We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

1

Optimum Fin Profile under Dry and Wet Surface Conditions

Balaram Kundu[1] and Somchai Wongwises[2]
[1]Department of Mechanical Engineering
Jadavpur University, Kolkata – 700 032
[2]Fluid Mechanics, Thermal Engineering and Multiphase
Flow Research Lab (FUTURE), Department of Mechanical Engineering
King Mongkut's University of Technology Thonburi (KMUTT)
Bangmod,Bangkok 10140
[1]India
[2]Thailand

1. Introduction

Fins or extended surfaces are frequently employed in heat exchangers for effectively improving the overall heat transfer performance. The simple design of fins and their stability in different surface conditions have created them a popular augmentation device. The different fin shapes are available in the literature. The geometry of the fin may be dependent upon the primary surface also. For circular primary surface, the attachment of circumferential fins is a common choice. The longitudinal and pin fins are generally used to the flat primary surface. However, due to attachment of fins with the primary surface, the heat transfer augments but the volume, weight, and cost of the heat exchanger equipments increase as well. Hence, it is a challenge to the designer to minimize the cost for the attachment of fins. This can be done by determining the optimum shape of a fin satisfying the maximization of heat transfer rate for a given fin volume. In general, two different approaches are considered for the optimization of any fin design problem. Through a rigorous technique, the profile of a fin for a particular geometry (flat or curved primary surface) may be obtained such that the criteria of the maximum heat transfer for a given fin volume or equivalently minimum fin volume for a given heat transfer duty is satisfied. In a parallel activity, the optimum dimensions of a fin of given profile (rectangular, triangular etc.) are determined from the solution of the optimality criteria. The resulting profile obtained from the first case of optimum design is superior in respect to heat transfer rate per unit volume and thus it is very much important in fin design problems. However, it may be limited to use in actual practice because the resulting profile shape would be slightly difficult to manufacture and fabricate. Alternatively, such theoretical shape would first be calculated and then a triangular profile approximating the base two thirds of the fin would be used. Such a triangular fin transfers heat per unit weight, which is closer to that of the analytical optimum value.

Under a convective environmental condition, Schmidt (1926) was the first researcher to forward a systematic approach for the optimum design of fins. He proposed heuristically that for an optimum shape of a cooling fin, the fin temperature must be a linear function with the fin length. Later, through the calculus of variation, Duffin (1959) exhibited a rigorous proof on the optimality criteria of Schmidt. Liu (1961) extended the variational principle to find out the optimum profile of fins with internal heat generation. Liu (1962) and Wilkins (1961) addressed for the optimization of radiating fins. Solov"ev (1968) determined the optimum radiator fin profile. The performance parameter of annular fins of different profiles subject to locally variable heat transfer coefficient had been investigated by Mokheimer (2002). From the above literature works, it can be indicated that the above works were formulated based on the "length of arc idealization (LAI)."

Maday (1974) was the first researcher to eliminate LAI and obtained the optimum profile through a numerical integration. It is interesting to note that an optimum convecting fin neither has a linear temperature profile nor possesses a concave parabolic shape suggested by Maday. The profile shape contains a number of ripples denoted as a "wavy fin". The same exercise was carried out for radial fins by Guceri and Maday (1975). Later Razelos and Imre (1983) applied Pontryagin's minimum principle to find out the minimum mass of convective fins with variable heat transfer coefficient. Zubair et al. (1996) determined the optimum dimensions of circular fins with variable profiles and temperature dependent thermal conductivity. They found an increasing heat transfer rate through the optimum profile fin by 20% as compare to the constant thickness fin.

A variational method was adopted by Kundu and Das (1998) to determine the optimum shape of three types of fins namely the longitudinal fin, spine and disc fin. A generalized approach of analysis based on a common form of differential equations and a set of boundary conditions had been described. For all the fin geometries, it was shown that the temperature gradient is constant and the excess temperature at the tip vanishes. By taking into account the LAI, Hanin and Campo (2003) forecasted a shape of a straight cooling fin for the minimum envelop. From the result, they have highlighted that the volume of the optimum circular fin with consideration of LAI found is 6.21-8 times smaller than the volume of the corresponding Schmidt's parabolic optimum fin. A new methodological determination for the optimum design of thin fins with uniform volumetric heat generation had been done by Kundu and Das (2005).

There are ample of practical applications in which extended surface heat transfer is involved in two-phase flow conditions. For example, when humid air encounter into a cold surface of cooling coils whose temperature is maintained below the dew point temperature, condensation of moisture will take place, and mass and heat transfer occur simultaneously. The fin-and-tube heat exchangers are widely used in conventional air conditioning systems for air cooling and dehumidifying. In the evaporator of air conditioning equipment, the fin surface becomes dry, partially or fully wet depending upon the thermogeometric and psychrometric conditions involved in the design process. If the temperature of the entire fin surface is lower than the dew point of the surrounding air, there may occur both sensible and latent heat transferred from the air to the fin and so the fin is fully wet. The fin is partially wet if the fin-base temperature is below the dew point while fin-tip temperature is above the dew point of the surrounding air. If the temperature of the entire fin surface is higher than the dew point, only sensible heat is transferred and so the fin is fully dry. For wet surface, the moisture is condensed on the fin surface, latent heat evolves and mass transfer occurs simultaneously with the heat transfer. Thermal performance of different surface conditions of a fin depends on the fin shape, thermophysical and psychrometric properties of air.

Many investigations have been devoted to analyze the effect of condensation on the performance of different geometric fins. It is noteworthy to mention that for each instance, a suitable fin geometry has been selected a priory to make the analysis. For the combined heat and mass transfer, the mathematical formulation becomes complex to determine the overall performance analysis of a wet fin. Based on the dry fin formula, Threlkeld (1970) and McQuiston (1975) determined the one-dimensional fin efficiency of a rectangular longitudinal fin for a fully wet surface condition. An analytical solution for the efficiency of a longitudinal straight fin under dry, fully wet and partially wet surface conditions was introduced elaborately by Wu and Bong (1994) first with considering temperature and humidity ratio differences as the driving forces for heat and mass transfer. For the establishment of an analytical solution, a linear relationship between humidity ratio and the corresponding saturation temperature of air was taken. Later an extensive analytical works on the performance and optimization analysis of wet fins was carried out by applying this linear relationship. A technique to determine the performance and optimization of straight tapered longitudinal fins subject to simultaneous heat and mass transfer has been established analytically by Kundu (2002) and Kundu and Das (2004). The performance and optimum dimensions of a new fin, namely, SRC profile subject to simultaneous heat and mass transfer have been investigated by Kundu (2007a; 2009a). In his work, a comparative study has also been made between rectangular and SRC profile fins when they are operated in wet conditions. Hong and Web (1996) calculated the fin efficiency for wet and dry circular fins with a constant thickness. Kundu and Barman (2010) have studied a design analysis of annular fins under dehumidifying conditions with a polynomial relationship between humidity ratio and saturation temperature by using differential transform method. In case of longitudinal fins of rectangular geometry, approximate analytic solution for performances has been demonstrated by Kundu (2009b). Kundu and Miyara (2009) have established an analytical model for determination of the performance of a fin assembly under dehumidifying conditions. Kundu et al. (2008) have described analytically to predict the fin performance of longitudinal triangular fins subject to simultaneous heat and mass transfer.

The heat and mass transfer analysis for dehumidification of air on fin-and-tube heat exchangers was done experimentally by the few authors. The different techniques, namely, new reduction method, tinny circular fin method, finite circular fin method and review of data reduction method used for analyzing the heat and mass transfer characteristics of wavy fin-and-tube exchangers under dehumidifying conditions had been investigated by Pirompugd et al. (2007a; 2007b; 2008; 2009).

The above investigations had been focused on determination of the optimum profile subjected to convective environment. However a thorough research works have already been devoted for analyzing the performance and optimization of wet fins. To carryout these analyses, suitable fin geometry has been chosen a priory. However, the optimum profile fin may be employed in air conditioning apparatus, especially, in aircrafts where reduction of weight is always given an extra design attention. Kundu (2008) determined an optimum fin profile of thin fins under dehumidifying condition of practical interest formulated with the treatment by a calculus of variation. Recently, Kundu (2010) focused to determine the optimum fin profile for both fully and partially wet longitudinal fins with a nonlinear saturation curve.

In this book chapter, a mathematical theory has been developed for obtaining the optimum fin shape of three common types of fins, namely, longitudinal, spine and annular fins by satisfying the maximizing heat transfer duty for a given either fin volume or both fin volume and length. The analysis was formulated for the dry, partially and fully wet surface conditions. For the analytical solution of a wet fin equation, a relationship between humidity ratio and temperature of the saturation air is necessary and it is taken a linear variation. The influence of wet fin surface conditions on the optimum profile shape and its dimensions has also been examined. From the analysis, it can be mentioned that whether a surface is dry, partially or fully wet at an optimum condition, the air relative humidity is a responsible factor. The optimum fin profile and design variables have been determined as a function of thermo-psychrometric parameters. The dry surface analysis can be possible from the present fully wet surface fin analysis with considering zero value of latent heat of condensation. From the analysis presented, it can be highlighted that unlike dry and partially wet fins, tip temperature for fully wet fins is below the ambient temperature for the minimum profile envelop fin.

2. Variational formulations for the optimum fin shape

For determination of the optimum fin shape, it can be assumed that the condensate thermal resistance to heat flow is negligibly small as the condensate film is much thinner than the boundary layer in the dehumidification process. Under such circumstances, it may follow that the heat transfer coefficient is not influenced significantly with the presence of condensation. The condensation takes place when fin surface temperature is below the dew point of the surrounding air and for its calculation, specific humidity of the saturated air on the wet surface is assumed to be a linear function with the local fin temperature. This assumption can be considered due to the smaller temperature range involved in the practical application between fin base and dew point temperatures and within this small range, saturation curve on the psychometric chart is possible to be an approximated by a straight line (Wu and Bong, 1994; Kundu, 2002; Kundu, 2007a; Kundu, 2007b; Kundu, 2008; Kundu, 2009). Owing to small temperature variation in the fin between fin-base and fin-tip, it can be assumed that the thermal conductivity of the fin material is a constant. The different types of fins, namely, longitudinal, spine and anuular fin are commonly used according to the shape of the primary surfaces. Depending upon the fin base, fin tip and dew point temperatures, fin-surface can be dry, partially and fully wet. The analysis for determination of an optimum profile of fully and partially wet fins for longitudinal, spine and annular fin geometries are described separately in the followings:

2.1 Fully wet longitudinal fins

The schematic diagram of an optimum shape of longitudinal fins is illustrated in Fig. 1. The governing energy equation for one-dimensional temperature distribution on fully wet surface fins can be written under steady state condition as follows:

$$\frac{d}{dx}\left(y\frac{dT}{dx}\right)=\frac{h}{k}\left[\left(T-T_a\right)+h_m\left(\omega-\omega_a\right)h_{fg}/h\right] \quad (1)$$

h_m is the average mass transfer coefficient based on the humidity ratio difference, ω is the humidity ratio of saturated air at temperature T, ω_a is the humidity ratio of the atmospheric air, and h_{jg} is the latent heat of condensation. For the mathematical simplicity, the following dimensionless variables and parameters can be introduced:

$$X=hx/k\ ;\ Y=hy/k\ ;\ L=hl/k\ ;\ \theta=\left(T_a-T\right)/\left(T_a-T_b\right)\ ;\ Le=\left(h/h_mC_p\right)^{3/2} \quad (2)$$

where, L_e is the Lewis number. The relationship between heat and mass transfer coefficients can be obtained from the Chilton-Colburn analogy (Chilton and Colburn, 1934). The relationship between the saturated water film temperature T and the corresponding saturated humidity ratio ω is approximated by a linear function (Wu and Bong, 1994; Kundu, 2002; Kundu, 2007a; Kundu, 2007b; Kundu, 2008; Kundu, 2009) in this study:

$$\omega = a + bT \tag{3}$$

where, a and b are constants determined from the conditions of air at the fin base and fin tip. Eq. (1) is written in dimensionless form by using Eqs. (2) and (3) as follows:

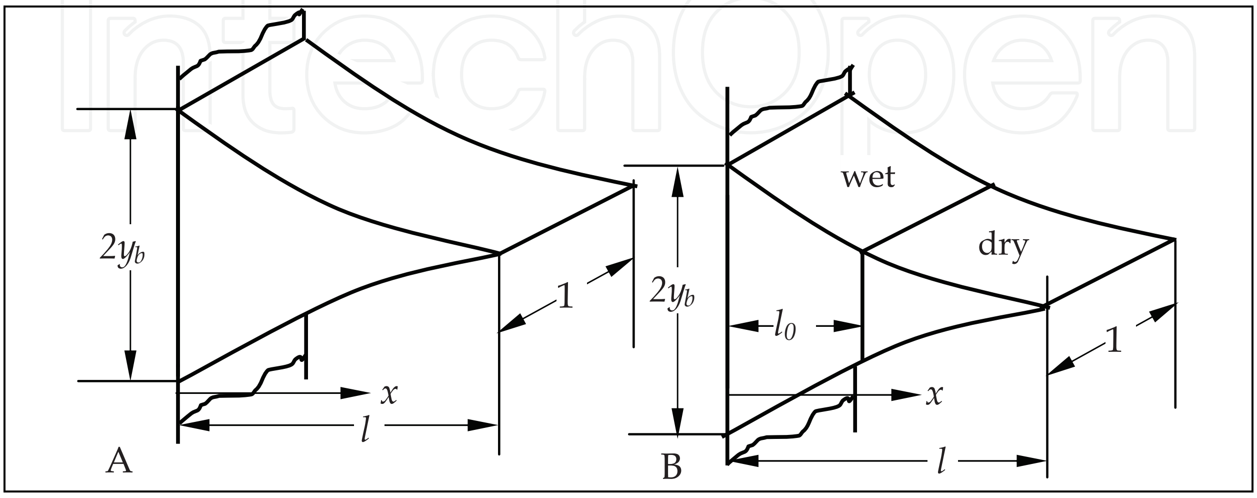


Fig. 1. Schematic diagram of an optimum longitudinal fin under dehumidifying conditions: A. Fully wet; and B. Partially wet.

$$d/dX\left(Y\,d\phi/dX\right) = \left(1 + b\xi\right)\phi \tag{4}$$

where

$$\phi = \theta + \theta_p\,;\ \theta_p = \left(\omega_a - a - bT_a\right)\Big/\left[\left(T_a - T_b\right)\left(1 + b\xi\right)\right];\ \xi = h_{fg}\Big/C_p Le^{2/3} \tag{5}$$

Eq. (4) is subjected to the following boundary conditions:

$$\text{at}\quad X = 0\,,\ \phi = 1 + \theta_p = \phi_0 \tag{6a}$$

$$\text{at}\quad X = L\,,\ Y d\phi/dX = 0 \tag{6b}$$

For determination of the heat transfer duty through fins, Eq. (4) is multiplied by ϕ, and then integrated, the following expression are obtained with the help of the corresponding boundary conditions:

$$-\left[Y\phi\,d\phi/dX\right]_{X=0} = \int_{X=0}^{L}\left[Y\left(d\phi/dX\right)^2 + \left(1 + b\xi\right)\phi^2\right]dX \tag{7}$$

The heat transfer rate through the fins can be calculated by applying the Fourier's law of heat conduction at the fin base:

$$Q = \frac{q}{2k\left(T_a - T_b\right)} = -\left[Y\,d\phi/dX\right]_{X=0} = \frac{1}{\phi_0}\int_{X=0}^{L}\left[Y\left(d\phi/dX\right)^2 + \left(1 + b\xi\right)\phi^2\right]dX \tag{8}$$

The fin volume is obtained from the following expression:

$$U = \frac{V(h/k)^2}{2} = \int_{X=0}^{L} Y\,dX \tag{9}$$

The profile shape of a fin has been determined from the variational principle after satisfying the maximization of heat transfer rate Q for a design condition. In the present study, either the fin volume or both the fin volume and length are considered as a constraint condition. A functional F may be constructed from Eqs. (8) and (9) by employing Lagrange multiplier λ:

$$F = Q - \lambda U = \frac{1}{\phi_0}\int_{X=0}^{L}\left[Y(d\phi/dX)^2 + (1+b\xi)\phi^2 - \lambda\phi_0 Y\right]dX \tag{10}$$

The relation between the variation of F and that of Y is obtained from the above equation and for maximum value of F, δF is zero for any admissible variation of δY. Thus

$$\delta F = \frac{1}{\phi_0}\int_{X=0}^{L} Y^{-1}\left[Y(d\phi/dX)^2 - \lambda\phi_0 Y\right]\delta Y\,dX = 0 \tag{11}$$

From the above equation, the following optimality criteria are obtained:

$$Y(d\phi/dX)^2 - \lambda\phi_0 Y = 0 \tag{12}$$

From Eq. (12), it is obvious that the temperature gradient in the longitudinal fin for the optimum condition is a constant.

2.1.1 Optimum longitudinal fin for the volume constraint

Here the fin length L is not a constant and thus it can be taken as a variable. From Eq. (10), the variation of function F with L is as follows:

$$\delta F = \frac{1}{\phi_0}\left[Y(d\phi/dX)^2 + (1+b\xi)\phi^2 - \lambda\,\phi_0\,Y\right]\delta X\Big|_{X=0}^{L} = 0 \tag{13}$$

At $X = 0$, the above term vanishes as δX=0. At X=L, δX is not zero; therefore, at the tip, the following optimality conditions can be obtained:

$$Y(d\phi/dX)^2 + (1+b\xi)\phi^2 - \lambda\phi_0 Y = 0 \tag{14}$$

Combining Eqs. (4), (6), (12) and (14) , yields the tip condition ϕ = 0. The tip thickness of a fin may be determined from the tip condition and the optimality criterion and boundary condition (Eqs. (12) and (6b)). It can be seen that the tip thickness is zero. The tip temperature for fully wet surface θ_t = -θ_p, which is slightly less than the ambient value and this temperature is obvious as a function of psychometric properties of the surrounding air. From Eqs. (4) (6), (12) and tip condition, the temperature distribution and fin profile are written as follows:

$$\theta = 1 - \left(1 + \theta_p\right) X \big/ L \tag{15}$$

and

$$Y = \frac{(1 + b\xi)}{2}(L - X)^2 \tag{16}$$

The optimum fin length L_{opt} can be obtained from Eqs. (9) and (16). The maximum heat transfer rate through the fin can be written by the design variables as follows:

$$\begin{bmatrix} L_{opt} \\ Q_{opt} \end{bmatrix} = \begin{bmatrix} \left\{6\,U/(1 + b\xi)\right\}^{1/3} \\ \phi_0 \left\{3U(1 + b\xi)^2/4\right\}^{1/3} \end{bmatrix} \quad \begin{matrix} (17a) \\ (17b) \end{matrix}$$

2.1.2 Optimum longitudinal fin for both length and volume constraints

In fin design, some times the length of the fin is required to specify due to restricted space and ease of manufacturing. Under this design consideration, both length (fixed L) and volume may be adopted as a constraint. For obtaining the temperature distribution and fin profile, Eqs. (6), (9) and (12) can be combined:

$$\theta = 1 - \alpha X \tag{18}$$

and

$$Y = \frac{(1 + b\xi)}{2\alpha}\left[2\phi_0 (L - X) - \alpha \left(L^2 - X^2\right)\right] \tag{19}$$

where

$$\alpha = \frac{3\,L^2 (1 + b\xi)\phi_0}{6U + 2L^3 (1 + b\xi)} \tag{20}$$

Here, it may be noted that the optimum fin shape for dry surface fins can be determined by using the above formula.

2.2 Partially wet longitudinal fins

There are two regions dry and wet in partially wet fins shown in Fig. 1B. For partially wet longitudinal fins, the energy equations are in the followings:

$$\begin{bmatrix} \dfrac{d}{dx}\left(y\dfrac{dT}{dx}\right) \\ \dfrac{d}{dx}\left(y\dfrac{dT}{dx}\right) \end{bmatrix} = \begin{bmatrix} \dfrac{h}{k}(T - T_a) \\ \dfrac{h}{k}\left\{(T - T_a) + h_m(\omega - \omega_a)h_{fg}/h\right\} \end{bmatrix} \quad \begin{matrix} \text{for dry surface } T > T_d \quad (21a) \\ \text{for wet surface } T \le T_d \quad (21b) \end{matrix}$$

By using Eqs. (2) and (3), Eq. (21) is made in normalized form and it can be expressed as follows:

$$\begin{bmatrix} \dfrac{d}{dX}\left(Y\dfrac{d\theta}{dX}\right) \\ \dfrac{d}{dX}\left(Y\dfrac{d\phi}{dX}\right) \end{bmatrix} = \begin{bmatrix} \theta \\ (1+b\xi)\phi \end{bmatrix} \quad \begin{matrix} \text{for dry domain } \theta > \theta_d & (22a) \\ \text{for wet domain } \theta \le \theta_d & (22b) \end{matrix}$$

The heat transfer through the tip is negligibly small in comparision to that through the lateral surfaces and fin base temperature is taken as a constant. In addition, continuity of temperature and heat conduction satisfies at the section where dry and wet separates. Thus, for solving Eq. (22) the following boundary conditions are taken:

$$\text{at} \quad X = 0\,,\ \phi = \phi_0 \tag{23a}$$

$$\text{at} \quad X = L_0\,,\ \begin{cases} \theta = \theta_d & (23b) \\ d\theta/dX = d\phi/dX & (23c) \end{cases}$$

$$\text{at} \quad X = L\,,\ Y d\theta/dX = 0 \tag{23d}$$

Eq. (22) are multiplied by respective variables θ and ϕ, and the following relationships are obtained by integration and using boundary conditions:

$$-\left[Y\phi\, d\phi/dX\right]_{X=0} = -\left[Y\phi\, d\phi/dX\right]_{X=L_0} + \int_{X=0}^{L_0}\left[Y\left(d\phi/dX\right)^2 + (1+b\xi)\phi^2\right]dX \tag{24a}$$

and

$$-\left[Y\theta\, d\theta/dX\right]_{X=L_0} = \int_{X=L_0}^{L}\left[Y\left(d\theta/dX\right)^2 + \theta^2\right]dX \tag{24b}$$

Combining Eqs. (24a) and (24b), one can get

$$-\left[Y\varphi\, d\varphi/dX\right]_{X=0} = \frac{\varphi_d}{\theta_d}\int_{X=L_0}^{L}\left[Y\left(d\theta/dX\right)^2 + \theta^2\right]dX + \int_{X=0}^{L_0}\left[Y\left(d\varphi/dX\right)^2 + (1+b\xi)\varphi^2\right]dX \tag{25}$$

The heat transfer rate through the fins is calculated by applying the Fourier's law of heat conduction at the fin base and it can be written by using Eq. (25) as

$$\begin{aligned} Q = \frac{q}{2k(T_a - T_b)} = -\left(Y\frac{d\phi}{dX}\right)_{X=0} &= \frac{\phi_d}{\phi_0\,\theta_d}\int_{X=L_0}^{L}\left[Y\left(d\theta/dX\right)^2 + \theta^2\right]dX \\ &+ \frac{1}{\phi_0}\int_{X=0}^{L_0}\left[Y\left(d\phi/dX\right)^2 + (1+b\xi)\phi^2\right]dX \end{aligned} \tag{26}$$

The fin volume per unit width can be obtained from the following expressions:

$$U = \frac{V\left(h/k\right)^2}{2} = \int_{X=0}^{L_0} Y dX + \int_{X=L_0}^{L} Y dX \tag{27}$$

The optimum profile shape of a fin can be determined from the variational principle by constructing a functional F from Eqs. (26) and (27) using Lagrange multiplier λ.

$$\begin{aligned} F = Q - \lambda U = \frac{1}{\phi_0} \int_{X=0}^{L_0} \left[Y\left(d\phi/dX\right)^2 + \left(1+b\xi\right)\phi^2 - \lambda \phi_0 Y \right] dX \\ + \frac{\phi_d}{\phi_0 \theta_d} \int_{X=L_0}^{L} \left[Y\left(d\theta/dX\right)^2 + \theta^2 - \lambda \phi_0 \theta_d Y/\phi_d \right] dX \end{aligned} \tag{28}$$

For maximizing value of F, the following condition is obtained from Eq. (28).

$$\begin{aligned} \delta F = \frac{1}{\phi_0} \int_{X=0}^{L_0} Y^{-1} \left[Y\left(d\phi/dX\right)^2 - \lambda \phi_0 Y \right] \delta Y dX \\ + \frac{\phi_d}{\phi_0 \theta_d} \int_{X=L_0}^{L} Y^{-1} \left[Y\left(d\theta/dX\right)^2 - \lambda \phi_0 \theta_d Y/\phi_d \right] \delta Y dX = 0 \end{aligned} \tag{29}$$

