23

Robust Sampled-Data Control Design of Uncertain Fuzzy Systems with Discrete and Distributed Delays

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1. Introduction

Nonlinear time-delay systems appear in many engineering systems and system formulations such as transportation systems, networked control systems, telecommunication systems, chemical processing systems, and power systems. Hence, it is important to analyze and synthesize such time-delay systems. Considerable research on nonlinear time-delay systems has been made via fuzzy system approach in (2), (6), (9), (12), (13) where stability conditions of fuzzy systems with discrete delays have been given in terms of Linear Matrix Inequalities (LMIs). Takagi-Sugeno fuzzy systems, described by a set of if-then rules which gives local linear models of an underlying system, represent a wide class of nonlinear systems. In the last two decade, Takagi-Sugeno fuzzy system has been extensively used for nonlinear control systems since it can universally approximate or exactly describe general nonlinear systems((8)). Theory has been extended to fuzzy systems with distributed delays in (7), (11), (15). Those results are based on continuous-time delay systems. From a practical point of view, sampled-data control is of importance. However, only a few results on sampled-data control for fuzzy system with discrete delays have been given in the literature ((1), (5), (14), (16)). Sampled-data controller design has been made for fuzzy systems with distributed delays in (3) and (4). To the best of our knowledge, no result for fuzzy sampled-data control systems with neutral and distributed delays has appeared yet.

In this paper, we propose a design method for robust sampled-data control of uncertain fuzzy systems with discrete, neutral and distributed delays. A zero-order sampled-data control can be regarded as a delayed control. Hence, a time-varying delay system approach is taken to design a sampled-data controller. We first obtain a stability condition by introducing an appropriate Lyapunov-Krasovskii functional with free weighting matrices, which reduce the conservatism in our stability condition. Then, based on such an LMI condition, we propose a robust sampled-data control design method of fuzzy uncertain systems with discrete, neutral and distributed delays. We also propose a sampled-data observer design method of fuzzy time-delay systems. A similar approach is taken for analysis of a sampled-data observer, and a condition for an existence of an observer is given by another LMI, which is a dual result of stabilizing controller. Finally, we give some illustrative examples to show our design procedures for sampled-data controller and observer.

2. Fuzzy time-delay systems

In this section, we introduce Takagi-Sugeno fuzzy systems with discrete, neutral and distributed delays. Consider the Takagi-Sugeno fuzzy time-delay model, described by the following IF-THEN rule:

$$
\begin{array}{ll}
\textbf{IF} & \xi_1 \text{ is } M_{i1} \text{ and } \cdots \text{ and } \xi_p \text{ is } M_{ip}, \\
\textbf{THEN} & \dot{x}(t) - (A_{ni} + \Delta A_{ni})\dot{x}(t-\gamma) = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-\alpha(t)) \\
& \qquad +(D_i + \Delta D_i)\displaystyle\int_{t-\beta}^{t} x(s)ds + (B_i + \Delta B_i)u(t), \\
& y(t) = C_i x(t),\ i = 1, \cdots, r
\end{array}
$$

where $\alpha(t), \beta$ and γ are time-varying discrete delay, constant distributed delay, and constant neutral delay, respectively. They may be unknown but they satisfy $0 \le \alpha(t) \le \alpha_M$, $\dot{\alpha}(t) \le d < 1$, $0 \le \beta \le \beta_M$, $0 \le \gamma \le \gamma_M$ where α_M, d, β_M and γ_M are known numbers. $x(t) \in \Re^n$ is the state and $u(t) \in \Re^m$ is the input. The matrices A_i, A_{di}, A_{ni}, B_i and D_i are of appropriate dimensions. r is the number of IF-THEN rule. M_{ij} is a fuzzy set and $\xi_1, \cdots, \xi_p$ are premise variables. We set $\xi = \left[\xi_1, \cdots, \xi_p\right]^T$ and $\xi(t)$ is assumed to be available. The uncertain matrices are of the form

$$
\begin{aligned}
&\left[\ \Delta A_i(t) \quad \Delta A_{di}(t) \quad \Delta A_{ni}(t) \quad \Delta B_i(t) \quad \Delta D_i(t)\ \right] \\
&\qquad = H_i F_i(t) \left[\ E_{1i} \quad E_{2i} \quad E_{3i} \quad E_{bi} \quad E_{di}\ \right],\ i = 1, \cdots, r
\end{aligned}
$$

