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Fibre Bundle Models and 3D Object Recognition

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1. Introduction

Surface modeling is a fundamental issue in 3D images, shape synthesis and recognition. Recently, a surface model called the fibre bundle model is proposed (Chao et al., 2000; Chao & Kim, 2004; Chao et al., 2004; Chao & Li, 2005; Suzuki & Chao, 2002). The fibre bundle model becomes powerful in its full generality however still needs much information, the information of these two curves at all points, which are roughly equivalent to the information of pointwise representation itself. On the other hand, to apply such a surface model efficiently in shape generation and representation, one needs to know the geometrical quantities of the model.

Object recognition techniques using 3D image data are expected to play an important role in recognition-synthesis image coding of 3D moving pictures or animations in virtual environments and image communications. Currently used 3D free object representations seem insufficient in the sense that, for models such as generalized cylinders or super-quadratics, it is usually difficult to find the invariant features, especially to find the complete set of invariants, which is defined as the smallest number of invariants in order to uniquely determine and reproduce the shapes (Chao & Ishii, 1999; Chao & Suzuki, 2002; Chao et al., 1999; Kawano et al., 2002; Sano et al., 2001).

From the view of differential geometry and Riemannian geometry, the authors obtained the geometric structures about fibre bundle models (Li et al., 2008, 1; 2). Meanwhile, an algorithm of 3D object recognition using the linear Lie algebra models is presented, including a convenient recognition method for the objects which are symmetrical about some axis (Li et al., 2009).

2. Fibre bundle model and its geometry

2.1 Fibre bundle model of surfaces

A surface $F = \{F(u,v)\}$ is called a fibre bundle on a given base curve $b = \{b(v), v \in \mathbb{R}\}$, if locally (i.e. at a neighborhood of any point) F is a direct product of b and another curve called a fibre curve. More specifically, as shown in Fig. 1, there is a projection map $\pi : F \to b$, such that for a point $x \in b$, there is a curve $f_x = \{f_x(u), u \in \mathbb{R}\}$ on F

$$f_x := \pi^{-1}(x) \subset F, \tag{1}$$

which is called a fibre at the base point x. For any $x \in b$, there is a neighborhood of x in $b : U_x \in b$ such that

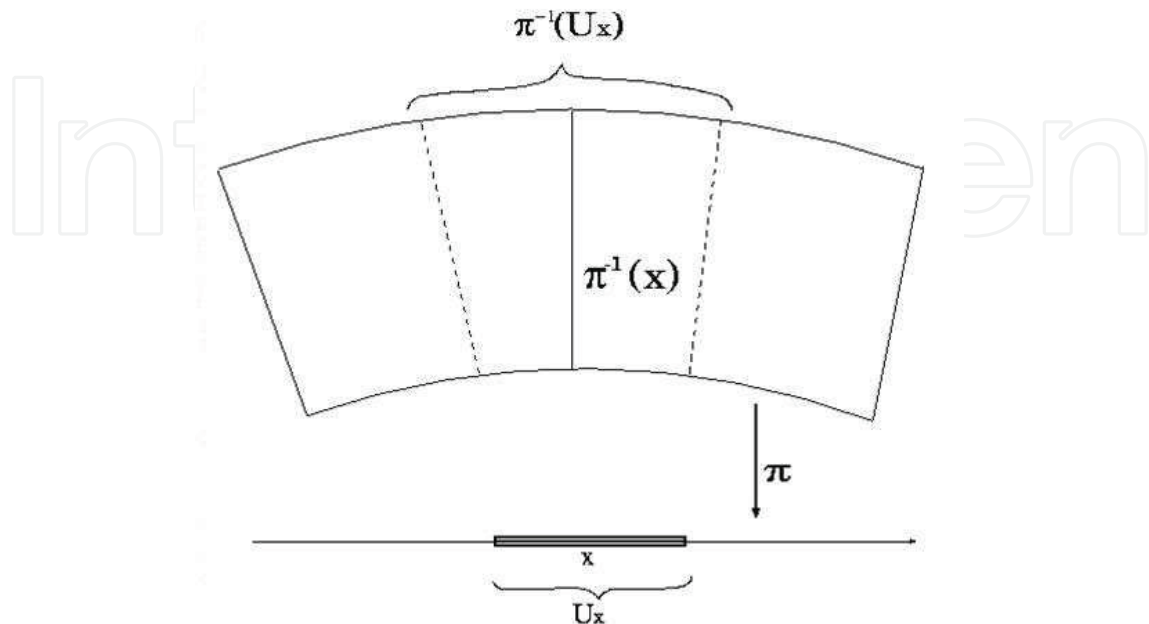$$\pi^{-1}(U_x) \cong U_x \times f_x. \tag{2}$$

Fig. 1. Fibre bundle model of surfaces.

The fibre bundle model can represent any surface F, e.g. generalized cylinders and ruled surfaces are special cases of this model.

On the other hand, the fibre bundle model is a local direction product, which means the fibre curves could be very different at each base point. Such a nontrivial fibre bundle thus can represent arbitrary complicated surface.

2.2 Fibre bundle model of 1-parameter lie groups of linear lie algebra and its geometry

Let the fibre curve of the fibre bundle model be a 1-parameter Lie group

$$g_v = \{g_v(u) = e^{Au}b(v), u \in \mathbb{R}\}, \tag{3}$$

where A is a 3×3 matrix called the representation matrix of the fibre curve. Therefore, the surface is defined as

$$F = \{x(u,v) = e^{Au}b(v), u,v \in \mathbb{R}\}. \tag{4}$$

The points $b(v)$ on the base curve b are initial points of integral flows of 1-parameter Lie groups g_v.

