_t M_c} \sum_{i=1}^{N_t M_c} P_{\text{IF},i} = \frac{1}{N_t M_c} \sum_{i=1}^{N_t M_c} Q(\sqrt{2\gamma_i}) \tag{14}$$

where γ_i is the average received SNR for the i-th bit when only the symbol x_l, $l = \lceil i/M \rceil$, is present in the system and other symbols are set to zero. Since $P_{\text{IF}} \leq P_{\text{ML}}$, using P_{IF} for P_{ML} in (13) ensures an error performance better than or equal to $(1 + \epsilon)P_{\text{ML}}$. The parameters μ_i and σ_i are obtained as described in (32).

5. Complexity analysis

In this section we describe the exact number of computations needed for T-LSS detector. The complexity depends on
1. MMSE detector
2. List generator
3. Extrinsic LLR calculator

5.1 MMSE detector
Let N_v be the number of V_{th} values where (13) is evaluated. The calculation of V_{th} requires about $(N_v + 1)N_t M_c + 3$ multiplications and $(N_v + 1)(N_t M_c - 1)$ additions. the Q function values can be obtained from tables or through a combination of tables and a few floating point operations.
Total number of multiplications for computing $\mathbf{H}\mathbf{R}_x\mathbf{H}^H + \sigma^2\mathbf{I}$ is equal to $(1/2)N_r(N_r + 1)(N_t + 1)$ assuming $\mathbf{R}_x = \sigma_s^2\mathbf{I}$, where $\sigma_s^2 = E[|x_i|^2]$. The number of addition operations is equal to $(1/2)N_r(N_r + 1)(N_t - 1) + N_r$. The number of multiplications and additions for calculating $(\mathbf{H}\mathbf{R}_x\mathbf{H}^H + \sigma^2\mathbf{I})^{-1}\mathbf{H}$ is equal to $N_r^2 N_t$ and $(N_r - 1)N_r N_t$ respectively. The generation of MMSE detector coefficients requires matrix inversion of size $N_r \times N_r$. From (23), matrix inversion operation requires N_r^3 multiplications and $N_r^3 - 2N_r^2 + N_r$ additions. Since the MMSE detector gives reliability of received code vector of $N_t M_c$ bits, the total number of multiplications per bit is equal to $((1/2)N_r(N_r + 1)(N_t + 1) + N_r^3 + N_r^2 N_t + (N_v + 1)N_t M_c + 3)/N_t M_c$ and addition operations is approximately equal to $(N_r^3 - 2N_r^2 + 2N_r + (1/2)N_r(N_r + 1)(N_t - 1) + (N_r - 1)N_r N_t + (N_v - 1)(N_t M_c - 1))/N_t M_c$. Note that in the case of block fading channels, these operations need to be done only once for the whole block. Then the number of operations per bit will decrease depending on the block size.

5.2 List generator
The sorting operation in Step 1 is needed over $N_t M_c$ elements. Hence the total number of comparison operations are $O(N_t M_c \log_2(N_t M_c))$. This can usually be neglected compared to the other comparison operations. Step 2 requires $\sum_{j=1}^{N_t M_c} 2^{\min(j-1, \log_2 \gamma^{(k)})} = 2\gamma^{(k)} - 1 + (N_t M_c - \log_2 \gamma^{(k)})\gamma^{(k)}$ additions, since the list is restricted to a maximum size of $\gamma^{(k)}$, and until that size is reached, the list size doubles for every increase in i value in $\mathbf{p}_i$. In Step 4, we compare the sum with V_{th} for each member in the list. Since the number of

