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Numerical Model for Multi-layered Tsunami Waves

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1. Introduction

In order to develop a generalized numerical model for multi-layered tsunami wave system, a three-layer system was considered. Six governing equations, two for each layer were derived from Euler equations of motion and continuity for three layers, assuming long wave approximation, negligible friction and interfacial mixing. From derived equations, it is found that only top layer equations are independent of number of intermediate layers; equations for all other layers are dependent on number, extent and density of intermediate layer(s). Momentum and continuity equations for the top layer are exactly same as in the case of earlier developed governing equations for two-layered system. Continuity equation for the bottom layer is also exactly same as in the case of two-layered system. Momentum equation for the bottom layer is dependent on extent and density of top layer as well as all intermediate layers. Continuity equation for intermediate layer is affected by levels of immediate bottom layer. Momentum equation for the intermediate layer is affected by extent and density of upper layer(s). Developed governing equations were converted to a numerical model using staggered Leap-Frog scheme for the computations of water level and discharge in each layer in one-dimensional propagation. Developed numerical model results were compared with an earlier developed model for two layers, which was rigorously verified by analytical solution. It was found that this three-layer model produces same results when it is converted to two-layer through mathematical manipulation (i.e. by assuming a negligible/zero depth or similar density of adjacent layer for any layer). The details properties of three-layer model were discussed through numerical simulations for different scenarios. The developed model can be easily converted to a multi-layer (any number) model and can be applied confidently to simulate the basic features of different practical tsunami problems similar to that investigated in this study.

2. Background

Multi-layered flow is related with many environmental phenomena. Thermally driven exchange flows through doorways to oceanic currents, salt water intrusion in estuaries, spillage of the oil on the sea surface, spreading of dense contaminated water, sediment laden discharges into lakes, generation of lee waves behind a mountain range and tidal

flows over sills of the ocean are examples of multi-layered flow. In hydraulics, this type of flow is often termed as gravity current. An extensive review on hydrodynamics of various gravity currents was provided by Simpson, J.E. (1982).

Tsunamis are generated due to disturbances of free surfaces caused by not only seismic fault motion, but also landslide and volcanic eruptions (Imamura and Imteaz, 1995). Tsunami waves are also affected by density differences along the depth of ocean. There are some studies on two-layered long waves or flows in the case of underwater landslide generated tsunamis (Hampton, 1972; Parker, 1982; Harbitz, 1991; Jiang & Leblond, 1992). Imamura & Imteaz (1995) developed a linear numerical model on two-layered long wave flow, which was successfully validated by a rigorous analytical solution. Later the linear model was extended to a non-linear model and effects of non-linearity were investigated (Imteaz & Imamura, 2001b). Madsen et al. (2002) developed a model of multi-layered flow based on Boussinesq-type equations, which are suitable for shallow depth flow. Lynett and Liu (2004) developed another model of multi-layered flow using piecewise integration of Laplace equation for each individual layer and expanded the model for deep water.

Choi and Camassa (1996) derived two-dimensional non-linear equations for two-layered fluid system and presented some numerical simulations of their model for one-dimensional unidirectional wave propagation. Later Choi and Camassa (1999) further developed governing equations for the unidirectional propagation of internal gravity waves at the interface of two immiscible inviscid fluids. They have compared their numerical results with available experimental data for solitary waves of large amplitude in two-fluid system. Liska and Wendroff (1997) derived one-layer and two-layer classic shallow water equations for flow over topography, as well as one-layer and two-layer non-hydrostatic equations. They have compared their numerical results with the numerical computations obtained by others. Percival et al. (2008) presented a multi-layer extension of Green-Naghdi equations (Green and Naghdi, 1976) using a special framework based on the Euler-Poincare theory. Through numerical simulations they have shown that free surface of a multi-layer model can exhibit intriguing differences compared to the results of single layer model. Cotter et al. (2010) modified the multi-layer Green-Naghdi equations to incorporate effects of shear stress. They have presented numerical simulations for the wave propagation and interactions between two layers, with and without shear considerations.

All the above mentioned models, presented their results in two-layer case only. However, interactions within the layers for a two-layer fluid are significantly different than a three (or more) layer fluid. Top surface is having effect from immediate lower layer only, which is same for both the models; however, intermediate layer is having effects from both the lower layer and upper layers and lower layer is having effects from all the upper layers (Imteaz et al., 2009). Imteaz et al. (2009) provided detailed derivation of multi-layered tsunami wave and flow equations based on Navier-Stokes equation. Also, properties of multi-layered equations were discussed in details. Present paper describes conversion of developed multi-layered tsunami equations in to numerical form, comparison of developed model with earlier validated model and several simulations scenario.

3. Governing equations

Figure 1 shows the schematic diagram of three layer propagation having different densities and depths. For a three layered one-dimensional propagation, considering Euler's equations of motion and continuity for each layer in a wide channel with non-horizontal bottom,