The Lie algebra of this fibre bundle (roughly can be regarded as its tangent vector field) is a linear Lie algebra

$$L := \frac{\partial x}{\partial u} = x_u = Ae^{Au}b(v) = Ax. \tag{5}$$

A major advantage of this model is that the Lie algebras of fibre curves are uniquely determined by a complete set of invariants I under Euclidean transformation

$$I = \{\sigma_1, \sigma_2, \sigma_3, \phi_1, \phi_2, \phi_3\}, \tag{6}$$

where $\{\sigma_i\}$ are the singular values of A, assuming the singular value decomposition of $A^R = R^T A R, R \in SO_3(\mathbb{R})$ is $A^R = U^T \Lambda V, \Lambda = diag\{\sigma_i\}$. $\{\phi_i\}$ are the Euler angles of $VU^T \in SO_3(\mathbb{R})$. Here, a complete set of invariants is the minimal set of invariants which can uniquely determine the curve. Thus, information to describe the fibre-bundle surface model is the base curve and the six invariants of the linear Lie algebra, i.e. of the representation matrix A.
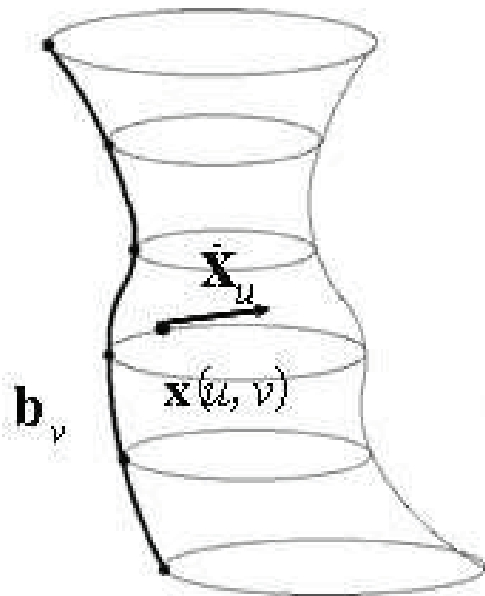

Fig. 2. Fibre bundle of 1-parameter groups of linear Lie algebra.

Lemma 2.1 For any $n \times n$ matrix $A = (a_{ij})$ where $a_{ij} \in \mathbb{R}$ and any $u \in \mathbb{R}$, one has

1. $\frac{d}{du}(e^{Au}) = Ae^{Au} = e^{Au}A$;

2. $\det(e^{Au}) = e^{\text{tr}(Au)} = e^{\text{tr}(A)u}$, where $\text{tr}(A)$ is the trace of A;

3. when $AB = BA$, $e^{A+B} = e^A e^B = e^B e^A$;

4. $(e^A)^T = e^{A^T}$.

Lemma 2.2 $A \in so_3(\mathbb{R}) = \{B|B^T = -B\}$, then $e^{Au} \in SO_3(\mathbb{R}) = \{B|B^T B = I, |B| = 1\}$, where $u \in \mathbb{R}$.

Proof. If $A \in so_3(\mathbb{R})$, we can get that $A^T = -A$ and $\text{tr}(A) = 0$. Then from Lemma 2.1, we get $(e^{Au})^T e^{Au} = e^{A^T u} e^{Au} = e^{(A^T + A)u} = I$ and $\det(e^{Au}) = e^{\text{tr}(A)u} = 1$. Therefore, $e^{Au} \in SO_3(\mathbb{R})$.

Theorem 2.1 For the fibre bundle surface defined by

$$F := \{x(u,v) = e^{Au}b(v), u, v \in \mathbb{R}\}, \tag{7}$$

where $A \in so_3(\mathbb{R})$, and $b(v)$ is a two order differentiable vector, the Gaussian curvature K and the mean curvature H of the fibre bundle surface are given by

$$K = \frac{\det(Ab, b', A^2 b) \det(Ab, b', b'') - (\det(Ab, b', Ab'))^2}{(|Ab|^2|b'|^2 - ((Ab) \cdot b')^2)^2} \tag{8}$$

and

$$H = \frac{1}{2} \frac{|b'|^2 \det(Ab, b', A^2 b) - 2((Ab) \cdot b') \det(Ab, b', Ab') + |Ab|^2 \det(Ab, b', b'')}{(|Ab|^2 |b'|^2 - ((Ab) \cdot b')^2)^{\frac{3}{2}}}, \quad (9)$$

respectively, where $(\cdot)$ denotes the inner product of vectors and det the determinant of an $n \times n$ matrix, respectively.

Proof. Firstly, from the Lemmas we can get

$$x_u = Ae^{Au}b, x_v = e^{Au}b'$$

and

$$x_{uu} = e^{Au}A^2 b, x_{uv} = x_{vu} = e^{Au}Ab', x_{vv} = e^{Au}b''.$$

Then we can get

$$E = (x_u \cdot x_u) = |Ab|^2, F = (x_u \cdot x_v) = ((Ab) \cdot b'), G = (x_v \cdot x_v) = |b'|^2,$$

$$| x_u \times x_v |^2 = (x_u \cdot x_u)(x_v \cdot x_v) - (x_u \cdot x_v)^2 = |Ab|^2 |b'|^2 - ((Ab) \cdot b')^2$$

and

$$L = \frac{(x_u, x_v, x_{uu})}{|x_u \times x_v|} = \frac{\det(Ab, b', A^2 b))}{\sqrt{|Ab|^2 |b'|^2 - ((Ab) \cdot b')^2}},$$

$$M = \frac{(x_u, x_v, x_{uv})}{|x_u \times x_v|} = \frac{\det(Ab, b', Ab')}{\sqrt{|Ab|^2 |b'|^2 - ((Ab) \cdot b')^2}},$$

$$N = \frac{(x_u, x_v, x_{vv})}{|x_u \times x_v|} = \frac{\det(Ab, b', b'')}{\sqrt{|Ab|^2 |b'|^2 - ((Ab) \cdot b')^2}}.$$

