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Hydraulic Conductivity of Layered Anisotropic Media

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1. Introduction

Calculating a flow in a multi-layered medium (material) requires entering geometric boundaries of all layers, boundary conditions in these areas and filtration parameters (Bear, 1972). In the case of numerous layers, entering these data is very laborious, and sometimes even impossible. What we do then is try to model the flow by adopting one medium with appropriate anisotropic filtration properties. In such a case, one should decide whether such an assumption is possible from a theoretical point of view, and what parameters of such a medium should be adopted. A full estimation of filtration properties of anisotropic media requires defining the values of hydraulic conductivity in the direction of the principal axis of permeability tensor (Bear, 1972; Batu, 1998; Rogoż, 2007). For two-dimensional orthotropic media, including media composed of parallel homogeneous isotropic layers, these will be two values – one in parallel direction ($k_{\|}$), and the other - perpendicular ($k_{\perp}$) to layer boundary (Snow, 1969; Cheng, 2000). These values can be determined by performing permeameter measurements, with a forced flow in the above directions. If this is not possible, measurements are performed by realising flow in the direction diagonal to layering, and then calculating $k_{\|}$ and $k_{\perp}$. However, this leads to errors in determining these coefficients. The aim of this work is to discuss the causes of these errors and estimate their value.

2. Theoretical basis

The possibility to replace a system of layers with different permeability parameters with one medium with specified parameters should be justified theoretically. Based on the flow model adopted for calculations, one should clearly state the conditions under which a system of layers can be treated as one medium, what parameters should be adopted for such a medium and how to determine them correctly. Hence, theoretical issues have received a lot of attention so that the problems of flow through layered media can be presented in a possibly full scope, which is essential for explaining the theses of the paper.

2.1 Flow through a single layer

Let us consider a simple case of an incompressible fluid flow through a homogeneous and anisotropic aquifer with constant thickness *M* (Fig. 1).

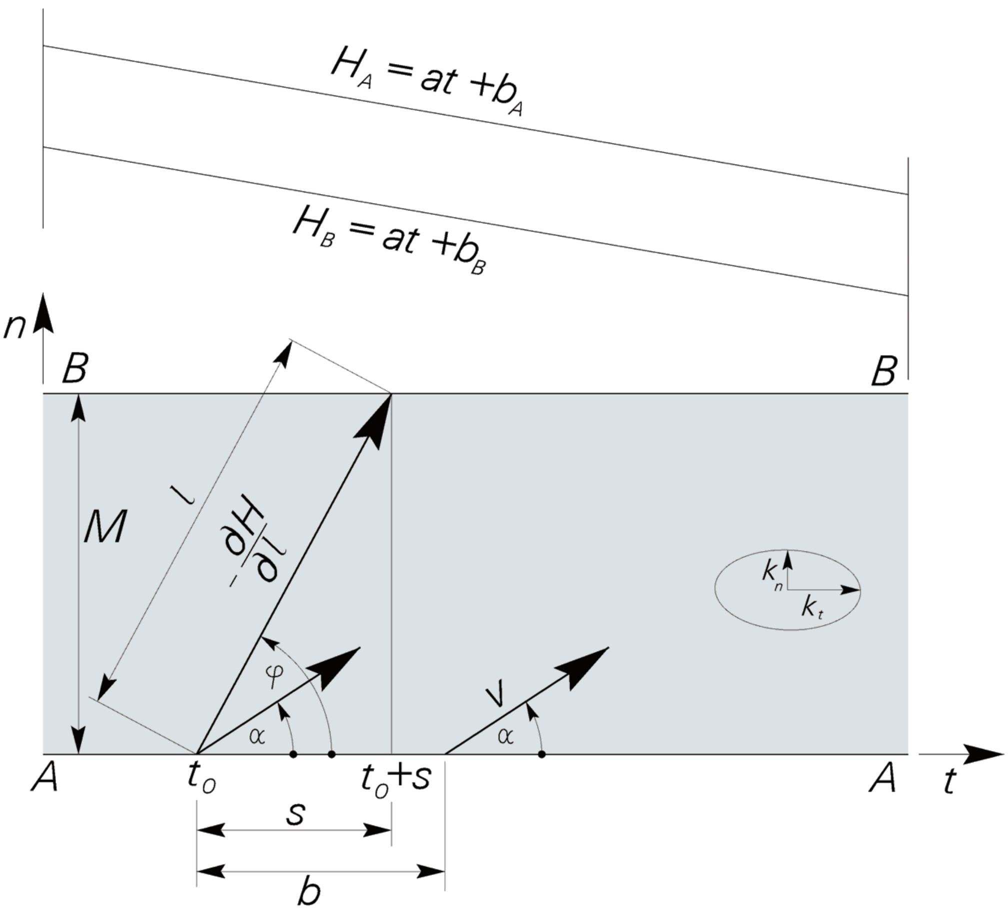

Fig. 1. Diagram of a flow through a single anisotropic layer.

Let us accept that the flow occurs according to Darcy's law. Let us designate the hydraulic conductivity in a plane tangential to the aquifer floor and roof as k_t, and in the direction normal to that plane as k_n. Let the hydraulic head change linearly along line A-A (axis *t*) according to the equation

$$H_A = at + b_A, \tag{1}$$

and along line B-B according to

$$H_B = at + b_B, \tag{2}$$

where *a*, b_A and b_B are constants, and $b_A > b_B$. Substituting $l = \frac{M}{\sin\varphi}$ and $s = \frac{M}{\tan\varphi}$, one can express hydraulic head drop along *l* in the form:

$$I = \frac{H_B(t_0 + s) - H_A(t_0)}{l} = -\frac{b_A - b_B}{M}\sin\varphi + a\cos\varphi \tag{3}$$

Filtration direction through an aquifer depends on the direction of hydraulic gradient. It can be determined by calculating the derivative $\frac{dI}{d\varphi}$ and comparing it to zero. Then

$$\varphi = \arctan\left(-\frac{b_A - b_B}{aM}\right) \tag{4}$$

The hydraulic head drop for the above angle is:

$$\begin{aligned} I_t &= a \\ I_n &= -\frac{\left(b_A - b_B\right)}{M} \\ I &= -\sqrt{\frac{\left(b_A - b_B\right)^2}{M^2} + a^2} \end{aligned} \tag{5}$$

while filtration velocity components are

$$\begin{aligned} v_t &= -k_t a \\ v_n &= k_n \frac{b_A - b_B}{M} \end{aligned} \tag{6}$$

The direction of filtration velocity can be determined from the formula

$$\tan\alpha = \frac{v_n}{v_t} = \frac{k_n \dfrac{\partial H}{\partial n}}{k_t \dfrac{\partial H}{\partial t}} = \frac{k_n \dfrac{\partial H}{\partial l}\sin\varphi}{k_t \dfrac{\partial H}{\partial l}\cos\varphi} = \frac{k_n}{k_t}\tan\varphi \tag{7}$$

Hence, for the discussed example

$$\alpha = \arctan\left(-\frac{k_n}{k_t}\frac{b_A - b_B}{aM}\right) \tag{8}$$

As we can see from relation (7), the more different the values of hydraulic conductivity k_n and k_t, the bigger the difference between hydraulic gradient direction and filtration velocity vector. If the layer is isotropic, angle φ will be equal to angle α.

