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# A Model of Adding Relations in Two Levels of a Linking Pin Organization Structure with Two Subordinates

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## 1. Introduction

A pyramid organization is a hierarchical structure based on the principle of unity of command (Koontz, 1980) that every member except the top in the organization should have a single immediate superior. On the other hand an organization characterized by System 4 (Likert, 1976) has a structure in which relations between members of the same section are added to the pyramid organization structure. Members of middle layers of System 4 which are both members of the upper units and chiefs of the lower units are called linking pins, and this type of organization is called a linking pin organization. In the linking pin organization there exist relations between each superior and his direct subordinates and those between members which have the same immediate superior.

The linking pin organization structure can be expressed as a structure where every pair of siblings which are nodes which have the same parent in a rooted tree is adjacent, if we let nodes and edges in the structure correspond to members and relations between members in the organization respectively. Then the linking pin organization structure is characterized by the number of subordinates of each member, that is, the number of children of each node and the number of levels in the organization, that is, the height of the rooted tree, and so on (Robbins, 2003; Takahara & Mesarovic, 2003). Moreover, the path between a pair of nodes in the structure is equivalent to the route of communication of information between a pair of members in the organization, and adding edges to the structure is equivalent to forming additional relations other than those between each superior and his direct subordinates and between members which have the same direct subordinate.

The purpose of our study is to obtain an optimal set of additional relations to the linking pin organization such that the communication of information between every member in the organization becomes the most efficient. This means that we obtain a set of additional edges to the structure minimizing the sum of lengths of shortest paths between every pair of all nodes.

We have obtained an optimal depth for a model of adding edges between every pair of nodes with the same depth to a complete  $K$ -ary linking pin structure of height  $H$  ( $H = 2, 3, \dots$ ) where every pair of siblings in a complete  $K$ -ary tree of height  $H$  is adjacent (Sawada, 2008). A complete  $K$ -ary tree is a rooted tree in which all leaves have the same depth and all internal nodes have  $K$  ( $K = 2, 3, \dots$ ) children (Cormen et al., 2001). Figure 1 shows an example

of a complete  $K$ -ary linking pin structure of  $K=2$  and  $H=5$ . In Fig.1 the value of  $N$  expresses the depth of each node.

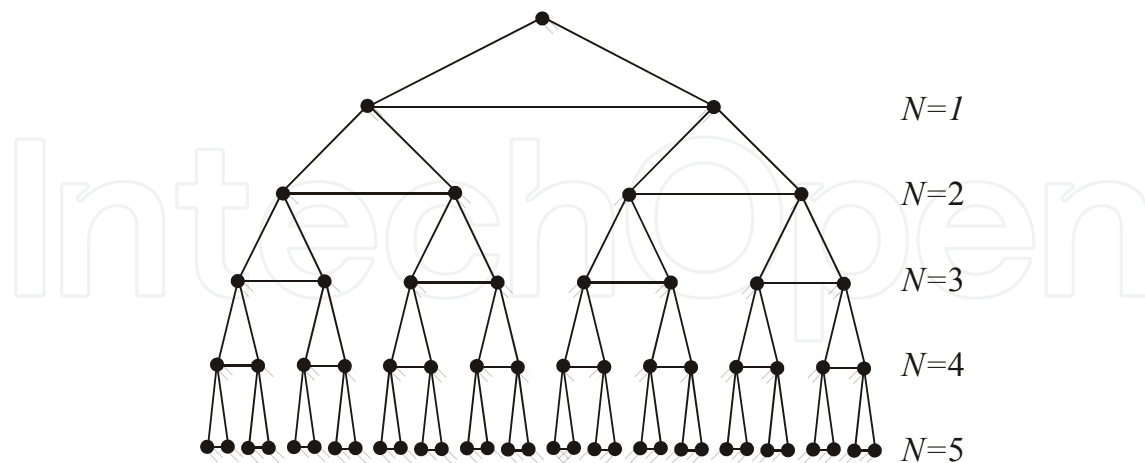


Fig. 1. An example of a complete  $K$ -ary linking pin structure of  $K=2$  and  $H=5$

This model gives us an optimal level when we add relations in one level to the organization structure which is a complete  $K$ -ary linking pin structure of height  $H$ , but this model cannot be applied to adding relations in two or more levels. This chapter expands the above model into the model of adding relations in two levels to the organization structure, which is that of adding edges between every pair of nodes at each depth of two depths to a complete binary ( $K = 2$ ) linking pin structure of height  $H$  ( $H = 3, 4, \dots$ ).