Therefore the Gaussian curvature is given by

$$K = \frac{LN - M^2}{EG - F^2}$$

$$= \frac{\det(Ab, b', A^2 b) \det(Ab, b', b'') - (\det(Ab, b', Ab'))^2}{(|Ab|^2 |b'|^2 - ((Ab) \cdot b')^2)^2}$$

and the mean curvature is given by

$$H = \frac{1}{2} \frac{LG - 2MF + NE}{EG - F^2}$$

$$= \frac{1}{2} \frac{|b'|^2 \det(Ab, b', A^2 b) - 2((Ab) \cdot b') \det(Ab, b', Ab') + |Ab|^2 \det(Ab, b', b'')}{(|Ab|^2 |b'|^2 - ((Ab) \cdot b')^2)^{\frac{3}{2}}}.$$

Example 2.1 Taking $b(v) = (0, \sin(v), \cos(v))^T$, $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, from Lemma 2.2, we know that $e^{Au} \in SO_3(\mathbb{R})$, then we can get $x(u, v) = e^{Au}b(v)$. Thus, we can get

$$b' = (0, \cos(v), -\sin(v))^T, b'' = (0, -\sin(v), -\cos(v))^T,$$

$$Ab = (-\sin(v), 0, 0)^T, Ab' = (-\cos(v), 0, 0)^T, A^2 b = (0, -\sin(v), 0)^T.$$

Then the Gaussian curvature is given by

$$K = 1$$

and the mean curvature is

$$H = \begin{cases} 1, & \sin(v) \geq 0 \\ -1, & \sin(v) < 0 \end{cases}.$$
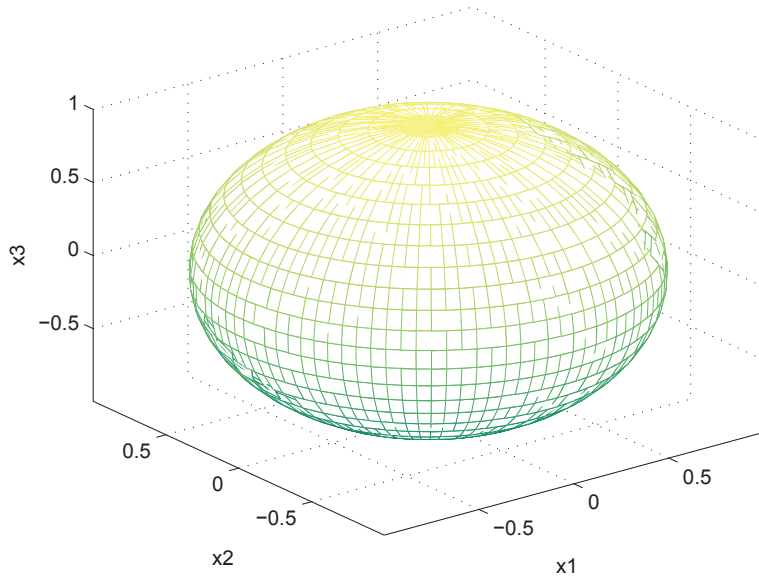
$$x(u,v)=(-\sin(v)\sin(u),\sin(v)\cos(u),\cos(v))$$

Fig. 3. The figure of $x(u,v)$ in Example 2.1.

Example 2.2 Taking $b(v) = (1,1,v)^T$, $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, we get $x(u,v) = e^{Au}b(v)$. Therefore we can get

$$b' = (0,0,1)^T, b'' = (0,0,0)^T,$$
$$Ab = (-1,1,0)^T, Ab' = (0,0,0)^T, A^2b = (-1,-1,0)^T.$$

Then the Gaussian curvature is given by

$$K = 0$$
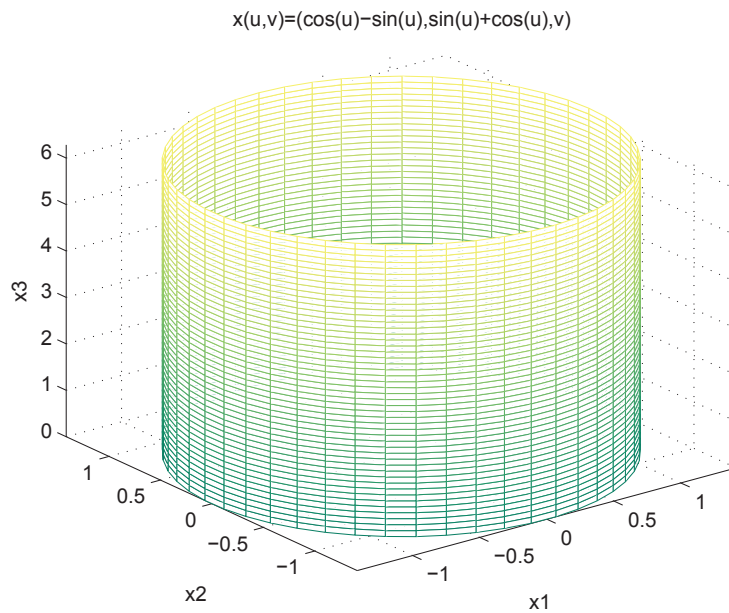
and the mean curvature is

$$H = -\frac{\sqrt{2}}{4}.$$

Fig. 4. The figure of $x(u,v)$ in Example 2.2.

Example 2.3 Taking $b(v) = (\cos^3(v), \sin^3(v), v)^T$, $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, we get $x(u,v) = e^{Au}b(v)$.