From Eq. (29), the optimality criterion is derived as follows:

$$\begin{bmatrix} Y\left(d\theta/dX\right)^2 - \lambda \phi_0 \theta_d Y/\phi_d \\ Y\left(d\phi/dX\right)^2 - \lambda \phi_0 Y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} \theta > \theta_d & (30a) \\ \theta \le \theta_d & (30b) \end{matrix}$$

2.2.1 Optimum longitudinal fin for volume constraint

The variation of F with a function of L and L_0 yields the following expressions from Eq. (29):

$$\begin{bmatrix} \delta F \\ \delta F \end{bmatrix} = \begin{bmatrix} \frac{1}{\phi_0}\left\{ Y\left(d\phi/dX\right)^2 + \left(1+b\xi\right)\phi^2 - \lambda \phi_0 Y \right\} \delta X \Big|_{X=0}^{X=L_0} \\ \frac{\varphi_d}{\theta_d \phi_0}\left\{ Y\left(d\theta/dX\right)^2 + \theta^2 - \lambda Y \phi_0 \theta_d / \phi_d \right\} \delta X \Big|_{X=L_0}^{L} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} \theta \le \theta_d & (31a) \\ \theta > \theta_d & (31b) \end{matrix}$$

At X = 0, the above term should vanish as δX=0. At X=L_0 and X=L, δX is nonzero; thus, the location for both dry and wet surfaces coexist and the fin tip satisfies are the optimality conditions:

$$\begin{bmatrix} Y\left(d\theta/dX\right)^2 + \theta^2 - \lambda \theta_d \phi_0 Y/\phi_d \\ Y\left(d\phi/dX\right)^2 + \left(1+b\xi\right)\phi^2 - \lambda \phi_0 Y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{at } X = L_0 \quad \begin{matrix} (32a) \\ (32b) \end{matrix}$$

and

$$Y\left(d\theta/dX\right)^2 + \theta^2 - \lambda \theta_d \phi_0 Y/\phi_d = 0 \quad \text{at } X = L \tag{33}$$

Combining Eqs. (4), (6b), (30), (32) and (33), the tip temperature vanishes. Using optimality criteria, the temperature distribution and fin profile can be expressed as

$$\theta = \begin{cases} 1-(1-\theta_d)X/L_0 & 0 \le X \le L_0 \quad (34a) \\ \theta_d(L-X)/(L-L_0) & L_0 \le X \le L \quad (34b) \end{cases}$$

and

$$Y = \frac{1}{2}\left[(L-L_0)^2 + \frac{(1+b\xi)}{(1-\theta_d)}\left\{2L_0\phi_0(L_0-X)-(1-\theta_d)\left(L_0^2-X^2\right)\right\}\right] \quad \text{for } 0 \le X \le L_0 \tag{35a}$$

$$Y = \frac{1}{2}(L-X)^2 \quad \text{for } L_0 \le X \le L \tag{35b}$$

The length of the wet region L_0 can be determined by using an energy balance at that length where dry and wet sections live together.

$$L_0 = L(1-\theta_d) \tag{36}$$

Here L is not a constraint. L can be obtained from Eqs. (27), (35) and (36). The optimum length and the maximum heat transfer rate through a fin can be written as

$$L_{opt} = \frac{(6U)^{1/3}}{\left[\theta_d^2(3-2\theta_d)+(1+b\xi)(1-\theta_d)^2(3\phi_0+2\theta_d-2)\right]^{1/3}} \tag{37a}$$

and

$$Q_{opt} = \frac{\left[\theta_d^2+(1+b\xi)(2\phi_0+\theta_d-1)(1-\theta_d)\right]^2(6U)^{1/3}}{\left[\theta_d^2(3-2\theta_d)+(1+b\xi)(1-\theta_d)^2(3\phi_0+2\theta_d-2)\right]^{1/3}} \tag{37b}$$

2.2.2 Optimum longitudinal fin for both length and volume constraints

The temperature distribution and fin profile can be determined by using Eqs. (4), (6) and (13):

$$\begin{bmatrix} \theta \\ \theta \end{bmatrix} = \begin{bmatrix} 1-\alpha X \\ \theta_d - \alpha(X-L_0) \end{bmatrix} \quad \begin{matrix} 0 \le X \le L_0 \quad (38a) \\ L_0 \le X \le L \quad (38b) \end{matrix}$$

and

$$Y = \frac{1}{2\alpha}\left[2(\theta_d+\alpha L_0)(L-L_0)-\alpha\left(L^2-L_0^2\right)+(1+b\xi)\left\{2\phi_0(L_0-X)-\alpha\left(L_0^2-X^2\right)\right\}\right. \quad (0 \le X \le L_0) \tag{39a}$$

$$Y = \frac{1}{2\alpha}\left[2(\theta_d+\alpha L_0)(L-X)-\alpha\left(L^2-X^2\right)\right] \quad (L_0 \le X \le L) \tag{39b}$$

where

$$\alpha = \frac{3\theta_d\left(L^2 - L_0^2\right) + 3\phi_0\left(1 + b\xi\right)L_0^2}{6U + 2L_0^3\left(1 + b\xi\right) + 2L^3 - L_0\left(3L^2 - L_0^2\right) + 4\left(L^3 - L_0^3\right)} \tag{40}$$

and

$$L_0 = \left(1 - \theta_d\right)/\alpha \tag{41}$$

2.3 Fully wet annular fins

Figure 2a is drawn for a schematic representation of an optimum annular fin under condensation of saturated vapor on its surfaces. The energy equation for one-dimensional temperature distribution in fully wet annular fins can be written under steady state condition as

$$\frac{d}{dx}\left[y\left(r_i + x\right)\frac{dT}{dx}\right] = \frac{h}{k}\left(r_i + x\right)\left[\left(T - T_a\right) + h_m\left(\omega - \omega_a\right)h_{fg}/h\right] \tag{42}$$

Eq. (42) is made in dimensionless form by using Eqs. (2) and (3) as

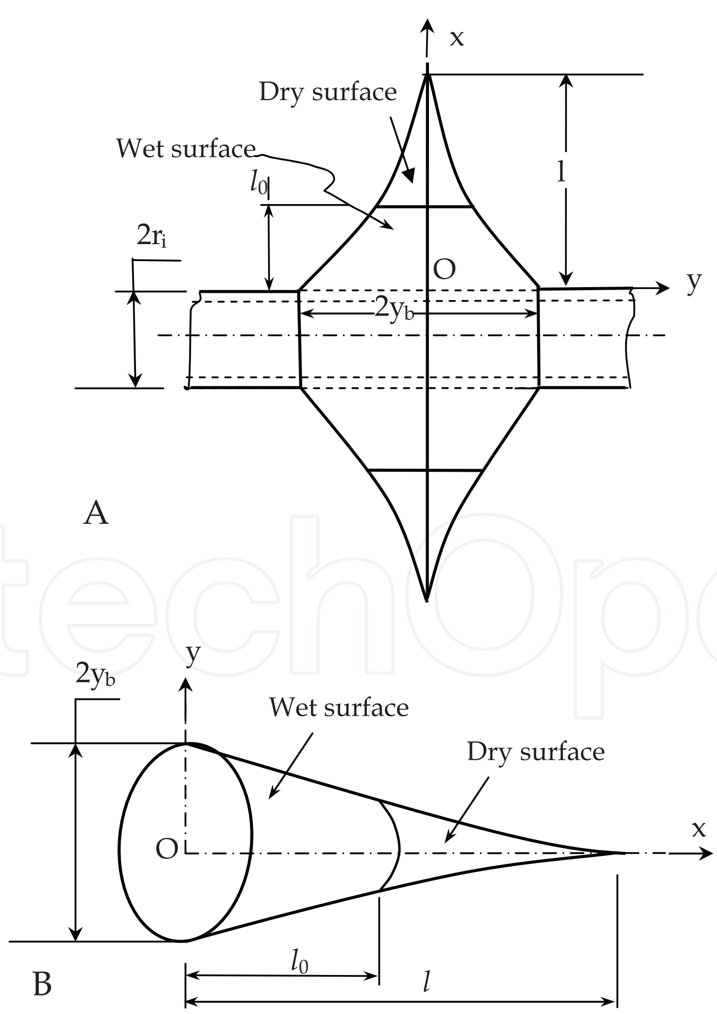


Fig. 2. Typical configuration of wet fins: A. Annular fin; and B. Spine

$$\frac{d}{dX}\left[Y\left(R_i+X\right)\frac{d\phi}{dX}\right]=\left(R_i+X\right)\left(1+b\xi\right)\phi; \quad R_i=hr_i/k \tag{43}$$

The boundary conditions for annular fins for temperature distribution are considered same as taken longitudinal fins described in Eq. (6). The actual nondimensional heat transfer rate is calculated from the following formula:

$$Q=\frac{q\left(h/\pi\right)}{4k^2\left(T_a-T_b\right)}=-\left[Y\left(R_i+X\right)\frac{d\phi}{dX}\right]_{X=0}=\frac{1}{\phi_0}\int_{X=0}^{L}\left(R_i+X\right)\left[Y\left(d\phi/dX\right)^2+\left(1+b\xi\right)\phi^2\right]dX \tag{44}$$

The fin volume in dimensionless form can be written as

$$U=\frac{V\left(h/k\right)^3}{4\pi^2}=\int_{X=0}^{L}\left(R_i+X\right)Y dX \tag{45}$$

For the application of variational principle, a function F can be constructed by using Lagrange multiplier λin the followings :

$$F=Q-\lambda U=\frac{1}{\phi_0}\int_{X=0}^{L}\left(R_i+X\right)\left[Y\left(d\phi/dX\right)^2+\left(1+b\xi\right)\phi^2-\lambda\phi_0 Y\right]dX \tag{46}$$

For maximizing value of F, Eq. (46) is differentiated with respect to Y and finally it becomes to zero.

$$\delta F=\frac{1}{\phi_0}\int_{X=0}^{L}Y^{-1}\left(R_i+X\right)\left[Y\left(d\phi/dX\right)^2-\lambda\phi_0 Y\right]\delta Y\,dX=0 \tag{47}$$

From Eq. (47), the following optimality condition for fully wet annular fins is obtained.

$$Y\left(d\phi/dX\right)^2-\lambda\phi_0 Y=0 \tag{48}$$

From the above optimality condition, it is to mention that the temperature gradient for fully wet anuular fins is a constant and it does not depend upon the wetness condition of the fin.

2.3.1 Optimum annular fin for volume constraint

For volume constraint, fin length is variable. The variation of function F with length L is determined from this expression.

$$\delta F=\frac{\left(R_i+X\right)}{\phi_0}\left[Y\left(d\phi/dX\right)^2+\left(1+b\xi\right)\phi^2-\lambda\,\phi_0\,Y\right]\delta X\Big|_{X=0}^{L}=0 \tag{49}$$

At $X = L$, the following optimality conditions are obtained:

$$Y\left(d\phi/dX\right)^2+\left(1+b\xi\right)\phi^2-\lambda\phi_0 Y=0 \tag{50}$$

Using Eqs. (6), (43), (48) and (50), the condition for temperature at the tip is $\phi = 0$. The thickness of fin profile at the tip is also zero. The temperature distribution and fin profile for the optimum annular fin for fully wet surface conditions is obtained from Eqs. (6), (43), (48) and optimum tip conditions as follows:

$$\theta = 1 - \left(1 + \theta_p\right) X / L \tag{51}$$

and

$$Y = \frac{(1 + b\xi)}{6(R_i + X)} \left[\left(L^3 - 3LX^2 + 2X^3 \right) + 3R_i \left(L - X\right)^2 \right] \tag{52}$$

The optimum length and the maximum heat transfer rate through the fin can be determined as a function of design variables in the followings:

$$\begin{bmatrix} L_{opt} \\ Q_{opt} \end{bmatrix} = \begin{bmatrix} L_{opt}^4 + 2R_i L_{opt}^3 - 12U/(1 + b\xi) = 0 \\ \phi_0 L_{opt}\left(3R_i - L_{opt}\right)(1 + b\xi)/6 \end{bmatrix} \begin{matrix} (53) \\ (54) \end{matrix}$$

The optimum length can be possible to calculate by a numerical technique. The Newton-Raphson method can be applied for the solution of Eq. (53) to determine the optimum fin length.

