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2

Stability of Linear Continuous Singular and Discrete Descriptor Systems over Infinite and Finite Time Interval

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1. Introduction

1.1 Classes of systems to be considered

It should be noticed that in some systems we must consider their character of dynamic and static state at the same time. Singular systems (also referred to as degenerate, descriptor, generalized, differential-algebraic systems or semi-state) are those, the dynamics of which are governed by a mixture of algebraic and differential (difference) equations. Recently many scholars have paid much attention to singular systems and have obtained many good consequences. The complex nature of singular systems causes many difficulties in the analytical and numerical treatment of such systems, particularly when there is a real need for their control.

It is well-known that singular systems have been one of the major research fields of control theory. During the past three decades, singular systems have attracted much attention due to the comprehensive applications in economics as the *Leontief* dynamic model (*Silva & Lima* 2003), in electrical (*Campbell* 1980) and mechanical models (*Muller* 1997), etc. Discussion of singular systems originated in 1974 with the fundamental paper of (*Campbell et al.* 1974) and latter on the anthological paper of (*Luenberger* 1977).

The research activities of the authors in the field of singular systems stability have provided many interesting results, the part of which were documented in the recent references. Still there are many problems in this field to be considered. This chapter gives insight into a detailed preview of the stability problems for particular classes of linear continuous and discrete time delayed systems. Here, we present a number of new results concerning stability properties of this class of systems in the sense of Lyapunov and non-Lyapunov and analyze the relationship between them.

1.2 Stability concepts

Numerous significant contributions have been made in the last sixty years in the area of Lyapunov stabilty for different classes of systems. Listing all contributions in this, always attractive area, at this point would represent a waste of time, since all necessary details and existing results, for so called normal systems, are very well known.

But in practice one is not only interested in system stability (e.g. in sense of Lyapunov), but also in bounds of system trajectories. A system could be stable but completely useless because it possesses undesirable transient performances. Thus, it may be useful to consider the stability of such systems with respect to certain sub-sets of state-space, which are *a priori* defined in a given problem.

Besides, it is of particular significance to concern the behavior of dynamical systems only over a finite time interval. These bound properties of system responses, i. e. the solution of system models, are very important from the engineering point of view.

Realizing this fact, numerous definitions of the so-called technical and practical stability were introduced. Roughly speaking, these definitions are essentially based on the predefined boundaries for the perturbation of initial conditions and allowable perturbation of system response. In the engineering applications of control systems, this fact becomes very important and sometimes crucial, for the purpose of characterizing in advance, in quantitative manner, possible deviations of system response. Thus, the analysis of these particular bound properties of solutions is an important step, which precedes the design of control signals, when finite time or practical stability concept are concerned.

2. Singular (descriptor) systems

2.1 Continuous singular systems
2.1.1 Continuous singular systems – stability in the sense of Lyapunov
Generally, the time invariant continuous singular control systems can be written, as:

$$E\dot{\mathbf{x}}(t) = A\mathbf{x}(t), \quad \mathbf{x}(t_0) = \mathbf{x}_0(t), \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is a generalized state space (co-state, semi-state) vector, $E \in \mathbb{R}^{n \times n}$ is a possibly singular matrix, with $rank\, E = r < n$.

Matrices E and A are of the appropriate dimensions and are defined over the field of real numbers.

System (1) is operatinig in a free regime and no external forces are applied on it. It should be stressed that, in a general case, the initial conditions for an autonomus and a system operating in the forced regime need not be the same.

System models of this form have some important advantages in comparison with models in the *normal form, e.g. when* $E = I$ and an appropriate discussion can be found in (*Debeljkovic et al*. 1996, 2004).

The complex nature of singular systems causes many difficultes *in analytical and numerical treatment* that do not appear when systems represented in the normal form are considered. In this sense questions of existence, solvability, uniqueness, and smothness are presented which must be solved in satisfactory manner. A short and concise, acceptable and understandable explanation of all these questions may be found in the paper of (*Debeljkovic* 2004).

STABILITY DEFINITIONS

Stability plays a central role in the theory of systems and control engineering. There are different kinds of stability problems that arise in the study of dynamic systems, such as *Lyapunov* stability, finite time stability, practical stability, technical stability and BIBO stability. The first part of this section is concerned with the asymptotic stability of the equilibrium points of *linear continuous singular systems.*

As we treat the linear systems this is equivalent to the study of the stability of the systems.
The *Lyapunov* direct method (LDM) is well exposed in a number of very well known references.

Here we present some different and interesting approaches to this problem, mostly based on the contributions of the authors of this paper.

Definition 2.1.1.1 System (1) is *regular* if there exist $s \in \mathbb{C}$, $\det(sE - A) \neq 0$, (*Campbell et al.* 1974).

Definition 2.1.1.2 System (1) with $A = I$ is exponentially stable if one can find two positive constants c_1, c_2 such that $\|\mathbf{x}(t)\| \leq c_2 \cdot e^{-c_1 \cdot t} \|\mathbf{x}(0)\|$ for every solution of (1), (*Pandolfi* 1980).

Definition 2.1.1.3 System (1) will be termed *asymptotically stable* if and only if, for all consistent initial conditions $\mathbf{x}_0$, $\mathbf{x}(t) \rightarrow \mathbf{0}$ as $t \rightarrow \infty$, (*Owens & Debeljkovic* 1985).