through integration and rigorous formulation assuming a hydrostatic pressure distribution, negligible friction, negligible interfacial mixing and proper boundary conditions, mathematical continuity and momentum equations were derived for each layer. Also uniform density and velocity distributions in each layer were assumed. Original derived equations are further simplified considering horizontal bottom (i.e. no variations of 'h' along x direction, $\partial h/\partial x = 0$).
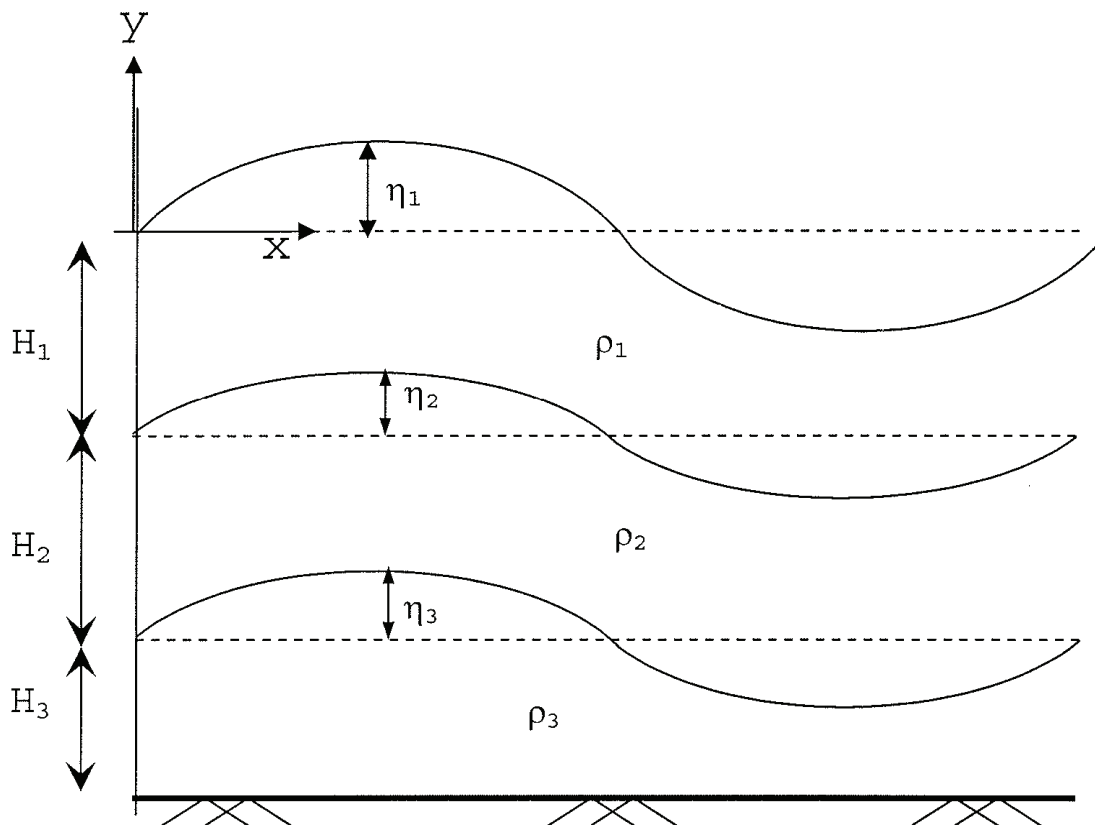

Fig. 1. Schematic diagram of three layer profile

Detailed derivations of mathematical equations are described by Imteaz et al. (2009). The equations are as follows:

For the upper layer-
Continuity equation,

$$\frac{\partial M_1}{\partial x} + \frac{\partial (\eta_1 - \eta_2)}{\partial t} = 0 \tag{1}$$

Momentum equation,

$$\frac{\partial M_1}{\partial t} + \frac{\partial (M_1^2 / D_1)}{\partial x} + gD_1\frac{\partial \eta_1}{\partial x} = 0 \tag{2}$$

For the intermediate layer-
Continuity equation,

$$\frac{\partial M_2}{\partial x} + \frac{\partial (\eta_2 - \eta_3)}{\partial t} = 0 \tag{3}$$

Momentum equation,

$$\frac{\partial M2}{\partial t} + \frac{\partial (M_2^2 / D2)}{\partial x} + gD2\left\{\frac{\alpha_1}{\alpha_2}\left(\frac{\partial \eta_1}{\partial x} - \frac{\partial \eta_2}{\partial x}\right) + \frac{\partial \eta_2}{\partial x}\right\} = 0 \tag{4}$$

For the lower layer-
Continuity equation,

$$\frac{\partial M3}{\partial x} + \frac{\partial \eta_3}{\partial t} = 0 \tag{5}$$

Momentum equation,

$$\frac{\partial M3}{\partial t} + \frac{\partial (M_3^2 / D_3)}{\partial x} + gD_3\left\{\alpha_1\left(\frac{\partial \eta_1}{\partial x} - \frac{\partial \eta_2}{\partial x}\right) + \frac{\partial \eta_3}{\partial x} + \alpha_2\left(\frac{\partial \eta_2}{\partial x} - \frac{\partial \eta_3}{\partial x}\right)\right\} = 0 \tag{6}$$

Where,
η_1 = Water surface elevation above still water level of layer '1'
η_2 = Water surface elevation above still water level of layer '2'
η_3 = Water surface elevation above still water level of layer '3'

$$D_1 = \eta_1 + h_1 - \eta_2,\ D_2 = h_2 + \eta_2 - \eta_3,\ D_3 = h_3 + \eta_3,\ \alpha_1 = \rho_1 / \rho_3,\ \alpha_2 = \rho_2 / \rho_3$$

h_1 = Still water depth of layer '1'
h_2 = Still water depth of layer '2'
h_3 = Still water depth of layer '3'

$$M_1 = \int_{-h_1 + \eta_2}^{\eta_1} u_1\ dy,\ \ M_2 = \int_{-h_1 - h_2 + \eta_3}^{-h_1 + \eta_2} u_2\ dy,\ \ M_3 = \int_{-h_1 - h_2 - h_3}^{-h_1 - h_2 + \eta_3} u_3\ dy$$

From the derived equations, it is found that momentum equation for upper layer is not affected by the properties of adjacent layer (layer underneath). However, continuity equation of upper layer is affected by surface elevation of intermediate layer. Continuity equation for intermediate layer is affected by the surface elevation of bottom layer. Momentum equation for intermediate layer is affected by density and spatial change of surface elevation of upper layer. Continuity equation for bottom layer is not affected by either uppermost layer or intermediate layer. However, momentum equation of bottom layer is affected by densities and spatial changes in surface elevations of all the layers above it. Properties of all these equations were described in detailed by Imteaz et al. (2009). Also, wave celerity of each layer was deduced as follows:

$$C_1 = \sqrt{gh_1\left(1 + \alpha_3\beta_1\right)}$$

$$C_2 = \sqrt{gh_2\left(1 + \beta_2(\alpha_2 - \alpha_1)\right)}$$

$$C_3 = \sqrt{gh_3\left(1 - \alpha_2\right)}$$

4. Numerical model

Developed governing equations are non-linear. It is very difficult to solve the non-linear governing equations analytically, but it can be solved numerically using proper finite difference scheme. Staggered Leap-Frog scheme has been used to solve the governing equations numerically, as it was found producing very good results in earlier developed models (Imamura & Imteaz, 1995; Imteaz & Imamura, 2001a and Imteaz & Imamura, 2001b). Figure 2 shows the schematic diagram of the staggered Leap-Frog scheme. This scheme is one of the explicit central difference schemes with the truncation error of second order. The staggered scheme considers that the computation point for one variable (η) does not coincide with the computation point for other variable (M). There are half step differences (½Δt and ½Δx) between computation points of two variables (as shown in Figure 2). Using this scheme, finite difference equations for the derived governing equations are as follows:

For the upper layer-
Continuity equation,

$$\frac{\eta_{1,i}^{n+1/2} - \eta_{1,i}^{n-1/2} - \eta_{2,i}^{n+1/2} + \eta_{2,i}^{n-1/2}}{\Delta t} + \frac{M_{1,i+1/2}^{n} - M_{1,i-1/2}^{n}}{\Delta x} = 0 \tag{7}$$

Momentum equation,

$$\frac{M_{1,i+1/2}^{n} - M_{1,i+1/2}^{n-1}}{\Delta t} + g\frac{D_{1,i+1}^{n-1/2} + D_{1,i}^{n-1/2}}{2}\frac{\eta_{1,i+1}^{n-1/2} - \eta_{1,i}^{n-1/2}}{\Delta x} +$$

$$\frac{\dfrac{(M_{1,i+1/2}^{n-1})^2}{(D_{1,i+1}^{n-1/2} + D_{1,i}^{n-1/2} + D_{1,i+1}^{n-3/2} + D_{1,i}^{n-3/2})/4} - \dfrac{(M_{1,i-1/2}^{n-1})^2}{(D_{1,i}^{n-1/2} + D_{1,i-1}^{n-1/2} + D_{1,i}^{n-3/2} + D_{1,i-1}^{n-3/2})/4}}{\Delta x} = 0 \tag{8}$$

For the intermediate layer-
Continuity equation,

$$\frac{\eta_{2,i}^{n+1/2} - \eta_{2,i}^{n-1/2} - \eta_{3,i}^{n+1/2} + \eta_{3,i}^{n-1/2}}{\Delta t} + \frac{M_{2,i+1/2}^{n} - M_{2,i-1/2}^{n}}{\Delta x} = 0 \tag{9}$$

Momentum equation,

$$\frac{M_{2,i+1/2}^{n} - M_{2,i+1/2}^{n-1}}{\Delta t} + g\frac{D_{2,i+1}^{n-1/2} + D_{2,i}^{n-1/2}}{2}\left[\frac{\alpha_1}{\alpha_2}\left(\frac{\eta_{1,i+1}^{n-1/2} - \eta_{1,i}^{n-1/2}}{\Delta x} - \frac{\eta_{2,i+1}^{n-1/2} - \eta_{2,i}^{n-1/2}}{\Delta x}\right) + \frac{\eta_{2,i+1}^{n-1/2} - \eta_{2,i}^{n-1/2}}{\Delta x}\right]$$

$$+\frac{\dfrac{(M_{2,i+1/2}^{n-1})^2}{(D_{2,i+1}^{n-1/2} + D_{2,i}^{n-1/2} + D_{2,i+1}^{n-3/2} + D_{2,i}^{n-3/2})/4} - \dfrac{(M_{2,i-1/2}^{n-1})^2}{(D_{2,i}^{n-1/2} + D_{2,i-1}^{n-1/2} + D_{2,i}^{n-3/2} + D_{2,i-1}^{n-3/2})/4}}{\Delta x} = 0 \tag{10}$$

For the lower layer-
Continuity equation,

$$\frac{\eta_{3,i}^{n+1/2} - \eta_{3,i}^{n-1/2}}{\Delta t} + \frac{M_{3,i+1/2}^{n} - M_{3,i-1/2}^{n}}{\Delta x} = 0 \tag{11}$$

Momentum equation,

$$g\frac{D_{3,i+1}^{n-1/2} + D_{3,i}^{n-1/2}}{2}\left[\alpha_1\left(\frac{\eta_{1,i+1}^{n-1/2} - \eta_{1,i}^{n-1/2}}{\Delta x} - \frac{\eta_{2,i+1}^{n-1/2} - \eta_{2,i}^{n-1/2}}{\Delta x}\right)\right]+$$

$$g\frac{D_{3,i+1}^{n-1/2} + D_{3,i}^{n-1/2}}{2}\left[\frac{\eta_{3,i+1}^{n-1/2} - \eta_{3,i}^{n-1/2}}{\Delta x} + \alpha_2\left(\frac{\eta_{2,i+1}^{n-1/2} - \eta_{2,i}^{n-1/2}}{\Delta x} - \frac{\eta_{3,i+1}^{n-1/2} - \eta_{3,i}^{n-1/2}}{\Delta x}\right)\right]+$$

$$\frac{\dfrac{(M_{3,i+1/2}^{n-1})^2}{(D_{3,i+1}^{n-1/2} + D_{3,i}^{n-1/2} + D_{3,i+1}^{n-3/2} + D_{3,i}^{n-3/2})/4} - \dfrac{(M_{3,i-1/2}^{n-1})^2}{(D_{3,i}^{n-1/2} + D_{3,i-1}^{n-1/2} + D_{3,i}^{n-3/2} + D_{3,i-1}^{n-3/2})/4}}{\Delta x}$$

$$+\frac{M_{2,i+1/2}^{n} - M_{2,i+1/2}^{n-1}}{\Delta t} = 0 \tag{12}$$

where, 'n' denotes the temporal grid points and 'i' denotes the spatial grid points as shown in Figure 2. To calculate 'D' values at the computation point of 'M', average of four surrounding 'D' values were taken.

In spatial direction all η_1, η_2 and η_3 at step 'n-1/2' and all M_1, M_2 and M_3 at step '(n-1)' are given as initial conditions. For all later time steps at right boundary all values of M_1, M_2 and M_3 are calculated by characteristic method, using the values of previous time step and wave celerity. By using deduced finite difference Momentum equations for upper, intermediate and lower layer all M_1, M_2 and M_3 values at step 'n' are calculated. Then using the latest values of M_3 and deduced finite difference continuity equation for the lower layer all the values of η_3 at step '(n+1/2)' are calculated. Then using the latest values of η_3, M_2 and deduced finite difference continuity equation for the intermediate layer, all the values of η_2 at step '(n+1/2)' are calculated. Again, using the latest values of η_2, M_1 and deduced finite difference continuity equation for the upper layer, all the values of η_1 at step '(n+1/2)' are calculated. Similarly, using new values of η_1, η_2, η_3, M_1, M_2 and M_3 as initial conditions calculations proceeded in time direction up to the desired time step.