2.2 Flow through layer boundary

If a flow occurs through the boundary between layers with different hydraulic conductivity values, streamline refraction occurs. Let us consider a flow through the boundary between two layers of soil media orthotropic in two dimensions (Fig. 2).

Two boundary conditions must be met in this area. The hydraulic head H_1 in layer 1 should be the same as H_2 in layer 2 and the normal velocity component to layer boundary v_{1n} in layer 1 and v_{2n} in layer 2 should be also the same.

$$\begin{aligned} H_1 &= H_2 \\ v_{1n} &= v_{2n} \end{aligned} \tag{9}$$

From the latter condition we obtain:

$$k_{1n}\frac{\partial H_1}{\partial n} = k_{2n}\frac{\partial H_2}{\partial n} \tag{10}$$

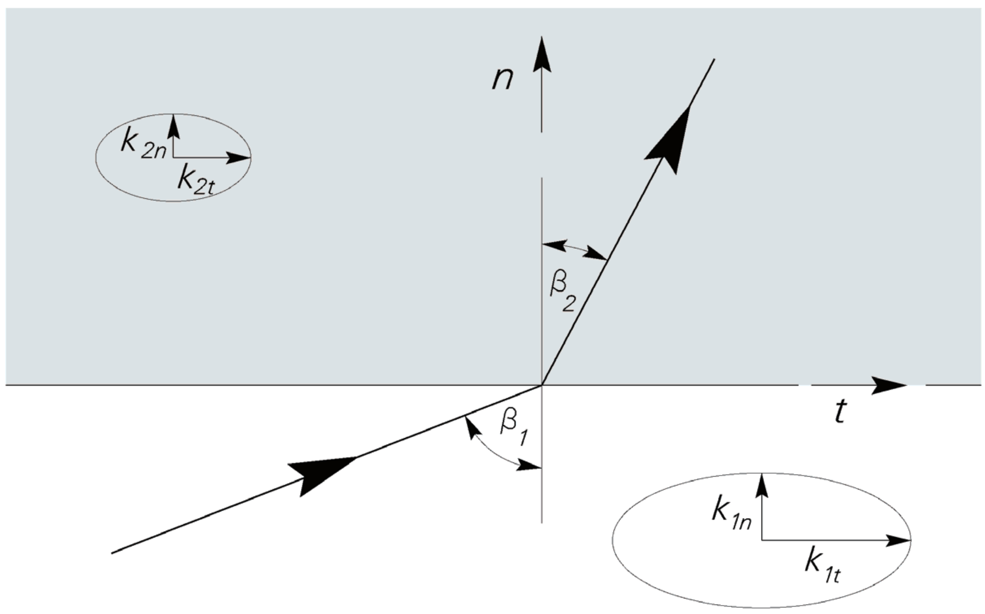

Fig. 2. Streamline refraction at the boundary of anisotropic layers.

But

$$\tan\beta_1 = \frac{k_{1t}\dfrac{\partial H_1}{\partial t}}{k_{1n}\dfrac{\partial H_1}{\partial n}} \quad \text{i} \quad \tan\beta_2 = \frac{k_{2t}\dfrac{\partial H_2}{\partial t}}{k_{2n}\dfrac{\partial H_2}{\partial n}} \tag{11}$$

After substituting relations (11), equation (10) assumes the form

$$\frac{k_{1t}\dfrac{\partial H_1}{\partial t}}{\tan\beta_1} = \frac{k_{2t}\dfrac{\partial H_2}{\partial t}}{\tan\beta_2} \tag{12}$$

Since H_1=H_2, we obtain

$$\frac{k_{1t}}{k_{2t}} = \frac{\tan\beta_1}{\tan\beta_2} \tag{13}$$

What emerges is that streamline refraction at the boundary of anisotropic media depends solely on the value of hydraulic conductivity in the direction tangential to layer boundary and does not depend on the values of hydraulic conductivity in the direction normal to layer boundary.

2.3 Flow through a multi-layered medium

Let us consider an incompressible fluid flow through an undeformable soil medium composed of N homogeneous and 2-D orthotropic parallel layers. Let us designate the hydraulic conductivity and thickness of the i^{th} layer as k_{it} , k_{in} and M_i respectively. Let us assume that a steady–state flow diagonal to layering takes place through such a system. (Fig. 3).

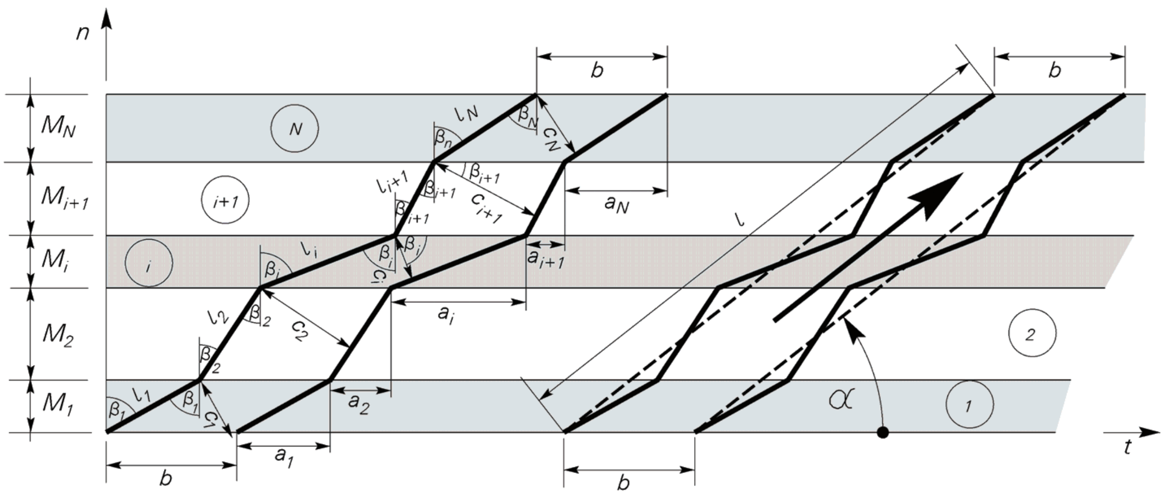

Fig. 3. Streamlines and filtration direction in a layered medium.

