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Advanced Attitude and Position MIMO Robust Control Strategies for Telescope-Type Spacecraft with Large Flexible Appendages

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1. Introduction

With extraordinary high priority science objectives to break the current barriers of our knowledge of the universe, and dealing with significant weight limitations of launch vehicle for cost-effective access to space, several NASA and ESA missions will involve both formation flying technology and satellites with large flexible structures in the next few decades: Terrestrial Planet Finder, Stellar and Planet Imager, Life Finder, Darwin and Lisa missions, etc.
This chapter deals with the design of multi-input multi-output (MIMO) robust control strategies to regulate simultaneously the position and attitude of a telescope-type spacecraft with large flexible appendages. Section 2 describes the main control challenges and dynamic characteristics of a MIMO system in general, and a spacecraft in particular; Section 3 presents advanced techniques to design MIMO robust controllers based on the quantitative feedback theory (QFT); and Section 4 shows some illustrative results achieved when applying the MIMO QFT control methodology to one of the telescope-type spacecraft (a 6-inputs/6-outputs MIMO system) of a multiple formation flying constellation of a European Space Agency (ESA) cornerstone mission (Fig. 1).
Control of spacecraft with large flexible structures and very demanding astronomical performance specifications, as the telescope-type satellite mission, involves significant difficulties due to the combination of a large number of flexible modes with small damping, model uncertainty and coupling among the inputs and outputs. The scientific objectives of such missions require very demanding control specifications, as micrometer accuracy for position and milli-arc-second precision for attitude, high disturbance rejection properties, loop-coupling attenuation and low controller complexity and order. The dynamics of such spacecraft usually present a complex 6-inputs/6-outputs MIMO plant, with 36 transfer functions with high order dynamics (50th order models in our example), large model uncertainty and high loop interactions introduced by the flexible modes of the low-stiffness appendages.
This chapter presents advanced tools and techniques to analyse and design MIMO robust control systems to regulate simultaneously the position and attitude of telescope-type spacecraft with large flexible appendages.

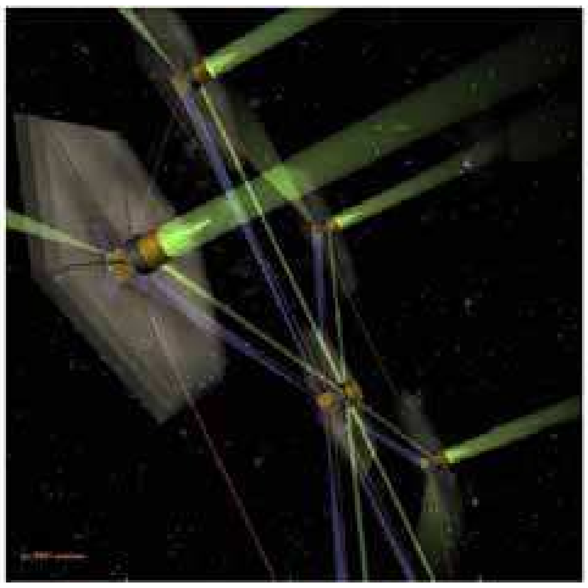


Fig. 1. Telescope-type spacecraft with large flexible appendages flying in formation (ESA courtesy)

2. MIMO systems – description and characteristics

Control of multivariable systems (multiple-input-multiple-output, MIMO) with model uncertainty are still one of the most difficult problems the control engineer has to face in real-world applications. Two of the main characteristics that define a MIMO system are the input and output directionality -different vectors to actuate $\boldsymbol{U}$ and to measure $\boldsymbol{Y}$-; and the coupling among control loops -each input u_i can affect some outputs y_i, and each output can be affected by one or several inputs. This problem, which is known as interaction or coupling, makes the control system design less intuitive since any change in one loop interferes with the rest of the plant loops.

The systems considered from now on are supposed to be linearizable, at least within a range of operating conditions, as we used to do with most of physical real problems. This type of systems can be described by means of an nxm matrix of transfer functions $\boldsymbol{P}(s) = [p_{ij}(s)]$, also called as the plant transfer function matrix (TFM), which relates the m input variables –manipulated variables- $[u_j(s)$ with j = 1,…, $m]$ with the n output variables –controlled variables- $[y_i(s)$ with i = 1,…, $n]$, so that $[y_i(s)] = \boldsymbol{P}(s)\ [u_j(s)]$.

In general, the MIMO transfer function matrix $\boldsymbol{P}(s)$ can be rectangular. However, most of the related literature deals with square systems –i.e., with the same number of inputs and outputs-. If it is not the case for the plant under study, there exist different procedures that can be followed, such as using weighting matrices which reduce the system to a square effective plant matrix (Houpis, Rasmussen & Garcia-Sanz, 2006), leaving some outputs (inputs) uncontrolled (not used), or looking for independent extra inputs or outputs, depending on which one is in excess (Dutton *et al.*, 1997).

Multivariable systems have aroused great interest within the control community and many design techniques have been developed. This is not only because of their mathematical and computational challenge –derived from the matrix representation-, but also due to inherent features that do not appear in SISO systems. The particular nature of MIMO systems poses additional difficulties to control design such as directionality, coupling, transmission zeros, etc.; and all with the intrinsic uncertainty of real-world applications.

2.1 Loops-Coupling and controller structure

The most distinctive aspect of MIMO plants is the existence of coupling among the different control loops. Thus, one input (manipulated variable) can affect various outputs (controlled variables), and the other way around, i.e., an output can be affected by one or several inputs. Consequently, applying a control signal to one of the plant inputs will cause responses at more than one output, which hampers the controller design. Then, it becomes hard to predict the type and amount of control action simultaneously needed at several inputs in order to get outputs to behave as desired.

The first and easiest way that comes to mind for dealing with a MIMO system is to reduce it to a set of SISO problems ignoring the system interactions, which is the so-called decentralized control (Skogestad & Postlethwaite, 2005). Then, each input is responsible for only one output and the resulting controller is diagonal. Finding a suitable input-output pairing becomes therefore essential for decentralized control. However, this approach is only valid provided the coupling among variables is not important, which unfortunately is not the case for many real applications, including our 6x6 spacecraft. In other approaches the goal is to remove, or at least greatly reduce, the effects of the interaction before performing a decentralized control of the somehow decoupled plant as if there were independent input-output pairs.

In any case, it is necessary to quantify the amount of coupling present in the system. Many of the MIMO design techniques, particularly the sequential ones, strongly depend on the correct selection and pairing of inputs and outputs at the beginning of the design procedure. Determining the controller structure is also crucial. This means deciding whether the multivariable system can be divided into several SISO or smaller MIMO subsystems, and establishing the off-diagonal compensators needed if a populated matrix controller is to be designed, avoiding non required extra controllers. This issue becomes extremely complex in the presence of large coupling and has generated great interest within the control community, as show the numerous related references, e.g. (Campo & Morari, 1994; Chiu & Arkun, 1990; Grosdidier *et al.*, 1985; Grosdidier & Morari, 1986; Manousiouthakis *et al.*, 1986; Mijares *et al.*, 1986; Morari & Zafiriou, 1989; Van de Wal & de Jager, 1995). Nevertheless, as (Nett & Spang, 1987) pointed out, too often only the extreme controller structures –the fully centralised (fully-populated matrix) and the fully decentralized (set of SISO loops)- are discussed.

2.1.1 Interaction analysis

An extensive amount of work on the way of quantifying the system interaction can be found in the literature (Maciejowski, 1989; Skogestad & Postlethwaite, 2005). One of the most popular techniques is the *Relative Gain Array* (RGA) defined by Bristol as a matrix of relative gains $\boldsymbol{\Lambda}$ based on the steady-state gains of the plant (Bristol, 1966):

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn} \end{bmatrix} \quad (1)$$

The elements λ_{ij} which constitute this matrix are dimensionless and represent the relation between the following gains of the system:

$$\lambda_{ij} = \frac{K_{OFF}}{K_{ON}} \quad (2)$$

where K_{OFF} is the open-loop gain between the output *i* and the input *j* when the rest of loops are open, while K_{ON} is the open-loop gain between the same output *i* and input *j* when the remaining loops are working in automatic mode, i.e. they are closed.
Another way of computing the RGA is through the following matrix expression:

$$\boldsymbol{\Lambda} = \boldsymbol{P}_0 \otimes (\boldsymbol{P}_0^{-1})^T \quad (3)$$

where $\boldsymbol{P}_0$ is an *nxn* matrix representing the steady-state process. Its elements are determined by applying the final value theorem to the transfer functions describing the system dynamics. The operator [⊗] denotes element-by-element multiplication (Hadamard or Schur product).
The RGA provides a scaling independent measure of the coupling among loops and useful information on how to achieve the best possible pairing of variables (McAvoy, 1983). Its elements λ_{ij} are closely related to the interaction among the different control loops. This is the meaning of the several possible values:

1. $\lambda_{ij} = 1 \Rightarrow$ The closure of the rest of loops does not change the influence of the input *j* on the output *i*. Hence the *ij* loop is decoupled from the rest of the system and can be treated as a SISO subsystem.
2. $\lambda_{ij} = 0 \Rightarrow$ There is no influence of the manipulated variable *j* over the control variable *i*.
3. $0 < \lambda_{ij} < 1 \Rightarrow$ When the rest of loops are closed, the gain between the input *j* and the output *i* increases, i.e., $K_{ON} > K_{OFF}$.
4. $\lambda_{ij} < 0 \Rightarrow$ At the closure of the remaining loops, the system gain changes its sign. Providing a controller with negative gain for the normal situation (all the loops closed and working), the system will react in the opposite direction if some of the remaining loops are open for any reason. Then, integrity is lost.
5. $\lambda_{ij} > 1 \Rightarrow$ When all the loops are closed, higher gains are required. The interaction reduces the gain in the *ij* control loop: $K_{OFF} > K_{ON}$.
6. $\lambda_{ij} > 10 \Rightarrow$ Pairings of variables with large RGA values are undesirable. They are sensitive to modelling errors and to small variations in the loop gain.