Definition 2.1.1.4 System (1) is *asymptotically stable* if all roots of $\det(sE - A)$, *i.e.* all finite eigenvalues of this matrix pencil, are in the open left-half complex plane, and system under consideration is *impulsive free* if there is no $\mathbf{x}_0$ such that $\mathbf{x}(t)$ exhibits discontinuous behaviour in the free regime, (*Lewis* 1986).

Definition 2.1.1.5 System (1) is called *asymptotically stable* if and only if all finite eigenvalues λ_i, $i = 1, \ldots, n_1$, of the matrix pencil $(\lambda E - A)$ have negative real parts, (*Muller* 1993).

Definition 2.1.1.6 The equilibrium $\mathbf{x} = \mathbf{0}$ of system (1) is said to be *stable* if for every $\varepsilon > 0$, and for any $t_0 \in \mathfrak{I}$, there exists a $\delta = \delta(\varepsilon, t_0) > 0$, such that $\|\mathbf{x}(t, t_0, \mathbf{x}_0)\| < \varepsilon$ holds for all $t \geq t_0$, whenever $\mathbf{x}_0 \in \mathcal{W}_k$ and $\|\mathbf{x}_0\| < \delta$, where $\mathfrak{I}$ denotes time interval such that $\mathfrak{I} = [t_0, +\infty[$, $t_0 \geq 0$, and $\mathcal{W}_k$ is the subspace of consistent intial conditions (*Chen & Liu* 1997).

Definition 2.1.1.7 The equilibrium $\mathbf{x} = \mathbf{0}$ of a system (1) is said to be *unstable* if there exist a $\varepsilon > 0$, and $t_0 \in \mathfrak{I}$, for any $\delta > 0$, such that there exists a $t^* \geq t_0$, for which $\|\mathbf{x}(t^*, t_0, \mathbf{x}_0)\| \geq \varepsilon$ holds, although $\mathbf{x}_0 \in \mathcal{W}_k$ [1] and $\|\mathbf{x}_0\| < \delta$, (*Chen & Liu* 1997).

Definition 2.1.1.8 The equilibrium $\mathbf{x} = \mathbf{0}$ of a system (1) is said to be *attractive* if for every $t_0 \in \mathfrak{I}$, there exists an $\eta = \eta(t_0) > 0$, such that $\lim_{t \rightarrow \infty} \mathbf{x}(t, t_0, \mathbf{x}_0) = \mathbf{0}$, whenever $\mathbf{x}_0 \in \mathcal{W}_k$ and $\|\mathbf{x}_0\| < \eta$, (*Chen & Liu* 1997).

Definition 2.1.1.9 The equilibrium $\mathbf{x} = \mathbf{0}$ of a singular system (1) is said to be *asymptotically stable* if it is *stable* and *attractive*, (*Chen & Liu* 1997).

    Definition **2.1.1.5** is equivalent to $\lim_{t \rightarrow +\infty} \mathbf{x}(t) = \mathbf{0}$.

Lemma 2.1.1.1 The equilibrium $\mathbf{x} = \mathbf{0}$ of a linear singular system (1) is *asymptotically stable* if and only if it is *impulsive-free*, and $\sigma(E, A) \subset \mathbb{C}^-$, (*Chen & Liu* 997).

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[1] The solutions of continuous singular system models in this investigation are continuously differentiable functions of time t which satisfy the considered equations of the model. Since for continuous singular systems not all initial values $\mathbf{x}_0$ of $\mathbf{x}(t)$ will generate smooth solution, those that generate such solutions (continuous to the right) we call consistent. Moreover, positive solvability condition guarantees uniqueness and closed form of solutions to (1).

Lemma 2.1.1.2 The equilibrium $\mathbf{x} = \mathbf{0}$ of a system (1) is *asymptotically stable* if and only if it is *impulsive-free*, and $\lim_{t \to \infty} \mathbf{x}(t) = \mathbf{0}$, (*Chen & Liu* 1997).

Due to the system structure and complicated solution, the regularity of the systems is the condition to make the solution to singular control systems exist and be unique.

Moreover if the consistent initial conditions are applied, then the closed form of solutions can be established.

STABILITY THEOREMS

Theorem 2.1.1.1 System (1), with $A = I$, I being the identity matrix, is *exponentially stable* if and only if the eigenvalues of E have non positive real parts, (*Pandolfi* 1980).

Theorem 2.1.1.2 Let $I_{\mathcal{W}_k}$ be the matrix which represents the operator on $\mathbb{R}^n$ which is the identity on $\mathcal{W}_k$ and the zero operator on $\mathcal{W}_k$.

System (1), with $A = I$, is stable if an $(n \times n)$ matrix P exist, which is the solution of the matrix equation:

$$E^T P + PE = -I_{\mathcal{W}_k} ,\tag{2}$$

with the following properties:

$$P = P^T ,\tag{3}$$

$$P\mathbf{q} = \mathbf{0}, \ \mathbf{q} \in \mathcal{V} ,\tag{4}$$

$$\mathbf{q}^T P \mathbf{q} > 0, \ \mathbf{q} \neq \mathbf{0}, \ \mathbf{q} \in \mathcal{W}_k ,\tag{5}$$

where:

$$\mathcal{W}_k = \aleph\left(I - EE^D\right)\tag{6}$$

$$\mathcal{V} = \aleph\left(EE^D\right),\tag{7}$$

where $\mathcal{W}_k$ is the subspace of consistent initial conditions, (*Pandolfi* 1980) and $\aleph(\ )$ denotes the kerrnel or null space of the matrix $(\ )$.