2.3.2 Optimum annular fin for both length and volume constraints

The temperature and fin profile for an optimum annular fin under length and volume constraints is determined from Eqs. (6), (43), (45) and (48) and they can be expressed as

$$\theta = 1 - \alpha X \tag{55}$$

and

$$Y = \frac{(1 + b\xi)}{6\alpha(R_i + X)} \left[6R_i \phi_0 (L - X) + 3\left(\phi_0 - \alpha R_i\right)\left(L^2 - X^2\right) - 2\alpha\left(L^3 - X^3\right) \right] \tag{56}$$

where

$$\alpha = \frac{2\left(3R_i + 2L\right) L^2 (1 + b\xi)\phi_0}{12U + L^3(1 + b\xi)\left(4R_i + 3L\right)} \tag{57}$$

2.4 Partially wet annular fins

The energy equations for dry and wet regions in partially wet fins can be written separately in the followings:

$$\begin{bmatrix} \frac{d}{dx}\left\{ y\left(r_i + x\right)\frac{dT}{dx} \right\} \\ \frac{d}{dx}\left\{ y\left(r_i + x\right)\frac{dT}{dx} \right\} \end{bmatrix} = \begin{bmatrix} \frac{h}{k}\left(r_i + x\right)\left(T - T_a\right) \\ \frac{h}{k}\left(r_i + x\right)\left\{ \left(T - T_a\right) + h_m\left(\omega - \omega_a\right)h_{fg}/h \right\} \end{bmatrix} \begin{matrix} \text{dry domain } T > T_d \quad (58a) \\ \text{wet domain } T \le T_d \quad (58b) \end{matrix}$$

Eq. (58) can be expressed in normalized form as

$$\begin{bmatrix} \dfrac{d}{dX}\left\{ Y\left(R_i + X\right)\dfrac{d\theta}{dX} \right\} \\ \dfrac{d}{dX}\left\{ Y\left(R_i + X\right)\dfrac{d\phi}{dX} \right\} \end{bmatrix} = \begin{bmatrix} \left(R_i + X\right)\theta \\ \left(R_i + X\right)\left(1 + b\xi\right)\phi \end{bmatrix} \quad \begin{matrix} \theta > \theta_d & (59a) \\ \theta \le \theta_d & (59b) \end{matrix}$$

For solving Eq. (59), four boundary conditions are required and these are already written in Eq. (23). Heat transfer equation is obtained from Eq. (59) and boundary conditions in the followings:

$$\begin{aligned} Q = \frac{q\left(h/\pi\right)}{4k^2\left(T_a - T_b\right)} &= -\left[Y\left(R_i + X\right)\frac{d\phi}{dX} \right]_{X=0} = \frac{\phi_d}{\phi_0\,\theta_d}\int_{X=L_0}^{L} \left(R_i + X\right)\left[Y\left(d\theta/dX\right)^2 + \theta^2 \right] dX \\ &+ \frac{1}{\phi_0}\int_{X=0}^{L_0} \left(R_i + X\right)\left[Y\left(d\phi/dX\right)^2 + \left(1 + b\xi\right)\phi^2 \right] dX \end{aligned} \quad (60)$$

The dimensionless fin volume for the partially wet fin can be formulated in the following:

$$U = \frac{V\left(h/k\right)^3}{4\pi} = \int_{X=0}^{L_0} \left(R_i + X\right)Y dX + \int_{X=L_0}^{L} \left(R_i + X\right)Y dX \quad (61)$$

To apply variational principle, a functional F is constructed by using heat transfer rate and fin volume expressions.

$$\begin{aligned} F = Q - \lambda U &= \frac{1}{\phi_0}\int_{X=0}^{L_0} \left(R_i + X\right)\left[Y\left(d\phi/dX\right)^2 + \left(1 + b\xi\right)\phi^2 - \lambda\,\phi_0 Y \right] dX \\ &+ \frac{\phi_d}{\phi_0\theta_d}\int_{X=L_0}^{L} \left(R_i + X\right)\left[Y\left(d\theta/dX\right)^2 + \theta^2 - \lambda\,\phi_0\theta_d Y/\phi_d \right] dX \end{aligned} \quad (62)$$

The optimality criterion can be derived by differentiating functional F with respect to Y and then equating to zero.

$$\begin{aligned} \delta F &= \frac{1}{\phi_0}\int_{X=0}^{L_0} Y^{-1}\left(R_i + X\right)\left[Y\left(d\phi/dX\right)^2 - \lambda\,\phi_0 Y \right] \delta Y\, dX \\ &+ \frac{\phi_d}{\phi_0\theta_d}\int_{X=L_0}^{L} Y^{-1}\left(R_i + X\right)\left[Y\left(d\theta/dX\right)^2 - \lambda\,\varphi_0\theta_d Y/\phi_d \right] \delta Y\, dX = 0 \end{aligned} \quad (63)$$

Therefore from Eq. (63), the following optimality criterion is found:

$$\begin{bmatrix} \left(d\theta/dX\right)^2 - \lambda\,\phi_0\,\theta_d/\phi_d \\ \left(d\phi/dX\right)^2 - \lambda\,\phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} \theta > \theta_d & (64a) \\ \theta \le \theta_d & (64b) \end{matrix}$$

2.4.1 Optimum annular fin for volume constraint

The following expression is obtained for variation of F with length L from Eq. (62)

$$\begin{bmatrix}\delta F\\ \\ \delta F\end{bmatrix}=\begin{bmatrix}\dfrac{(R_i+X)}{\phi_0}\left\{Y(d\phi/dX)^2+(1+b\xi)\phi^2-\lambda\,\phi_0\,Y\right\}\delta X\Big|_{X=0}^{X=L_0}\\ \dfrac{(R_i+X)\phi_d}{\theta_d\phi_0}\left\{Y(d\theta/dX)^2+\theta^2-\lambda Y\phi_0\theta_d/\phi_d\right\}\delta X\Big|_{X=L_0}^{L}\end{bmatrix}=\begin{bmatrix}0\\ \\ 0\end{bmatrix}\quad\begin{matrix}\theta\le\theta_d\\ \\ \theta>\theta_d\end{matrix}\qquad\begin{matrix}(65a)\\ \\ (65b)\end{matrix}$$

From Eq. (65), the following conditions are determined:

$$\begin{bmatrix}Y(d\theta/dX)^2+\theta^2-\lambda\,\theta_d\phi_0 Y/\phi_d\\ Y(d\phi/dX)^2+(1+b\xi)\phi^2-\lambda\,\phi_0 Y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\quad\text{at }\ X=L_0\qquad\begin{matrix}(66a)\\(66b)\end{matrix}$$

and

$$Y(d\theta/dX)^2+\theta^2-\lambda\,\theta_d\varphi_0 Y/\phi_d=0\quad\text{at }\ X=L\tag{67}$$

From the above analysis, one can determine the dimensionless tip temperature and tip thickness, both are zero for the optimum design condition. The fin temperature and fin profile are obtained as follows:

$$\theta=\begin{cases}1-(1-\theta_d)X/L_0 & 0\le X\le L_0\\ \theta_d(L-X)/(L-L_0) & L_0\le X\le L\end{cases}\qquad\begin{matrix}(68a)\\(68b)\end{matrix}$$

and

$$\begin{aligned}Y=&\frac{1}{6(R_i+X)}\Big[6LR_i\,(L-L_0)+3(L-R_i)\left(L^2-L_0^2\right)-2\left(L^3-L_0^3\right)\\&+\frac{(1+b\xi)}{(1-\theta_d)}\left\{6R_i\,L_0\varphi_0\,(L_0-X)+3\left(\varphi_0L_0-(1-\theta_d)R_i\right)\left(L_0^2-X^2\right)-2(1-\theta_d)\left(L_0^3-X^3\right)\right\}\Big]\\&\text{for }\ (0\le X\le L_0)\end{aligned}\tag{69a}$$

$$Y=\frac{1}{6(R_i+X)}\left[6LR_i\,(L-X)+3(L-R_i)\left(L^2-X^2\right)-2\left(L^3-X^3\right)\right]\quad\text{for }L_0\le X\le L\tag{69b}$$

The length of the optimum annular fin can be determined from the following equation given below:

$$\begin{aligned}&\left[2\theta_d^2(1-\theta_d)+(1+b\xi)\left\{4\varphi_0-4\theta_d\varphi_0-3\left(1-\theta_d\right)^2\right\}(1-\theta_d)^2+\theta_d^3(2-\theta_d)\right]L^4\\&+\left[6\theta_d^2(1-\theta_d)+2(1+b\xi)(3\varphi_0+2\theta_d-2)(1-\theta_d)^2+2\theta_d^3\right]R_iL^3-12U=0\end{aligned}\tag{70}$$

Eq. (70) can be solved numerically. The Newton-Raphson iterative method can be employed to determine the optimum length of the fin after satisfying the necessary convergence criterion. After estimating L_{opt} value, one can calculate the maximum or optimum actual heat transfer rate which can be determined from the expression give below:

$$Q_{opt} = \frac{L_{opt}}{6}\Big[6R_i\theta_d + 3\left(L_{opt} - R_i\right)\left\{1-(1-\theta_d)^2\right\} - 2L_{opt}\left\{1-(1-\theta_d)^3\right\} + (1-\theta_d)(1+b\xi)L_{opt}^2\left\{6R_i\phi_0 + 3\left(\phi_0 L_{opt} - R_i\right)(1-\theta_d) - 2L_{opt}(1-\theta_d)^2\right\}\Big] \tag{71}$$

2.4.2 Optimum annular fin for both length and volume constraints

The temperature distribution and fin profile for the annular fin can be determined by using Eqs. (6), (59), (61) and (64) as

$$\begin{bmatrix}\theta\\ \theta\end{bmatrix} = \begin{bmatrix}1-\alpha X\\ \theta_d - \alpha(X-L_0)\end{bmatrix} \quad \begin{matrix}0\le X\le L_0 & (72a)\\ L_0\le X\le L & (72b)\end{matrix}$$

and

$$Y = \frac{1}{6\alpha(R_i+X)}\Big[6R_i(\theta_d+\alpha L_0)(L-L_0) + 3\left(\theta_d - \alpha R_i + \alpha L_0\right)\left(L^2-L_0^2\right) - 2\alpha\left(L^3-L_0^3\right) + (1+b\xi)\left\{6\phi_0 R_i(L_0-X) + 3\left(\phi_0 - \alpha R_i\right)\left(L_0^2-X^2\right) - 2\alpha\left(L_0^3-X^3\right)\right\}\Big] \quad (0\le X\le L_0) \tag{73a}$$

$$Y = \frac{1}{6\alpha(R_i+X)}\Big[6\left(R_i\theta_d + \alpha R_i L_0\right)(L-X) + 3\left(\alpha L_0 - \alpha R_i + \theta_d\right)\left(L^2-X^2\right) - 2\alpha\left(L^3-X^3\right)\Big] \quad (L_0\le X\le L) \tag{73b}$$

where

$$\alpha = \frac{6R_i\theta_d\left(L^2-L_0^2\right) + 2\phi_0(1+b\xi)(3R_i+2L_0)L_0^2 + 4\theta_d\left(L^3-L_0^3\right)}{12U + L_0^3(1+b\xi)(4R_i+3L_0) - 2R_iL_0\left(3L^2-L_0^2\right) + L^3(4R_i+3L) - L_0\left(4L^3-L_0^3\right)} \tag{74a}$$

and

$$L_0 = (1-\theta_d)/\alpha \tag{74b}$$

Combining Eqs. (74a) and (74b), the following transcendental equation is obtained:

$$\left[(1+b\xi)(3-3\theta_d-4\phi_0)+3\theta_d+1\right]L_0^4 + 2R_i\left[(1-\theta_d)(3+2b\xi) - 3\phi_0(1+b\xi)+3\theta_d\right]L_0^3 - 2L^2L_0 + (1-\theta_d)\left(12U+4R_iL^3+3L^4\right) = 0 \tag{75}$$

In order to determine the wet length in the fin L_0, Eq. (75) can be solved by using Newton-Raphson iterative technique.