members doubles for every increase in i value in $\mathbf{p}_i$ and half of the members in the list each time have already been compared, we have a worst case number of comparisons as $2\gamma^{(1)} - 1 + (N_t M_c - \log_2 \gamma^{(1)})\gamma^{(1)}$. In Step 5, until i exceeds $\log_2 \gamma^{(k)}$, only $(2^i - 1)$ comparisons are needed for a given i. This is due the special structure of the algorithm where the members after merging in Step 3 remain in the form of two sorted lists. Therefore, the cumulative number of comparisons till i exceeds $\log_2 \gamma^{(k)}$ can be obtained by summing $(2^i - 1)$ over i. Defining $\gamma^{(k)'} = 2^n$, where $n = \lceil \log_2 \gamma^{(k)} \rceil$, the number of comparisons is about $2\gamma^{(k)'} - \log_2 \gamma^{(k)'}$. When $i > \log_2 \gamma^{(k)}$ then the list doubles (after Step 3) for every increase in the i value, but members remain in two sorted lists. We thus need to select $\gamma^{(k)}$ elements from $2\gamma^{(k)}$ elements available in two sorted lists. Thus, $\gamma^{(k)}$ comparisons are needed for each i when $i > \log_2 \gamma^{(k)}$. Thus, Step 5 requires $2\gamma^{(k)'} - \log_2 \gamma^{(k)'} + (N_t M_c - \log_2 \gamma^{(k)})\gamma^{(k)}$ comparisons. In summary, if η iterations are used, the total number of additions per bit is approximately $(1/(N_t M_c))\sum_{k=1}^{\eta}(2\gamma^{(k)} - 1 + (N_t M_c - \log_2 \gamma^{(k)})\gamma^{(k)})$. The total number of comparisons per bit is about $\eta \log_2(N_t M_c) + (1/(N_t M_c))(2\gamma^{(1)} - 1 + (N_t M_c - \log_2 \gamma^{(1)})\gamma^{(1)}) + (1/(N_t M_c))\sum_{k=2}^{\eta}(2\gamma^{(k)'} - \log_2 \gamma^{(k)'} + (N_t M_c - \log_2 \gamma^{(k)})\gamma^{(k)})$.

5.3 Extrinsic LLR calculator

The extrinsic LLR values are calculated using (5). We can see that $\mathbf{r} - \mathbf{Hx}$ requires $N_r N_t$ multiplications and $N_r N_t$ addition operations for each member in the list. The norm operation $||\mathbf{r} - \mathbf{Hx}||^2$ requires N_r multiplications and $N_r - 1$ additions. Multiplication by $1/\sigma^2$ requires 1 multiplication operation. The term $\mathbf{c'}_{[i]}^T \mathbf{L}_{A,[i]}$ require $N_t M_c - 2$ addition operations and the summation of the two terms require 1 addition operation. There are $\gamma^{(k)}$ comparison operations. Thus there are $\frac{1}{N_t M_c}(N_r N_t + N_r + 1)\sum_{k=1}^{\eta}\gamma^{(k)}$ multiplications per bit, $\frac{1}{N_t M_c}(N_r N_t + N_t M_c + N_r - 2)\sum_{k=1}^{\eta}\gamma^{(k)}$ additions per bit and $\frac{1}{N_t M_c}\sum_{k=1}^{\eta}\gamma^{(k)}$ comparisons per bit.

6. Iterative list detection for turbo MIMO systems

For MIMO systems concatenated with a channel code, the list size is controlled using V_{th} during the first iteration. Equation (5) is used to generate extrinsic LLR values that are passed to the channel decoder via the de-interleaver. Since $Ł^{(k)}$ does not contain all the code bit vectors, it may so happen for a few bits that they are either 0 or 1 throughout the list. In these cases, we can set the LLR of that particular bit to some fixed value such as +8 or -8 depending on the bit reliability, or we can use the bit flipping algorithm as in (28). In bit flipping method, there is a list size penalty. Once we obtain the channel decoder output information of all the coded bits, they are passed through the interleaver to the MIMO detector. Since the channel decoder outputs are more reliable than the initial reliability detector, the detector uses them for subsequent iterations. In that case, the list size can simply be fixed at the value provided by the first iteration. Fixed list sizes have also been considered in (20). However, further reduction in list sizes can be obtained using an EXIT chart as described below.

6.1 List detector transfer curves

The mutual information transfer curve of List MIMO detector can be computed by assuming *a priori* information L from the channel decoder to be Gaussian distributed with variance σ_A^2 and mean $\sigma_A^2/2$. The measure of information is computed between the *a priori* LLR and the

true transmitted bits as well as between extrinsic output LLR and the transmitted bits. The mutual information at the input of the MIMO detector is denoted as I_{A_1} and at the output as I_{E_2}. The mutual information between transmitted bit x and the LLR L is given by (35), (36)

$$I(B,L) = \frac{1}{2} \sum_{d_i=-1,1} \int_{-\infty}^{\infty} p(l|B=b) \times$$

$$\log_2 \frac{2p(l|B=b)}{p(l|B=-1) + p(l|B=1)} dl \qquad (15)$$

The measure of a-priori information is characterized by σ_A^2. The value σ_A for that particular measure of input mutual information I_A is calculated using the relationship

$$J(\sigma) = I_{A_1}(\sigma_A = \sigma) \qquad (16)$$

where

$$J(\sigma_A) = 1 - \int_{-\infty}^{\infty} \frac{e^{-(l-\sigma_A^2/2)^2/2\sigma_A^2}}{\sqrt{2\pi\sigma_A^2}} \log_2(1 + e^{-l}) dl \qquad (17)$$

For every given value of I_A, the MIMO detector is simulated to obtain histogram based pdf, which is used in 15 to compute the mutual output information I_{E_1}.

