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Fast BEM Based Methods for Heat Transfer Simulation

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1. Introduction

Development of numerical techniques for simulation of fluid flow and heat transfer has a long standing tradition. Computational fluid dynamics has evolved to a point where new methods are needed only for special cases. In this chapter we introduce a Fast Boundary Element Method (BEM), which enables accurate prediction of vorticity fields. Vorticity field is defined as a curl of the velocity field and is an important quantity in wall bounded flows. Vorticity is generated on the walls and diffused and advected into the flow field. Using BEM, we are able to accurately predict boundary values of vorticity as a part of the nonlinear system of equations, without the use of finite difference approximations of derivatives of the velocity field. The generation of vorticity on the walls is important for the development of the flow field, shear strain, shear velocity and heat transfer.

The developed method will be used to simulate natural convection of pure fluids and nanofluids. Over the last few decades buoyancy driven flows have been widely investigated. Cavities under different inclination angles with respect to gravity, heated either differentially on two opposite sides or via a hotstrip in the centre, are usually the target of research. Natural convection is used in many industrial applications, such as cooling of electronic circuitry, nuclear reactor insulation and ventilation of rooms.

Research of the natural convection phenomena started with the two-dimensional approach and has been recently extended to three dimensions. A benchmark solution for two-dimensional flow and heat transfer of an incompressible fluid in a square differentially heated cavity was presented by Davies (1983). Stream function-vorticity formulation was used. Vierendeels et al. (2001; 2004) and Škerget & Samec (2005) simulated compressible fluid in a square differentially heated cavity using multigrid and BEM methods. Rayleigh numbers between $Ra = 10^2$ and $Ra = 10^7$ were considered. Weisman et al. (2001) studied the transition from steady to unsteady flow for compressible fluid in a 1 : 4 cavity. They found that the transition occurs at $Ra \approx 2 \times 10^5$. Ingber (2003) used the vorticity formulation to simulate flow in both square and 1 : 8 differentially heated cavities. Tric et al. (2000) studied natural convection in a 3D cubic cavity using a pseudo-spectra Chebyshev algorithm based on the projection-diffusion method with spatial resolution supplied by polynomial expansions. Lo et al. (2007) also studied a 3D cubic cavity under five different inclinations $\vartheta = 0^o, 15^o, 30^o, 45^o, 60^o$. They used a differential quadrature method to solve the velocity-vorticity formulation of Navier-Stokes equations employing higher order polynomials to approximate differential operators. Ravnik et al. (2008) used a combination of single domain and sub domain BEM to solve the velocity-vorticity formulation of Navier-Stokes equations for fluid