As initial condition (i.e. at t=0) all η_1, η_2, M_1 and M_2 values are taken as zero. For interface (between bottom layer and intermediate layer), assumed initial conditions are shown in Equations 13 & 14, which are based on the initial formation of tsunami wave. Expression of η_3,

$$\eta_3 = a_3\text{Sin}(kx) = a_3\text{Sin}(\frac{2\pi}{L}x) \tag{13}$$
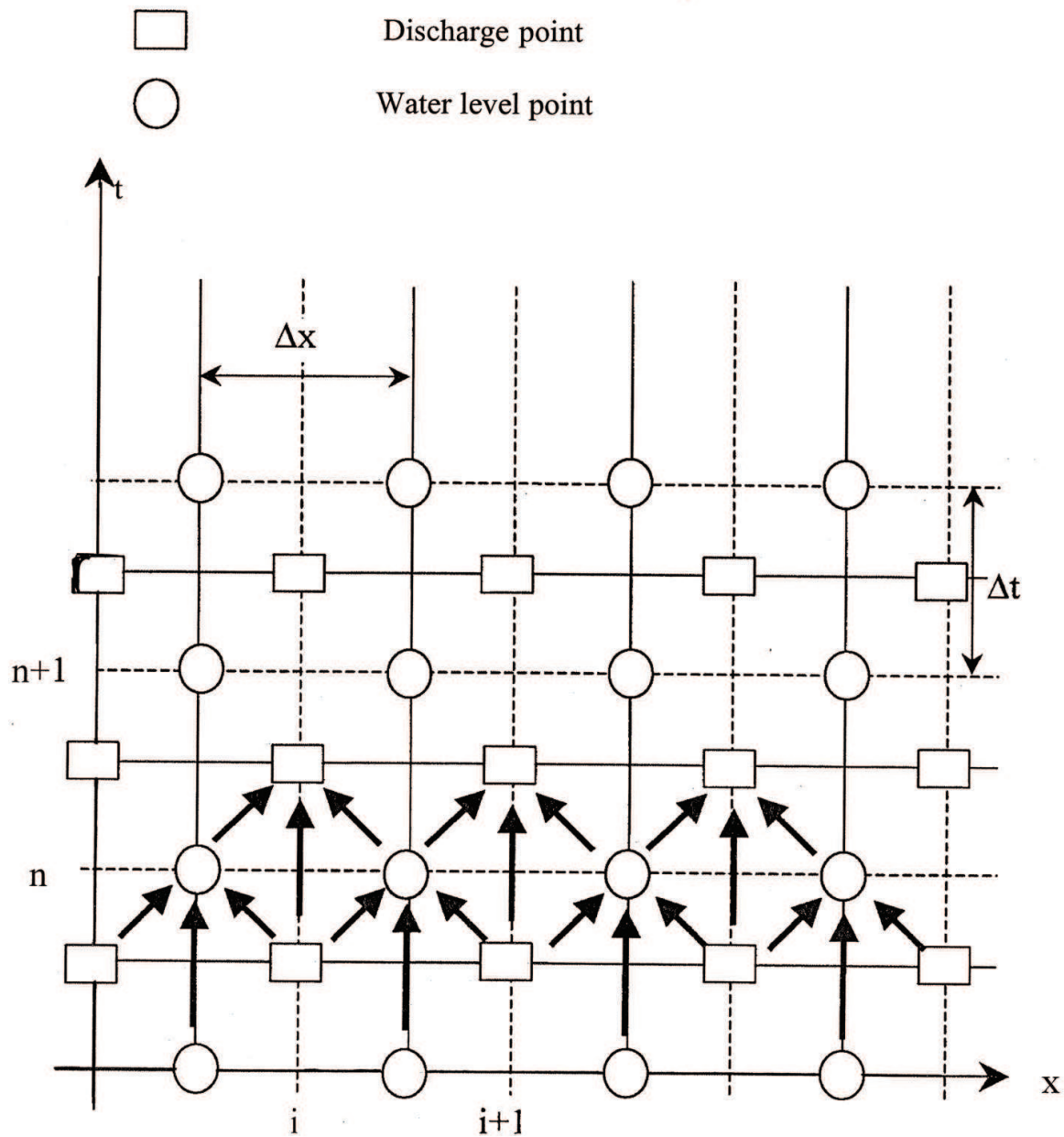
Fig. 2. Schematic diagram of the staggered Leap-Frog scheme

and corresponding discharge,

$$M_3 = \sqrt{(g / h_3)}\,\eta_3(h_3 + \eta_3)$$ (14)

where, a_3, k and L are the wave amplitude, wave number and wave length for the interfacial surface of intermediate layer and the bottom layer.

While computing water levels and discharges using Staggered Leap-Frog scheme, it is required to calculate boundary values (right and left boundaries) of water levels and discharges, using appropriate boundary conditions. Imamura and Imteaz (1995) discussed

several possible boundary conditions. Among all the possible boundary conditions, it was found that the following provides good results for non-linear model simulations (Imteaz & Imamura, 2001b): M_1, M_2 and M_3 at the right boundary are calculated using characteristic method; constant wave celerities (which were estimated using analytical expressions) were used throughout the computational domain. Finally, using periodic condition, water levels at left boundary was used same as the right boundary.