Theorem 2.1.1.3 System (1) is *asymptotically stable* if and only if (*Owens & Debeljkovic* 1985):

a. A is invertible.

b.   A positive-definite, self-adjoint operator P on $\mathbb{R}^n$ exists, such that:

$$A^T PE + E^T PA = -Q ,\tag{8}$$

where Q is self-adjoint and positive in the sense that:

$$\mathbf{x}^T(t) Q \mathbf{x}(t) > 0 \ \text{for all} \ \mathbf{x}(t) \in \mathcal{W}_{k^*} \setminus \{\mathbf{0}\} .\tag{9}$$

Theorem 2.1.1.4 System (1) is *asymptotically stable* if and only if (*Owens & Debeljkovic* 1985):

a. A is invertible,

b. there exists a positive-definite, self-adjoin operator P, such that:

$$\mathbf{x}^T(t)\left(A^T PE + E^T PA\right)\mathbf{x}(t) = -\mathbf{x}^T(t)I\mathbf{x}(t),\tag{10}$$

for all $\mathbf{x} \in \mathcal{W}_{k^*}$, where $\mathcal{W}_{k^*}$ denotes the subspace of consistent initial conditions.

2.1.2 Continuous singular systems – stability over finite time interval

Dynamical behaviour of the system (1) is defined over time interval $\mathfrak{I} = \{t : t_0 \le t \le t_0 + T\}$, where quantity T may be either a positive real number or symbol $+\infty$, so finite time stability and practical stability can be treated simultaneously. Time invariant sets, used as bounds of system trajectories, are assumed to be open, connected and bounded.

Let index β stand for the set of all allowable states of system and index α for the set of all initial states of the system, such that $\mathcal{S}_\alpha \subseteq \mathcal{S}_\beta$.

In general, one may write:

$$\mathcal{S}_\wp = \left\{\mathbf{x} : \|\mathbf{x}(t)\|_Q < \wp\right\}, \ \ \mathbf{x}(t) \in \mathcal{W}_k \setminus \{0\},\tag{11}$$

where Q will be assumed to be symmetric, positive definite, real matrix and where $\mathcal{W}_k$ denotes the sub-space of consistent initial conditions generating the smooth solutions.

A short and concise, acceptable and understandable explanation of all these questions can be found in the paper of (*Debeljkovic* 2004). Vector of initial conditions is consistent if there exists continuous, differentiable solution to (1).

A geometric treatment (*Owens & Debeljkovic* 1985) yields $\mathcal{W}_k$ as the limit of the sub-space algorithm:

$$\mathcal{W}_0 = \mathbb{R}^n, \ \ \ \mathcal{W}_{j+1} = A^{-1}\left(E\mathcal{W}_j\right), \ j \ge 0,\tag{12}$$

where $A^{-1}(\cdot)$ denotes inverse image of $(\cdot)$ under the operator A.

Campbell et al. (1974) have shown that sub-space $\mathcal{W}_k$ represents the set of vectors satisfying:

$$\left(I - \hat{E}^D\hat{E}\right)\mathbf{x}_0 = \mathbf{0}, \ \text{ or } \ \mathcal{W}_k = \aleph\left(I - \hat{E}^D\hat{E}\right),\tag{13}$$

where $\hat{E} = \left(\lambda E - A\right)^{-1}E$. c is any complex scalar such that:

$$\det\left(\lambda E - A\right) \ne 0 \ \text{ or } \ \mathcal{W}_k \cap \aleph(E) = \{0\}.\tag{14}$$

This condition guarantees the uniqueness of solutions that are generated by $\mathcal{W}_k$ and $\left(\lambda E - A\right)$ is invertible for some $\lambda \in \mathbb{R}$. The null space of matrix F is denoted by $\aleph(F)$, range space with $\Re(F)$ and superscript "D" is used to indicate Drazin inverse. Let $\|\mathbf{x}(t)\|_{(\cdot)}$ be any vector norm (i. g. $\cdot = 1, 2, \infty$) and $\|(\cdot)\|$ the matrix norm induced by this vector.

The matrix measure, for our purposes, is defined as follows:

$$\mu(F) = \frac{1}{2} \max_i \lambda_i (F^* + F), \tag{15}$$

for any matrix $F \in \mathbb{C}^{n \times n}$. Upper index $*$ denotes transpose conjugate. In case of $F \in \mathbb{R}^{n \times n}$ it follows $F^* = F^T$, where superscript T denotes transpose.

The value of a particular solution at the moment t, which at the moment $t = 0$ passes through the point $\mathbf{x}_0$, is denoted as $\mathbf{x}(t, \mathbf{x}_0)$, in abbreviated notation $\mathbf{x}(t)$.

The set of all points S_i, in the phase space $\mathbb{R}^n$, $S_i \subseteq \mathbb{R}^n$, which generate smooth solutions can be determined via the *Drazin* inverse technique.

STABILITY DEFINITIONS

Definition 2.1.2.1 System (1) *is finite time stable* w.r.t. $\{\alpha, \beta, Q, \Im\}$, $\alpha < \beta$, iff $\forall \mathbf{x}(t_0) = \mathbf{x}_0 \in \mathcal{W}_k$, satisfying $\|\mathbf{x}_0\|_Q^2 < \alpha$, implies $\|\mathbf{x}(t)\|_Q^2 < \beta$, $\forall t \in \Im$, (*Debeljkovic & Owens* 1985).