Let us also assume that the gradient of hydraulic head along lines A-A and B-B changes according to equations (1) and (2). In such a case, streamlines will take the shape of a polygonal chain composed of straight line segments. The differences between hydraulic heads along each streamline during the flow through particular layers will have the constant value ΔH_i. At layer boundaries, according to relation (13), streamline refraction will take place, which, after adopting the symbols from Fig. 3, will be expressed as

$$\frac{k_{it}}{k_{(i+1)t}} = \frac{\tan(\beta_i)}{\tan(\beta_{i+1})} \tag{14}$$

Water discharge between two streamlines through each layer must be the same. For a flow through the i^{th} layer it is:

$$q = \sqrt{k_{it}^2\left(\frac{\partial H_i}{\partial t}\right)^2 + k_{in}^2\left(\frac{\partial H_i}{\partial n}\right)^2}\int_0^b \cos(\beta_i)\,dt = b\cos(\beta_i)\sqrt{k_{it}^2\left(\frac{\partial H_i}{\partial t}\right)^2 + k_{in}^2\left(\frac{\partial H_i}{\partial n}\right)^2} \tag{15}$$

Taking into account the relations

$$\frac{\partial H_i}{\partial t} = \frac{\Delta H_i}{l_i}\cos(\varphi_i) \tag{16}$$

$$\frac{\partial H_i}{\partial n} = \frac{\Delta H_i}{l_i}\sin(\varphi_i) \tag{17}$$

$$l_i = \frac{M_i}{\cos(\beta_i)} \tag{18}$$

We obtain the formula

$$q = b\cos(\beta_i)\frac{\Delta H_i}{l_i}\sqrt{k_{it}^2\cos^2(\varphi_i) + k_{in}^2\sin^2(\varphi_i)} \tag{19}$$

For the symbols used in Fig. 3, the relation corresponding to equation (13) between β_i and φ_i assumes the form

$$\tan(\varphi_i) = \frac{k_{it}}{k_{in}} \frac{1}{\tan(\beta_i)} \tag{20}$$

On can also prove that

$$\tan(\beta_i) = \frac{k_{it}}{k_{1t}} \tan(\beta_1) \tag{21}$$

Also, considering the relations

$$1 + \tan^2(\beta_i) = \frac{1}{\cos^2(\beta_i)} \tag{22}$$

And

$$\tan(\alpha) = \frac{\sum_{i=1}^{n} M_i}{\sum_{i=1}^{n} a_i} = \frac{\sum_{i=1}^{n} M_i}{\sum_{i=1}^{n} M_i \tan(\beta_i)} = \frac{k_{1t} \sum_{i=1}^{n} M_i}{\tan(\beta_1) \sum_{i=1}^{n} M_i\, k_{it}} \tag{23}$$

One can calculate the sum of hydraulic losses along all the flow path through N layers. It comes to

$$\Delta H = \sum_{i=1}^{N} \Delta H_i = \frac{q}{b \sin^2(\alpha)} \sum_{i=1}^{N} \frac{M_i}{k_{in}} \sqrt{\sin^2(\alpha) + \frac{k_{it}^2 \left(\sum_{i=1}^{N} M_i \right)^2}{\left(\sum_{i=1}^{N} M_i k_{it} \right)^2} \cos^2(\alpha)} \quad \sqrt{\sin^2(\alpha) + \frac{k_{in}^2 \left(\sum_{i=1}^{N} M_i \right)^2}{\left(\sum_{i=1}^{N} M_i k_{it} \right)^2} \cos^2(\alpha)} \tag{24}$$

In order to determine the hydraulic conductivity of all the layered structure, let us replace the system of layers between the streamlines spaced at b intervals with one homogeneous layer with the same width b and length l, in accordance with Fig. 3. If we assume that the hydraulic conductivity of this layer in the flow direction determined by angle α is k_e, then the head loss will be

$$\Delta H = \frac{q \sum_{i=1}^{n} M_i}{k_e\, b \sin^2(\alpha)} \tag{25}$$

Comparing the size of hydraulic losses in both cases, one obtains the sought formula for the value of equivalent hydraulic conductivity k_e for a flow through a layered medium at any angle α.

$$\frac{1}{k_e}=\frac{\sum_{i=1}^{N}\frac{M_i}{k_{in}}\sqrt{\sin^2(\alpha)+\frac{k_{it}^2\left(\sum_{i=1}^{N}M_i\right)^2}{\left(\sum_{i=1}^{N}M_ik_{it}\right)^2}\cos^2(\alpha)}\sqrt{\sin^2(\alpha)+\frac{k_{in}^2\left(\sum_{i=1}^{N}M_i\right)^2}{\left(\sum_{i=1}^{N}M_ik_{it}\right)^2}\cos^2(\alpha)}}{\sum_{i=1}^{N}M_i} \tag{26}$$

For a flow parallel to layering (α = 0˚ or α = 180˚), hydraulic conductivity k_e will be designated as $k_{\parallel}$ and, according to the above formula it is:

$$k_{\parallel}=\frac{\sum_{i=1}^{n}k_{it}M_i}{\sum_{i=1}^{n}M_i}\,, \tag{27}$$

while in perpendicular direction (α = 90˚ or α = 270˚), hydraulic conductivity k_e will be designated as $k_{\perp}$ and in this case, according to formula (26), it is

$$k_{\perp}=\frac{\sum_{i=1}^{n}M_i}{\sum_{i=1}^{n}\frac{M_i}{k_{in}}} \tag{28}$$

It is worth emphasizing that equations (27) and (28) can be easily obtained by directly analysing a flow in two directions: parallel and perpendicular to layering.
After substituting relation (27) in equation (26), we obtain

$$\frac{1}{k_e}=\frac{\sum_{i=1}^{N}\frac{M_i}{k_{in}}\sqrt{\sin^2(\alpha)+\frac{k_{it}^2}{k_{\parallel}^2}\cos^2(\alpha)}\sqrt{\sin^2(\alpha)+\frac{k_{in}^2}{k_{\parallel}^2}\cos^2(\alpha)}}{\sum_{i=1}^{N}M_i} \tag{29}$$