Given its importance, the RGA method has been the subject of multiple revisions and research. For instance, although originally defined for the steady-state gain, the RGA was extended to a frequency-dependent definition and used to assess the interaction at frequencies other than zero (McAvoy, 1983; Skogestad & Postlethwaite, 2005; Slaby & Rinard, 1986; Witcher & McAvoy, 1977). In most cases, it is the value of RGA at frequencies close to crossover which is the most important one, and both the gain and the phase are to be taken into account. For a detailed analysis of the plant we consider RGA as a function of frequency:

$$RGA(\omega) = \boldsymbol{P}(\omega) \otimes \left(\boldsymbol{P}^{-1}(\omega)\right)^T \quad (4)$$

where $\boldsymbol{P}(\omega)$ is a frequency-dependent matrix.
According to the meaning of the RGA elements outlined above, it is desired to pair variables so that λ_{ij} is positive and close to one, because this means that the gain from the input u_j to output y_i is not very much affected by closing the other loops. On the other hand, a pairing

corresponding to $0 < \lambda_{ij} < 1$ values means that the other loops reinforce the gain of our given loop; corresponding to $1 < \lambda_{ij}$ values means that the other loops reduce the gain of our given loop; and negative values of λ_{ij} are undesirable because it means that the steady-state gain in our given loop changes sign when the other loops are closed.

As a conclusion, to avoid instability caused by interactions, in the crossover region one should prefer pairings for which the RGA-matrix in this frequency range is close to identity. In the same way, to avoid instability caused by poor integrity, one should avoid pairings with negative steady-state RGA elements.

Further information on how to perform the pairing is available in (McAvoy, 1983). And different properties of the RGA can be consulted at (Bristol, 1966; McAvoy, 1983; Grosdidier *et al.*, 1985; Skogestad & Morari, 1987 a & b; Skogestad & Postlethwaite, 2005; Hovd & Skogestad, 1992, Skogestad & Havre, 1996).

Other measures of interaction that exist in the literature are: the *Block Relative Gain* (Manousiouthakis *et al.*, 1986; Grosdidier & Morari, 1987; Yu & Fan, 1990); the *Relative Disturbance Gain* (Stanley et al., 1985; Marino-Galarraga *et al.*, 1985; Skogestad & Morari, 1987 a & b); or the *Generalized Relative Disturbance Gain* (Chang & Yu, 1992).

2.2 Multivariable poles and zeros

Due to the abovementioned interaction among loops, the poles and zeros of a multivariable system may differ from what could be deduced from observation of the elements of the plant transfer function matrix –TFM- (Maciejowski, 1989). In fact, the pole positions can be inferred from the matrix elements $p_{ij}(s)$, but not their multiplicity, which is of great importance when applying Nyquist-like stability theorems in the presence of right-half plane (RHP) poles. Regarding the multivariable zeros –also known as transmission zeros-, neither the position nor the multiplicity can be derived from direct observation of $p_{ij}(s)$. These multivariable zeros present a transmission-blocking property, since they provoke the loss of rank of the plant TFM.

Thus, it is necessary to determine the effective poles and zeros of a MIMO system, e.g., by using the so-called Smith-McMillan form (McMillan, 1952), as Rosenbrock first suggested (Rosenbrock, 1970; 1973; 1974). Alternative definitions for transmission zeros can be found in (Davison & Wang, 1974; Desoer & Schulman, 1974; MacFarlane & Karcanias, 1976; MacFarlane & Karcanias, 1978; Wolovich, 1974). Further information on this issue is available in (Hsu & Chen, 1968; Kailath, 1980; Maciejowski, 1989; Rosenbrock, 1970).

2.3 Directionality

Among the main reasons why SISO analysis and design tools are difficult to translate to the MIMO case is the existence of directionality, which is one of the most important differences between MIMO and SISO plants (Freudenberg & Looze, 1988; Skogestad & Postlethwaite, 2005). A given direction is a combination of input signal values: for instance $[u_1, u_2, u_3]$ = [4 1 3] has the same direction as $[u_1, u_2, u_3]$ = [8 2 6], which is 2x[4 1 3]. Inherently, MIMO systems present spatial –directional- and frequency dependency. Basically, such systems respond differently to input signals lying in distinct directions. As a result, the relationship between the open-loop and closed-loop properties of the feedback system is less obvious. This directionality is completely in accordance with the TFM representation for MIMO systems.

2.3.1 Gain and phase

The concept of gain of a system is somehow easy to translate to MIMO plants through the *Singular Value Decomposition* (SVD) of the TFM (Deshpande, 1989; Doyle, 1978; MacFarlane & Scott-Jones, 1979; Skogestad & Postlethwaite, 2005), which provides the plant gain at each particular frequency with respect to the main directions –determined by the corresponding singular vectors-.

However, the extension of the notion of phase, as understood in scalar systems, is not so straightforward. Several attempts have been made to define a multivariable phase, such as (Freudenberg & Looze, 1988; Hung & MacFarlane, 1982; MacFarlane & Hung, 1981; Postlethwaite *et al.*, 1981). On the other hand, as (Wall *et al.*, 1980) showed, transmission zeros contribute with extra phase lag in some directions, but not in others. Generally speaking, the change imposed by a MIMO system upon a vector signal can be observed in the magnitude, the direction and the phase (Freudenberg & Looze, 1988).

2.3.2 Effect of poles and zeros

The effect of multivariable poles and zeros –see Section 2.2- strongly depends on directionality as well. That is, their nature is only perceptible for particular directions. So, the TFM transmittance gets unbounded when the matrix is evaluated at a pole, but only in the directions determined by the residue matrix at the pole. Likewise, transmission zeros exert their blocking influence provided the TFM is evaluated at the zero, and the input signal lies in the corresponding null-space (Freudenberg & Looze, 1988).

2.3.3 Disturbance and noise signals

Because of directionality, disturbance and noise signals generally do not equally affect all the loops. In general, they have more influence on some loops than on others. Depending on the disturbance direction –i.e., the direction of the system output vector resulting from a specific disturbance-, some disturbances may be easily rejected, while others may not. The disturbance direction can influence in two ways: through the magnitude of the manipulated variables needed to cancel the effect of the disturbance at steady-state, independently of the designed controllers, and through its effect on closed-loop performance of the controlled outputs (Skogestad & Morari, 1987 a & b). To address this issue, Skogestad and Morari defined the *Disturbance Condition Number*. It measures the magnitude of the manipulated variables needed to counteract a disturbance acting in a particular direction relative to the "best" possible direction.

2.4 Uncertainty

Uncertainty, present in all real-world systems, adds a bigger complexity to MIMO systems, especially in the crossover frequency region. Indeed, uncertainty is one of the reasons – together with the presence of disturbances, and the original instability of the plant if that is the case- why feedback is necessary in control systems.

There exist multiple sources of uncertainty (model/plant mismatch), for instance:

- The model is known only approximately or have been inaccurately identified,
- The model varies because of a change in the operating conditions (experimental models are accurate for a limited range of operating conditions), wear of components, non-linearities, etc.
- Measurement devices are not perfect and their resolution range may be limited.

- The structure or order of the system are unknown at high frequencies.
- The plant model is sometimes simplified to carry out the controller design, being the neglected dynamics considered as uncertainty.
- Other events such as sensor and actuator failures, changes in the control objectives, the switch from automatic to manual –or the other way around- in any loop, inaccuracy in the implementation of the control laws, etc.

The uncertainty can be characterised as *unstructured* when the only available knowledge is the loop location, the stability and a frequency-dependent magnitude of the uncertainty. The weights used for that magnitude (or bound) are generally stable and minimum-phase to avoid additional problems, and multiplicative –relative- weights are usually preferred. This description is useful for representing unmodeled dynamics, particularly in the high frequency range, and small nonlinearities. Different ways of expressing the unstructured uncertainty mathematically and their corresponding properties are available in (Skogestad & Postlethwaite, 2005).

Nevertheless, unstructured uncertainty is often a poor assumption for MIMO plants. It can sometimes lead to highly conservative designs since the controller has to face events that, in fact, are not likely to exist. On the one hand, errors on particular model parameters, such as mode shapes, natural frequencies, damping values, etc., are highly structured. This is the so-called *parametric uncertainty*. Likewise, parameters errors arising in linearised models are correlated, i.e., they are not independent. On the other hand, uncertainty that is unstructured at a component level becomes structured when analysed at a system level.

Thus, in all those cases, it is more convenient to use *structured uncertainty*. Several approaches can be followed to represent this type of uncertainty. For example, a diagonal block can be utilised (Doyle, 1982; Doyle *et al.*, 1982), or a straightforward and accurate representation of the uncertain elements can be performed by means of the plant templates – which are particularly useful for parametric uncertainty-. Introduced by Horowitz in the Quantitative Feedback Theory (QFT) framework (Houpis, Rasmussen & Garcia-Sanz, 2006), the templates describe the set of possible frequency responses of a plant at each frequency. Indeed, the QFT robust control theory can quantitatively handle both types of uncertainty, structured and unstructured.

Alternative approaches for describing uncertainty are also available, but so far its practicality is somehow limited for controller design. An example is the assumption of a probabilistic distribution (e.g. normal, uniform) for parametric uncertainty.