6.2 List optimization using mutual information approach

Since we would like to ensure convergence, the EXIT optimization is done under two worse scenarios: (1) A low SNR value is used so that the list size found from the chart also works at high SNR. In practice, charts pre-computed at several SNR values can be used. (2) The EXIT chart assumes that extrinsic LLR is used both in the list generator as well as in the extrinsic LLR calculator. This is required to simplify the EXIT plots. This, however, is a worse scenario compared to the actual algorithm where full LLR is used in the list generator while extrinsic LLR is used in the extrinsic LLR calculator.

Fig. 3 shows the EXIT characteristics for the MIMO detector using T-LSS and a rate 1/2 convolutional decoder with constraint length 7 with polynomials [147, 117]. From the numerical simulations, the average list size for T-LSS is found to be 35. This is the list size in the first iteration. For subsequent iterations, the figure shows that the list size can be reduced to 32 and 24, that can provide performance very close to a full list with 256 members.

Fig. 4 shows the EXIT characteristics for the case of $N_t > N_r$, In this case, the average list size for the T-LSS detector is 81. The list size can be reduced to 48 and then to 32 for the second and higher iterations. This can reduce numerical complexity significantly.

7. Numerical Results and Discussions

A MIMO system with N_t transmit and N_r receive antennas is considered. We use a complex Gaussian random MIMO channel. In our work, the channel changes for each code bit vector although our technique applies to block fading channels as well. The channel information is assumed to be known at the receiver only. QPSK modulation is considered for all the cases. Unless specified, ϵ is set to 0.01.

Fig 5 shows the BER plot of the MIMO detector (uncoded system) with symmetrical configurations for 2×2, 4×4, 6×6 and 8×8 systems. Full search maximum likelihood detector (MLD) results are also shown. It can be seen that with T-LSS we can reach the
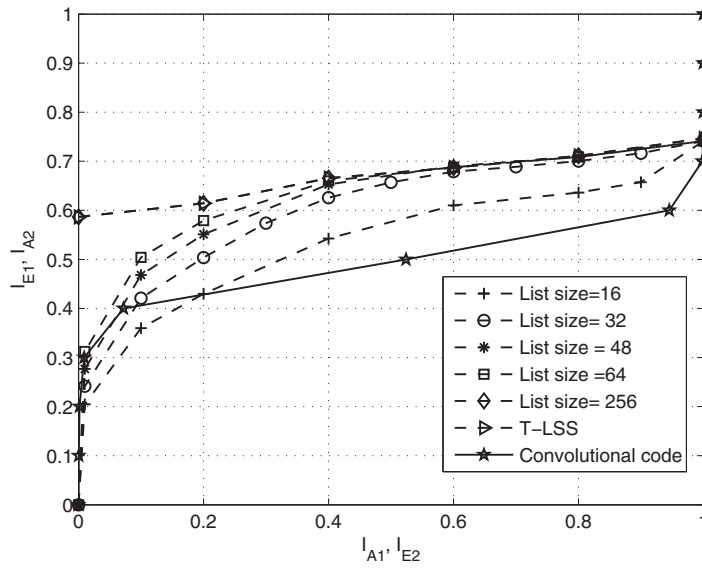
Fig. 3. EXIT characteristics for 4×4 MIMO system with LSS detector at $E_b/N_0 = 4.0$ dB. Note that average complexity for T-LSS is 35.
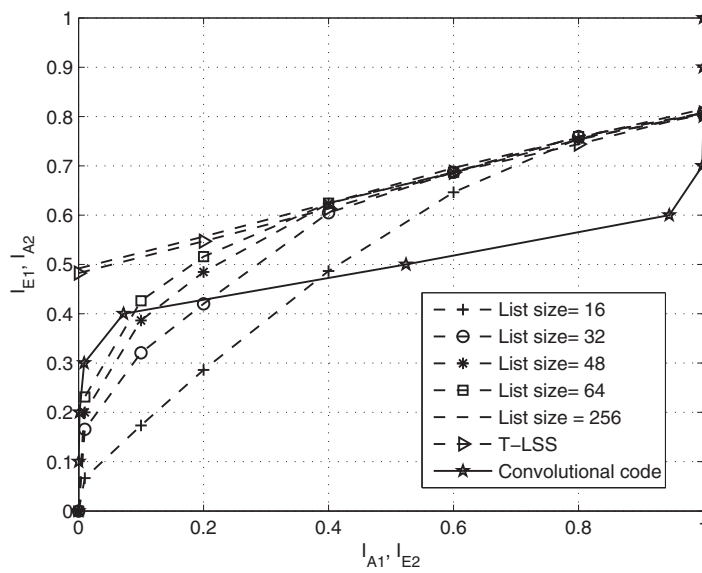