5. Model comparison

It is found that the developed governing equations are complicated having influence from upper and/or lower layer(s) flow. Analytical solutions for such complicated differential equations are yet to be achieved. However, numerical models for two layer tsunami wave were validated with analytical solutions for known interface propagation with unknown top surface propagation by Imamura & Imteaz (1995) and for known top surface propagation with unknown interface propagation by Imteaz & Imamura (2001a). As the present developed model could not be verified with an analytical solution, it is indirectly compared with an earlier validated numerical model. Developed model was converted to a pseudo two-layer by assuming very close densities (1.0 and 0.99) for the intermediate and bottom layers respectively, with an upper layer density of 0.90. This numerical manipulation is supposed to produce similar results with a two-layer model having densities of 0.90 and 1.0 for the upper layer and lower layer respectively, provided depth of upper layer in both the two-layer and three-layer models are same and total depth of lower layers (intermediate layer and bottom layer) in three-layer model is same as the bottom layer of the two-layer model. With the above-mentioned conditions two separate models (two-layer model and three-layer model) were simulated with the same boundary conditions and having other properties as:

Wave length: 395.0 m
Wave amplitude: 2.0 m
DX = 10.0 m
DT = 0.20 sec

Models were simulated for a period of 4 seconds. Figures 3 & 4 show the pseudo three-layer model results comparison with the two-layer model results for similar input data. Figure 3 shows the comparison of model results for the top surface. It is found that the pseudo three-layer model simulates exactly same top surface level as the original two-layer model. Figure 4 shows the comparison of model results for the interface (interface between the intermediate layer and the upper layer for three-layer model). Again it is found that the pseudo three-layer model simulates almost same interface level as the original two-layer model. However, there are slight variations in the simulation of interface level, compared to the original two-layer model simulation. The reason for this variation is the fact that the lower layer density for the two-layer model is 1.0; however for a three-layer model this is not a single layer having a same density of 1.0, rather it is a combination of two layers having densities of 1.0 and 0.99. However, the variations are very insignificant and agreements of models results can be termed as very good.
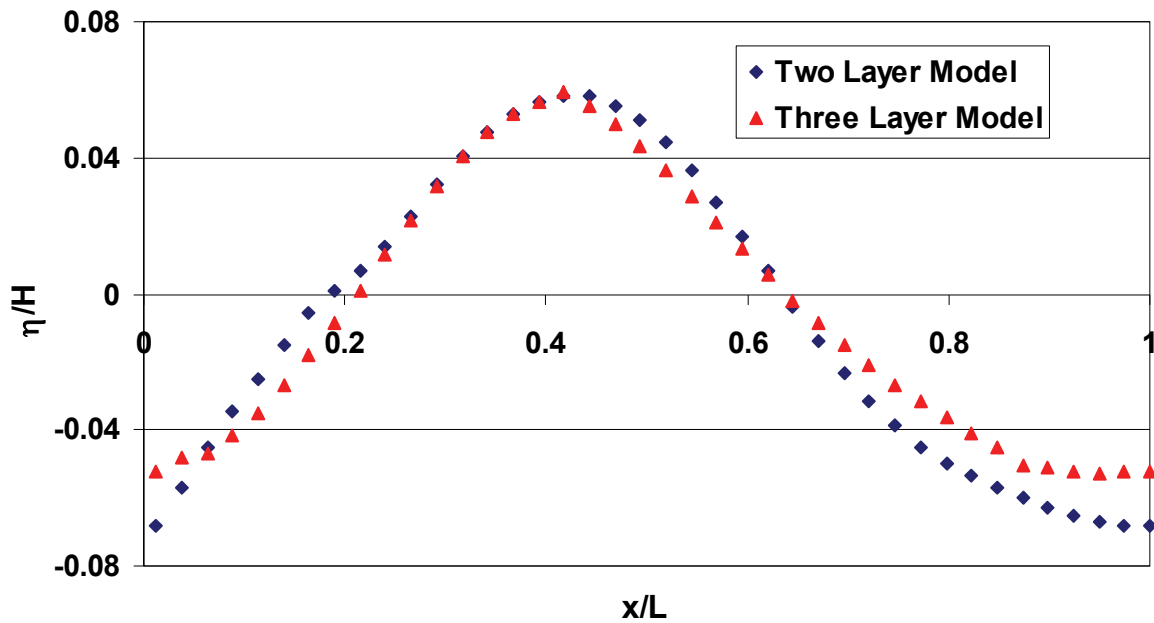
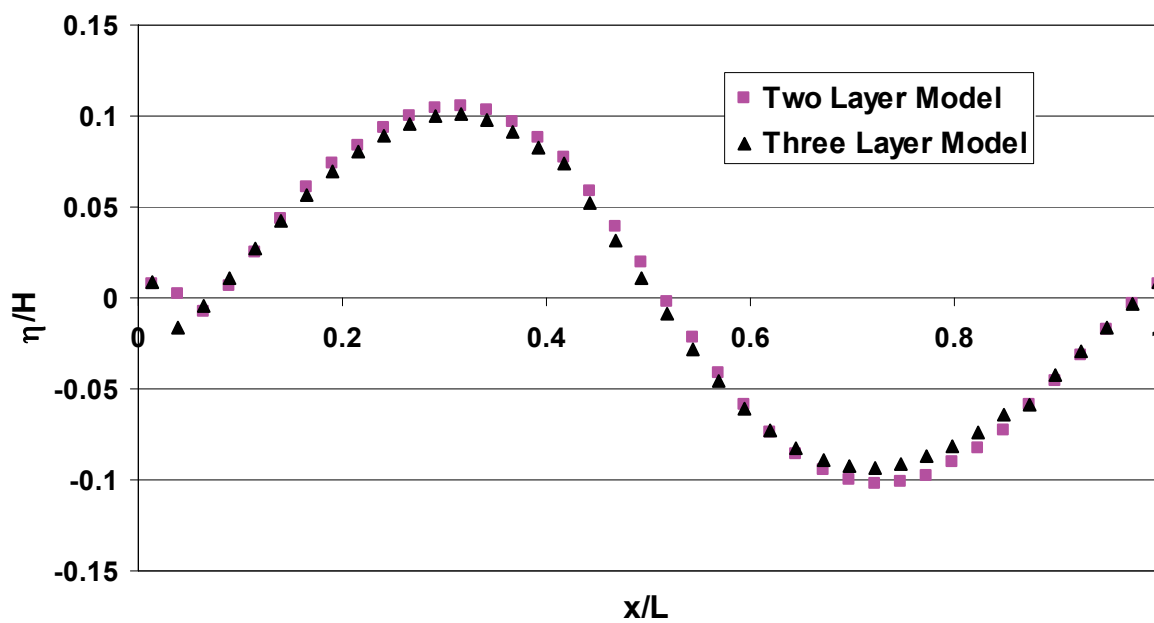Fig. 3. Comparison of model results for the top surface

Fig. 4. Comparison of model results for the interface

6. Scenario simulations

The developed model was used for several scenario simulations. Model was simulated for the following conditions:

Wave length: 395.0 m
Lower layer wave amplitude: 2.0 m
DX = 10.0 m, DT = 0.20 seconds

$h_1 = h_2 = h_3 = 25.0$ m
$\rho_1 = 0.8$, $\rho_2 = 0.9$, $\rho_3 = 1.0$

For simplicity each layer depth was assumed as 25.0m, however the model can simulate for any layer depth. For the previous two-layered models, it was found that with the assumed DX/DT ratio, model's stability condition is good. Imteaz (1994) has presented detailed stability criteria analysis for the two-layered tsunami waves with Staggered Leap-Frog scheme. Eventually, Imteaz (1994) has proposed several regimes of DX and DT for the numerical stability of the model. A separate stability criteria analysis for the current three-layered model needs to be performed to achieve best numerical stability.