If  $l_{i,j}$  ( $= l_{j,i}$ ) denotes the distance, which is the number of edges in the shortest path from a node  $v_i$  to a node  $v_j$  ( $i, j = 1, 2, \dots, 2^{H+1}-1$ ) in the complete binary linking pin structure of height  $H$ , then  $\sum_{i<j} l_{i,j}$  is the total distance. Furthermore, if  $l'_{i,j}$  denotes the distance from  $v_i$  to  $v_j$  after adding edges in this model,  $l_{i,j} - l'_{i,j}$  is called the shortening distance between  $v_i$  and  $v_j$ , and  $\sum_{i<j} (l_{i,j} - l'_{i,j})$  is called the *total shortening distance*.

In Section 2 for the model of adding relations in one level we show formulation of a total shortening distance and an optimal depth which maximizes the total shortening distance. In Section 3 for the model of adding relations in two levels we formulate a total shortening distance and obtain an optimal pair of depths which maximizes the total shortening distance.

## 2. A model of adding relations in one level

This section shows an optimal depth  $L^*$  by maximizing the total shortening distance, when edges between every pair of nodes at one depth  $L$  ( $L = 2, 3, \dots, H$ ) are added to a complete binary linking pin structure of height  $H$  ( $H = 2, 3, \dots$ ) (Sawada, 2008).

### 2.1 Formulation of total shortening distance

Let  $\sigma_H(L)$  denote the total shortening distance, when we add edges between every pair of nodes with a depth of  $L$ .

The total shortening distance  $\sigma_H(L)$  can be formulated by adding up the following three sums of shortening distances: (i) the sum of shortening distances between every pair of nodes whose depths are equal to or greater than  $L$ , (ii) the sum of shortening distances between every pair of nodes whose depths are less than  $L$  and those whose depths are equal

to or greater than  $L$  and (iii) the sum of shortening distances between every pair of nodes whose depths are less than  $L$ .

The sum of shortening distances between every pair of nodes whose depths are equal to or greater than  $L$  is given by

$$\alpha_H(L) = \{W(H-L)\}^2 2^L \sum_{i=1}^{L-1} i2^i, \quad (1)$$

where  $W(h)$  denotes the number of nodes of a complete binary tree of height  $h$  ( $h = 0, 1, 2, \dots$ ). The sum of shortening distances between every pair of nodes whose depths are less than  $L$  and those whose depths are equal to or greater than  $L$  is given by

$$\beta_H(L) = W(H-L)2^{L+1} \sum_{i=1}^{L-2} \sum_{j=1}^i j2^j, \quad (2)$$

and the sum of shortening distances between every pair of nodes whose depths are less than  $L$  is given by

$$\gamma(L) = 2^L \sum_{i=1}^{L-3} \sum_{j=1}^i j(i-j+1)2^j, \quad (3)$$

where we define

$$\sum_{i=1}^0 \cdot = 0 \quad (4)$$

and

$$\sum_{i=1}^{-1} \cdot = 0. \quad (5)$$

From these equations, the total shortening distance  $\sigma_H(L)$  is given by

$$\begin{aligned} \sigma_H(L) &= \alpha_H(L) + \beta_H(L) + \gamma(L) \\ &= \{W(H-L)\}^2 2^L \sum_{i=1}^{L-1} i2^i + W(H-L)2^{L+1} \sum_{i=1}^{L-2} \sum_{j=1}^i j2^j + 2^L \sum_{i=1}^{L-3} \sum_{j=1}^i j(i-j+1)2^j. \end{aligned} \quad (6)$$

Since the number of nodes of a complete binary tree of height  $h$  is

$$W(h) = 2^{h+1} - 1, \quad (7)$$

$\sigma_H(L)$  of Eq.(6) becomes

$$\sigma_H(L) = (L-2)2^{2H+2} + 2^{2H-L+3} - 2^{H+L+3} + (L+1)2^{H+3} + L(L-1)2^L. \quad (8)$$

## 2.2 An optimal depth $L^*$

In this subsection, we seek  $L = L^*$  which maximizes  $\sigma_H(L)$  of Eq.(8).