Fig. 4. EXIT characteristics for 4×2 MIMO system with T-LSS detector at $E_b/N_0 = 6.0$ dB. Note that average complexity for T-LSS is 81.

same performance as that of MLD. The average list S_{avg} required for T-LSS symmetrical configuration is shown in Fig 6. We observe that we can get significant reduction in the list size at high E_b/N_o. For example, for a 4×4 system at $E_b/N_o = 14dB$, the average list size equal to 18, as compared to 256 for MLD.
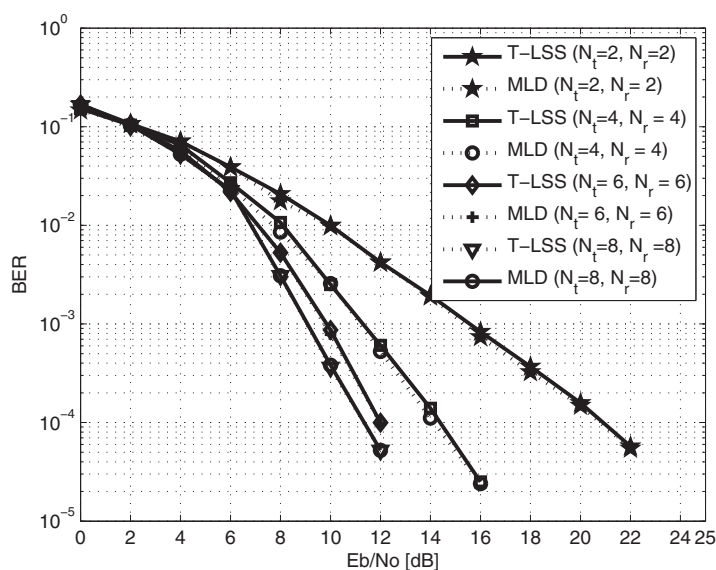
Fig. 5. BER performance of T-LSS for symmetrical configuration.

Fig 7 shows the BER results for a $4 \times N_r$ system for different values of N_r. We note that the T-LSS detector works for any of the combinations of N_t and N_r unlike the SD, or the original vertical Bell Labs layered space-time (V-BLAST) algorithm. Fig 8 shows the average list sizes of the T-LSS for these configurations. Reduction in the number of receiver antennas results in a loss in diversity and, therefore, an equivalent SNR loss. Therefore, the benefit does
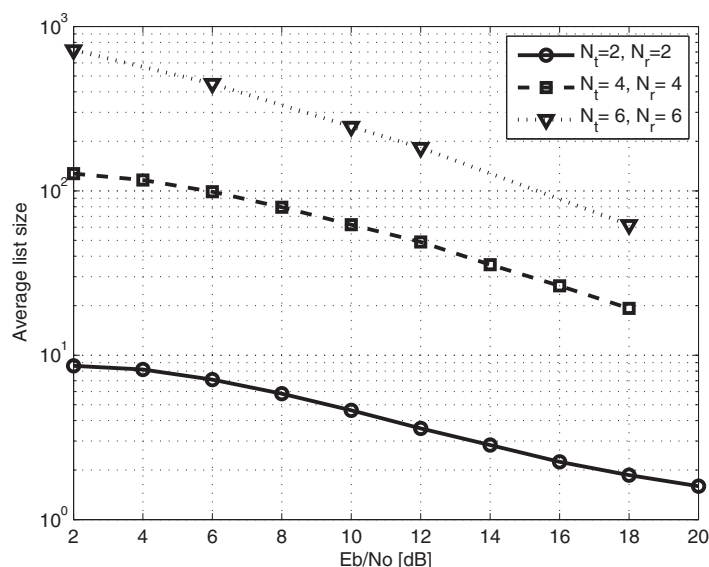
Fig. 6. Average list size per bit of T-LSS for symmetrical configuration.