Figure 5 & 6 show the model simulations for all the three layer surfaces after 4 seconds and 12 seconds respectively. From the figures it is found that the surface of the lower layer gets amplified with the course of time, whereas the surface of the intermediate layer gets dampen down with the course of time. These are because of the complicated interactions from the adjacent layers.

Figures 7, 8 & 9 show the wave propagation patterns for lower surface, intermediate surface and top surface respectively. From the figures it is clear that propagations of lower layer and intermediate layers are relatively smooth. However, propagation of top surface is faster and having dramatic changes. For the top surface, wave phase changes from 4 seconds to 12 seconds, i.e. wave moves a distance of half of the wave length within 8 seconds. Also, within 4 seconds to 12 seconds, two wave crests of smaller amplitudes were formed.

Till now analytical solution of such complicated flow was not successfully achieved. However, in the future if such analytical solution is achieved, numerical results of the current model can be verified.
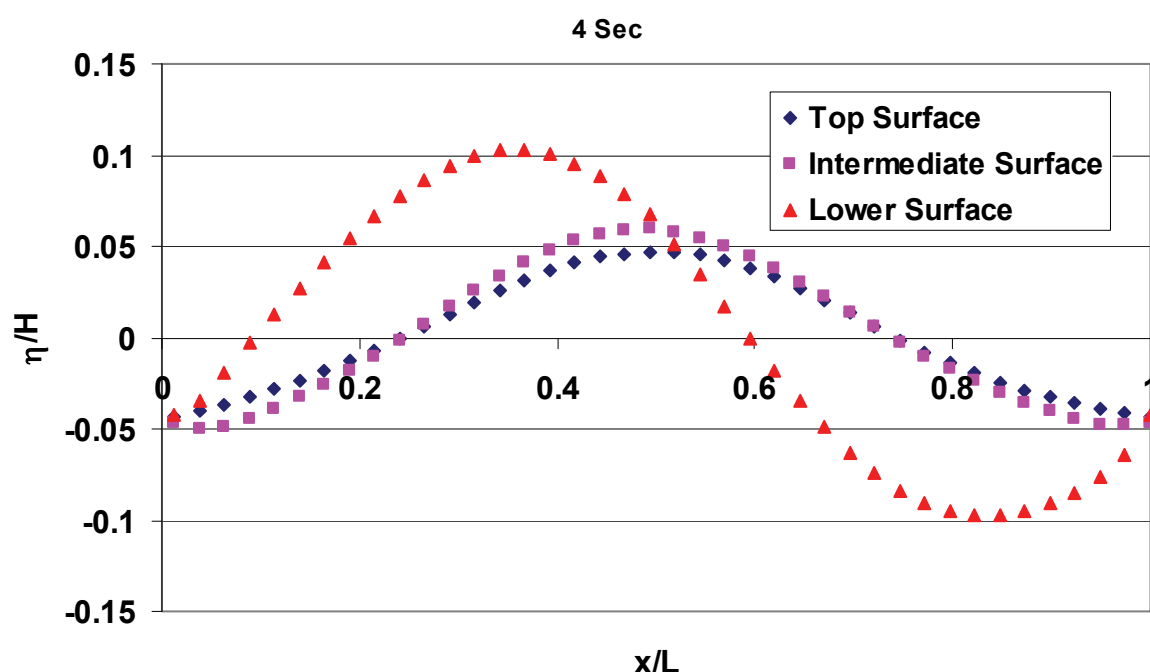

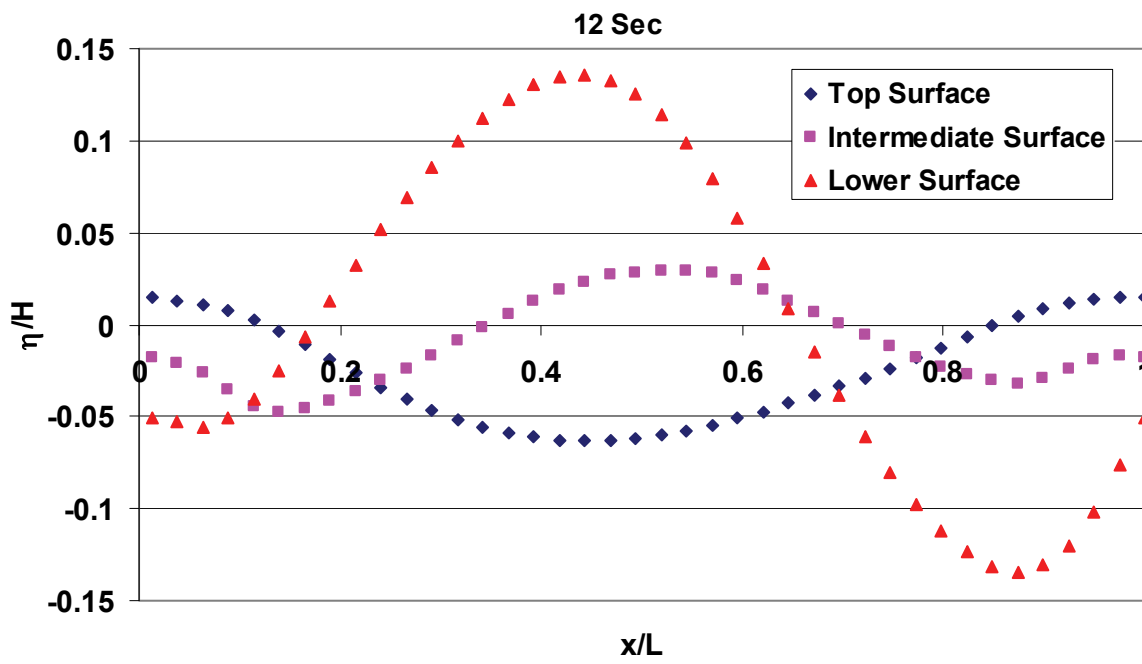Fig. 5. Three layer model simulation results after 4 seconds