Hence

$$b' = (-3\cos^2(v)\sin(v), 3\sin^2(v)\cos(v), 1)^T,$$
$$b'' = (-3\cos^3(v) + 6\sin^2(v)\cos(v), -3\sin^3(v) + 6\sin(v)\cos^2(v), 0)^T,$$
$$Ab = (-\sin^3(v), \cos^3(v), 0)^T, Ab' = (-3\sin^2(v)\cos(v), -3\cos^2(v)\sin(v), 0)^T,$$
$$A^2b = (-\cos^3(v), -\sin^3(v), 0)^T.$$

Then the Gaussian curvature is given by

$$K = \frac{48 - 96\sin^2(2v) + 36\sin^4(2v)}{(4 + 6\sin^2(2v) - 9\sin^4(2v))^2}$$

and the mean curvature is given by

$$H = \frac{18\sin^2(2v) - 16}{(4 + 6\sin^2(2v) - 9\sin^4(2v))^{3/2}}.$$

We can see that

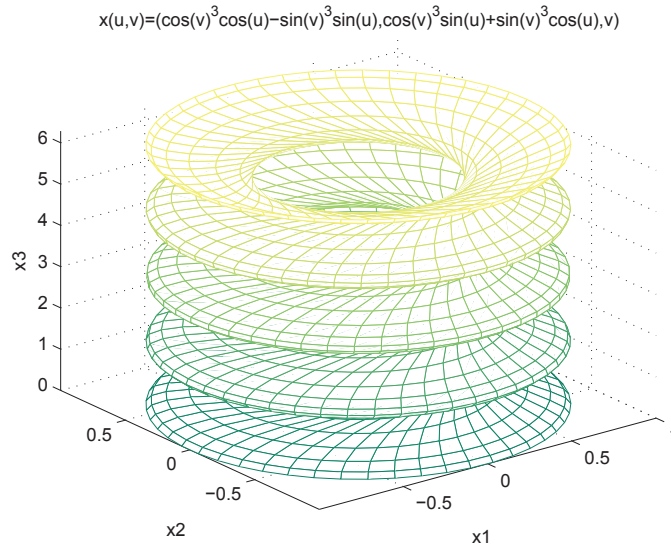$$K = \begin{cases} -12, & \sin(2v) = 1 \\ 3, & \sin(2v) = 0 \\ 0, & \sin(2v) = \frac{\sqrt{6}}{3} \end{cases}.$$

$$x(u,v)=(\cos(v)^3\cos(u)-\sin(v)^3\sin(u),\cos(v)^3\sin(u)+\sin(v)^3\cos(u),v)$$

Fig. 5. The figure of $x(u,v)$ in Example 2.3.

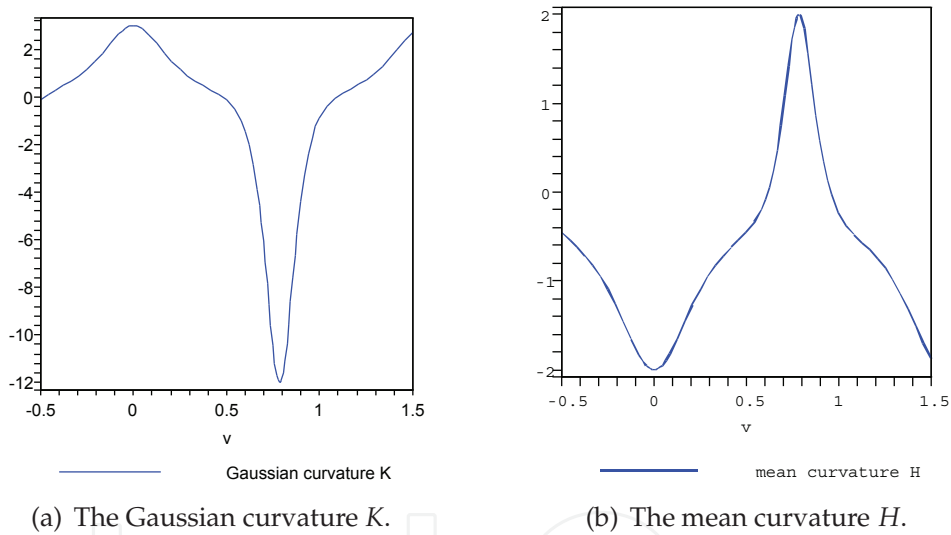(a) The Gaussian curvature K. (b) The mean curvature H.

Fig. 6. Curvatures of $x(u,v)$ in Example 2.3.

2.3 Fibre bundle model of 1-parameter lie groups of Hamiltonian lie algebra and its geometry

Consider a spatial curve on a surface M as

$$x(t) := \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} \in M \subset \mathbb{R}^3 \tag{10}$$

and a state vector as $y(t) = (y_1(t), y_2(t), \cdots, y_6(t))^T \in \mathbb{R}^6$

$$y(t) := \begin{pmatrix} \dot{x}(t) \\ x(t) \end{pmatrix} \in M \otimes T_x M, \tag{11}$$

where $T_x M$ is the tangent space of M at point x. A Hamiltonian Lie algebra of tangent vector fields is defined by the infinitesimal generator

$$\begin{pmatrix} \ddot{x}(t) \\ \dot{x}(t) \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \dot{x}(t) \\ x(t) \end{pmatrix} \tag{12}$$

or $\dot{y} = Hy$, where $H := \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is called a representation matrix.