flow and heat transfer.
Simulations as well as experiments of turbulent flow were also extensively investigated. Hsieh & Lien (2004) considered numerical modelling of buoyancy-driven turbulent flows in cavities using RANS approach. 2D DNS was performed by Xin & Quéré (1995) for an cavity with aspect ratio 4 up to Rayleigh number, based on the cavity height, 10^{10} using expansions in series of Chebyshev polynomials. Ravnik et al. (2006) confirmed these results using a 2D LES model based on combination of BEM and FEM using the classical Smagorinsky model with Van Driest damping. Peng & Davidson (2001) performed a LES study of turbulent buoyant flow in a $1:1$ cavity at $Ra = 1.59 \cdot 10^9$ using a dynamic Smagorinsky model as well as the classical Smagorinsky model with Van Driest damping.
Low thermal conductivity of working fluids such as water, oil or ethylene glycol led to the introduction of nanofluids. Nanofluid is a suspension consisting of uniformly dispersed and suspended nanometre-sized (10–50 nm) particles in base fluid, pioneered by Choi (1995). Nanofluids have very high thermal conductivities at very low nanoparticle concentrations and exhibit considerable enhancement of convection (Yang et al., 2005). Intensive research in the field of nanofluids started only recently. A wide variety of experimental and theoretical investigations have been performed, as well as several nanofluid preparation techniques have been proposed (Wang & Mujumdar, 2007).
Several researchers have been focusing on buoyant flow of nanofluids. Oztop & Abu-Nada (2008) performed a 2D study of natural convection of various nanofluids in partially heated rectangular cavities, reporting that the type of nanofluid is a key factor for heat transfer enhancement. They obtained best results with Cu nanoparticles. The same researchers (Abu-Nada & Oztop, 2009) examined the effects of inclination angle on natural convection in cavities filled with Cu–water nanofluid. They reported that the effect of nanofluid on heat enhancement is more pronounced at low Rayleigh numbers. Hwang et al. (2007) studied natural convection of a water based Al_2O_3 nanofluid in a rectangular cavity heated from below. They investigated convective instability of the flow and heat transfer and reported that the natural convection of a nanofluid becomes more stable when the volume fraction of nanoparticles increases. Ho et al. (2008) studied effects on nanofluid heat transfer due to uncertainties of viscosity and thermal conductivity in a buoyant cavity. They demonstrated that usage of different models for viscosity and thermal conductivity does indeed have a significant impact on heat transfer. Natural convection of nanofluids in an inclined differentially heated square cavity was studied by Ögüt (2009), using polynomial differential quadrature method. Stream function-vorticity formulation was used for simulation of nanofluids in two dimensions by Gümgüm & Tezer-Sezgin (2010).
Forced and mixed convection studies were also performed. Abu-Nada (2008) studied the application of nanofluids for heat transfer enhancement of separated flows encountered in a backward facing step. He found that the high heat transfer inside the recirculation zone depends mainly on thermophysical properties of nanoparticles and that it is independent of Reynolds number. Mirmasoumi & Behzadmehr (2008) numerically studied the effect of nanoparticle mean diameter on mixed convection heat transfer of a nanofluid in a horizontal tube using a two-phase mixture model. They showed that the convective heat transfer could be significantly increased by using particles with smaller mean diameter. Akbarinia & Behzadmehr (2007) numerically studied laminar mixed convection of a nanofluid in horizontal curved tubes. Tiwari & Das (2007) studied heat transfer in a lid-driven differentially heated square cavity. They reported that the relationship between heat transfer and the volume fraction of solid particles in a nanofluid is nonlinear. Torii (2010) experimentally

studied turbulent heat transfer behaviour of nanofluid in a circular tube, heated under constant heat flux. He reported that the relative viscosity of nanofluids increases with concentration of nanoparticles, pressure loss of nanofluids is slightly larger than that of pure fluid and that heat transfer enhancement is affected by occurrence of particle aggregation.

Development of numerical algorithms capable of simulating fluid flow and heat transfer has a long standing tradition. A vast variety of methods was developed and their characteristics were examined. In this work we are presenting an algorithm, which is able to simulate 3D laminar viscous flow coupled with heat transfer by solving the velocity-vorticity formulation of Navier-Stokes equations using fast BEM. The velocity-vorticity formulation is an alternative form of the Navier-Stokes equation, which does not include pressure. The unknown field functions are the velocity and vorticity. In an incompressible flow, both are divergence free. Daube (1992) pointed out that the correct evaluation of boundary vorticity values is essential for conservation of mass. Thus, the main challenge of velocity-vorticity formulation lies in the determination of boundary vorticity values. Several different approaches have been proposed for the determination of vorticity on the boundary. Wong & Baker (2002) used a second-order Taylor series to determine the boundary vorticity values explicitly. Daube (1992) used an influence matrix technique to enforce both the continuity equation and the definition of the vorticity in the treatment of the 2D incompressible Navier-Stokes equations. Liu (2001) recognised that the problem is even more severe when he extended it to three dimensions. Lo et al. (2007) used the differential quadrature method. Sellountos & Sequeira (2008) proposed a hybrid multi BEM scheme in combination with local boundary integral equations and radial basis functions for 2D fluid flow. Škerget et al. (2003) proposed the usage of single domain BEM to obtain a solution of the kinematics equation in tangential form for the unknown boundary vorticity values and used it in 2D. This work was extended into 3D using a linear interpolation in combination with FEM by Žunič et al. (2007) and using quadratic interpolation by Ravnik et al. (2009a) for uncoupled flow problems.