As for the rest of system features, uncertainty in MIMO systems also displays directionality properties. One loop may contain substantially more uncertainty due to unmodeled dynamics or parameter variations than do other loops. Added to this, and again because of directionality, uncertainty at the plant input or output has a different effect –see Section 2.3-. Primarily, input uncertainty is usually a diagonal perturbation, since in principle there is no reason to assume that the perturbations in the manipulated variables are correlated. This uncertainty represents errors on the change rather than on the absolute value (Skogestad & Morari, 1987 a & b).

2.5 Stability

Stability of MIMO systems is also a crucial point in the design process. In the literature, and depending on the design methodology applied, there exist different ways of assessing the feedback system stability.

One of the main approaches is the *generalized Nyquist stability criterion*, in its direct and inverse version (Postlethwaite, 1977; Rosenbrock, 1970). It places an encirclement condition on the Nyquist plot of the determinant of the return difference matrix (Rosenbrock, 1974). However, it is necessary to get a diagonally dominant system for this criterion to be practical because of loop interaction. This is achieved by means of pre-compensation. The designer is helped in this task by the Gershgorin and Ostrowski bands –see (Maciejowski, 1989; Rosenbrock, 1970; Rosenbrock, 1974)-, or by Mees' theorem (Mees, 1981). This stability criterion is mainly used in non-sequential classical methodologies –e.g. the Inverse Nyquist Array (Rosenbrock, 1969) and Direct Nyquist Array (Rosenbrock, 1970; 1974). By contrast, sequential classical techniques do not make a direct use of it. Proofs of the multivariable Nyquist stability criterion have been given from different viewpoints, e.g. (Barman & Katzenelson, 1974; Desoer & Wang, 1980; MacFarlane & Postlethwaite, 1977; Postlethwaite & MacFarlane, 1979).

An alternative way of checking stability is by means of the *Smith-McMillan poles* (McMillan, 1952). This approach is applied in classical sequential methodologies through stability conditions such as those defined by De Bedout and Franchek (De Bedout & Franchek, 2002) for non-diagonal sequential techniques.

A completely different strategy is adopted by synthesis techniques, which make use of stability robustness results such as the *small-gain theorem* (Desoer, C.A. & Vidyasagar, 1975). This states that a feedback loop composed of stable operators will remain stable if the product of all the operator gains is smaller than unity. The theorem is applied to systems with unstructured uncertainty. When the phases of perturbations, rather than their gains, can be bounded, the *small-phase theorem* (Postlethwaite *et al*., 1981) can be used. However, the main drawback of this approach is the highly conservative results it may provide. In the presence of structured uncertainty, results based on the *structured singular value* SSV (Doyle *et al*., 1982) can be used instead.

3. MIMO QFT control

3.1 Overview

The Quantitative Feedback Theory (QFT), first introduced by Prof. Isaac Horowitz in 1959, is an engineering control design methodology, which explicitly emphasizes the use of feedback to simultaneously reduce the effects of plant uncertainty and satisfy performance specifications (Horowitz, 1993; Yaniv, 1999; Sidi, 2002; Houpis, Rasmussen & Garcia-Sanz, 2006). Horowitz's work is deeply rooted in classical frequency response analysis involving Bode diagrams, template manipulations and Nichols Charts. It relies on the observation that the feedback is needed principally when the plant presents model uncertainty or when there are uncertain disturbances acting on the plant.

Model uncertainty, frequency domain specifications and desired time-domain responses translated into frequency domain tolerances, lead to the so-called Horowitz-Sidi bounds (or constraints). These bounds serve as a guide for shaping the nominal loop transfer function $L(s) = G(s)\ P(s)$, which involves the selection of gain, poles and zeros to design the appropriate controller $G(s)$. On the whole, the QFT main objective is to synthesize (loop-shape) a simple, low-order controller with minimum bandwidth, which satisfies the desired performance specifications for all the possible plants due to the model uncertainty. The use of CAD tools have made the QFT controller design much simpler –see for instance the QFT Control MATLAB Toolbox developed by (Garcia-Sanz, Mauch & Philippe, 2009) for the

European Space Agency; the popular QFT Control Design MATLAB Toolbox developed by (Borghesani, Chait, & Yaniv, 2002); the pioneer AFIT CAD tool developed by (Sating, 1992; Houpis & Sating, 1997; also at Houpis, Rasmussen & Garcia-Sanz, 2006); and the Qsyn CAD tool developed by (Gutman, 1996).

The first proposal for MIMO QFT design was made by Horowitz in his first book (Horowitz, 1963), where he pointed out the possibility of using diagonal controllers for quantitative design. This was divided into different frequency ranges: for the low-frequency interval the controller gain generally needs to be high and is easily determined. As for the medium and high-frequency bands, he suggested the progressive tuning loop by loop sorted in increasing order. A more systematic and precise approach was later introduced by (Shaked *et al.*, 1976). However, no proof of convergence to a solution was provided.

The first rigorous MIMO QFT methodology was again developed by Horowitz (Horowitz, 1979). This non-sequential technique translates the original nxn MIMO problem with uncertainty into n MISO systems with uncertainty, disturbances and specifications derived from the initial problem. The coupling is then treated as a disturbance at the plant input, and the individual solutions guarantee the whole multivariable solution. This is assured by the application of the Schauder's fixed point theorem (Kantorovich & Akilov, 1964). This theory maps the desired fixed point on the basis of unit impulse functions.

As before, there exist differentiated frequency ranges in the design procedure. Loops are designed as basically non-interacting (BNI) at low frequency, whereas in the middle and high-frequency range attention must be paid to the effect of the noise at the plant input, especially in problems with significant uncertainty.

On the whole, first Horowitz's method is a direct technique oriented towards MIMO plants with uncertainty. It also allows the trade-off among loops in the ranges of frequency. Nevertheless, the type of plant which can be dealt with is constrained in several ways, and the method places necessary conditions depending on the system size, which hampers its application to high-order systems. In addition, it presents potential overdesign and may generate highly conservative designs. Additional references on this methodology and its applications are available in (Horowitz & Sidi, 1980; Horowitz & Loecher, 1981; Horowitz *et al.*, 1981; Horowitz *et al.*, 1982).

An improvement of the preceding technique was also provided by Horowitz with a sequential procedure (Horowitz, 1982), also called *Second Method* in (Houpis, Rasmussen & Garcia-Sanz, 2006). There exist some similarities between this technique and the SRD method from Mayne (Mayne, 1973; 1979), such as the fact that the resulting controller is diagonal or that they proceed as if each input-output pair was a standard SISO system with loop interaction behaving as an external disturbance. Besides, both methods incorporate the effects of each loop once it is designed into the subsequent loop designs.

Nevertheless, the main difference is that Horowitz's methodology relies on a factorisation of the return difference matrix which is based on the inverse of the plant TFM. By using the inverse plant, a much simpler relationship between the closed-loop and the open-loop TFMs is obtained. One of Horowitz's major contributions with this technique is that he dealt with the problem of robust stability by considering parametric uncertainty.

The stability proof for Horowitz's Second Method was provided in (Yaniv & Horowitz, 1986) and (De Bedout & Franchek, 2002). By and large, the method constituted a great step forward in MIMO QFT design techniques. First, as abovementioned, parametric uncertainty was considered. Second, the Schauder's fixed point theorem was no longer needed. Third, the limitation related to the system size from the first method was avoided. Finally, it reduced the conservativeness of the former method by using the concept of equivalent plant –which takes into account the controllers previously designed-. All in all, the second method is a much more powerful technique –although obviously more complicated than other classical approaches-, and the physical sense is kept all along the procedure.

Different authors made some improvements of these first two MIMO QFT design methods by Horowitz in subsequent works (Nwokah, 1984; 1988; Yaniv & Horowitz, 1986). A detailed compilation of the above techniques is presented in (Houpis, Rasmussen & Garcia-Sanz, 2006).

An alternative approach to MIMO QFT methodologies was presented by (Park *et al.*, 1994), who developed a direct technique. In other words, the inversion of the plant matrix was not required anymore, which therefore simplified the design process to some extent.

The methodologies outlined so far only deal with the problem of designing a diagonal controller. Nevertheless, there exist potential benefits in the use of full-matrix compensators. Horowitz (Horowitz, 1979) already commented that the use of diagonal controllers was established just to simplify the theoretical development, but that in practice it could be convenient to consider the off-diagonal elements as well. These terms could then be used to reduce the level of coupling in open loop, and therefore reduce the amount of feedback needed in the diagonal compensators to fulfil the required specifications (Horowitz, 1982).

Furthermore, as (Franchek *et al.*, 1997) demonstrated, non-diagonal compensators can be used for ensuring that no SISO loop introduces extra unstable poles into the subsequent loops in sequential procedures based on the inverse plant domain, e.g. Horowitz's second method (Horowitz, 1982), -accordingly, this is not possible in Mayne's (Mayne, 1973; 1979), or Park's (Park *et al.*, 1994) framework-. As a result, it can be reduced the minimum cross-over frequency needed to achieve closed-loop stability in these succeeding loops. In other words, the actuation bandwidth requirements can be relaxed. Additionally, specific integrity objectives can be achieved, allowing the design of fault-tolerant MIMO systems. In the case of Horowitz's diagonal sequential method (Horowitz, 1982), however, it is not possible to remove the unstable poles originally present in those subsequent loops, but a more general design technique could be developed for that purpose (De Bedout & Franchek, 2002). On the other hand, diagonal compensators are limited for the correction of the plant directionality when needed. There even exist cases where a diagonal or triangular controller cannot stabilise the system (De Bedout & Franchek, 2002).