It follows from equation (29) that in order to determine the equivalent hydraulic conductivity of a medium composed of anisotropic layers for diagonal direction α, it is essential to know all the values of hydraulic conductivity k_{it} and k_{in} of particular layers and their thickness M_i. Therefore, from the theoretical point of view, it is not possible to model a flow through such a structure like that through a single layer.
On the other hand, in the case when particular layers in this structure have isotropic properties, i.e. one can adopt $k_{it}=k_{in}=k_i$ for all the layers, relation (29) can be expressed in the following form (Bear, 1972):

$$\frac{1}{k_e}=\frac{\sum_{i=1}^{n}M_i}{\sum_{i=1}^{n}M_ik_i}\cos^2(\alpha)+\frac{\sum_{i=1}^{n}\frac{M_i}{k_i}}{\sum_{i=1}^{n}M_i}\sin^2(\alpha)=\frac{1}{k_{\parallel}}\cos^2(\alpha)+\frac{1}{k_{\perp}}\sin^2(\alpha) \tag{30}$$

Then, in order to calculate the flow in any direction α, it is enough to know the value of hydraulic conductivity for all the layered structure in the direction parallel and perpendicular to the surface constituting layer boundary.
As expected, the form of relation (30) is identical to the function representing hydraulic conductivity in orthotropic soils, where two-dimensional anisotropy occurs. (Batu, 1998; Cheng, 2000; Snow, 1969; Wieczysty, 1982).

$$\frac{1}{k_e} = \frac{1}{k_{xx}} \cos^2(\alpha) + \frac{1}{k_{yy}} \sin^2(\alpha) \tag{31}$$

In equation (31), k_{xx} and k_{yy} correspond to the values of hydraulic conductivity in the principal directions of hydraulic conductivity tensor and $k_{\parallel} = k_{xx}$, while $k_{\perp} = k_{yy}$.
If based on equation (30) or (31) one determines the values of $\sqrt{k_e}$ for various α values, then plotting them in a circle diagram (a graph in polar coordinates) will produce an ellipse. It is referred to as hydraulic conductivity ellipse.

2.4 Angle between hydraulic gradient and filtration velocity

Angle γ between hydraulic gradient (grad$H$) and filtration velocity $\mathbf{v}$ can be determined from the ratio of the dot products of vectors grad$H$ and $\mathbf{v}$ to the product of their lengths. In the case of a 2D flow, and adopting a Cartesian coordinate system whose axes are directed along the principal axes of hydraulic conductivity tensor, angle γ is (Fig. 4)

$$\gamma = \arccos\left(\frac{\text{grad}H \bullet \mathbf{v}}{|\text{grad}H||\mathbf{v}|}\right) = \arccos\left[\frac{\frac{\partial H}{\partial x} k_{xx} \frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} k_{yy} \frac{\partial H}{\partial y}}{\sqrt{\left(\frac{\partial H}{\partial x}\right)^2 + \left(\frac{\partial H}{\partial y}\right)^2} \sqrt{k_{xx}^2 \left(\frac{\partial H}{\partial x}\right)^2 + k_{yy}^2 \left(\frac{\partial H}{\partial y}\right)^2}}\right] \tag{32}$$

where k_{xx} and k_{yy} denote the values of hydraulic conductivity tensor k:

$$k = \begin{pmatrix} k_{xx} & 0 \\ 0 & k_{yy} \end{pmatrix} \tag{33}$$

Remembering that

$$\tan(\alpha) = \frac{k_{yy} \frac{\partial H}{\partial y}}{k_{xx} \frac{\partial H}{\partial x}} \tag{34}$$

one obtains

$$\gamma = \arccos\left(\frac{k_{yy} \cos^2(\alpha) + k_{xx} \sin^2(\alpha)}{\sqrt{k_{yy}^2 \cos^2(\alpha) + k_{xx}^2 \sin^2(\alpha)}}\right) = \arccos\left(\frac{\cos^2(\alpha) + \lambda \sin^2(\alpha)}{\sqrt{\cos^2(\alpha) + \lambda^2 \sin^2(\alpha)}}\right). \tag{35}$$

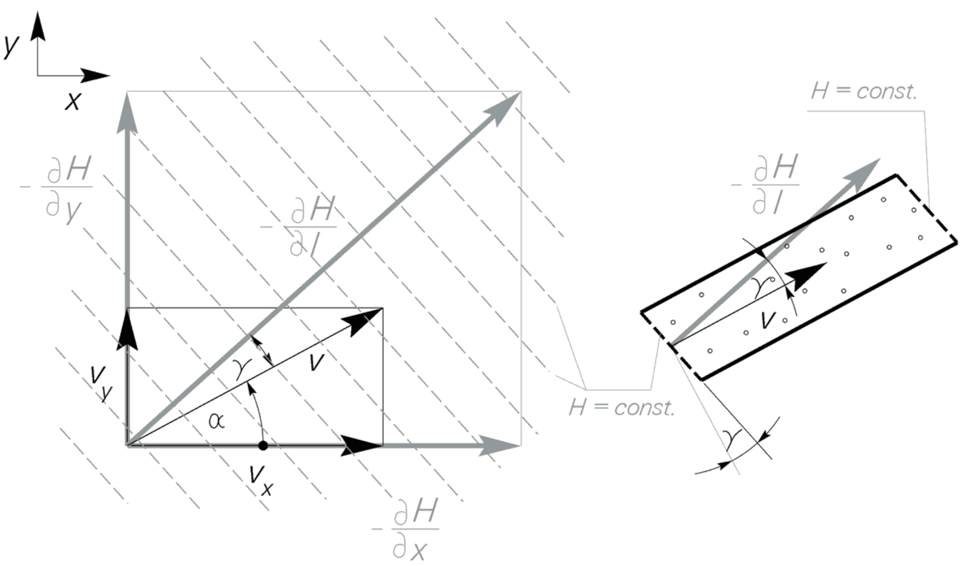

Fig. 4. Direction differences between hydraulic gradient and filtration velocity.

where α denotes the angle between velocity vector and the principal axis of hydraulic conductivity tensor, along which value k reaches the maximum values.
From the above relation, one can determine the value of angle α, for which angle γ has the maximum value. In order to do this, one must determine the derivative $\frac{d\gamma}{d\alpha}$ and compare it to zero. The angle determined in this way is:

$$\alpha = \pm \arctan \sqrt{\frac{k_{yy}}{k_{xx}}} = \pm \arctan \sqrt{\frac{1}{\lambda}} \tag{36}$$

Assuming that $k_{xx} > k_{yy}$, $\lambda = \frac{k_{xx}}{k_{yy}}$ denotes anisotropy factor. For layered media composed of homogeneous isotropic and parallel layers, value k_{xx} corresponds to $k_{\parallel}$, and value k_{yy} – to $k_{\perp}$.