For a special fibre bundle model of 1-parameter Lie groups of Hamiltonian Lie algebra defined by

$$F := \{ y(u,v) = e^{Hu} b(v), u, v \in \mathbb{R} \},$$

which is an embedded surface of $\mathbb{R}^6$, where $e^{Hu} \in SO_6(\mathbb{R})$ and $b(v)$ is a two order differentiable vector, one can get the following

Theorem 2.2 The Gaussian curvature $K(u,v)$ of the model is given by

$$\begin{aligned}
K(u,v) = &\frac{1}{\det(g_{ij})} ((Hb \cdot Hb'') + |Hb'|^2) \\
&+ \frac{(Hb \cdot Hb')}{\det^2(g_{ij})} ((Hb \cdot b')(Hb \cdot b'') + (Hb \cdot b')(Hb' \cdot b') \\
&- (Hb \cdot Hb')|b'|^2 - (b' \cdot b'')|Hb|^2),
\end{aligned} \tag{13}$$

where $(\cdot)$ denotes the inner product of vectors and det the determinant of an $n \times n$ matrix, respectively.

Proof. Using Lemma 2.1, we can get

$$x_u = He^{Hu}b, \quad x_v = e^{Hu}b'$$

and

$$x_{uu} = H^2 e^{Hu} b, \quad x_{uv} = x_{vu} = He^{Hu}b', \quad x_{vv} = e^{Hu}b''.$$

Then from $g_{ij} = (x_i \cdot x_j)$, we can get the Riemannian metric as

$$(g_{ij}) = \begin{pmatrix} |Hb|^2 & (Hb \cdot b') \\ (Hb \cdot b') & |b'|^2 \end{pmatrix}.$$

The determinant and inverse of (g_{ij}) are respectively given by

$$\det(g_{ij}) = |Hb|^2 |b'|^2 - (Hb \cdot b')^2$$

and

$$(g^{ij}) = \frac{1}{\det(g_{ij})} \begin{pmatrix} |b'|^2 & -(Hb \cdot b') \\ -(Hb \cdot b') & |Hb|^2 \end{pmatrix}.$$

From

$$\Gamma_{ij}^k = \frac{1}{2} g^{kl} \left(\partial_j g_{li} + \partial_i g_{lj} - \partial_l g_{ij} \right),$$

one can get

$$\Gamma_{11}^1 = \frac{1}{\det(g_{ij})}(Hb \cdot b')(Hb \cdot Hb'), \ \ \Gamma_{11}^2 = -\frac{1}{\det(g_{ij})}(Hb \cdot Hb')|Hb|^2,$$

$$\Gamma_{12}^1 = \Gamma_{21}^1 = \frac{1}{\det(g_{ij})}(Hb \cdot Hb')|b'|^2, \ \ \Gamma_{21}^2 = \Gamma_{12}^2 = \frac{1}{\det(g_{ij})}(Hb \cdot b')(Hb \cdot Hb'),$$

$$\Gamma_{22}^1 = \frac{1}{\det(g_{ij})}(((Hb' \cdot b') + (Hb \cdot b''))|b'|^2 - (Hb \cdot b')(b' \cdot b'')),$$

$$\Gamma_{22}^2 = -\frac{1}{\det(g_{ij})}(((Hb' \cdot b') + (Hb \cdot b''))(Hb \cdot b') - (b' \cdot b'')|Hb|^2).$$

Using

$$R_{ijkl} = R_{ikl}^h g_{hj},$$

where

$$R_{ikl}^h = \partial_k \Gamma_{il}^h - \partial_l \Gamma_{ik}^h + \Gamma_{il}^j \Gamma_{jk}^h - \Gamma_{ik}^j \Gamma_{jl}^h,$$

we can get the nonzero component of the curvature tensors

$$\begin{aligned} R(u,v,u,v) = & (Hb \cdot Hb'') + |Hb'|^2 \\ & + \frac{(Hb \cdot Hb')}{\det(g_{ij})}((Hb \cdot b')(Hb \cdot b'') + (Hb \cdot b')(Hb' \cdot b') \\ & - (Hb \cdot Hb')|b'|^2 - (b' \cdot b'')|Hb|^2). \end{aligned}$$

Then from the definition of the Gaussian curvature

$$K(u,v) = \frac{R(u,v,u,v)}{\det(g_{ij})},$$

one can the the conclusion directly.

In all of the following examples, $A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $C = -B$ and $D = -A^T$,

then $H = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$.

Example 2.4 Taking $b(v) = (v, 1, 1, 2, 3, 4)^T$, therefore, $b'(v) = (1, 0, 0, 0, 0, 0)^T, b''(v) = (0, 0, 0, 0, 0, 0)^T$. We can get

$$K = \frac{109}{(2v^2 - 6v + 59)^2} > 0.$$

Example 2.5 Taking $b(v) = (\cos v, \sin v, 0, 1, 1, 0)^T$, then $b'(v) = (-\sin v, \cos v, 0, 0, 0, 0)^T, b''(v) = (-\cos v, -\sin v, 0, 0, 0, 0)^T$. We can get
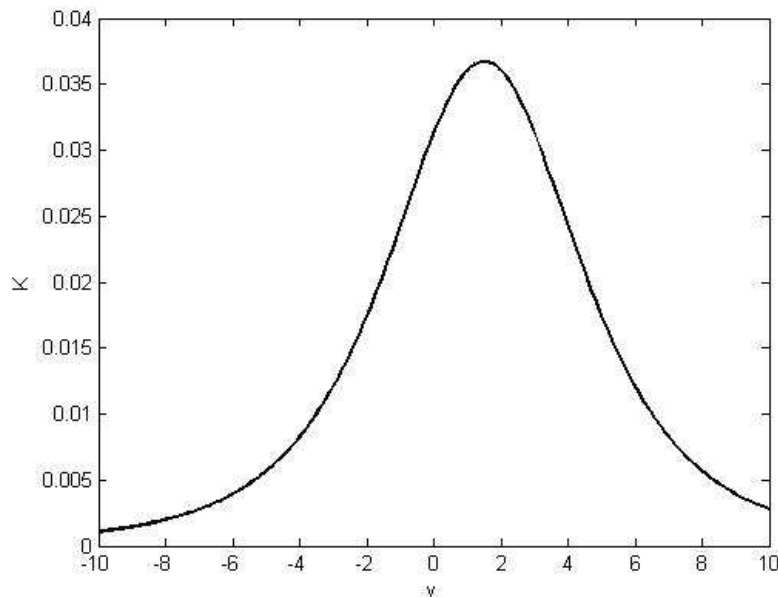
$$K = -\frac{2\sqrt{2} + 1}{144} < 0.$$

Fig. 7. Gaussian curvature of the surface in Example 2.4.