On balance, the designer has greater flexibility to design the MIMO feedback control system when using fully populated controllers. But the introduction of such non-diagonal controllers poses two main issues: the way of determining the off-diagonal compensators and the need for suitable stability conditions. In systems controlled by a full-matrix compensator, the property of diagonal dominance is not assured. The Gershgorin circles become too conservative in that case and the stability test gets more complicated. As a result, different stability results are needed. Sufficient stability conditions for non-diagonal sequential procedures have been defined by (De Bedout & Franchek, 2002).

Regarding the determination of the needed off-diagonal compensators, different techniques have arisen to deal with. The first attempt in non-diagonal MIMO QFT was proposed by Horowitz and co-workers (Horowitz *et al.*, 1981; Horowitz, 1991), who suggested the pre-multiplication of the plant by a full matrix. Yaniv (Yaniv, 1995) presented a procedure where a non-diagonal decoupler is applied as a pre-compensator and a classical diagonal controller is designed afterwards. Therein, the main objective becomes the improvement of the system bandwidth.

A different approach was adopted by Boje and Nwokah (Boje & Nwokah, 1999; 2001). They used the Perron-Frobenius root as a measure of interaction and of the level of triangularization of the uncertain plant. The full-matrix pre-compensator is accordingly designed to reduce the coupling before designing a diagonal QFT controller.
On the other hand, Franchek and collaborators (Franchek *et al.*, 1995), (Franchek *et al.*, 1997) introduced a non-diagonal sequential procedure. They made use of the Gauss elimination technique (Bryant , 1985) to introduce the effects of the controllers previously designed by means of a recursive expression. Integrity considerations are also included. The controller is then divided into three parts with differentiated roles in the design process. The technique achieves the reduction of the required bandwidth with respect to previous classical sequential techniques. Additionally, De Bedout and Franchek established sufficient stability conditions for non-diagonal sequential procedures (De Bedout & Franchek, 2002).
Another important sequential technique to be considered is the one presented by Garcia-Sanz and collaborators (Garcia-Sanz & Egana, 2002; Garcia-Sanz *et al.*, 2005; Garcia-Sanz & Eguinoa, 2005; Garcia-Sanz & Barreras, 2006; Garcia-Sanz *et al.*, 2006; Garcia-Sanz & Hadaegh, 2007; Garcia-Sanz *et al.*, 2008; Houpis, Rasmussen & Garcia-Sanz, 2006). Following Horowitz's ideas, they extended Horowitz's sequential methodology (Horowitz, 1982) to the design of fully populated MIMO controllers. The role of the non-diagonal terms is simultaneously analysed for the fundamental cases of reference tracking, disturbance rejection at plant input and disturbance rejection at plant output. The compensators are aimed at the reduction of the coupling on the basis of defined coupling matrices, which are accordingly minimised. This method has proved to be a convincing design tool in real applications from different fields (Barreras & Garcia-Sanz, 2004; Garcia-Sanz *et al.*, 2005; Barreras, 2005; Barreras *et al.*, 2006; Garcia-Sanz & Barreras, 2006; Garcia-Sanz *et al.*, 2006), including control of spacecraft flying in formation (Garcia-Sanz & Hadaegh, 2007) or spacecraft with flexible appendages (Garcia-Sanz *et al.*, 2008) at NASA and ESA respectively.
In 2009, Garcia-Sanz and Eguinoa (Garcia-Sanz *et al.*, 2009) introduced a reformulation of the full-matrix QFT robust control methodology for MIMO plants with uncertainty. The methodology includes a generalization of their previous non-diagonal MIMO QFT techniques; avoiding former hypotheses of diagonal dominance; simplifying the calculations for the off-diagonal elements, and then the method itself; reformulating the classical matrix definition of MIMO specifications by designing a new set of loop-by-loop QFT bounds on the Nichols Chart which establish necessary and sufficient conditions; giving explicit expressions to share the load among the loops of the MIMO system to achieve the matrix specifications; and all for stability, reference tracking, disturbance rejection at plant input and output, and noise attenuation problems. The new methodology was also applied to the design of a MIMO controller for a spacecraft flying in formation in a low Earth orbit.
Regarding the field of non-sequential MIMO QFT techniques, it is to be remarked the approach by Kerr, Jayasuriya and co-workers in (Kerr, 2004; Kerr *et al.*, 2005b; Kerr & Jayasuriya, 2006; Kerr *et al.*, 2007; Lan *et al.*, 2004). Stability conditions have also been established within this framework (Kerr & Jayasuriya, 2003; Kerr *et al.*, 2005a).
Other approaches have also been introduced for particular types of MIMO systems. For example, there are results on NMP MIMO plants (Horowitz *et al.*, 1986). It is noted that not all the nxn transfer functions have to suffer the limitations imposed by the NMP behaviour (Horowitz & Liao, 1984). The MIMO system has the capacity to relocate the RHP zeros in those outputs which are not so determining, while the critical outputs are kept as minimum-phase loops. Likewise, some research has been done for unstable and strongly NMP MIMO

systems, e.g. the X-29 aircraft (Horowitz, 1986; Kerr *et al.*, 2007; Walke et al, 1984). One interesting suggestion is the *singular-G method* (Horowitz, 1986; Walke et al, 1984), which makes use of a singular compensator –i.e., with a determinant equal to zero, which implies that one output is dependent from the rest of outputs-. In this way, the technique allows easing the NMP problem and the instability in the MIMO system, and simultaneously achieving good performance.

3.2 Non-diagonal MIMO QFT Technique

This section describes the main characteristics of the non-diagonal MIMO QFT technique introduced by (Garcia-Sanz & Egaña, 2002; Garcia-Sanz *et al.*, 2005; and Garcia-Sanz & Eguinoa, 2005; Houpis, Rasmussen & Garcia-Sanz, 2006).

3.2.1 System definition

Let us consider the generic *nxn* linear multivariable system shown in Fig. 2. The plant is represented by the matrix $\boldsymbol{P}(s) = [p_{ij}(s)]$, $[y_i(s)] = \boldsymbol{P}(s)\,[u_j(s)]$, where $\boldsymbol{P} \in \mathcal{P}$, and $\mathcal{P}$ is the set of possible plants due to uncertainty. The compensator matrix is $\boldsymbol{G}(s) = [g_{ij}(s)]$ and $\boldsymbol{F}(s) = [f_{ij}(s)]$ is the prefilter. All these matrices are of dimension *n*x*n*. The transfer function matrices of disturbances at plant input and plant output are represented by $\boldsymbol{P}_{di}(s)$ and $\boldsymbol{P}_{do}(s)$ respectively. The reference vector $\boldsymbol{r}'(s)$, the external disturbance vectors at plant input $\boldsymbol{d_i}'(s)$ and plant output $\boldsymbol{d_o}'(s)$, and the noise $\boldsymbol{n}(s)$ are the inputs of the system. The output vector $\boldsymbol{y}(s)$ represents the variables to be controlled.

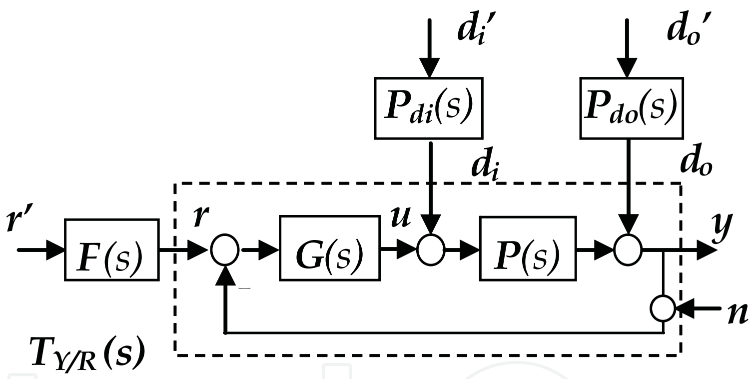


Fig. 2. Two degree of freedom MIMO control system

The plant inverse, denoted by $\boldsymbol{P}^*(s)$, and the compensator $\boldsymbol{G}(s)$ can be respectively expressed as the sum of their diagonal part and their balance:

$$\boldsymbol{P}^{-1} = \boldsymbol{P}^{*} = \left[p^{*}_{ij} \right] = \boldsymbol{\Lambda} + \boldsymbol{B} = \begin{bmatrix} p^{*}_{11} & 0 & 0 \\ 0 & \cdots & 0 \\ 0 & 0 & p^{*}_{nn} \end{bmatrix} + \begin{bmatrix} 0 & \ldots & p^{*}_{1n} \\ \ldots & 0 & \ldots \\ p^{*}_{n1} & \ldots & 0 \end{bmatrix} \tag{5}$$

$$\boldsymbol{G} = \boldsymbol{G}_d + \boldsymbol{G}_b = \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & \cdots & 0 \\ 0 & 0 & g_{nn} \end{bmatrix} + \begin{bmatrix} 0 & \ldots & g_{1n} \\ \ldots & 0 & \ldots \\ g_{n1} & \ldots & 0 \end{bmatrix} \tag{6}$$

3.2.2 Hypothesis

Given the transfer function t_{ij}, an element of the Transfer matrix $\boldsymbol{T_{Y/R}} = \boldsymbol{y/r'}$, the sole necessary hypothesis that the compensator design methodology needs to meet is:

$$\left| \left(p_{ij}^{*} + g_{ij}\right) t_{jj} \right| >> \left| \left(p_{ik}^{*} + g_{ik}\right) t_{kj} \right| \tag{7}$$

for k≠j and in the bandwidth of t_{jj}, which is usually satisfied once the matrix is ordered with the RGA procedure -Relative Gain Analysis (Bristol, 1966)-.

3.2.3 Methodology steps

The design methodology consists in four steps. Step *A* arranges the system to apply afterwards the sequential procedure closing *n* loops with steps *B* and *C*, which are repeated for every column of the compensator matrix $\boldsymbol{G}(s)$ (Fig. 3). Step *D* designs the prefilter.