3. Problems with determining the hydraulic conductivity of anisotropic media.

Defining water permeability of anisotropic media requires determining values of hydraulic conductivity in the direction of principal axes of hydraulic conductivity (Renard, 2001; Mozely et al., 1996). For media characterized by two-dimensional orthotropy, including layered media, these will be two values – one in parallel direction, and the other – perpendicular to layering. In accordance with equation (30), determining hydraulic conductivity in these directions is possible based on tests of hydraulic conductivity k_e for any two known angles α. Then one obtains two equations with two unknowns $k_{\parallel}$ and $k_{\perp}$. When determining these values, one can later calculate equivalent hydraulic conductivity k_e in any direction α. However, if there are any difficulties defining angles α, for which water permeability measurements are performed, $k_{\parallel}$ and $k_{\perp}$ can be determined by performing a larger number of measurements for various angles (at least three different angles), and then matching the obtained results with the ellipse equation (Cheng, 2000).

This is the theory. However, determining hydraulic conductivity of layered media in permeameters involves additional problems. Analysing relation (35), one can observe that the angle between grad$H$ and **v** is equal to zero, i.e. both directions coincide only when angle α is equal to 0, 90, 180 and 270 degrees, then if the flow occurs in the direction parallel or perpendicular to layering. When performing measurements along these directions, one does not make additional errors resulting from anisotropy. However, if such a measurement is not possible, (e.g. samples are obtained from hole coring and the hole axis is diagonal to layering), determining the value of hydraulic conductivity in the direction diagonal to layering entails an error. It results from the fact that during measurements inside permeameters the appropriate geometric shapes of the sample, the proper size and the difference between filtration velocity **v** and grad$H$ are not maintained.

Geometric dimensions should allow for maintaining the angle between filtration direction and the plane through which water flows to the sample, and the lateral surfaces of the sample. The inflow and outflow surfaces should be perpendicular to the gradient of hydraulic head, and the lateral surfaces – parallel to streamlines composed of straight line segments (Fig. 3).

Meeting the above conditions is very difficult technically, moreover it would require prior knowledge of filtration parameters of particular layers of the medium. The knowledge of these parameters would enable theoretical calculation of hydraulic conductivity without a need to analyse all the layer structure. If we assume however that we do not know these parameters, maintaining appropriate shapes of the sample is not possible. In such a case, as an approximation of the abovementioned theoretical solution, reflecting a flow diagonal to layering, one can adopt a flow through a medium sample whose geometric dimensions and flow conditions are consistent with the idea presented in Fig. 5. One should remember here that the cross-section of the sample in the direction perpendicular to filtration direction should be a rectangle.

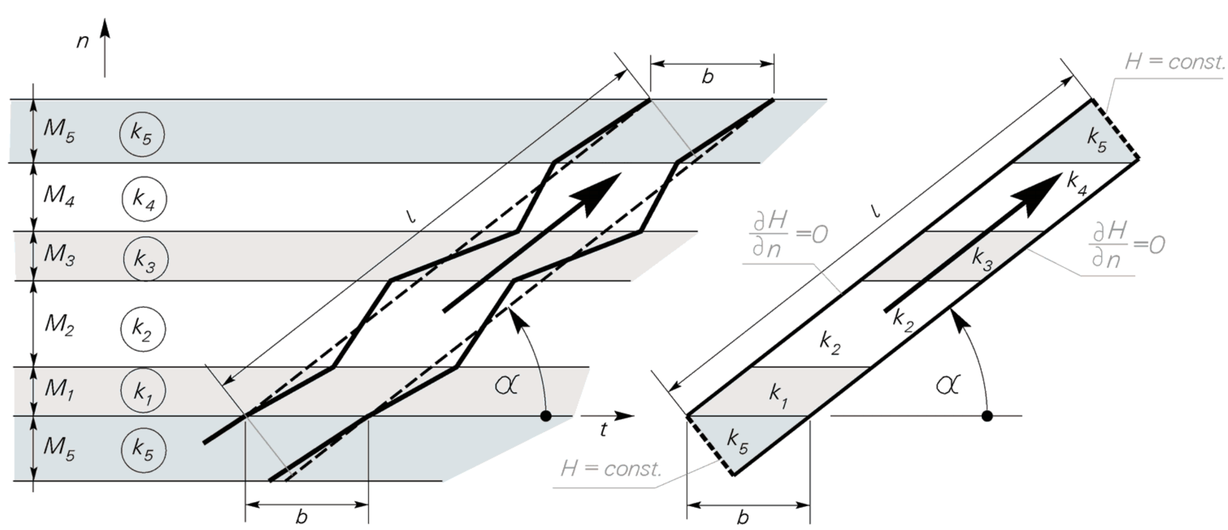

Fig. 5. Diagram of the pattern of adopting a geometric shape for a layered medium sample for permeameter analysis.

Unfortunately, when performing permeameter tests on samples with the shape presented in Fig. 5, one cannot avoid errors in determination of hydraulic conductivity.

4. Model calculations

In order to estimate error values connected with determining hydraulic conductivity of layered media tested in a permeameter, water flow through an imaginary soil medium composed of layers with known properties was calculated. The calculations consisted of a numerical solution of a flow through a specific arrangement of layers in a permeameter and then defining equivalent hydraulic conductivity. The hydraulic conductivity defined in this way was then compared to theoretically calculated hydraulic conductivity of a layered structure corresponding to the layer pattern in the permeameter.

4.1 Characteristics of the medium

Model calculations were performed on a soil sample composed of five homogeneous and isotropic layers with constant thickness (Fig. 3). The thickness of particular layers M_i, and hydraulic conductivity k_i are presented in Tab. 1.