The BEM uses the fundamental solution of the differential operator and the Green's theorem to rewrite a partial differential equation into an equivalent boundary integral equation. After discretization of only the boundary of the problem domain, a fully populated system of equations emerges. The number of degrees of freedom is equal to the number of boundary nodes. This reduction of the dimensionality of the problem is a major advantage over the volume based methods. Fundamental solutions are known for a wide variety of differential operators (Wrobel, 2002), making BEM applicable for solving a wide range of problems.

Unfortunately, integral equations of nonhomogeneous and nonlinear problems, such as heat transfer in fluid flow, include a domain term. In this work, we solve the velocity-vorticity formulation of incompressible Navier-Stokes equations. The formulation joins the Poisson type kinematics equation with diffusion advection type equations of vorticity and heat transport. These equations are nonhomogenous and nonlinear. In order to write discrete systems of linear equations for such equations, matrices of domain integrals must be evaluated. Such domain matrices, since they are full and unsymmetrical, require a lot of storage space and algebraic operations with them require a lot of CPU time. Thus the domain matrices present a bottleneck for any BEM based algorithm effectively limiting the maximal usable mesh size through their cost in storage and CPU time.

The dual reciprocity BEM (Partridge et al. (1992), Jumarhon et al. (1997)) is one of the most popular techniques to eliminate the domain integrals. It uses expansion of the nonhomogenous term in terms of radial basis functions. Several other approaches that enable construction of data sparse approximations of fully populated matrices are also known. Hackbusch & Nowak (1989) developed a panel clustering method, which also enables approximate matrix vector multiplications with decreased amount of arithmetical work. A class of hierarchical matrices was introduced by Hackbusch (1999) with the aim of reducing the complexity of matrix-vector multiplications. Bebendorf & Rjasanow (2003) developed an algebraic approach for solving integral equations using collocation methods with almost linear complexity. Methods based on the expansion of the integral kernel (Bebendorf, 2000) have been proposed as well. Fata (2010) proposed treatment of domain integrals by rewriting them as a combination of surface integrals whose kernels are line integrals. Ravnik et al. (2004) developed a wavelet compression method and used it for compression of single domain BEM in 2D. Compression of single domain full matrices has also been the subject of research of Eppler & Harbrecht (2005).

The algorithm proposed in this chapter tackles the domain integral problem using two techniques: a kernel expansion method based single domain BEM is employed for fast solution of the kinematics equation and subdomain BEM is used for diffusion-advection type equations.

In the subdomain BEM (Popov et al., 2007), integral equations are written for each subdomain (mesh element) separately. We use continuous quadratic boundary elements for the discretization of function and discontinuous linear boundary element for the discretization of flux. By the use of discontinuous discretization of flux, all flux nodes are within boundary elements where the normal and the flux are unambiguously defined. The corners and edges, where the normal is not well defined, are avoided. The singularities of corners and edges were dealt with special singular shape functions by Ong & Lim (2005) and by the use of additional nodes by Gao & Davies (2000). By the use of a collocation scheme, a single linear equation is written for every function and flux node in every boundary element. By using compatibility conditions between subdomains, we obtain an over-determined system of linear equations, which may be solved in a least squares manner. The governing matrices are sparse and have similar storage requirements as the finite element method. Subdomain BEM was applied on the Laplace equation by Ramšak & Škerget (2007) and on the velocity-vorticity formulation of Navier-Stokes equations by Ravnik et al. (2008; 2009a).

The second part of the algorithm uses fast kernel expansion based single domain BEM. The method is used to provide a sparse approximation of the fully populated BEM domain matrices. The storage requirements of the sparse approximations scale linearly with the number of nodes in the domain, which is a major improvement over the quadratic complexity of the full BEM matrices. The technique eliminates the storage and CPU time problems associated with application of BEM on nonhomogenous partial differential equations.