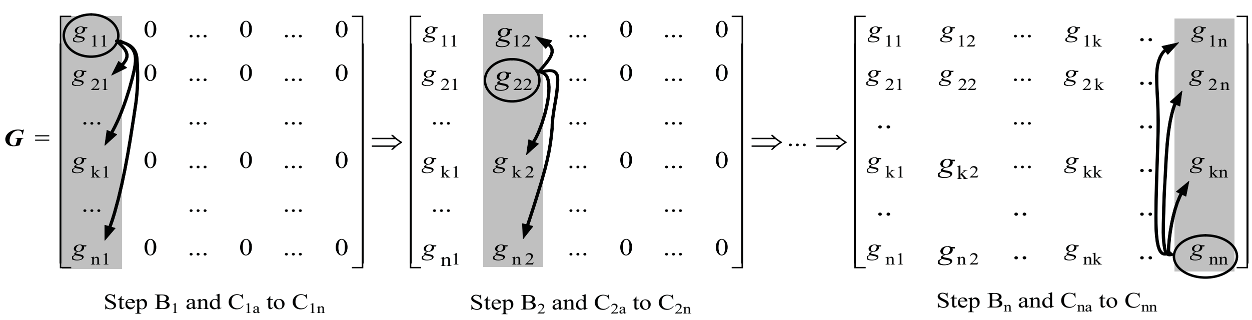


Fig. 3. Sequential steps for $\boldsymbol{G}(s)$ controller design

Step 1. *Input-Output pairing and loop ordering*. First, the methodology identifies input-output pairings by using the RGA (Bristol, 1966). Then, the matrix $\boldsymbol{P}^*$ is reorganized so that $(p_{11}^*)^{-1}$ has the smallest bandwidth, $(p_{22}^*)^{-1}$ the next smallest bandwidth, and so on (Houpis, Rasmussen & Garcia-Sanz, 2006).

Step 2. *Design of the diagonal compensator* g_{kk}. The diagonal element g_{kk} is calculated through standard QFT loop-shaping (Horowitz, 1982; Houpis, Rasmussen & Garcia-Sanz, 2006) for the inverse of the equivalent plant $(p_{kk}^{*e})^{-1}$ in order to achieve robust stability and robust performance specifications (Franchek *et al.*, 1997; De Bedout and Franchek, 2002). The equivalent plant satisfies the recursive relationship of Eq. (8) (Franchek *et al.*, 1997), which is an extension for the non-diagonal case of the recursive expression proposed by (Horowitz, 1982).

$$\left[p_{ii}^{*e}\right]_{k} = \left[p_{ii}^{*}\right]_{k-1} - \frac{\left(\left[p_{i(i-1)}^{*}\right]_{k-1} + \left[g_{i(i-1)}\right]_{k-1}\right)\left(\left[p_{(i-1)i}^{*}\right]_{k-1} + \left[g_{(i-1)i}\right]_{k-1}\right)}{\left[p_{(i-1)(i-1)}^{*}\right]_{k-1} + \left[g_{(i-1)(i-1)}\right]_{k-1}} \tag{8}$$

$$i \geq k; \quad \left[\boldsymbol{P}^{*}\right]_{k=1} = \boldsymbol{P}^{*}$$

Step 3. *Design of the (n-1) non-diagonal elements* g_{ik} ($i \neq k$, $i = 1,2,\ldots n$). These elements are designed to minimize the cross-coupling terms c_{ik} according to the type problem case: for reference tracking (Eq. 9), for disturbance rejection at plant input (Eq. 10) or at plant output (Eq. 11).

$$c_{1ik} = g_{ik} - \frac{g_{kk}\left(p_{ik}^{*} + g_{ik}\right)}{\left(p_{kk}^{*} + g_{kk}\right)} \quad ; \quad i \neq k \tag{9}$$

$$c_{2ik} = \frac{\left(p_{ik}^{*} + g_{ik}\right)}{\left(p_{kk}^{*} + g_{kk}\right)} \quad ; \quad i \neq k \tag{10}$$

$$c_{3ik} = p_{ik}^{*} - \frac{p_{kk}^{*}\left(p_{ik}^{*} + g_{ik}\right)}{\left(p_{kk}^{*} + g_{kk}\right)} \quad ; \quad i \neq k \tag{11}$$

Step 4. *Design of the prefilter*. The final $\boldsymbol{T}_{y/r}(s)$ function shows less loop interaction thanks to the fully populated compensator design. Therefore, the prefilter $\boldsymbol{F}(s)$ can generally be a diagonal matrix.

3.2.4 Stability conditions

Closed-loop stability of a MIMO system with a non-diagonal controller, designed by using a sequential procedure as the one presented above, is guaranteed by the following sufficient conditions (De Bedout and Franchek, 2002):

1. each $L_i(s) = g_{ii}(s)\,(p_{ii}^{*e})^{-1}$, i=1, ..., n, satisfies the Nyquist encirclement condition,
2. no RHP pole-zero cancellations occur between $g_{ii}(s)$ and $(p_{ii}^{*e})^{-1}$, i=1, ..., n,
3. no Smith-McMillan pole-zero cancellations occur between $\boldsymbol{P}(s)$ and $\boldsymbol{G}(s)$, and
4. no Smith-McMillan pole-zero cancellations occur in $\left|\boldsymbol{P}^*(s) + \boldsymbol{G}(s)\right|$

3.2.5 Non-minimum phase aspects

Although it is very remote, theoretically there exists the possibility of introducing right-half plane (RHP) transmission zeros in the controller design procedure. This undesirable situation cannot be detected until the multivariable system design is completed. To avoid it, the proposed methodology –Steps *A*, *B* and *C*– is inserted in an additional procedure (Garcia-Sanz & Eguinoa, 2005). Once the matrix compensator $\boldsymbol{G}(s)$ is designed, the transmission zeros of $\boldsymbol{P}(s)$ $\boldsymbol{G}(s)$ are determined using the Smith-McMillan form and over the set of possible plants P due to uncertainty. If there exist new RHP zeros apart from those initially present in $\boldsymbol{P}(s)$, they can be removed by using the non-diagonal elements of the last column of the $\boldsymbol{G}(s)$ matrix.

4. Application to control a telescope-type spacecraft

4.1 System description

This Section shows some illustrative results achieved when applying the non-diagonal MIMO QFT control methodology introduced in Section 3 to one of the telescope-type spacecraft (a 6-inputs/6-outputs MIMO system) of a multiple formation flying constellation of a European Space Agency (ESA) cornerstone mission: the Darwin mission (Garcia-Sanz *et al.*, 2008). It consists of one master satellite (central hub) and three to six telescopes arranged in a symmetric configuration flying in formation (Fig. 1). They will operate together to analyze the atmosphere of remote planets through appropriate spectroscopy techniques. The mission will employ nulling interferometry to detect dim planets close to bright stars. The infrared light

collected by the free flying telescopes will be recombined inside the hub-satellite in such a way that the light from the central star will suffer destructive interference and will be cancelled out, allowing the much fainter planet to stand out. That interferometry technology requires very accurate and stable positioning of the spacecraft in the constellation, which puts high demands on the attitude and position control system. Instead of an orbit around the Earth, the mission will be placed further away, at a distance of 1.5 million kilometers from Earth, in the opposite direction from the Sun (Earth-Sun Lagrangian Point L2).
The present Section shows the control of one of the telescope flyers. Each telescope flyer is cylindrically shaped (2 m diameter, 2 m height) and weighs 500 kg. In order to protect the instrument from the sunlight, it is equipped with a sunshield modeled with 6 large flexible beams (4 m long and 7 kg) attached to the rigid structure (see Fig. 3; beam end-point coordinates in brackets).

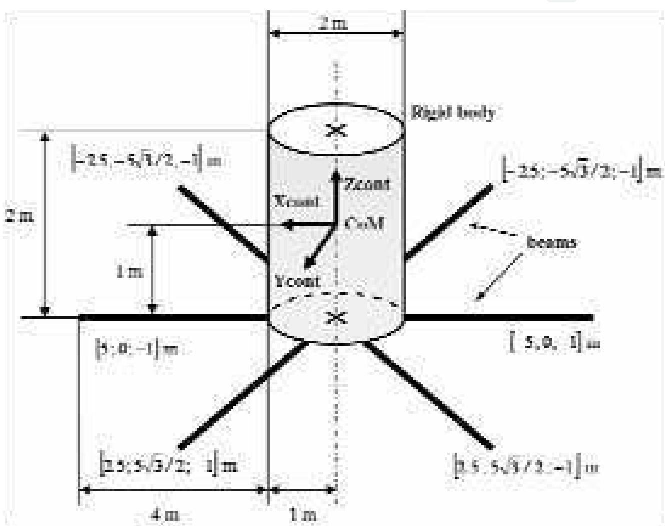


Fig. 3. Spacecraft description

For every beam, two different frequencies for the first modes along Y and Z beam axes are considered. Their frequency can vary from 0.05 Hz to 0.5 Hz, with a nominal value of 0.1 Hz, and their damping can vary from 0.1% to 1%, with a nominal value of 0.5%. As regards spacecraft mass and inertia, the corresponding uncertainty around their nominal value is of 5%.