Layer No.	M_i	k_i	$k_{\parallel}$	$k_{\perp}$	λ
	cm	$m\cdot s^{-1}$	$m\cdot s^{-1}$	$m\cdot s^{-1}$	-
1	21	$7{,}0\cdot10^{-4}$	$2{,}06\cdot10^{-4}$	$1{,}98\cdot10^{-5}$	10,37
2	20	$1{,}0\cdot10^{-5}$			
3	19	$6{,}0\cdot10^{-6}$			
4	20	$8{,}0\cdot10^{-5}$			
5	31	$2{,}0\cdot10^{-4}$			

Table 1. Basic properties of a layered medium.

For such a layer pattern, formula (30) was used to calculate theoretical values of hydraulic conductivity $k_{e(t)}$ and the angle between vectors grad$H$ and **v** for different values of angle α between filtration velocity and the direction of principal axis t. Based on the obtained results, hydraulic conductivity ellipse ($\sqrt{k_e}$), and the values of angles between grad$H$ and filtration velocity **v** (angles γ) depending on filtration direction, were drawn (Figs. 6 and 7). Table 1 also contains calculated hydraulic conductivity values for all the layered structure in the direction parallel and perpendicular to layer boundaries, as well as the anisotropy factor.

4.2 Numerical solution of permeameter flow

The problem of permeameter flow was solved based on Laplace's equation, assuming the steady flow of an incompressible fluid through an undeformed medium. On the surfaces forming the filtration area boundary, boundary conditions were adopted according to Fig. 5., while on the boundary of layers with different water permeability, $H_i = H_{i+1}$ and $v_{ni} = v_{ni+1}$ were adopted, with H_i, H_{i+1} denoting the hydraulic heads, and v_{ni} and v_{ni+1} – filtration velocity components normal to the boundary of i^{th} and $i+1^{th}$ layers. All the filtration area was divided into 110x14 blocks. The calculations were performed with finite difference method in Excel spreadsheet, using Gauss-Seidel iteration. The calculation results for different angles α are shown in Tab. 2.

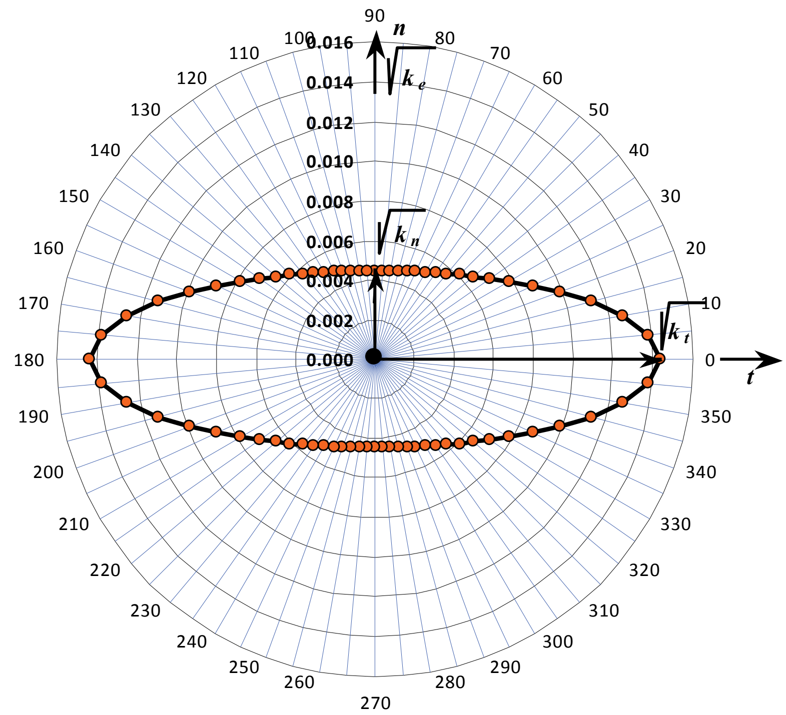

Fig. 6. Hydraulic conductivity ellipse.

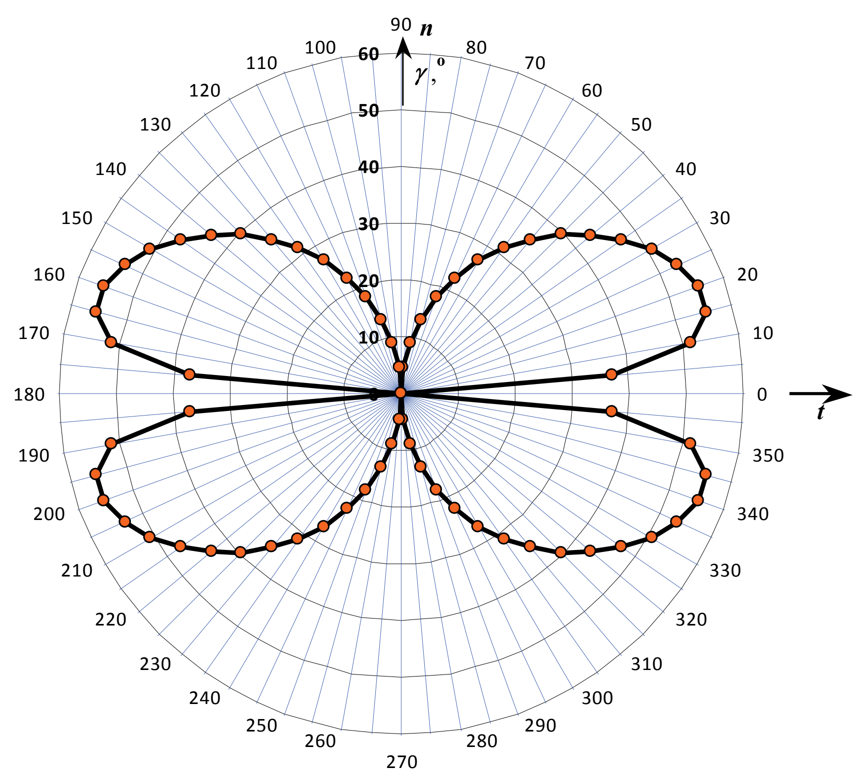

Fig. 7. Angle between hydraulic gradient and filtration velocity depending on filtration direction.