Definition 2.1.2.2 System (1) is *finite time instable* w.r.t. $\{\alpha, \beta, Q, \Im\}$, $\alpha < \beta$, iff for $\forall \mathbf{x}(t_0) = \mathbf{x}_0 \in \mathcal{W}_k$, satisfying $\|\mathbf{x}_0\|_Q^2 < \alpha$, exists $t^* \in \Im$ implying $\|\mathbf{x}(t^*)\|_Q^2 \geq \beta$, (*Debeljkovic & Owens* 1985).

Preposition 2.1.2.1 If $\varphi(\mathbf{x}) = \mathbf{x}^T(t) M \mathbf{x}(t)$ is quadratic form on $\mathbb{R}^n$ then it follows that there exist numbers $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$, satisfying $-\infty < \lambda_{\min}(M) \leq \lambda_{\max}(M) < +\infty$, such that:

$$\lambda_{\min}(M) \leq \frac{\mathbf{x}^T(t) M \mathbf{x}(t)}{V(\mathbf{x})} \leq \lambda_{\max}(M), \quad \forall \mathbf{x} \in \mathcal{W}_k \setminus \{\mathbf{0}\}. \tag{16}$$

If $M = M^T$ and $\mathbf{x}^T(t) M \mathbf{x}(t) > 0$, $\forall \mathbf{x} \in \mathcal{W}_k \setminus \{\mathbf{0}\}$, then $\lambda_{\min}(M) > 0$ and $\lambda_{\max}(M) > 0$, where $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ are defined in such way:

$$\begin{aligned} \lambda_{\min}(M) &= \min \left\{ \begin{matrix} \mathbf{x}^T(t) M \mathbf{x}(t), \ \mathbf{x} \in \mathcal{W}_k \setminus \{\mathbf{0}\}, \\ \mathbf{x}^T(t) E^T P E \mathbf{x}(t) = 1 \end{matrix} \right\}, \\ \lambda_{\max}(M) &= \max \left\{ \begin{matrix} \mathbf{x}^T(t) M \mathbf{x}(t), \ \mathbf{x} \in \mathcal{W}_k \setminus \{\mathbf{0}\}, \\ \mathbf{x}^T(t) E^T P E \mathbf{x}(t) = 1 \end{matrix} \right\}. \end{aligned} \tag{17}$$

It is convenient to consider, for the purposes of this exposure, the aggregation function for the system (1) in the following manner:

$$V(\mathbf{x}(t)) = \mathbf{x}^T(t) E^T P E \mathbf{x}(t), \tag{18}$$

with particular choice $P = I$, I being identy matrix.

STABILITY THEOREMS

Theorem 2.1.2.1 The system is *finite stable* with respect to $\{\alpha, \beta, \Im\}$, $\alpha < \beta$, if the following conditiones are satisfied:

$$(i) \qquad\qquad \beta/\alpha > \frac{\gamma_2(Q)}{\gamma_1(Q)} \qquad\qquad\qquad\qquad (19)$$

$$(ii) \qquad\qquad \ln \beta/\alpha > \Lambda(Q) + \ln \frac{\gamma_2(Q)}{\gamma_1(Q)}, \quad \forall t \in \Im . \qquad\qquad (20)$$

with $\lambda_{\max}(Q)$ as in Preposition 2.1.2.1, (*Debeljkovic & Owens* 1985).

Preposition 2.1.2.2 There exists matrix $P = P^T > 0$, such that $\gamma_1(Q) = \gamma_2(Q) = 1$, (*Debeljkovic & Owens* 1985).

Corollary 2.1.2.1 If $\beta/\alpha > 1$, there exist choice of P such that

$$\frac{\beta}{\alpha} > \frac{\gamma_2(Q)}{\gamma_1(Q)} . \qquad\qquad\qquad\qquad (21)$$

The practical meaning of this result is that condition (i) of *Definition* 2.1.2.1 can be satisfied by initial choice of free parameters of matrix P. Condition (*ii*) depends also on the system data and hence is more complex but it is also natural to ask whether we can choose P such that $\lambda_{\max}(Q) < 0$, (*Debeljkovic & Owens* 1985).

Theorem 2.1.2.2 System (1) is *finite time stable* w.r.t. $\{\alpha, \beta, I, \Im\}$ if the following condition is satisfied

$$\Phi_{CSS}(t) < \sqrt{\frac{\beta}{\alpha}}, \quad \forall t \in \Im , \qquad\qquad\qquad (22)$$

$\Phi_{CSS}(t)$ being the *fundamental matrix* of linear singular system (1), (*Debeljkovic et al.* 1997).

Now we apply matrix mesure approach.

Theorem 2.1.2.3 System (1) is *finite time stable* w.r.t. $\{\alpha, \beta, I, \Im\}$, if the following condition is satisfied (*Debeljkovic et al.* 1997).

$$e^{\mu(\Upsilon) \cdot t} < \frac{\beta}{\alpha}, \quad \forall t \in \Im , \qquad\qquad\qquad (23)$$

where:

$$\Upsilon = \hat{E}^D \hat{A}, \quad \hat{A} = (sE - A)^{-1} A, \quad \hat{E} = (sE - A)^{-1} E . \qquad\qquad (24)$$

Starting with explicit solution of system (1), derived in (*Campbell* 1980).

$$\mathbf{x}(t) = e^{\hat{E}^D \hat{A}(t - t_0)} \mathbf{x}_0, \qquad \mathbf{x}_0 = \hat{E} \hat{E}^D \mathbf{x}_0 , \qquad\qquad (25)$$

and differentiating equitation (25), one gets:

$$\dot{\mathbf{x}}(t) = \hat{E}^D \hat{A} e^{\hat{E}^D \hat{A} \cdot t} \cdot \mathbf{x}_0 = \hat{E}^D \hat{A} \mathbf{x}(t) , \qquad\qquad (26)$$

so only the *regular* singular systems are treated with matrices given in (24).