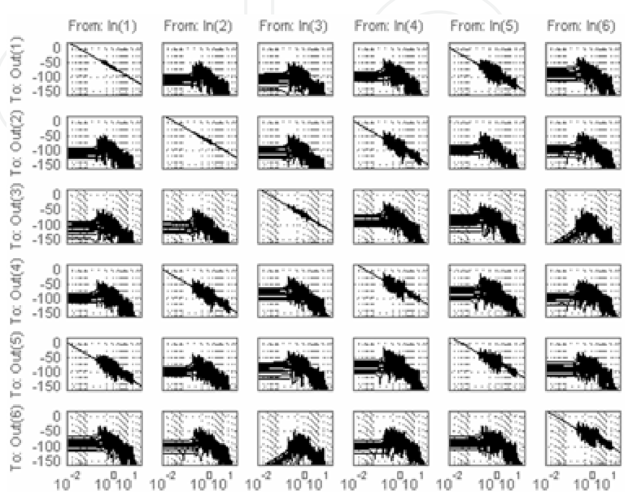


Fig. 4. Spacecraft model dynamics

Based on that description, and using a mechanical modeling formulation for multiple flexible appendages of a rigid body spacecraft, the open-loop transfer function matrix representation of the Flyer is given in (12) and Fig. 4, where x, y, z are the position coordinates; φ, θ, ψ are the corresponding attitude angles; Fx, Fy, Fz are the force inputs; T_φ, T_θ, T_ψ are the torque inputs; and where each $p_{ij}(s)$, i, j = 1,…,6, is a 50th order Laplace transfer function with uncertainty.

$$\begin{bmatrix} x(s) \\ y(s) \\ z(s) \\ \phi(s) \\ \theta(s) \\ \psi(s) \end{bmatrix} = \boldsymbol{P}(s)\ \boldsymbol{U}(s) = \begin{bmatrix} p_{11}(s) & p_{12}(s) & p_{13}(s) & p_{14}(s) & p_{15}(s) & p_{16}(s) \\ p_{21}(s) & p_{22}(s) & p_{23}(s) & p_{24}(s) & p_{25}(s) & p_{26}(s) \\ p_{31}(s) & p_{32}(s) & p_{33}(s) & p_{34}(s) & p_{35}(s) & p_{36}(s) \\ p_{41}(s) & p_{42}(s) & p_{43}(s) & p_{44}(s) & p_{45}(s) & p_{46}(s) \\ p_{51}(s) & p_{52}(s) & p_{53}(s) & p_{54}(s) & p_{55}(s) & p_{56}(s) \\ p_{61}(s) & p_{62}(s) & p_{63}(s) & p_{64}(s) & p_{65}(s) & p_{66}(s) \end{bmatrix} \begin{bmatrix} F_x(s) \\ F_y(s) \\ F_z(s) \\ T_\phi(s) \\ T_\theta(s) \\ T_\psi(s) \end{bmatrix} \quad (12)$$

The Bode diagram of the plant (Fig. 4) shows the dynamics of the 36 matrix elements. Each of them and the MIMO system (matrix) itself are minimum phase. The flexible modes introduced by the appendages (second-order dipoles) affect all the elements around the frequencies ω = [0.19, 10] rad/sec. The diagonal elements $p_{ii}(s)$, i = 1,…,6, and the elements $p_{15}(s)$, $p_{51}(s)$, $p_{24}(s)$ and $p_{42}(s)$ are mainly double integrators plus the flexible modes.

4.2 Performance specifications

The main objective of the spacecraft control system is to fulfill some astronomical requirements that demand to keep the flying telescope pointing at both the observed space target and the central hub-satellite. This set of specifications leads to some additional engineering requirements (bandwidth, saturation limits, noise rejection, etc.) and also needs some complementary control requirements (stability, low loop interaction, low controller complexity and order, etc.). In other words, the requirements are:

A. *Astronomical specifications:*
   A1. Position accuracy: maximum absolute value: 1 μm (micro-meter) for all axes, and standard deviation: 0.33 μm for all axes.
B. A2. Pointing accuracy: maximum absolute value: 25 mas (milli-arc-second) for all axes, and standard deviation: 8.5 mas for all axes.
C. *Engineering specifications*:
   B1. Banthwidth: ~ 0.01 Hz for all axes.
   B2. Saturation limits: maximum force: 150 μN, maximum torque: 150 μNm.
   B3. High frequency noise rejection: high roll-off after the bandwidth.
D. *Control specifications*:
   C1. Loop interaction: minimum.
   C2. Rejection of flexible modes: maximum.
   C3. Controller complexity and order: minimum.

To achieve these goals, the astronomical, engineering and control specifications are translated into frequency domain requirements (see some examples at D'Azzo, Houpis, & Sheldon, 2003), defined as shown in Table I, where $(p_{ii}^{*e})^{-1}$ is the inverse of the equivalent plant, which corresponds to $p_{ii}(s)$ in the SISO case (Garcia-Sanz *et al.*, 2008).

		Value $(\forall\omega)$	*Loops*
1	$\left\|\dfrac{\left(p_{ii}^{*e}\right)^{-1} g_{ii}(s)}{1+\left(p_{ii}^{*e}\right)^{-1} g_{ii}(s)}\right\| = \left\|\dfrac{y_i(s)}{r_i(s)}\right\| = \left\|\dfrac{y_i(s)}{n_i(s)}\right\| = \left\|\dfrac{u_i(s)}{v_i(s)}\right\| \le \delta_1(\omega)$ (13)	$\delta_1(\omega)=1.85$ (14)	*1,2,3*
		$\delta_1(\omega)=\left\|\dfrac{0.1687}{s^2+0.4s+0.0912}\right\|$ (15)	*4,5,6*
2	$\left\|\dfrac{1}{1+\left(p_{ii}^{*e}\right)^{-1} g_{ii}(s)}\right\| = \left\|\dfrac{e_i(s)}{n_i(s)}\right\| = \left\|\dfrac{y_i(s)}{d_i(s)}\right\| \le \delta_2(\omega)$ (16)	$\delta_2(\omega)=2$ (17)	*1,2,3, 4,5,6*
3	$\left\|\dfrac{\left(p_{ii}^{*e}\right)^{-1}}{1+\left(p_{ii}^{*e}\right)^{-1} g_{ii}(s)}\right\| \le \delta_3(\omega)$ (18)	$\delta_3(\omega)=\left\|\dfrac{0.21553\,(s+0.385)}{(s+0.307)\,(s+6.18)\,\left(s^2+0.4s+0.0912\right)}\right\|$ (19)	*1,2*
		$\delta_3(\omega)=\left\|\dfrac{0.313\,(s-0.01705)\left(s^2+0.009974s+5.104\cdot 10^{-5}\right)}{(s-0.01813)\left(s^2+0.02554s+0.0004754\right)}\right\|$ (20)	*3*
	$\delta_3(\omega)=\left\|\dfrac{(s+0.2)\,(s+0.186)\,(s+0.2044)\,(s+0.003892)\left(s^2+0.06014s+0.02736\right)}{(s+0.007333)\,(s+0.445)\left(s^2+0.07904s+0.00326\right)\left(s^2+0.2352s+0.0981\right)}\right\|$ (21)		*4,5,6*
4	$\left\|\dfrac{g_{ii}(s)}{1+\left(p_{ii}^{*e}\right)^{-1} g_{ii}(s)}\right\| \le \delta_4(\omega)$ (22)	$\delta_4(\omega)=\left\|\dfrac{557.1\,(s+5)}{\left(s^2+3.23s+6.5\right)}\right\|$ (23)	*1,2*
		$\delta_4(\omega)=\left\|\dfrac{106.9210\,(s+0.55)\left(s^2+0.04s+0.13\right)}{(s+1.4)^2\left(s^2+0.1227s+0.097\right)}\right\|$ (24)	*3*
		$\delta_4(\omega)=\left\|\dfrac{4.026\left(s^2-0.1854s+0.203\right)\left(s^2+0.04s+0.504\right)}{\left(s^2+0.305s+0.056\right)\left(s^2+0.115s+0.095\right)}\right\|$ (25)	*4,5,6*

Table I. Transfer function for Frequency Domain Specifications

4.3 Applying the non-diagonal MIMO QFT control methodology

The MIMO QFT methodology explained in Section 3.2 is applied here to design the 6x6 robust control system for the telescope-type spacecraft described in Section 4.1, and with the performance specifications defined in Section 4.2.

Step A. *Input-Output pairing and loop ordering.*
An illustrative result of the Relative Gain Array for all the uncertainty, at low frequency (steady state), and up to 0.19 rad/sec, is shown in Eq. (26). According to it, the pairing should be done through the main diagonal of the matrix, which contains positive RGA elements, and the elements $g_{15}(s)$, $g_{24}(s)$, $g_{42}(s)$, $g_{51}(s)$ should also be considered relevant.

$$\mathbf{RGA}_{\left(\omega=6.28\cdot10^{-4}\,\mathrm{rad/sec}\right)}=\begin{bmatrix}1.0064 & 0 & 0 & 0 & 0.0064 & 0\\ 0 & 1.0064 & 0 & 0.0064 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0.0064 & 0 & 1.0064 & 0 & 0\\ 0.0064 & 0 & 0 & 0 & 1.0064 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix} \tag{26}$$

In accordance with the above RGA results and taking into account the requirement of minimum controller complexity and order (Section 4.2, Specification C3), the compensator structure consisting of six diagonal elements and four off-diagonal elements is chosen as the most suitable one (27).

$$\boldsymbol{G}(s)=\begin{bmatrix}g_{11}(s) & 0 & 0 & 0 & g_{15}(s) & 0\\ 0 & g_{22}(s) & 0 & g_{24}(s) & 0 & 0\\ 0 & 0 & g_{33}(s) & 0 & 0 & 0\\ 0 & g_{42}(s) & 0 & g_{44}(s) & 0 & 0\\ g_{51}(s) & 0 & 0 & 0 & g_{55}(s) & 0\\ 0 & 0 & 0 & 0 & 0 & g_{66}(s)\end{bmatrix} \tag{27}$$

From this, four independent compensator design problems have been adopted, two SISO -[$g_{33}(s)$] and [$g_{66}(s)$]- and two 2x2 MIMO -[$g_{11}(s)$ $g_{15}(s)$; $g_{51}(s)$ $g_{55}(s)$] and [$g_{22}(s)$ $g_{24}(s)$; $g_{42}(s)$ $g_{44}(s)$]- problems. The SISO problems are considered as a classical SISO QFT problem, while the two 2x2 MIMO subsystems are studied through the non-diagonal MIMO QFT methodology.