2.5 Fully wet pin fins

A schematic diagram of a pin fin is shown in Fig. 2B. The energy equation for pin fins subject to condensation of vapor under fully wet condition is written below:

$$\frac{d}{dx}\left(y^2\frac{dT}{dx}\right)=\frac{2h}{k}y\left[\left(T-T_a\right)+h_m\left(\omega-\omega_a\right)h_{fg}/h\right] \tag{76}$$

Eq. (76) is expressed in nondimensional form as

$$\frac{d}{dX}\left(Y^2\frac{d\phi}{dX}\right)=2Y\left(1+b\xi\right)\phi \tag{77}$$

For the solution of Eq. (77), the boundary conditions expressed in Eq. (7) are taken. The actual heat transfer rate is calculated from the following expression:

$$Q=\frac{2q\left(h/\pi\right)}{2k^2\left(T_a-T_b\right)}=-\left[Y^2\frac{d\phi}{dX}\right]_{X=0}=\frac{1}{\phi_0}\int_{X=0}^{L}Y\left[Y\left(d\phi/dX\right)^2+2\left(1+b\xi\right)\phi^2\right]dX \tag{78}$$

The fin volume of a pin fin is written in dimensionless form as

$$U=\frac{V\left(h/k\right)^3}{\pi}=\int_{X=0}^{L}Y^2dX \tag{79}$$

For determination of the optimum shape, a functional F is defined as

$$F=Q-\lambda U=\frac{1}{\phi_0}\int_{X=0}^{L}Y\left[Y\left(d\phi/dX\right)^2+2\left(1+b\xi\right)\phi^2-\lambda\phi_0 Y\right]dX \tag{80}$$

The above equation gives a relationship between F and Y. The optimum condition can be obtained by differentiating functional F with respect to Y.

$$\delta F=\frac{1}{\phi_0}\int_{X=0}^{L}2\left[Y\left(d\phi/dX\right)^2+\left(1+b\xi\right)\phi^2-\lambda\phi_0 Y\right]\delta Y\,dX=0 \tag{81}$$

The following expression is obtained from Eq. (81).

$$Y\left(d\phi/dX\right)^2+\left(1+b\xi\right)\phi^2-\lambda\phi_0 Y=0 \tag{82}$$

Multiplying on both sides in Eq. (77) by $d\phi/dX$ yields the following expression after some manipulations:

$$d\left[Y^2\left(d\phi/dX\right)^2\right]/dX+\left(d\phi/dX\right)^2 d\left(Y^2\right)/dX=2\left(1+b\xi\right)Y\left(d\phi^2\right)/dX \tag{83}$$

Eliminating ϕ^2 from Eq. (82) by using Eq. (83) and then integrating, the following expression is obtained.

$$3Y^2\left(d\phi/dX\right)^2 - \lambda\phi_0 Y^2 = C \tag{84}$$

where, C is an integration constant determined by using boundary conditions.

2.5.1 Optimum fully wet pin fin for volume constraint

The variation of F with a function of L yields from Eq. (80) as

$$\delta F = \frac{Y}{\phi_0}\left[Y\left(d\phi/dX\right)^2 + 2\left(1+b\xi\right)\phi^2 - \lambda\,\phi_0\,Y\right]\delta X\Big|_{X=0}^{L} = 0 \tag{85}$$

As δX is nonzero at $X = L$, the following tip condition can be achieved :

$$Y\left(d\phi/dX\right)^2 + 2\left(1+b\xi\right)\phi^2 - \lambda\phi_0 Y = 0 \tag{86}$$

From Eqs. (6), (77), (82) and (86) the dimensionless temperature ϕ at the tip vanishes and it can be indicated that the same condition is obtained in the case of longitudinal and annular fins with fully wet condition. Using Eqs. (6), (77), (82) and (86), temperature distribution and fin profile for the optimum fin can be written as

$$\theta = 1 - \left(1+\theta_p\right)X/L \tag{87a}$$

and

$$Y = \frac{\left(1+b\xi\right)\left(L-X\right)^2}{2} \tag{87b}$$

The optimum fin length and heat transfer rate are determined as follows:

$$\begin{bmatrix} L_{opt} \\ Q_{\max} \end{bmatrix} = \begin{bmatrix} \left\{20\,U/\left(1+b\xi\right)^2\right\}^{1/5} \\ \phi_0\left\{125U^3\left(1+b\xi\right)^4/8\right\}^{1/5} \end{bmatrix} \quad \begin{matrix} (88a) \\ (88b) \end{matrix}$$

2.5.2 Optimum fully wet pin fin for both length and volume constraints

The temperature distribution and fin profile can be found from Eqs. (6), (77) and (84) as

$$\phi = \phi_0\sqrt{Y_b\left(Y_t^2 Y^{-1} + 2Y\right)/\left(Y_t^2 + 2Y_b^2\right)}, \quad Y_t \le Y \le Y_b \tag{89}$$

and

$$\int_{Y_t}^{Y} \frac{\left(2Y^2 - Y_t^2\right)dY}{\sqrt{\left(Y^2 - Y_t^2\right)\left(Y_t^2 + 2Y^2\right)Y}} = 2\sqrt{\left(1+b\xi\right)}\,\left(L-X\right) \tag{90}$$

In order to determine the temperature profile and fin profile from the above two equations, the unknown variables Y_t and Y_b are required to calculate apriori. These can be determined by constructing two constraint equations as follows:

$$U=\int_{Y=Y_t}^{Y_b}\frac{\left(2Y^2-Y_t^2\right)Y^2dY}{2\sqrt{(1+b\xi)}\sqrt{\left(Y^2-Y_t^2\right)\left(Y_t^2+2Y^2\right)Y}} \tag{91a}$$

and

$$\int_{Y=Y_t}^{Y_b}\frac{\left(2Y^2-Y_t^2\right)dY}{\sqrt{\left(Y^2-Y_t^2\right)\left(Y_t^2+2Y^2\right)Y}}=2\sqrt{(1+b\xi)}\,L \tag{91b}$$

A simultaneous solution of Eqs. (91a) and (91b) is provided to get the fin thickness at the base and tip. The above all integration can be performed by Simson's 1/3 rule.

2.6 Partially wet pin fins

The energy equations for partially wet pin fins are written as

$$\begin{bmatrix}\dfrac{d}{dx}\left(y^2\dfrac{dT}{dx}\right)\\ \dfrac{d}{dx}\left(y^2\dfrac{dT}{dx}\right)\end{bmatrix}=\begin{bmatrix}\dfrac{2h}{k}y\left(T-T_a\right)\\ \dfrac{2h}{k}y\left\{\left(T-T_a\right)+h_m\left(\omega-\omega_a\right)h_{fg}/h\right\}\end{bmatrix}\quad\begin{matrix}\text{for dry domain } T>T_d & (92a)\\ \text{for wet domain } T\le T_d & (92b)\end{matrix}$$

Eq. (92) can be expressed in dimensionless form as

$$\begin{bmatrix}d\left(Y^2\,d\theta/dX\right)/dX\\ d\left(Y^2\,d\varphi/dX\right)/dX\end{bmatrix}=\begin{bmatrix}2Y\theta\\ 2Y(1+b\xi)\phi\end{bmatrix}\quad\begin{matrix}\text{dry domain } \theta>\theta_d & (93a)\\ \text{wet domain } \theta\le\theta_d & (93b)\end{matrix}$$

Boundary conditions are reguired to solve Eq. (93) which can be taken as longitudinal fins expressed in Eq. (23). The actual heat transfer rate is calculated from the following equation:

$$\begin{aligned}Q=\frac{q\,(h/\pi)}{k^2\left(T_a-T_b\right)}=-\left[Y^2\frac{d\phi}{dX}\right]_{X=0}=\frac{\phi_d}{\phi_0\,\theta_d}\int_{X=L_0}^{L}Y\left[Y\left(d\theta/dX\right)^2+2\theta^2\right]dX\\ +\frac{1}{\phi_0}\int_{X=0}^{L_0}Y\left[Y\left(d\phi/dX\right)^2+2\,(1+b\xi)\phi^2\right]dX\end{aligned} \tag{94}$$

The fin volume for a partially wet pin fin is written in an integral form as

$$U=\frac{V\left(h/k\right)^3}{\pi}=\int_{X=0}^{L_0}Y^2dX+\int_{X=L_0}^{L}Y^2dX \tag{95}$$

For the application of variational method, a functional F is constructed from heat transfer rate and fin volume expressions:

$$F = Q - \lambda U = \frac{1}{\phi_0}\int_{X=0}^{L_0} Y\left[Y\left(d\phi/dX\right)^2 + 2\left(1+b\xi\right)\phi^2 - \lambda\phi_0 Y\right]dX$$
$$+\frac{\phi_d}{\phi_0\theta_d}\int_{X=L_0}^{L} Y\left[Y\left(d\theta/dX\right)^2 + 2\theta^2 - \lambda\phi_0\theta_d Y/\phi_d\right]dX \tag{96}$$

For maximizing value of F condition, the following expression is obtained:

$$\delta F = \frac{1}{\phi_0}\int_{X=0}^{L_0}\left[Y\left(d\phi/dX\right)^2 + \left(1+b\xi\right)\phi^2 - \lambda\phi_0 Y\right]\delta Y\, dX$$
$$+\frac{\phi_d}{\phi_0\theta_d}\int_{X=L_0}^{L}\left[Y\left(d\theta/dX\right)^2 + \theta^2 - \lambda\phi_0\theta_d Y/\phi_d\right]\delta Y\, dX = 0 \tag{97}$$

Therefore, from Eq. (97), the following optimality criterion is derived:

$$\begin{bmatrix} Y\left(d\theta/dX\right)^2 + \theta^2 - \lambda\phi_0\theta_d Y/\phi_d \\ Y\left(d\phi/dX\right)^2 + \left(1+b\xi\right)\phi^2 - \lambda\phi_0 Y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} \theta > \theta_d & (98a) \\ \theta \le \theta_d & (98b) \end{matrix}$$

The following expressions is obtained from Eq. (93) after multiplying on both sides of Eqs. (93a) and (93b) by $d\theta/dX$ and $d\phi/dX$, respectively.

$$\begin{bmatrix} d\left\{Y^2\left(d\theta/dX\right)^2\right\}/dX + \left(d\theta/dX\right)^2 d\left(Y^2\right)/dX \\ d\left\{Y^2\left(d\phi/dX\right)^2\right\}/dX + \left(d\phi/dX\right)^2 d\left(Y^2\right)/dX \end{bmatrix} = \begin{bmatrix} 2\,Y d\left(\theta^2\right)/dX \\ 2\left(1+b\xi\right)Y d\left(\phi^2\right)/dX \end{bmatrix} \quad \begin{matrix} \theta > \theta_d & (99a) \\ \theta \le \theta_d & (99b) \end{matrix}$$

Eleminating θ^2 and ϕ^2 from Eq. (99) with the help of Eq. (98), the following equations are obtained:

$$\begin{bmatrix} 3Y^2\left(d\theta/dX\right)^2 - \lambda\phi_0\theta_d Y^2/\phi_d \\ 3Y^2\left(d\phi/dX\right)^2 - \lambda\phi_0 Y^2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad \begin{matrix} \theta > \theta_d & (100a) \\ \theta \le \theta_d & (100b) \end{matrix}$$

The integration constants C_1 and C_2 are determined from the boundary conditions.

2.6.1 Optimum partially wet pin fin for volume constraint only

The variation of functional F with length parameter L for partially wet pin fins is given below:

$$\begin{bmatrix} \delta F \\ \delta F \end{bmatrix} = \begin{bmatrix} \left.\frac{Y}{\phi_0}\left\{Y\left(d\phi/dX\right)^2 + 2\left(1+b\xi\right)\phi^2 - \lambda\phi_0 Y\right\}\delta X\right|_{X=0}^{X=L_0} \\ \left.\frac{Y\phi_d}{\theta_d\phi_0}\left\{Y\left(d\theta/dX\right)^2 + 2\theta^2 - \lambda Y\varphi_0\theta_d/\phi_d\right\}\delta X\right|_{X=L_0}^{L} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} \theta \le \theta_d & (101a) \\ \theta > \theta_d & (101b) \end{matrix}$$

For X=0, the above terms vanish as δX=0. For X=L_0 and X=L, δX is not zero; therefore, the location where dry and wet surfaces present, and the tip, the following optimality conditions are achieved:

$$\begin{bmatrix} Y\left(d\theta/dX\right)^2 + 2\theta^2 - \lambda\,\theta_d\phi_0 Y/\phi_d \\ Y\left(d\phi/dX\right)^2 + 2\left(1+b\xi\right)\phi^2 - \lambda\,\phi_0 Y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{at } X = L_0 \tag{102a, 102b}$$

$$Y\left(d\theta/dX\right)^2 + 2\theta^2 - \lambda\,\theta_d\phi_0 Y/\phi_d = 0 \quad \text{at } X = L \tag{102c}$$

The temperature distribution and fin profile for an optimum pin fin for volume constraint only are same with that of the longitudinal fin. The fin length and heat transfer rate for the optimum profile fin is obtained from the following equations :

$$L_{opt} = \frac{\left(60U\right)^{1/5}}{\left[3\theta_d^4\left(5-2\theta_d\right) + Z_1\left\{Z_2 + 10\theta_d^2 + 1\right\}\left(3\phi_0 + 2\theta_d - 2\right)\right]^{1/5}} \tag{103a}$$

$$Q_{opt} = \frac{\left[\theta_d^2 + \left(1+b\xi\right)\left(2\varphi_0 + \theta_d - 1\right)\left(1-\theta_d\right)\right]^2\left(60U\right)^{3/5}}{\left[3\theta_d^4\left(5-2\theta_d\right) + Z_1\left\{Z_2 + 10\theta_d^2 + 1\right\}\left(3\phi_0 + 2\theta_d - 2\right)\right]^{3/5}} \tag{103b}$$

where

$$Z_1 = \left(1+b\xi\right)\left(1-\theta_d\right)^2 \tag{104a}$$

$$Z_2 = \left(1-\theta_d\right)\left(1+b\xi\right)\left\{5\left(2\phi_0 + \theta_d - 1\right)\left(5 - 6\phi_0 - 2\theta_d\right) + 5\phi_0\left(4\phi_0 + 3\theta_d - 3\right) + 3\left(1-\theta_d\right)^2\right\} \tag{104b}$$