Theorem 2.1.2.4 For given constant matrix $\hat{E}^D \hat{A}$ any solution of (1) satisfies the following inequality (*Kablar & Debeljkovic* 1998).

$$\left\| \mathbf{x}(t_0) \right\| e^{-\mu\left(-\hat{E}^D \hat{A}\right)(t-t_0)} \leq \left\| \mathbf{x}(t) \right\| \leq \left\| \mathbf{x}(t_0) \right\| e^{\mu\left(\hat{E}^D \hat{A}\right)(t-t_0)}, \quad \forall t \in \mathfrak{I} \tag{27}$$

Theorem 2.1.2.5 In order for the system (1) to *be finite time stable* w.r.t. $\{\alpha, \beta, I, \mathfrak{I}\}$, $\alpha < \beta$, it is *necessary* that the following condition is satisfied:

$$e^{-\mu\left(-\hat{E}^D \hat{A}\right)\cdot(t-t_0)} < \sqrt{\frac{\beta}{\delta}}, \quad \forall t \in \mathfrak{I}, \tag{28}$$

where $0 < \delta \leq \alpha$, (*Kablar & Debeljkovic* 1998).

Theorem 2.1.2.6 In order for system (1) to be *finite time instable* w.r.t. $\{\alpha, \beta, I, \mathfrak{I}\}$, $\alpha < \beta$, it is *necessary* that there exists $t^* \in \mathfrak{I}$ such that the following condition is satisfied:

$$e^{\mu\left(\hat{E}^D \hat{A}\right)\cdot\left(t^*-t_0\right)} \geq \sqrt{\frac{\beta}{\alpha}}, \quad t^* \in \mathfrak{I}. \tag{29}$$

Theorem 2.1.2.7 System (1) *is finite time instable* w.r.t. $\{\alpha, \beta, I, \mathfrak{I}\}$, $\alpha < \beta$, *if* $\exists \delta, 0 < \delta \leq \alpha$ and $t^* \in \mathfrak{I}$ such that the following condition is satisfied:

$$e^{-\mu\left(-\hat{E}^D \hat{A}\right)\cdot\left(t^*-t_0\right)} < \sqrt{\frac{\beta}{\delta}}, \quad t^* \in \mathfrak{I}. \tag{30}$$

Finally, we present *Bellman–Gronwall approach* to derive our results, earlier given in *Theorem 2.1.2.7.*

Lemma 2.1.2.1 Suppose the vector $\mathbf{q}(t, t_0)$ is defined in the following manner (*Debeljkovic & Kablar* 1999):

$$\mathbf{q}(t, t_0) = \Phi(t, t_0) \hat{E}^D \hat{E} \mathbf{v}(t_0). \tag{31}$$

So if:

$$E\mathbf{q}(t, t_0) = E\Phi(t, t_0) \hat{E}^D \hat{E} \mathbf{v}(t_0), \tag{32}$$

then:

$$\left\| \mathbf{q}(t, t_0) \right\|_{E^T E}^2 \leq \left\| \mathbf{v}(t_0) \right\|_{E^T E}^2 e^{\lambda_{\max}(M)(t-t_0)}, \tag{33}$$

where:

$$\lambda_{\max}(M) = \max\{\mathbf{q}^T(t, t_0) \Xi \mathbf{q}(t, t_0) : \mathbf{q}(t, t_0) \in \mathcal{W}_k \setminus \{0\},$$
$$\mathbf{q}^T(t, t_0) E^T E \mathbf{q}(t, t_0) = 1, \quad \Xi = A^T E + E^T A\} \tag{34}$$

$$\mathbf{v}(t_0) = \mathbf{q}(t_0, t_0). \tag{35}$$

Using this approach the results of *Theorem* 2.1.2.1 can be reformulate in the following manner.

Theorem 2.1.2.8 System (1) is *finite time stable* w.r.t. $\left\{\alpha, \beta, \left\|(\cdot)\right\|_Q^2, \Im\right\}$, $a < \beta$, if the following condition is satisfied:

$$e^{\lambda_{\max}(\Xi)\cdot(t-t_0)} < \frac{\beta}{\alpha}, \quad \forall t \in \Im, \tag{36}$$

with $\lambda_{\max}(M)$ given (34), (*Debeljkovic & Kablar* 1999).

2.2 Discrete descriptor system
2.2.1 Discrete descriptor system – stability in sense of Lyapunov
Generally, the time invariant linear discrete descriptor control systems can be written, as:

$$E\mathbf{x}(k+1) = A\mathbf{x}(k), \quad \mathbf{x}(k_0) = \mathbf{x}_0, \tag{37}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is a generalized state space (co-state, semi-state) vector, $E \in \mathbb{R}^{n \times n}$ is a possibly singular matrix, with $rank\, E = r < n$. Matrices E and A are of the appropriate dimensions and are defined over the field of real numbers.