where H_i, E_{1i}, E_{2i}, E_{3i}, E_{bi} and E_{di} are known matrices of appropriate dimensions, and each $F_i(t)$ is unknown real time varying matrices satisfying

$$F_i^T(t)F_i(t) \le I,\ \ i = 1, \cdots, r.$$

The system is defined as follows:

$$
\begin{aligned}
\dot{x}(t) \quad - \quad & \sum_{i=1}^{r} \lambda_i(\xi(t))(A_{ni} + \Delta A_{ni})\dot{x}(t-\gamma) = \sum_{i=1}^{r} \lambda_i(\xi(t))\{(A_i + \Delta A_i)x(t) \\
& +(A_{di} + \Delta A_{di})x(t-\alpha(t)) + (D_i + \Delta D_i)\int_{t-\beta}^{t} x(s)ds + (B_i + \Delta B_i)u(t)\}, \qquad (1) \\
y(t) \quad = \quad & \sum_{i=1}^{r} \lambda_i(\xi(t))C_i x(t)
\end{aligned}
$$

where $\lambda_i(\xi) = \frac{\mu_i(\xi)}{\sum_{i=1}^{r}\mu_i(\xi)}$, $\mu_i(\xi) = \prod_{j=1}^{q} M_{ij}(\xi_j)$ and $M_{ij}(\cdot)$ is the grade of the membership function of M_{ij}. We assume $\mu_i(\xi(t)) \ge 0$, $i = 1, \cdots, r$, $\sum_{i=1}^{r}\mu_i(\xi(t)) > 0$ for any $\xi(t)$. Hence $\lambda_i(\xi(t))$ satisfy $\lambda_i(\xi(t)) \ge 0$, $i = 1, \cdots, r$, $\sum_{i=1}^{r}\lambda_i(\xi(t)) = 1$ for any $\xi(t)$. We consider the sampled-data control input. It may be represented as delayed control as follows:

$$u(t) = u_d(t_k) = u_d(t - (t - t_k)) = u_d(t - h(t)),\ t_k \le t \le t_{k+1}$$

where u_d is a zero-order control signal and the time-varying delay $0 \le h(t) = t - t_k$ is piecewise linear with the derivative $\dot{h}(t) = 1$ for $t \ne t_k$. A sampling time t_k is the time-varying sampling instant satisfying $0 < t_1 < t_2 < \cdots < t_k < \cdots$. Sampling interval $h_k = t_{k+1} - t_k$ may vary but it is bounded. Thus, we assume $h(t) \le t_{k+1} - t_k = h_k \le h_M$ for all t_k where h_M is known constant. We consider the following rules for a controller:

$$
\begin{array}{ll}
\textbf{IF} & \xi_1(t_k) \text{ is } M_{i1} \text{ and } \cdots \text{ and } \xi_p(t_k) \text{ is } M_{ip}, \\
\textbf{THEN} & u(t) = K_i x(t_k),\ i = 1, \cdots, r
\end{array}
$$

where K_i is to be determined. Then, the natural choice of a controller is given by

$$u(t) = \sum_{i=1}^{r} \lambda_i(\xi(t_k))K_i x(t_k). \tag{2}$$

We represent a piecewise control law as a continuous-time one with a time-varying piecewise continuous(continuous from the right) delay $h(t)$. Hence, we look for a state feedback controller of the form

$$u(t) = \sum_{i=1}^{r} \lambda_i(\xi(t_k))K_i x(t - h(t)). \tag{3}$$

that robustly stabilizes the system (1).The system is said to be robustly stable if it is asymptotically stable for all admissible uncertainties. The closed-loop system (1) with (3) becomes

$$\begin{aligned} \dot{x}(t) \quad - \quad & \sum_{i=1}^{r} \lambda_i(\xi(t))(A_{ni} + \Delta A_{ni})\dot{x}(t-\gamma) = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\big\{(A_i + \Delta A_i)x(t) \\ & +(A_{di} + \Delta A_{di})x(t - \alpha(t)) + (D_i + \Delta D_i)\int_{t-\beta}^{t} x(s)ds + (B_i + \Delta B_i)K_j x(t - h(t))\big\}. \end{aligned}$$