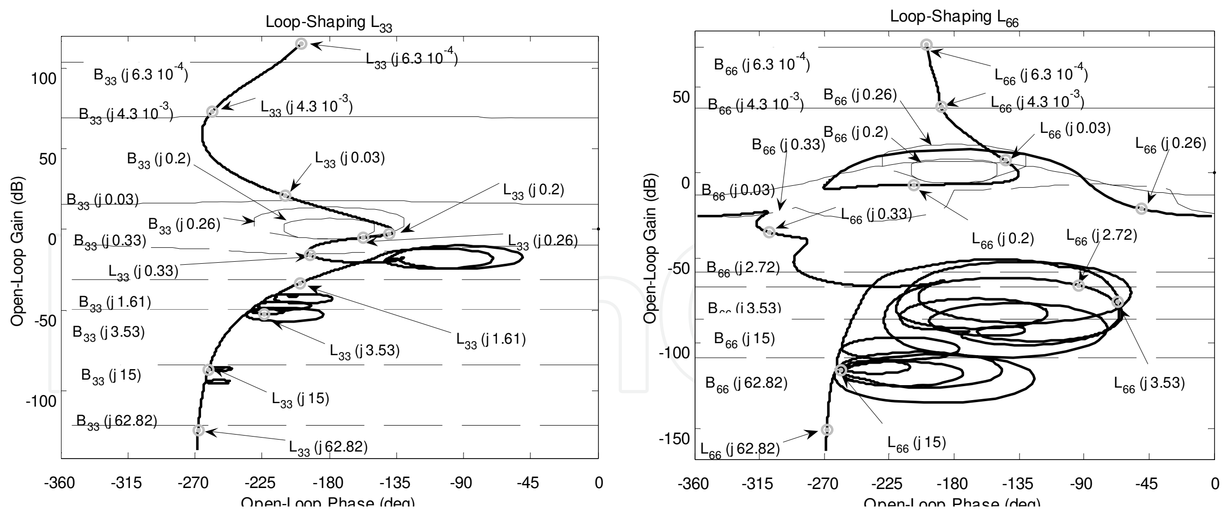
Step B0. *Design of the diagonal compensator $g_{kk}(s)$, k = 3, 6. SISO cases.*
Compensators $g_{33}(s)$ and $g_{66}(s)$ are independently designed by using classical single-input single-output SISO QFT (Houpis, Rasmussen & Garcia-Sanz, 2006) to satisfy the performance specifications stated in Table I for every plant within the uncertainty. The corresponding QFT bounds and the nominal open-loop transfer functions $L_{ii}(s) = p_{ii}(s)\, g_{ii}(s)$, i = 3, 6, are plotted on the Nichols Charts shown in Fig. 5.

Step B1. *Design of the diagonal compensator $g_{11}(s)$. First MIMO problem.*
The compensator $g_{11}(s)$ is designed according to the non-diagonal MIMO QFT methodology explained in Section 3.2, for the inverse of the equivalent plant $[p_{11}^{*e}(s)]_1 = p_{11}^{*}(s)$. See Fig. 6a.

Step C1. *Design of the non-diagonal compensator $g_{51}(s)$. First MIMO problem.*
The non-diagonal compensator $g_{51}(s)$ is designed to minimize the (5,1) element of the coupling matrix in the case of disturbance rejection at plant input, which gives the following expression:

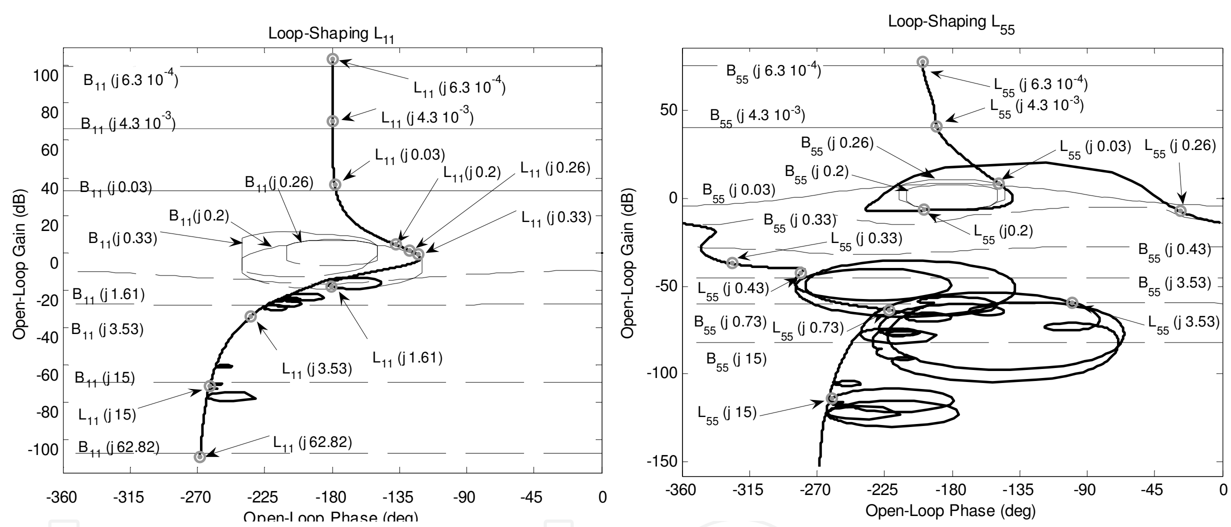
$$g_{51}^{opt}(s)=-p_{51}^{*\,\mathrm{N}}(s) \tag{28}$$

where N denotes the middle plant that interpolates the expression [$-p_{51}^{*}(s)$] from 0 to 10^{-1} rad/s, as shown in Fig. 7, (Garcia-Sanz *et al.*, 2008).



(a) (b)

Fig. 5. Loop-shaping: (a) $L_{33}(s) = p_{33}(s)\, g_{33}(s)$, (b) $L_{66}(s) = p_{66}(s)\, g_{66}(s)$



(a) (b)

Fig. 6. Loop-shaping (a) $L_{11}(s) = [p_{11}^{*e}(s)]_1^{-1}\, g_{11}(s)$. (b) $L_{55}(s) = [p_{55}^{*e}(s)]_2^{-1}\, g_{55}(s)$

Step B2. *Design of the diagonal compensator $g_{55}(s)$. First MIMO problem.*

The compensator $g_{55}(s)$ is designed according to the non-diagonal MIMO QFT methodology explained in Section 3.2, for the inverse of the equivalent plant $[p_{55}^{*e}(s)]_2$, which is:

$$\left[p_{55}^{*e}(s)\right]_2 = \left[p_{55}^{*e}(s)\right]_1 - \left(\left[p_{51}^{*e}(s)\right]_1 + \left[g_{51}(s)\right]_1\right)\left(\left[p_{15}^{*e}(s)\right]_1\right) \Big/ \left(\left[p_{11}^{*e}(s)\right]_1 + \left[g_{11}(s)\right]_1\right)$$. See Fig. 6b.

Step C2. *Design of the non-diagonal compensator $g_{15}(s)$. First MIMO problem.*

The non-diagonal compensator $g_{15}(s)$ is designed to minimize the (1,5) element of the coupling matrix in the case of disturbance rejection at plant input which, taking the 4.2-C3 performance specification also into account gives the following expression:

$$g_{15}^{opt}(s) = -p_{15}^{*\,N}(s) = 0 \tag{29}$$

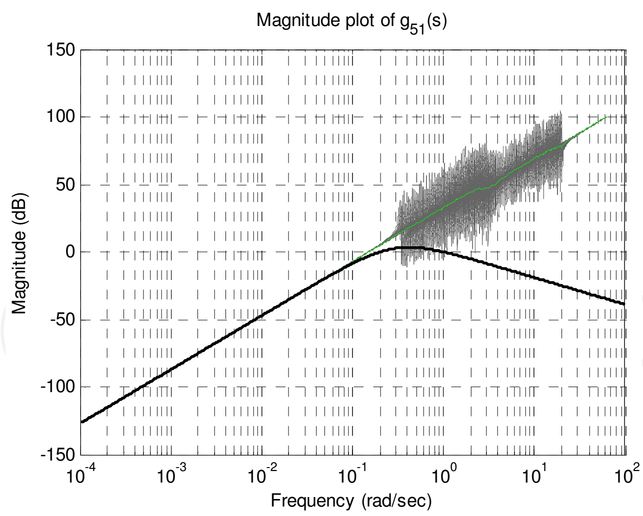
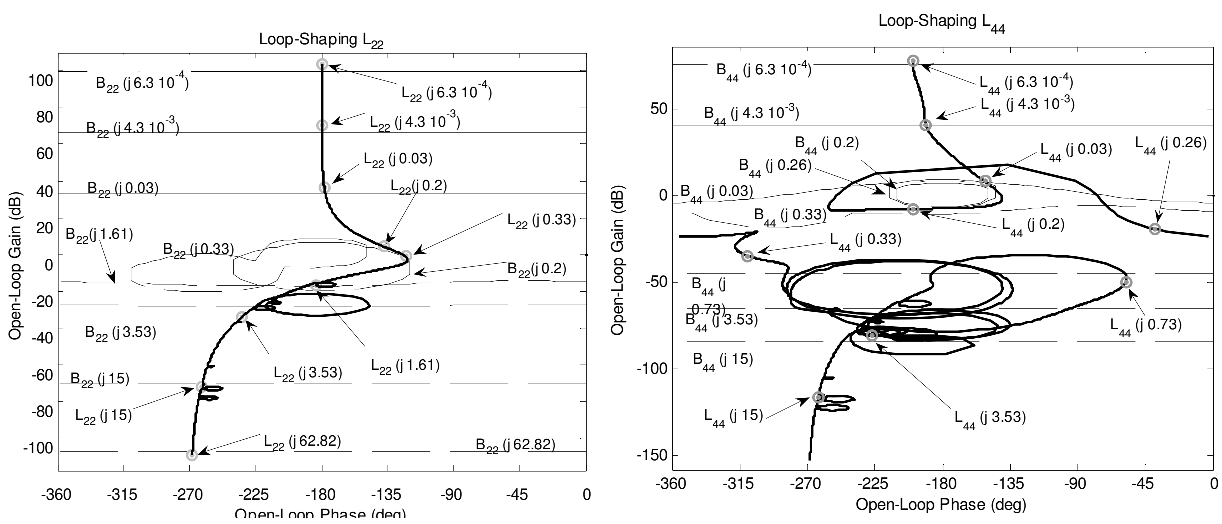