NECESSARY CONSIDERATIONS

In the discrete case, the concept of smoothness has little meaning but the idea of consistent initial conditions being these initial conditions $\mathbf{x}_0$, that generate solution sequences $(\mathbf{x}(k): k \geq 0)$ has a physical meaning.

The fundamental geometric tool in the characterization of the subspace of consistent initial conditions $\mathcal{W}_d$, is the subspace sequence:

$$W_{d,0} = R^n, \quad \mathcal{W}_{d,j+1} = A^{-1}\left(E\mathcal{W}_{d,j}\right), \quad (j \geq 0). \tag{38}$$

Here $A^{-1}(\cdot)$ denotes the inverse image of $(\cdot)$ under the operator A and we will denote by $\aleph(F)$ and $\Re(F)$ the kernel and range of any operator F, respectively.

Lemma 2.2.1.1 The subspace sequence $\left\{\mathcal{W}_{d,0}, \mathcal{W}_{d,1}, \mathcal{W}_{d,2}, \ldots\right\}$ is nested in the sense that:

$$\mathcal{W}_{d,0} \supset \mathcal{W}_{d,1} \supset \mathcal{W}_{d,2} \supset \mathcal{W}_{d,3} \supset \cdots. \tag{39}$$

Moreover:

$$\aleph(A) \subset \mathcal{W}_{d,j}, \, (j \geq 0), \tag{40}$$

and there exists an integer $k \geq 0$, such that:

$$\mathcal{W}_{d,k+1} = \mathcal{W}_{d,k}, \tag{41}$$

and hence $\mathcal{W}_{d,k+1} = \mathcal{W}_{d,k}$ for $j \geq 1$.

If k^* is the smallest such integer with this property, then:

$$\mathcal{W}_{d,k} \cap \aleph(E) = \{0\}, \left(k \geq k^*\right),$$ (42)

provided that $(\lambda E - A)$ is invertible for some $\lambda \in R$, (*Owens & Debeljkovic* 1985).

Theorem 2.2.1.1 Under the conditions of *Lemma* 2.2.1.1, $\mathbf{x}_0$ is a consistent initial condition for (37) if $\mathbf{x}_0 \in \mathcal{W}_{d,k^*}$. Moreover $\mathbf{x}_0$ generates a unique solution $\mathbf{x}(t) \in \mathcal{W}_{d,k^*}$, $(k \geq 0)$ that is real - analytic on $\{k : k \geq 0\}$, (*Owens & Debeljkovic* 1985).

Theorem 2.2.1.1 is the geometric counterpart of the algebraic results of *Campbell* (1980). A short and concise, acceptable and understandable explanation of all these questions can be found in the papers of (*Debeljkovic* 2004).

Definition 2.2.1.1 The linear discrete descriptor system (37) is said to be regular if $\det(sE - A)$ is not identically equal to zero, (*Dai* 1989).

Remark 2.2.1.1 Note that the regularity of matrix pair (*E*, *A*) guarantees the existence and uniqueness of solution $\mathbf{x}$ (·) for any specified initial condition, and the impulse immunity avoids impulsive behavior at initial time for inconsistent initial conditions. It is clear that, for nontrivial case, $\det E \neq 0$, impulse immunity implies regularity.

Definition 2.2.1.2 The linear discrete descriptor system (37) is assumed to be *non-degenerate* (or regular), i.e. $\det(zE - A) \neq 0$. Otherwise, it will be called *degenerate*, (*Syrmos et al.* 1995).

If $(zE - A)$ is non-degenerate, we define the *spectrum* of $(zE - A)$, denoted as $\sigma\{E, A\}$ as those isolated values of z where $\det(zE - A) \neq 0$ fails to hold. The usual spectrum of $(zI - A)$ will be denoted as $\sigma\{A\}$.

Note that owing to (possible) singularity of E, $\sigma\{E, A\}$ may contain finite and infinite values of z.

Definition 2.2.1.3 The linear discrete descriptor system (37) is said to be *causal* if (37) is regular and degree $\det(zE - A) = \text{rank } E$, (*Dai* 1989).

Definition 2.2.1.4 A pair (*E*, *A*) is said to be *admissible* if it is regular, impulse-free and stable, *Hsiung*, *Lee* (1999).

Lemma 2.2.1.2 The linear discrete-time descriptor system (37) is regular, causal and stable if and only if there exists an invertible symmetric matrix $H \in R^{n \times n}$ such that the following two inequalities holds (*Xu & Yang* 1999):

$$E^T H E \geq 0,$$ (43)

$$A^T H A - E^T H E < 0.$$ (44)

STABILITY DEFINITIONS

Definition 2.2.1.5 Linear discrete descriptor system (37) is said to be stable if and only if (37) is regular and all of its finite poles are within region $\Omega(0,1)$, (*Dai* 1989).

Definition 2.2.1.6 The system in (37) is asymptotically stable if all the finite eigenvalues of the pencil $(zE - A)$ are inside the unit circle, and anticipation free if every admissible $\mathbf{x}(0)$ in (37) admits one-sided solutions, (*Syrmos et al.* 1995).

Definition 2.2.1.7 Linear discrete descriptor system (37) is said to be asymptotically stable if, for all consistent initial conditions $\mathbf{x}_0$, we have that $\mathbf{x}(t) \to \mathbf{0}$ as $t \to +\infty$, (*Owens & Debeljkovic* 1985).