2.6.2 Optimum partially wet pin fin for both length and volume constraints

The temperature distribution and fin profile can be found from Eqs. (6), (93), (95) and (100) for the partially wet conditions as follows:

$$\phi = \phi_0\sqrt{\frac{Y_b}{Y}\left[\frac{2\phi_d Y^2 + \left(\phi_d - \theta_d\right)Y_0^2 + \theta_d Y_t^2}{2\phi_d Y_b^2 + \left(\phi_d - \theta_d\right)Y_0^2 + \theta_d Y_t^2}\right]}, \quad Y_0 \le Y \le Y_b \tag{105a}$$

$$\theta = \theta_d\sqrt{Y_0\left(Y_t^2 Y^{-1} + 2Y\right)\Big/\left(Y_t^2 + 2Y_0^2\right)}, \quad Y_0 \le Y \le Y_t \tag{105b}$$

and

$$\int_{Y_0}^{Y}\frac{\left(2Y^2 - Y_0^2\right)dY}{\sqrt{\left(Y^2 - Y_0^2\right)\left(Y_0^2 + 2Y^2\right)Y}} = 2\sqrt{\left(1+b\xi\right)}\,\left(L_0 - X\right) \tag{106a}$$

$$\int_{Y_t}^{Y} \frac{\left(2Y^2 - Y_t^2\right)dY}{\sqrt{\left(Y^2 - Y_t^2\right)\left(Y_t^2 + 2Y^2\right)Y}} = 2\left(L - X\right) \tag{106b}$$

From the above equations for obtaining temperature distribution and fin profile, it is worthy to mention that the design variables Y_0, Y_t, Y_b and L_0 are required to determine first. The Y_t and Y_b can be determined from the constraint equations taken in the design as

$$U = \int_{Y=Y_0}^{Y_b} \frac{\left(2Y^2 - Y_0^2\right)Y^2 dY}{2\sqrt{(1+b\xi)}\sqrt{\left(Y^2 - Y_0^2\right)\left(Y_0^2 + 2Y^2\right)Y}} + \int_{Y=Y_t}^{Y_0} \frac{\left(2Y^2 - Y_t^2\right)Y^2 dY}{2\sqrt{\left(Y^2 - Y_t^2\right)\left(Y_t^2 + 2Y^2\right)Y}} \tag{107a}$$

$$\int_{Y=Y_0}^{Y_b} \frac{\left(2Y^2 - Y_0^2\right)dY}{\sqrt{\left(Y^2 - Y_0^2\right)\left(Y_0^2 + 2Y^2\right)Y}} + \int_{Y=Y_t}^{Y_0} \frac{\left(2Y^2 - Y_t^2\right)dY}{\sqrt{\left(Y^2 - Y_t^2\right)\left(Y_t^2 + 2Y^2\right)Y}} = 2\sqrt{(1+b\xi)}\,L_0 + 2\left(L - L_0\right) \tag{107b}$$

The parameters L_0 and Y_0 are determined from the dewpoint temperature section where dry and wet part coexist and one can take the continuity of temperature and heat conduction at this section.

3. Results and discussion

The humid air is a mixture of dry air and water vapor. Three properties of air are used to calculate a thermodynamic state point to show the influence of dehumidification of air on the optimizing shape of three common type of fins, namely, Longitudinal, annular and pin fins. The air properties such as pressure, temperature, and relative humidity of air are used to determine a pychrometric state. The effect of the psychometric properties of air on the optimization study is investigated. Two constraints, namely, fin volume, and both fin volume and length have been adopted to furnish the result for any designed application.

Fig. 3 is depicted the profile shape and temperature distribution in an optimum fin under different surface conditions as a function of fin length for a constraint fin volume. For the dry surface fin, the optimum fin shape can be determined from the fully wet fin analysis presented above by taking zero value of latent heat parameter ξ. To make a comparison between dry and wet surface fins, the results for the dry surface fin has been plotted in the same figure. From the figure, it can be noticeable that the temperature distribution in the fin at an optimum condition under dry, partially wet, and fully wet surfaces varies linearly (Fig. 3A). For the same fin base temperature for all the surface conditions adopted here, temperature on the fin surface of wet fins differs from that of the dry surface fin and the difference increases gradually from the fin base to fin tip. The discrepancy in temperature occurs due to evolving of latent heat of condensation of moisture on the fin surface in the case of wet fins. This difference in temperature becomes maximum for the 100% relative humidity of air as the maximum value of latent heat released for this relative humidity. From the analysis, it can be highlighted that for the dry and partially wet fin at an optimum condition, the tip temperature equalizes to the ambient temperature. However, for fully wet fin, the tip temperature is sightly less than the ambient value. Figure 3B is illustrated the optimum fin profile for dry, partially wet and fully wet conditions. For the dry fin, the optimum length and optimum base thickness are larger and smaller, respectively than that for the any wet fin for the same fin volume and the difference is maximum for the fully wet fin of 100% relative humidity of air. Nevertheless, the effect of relative humidity on the optimum fin profile for a wet fin is marginal. The same trend for temperature distribution and the optimum fin profile of the annular and pin fins, are found and they are displayed in Figs. 4 and 5, respectively. In the case of annular fins, it can be mentioned that the optimum temperature distribution and fin profile are also function of the tube outer radius parameter R_i.

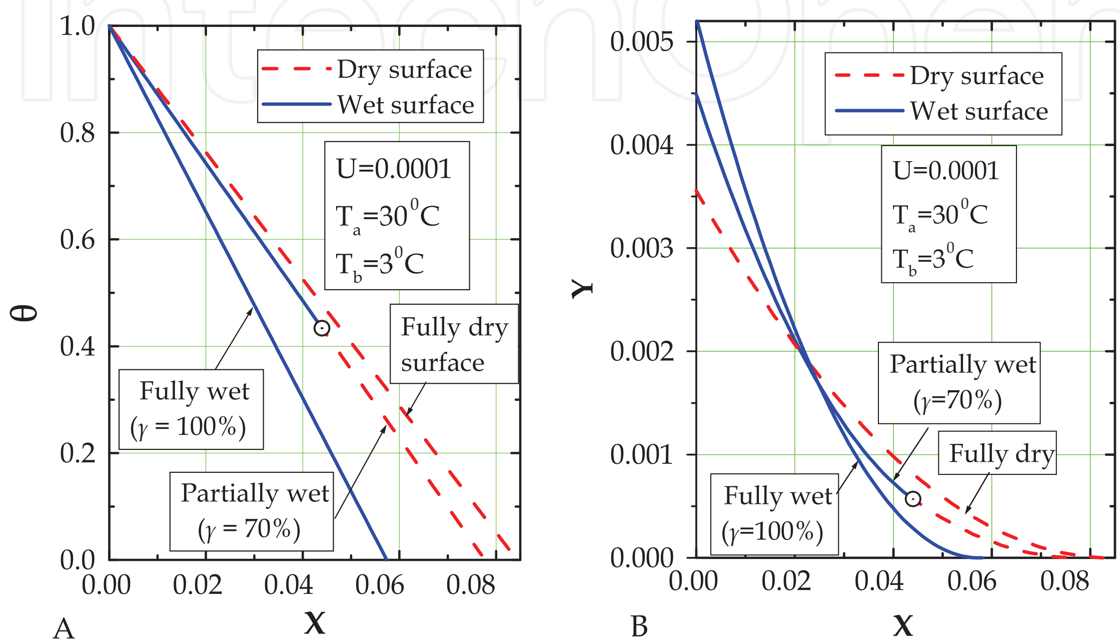


Fig. 3. Variation of temperature distribution and fin profile of an optimum longitudinal fin for the different surface conditions under a volume constraint: A. fin temperature; and B. fin profile

With satisfying either maximizing heat transfer rate for a given fin volume or minimizing fin volume for a given heat transfer duty, the optimization of any fin is studied. Depending upon the requirement of design, any one of these two constraints is used but they give the same result. The optimum profile shape is determined from the solution of the optimality criteria of the fin design and the constraint condition taken. The result from the optimization study of wet fins is depicted in Figs. 6, 7 and 8 as a function of fin volume for longitudinal, annular and spine, respectively. The optimum result for the dry surface condition of each fin, in comparison, is plotted in the corresponding figure. From these illustrations, it is understandable that the optimum parameters, namely, heat transfer rate, fin length, and fin thickness at the base, enhance continually with the fin volume. The maximum rate of heat transfer is not only as a function of the fin material but also as a function of the condition of the surface. A fully wet surface with 100% relative humidity predicts a maximum optimum heat transfer rate per unit volume in comparison with that transferred by any other surfaces. A partially wet surface transfers a less amount of heat per unit volume in comparison to that by fully wet surface fin and heat transfer rate decreases gradually with the decremented relative humidity. A dry surface fin at an optimum condition transfers least amount of heat per unit fin volume in comparison with the wet surface fin. This is happen due to latent heat evolved in wet fin heat transfer mechanism. For a lower value of fin volume, difference in heat transfer among fully wet, partially wet and dry surfaces may not so much important

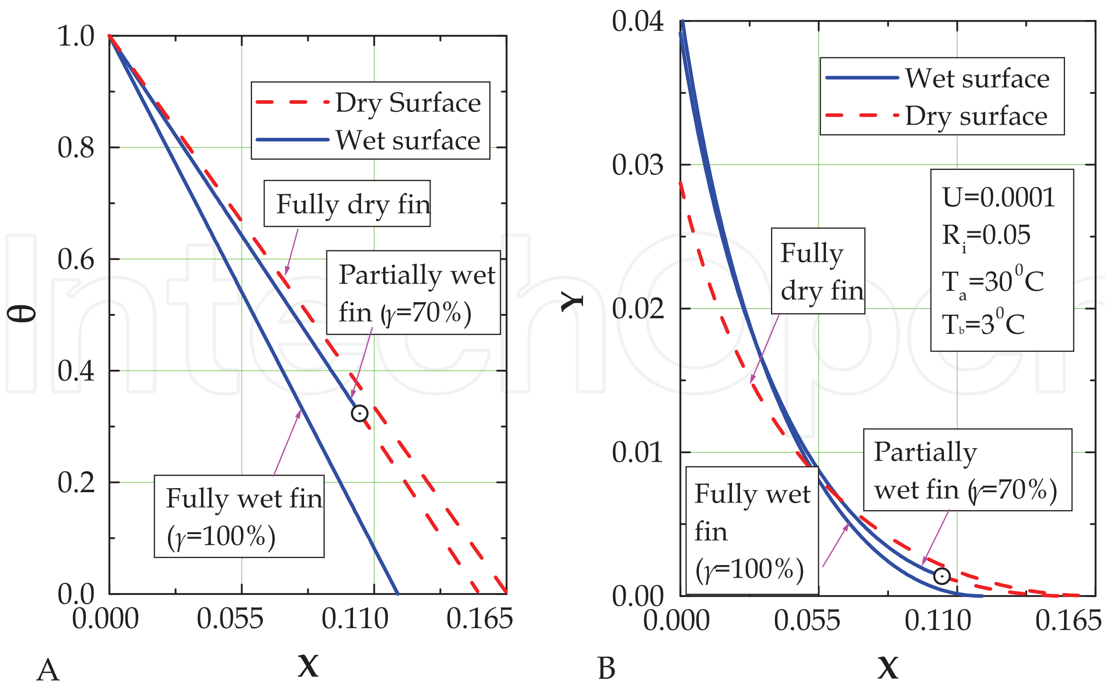


Fig. 4. Variation of temperature distribution and fin profile of an optimum annular fin for the different surface conditions for a volume constraint: A. fin temperature; and B. fin profile.