Fig. 6. Three layer model simulation results after 12 seconds
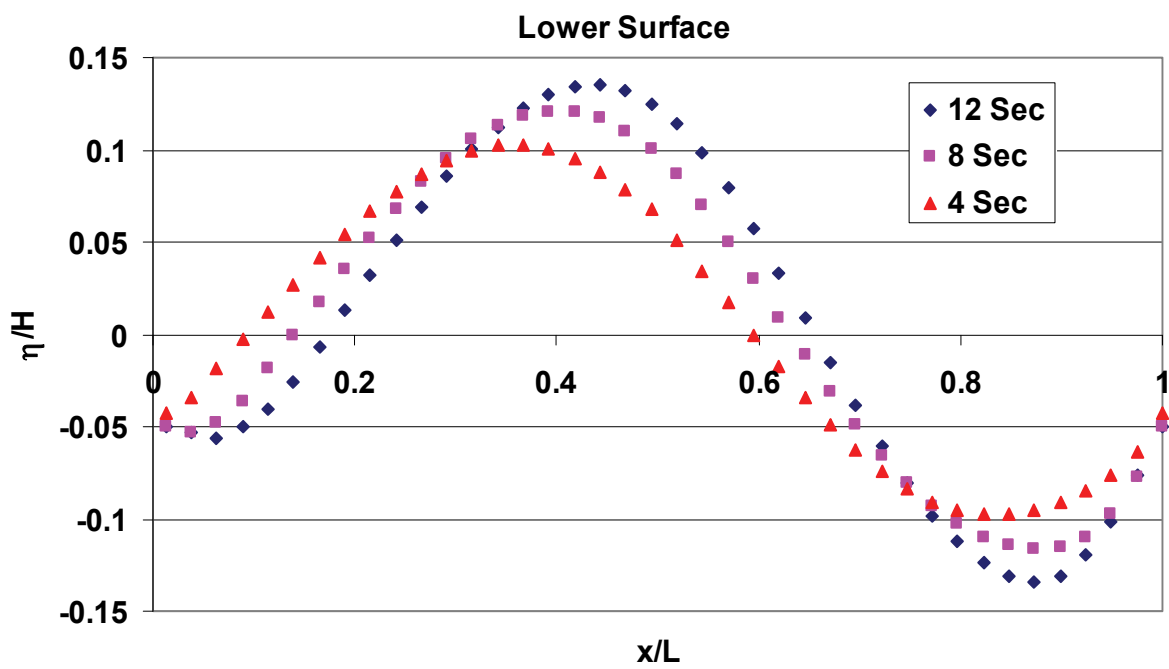

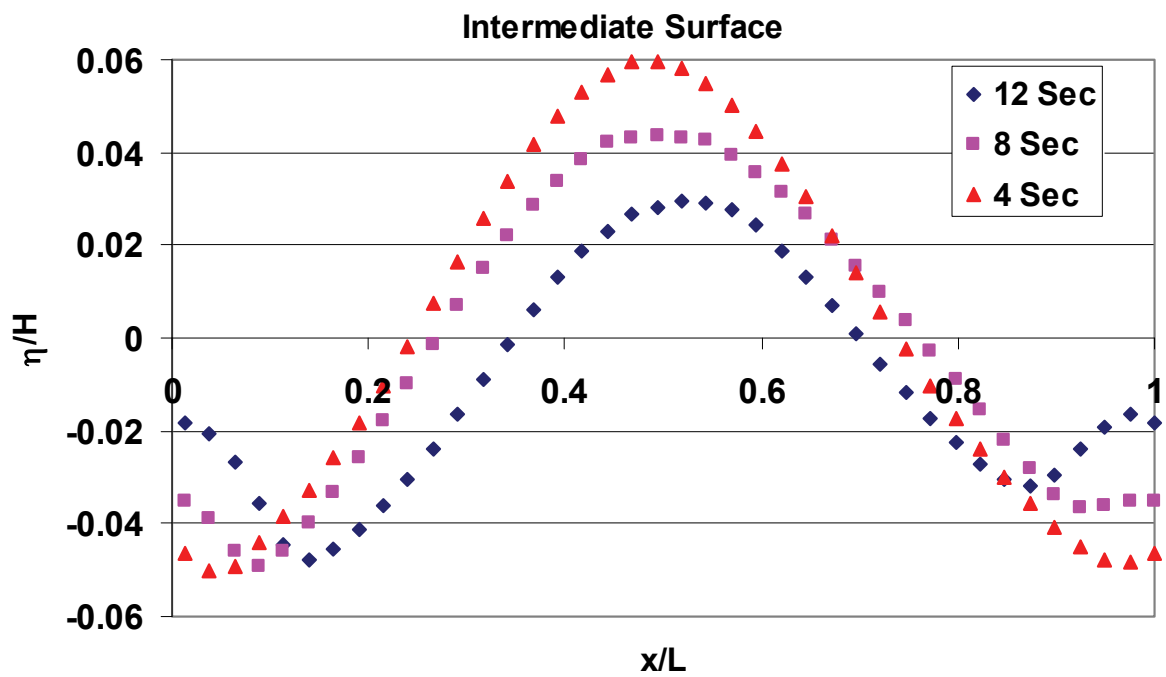Fig. 7. Simulation results for the propagation of lower surface