Let  $\Delta \sigma_H(L) \equiv \sigma_H(L+1) - \sigma_H(L)$ , so that we have

$$\Delta\sigma_H(L) = 4(1 - 2^{-L})2^{2H} + 8(1 - 2^{-L})2^H + L(L + 3)2^L \quad (9)$$

for  $L = 2, 3, \dots, H-1$ . Let us define  $x$  as

$$x = 2^H, \quad (10)$$

then  $\Delta\sigma_H(L)$  in Eq.(9) becomes

$$\tau_L(x) = 4(1 - 2^{-L})x^2 + 8(1 - 2^{-L})x + L(L + 3)2^L \quad (11)$$

which is a quadratic function of the continuous variable  $x$ . By differentiating  $\tau_L(x)$  in Eq.(11) with respect to  $x$ , we obtain

$$\tau'_L(x) = 8(1 - 2^{-L})x + 8(1 - 2^{-L}). \quad (12)$$

Since  $\tau_L(x)$  is convex downward from

$$4(1 - 2^{-L}) > 0, \quad (13)$$

and

$$\tau_L(2^{L+1}) = L(L + 3)2^L > 0 \quad (14)$$

and

$$\tau'_L(2^{L+1}) = 8(2^L - 1) > 0, \quad (15)$$

we have  $\tau_L(x) > 0$  for  $x \geq 2^{L+1}$ . Hence, we have  $\Delta\sigma_H(L) > 0$  for  $H \geq L+1$ ; that is,  $L = 2, 3, \dots, H-1$ .

From the above results, the optimal depth of this model is  $L^* = H$ .

### 2.3 Numerical examples

Table 1 shows the optimal depths  $L^*$  and the total shortening distances  $\sigma_H(L^*)$  in the case of  $H=2, 3, \dots, 20$ .

## 3. A model of adding relations in two levels

This section obtains an optimal pair of depths  $(M, N)^*$  by maximizing the total shortening distance, when edges between every pair of nodes with depth  $M(M = 2, 3, \dots, H-1)$  and those between every pair of nodes with depth  $N(N = M+1, M+2, \dots, H)$  which is greater than  $M$  are added to a complete binary linking pin structure of height  $H(H = 3, 4, \dots)$ .

### 3.1 Formulation of total shortening distance

Using formulation of the model of adding relations in one level shown in Subsection 2.1, we formulate the total shortening distance of the model of adding relations in two levels  $S_H(M, N)$ .

$H$	$L^*$	$\sigma_H(L^*)$
2	2	8
3	3	112
4	4	960
5	5	6528
6	6	38784
7	7	211200
8	8	1083392
9	9	5324800
10	10	25356288
11	11	117878784
12	12	537870336
13	13	2418180096
14	14	10742497280
15	15	47255977984
16	16	206183596032
17	17	893408772096
18	18	3848412856320
19	19	16492941803520
20	20	70369327185920

Table 1. Optimal depths  $L^*$  and total shortening distances  $\sigma_H(L^*)$

Let  $V_1$  denote the set of nodes whose depths are less than  $M$ . Let  $V_2$  denote the set of nodes whose depths are equal to or greater than  $M$  and are less than  $N$ . Let  $V_3$  denote the set of nodes whose depths are equal to or greater than  $N$ .

The sum of shortening distances between every pair of nodes in  $V_3$  is given by

$$A_H(N) = \alpha_H(N) \tag{16}$$

from Eq.(1). The sum of shortening distances between every pair of nodes in  $V_3$  and nodes in  $V_1$  and  $V_2$  is given by

$$B_H(N) = \beta_H(N) \tag{17}$$

from Eq.(2). The sum of shortening distances between every pair of nodes in  $V_1$  is given by

$$C(M) = \gamma(M) \tag{18}$$

from Eq.(3), and the sum of shortening distances between every pair of nodes in  $V_1$  and nodes in  $V_2$  is given by

$$D(M, N) = \beta_{N-1}(M) \tag{19}$$

from Eq.(2). The sum of shortening distances between every pair of nodes in  $V_2$  is formulated as follows.