not significantly improve with an increase in SNR in the low SNR region. Nevertheless, the average list size for the 4×2 system is reduced to 90 in contrast with the ML size of 256.
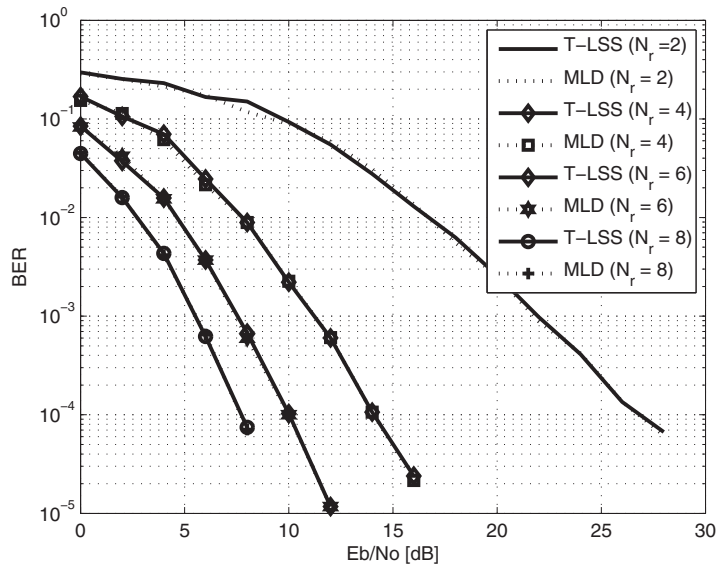

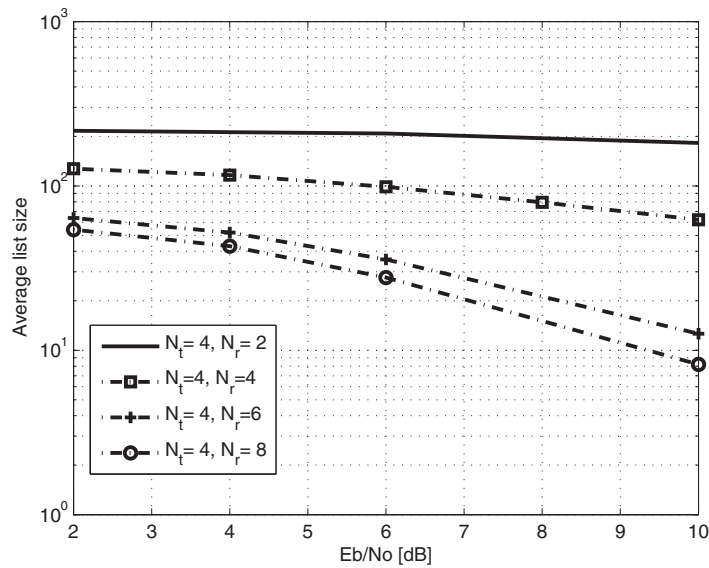Fig. 7. BER performance of T-LSS for $4 \times N_r$ systems.

Fig. 8. Average list size per bit of T-LSS for $4 \times N_r$ systems.

The role of the parameter ϵ on the average list size for a 4×4 system is shown in Fig 9. Note that to get exact ML performance, we need a list size of 256 in this case. However, by relaxing the requirement to $(1 + \epsilon)P_{ML}$, the list size is significantly reduced. Thus at a 15 dB SNR, we reduce the list size 17 times by targeting for a BER of $1.01P_{ML}$ instead of exact P_{ML}. The complexity reduction depends on the SNR.