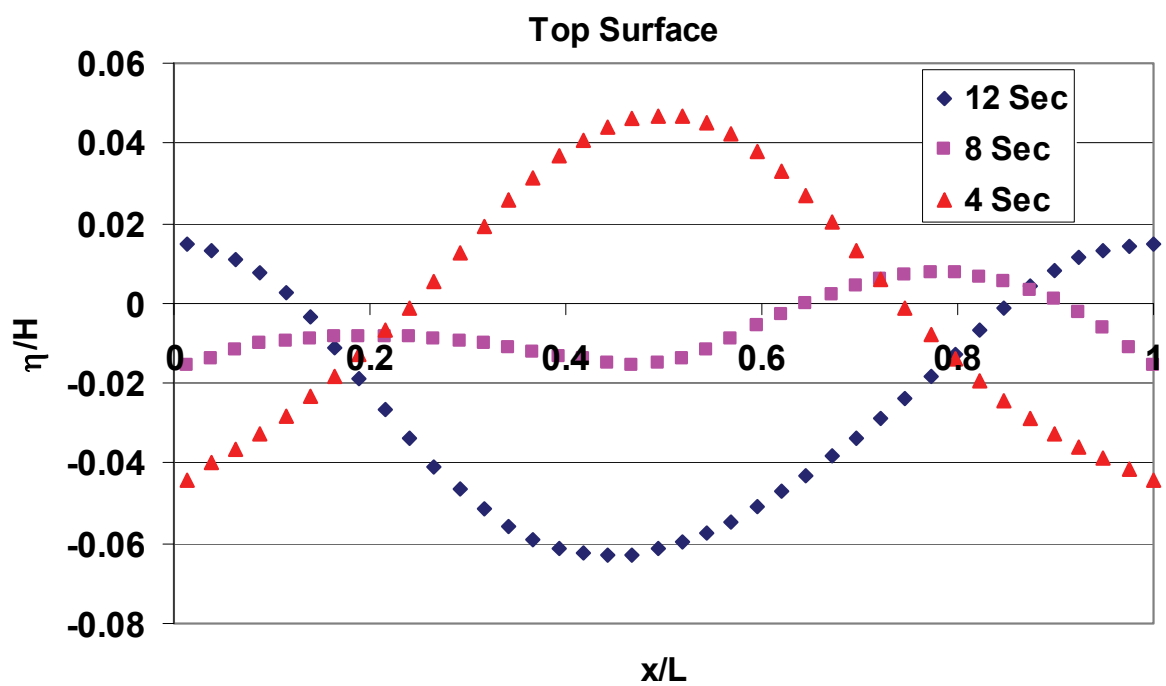Fig. 8. Simulation results for the propagation of intermediate surface

Fig. 9. Simulation results for the propagation of top surface

7. Conclusion

Earlier developed governing equations for three layered long waves are complex having several interactions from adjacent/all layers, which make those equations very difficult to solve analytically. Analytical solutions for these sorts of equations are yet to be succeeded by anyone. To achieve a numerical solution, governing equations were transformed into numerical formulations. Original derived equations were simplified considering horizontal bottom (i.e. no variations of 'h' along x direction, $\partial h/\partial x = 0$). Numerical model was developed using staggered Leap-Frog scheme, as the same scheme produced good results for two layer numerical models.

At the beginning of the computations, all the initial values of η_1, η_2, η_3, M_1, M_2 and M_3 were given as initial conditions. All the variables for the later time steps were computed as follows:

- using deduced finite difference Momentum equations for the upper, intermediate and lower layer all the M_1, M_2 and M_3 values were calculated
- then using the latest values of M_3 and deduced finite difference continuity equation for the lower layer all the values of η_3 were calculated
- then using the latest values of η_3, M_2 and deduced finite difference continuity equation for the intermediate layer, all the values of η_2 were calculated
- using the latest values of η_2, M_1 and deduced finite difference continuity equation for the upper layer, all the values of η_1 were calculated
- at right boundary all the values of M_1, M_2 and M_3 were calculated by characteristic method, using the values of previous time step and wave celerity
- using periodic condition, water levels at left boundary was used same as the right boundary

Model results for a pseudo two-layer case were compared with an earlier validated model for real two–layer mode. Agreements are very good and it can be concluded that developed three-layer model is capable to produce realistic results. Using the developed model, some scenarios were presented. From scenario simulations it is found that the lower surface gets amplified with the course of time, whereas the intermediate surface and top surface get dampen down with the course of time. Also, it is found that the propagations of lower and intermediate surfaces are relatively smooth. However, propagation of top surface is faster and having dramatic changes. These are because of the interactions from the adjacent layers.

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NOTATIONS

ρ = Density of fluid

M = Discharge per unit width of flow

η = Water surface elevation above still water level

h = Still water depth for a particular layer

D = Total depth of layer

α = Ratio of density of upper layer fluid to lower layer fluid

x = Distance along downstream direction

y = Distance perpendicular to x-direction

u = Uniform velocity over the depth along x-direction

v = Uniform velocity along y-direction

P = Hydrostatic pressure of fluid

β = Ratio of depths of lower layer to upper layer

C = Wave celerity

't' represents for time and subscripts '1', '2' and '3' denotes for upper layer, intermediate layer and bottom layer respectively.

The Tsunami Threat - Research and Technology

Edited by Nils-Axel Mörner

Submarine earthquakes, submarine slides and impacts may set large water volumes in motion characterized by very long wavelengths and a very high speed of lateral displacement, when reaching shallower water the wave breaks in over land - often with disastrous effects. This natural phenomenon is known as a tsunami event. By December 26, 2004, an event in the Indian Ocean, this word suddenly became known to the public. The effects were indeed disastrous and 227,898 people were killed. Tsunami events are a natural part of the Earth's geophysical system. There have been numerous events in the past and they will continue to be a threat to humanity; even more so today, when the coastal zone is occupied by so much more human activity and many more people. Therefore, tsunamis pose a very serious threat to humanity. The only way for us to face this threat is by increased knowledge so that we can meet future events by efficient warning systems and aid organizations. This book offers extensive and new information on tsunamis; their origin, history, effects, monitoring, hazards assessment and proposed handling with respect to precaution. Only through knowledge do we know how to behave in a wise manner. This book should be a well of tsunami knowledge for a long time, we hope.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Monzur Alam Imteaz, Fumihiko Imamura and Jamal Naser (2011). Numerical Model for Multi-Layered Tsunami Waves, The Tsunami Threat - Research and Technology, Nils-Axel Mörner (Ed.), ISBN: 978-953-307-552-5, InTech, Available from: http://www.intechopen.com/books/the-tsunami-threat-research-and-technology/numerical-model-for-multi-layered-tsunami-waves

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