Example 2.6 Taking $b(v) = (v, 0, a, -a, 0, a)^T$, since $b'(v) = (1, 0, 0, 0, 0, 0)^T, b''(v) = (0, 0, 0, 0, 0, 0)^T$. We can get

$$K = 0.$$

3. 3D object recognition based on fibre bundle models

Recall the fibre bundle model in (4) as

$$F = \{x(u, v) = e^{Au}b(v), u, v \in \mathbb{R}\}, \tag{14}$$

where A is the representation matrix of the fibre curve. The base curve $b(v)$ can also be described by a voluntariness initial point $x_0 \in \mathbb{R}$ and its representation matrix B given by

$$b(v) = e^{Bv}x_0. \tag{15}$$

And its tangent vector field is also a tangent vector field shown as

$$\frac{\partial x}{\partial v} := Be^{Bv}x_0 = Bb(v). \tag{16}$$

Therefore, the information to describe the fibre bundle model is the base curve and the invariants of the linear Lie algebra, i.e., the representation matrix A or representation matrix A, B and the initial point x_0.

3.1 Simulations with known representation matrixes and the initial points
Next simulation results of shape synthesis using the proposed models are shown together with the invariants of representation matrix A or representation matrixes A, B and the initial point x_0.

(a) Base curve $(\sin v, \sin v, \cos v)^T$.   (b) Base curve $(\cos^3 v, \sin v \cos^3 v, 1)^T$.
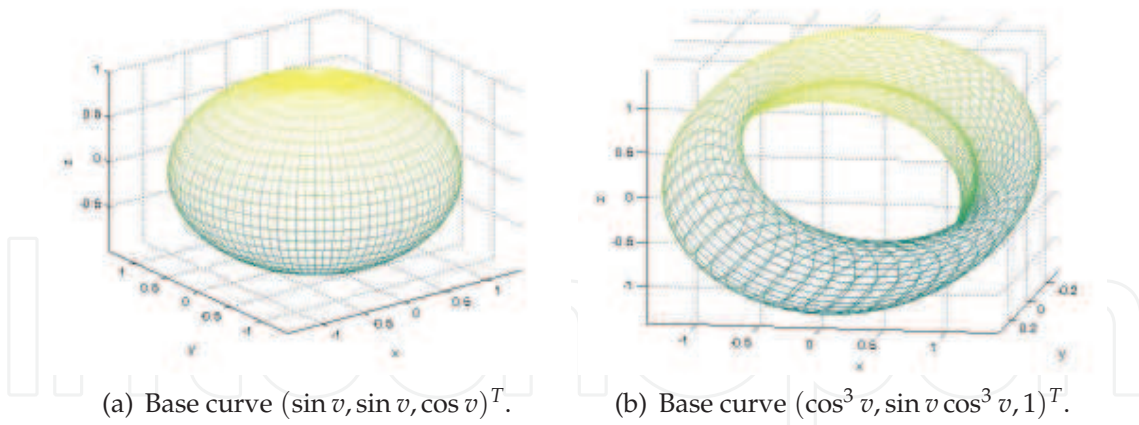
Fig. 8. Simulations with representation matrixes A and base curves $b(v)$. The representation matrixes A are $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ in (a) and (b), respectively.
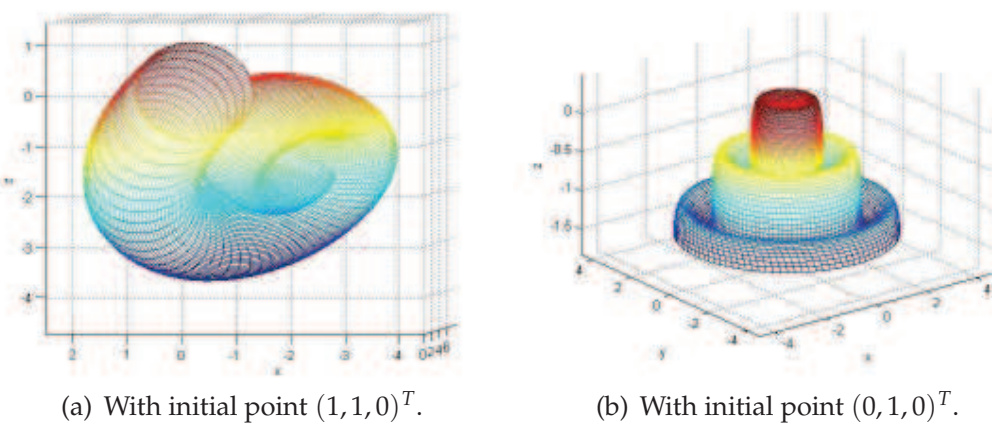

(a) With initial point $(1, 1, 0)^T$.   (b) With initial point $(0, 1, 0)^T$.

Fig. 9. Simulations with representation matrixes A, B and initial points x_0. In (a), the representation matrixes are $A = \begin{pmatrix} 2 & 1.5 & 2 \\ -2 & -1.5 & -2 \\ -3 & -1.5 & -1 \end{pmatrix}$, and $B = \begin{pmatrix} 0 & 0 & -2 \\ -2 & 0 & 2 \\ 2 & 0 & 0 \end{pmatrix}$. And in (b), $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & -1 & -1 \\ 2 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix}$.