When we consider a nominal system, we have

$$\begin{aligned} \dot{x}(t) - \sum_{i=1}^{r} \lambda_i(\xi(t))A_{ni}\dot{x}(t-\gamma) \quad = \quad & \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\big\{A_i x(t) + A_{di}x(t-\alpha(t)) \\ & +D_i \int_{t-\beta}^{t} x(s)ds + B_i K_j x(t - h(t))\big\}. \end{aligned} \tag{4}$$

3. Stability analysis

First, we make stability analysis of the nominal closed-loop system (4).

Theorem 3.1 *Given control gain matrices K_i, $i = 1, \cdots, r$, the closed-loop system (4) is asymptotically stable if there exist matrices $P_i > 0$, $R \geq 0$, $X > 0$, $Y_i > 0$, $i = 1,2,3$, $Q_i \geq 0$, $Z_i > 0$, $i = 1,2$, and*

$$\begin{aligned} N_{ij} &= \begin{bmatrix} N_{1ij}^T & N_{2ij}^T & N_{3ij}^T & N_{4ij}^T & N_{5ij}^T & N_{6ij}^T & N_{7ij}^T & N_{8ij}^T & N_{9ij}^T \end{bmatrix}^T, \\ S_{ij} &= \begin{bmatrix} S_{1ij}^T & S_{2ij}^T & S_{3ij}^T & S_{4ij}^T & S_{5ij}^T & S_{6ij}^T & S_{7ij}^T & S_{8ij}^T & S_{9ij}^T \end{bmatrix}^T, \\ M_{ij} &= \begin{bmatrix} M_{1ij}^T & M_{2ij}^T & M_{3ij}^T & M_{4ij}^T & M_{5ij}^T & M_{6ij}^T & M_{7ij}^T & M_{8ij}^T & M_{9ij}^T \end{bmatrix}^T, \\ V_{ij} &= \begin{bmatrix} V_{1ij}^T & V_{2ij}^T & V_{3ij}^T & V_{4ij}^T & V_{5ij}^T & V_{6ij}^T & V_{7ij}^T & V_{8ij}^T & V_{9ij}^T \end{bmatrix}^T, \\ W_{ij} &= \begin{bmatrix} W_{1ij}^T & W_{2ij}^T & W_{3ij}^T & W_{4ij}^T & W_{5ij}^T & W_{6ij}^T & W_{7ij}^T & W_{8ij}^T & W_{9ij}^T \end{bmatrix}^T, \\ O_{ij} &= \begin{bmatrix} O_{1ij}^T & O_{2ij}^T & O_{3ij}^T & O_{4ij}^T & O_{5ij}^T & O_{6ij}^T & O_{7ij}^T & O_{8ij}^T & O_{9ij}^T \end{bmatrix}^T, \ i,j = 1, \cdots, r, \end{aligned}$$

$$T = \begin{bmatrix} T_1^T & T_2^T & T_3^T & T_4^T & T_5^T & T_6^T & T_7^T & T_8^T & T_9^T \end{bmatrix}^T$$

such that

$$\Phi_{ij} = \begin{bmatrix} \Phi_{11ij} & \Phi_{12ij} \\ \Phi_{12ij}^T & \Phi_{22} \end{bmatrix} < 0, \ i, j = 1, \cdots, r \tag{5}$$