Fig. 7. Magnitude plot of [-$p_{51}^*(s)$] with uncertainty and $g_{51}(s)$ –bold solid line-



(a) (b)

Fig. 8. Loop-shaping (a) $L_{22}(s) = [p_{22}^{*e}(s)]_1^{-1}\, g_{22}(s)$. (b) $L_{44}(s) = [p_{44}^{*e}(s)]_2^{-1}\, g_{44}(s)$

The second MIMO problem is shown in the following Steps. It consists of the design of the elements $g_{22}(s)$, $g_{42}(s)$, $g_{44}(s)$ and $g_{24}(s)$, which are equivalently performed as in the previous Steps *B1*, *C1*, *B2* and *C2* respectively.

Step B3. *Design of the diagonal compensator $g_{22}(s)$. Second MIMO problem.*

The compensator $g_{22}(s)$ is designed according to the non-diagonal MIMO QFT methodology explained in Section 3.2, for the inverse of the equivalent plant $[p_{22}^{*e}(s)]_1 = p_{22}^*(s)$. See Fig. 8a.

Step C3. *Design of the non-diagonal compensator $g_{42}(s)$. Second MIMO problem.*

The non-diagonal compensator $g_{42}(s)$ is designed to minimize the (4,2) element of the coupling matrix in the case of disturbance rejection at plant input, which gives the following expression:

$$g_{42}^{opt}(s) = -p_{42}^{*\,\mathrm{N}}(s) \tag{30}$$

where N denotes the middle plant that interpolates the expression [-$p_{42}^*(s)$] from 0 to 10^{-1} rad/s, as shown in Fig. 9, (Garcia-Sanz *et al.*, 2008).

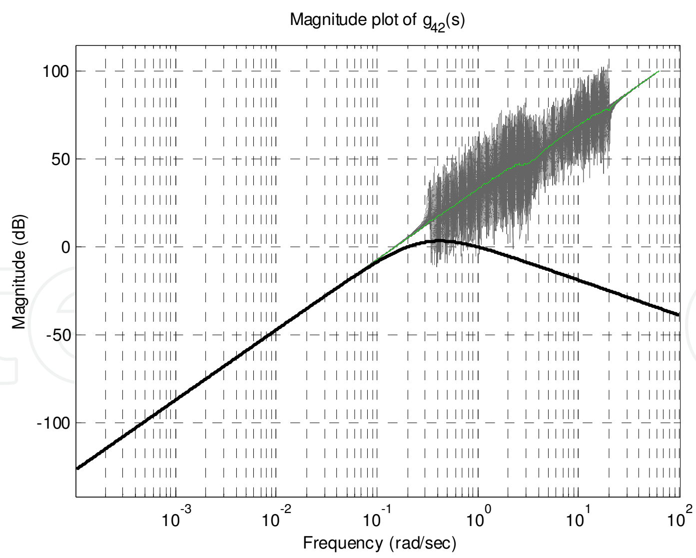


Fig. 9. Magnitude plot of [-$p_{42}^{*}(s)$] with uncertainty and $g_{42}(s)$ –bold solid line-

Step B4. *Design of the diagonal compensator $g_{44}(s)$. Second MIMO problem.*
The compensator $g_{44}(s)$ is designed according to the non-diagonal MIMO QFT methodology explained in Section 3.2, for the inverse of the equivalent plant $[p_{44}^{*e}(s)]_2$, which is:

$$\left[p_{44}^{*e}(s)\right]_2=\left[p_{44}^{*e}(s)\right]_1-\left(\left[p_{42}^{*e}(s)\right]_1+\left[g_{42}(s)\right]_1\right)\left(\left[p_{24}^{*e}(s)\right]_1\right)\Big/\left(\left[p_{22}^{*e}(s)\right]_1+\left[g_{22}(s)\right]_1\right).$$

See Fig. 8b.
Step C4. *Design of the non-diagonal compensator g24(s). Second MIMO problem.*
The non-diagonal compensator $g_{24}(s)$ is designed to minimize the (2,4) element of the coupling matrix in the case of disturbance rejection at plant input which, taking the 4.2-C3 performance specification also into account gives the following expression:

$$g_{24}^{opt}(s)=-p_{24}^{*\,\mathrm{N}}(s)=0 \quad (31)$$

Step D. *Design of the prefilter $f_{kk}(s)$, k = 1, 2,...6.*
There is not prefilter required in this example, because we do not have reference tracking specifications (See Section 4.2).

4.4 Validation

Time domain simulations were performed for 300 random mode dynamics within the uncertainty range (MonteCarlo analysis) in the ESA telescope-type benchmark simulator (Fig. 10).

The position and attitude performance obtained by the non-diagonal MIMO QFT was excellent, fulfilling easily all the required specifications (Section 4.2, A, B and C), improving also by two order of magnitude the results obtained by other robust control techniques on the maximum and standard deviation error results. At the same time, while these other robust control techniques (H-infinity type) required controller structures with full-matrices of 36 elements of 42nd order, the non-diagonal MIMO QFT design consists of only eight compensators going from 3rd to 14th order, dividing by more than 20 the number of operations per second needed (see Table II).

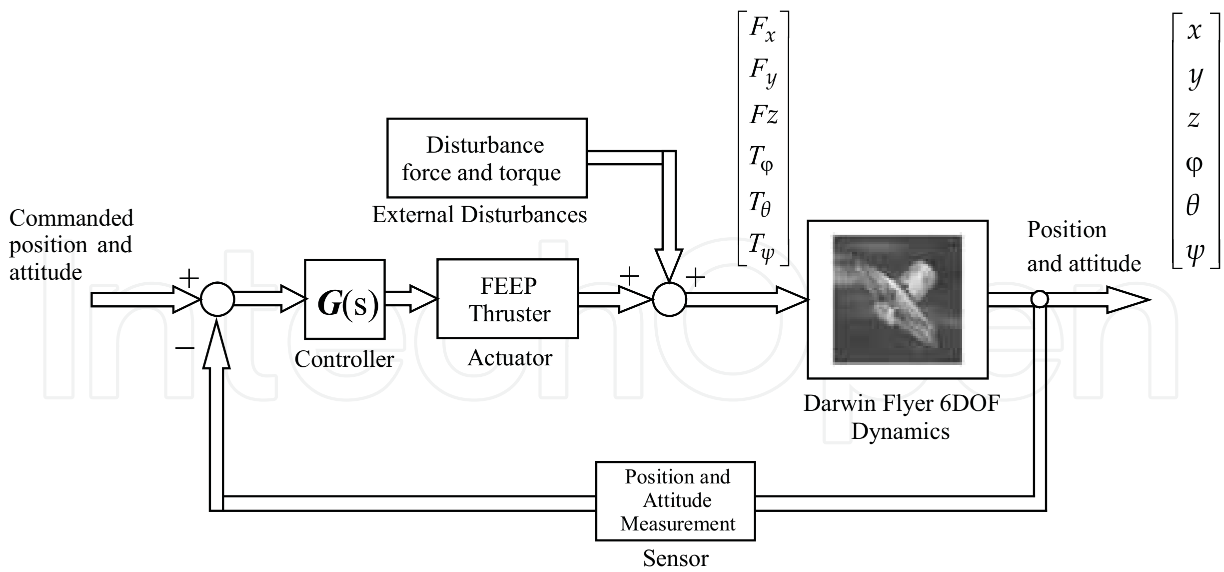


Fig. 10. ESA Telescope-type Spacecraft Simulator

Controller	*Number of Multiplications*	*Number of Sums*
Non-diagonal MIMO QFT	130	124
Other robust control technique	2994	2988

Table II. Number of operations per second required by the controllers

5. Conclusions

This chapter demonstrated the feasibility of sequential non-diagonal multi-input multi-output –MIMO- robust QFT control strategies to regulate simultaneously the position and attitude of a telescope-type spacecraft with large flexible appendages. The chapter described: 1) the main control challenges and dynamic characteristics of MIMO systems in general; 2) advanced MIMO techniques to design robust controllers based on the quantitative feedback theory –QFT-; and 3) some illustrative results achieved when applying the MIMO QFT control methodology to one of the telescope-type spacecraft of a multiple formation flying constellation of a European Space Agency cornerstone mission, fulfilling satisfactory the astronomical, engineering and control requirements of the spacecraft.

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