3.2 Algorithm of 3D object recognition

Here we introduce an algorithm to obtain the invariants of a linear Lie algebra model from local image data of 3D objects as follows.

1. Take more than $N > 9$ position vectors $\{x_i\}_{i=1}^N$ on the object. Calculate the normal vectors at all positions $\{n_i\}_{i=1}^N$ from geometric relation, and normalize them.

2. From the position and normal vectors to build the following equations in entries of the representation matrix A under a rotation, where A stems from normalization

$$L := \frac{\partial x}{\partial u} = x_u = Ae^{Au}b(v) = Ax.$$

According to that the tangent vector field perpendicular to the normal vectors, we can get that n_i perpendiculars to $\frac{\partial x}{\partial u}|_{x_i} = Ax|_{x_i}$, then we can get

$$n_i^T A x_i,$$

namely,

$$n_i^T \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} x_i.$$

3. From linear system, we get the components of fibre representation matrix A.

4. Just liking the fibre representation matrix extraction, we can take more than nine position vectors y_i and the corresponding normal vectors to build this equation to solve the coefficient of the base curve representation matrix B.

5. Given a discretional initialization point x_0 and using the obtained representation matrix A and B to restore the primary 3D objects.

When we give the fibre representation matrix $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, then we can use Taylor series

to calculate its linear Lie algebra

$$\exp(Au) = \begin{pmatrix} \cos u & -\sin u & 0 \\ \sin u & \cos u & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{17}$$

So if we know the initialization point x_0 and the base curve representation matrix B, we can use this representation matrix A to express a form symmetry about an axis without solve the representation matrix A. That can help us easy to express a large number of shapes of 3D objects. (Fig. 8 and Fig. 9)

3.3 Practicality recognition

Next we consider the recognition of two objects, the sphere and vase. Our target sphere is shown in Fig. 10.

Firstly we give a base curve data (easy to confirm) and take more than nine position vectors from the sphere to recognize the object. Here we take data number as 30. From the algorithm, we can get the representation matrix as

$$A = \begin{pmatrix} -0.01 & 1 & -0.0043 \\ -0.9825 & -0.0032 & 0.0204 \\ 0.0202 & 0.0013 & -0.0001 \end{pmatrix}. \tag{18}$$

For the target vase shown in Fig. 12, we have to consider it as three parts, capsule, middle part and bottom, and do the simulations, respectively.

The target vase is a form symmetry about an axis, as presented we can use matrix $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ as the fibre representation matrix, then we only need to extract the base curve representation matrix B.
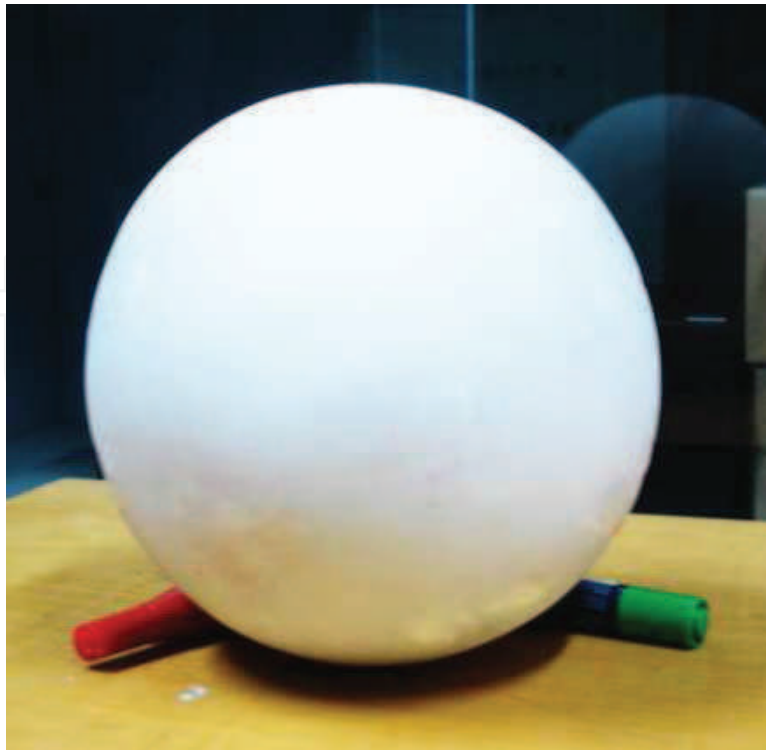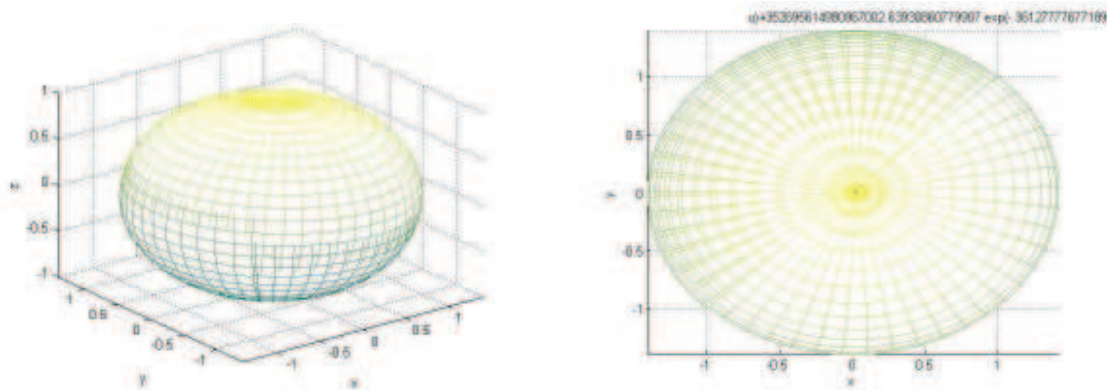
Fig. 10. Recognize target sphere.