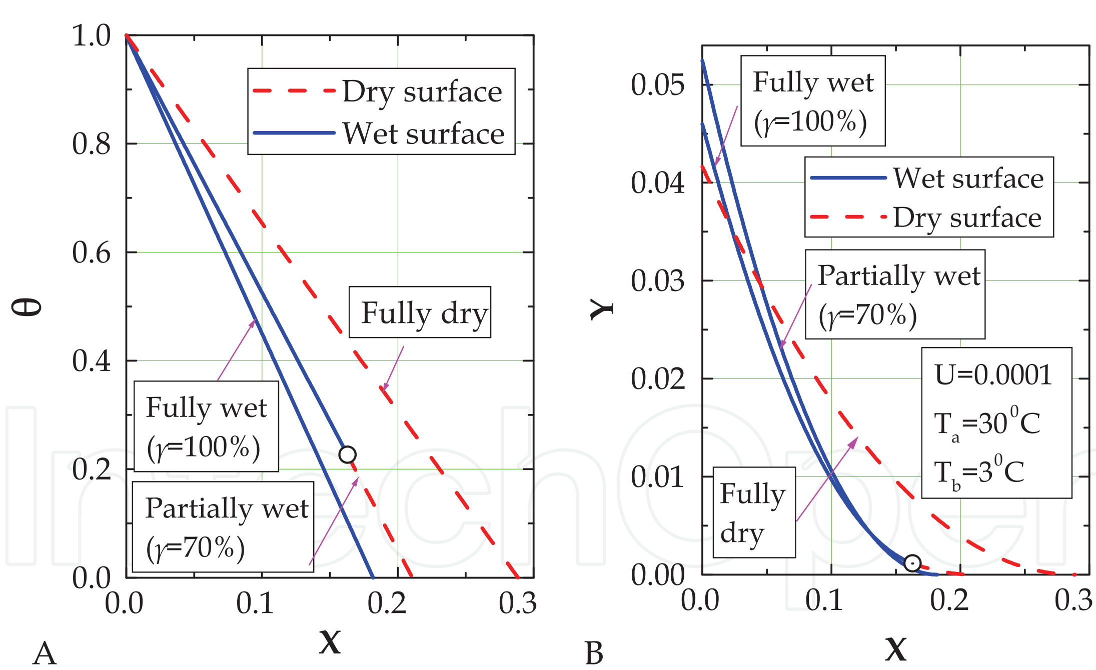


Fig. 5. Variation of temperature distribution and fin profile of an optimum spine for the different surface conditions under a volume constraint: A. fin temperature; and B. fin profile

than that for a higher value of fin volume as shown in Fig. 6A. Irrespective of any surface condition, the length and fin thickness at the base of an optimum fin increases with the fin volume. The optimum fin length for fully wet fins is always shorter than that for the partially wet as well as dry fins. The optimum length is a maximum for the dry surface condition under the same fin volume (Fig. 6B). Nevertheless, an opposite trend is noticed for

the variation of fin thickness at the base with the fin volume in comparison with the variation of fin length with volume for different surface condition as shown in Fig. 6C. A similar exercise has been made for the annular fin and spine by plotting Figs. 7 and 8. In the case of annular fin, the above parameters also function of the thermogeometric parameter R_i. With the increase in R_i, the optimum heat transfer rate increases as well as the optimum length and optimum base thickness decreases, separately with the same fin volume.
From the above optimum results, it can be emphasized to highlight that the optimum fin shape obtained from an optimization technique with the consideration of only one constraint either fin volume or heat transfer rate, is a complex in nature and fragile shape at the tip as already shown in Fig. 3 and hence, it is difficult surely in manufacturing process. To overcome this problem and to restrict the length of the fin, fin length can be taken as an additional constraint with the fin volume. In this case, the shape of the fin profile and fragile

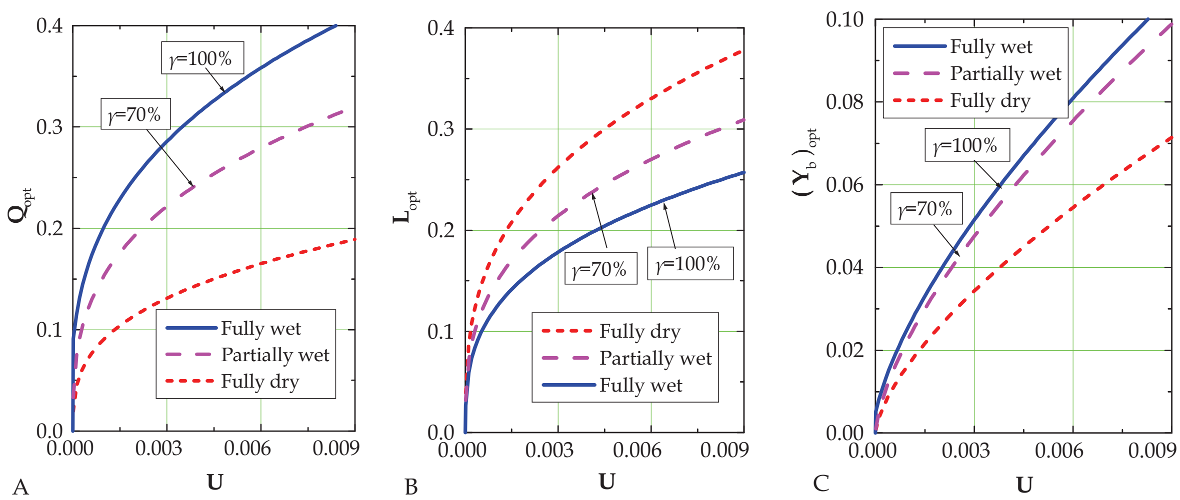


Fig. 6. Design parameters of an optimum longitudinal fin as a function of fin volume: A. maximum heat transfer rate; B. optimum length; and C. optimum semi-fin thickness at the base

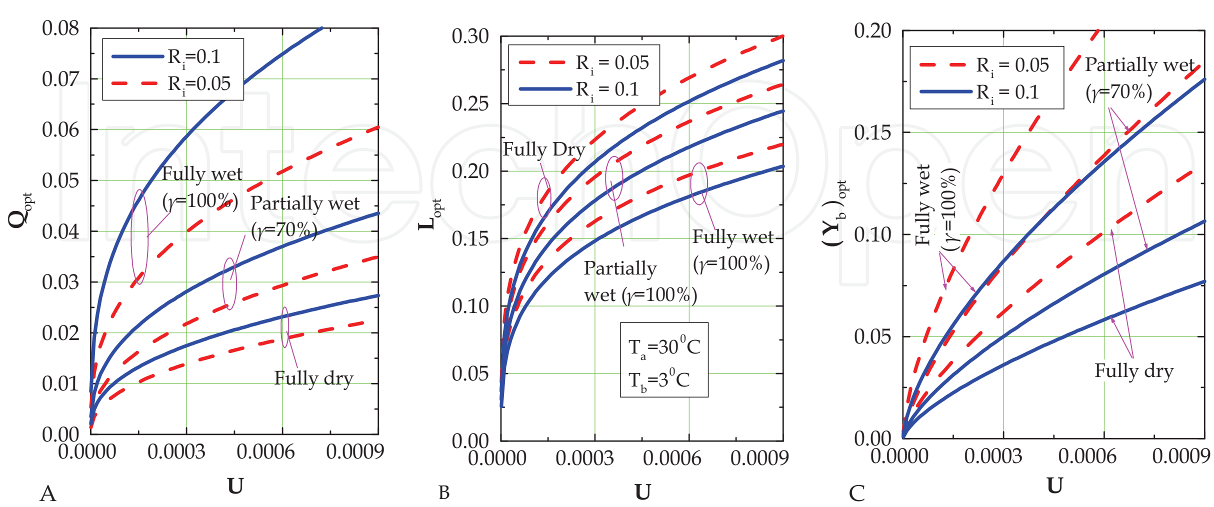


Fig. 7. Design parameters of an optimum annular fin as a function of fin volume: A. maximum heat transfer rate; B. optimum length; and C. optimum semi-fin thickness at the base

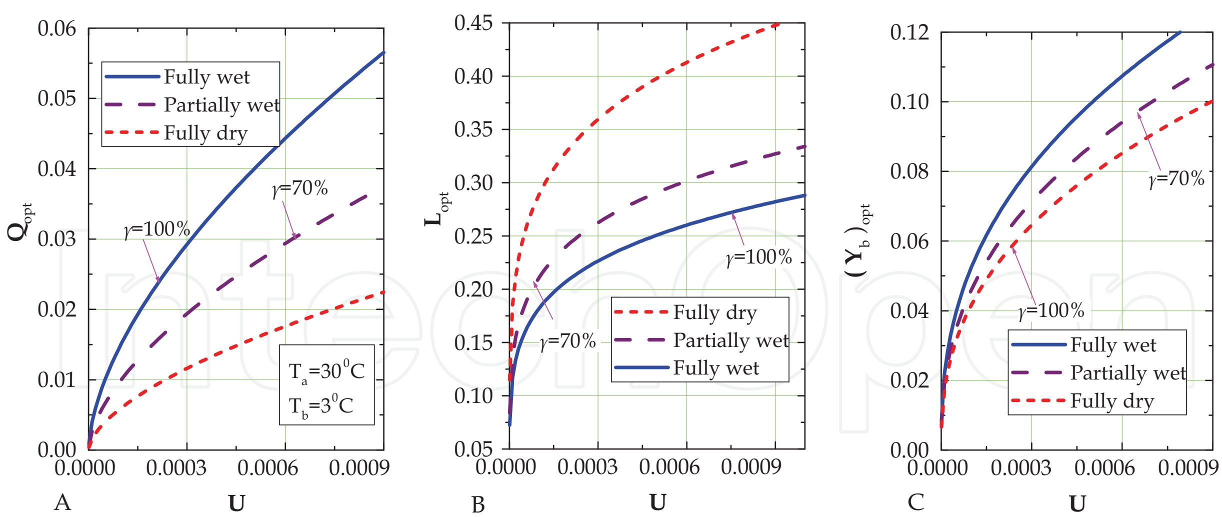


Fig. 8. Design parameters of an optimum pin fin as a function of fin volume: A. maximum heat transfer rate; B. optimum length; and C. optimum fin thickness at the base

geometry at the tip can be improved significantly. To avoid the same nature of the result, the optimum result under both volume and length constraint is illustrated only for the longitudinal fin. The variation of temperature and fin profile is determined under the aforementioned constraint, which is displayed in Fig. 9. From the temperature distribution, it can be mentioned that temperature at the tip of an optimum fin does not vanish and depends upon the magnitude of constraints chosen. For a fully wet surface, temperature at the tip may be closer to the ambient value in comparison with that for the partially and dry surface conditions. With the increase in relative humidity of air, condensation of moisture increases and as a result fin surface temperature increases. This observation can be found in Fig. 9A. The profile shape under both volume and length constraints for various surface conditions is illustrated as a function of dimensionless fin length shown in Fig. 9B. From this figure, it is clear that the profile shape is improved significantly with respect to a profile obtained from only volume constraint chosen, with the consideration of a suitable compatibility in the manufacturing technique. However, there is slightly different in shape between dry and wet surface optimum fin profiles under the same design constants.

4. Conclusions

The fin surface may be dry, fully wet or partially wet depending upon the psychometric conditions of the surrounding air participated as well as the constraints taken in the design. Owing to mass transfer occured with the heat transfer mechanism, the wet surface fin differs from that of dry surface fin. The optimum-envelop shape of wet fins is different with respect to that of dry surface fins. This deviation may be increased with the increase in relative humidity of air. In this chapter, the optimum profile shape of different fins, namely, longitudinal, spine and annular are evaluated for dry, fully wet, and partially wet surface conditions using variational principle. The analysis has also included for different constraint conditions, namely, fin volume and both fin length and fin volume presented for possible requirements of an optimum design. From the results, the optimum design variable for wet fins is not only function of the design constraints but also is a function of psychometric properties of air. Unlike dry and partially wet surface fins, tip temperature for the fully wet optimum fins under the volume constraint is less than the surrounding temperature. A significant change in optimum design variables has been noticed with the design constants such as fin volume and surface conditions. In order to reduce the complexcity of the optimum profile fins under different surface conditions, the constraint fin length can be selected suitably with the constraint fin volume.