where

$$\begin{aligned}
\Phi_{11ij} &= \Phi_1 + \Phi_{2ij} + \Phi_{2ij}^T + \Phi_{3ij} + \Phi_{3ij}^T, \\
\Phi_1 &= \begin{bmatrix}
\Phi_{111} & 0 & 0 & 0 & 0 & 0 & 0 & P_2 & P_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -R & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1-d)Q_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -P_2 & 0 \\
0 & 0 & 0 & 0 & 0 & -Q_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
P_2 & 0 & 0 & 0 & -P_2 & 0 & 0 & -X - \frac{1}{\beta_M}U & 0 \\
P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{199}
\end{bmatrix}, \\
\Phi_{111} &= Q_1 + R + \beta_M U + \beta_M^2 X, \\
\Phi_{199} &= Q_2 + \alpha_M Y_1 + \beta_M Y_2 + \gamma_M Y_3 + h_M(Z_1 + Z_2), \\
\Phi_{2ij} &= \begin{bmatrix} N_{ij} + M_{ij} + W_{ij} + O_{ij} + V_{ij} & -N_{ij} + S_{ij} & -M_{ij} - S_{ij} & -W_{ij} \end{bmatrix. \\
&\qquad\qquad \begin{matrix} -O_{ij} & 0 & -V_{ij} & 0 & 0 \end{bmatrix}, \\
\Phi_{3ij} &= \begin{bmatrix} -TA_i & -TB_iK_j & 0 & -TA_{di} & 0 & -TA_{ni} & 0 & -TD_i & T \end{bmatrix}, \\
\Phi_{12ij} &= \begin{bmatrix} h_M N_{ij} & h_M S_{ij} & h_M M_{ij} & \alpha_M W_{ij} & \beta_M O_{ij} & \gamma_M V_{ij} \end{bmatrix}, \\
\Phi_{22} &= \mathrm{diag}\begin{bmatrix} -h_M Z_1 & -h_M Z_1 & -h_M Z_2 & -\alpha_M Y_1 & -\beta_M Y_2 & -\gamma_M Y_3 \end{bmatrix}.
\end{aligned}$$

Proof: First, it follows from the Leibniz-Newton formula that the following equations hold for any matrices N_{ij}, S_{ij}, M_{ij}, V_{ij}, W_{ij} and O_{ij}, the forms of which are given in Theorem 3.1.

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)N_{ij}\left[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t}\dot{x}(s)ds\right] = 0, \tag{6}$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)S_{ij}\left[x(t-h(t)) - x(t-h_M) - \int_{t-h_M}^{t-h(t)}\dot{x}(s)ds\right] = 0, \tag{7}$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)M_{ij}\left[x(t) - x(t-h_M) - \int_{t-h_M}^{t}\dot{x}(s)ds\right] = 0, \tag{8}$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)V_{ij}\left[x(t) - x(t-\gamma) - \int_{t-\gamma}^{t}\dot{x}(s)ds\right] = 0, \tag{9}$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)W_{ij}\left[x(t) - x(t-\alpha(t)) - \int_{t-\alpha(t)}^{t}\dot{x}(s)ds\right] = 0, \tag{10}$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)O_{ij}\left[x(t) - x(t-\beta) - \int_{t-\beta}^{t}\dot{x}(s)ds\right] = 0 \tag{11}$$

where

$$\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t-h(t)) & x^T(t-h_M) & x^T(t-\alpha(t)) & x^T(t-\beta) \\ & & \dot{x}(t-\gamma) & x(t-\gamma) & \int_{t-\beta}^{t} x^T(s)ds & \dot{x}^T(t) \end{bmatrix}^T.$$

It is also clear from the closed-loop system (4) that the following is true for any matrix T.

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)T\big[\dot{x}(t) - A_i x(t) \\ -A_{di}x(t-\alpha(t)) - A_{ni}\dot{x}(t-\gamma) - D_i\int_{t-\beta}^{t} x(s)ds - B_iK_jx(t-h(t))\big] = 0. \quad (12)$$

Now, we consider the following Lyapunov-Krasovskii functional:

$$V(x_t) = V_1(x) + V_2(x_t) + V_3(x_t) + V_4(x_t)$$

where $x_t = x(t+\theta)$, $-\max(h_M, \alpha_M, \beta_M) \le \theta \le 0$,

$$V_1(x) = x^T(t)P_1x(t) + \left[\int_{t-\beta}^{t} x(s)ds\right]^T P_2 \int_{t-\beta}^{t} x(s)ds,$$

$$V_2(x_t) = \int_{t-\alpha(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-\gamma}^{t} \dot{x}^T(s)Q_2\dot{x}(s)ds + \int_{t-h_M}^{t} x^T(s)Rx(s)ds,$$