STABILITY THEOREMS

First, we present the fundamental work in the area of stability in the sense of Lyapunov applied to the linear discrete descriptor systems, (*Owens & Debeljkovic* 1985).

Our attention is restricted to the case of singular (i.e. noninvertible) E and the construction of geometric conditions on $\mathbf{x}_0$ for the existence of causal solutions of (37) in terms of the relative subspace structure of matrices E and A. The results are hence a geometric counterpart of the algebraic theory of (*Campbell* 1980) who established the required form of $\mathbf{x}_0$ in terms of the Drazin inverse and the technical trick of replacing E and A by commuting operators.

The ideas in this paper work with E and A directly and commutability is not assumed. The geometric theory of consistency leads to a natural class of positive-definite quadratic forms on the subspace containing all solutions. This fact makes possible the construction of a Lyapunov stability theory for linear discrete descriptor systems in the sense that asymptotic stability is equivalent to the existence of symmetric, positive-definite solutions to *a weak form of Lyapunov equation.*

Throughout this exposure it is assumed that $(\lambda E - A)$ is invertible at all but a finite number of points $\lambda \in C$ and hence that if a solution $\mathbf{x}(k), (k \geq 0)$ of $(\mathbf{x}(k) : k = 0,1,...)$ exists for a given choice of $\mathbf{x}_0$, it is unique, (*Campbell* 1980).

The linear discrete descriptor system is said to be stable if (37) is regular and all of its finite poles are within region $\Omega(0,1)$, (*Dai* 1989), so careful investigation shows there is no need for the matrix A to be invertible, in comparison with continuous case, see (*Debeljkovic et al.* 2007) so it could be noninvertible.

Theorem 2.2.1.2 The linear discrete descriptor system (37) is asymptotically stable if, and only if, there exists a real number $\lambda^* > 0$ such that, for all λ in the range $0 < |\lambda| < \lambda^*$, there exists a self-adjoint, positive-definite operator H_λ in R^n satisfying:

$$(A - \lambda E)^T H_\lambda (A - \lambda E) - E^T H_\lambda E = -Q_\lambda, \tag{45}$$

for some self-adjoint operator Q_λ satisfying the positivity condition (*Owens & Debeljkovic* 1985):

$$\mathbf{x}^T(t) Q_\lambda \mathbf{x}(t) > 0, \quad \forall \mathbf{x}(t) \in \mathcal{W}_{d,k^*} \setminus \{0\}. \tag{46}$$

Theorem 2.2.1.3 Suppose that matrix A is invertible. Then the linear discrete descriptor system (37) is asymptotically stable if, and only if, there exists a self-adjoint, positive-definite solution H in R^n satisfying

$$A^T H A - E^T H E = -Q, \tag{47}$$

where Q is self-adjoint and positive in the sense that (*Owens & Debeljkovic* 1985):

$$\mathbf{x}^T(t)Q\mathbf{x}(t) > 0, \quad \forall \mathbf{x}(t) \in \mathcal{W}_{d,k^*} \setminus \{0\}. \tag{48}$$

Theorem 2.2.1.4 The linear discrete descriptor system (37) is asymptotically stable if and only if there exists a real number $\lambda^* > 0$ such that, for all λ in the range $0 < |\lambda| < \lambda^*$, there exists a self-adjoint, positive-definite operator H_λ in R^n satisfying *Owens, Debeljkovic* (1985):

$$\mathbf{x}^T(t)\left((A - \lambda E)^T H_\lambda (A - \lambda E) - E^T H_\lambda E\right)\mathbf{x}(t) = -\mathbf{x}^T(t)\mathbf{x}(t), \quad \forall \mathbf{x}(t) \in \mathcal{W}_{d,k^*}. \tag{49}$$

Corollary 2.2.1.4 If matrix A is invertible, then the linear discrete descriptor system (37) is asymptotically stable if and only if (49) holds for $\lambda = 0$ and some self-adjoint, positive-definite operator H_0, (*Owens & Debeljkovic* 1985).

2.2.2 Discrete descriptor system – stability over infinite time interval

Dynamical behaviour of system (37) is defined over time interval $\mathcal{K} = \left\{ k_0, (k_0 + k_N) \right\}$, where quantity k_N may be either a positive real number or symbol $+\infty$, so finite time stability and practical stability can be treated simultaneously.

Time invariant sets, used as bounds of system trajectories, are assumed to be open, connected and bounded.

Let index β stands for the set of all allowable states of system and index α for the set of all initial states of the system, such that $\forall \mathbf{x}(k_0) = \mathbf{x}_0 \in \mathcal{W}_d$.

Sets are assumed to be open, connected and bounded and defined by (11) in discrete case sense.

Under assumption that *discrete version of* the Preposition 2.1.2.1 is acceptable here, without any limitation, we can give the following *Definitions*.

STABILITY DEFINITIONS

Definition 2.2.2.1 System (37) is finite time stable w.r.t $\left\{ \alpha, \beta, G, \mathcal{K}, \mathcal{W}_d \right\}$, if and only if a *consistent initial condition*, $\mathbf{x}_0 \in \mathcal{W}_d$, satisfying $\|\mathbf{x}_0\|_G^2 < \alpha$, $G = E^T PE$, implies $\|\mathbf{x}(k)\|_G^2 < \beta$, $\forall k \in \mathcal{K}$. G is chosen to represent physical constraints on the system variables and it is assumed, as before, to satisfy $G = G^T$, $\mathbf{x}^T(k)G\mathbf{x}(k) > 0, \forall \mathbf{x}(k) \in \mathcal{W}_d \setminus \{0\}$, (*Debeljkovic* 1985, 1986), (*Debeljkovic, Owens* 1986), (*Owens, Debeljkovic* 1986).