No.	α	γ	$k_{e(t)}$	$k_{e(num)}$	$\frac{\Delta k}{k_{e(t)}}$	$\frac{\Delta Q}{Q_{sr}}$
	°	°	$m \cdot s^{-1}$	$m \cdot s^{-1}$	%	%
1	0	0,00	$2{,}06 \cdot 10^{-4}$	$2{,}06 \cdot 10^{-4}$	0,00	0,00
2	45	39,49	$3{,}61 \cdot 10^{-5}$	$2{,}31 \cdot 10^{-5}$	36,15	-0,03
3	50	35,37	$3{,}16 \cdot 10^{-5}$	$2{,}15 \cdot 10^{-5}$	32,01	3,82
4	55	31,14	$2{,}82 \cdot 10^{-5}$	$2{,}13 \cdot 10^{-5}$	24,39	3,04
5	60	26,81	$2{,}56 \cdot 10^{-5}$	$2{,}11 \cdot 10^{-5}$	17,40	2,43
6	65	22,43	$2{,}36 \cdot 10^{-5}$	$2{,}09 \cdot 10^{-5}$	11,45	1,68
7	70	17,99	$2{,}22 \cdot 10^{-5}$	$2{,}07 \cdot 10^{-5}$	6,64	1,09
8	75	13,52	$2{,}11 \cdot 10^{-5}$	$2{,}04 \cdot 10^{-5}$	3,18	0,61
9	80	9,03	$2{,}04 \cdot 10^{-5}$	$2{,}01 \cdot 10^{-5}$	1,08	0,27
10	85	4,52	$2{,}00 \cdot 10^{-5}$	$1{,}99 \cdot 10^{-5}$	0,17	0,06
11	90	0,00	$1{,}98 \cdot 10^{-5}$	$1{,}98 \cdot 10^{-5}$	0,09	0,00

Table 2. Results of numerical calculations of angle γ between hydraulic gradient and filtration velocity and equivalent hydraulic conductivity $k_{e(num)}$.

It contains theoretically and numerically defined values of equivalent hydraulic conductivity ($k_{e(t)}$ and $k_{e(num)}$), theoretical values of angles γ between grad$H$ and $\mathbf{v}$, error in numerical determination of conductivity in relation to the theoretical value $\frac{\Delta k}{k_{e(t)}} = \frac{k_{e(t)} - k_{e(num)}}{k_{e(t)}} 100\%$, as well as error values $\frac{\Delta Q}{Q_{sr}} = \frac{Q_w - Q_d}{0,5(Q_w + Q_d)} \cdot 100\%$ resulting from a numerical comparison of the quantity of water flowing into (Q_d) and out of (Q_w) the analysed sample. This error was regarded as representative for estimating the accuracy of numerical calculation results.

4.3 Error of hydraulic conductivity determination

In order to estimate the error in determining hydraulic conductivity of a layered medium, it was assumed that the analyses had been performed in a permeameter. The tests were carried out on a layered medium presented in Fig. 5 for two flow directions in relation to the principal axis of permeability tensor. One direction corresponded to angle α of 45˚, while the other - 80˚. The values of equivalent hydraulic conductivity determined correctly through research should be consistent with Tab. 2 and amount to $2{,}31 \cdot 10^{-5}$ $m \cdot s^{-1}$ and $2{,}01 \cdot 10^{-5}$ $m \cdot s^{-1}$ respectively. Using them later to calculate the hydraulic conductivity in the direction of the principal axis of permeability tensor from formula (30), one obtains $k_{\|} = 2{,}75 \cdot 10^{-5}$ $m \cdot s^{-1}$ and $k_{\perp} = 1{,}99 \cdot 10^{-5}$ $m \cdot s^{-1}$. The values calculated in this way were compared with the theoretical properties of a layered medium presented in Tab.1. On can see from the comparison that the values of $k_{\perp}$ are only slightly different (the theoretical value is $1{,}98\ 10^{-5}$ $m\ s^{-1}$), while in the case of $k_{\|}$, the difference is very distinct, reaching 7,5 times. One should also emphasize that such a big difference between these results is not caused by the low accuracy of numerical flow calculations. This is proved by a very small difference in the discharge of water flowing into and out of the sample, no more than 0,3% (Tab. 2).

The error in hydraulic conductivity determination in a permeameter may be also caused by the order of layering. In order to estimate its value, numerical calculations were performed for the adopted flow through a layered medium composed of the same layers but arranged in a different order. One should emphasize that according to formula (30), the theoretical value of equivalent hydraulic conductivity k_t does not depend on the layering order. The calculation results for a flow at the angle $\alpha = 45°$ and for four layering variants are shown in Tab. 3.

Variant No	Layering order	$k_{e(t)}$	$k_{e(num)}$	$\frac{\Delta k}{k_{e(t)}}$	$\frac{\Delta Q}{Q_{sr}}$
		$m \cdot s^{-1}$	$m \cdot s^{-1}$	%	%
1	5-1-2-3-4-5	$3{,}61 \cdot 10^{-5}$	$2{,}31 \cdot 10^{-5}$	36,15	-0,027
2	5-3-2-4-1-5		$2{,}16 \cdot 10^{-5}$	40,14	-0,058
3	5-1-4-2-3-5		$2{,}27 \cdot 10^{-5}$	37,07	0,033
4	5-2-4-3-1-5		$2{,}55 \cdot 10^{-5}$	29,54	0,012

Tab 3. Calculation results of hydraulic conductivity of a layered medium for various layering orders.

One can see that depending on the considered variant, different results are obtained, and anisotropy-related determination error is contained within wide limits. In the analysed examples it oscillates between 30 and over 40% in relation to the theoretical value. In order to illustrate the causes of differences in hydraulic conductivity, calculations of streamline pattern were also performed. For the layering in variants 2 and 4, streamline patterns are shown in Figs. 6 and 7.