$$V_3(x_t) = \int_{-\beta}^{0}\int_{t+\theta}^{t} x^T(s)Ux(s)dsd\theta + \int_{-\alpha_M}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Y_1\dot{x}(s)dsd\theta \\ + \int_{-\beta}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Y_2\dot{x}(s)dsd\theta + \int_{-\gamma}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Y_3\dot{x}(s)dsd\theta \\ + \int_{-h_M}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)dsd\theta,$$

$$V_4(x_t) = \int_{t-\beta}^{t}\left[\int_{\theta}^{t} x^T(s)ds\right]X\left[\int_{\theta}^{t} x(s)ds\right]d\theta + \int_{0}^{\beta}\int_{t-\theta}^{t}(s-t+\theta)x^T(s)Xx(s)dsd\theta,$$

and $P_i > 0$, $R \ge 0$, $U > 0$, $X > 0$, $Y_i > 0$, $i = 1,2,3$, $Q_i \ge 0$, $Z_i > 0$, $i = 1,2$ are to be determined. We take the derivative of $V(x_t)$ with respect to t along the solution of the system (4) and add the left-hand-sides of (6)-(12):

$$
\begin{aligned}
\dot{V}(x_t) \leq\ & 2\dot{x}^T(t)P_1x(t) + 2x^T(t)P_2\int_{t-\beta}^{t} x(s)ds - 2x^T(t-\beta)P_2\int_{t-\beta}^{t} x(s)ds \\
& + x^T(t)(Q_1 + R + \beta_M U + \beta_M^2 X)x(t) - (1-d)x^T(t-\alpha(t))Q_1x(t-\alpha(t)) \\
& - \dot{x}^T(t-\gamma)Q_2\dot{x}(t-\gamma) - x^T(t-h_M)Rx(t-h_M) \\
& - \left[\int_{t-\beta}^{t} x(s)ds\right]^T \frac{1}{\beta_M} U\left[\int_{t-\beta}^{t} x(s)ds\right] \\
& + \dot{x}^T(t)[Q_2 + \alpha_M Y_1 + \beta_M Y_2 + \gamma_M Y_3 + h_M(Z_1 + Z_2)]\dot{x}(t) \\
& - \int_{t-\alpha_M}^{t} x^T(s)Y_1x(s)ds - \int_{t-\beta}^{t} \dot{x}^T(s)Y_2\dot{x}(s)ds - \int_{t-\gamma}^{t} \dot{x}^T(s)Y_3\dot{x}(s)ds \\
& - \int_{t-h(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-h_M}^{t-h(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds \\
& - \int_{t-h_M}^{t} \dot{x}^T(s)Z_2\dot{x}(s)ds - \left[\int_{t-\beta}^{t} x^T(s)ds\right] X\left[\int_{t-\beta}^{t} x(s)ds\right] \\
& + 2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)N_{ij}\left[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} \dot{x}(s)ds\right] \\
& + 2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)S_{ij}\left[x(t-h(t)) - x(t-h_M) - \int_{t-h_M}^{t-h(t)} \dot{x}(s)ds\right] \\
& + 2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)M_{ij}\left[x(t) - x(t-h_M) - \int_{t-h_M}^{t} \dot{x}(s)ds\right] \\
& + 2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)V_{ij}\left[x(t) - x(t-\gamma) - \int_{t-\gamma}^{t} \dot{x}(s)ds\right] \\
& + 2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)W_{ij}\left[x(t) - x(t-\alpha(t)) - \int_{t-\alpha(t)}^{t} \dot{x}(s)ds\right] \\
& + 2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)O_{ij}\left[x(t) - x(t-\beta) - \int_{t-\beta}^{t} \dot{x}(s)ds\right] \\
& + 2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)T[\dot{x}(t) - A_ix(t) - A_{di}x(t-\alpha(t)) \\
& - A_{ni}\dot{x}(t-\gamma) - D_i \textstyle\int_{t-\beta}^{t} x(s)ds - B_iK_jx(t-h(t))\Big] \\
\leq\ & \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\Big\{\zeta^T(t)\Psi_{ij}\zeta(t) \\
& - \int_{t-h(t)}^{t} \left[\zeta^T(t)N_{ij} + \dot{x}^T(s)Z_1\right] Z_1^{-1}\left[N_{ij}^T\zeta(t) + Z_1\dot{x}(s)\right] ds \\
& - \int_{t-h_M}^{t-h(t)} \left[\zeta^T(t)S_{ij} + \dot{x}^T(s)Z_1\right] Z_1^{-1}\left[S_{ij}^T\zeta(t) + Z_1\dot{x}(s)\right] ds \\
& - \int_{t-h_M}^{t} \left[\zeta^T(t)M_{ij} + \dot{x}^T(s)Z_2\right] Z_2^{-1}\left[M_{ij}^T\zeta(t) + Z_2\dot{x}(s)\right] ds \\
& - \int_{t-\alpha(t)}^{t} \left[\zeta^T(t)W_{ij} + \dot{x}^T(s)Y_1\right] Y_1^{-1}\left[W_{ij}^T\zeta(t) + Y_1\dot{x}(s)\right] ds \\
& - \int_{t-\beta}^{t} \left[\zeta^T(t)O_{ij} + \dot{x}^T(s)Y_2\right] Y_2^{-1}\left[O_{ij}^T\zeta(t) + Y_2\dot{x}(s)\right] ds
\end{aligned}
$$