The origins of the method can be found in a fast multipole algorithm (FMM) for particle simulations developed by Greengard & Rokhlin (1987). The algorithm decreases the amount of work required to evaluate mutual interaction of particles by reducing the complexity of the problem from quadratic to linear. Ever since, the method was used by many authors for a wide variety of problems using different expansion strategies. Recently, Bui et al. (2006) combined FMM with the Fourier transform to study multiple bubbles dynamics. Gumerov & Duraiswami (2006) applied the FMM for the biharmonic equation in three dimensions. The boundary integral Laplace equation was accelerated with FMM by Popov et al. (2003). In contrast to the contribution of this paper, where the subject of study is the application of FMM to obtain a sparse approximation of the domain matrix, the majority of work done by other authors dealt with coupling BEM with FMM for the boundary matrices. Ravnik et al. (2009b) compared wavelet and fast data sparse approximations for boundary - domain integral equations of Poisson type.

2. Governing equations

In this work, we will present a numerical algorithm and simulation results for heat transfer in pure fluids and in nanofluids. We present the governing equations for nanofluids, since they can be, by choosing the correct parameter values, used for pure fluids as well. We assume the pure fluid and nanofluid to be incompressible. Flow in our simulations is laminar and steady. Effective properties of the nanofluid are: density ρ_{nf}, dynamic viscosity μ_{nf}, heat capacitance $(c_p)_{nf}$, thermal expansion coefficient β_{nf} and thermal conductivity k_{nf}, where subscript *nf* is used to denote effective i.e. nanofluid properties. The properties are all assumed constant throughout the flow domain. The mass conservation law for an incompressible fluid may be stated as

$$\vec{\nabla} \cdot \vec{v} = 0. \tag{1}$$

Considering constant nanofluid material properties and taking density variation into account within the Boussinesq approximation we write the momentum equation as

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\beta_{nf}(T - T_0)\vec{g} - \frac{1}{\rho_{nf}}\vec{\nabla} p + \frac{\mu_{nf}}{\rho_{nf}}\nabla^2 \vec{v}. \tag{2}$$

We assume that no internal energy sources are present in the fluid. We will not deal with high velocity flow of highly viscous fluid, hence we will neglect irreversible viscous dissipation. With this, the internal energy conservation law, written with temperature as the unknown variable, reads as:

$$\frac{\partial T}{\partial t} + (\vec{v} \cdot \vec{\nabla})T = \frac{k_{nf}}{(\rho c_p)_{nf}}\nabla^2 T. \tag{3}$$

Relationships between properties of nanofluid to those of pure fluid and pure solid are provided with the models. Density of the nanofluid is calculated using particle volume fraction φ and densities of pure fluid ρ_f and of solid nanoparticles ρ_s as:

$$\rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_s \tag{4}$$

The effective dynamic viscosity of a fluid of dynamic viscosity μ_f containing a dilute suspension of small rigid spherical particles, is given by Brinkman (1952) as

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}}. \tag{5}$$

The effective viscosity is independent of nanoparticle type, thus the differences in heat transfer between different nanofluids will be caused by heat related physical parameters only. The heat capacitance of the nanofluid can be expressed as (Khanafer et al., 2003):

$$(\rho c_p)_{nf} = (1 - \varphi)(\rho c_p)_f + \varphi(\rho c_p)_s. \tag{6}$$

Similarly, the nanofluid thermal expansion coefficient can be written as $(\rho\beta)_{nf} = (1 - \varphi)(\rho\beta)_f + \varphi(\rho\beta)_s$, which may be, by taking into account the definition of ρ_{nf} in equation (4), written as:

$$\beta_{nf} = \beta_f \left[\frac{1}{1 + \frac{(1-\varphi)\rho_f}{\varphi\rho_s}} \frac{\beta_s}{\beta_f} + \frac{1}{1 + \frac{\varphi}{1-\varphi}\frac{\rho_s}{\rho_f}} \right]. \tag{7}$$

ϑ	Lo et al. (2007)			present		
	$Ra = 10^3$	$Ra = 10^4$	$Ra = 10^5$	$Ra = 10^3$	$Ra = 10^4$	$Ra = 10^5$
15^o	1.0590	1.8425	3.7731	1.0592	1.8464	3.7881
30^o	1.0432	1.5894	2.9014	1.0433	1.5916	2.9071
45^o	1.0268	1.3434	1.9791	1.0268	1.3443	1.9782
60^o	1.0127	1.1524	1.3623	1.0127	1.1526	1.3600