The sum of shortening distances between every pair of nodes in each linking pin structure whose root is a node with depth  $M$  is given by

$$E(M, N) = \gamma(N - M)2^M \quad (20)$$

from Eq.(3). The sum of shortening distances between every pair of nodes in two different linking pin structures whose roots are nodes with depth  $M$  is given by summing up  $F(M, N)$  and  $G(M, N)$ .  $F(M, N)$  which is the sum of shortening distances by adding edges only between nodes with depth  $M$  is given by

$$F(M, N) = \alpha_{N-1}(M) \quad (21)$$

from Eq.(1).  $G(M, N)$  which is the sum of additional shortening distances by adding edges between nodes with depth  $N$  after adding edges between nodes with depth  $M$  is expressed by

$$G(M, N) = (2^M - 1) \sum_{i=1}^{N-M-2} 2^{N-i} \sum_{j=1}^{N-M-i-1} 2^{N-M-j} (N - M - i - j), \quad (22)$$

where we define

$$\sum_{i=1}^0 \cdot = 0 \quad (23)$$

and

$$\sum_{i=1}^{-1} \cdot = 0. \quad (24)$$

From these equations, the total shortening distances  $S_H(M, N)$  is given by

$$\begin{aligned} S_H(M, N) &= A_H(N) + B_H(N) + C(M) + D(M, N) + E(M, N) + F(M, N) + G(M, N) \\ &= \{W(H - N)\}^2 2^N \sum_{i=1}^{N-1} i 2^i + W(H - N) 2^{N+1} \sum_{i=1}^{N-2} \sum_{j=1}^i j 2^j \\ &\quad + 2^M \sum_{i=1}^{M-3} \sum_{j=1}^i j(i - j + 1) 2^j + W(N - M - 1) 2^{M+1} \sum_{i=1}^{M-2} \sum_{j=1}^i j 2^j \\ &\quad + 2^N \sum_{i=1}^{N-M-3} \sum_{j=1}^i j(i - j + 1) 2^j + \{W(N - M - 1)\}^2 2^M \sum_{i=1}^{M-1} i 2^i \\ &\quad + (2^M - 1) \sum_{i=1}^{N-M-2} 2^{N-i} \sum_{j=1}^{N-M-i-1} 2^{N-M-j} (N - M - i - j). \end{aligned} \quad (25)$$

Since the number of nodes of a complete binary tree of height  $h$  is

$$W(h) = 2^{h+1} - 1, \quad (26)$$

$S_H(M, N)$  of Eq.(25) becomes

$$\begin{aligned} S_H(M, N) &= (N - 2)2^{2H+2} + 2^{2H-N+3} - 2^{H+N+3} + (N + 1)2^{H+3} + (N - M)2^{N+M+1} \\ &\quad + (N - M)(N - M - 3)2^N + M(M - 1)2^M. \end{aligned} \quad (27)$$

### 3.2 An optimal depth $N^*$ for a fixed value of $M$

In this subsection, we seek  $N = N^*$  which maximizes  $R_{H,M}(N) = S_H(M, N)$  for a fixed value of  $M(M = 2, 3, \dots, H-1)$ .