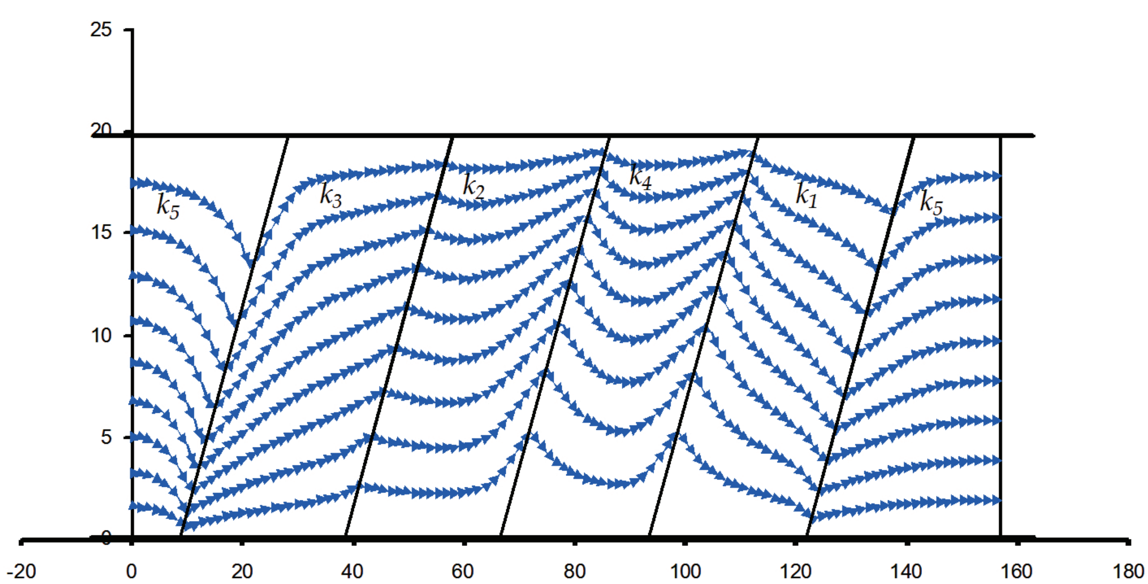

Fig. 6. Streamlines in a sample for layering variant 2.

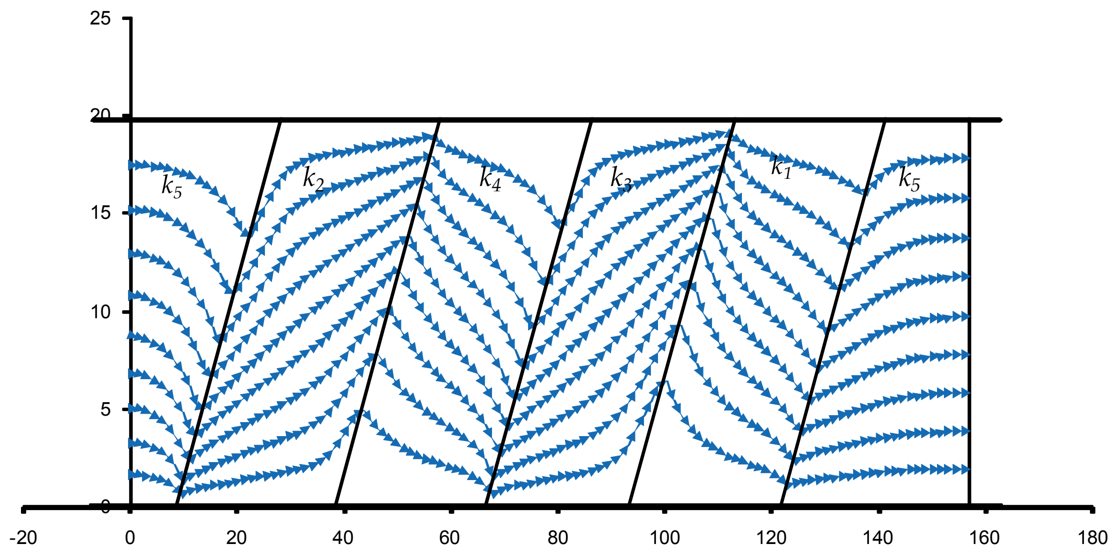

Fig. 7. Streamlines in a sample for layering variant 4.

A distinct difference in streamline patterns is noticeable. Certainly the streamline pattern is basically different from the theoretical streamlines shown in Fig. 3, which are polygonal chains composed of straight line segments.

5. Conclusion

The presented theoretical discussion has demonstrated that flows in layered media composed of homogeneous and anisotropic layers cannot be modelled as flows in a homogeneous medium. However, it is possible in the case of a structure composed of parallel layers characterised by homogeneity and orthotropy in two dimensions. A flow in such a structure can be modelled as a flow in a single anisotropic layer. In such a case one does not have to know the permeability parameters and the thickness of particular layers. In order to calculate the flow, it is then enough to know the hydraulic conductivity in the direction parallel and perpendicular to layering. These values can be determined theoretically by analysing a flow through such a layered structure in any two directions to the boundary surfaces of the layers. However, the calculation results point to a possibility of vital errors occurring while determining the permeability of layered media in a permeameter. This regards those tests where flow occurs in the direction diagonal to layering. The occurrence of major determination errors is connected with a failure to maintain the appropriate directions of grad$H$ and $\mathbf{v}$ and the proper geometric shape of the sample in a permeameter. The value of those errors depends on the value of anisotropy factor λ and the angular difference between grad$H$ and $\mathbf{v}$. In the performed model calculations of a flow through a layered medium with anisotropy factor $\lambda = 10{,}37$, the obtained value of hydraulic conductivity in the direction perpendicular to layer boundary was very close to the theoretical one, while in the parallel direction it was 7,5 times underrated. Clearly then, such analyses result in very serious underrating of anisotropy factor.

The conducted calculations also reveal a considerable influence of layering order on the determination results of hydraulic conductivity in a permeameter. The differences obtained from model calculations oscillated between c. 30 to 40 % in relation to the theoretical value.

To sum up, one should clearly state that the correct determination results of hydraulic conductivity of layered media can be obtained by forcing a flow in the direction parallel and perpendicular to layering. Any departures from this rule may lead to very large determination errors.

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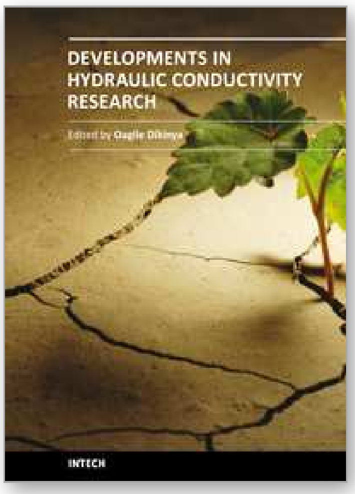

Developments in Hydraulic Conductivity Research
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This book provides the state of the art of the investigation and the in-depth analysis of hydraulic conductivity from the theoretical to semi-empirical models perspective as well as policy development associated with management of land resources emanating from drainage-problem soils. A group of international experts contributed to the development of this book. It is envisaged that this thought provoking book will excite and appeal to academics, engineers, researchers and University students who seek to explore the breadth and in-depth knowledge about hydraulic conductivity. Investigation into hydraulic conductivity is important to the understanding of the movement of solutes and water in the terrestrial environment. Transport of these fluids has various implications on the ecology and quality of environment and subsequently sustenance of livelihoods of the increasing world population. In particular, water flow in the vadose zone is of fundamental importance to geoscientists, soil scientists, hydrogeologists and hydrologists and allied professionals.

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