Table 3. Natural convection of air in a cubic cavity under inclination ϑ. Present Nusselt number values are compared with the benchmark results of Lo et al. (2007).

and thus resulting in high heat transfer. Upon reaching the top of the hotstrip the fluid flows over the top ultimately colliding with the fluid from the other side and rising upwards. When the hotstrip is located in the centre of the cavity, the flow field is symmetric and the fluid rises from the centre of the hotstrip. If the hotstrip is placed off-centre, the flow symmetry is lost. The sizes of large vortices on each side of the hotstrip are different. The flow does not rise above the centre of the hotstrip. Mixing of the fluid from both sides of the hotstrip occurs, which does not happen in the symmetric case.

We measured heat transfer in terms of the Nusselt number values over the whole surface of the hostrip (two vertical wall and a top wall). The values are shown in Table 6 for Rayleigh number values ranging from $Ra = 10^3$ to $Ra = 10^5$. We observe that the addition of nanoparticles increases heat transfer in all cases. The increase is the largest in the low Rayleigh number case and the smallest in the case of high Rayleigh values. This is expected, since at low Rayleigh number values conduction is the dominating heat transfer mechanism

ϑ	$Ra = 10^3$	$Ra = 10^4$	$Ra = 10^5$
0^o	1.111	2.163	4.177
15^o	1.096	2.029	3.892
30^o	1.073	1.839	3.430
45^o	1.047	1.594	2.774
60^o	1.023	1.317	1.915

Table 4. Natural convection of air in a $H/L = 2$ cavity. The Nusselt number values representing the heat flux through walls are shown for different inclination angles and Rayleigh numbers.

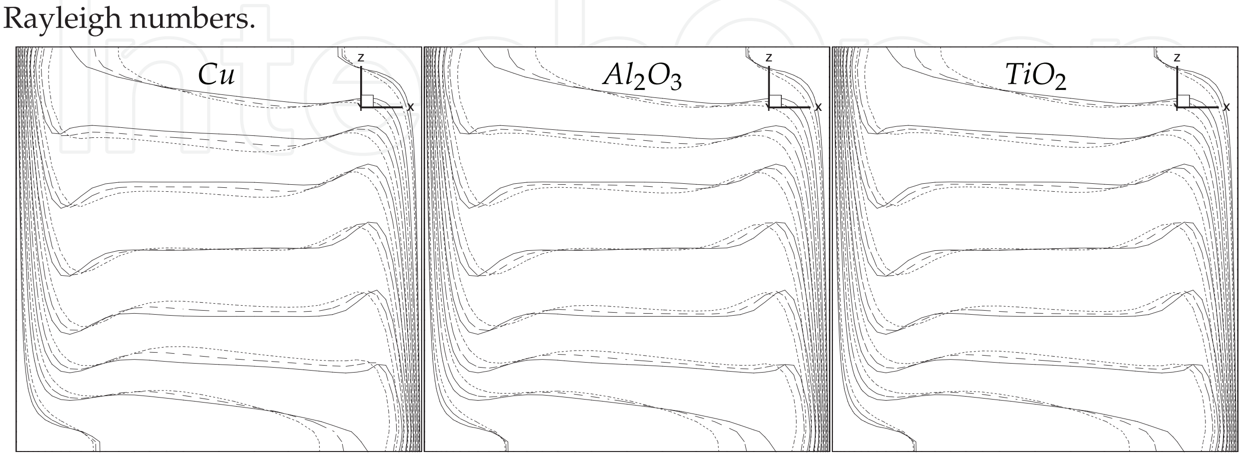

Fig. 6. Temperature contours on the central $y = 0.5H$ plane for natural convection in a differentially heated cubic cavity. Contour values are -0.4(0.1)0.4; $Ra = 10^6$. Solid line denotes pure water, dashed line $\varphi = 0.1$ nanofluid and dotted line $\varphi = 0.2$ nanofluid.

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