Definition 2.2.2.2 System (37) is finite time unstable w.r.t respect to $\left\{ K, \alpha, \beta, G, W_q \right\}$, if and only if there is a *consistent initial condition*, satisfying $\|\mathbf{x}_0\|_G^2 < \alpha$, $G = E^T PE$, and there exists discrete moment $k^* \in K$, such that the next condition is fulfilled $\|x(k^*)\|_G^2 > \beta$, for some $k^* \in \mathcal{K}$, (*Debeljkovic & Owens* 1986), (*Owens & Debeljkovic* 1986).

STABILITY THEOREMS

Theorem 2.2.2.1 System (37) is finite time stable w.r.t $\left\{ \alpha, \beta, \mathcal{K} \right\}$, $\beta > \alpha$, if the following condition is satisfied:

$$\lambda_{\max}^k(Q) < \beta \,/\, \alpha, \quad \forall k \in \mathcal{K}, \tag{50}$$

where $\lambda_{\max}^k(Q)$ is defined by:

$$\lambda_{\max}^k(Q) = \max_{\mathbf{x}}\left\{\mathbf{x}^\mathrm{T}(k)A^\mathrm{T}PA\mathbf{x}(k): \ \mathbf{x}(k) \in \mathcal{W}_d \setminus \{0\}, \ \mathbf{x}^\mathrm{T}(k)E^\mathrm{T}PE\mathbf{x}(k) = 1\right\} \tag{51}$$

with matrix $P = P^T > 0$, (*Debeljkovic* 1986), (*Debeljkovic & Owens* 1986).

Theorem 2.2.2.2 System (37) is finite time unstable w.r.t $\{\alpha, \beta, \mathcal{K}\}$, $\beta > \alpha$ if there exists a positive scalar $\gamma \in \,]0, \alpha[$ and a discrete moment k^*, $\exists(k^* > k_0) \in \mathcal{K}$ such that the following condition is satisfied (*Debeljkovic & Owens* 1986):

$$\lambda_{\min}^{k^*}(Q) > \beta \,/\, \gamma, \ \text{for some } k^* \in \mathcal{K} \tag{52}$$

where $\lambda^k(Q)$ being defined by:

$$\lambda_{\min}^k(Q) = \min_{\mathbf{x}}\left\{\mathbf{x}^\mathrm{T}(k)A^\mathrm{T}PA\mathbf{x}(k): \ \mathbf{x}(k) \in \mathcal{W}_d \setminus \{0\}, \ \mathbf{x}^\mathrm{T}(k)E^\mathrm{T}PE\mathbf{x}(k) = 1\right\}. \tag{53}$$

Theorem 2.2.2.3. System (37) is finite time stable w.r.t $\{\alpha, \beta, \mathcal{K}\}$, $\beta > \alpha$, if the following condition is satisfied:

$$\left\|\Psi(k)\right\| < \beta \,/\, \alpha, \quad \forall k \in K. \tag{54}$$

where: $\Psi(k) = \left(\hat{E}^D\hat{A}\right)^k$ and $\hat{E} = (cE - A)^{-1}E$, $\hat{A} = (cE - A)^{-1}A$, (*Debeljkovic* 1986).

3. Conclusion

This chapter considers important stability issues of linear continuous singular and discrete descriptor systems over infinite and finite time interval. Here, we present a number of new results concerning stability properties of this class of systems *in the sense of Lyapunov and non-Lyapunov* and analyze the relationship between them over finite and infinite time interval.

In the first part of the chapter continuous singular systems were considered. Basic stability concepts were introduced, starting with a preview of important stability definitions. Stability in the sense of Lyapunov, as well as the stability over finite time interval were addressed in detail.

Second part of this chapter deals with stability issues for discrete descriptor systems in the sense of Lyapunov and over infinite and finite time interval.

The chapter also represents a comprehensive survey on important stability theorems which apply to studied classes of systems.

The geometric theory of consistency leads to the natural class of positive definite quadratic forms on the subspace containing all solutions. This fact makes possible the construction of Lyapunov stability theory even for the *time delay systems* in that sense that asymptotic

stability is equivalent to the existence of symmetric, positive definite solutions to a *weak* form of Lyapunov continuous (discrete) algebraic matrix equation (*Owens*, *Debeljkovic* 1985) respectively, incorporating condition which refers to time delay term.

Time delay systems represent a special and very important class of systems and therefore their investigation deserves special attention. Detailed consideration of time delayed systems, together with important new results of the authors, will be presented in the subsequent chapter, which concerns continuous singular as well as discrete descriptor time delay systems. Presented chapter is therefore a necessary premise as an introduction to the stability issues of continuous singular and discrete descriptor time delay system, which provides consistency and comprehensibility of the presented topics.

4. Acknowledgment

5. References

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Time-Delay Systems

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Time delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, robotics, etc. The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. Consequently, the problem of controllability, observability, robustness, optimization, adaptive control, pole placement and particularly stability and robustness stabilization for this class of systems, has been one of the main interests for many scientists and researchers during the last five decades.

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