Let  $\Delta R_{H,M}(N) \equiv R_{H,M}(N+1) - R_{H,M}(N)$ , so that we have

$$\Delta R_{H,M}(N) = 4(1 - 2^{-N})2^{2H} + 8(1 - 2^N)2^H + (N - M + 2)2^{N+M+1} + \{(N - M)(N - M + 1) - 4\}2^N \quad (28)$$

for  $N = M+1, M+2, \dots, H-1$ . Let us define  $x$  as

$$x = 2^H, \quad (29)$$

then  $\Delta R_{H,M}(N)$  in Eq.(28) becomes

$$T_{M,N}(x) = 4(1 - 2^{-N})x^2 + 8(1 - 2^N)x + (N - M + 2)2^{N+M+1} + \{(N - M)(N - M + 1) - 4\}2^N \quad (30)$$

which is a quadratic function of the continuous variable  $x$ . By differentiating  $T_{M,N}(x)$  in Eq.(30) with respect to  $x$ , we obtain

$$T'_{M,N}(x) = 8(1 - 2^{-N})x + 8(1 - 2^N). \quad (31)$$

Since  $T_{M,N}(x)$  is convex downward from

$$4(1 - 2^{-N}) > 0, \quad (32)$$

and

$$T_{M,N}(2^{N+1}) = (N - M)2^{N+M+1} + (N - M)(N - M + 1)2^N + (2^M - 1)2^{N+2} > 0 \quad (33)$$

and

$$T'_{M,N}(2^{N+1}) = 8(2^N - 1) > 0, \quad (34)$$

we have  $T_{M,N}(x) > 0$  for  $x \geq 2^{N+1}$ . Hence, we have  $\Delta R_{H,M}(N) > 0$  for  $H \geq N+1$ ; that is,  $N = M+1, M+2, \dots, H-1$ .

From the above results, the optimal depth  $N^*$  for a fixed value of  $M(M = 2, 3, \dots, H-1)$  is  $N^* = H$ .

### 3.3 An optimal pair of depths $(M, N)^*$

In this subsection, we seek  $(M, N) = (M, N)^*$  which maximizes  $S_H(M, N)$  in Eq.(27).

Let  $Q_H(M)$  denote the total shortening distance when  $N = H$ , so that we have

$$\begin{aligned} Q_H(M) &\equiv S_H(M, H) \\ &= (H - 4)2^{2H+2} + (H - M)2^{H+M+1} + (H + 2)2^{H+3} + (H - M)(H - M - 3)2^H \\ &\quad + M(M - 1)2^M. \end{aligned} \quad (35)$$



Let  $\Delta Q_H(M) \equiv Q_H(M+1) - Q_H(M)$ , so that we have

$$\Delta Q_H(M) = (H - M - 2)(2^M - 1)2^{H+1} + M(M+3)2^M > 0 \quad (36)$$

for  $M = 2, 3, \dots, H-2$ .

From the results in Subsection 3.2 and 3.3, the optimal pair of depths is  $(M, N)^* = (H-1, H)$ .

### 3.4 Numerical examples

Tables 2-19 show the optimal depths  $N^*$  for a fixed value of  $M$  ( $M = 2, 3, \dots, H-1$ ) and the total shortening distances  $S_H(M, N^*)$  in the case of  $H=3, 4, \dots, 20$ .

$M$	$N^*$	$S_H(M, N^*)$
2	3	120

Table 2. Optimal depth  $N^*$  and total shortening distance  $S_H(M, N^*)$  in the case of  $H=3$

$M$	$N^*$	$S_H(M, N^*)$
2	4	1000
3	4	1040

Table 3. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=4$

$M$	$N^*$	$S_H(M, N^*)$
2	5	6664
3	5	6896
4	5	7040

Table 4. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=5$

$M$	$N^*$	$S_H(M, N^*)$
2	6	39176
3	6	39984
4	6	41024
5	6	41472

Table 5. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=6$

$M$	$N^*$	$S_H(M, N^*)$
2	7	212232
3	7	214576
4	7	218304
5	7	222592
6	7	223872

Table 6. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=7$

$M$	$N^*$	$S_H(M, N^*)$
2	8	1085960
3	8	1092144
4	8	1103040
5	8	1118848
6	8	1136000
7	8	1139456

Table 7. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=8$

$M$	$N^*$	$S_H(M, N^*)$
2	9	5330952
3	9	5346352
4	9	5375168
5	9	5421696
6	9	5486464
7	9	5554432
8	9	5563392

Table 8. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=9$

$M$	$N^*$	$S_H(M, N^*)$
2	10	25370632
3	10	25407536
4	10	25479360
5	10	25602688
6	10	25794432
7	10	26055936
8	10	26324992
9	10	26347520