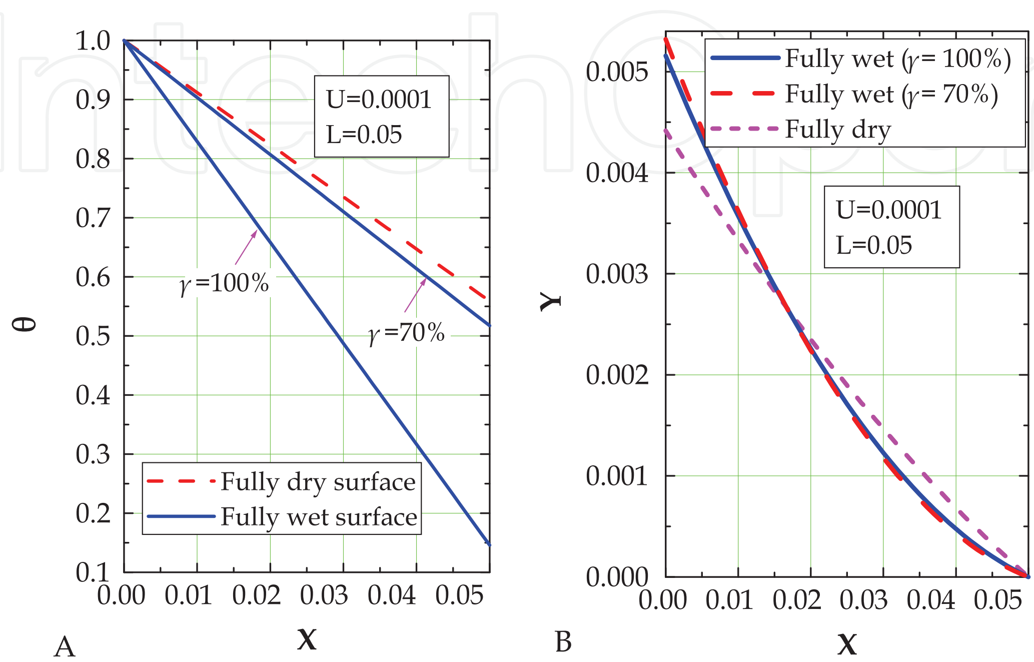


Fig. 9. Variation of temperature and fin profile in a longitudinal fin as a function of length for both volume and length constraints: A. Temperature distribution; and B. Fin profile

5. Acknowledgement

The authors would like to thank King Mongkut's University of Technology Thonburi (KMUTT), the Thailand Research Fund, the Office of Higher Education Commission and the National Research University Project for the financial support.

6. Nomenclatures

a	constant determined from the conditions of humid air at the fin base and fin tip
b	slop of a saturation line in the psychometric chart, K^{-1}
C	non-dimensional integration constant used in Eq. (84)
C_p	specific heat of humid air, $\mathrm{J\,kg^{-1}\,K^{-1}}$
F	functional defined in Eqs. (10), (28), (46), (62), (80) and (96)
h	convective heat transfer coefficient, $\mathrm{W\,m^{-2}}K^{-1}$
h_m	mass transfer coefficient, $\mathrm{kg\,m^{-2}\,S^{-1}}$
h_{fg}	latent heat of condensation, $\mathrm{J\,kg^{-1}}$
k	thermal conductivity of the fin material, $\mathrm{W\,m^{-1}}K^{-1}$
l	fin length, m

l_0 wet length in partially wet fins, m
L dimensionless fin length, hl/k
L_0 dimensionless wet length in partially wet fins, hl_0/k
Le Lewis number
q heat transfer rate through a fin, W
Q dimensionless heat transfer rate
r_i base radius for annular fins, m
R_i dimensionless base radius, hr_i/k
T temperature, K
U dimensionless fin volume, see Eqs. (9), (27), (45), (61), (79), (91a) and (95)
V fin volume (volume per unit width for longitudinal fins), m^3
x, y coordinates, see Figs. 1 and 2, m
X, Y dimensionless coordinates, hx/k and hy/k, respectively
y_0 semi-thickness of a fin at which dry and wet parts separated, m
Y_0 dimensionless thickness, hy_0/k
Z_1, Z_2 dimensionless parameters defined in Eqs. (104a) and (104b), respectively

Greek Letters

α parameter defined in Eqs. (20), (40), (57) and (74a)
λ Lagrange multiplier
ω specific humidity of air, kg w. v. / kg. d. a.
ξ Latent heat parameter
ϕ dimensionless temperature, $\theta + \theta_p$
ϕ_0 dimensionless temperature at the fin base, $1 + \theta_p$
θ dimensionless fin temperature, $(T_a - T)/(T_a - T_b)$
θ_p dimensionless temperature parameter, see Eq. (5)
γ Relative humidity

Subscripts

a ambient
b base
d dewpoint
opt optimum
t tip

7. References

Chilton, T.H. & Colburn, A.P. (1934). Mass transfer (absorption) coefficients–prediction from data on heat transfer and fluid friction. *Ind. Eng. Chem.*, Vol. 26, 1183.

Duffin, R. J. (1959). A variational problem relating to cooling fins with heat generation. *Q. Appl. Math.*, Vol. 10, 19-29.

Guceri, S. & Maday, C. J. (1975). A least weight circular cooling fin. *ASME J. Eng. Ind.*, Vol. 97, 1190-1193.

Hanin, L. & Campo, A. (2003). A new minimum volume straight cooling fin taking into account the length of arc. *Int. J. Heat Mass Transfer*, Vol. 46, 5145-5152.

Hong, K. T. & Webb, R. L. (1996). Calculation of fin efficiency for wet and dry fins. *HVAC&R Research*, Vol. 2, 27-40.

Kundu, B. & Das, P.K. (1998). Profiles for optimum thin fins of different geometry - A unified approach. *J. Institution Engineers (India): Mechanical Engineering Division*, Vol. 78, No. 4, 215-218.

Kundu, B. (2002). Analytical study of the effect of dehumidification of air on the performance and optimization of straight tapered fins. *Int. Comm. Heat Mass Transfer*, Vol. 29, 269-278.

Kundu, B. & Das, P.K. (2004). Performance and optimization analysis of straight taper fins with simultaneous heat and mass transfer. *ASME J. Heat Transfer*, Vol. 126, 862-868.

Kundu, B. & Das, P. K. (2005). Optimum profile of thin fins with volumetric heat generation: a unified approach. *J. Heat Transfer*, Vol. 127, 945-948.

Kundu, B. (2007a). Performance and optimization analysis of SRC profile fins subject to simultaneous heat and mass transfer. *Int. J. Heat Mass Transfer*, Vol. 50, 1645-1655.

Kundu, B. (2007b). Performance and optimum design analysis of longitudinal and pin fins with simultaneous heat and mass transfer: Unified and comparative investigations. *Applied Thermal Engg.*, Vol. 27, Nos. 5-6, 976-987.

Kundu, B. (2008). Optimization of fins under wet conditions using variational principle. *J. Thermophysics Heat Transfer*, Vol. 22, No. 4, 604-616.

Kundu, B., Barman, D. & Debnath, S. (2008). An analytical approach for predicting fin performance of triangular fins subject to simultaneous heat and mass transfer, *Int. J. Refrigeration*, Vol. 31, No. 6, 1113-1120.

Kundu, B. (2009a). Analysis of thermal performance and optimization of concentric circular fins under dehumidifying conditions, *Int. J. Heat Mass Transfer*, Vol. 52, 2646-2659.

Kundu, B. (2009b). Approximate analytic solution for performances of wet fins with a polynomial relationship between humidity ratio and temperature, *Int. J. Thermal Sciences*, Vol. 48, No. 11, 2108-2118.

Kundu, B. & Miyara, A. (2009). An analytical method for determination of the performance of a fin assembly under dehumidifying conditions: A comparative study, *Int. J. Refrigeration*, Vol. 32, No. 2, 369-380.

Kundu, B. (2010). A new methodology for determination of an optimum fin shape under dehumidifying conditions. *Int. J. Refrigeration*, Vol. 33, No. 6, 1105-1117.

Kundu, B. & Barman, D. (2010). Analytical study on design analysis of annular fins under dehumidifying conditions with a polynomial relationship between humidity ratio and saturation temperature, *Int. J. Heat Fluid Flow*, Vol. 31, No. 4, 722-733.

Liu, C. Y. (1961). A variational problem relating to cooling fins with heat generation. *Q. Appl. Math.*, Vol. 19, 245-251.

Liu, C. Y. (1962). A variational problem with application to cooling fins. *J. Soc. Indust. Appl. Math.*, Vol. 10, 19-29.

Maday, C. J. (1974). The minimum weight one-dimensional straight fin. *ASME J. Eng. Ind.*, Vol. 96, 161-165.

McQuiston, F. C. (1975). Fin efficiency with combined heat and mass transfer. *ASHRAE Transaction*, Vol. 71, 350-355.

Mokheimer, E. M. A. (2002). Performance of annular fins with different profiles subject to variable heat transfer coefficient. *Int. J. Heat Mass Transfer*, Vol. 45, 3631-3642.

Pirompugd, W., Wang, C. C. & Wongwises, S. (2007a). Heat and mass transfer characteristics of finned tube heat exchangers with dehumidification. *J. Thermophysics Heat transfer*, Vol. 21, No. 2, 361-371.

Pirompugd, W., Wang, C. C. & Wongwises, S. (2007b). A fully wet and fully dry tiny circular fin method for heat and mass transfer characteristics for plain fin-and-tube heat exchangers under dehumidifying conditions. J. Heat Transfer, Vol. 129, No. 9, 1256-1267.

Pirompugd, W., Wang, C. C. & Wongwises, S. (2008). Finite circular fin method for wavy fin-and-tube heat exchangers under fully and partially wet surface conditions. *Int. J. Heat Mass Transfer*, Vol. 51, 4002-4017.

Pirompugd, W., Wang, C. C. & Wongwises, S. (2009). A review on reduction method for heat and mass transfer characteristics of fin-and-tube heat exchangers under dehumidifying conditions. *Int. J. Heat Mass Transfer*, Vol. 52, 2370-2378.

Razelos, P. & Imre, K. (1983). Minimum mass convective fins with variable heat transfer coefficient. *J. Franklin Institute*, Vol. 315, 269-282.

Schmidt, E. (1926). *Warmeubertragung durch Rippen. Z. Deustsh Ing.*, Vol. 70, 885-951.

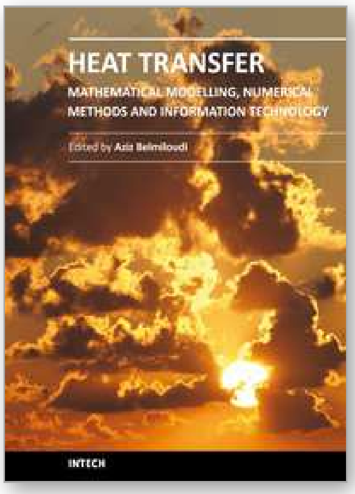
Solov"ev, B.A. (1968). An optimum radiator-fin profile. *Inzhenerno Fizicheskii Jhurnal*, Vol. 14, No. 3, 488-492.

Threlkeld, J. L. (1970). *Thermal environment engineering*. Prentice-Hall, New York.

Wilkins, J. E. Jr. (1961). Minimum mass thin fins with specified minimum thickness. *J. Soc. Ind. Appl. Math.*, Vol. 9, 194-206.

Wu, G. & Bong, T. Y. (1994). Overall efficiency of a straight fin with combined heat and mass transfer. *ASRAE Transation*, Vol. 100, No. 1, 365-374.

Zubair, S. M.; Al-Garni, A. Z. & Nizami, J. S. (1996). The optimum dimensions of circular fins with variable profile and temperature-dependent thermal conductivity. Vol. 39, No. 16, 3431-3439.



Heat Transfer - Mathematical Modelling, Numerical Methods and Information Technology
Edited by Prof. Aziz Belmiloudi

ISBN 978-953-307-550-1
Hard cover, 642 pages
Publisher InTech
Published online 14, February, 2011
Published in print edition February, 2011

Over the past few decades there has been a prolific increase in research and development in area of heat transfer, heat exchangers and their associated technologies. This book is a collection of current research in the above mentioned areas and describes modelling, numerical methods, simulation and information technology with modern ideas and methods to analyse and enhance heat transfer for single and multiphase systems. The topics considered include various basic concepts of heat transfer, the fundamental modes of heat transfer (namely conduction, convection and radiation), thermophysical properties, computational methodologies, control, stabilization and optimization problems, condensation, boiling and freezing, with many real-world problems and important modern applications. The book is divided in four sections : "Inverse, Stabilization and Optimization Problems", "Numerical Methods and Calculations", "Heat Transfer in Mini/Micro Systems", "Energy Transfer and Solid Materials", and each section discusses various issues, methods and applications in accordance with the subjects. The combination of fundamental approach with many important practical applications of current interest will make this book of interest to researchers, scientists, engineers and graduate students in many disciplines, who make use of mathematical modelling, inverse problems, implementation of recently developed numerical methods in this multidisciplinary field as well as to experimental and theoretical researchers in the field of heat and mass transfer.

Fax: +385 (51) 686 166
www.intechopen.com

Fax: +86-21-62489821