$$-\int_{t-\gamma}^{t}\left[\zeta^T(t)V_{ij}+\dot{x}^T(s)Y_3\right]Y_3^{-1}\left[V_{ij}^T\zeta(t)+Y_3\dot{x}(s)\right]ds\Bigg\} \tag{13}$$

where

$$\begin{aligned}\Psi_{ij} &= \Phi_{11ij}+h_M(N_{ij}Z_1^{-1}N_{ij}^T+S_{ij}Z_1^{-1}S_{ij}^T+M_{ij}Z_2^{-1}M_{ij}^T)+\alpha_M W_{ij}Y_1^{-1}W_{ij}^T\\ &\quad +\beta_M O_{ij}Y_2^{-1}O_{ij}^T+\gamma_M V_{ij}Y_3^{-1}V_{ij}^T.\end{aligned}$$

Now, if (5) is satisfied, then by Schur complement formula we have

$$\Psi_{ij}<0,\ i,j=1,\cdots,r. \tag{14}$$

If (14) holds, we have $\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)\Psi_{ij}\zeta(t)<0$, which implies that $\dot{V}(x_t)$ <0 because $Y_i>0$, $Z_i>0$, $i=1,2$ and the last five terms in (13) are all less than 0. This proves that conditions (5) suffice to show the asymptotic stability of the system (4).

4. Sampled-data control design

In this section, we seek a design method of a sampled-data control for fuzzy time-delay systems based on Theorem 3.1. Unfortunately, however, Theorem 3.1 does not give feasible LMI conditions for obtaining state feedback control gain matrices K_i. To this end, we take an appropriate congruence transformation to obtain feasible LMI conditions and a design method of a sampled-data state feedback controller.

Theorem 4.1 *Given scalars ρ_i, $i=1,\cdots,9$, the sampled-data controller (2) asymptotically stabilizes the nominal system (4) if there exist matrices $\bar{P}_i>0$, $\bar{R}\geq 0$, $\bar{U}>0$, $\bar{X}>0$, $\bar{Y}_i>0$, $i=1,2,3$, $\bar{Q}_i\geq 0$, $\bar{Z}_i>0$, $i=1,2$, L, G_j, $j=1,\cdots,r$,*