Table 9. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=10$

$M$	$N^*$	$S_H(M, N^*)$
2	11	117911560
3	11	117997616
4	11	118169792
5	11	118477440
6	11	118986624
7	11	119764224
8	11	120813568
9	11	121880576
10	11	121935872

Table 10. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=11$

$M$	$N^*$	$S_H(M, N^*)$
2	12	537944072
3	12	538140720
4	12	538542272
5	12	539280000
6	12	540551040
7	12	542618880
8	12	545748992
9	12	549949440
10	12	554190848
11	12	554323968

Table 11. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=12$ 

$M$	$N^*$	$S_H(M, N^*)$
2	13	2418343944
3	13	2418786352
4	13	2419704000
5	13	2421424768
6	13	2424473472
7	13	2429637888
8	13	2437969920
9	13	2450526208
10	13	2467325952
11	13	2484219904
12	13	2484535296

Table 12. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=13$ 

$M$	$N^*$	$S_H(M, N^*)$
2	14	10742857736
3	14	10743840816
4	14	10745905344
5	14	10749837952
6	14	10756949888
7	14	10769339648
8	14	10790156288
9	14	10823602176
10	14	10873890816
11	14	10941067264
12	14	11008458752
13	14	11009196032

Table 13. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=14$

M	$N^*$	$S_H(M, N^*)$
2	15	47256764424
3	15	47258927152
4	15	47263514816
5	15	47272362624
6	15	47288616832
7	15	47317521664
8	15	47367469056
9	15	47451049984
10	15	47585060864
11	15	47786323968
12	15	48054943744
13	15	48324050944
14	15	48325754880

Table 14. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=15$

M	$N^*$	$S_H(M, N^*)$
2	16	206185299976
3	16	206190018608
4	16	206200111296
5	16	206219772544
6	16	206256342912
7	16	206322406656
8	16	206438938624
9	16	206639501312
10	16	206974445568
11	16	207510925312
12	16	208316153856
13	16	209390370816
14	16	210465685504
15	16	210469584896

Table 15. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=16$

$M$	$N^*$	$S_H(M, N^*)$
2	17	893412442120
3	17	893422665776
4	17	893444686016
5	17	893487940224
6	17	893569206144
7	17	893717845248
8	17	893984192512
9	17	894452142080
10	17	895255930880
11	17	896596930560
12	17	898743681024
13	17	901964857344
14	17	906261004288
15	17	910559608832
16	17	910568456192

Table 16. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=17$

$M$	$N^*$	$S_H(M, N^*)$
2	18	3848420720648
3	18	3848442740784
4	18	3848490451136
5	18	3848584823424
6	18	3848763606912
7	18	3849093911808
8	18	3849693181952
9	18	3850762752000
10	18	3852638185472
11	18	3855856398336
12	18	3861222801408
13	18	3869811376128
14	18	3882696409088
15	18	3899879129088
16	18	3917067321344
17	18	3917087244288

Table 17. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=18$

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$M$	$N^*$	$S_H(M, N^*)$
2	19	16492958580744
3	19	16493005766704
4	19	16493108527296
5	19	16493313000064
6	19	16493703071616
7	19	16494429738240
8	19	16495761438720
9	19	16498167943168
10	19	16502454577152
11	19	16509963563008
12	19	16522842488832
13	19	16544312819712
14	19	16578670067712
15	19	16630210428928
16	19	16698936655872
17	19	16767675006976
18	19	16767719571456

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Table 18. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=19$

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$M$	$N^*$	$S_H(M, N^*)$
2	20	70369362837512
3	20	70369463500848
4	20	70369683701952
5	20	70370124104320
6	20	70370969257856
7	20	70372554708224
8	20	70375484438528
9	20	70380832198656
10	20	70390477056000
11	20	70407640281088
12	20	70437690687488
13	20	70489218449408
14	20	70575109013504
15	20	70712543477760
16	20	70918704463872
17	20	71193598099456
18	20	71468518473728
19	20	71468617564160

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Table 19. Optimal depths  $N^*$  and total shortening distances  $S_H(M, N^*)$  in the case of  $H=20$

Table 20 shows the optimal pairs of depths  $(M, N)^*$  and the total shortening distances  $S_H(M, N)^*$  in the case of  $H=3, 4, \dots, 20$ .