Fig. 10 shows the BER plot for the iterative MIMO system. A convolutional code with polynomials as described in Section VI is considered. In the first iteration, the list is generated from the T-LSS algorithm. Subsequent iterations use $\gamma^{(k)}$ obtained from the EXIT chart at 4.0

dB (Fig 3). We observe that the BER performance of the proposed method closely matches with full search MLD where a list size of 256 is used. For the proposed method, using list sizes of 35, 32, 24 and 24 respectively in the four iterations, the number of multiplications per bit is about 420 using $N_v = 100$. For the full list MLD, the number of multiplications per bit is about 2688 that comes from the extrinsic LLR calculator. For larger number of antennas or constellation sizes, the potential benefit in complexity will be much more significant.
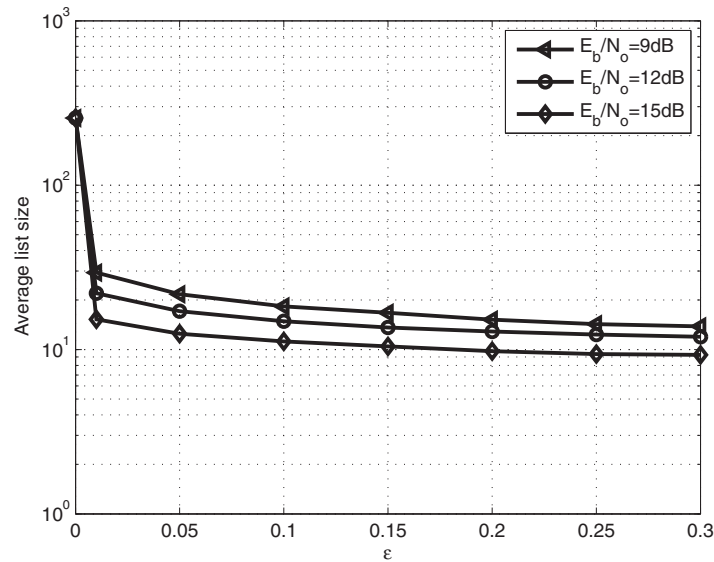
Fig. 9. Effects of ϵ on the complexity.
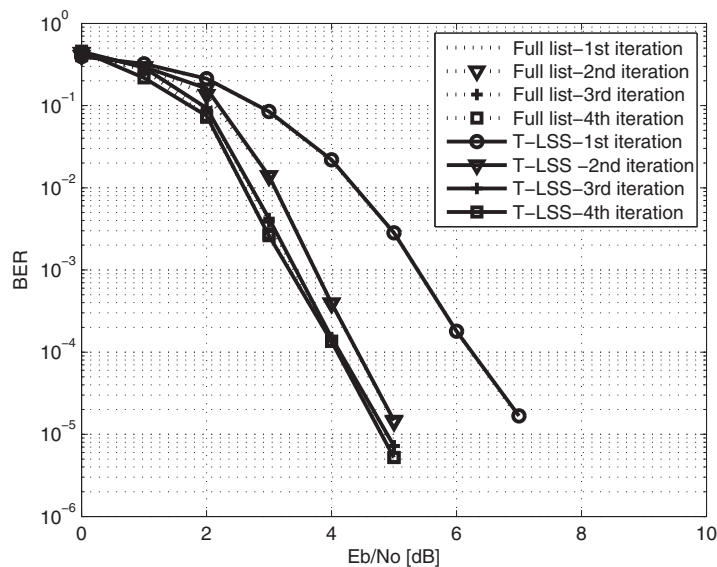
Fig. 10. BER performance of iterative T-LSS for 4×4 MIMO system.

8. Conclusions

A soft input soft output MIMO detector for turbo MIMO systems is presented. The detector searches over a list of candidate codewords to find the most likely codeword vector. The

BER performance of the detector is analyzed, and a method to choose the optimal list size is described. During the first iteration, the detector uses a threshold value to limit the number of list members, while subsequent iterations use list sizes obtained through EXIT analysis. The numerical complexity of the detector is analyzed. We show that the detector provides BER performance very close to the use of ML detector based receiver at significantly low complexity. Possible future work includes design of capacity approaching irregular LDPC codes based on the proposed T-LSS detector.

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MIMO Systems, Theory and Applications

Edited by Dr. Hossein Khaleghi Bizaki

In recent years, it was realized that the MIMO communication systems seems to be inevitable in accelerated evolution of high data rates applications due to their potential to dramatically increase the spectral efficiency and simultaneously sending individual information to the corresponding users in wireless systems. This book, intends to provide highlights of the current research topics in the field of MIMO system, to offer a snapshot of the recent advances and major issues faced today by the researchers in the MIMO related areas. The book is written by specialists working in universities and research centers all over the world to cover the fundamental principles and main advanced topics on high data rates wireless communications systems over MIMO channels. Moreover, the book has the advantage of providing a collection of applications that are completely independent and self-contained; thus, the interested reader can choose any chapter and skip to another without losing continuity.

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