(a) Recognition result (date number 30).   (b) Recognition result (view from the top).

Fig. 11. Recognition for a sphere.

For all the simulations in Fig. 13, the initial points are the same as $x_0 = (1, -1, 0)^T$. Meanwhile, from the algorithm, we obtain the base curve representation matrixes B in (a), (b) and (c) as $\begin{pmatrix} -6.0859 & 1 & -0.0689 \\ 0.7695 & -0.0865 & 0.0066 \\ -1.8276 & 0.3112 & -0.0212 \end{pmatrix}$, $\begin{pmatrix} -6.5247 & 1 & 0.2672 \\ -3.9099 & 0.6090 & 0.1608 \\ -0.6605 & 0.1122 & 0.0277 \end{pmatrix}$ and $\begin{pmatrix} -0.3293 & 1 & -0.0442 \\ -3.8194 & 3.0828 & -0.3170 \\ 13.1096 & -5.3890 & 0.8316 \end{pmatrix}$, respectively.

By the way, all the recognition results are exactly calculated and restored under strong noisy environments.

Fig. 12. Recognize target vase.

4. Conclusion

As we know, differential geometry and Riemannian geometry are powerful in applications in kinds of fields, and many methods and subjects are proposed, e.g. geometric mechanics, human face recognition using the method of manifold and geometry in statistics. In this chapter, we show the beauty of the geometry of the fibre bundle models of 1-parameter Lie groups of linear Lie algebra and Hamiltonian Lie algebra.

Fibre bundle model is effective in its representation for objects. Any object can represent as the form of a fibre bundle model, theoretically. Nevertheless, in practice, it's a challenging task for one to get the parameters of a special object. When the representation matrixes and the initial points are given, one can obtain beautiful photos for 3D objects. However, once one have the object, how to recognize it, namely, how to realize it in one's computer is the special challenging but the most important work. We propose an algorithm for 3D object recognition mainly based on the geometric relationship of the positions and normal vectors. The recognition results of sphere and vase demonstrate the algorithm perfectly.

5. Acknowledgements

(a) Capsule recognition result.

(b) Middle part of the vase's recognition result.

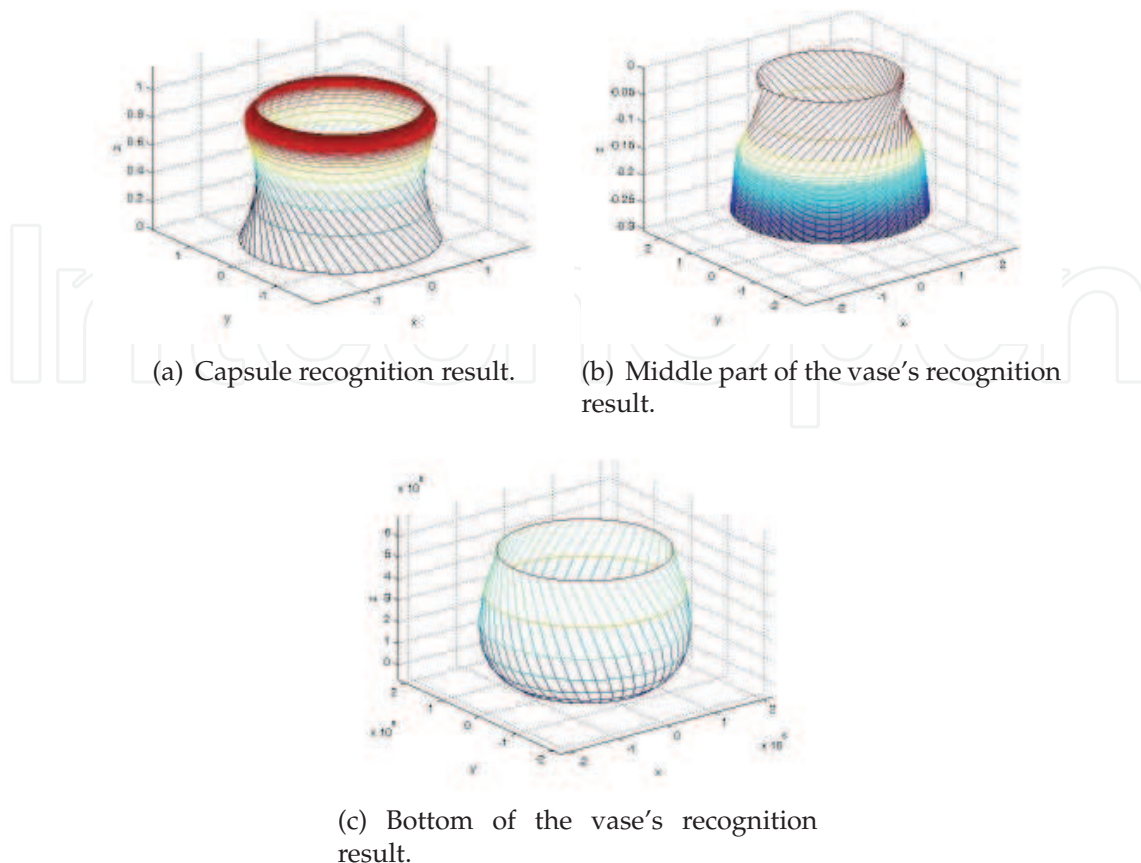(c) Bottom of the vase's recognition result.

Fig. 13. Recognition for a vase.

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Object Recognition

Edited by Dr. Tam Phuong Cao

Vision-based object recognition tasks are very familiar in our everyday activities, such as driving our car in the correct lane. We do these tasks effortlessly in real-time. In the last decades, with the advancement of computer technology, researchers and application developers are trying to mimic the human's capability of visually recognising. Such capability will allow machine to free human from boring or dangerous jobs.

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