$H$	$(M, N)^*$	$S_H(M, N)^*$
3	(2, 3)	120
4	(3, 4)	1040
5	(4, 5)	7040
6	(5, 6)	41472
7	(6, 7)	223872
8	(7, 8)	1139456
9	(8, 9)	5563392
10	(9, 10)	26347520
11	(10, 11)	121935872
12	(11, 12)	554323968
13	(12, 13)	2484535296
14	(13, 14)	11009196032
15	(14, 15)	48325754880
16	(15, 16)	210469584896
17	(16, 17)	910568456192
18	(17, 18)	3917087244288
19	(18, 19)	16767719571456
20	(19, 20)	71468617564160

Table 20. Optimal pairs of depths  $(M, N)^*$  and total shortening distances  $S_H(M, N)^*$



## 4. Conclusions

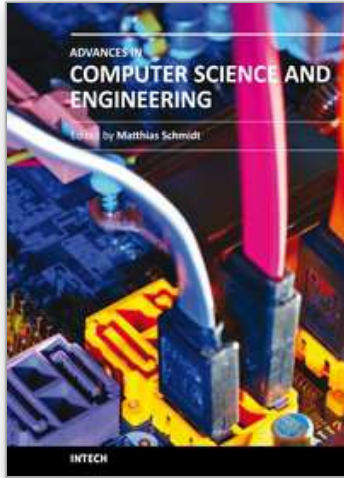
This study considered obtaining optimal depths of adding edges to a complete binary linking pin structure where every pair of siblings in a complete binary tree is adjacent maximizing the total shortening distance which is the sum of shortening lengths of shortest paths between every pair of all nodes in the complete binary linking pin structure. This means to obtain optimal levels of adding relations to a linking pin organization structure in which relations between members of the same section are added to the pyramid organization structure such that the communication of information between every member in the organization becomes the most efficient.

For the model of adding edges between every pair of nodes at one depth  $L$  to a complete binary linking pin structure of height  $H$ , we had already obtained an optimal depth  $L^* = H$  in our paper (Sawada, 2008). This result shows that the most efficient way of adding relations between all members in one level is to add relations at the lowest level, irrespective of the number of levels in the organization structure.

This chapter expanded the above model into the model of adding relations in two levels of the organization structure, which is that of adding edges between every pair of nodes with depth  $M$  and those between every pair of nodes with depth  $N$  which is greater than  $M$  to a complete binary linking pin structure of height  $H$ . We obtained an optimal pair of depth  $(M, N)^* = (H-1, H)$  which maximizes the total shortening distances. In the case of  $H = 5$  illustrated with the example in Fig.1 an optimal pair of depths is  $(M, N)^* = (4, 5)$ . This result means that the most efficient manner of adding relations between all members in each level of two levels is to add relations at the lowest level and the second lowest level, irrespective of the number of levels in the organization structure.

## 5. References

- Cormen, T. H.; Leiserson, C. E.; Rivest, R. L. & Stein, C. (2001). *Introduction to Algorithms*, 2nd Edition, MIT Press
- Koontz, H.; O'Donnell, C. & Weihrich, H. (1980). *Management*, 7th Edition, McGraw-Hill
- Likert, R. & Likert, J. G. (1976). *New Ways of Managing Conflict*, McGraw-Hill
- Robbins, S. P. (2003). *Essentials of Organizational Behavior*, 7th Edition, Prentice Hall
- Sawada, K. & Wilson, R. (2006). Models of adding relations to an organization structure of a complete  $K$ -ary tree. *European Journal of Operational Research*, Vol.174, pp.1491-1500
- Sawada, K. (2008). Adding relations in the same level of a linking pin type organization structure. *IAENG International Journal of Applied Mathematics*, Vol.38, pp.20-25
- Takahara, Y. & Mesarovic, M. (2003). *Organization Structure: Cybernetic Systems Foundation*, Kluwer Academic/Plenum Publishers



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