$$\begin{aligned}
\bar{N}_{ij} &= \begin{bmatrix}\bar{N}_{1ij}^T & \bar{N}_{2ij}^T & \bar{N}_{3ij}^T & \bar{N}_{4ij}^T & \bar{N}_{5ij}^T & \bar{N}_{6ij}^T & \bar{N}_{7ij}^T & \bar{N}_{8ij}^T & \bar{N}_{9ij}^T\end{bmatrix}^T,\\
\bar{S}_{ij} &= \begin{bmatrix}\bar{S}_{1ij}^T & \bar{S}_{2ij}^T & \bar{S}_{3ij}^T & \bar{S}_{4ij}^T & \bar{S}_{5ij}^T & \bar{S}_{6ij}^T & \bar{S}_{7ij}^T & \bar{S}_{8ij}^T & \bar{S}_{9ij}^T\end{bmatrix}^T,\\
\bar{M}_{ij} &= \begin{bmatrix}\bar{M}_{1ij}^T & \bar{M}_{2ij}^T & \bar{M}_{3ij}^T & \bar{M}_{4ij}^T & \bar{M}_{5ij}^T & \bar{M}_{6ij}^T & \bar{M}_{7ij}^T & \bar{M}_{8ij}^T & \bar{M}_{9ij}^T\end{bmatrix}^T,\\
\bar{V}_{ij} &= \begin{bmatrix}\bar{V}_{1ij}^T & \bar{V}_{2ij}^T & \bar{V}_{3ij}^T & \bar{V}_{4ij}^T & \bar{V}_{5ij}^T & \bar{V}_{6ij}^T & \bar{V}_{7ij}^T & \bar{V}_{8ij}^T & \bar{V}_{9ij}^T\end{bmatrix}^T,\\
\bar{W}_{ij} &= \begin{bmatrix}\bar{W}_{1ij}^T & \bar{W}_{2ij}^T & \bar{W}_{3ij}^T & \bar{W}_{4ij}^T & \bar{W}_{5ij}^T & \bar{W}_{6ij}^T & \bar{W}_{7ij}^T & \bar{W}_{8ij}^T & \bar{W}_{9ij}^T\end{bmatrix}^T,\\
\bar{O}_{ij} &= \begin{bmatrix}\bar{O}_{1ij}^T & \bar{O}_{2ij}^T & \bar{O}_{3ij}^T & \bar{O}_{4ij}^T & \bar{O}_{5ij}^T & \bar{O}_{6ij}^T & \bar{O}_{7ij}^T & \bar{O}_{8ij}^T & \bar{O}_{9ij}^T\end{bmatrix}^T,\ i,j=1,\cdots,r
\end{aligned}$$

such that

$$\Theta_{ij}=\begin{bmatrix}\Theta_{11ij} & \Theta_{12ij}\\ \Theta_{12ij}^T & \Theta_{22}\end{bmatrix}<0,\ i,j=1,\cdots,r \tag{15}$$

where

$$
\begin{aligned}
\Theta_{11ij} &= \Theta_1 + \Theta_{2ij} + \Theta_{2ij}^T + \Theta_{3ij} + \Theta_{3ij}^T, \\
\Theta_1 &= \begin{bmatrix}
\Theta_{111} & 0 & 0 & 0 & 0 & 0 & 0 & \bar{P}_2 & \bar{P}_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\bar{R} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1-d)\bar{Q}_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{P}_2 & 0 \\
0 & 0 & 0 & 0 & 0 & -Q_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{P}_2 & 0 & 0 & 0 & -\bar{P}_2 & 0 & 0 & -\bar{X} - \frac{1}{\beta_M}\bar{U} & 0 \\
\bar{P}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Theta_{199}
\end{bmatrix}, \\
\Theta_{111} &= \bar{Q}_1 + \bar{R} + \beta_M \bar{U} + \beta_M^2 \bar{X}, \\
\Theta_{199} &= Q_2 + \alpha_M \bar{Y}_1 + \beta_M \bar{Y}_2 + \gamma_M \bar{Y}_3 + h_M(\bar{Z}_1 + \bar{Z}_2), \\
\Theta_{2ij} &= \big[\ \bar{N}_{ij} + \bar{M}_{ij} + \bar{W}_{ij} + \bar{O}_{ij} + \bar{V}_{ij} \quad -\bar{N}_{ij} + \bar{S}_{ij} \quad -\bar{M}_{ij} - \bar{S}_{ij} \quad -\bar{W}_{ij} \\
&\qquad\qquad -\bar{O}_{ij} \quad 0 \quad -\bar{V}_{ij} \quad 0 \quad 0\ \big],
\end{aligned}
$$

$$
\begin{aligned}
\Theta_{3ij} &= -\begin{bmatrix}
\rho_1 A_i L^T & \rho_1 B_i G_j & 0 & \rho_1 A_{di} L^T & 0 & \rho_1 A_{ni} L^T & 0 & \rho_1 D_i L^T & -\rho_1 L^T \\
\rho_2 A_i L^T & \rho_2 B_i G_j & 0 & \rho_2 A_{di} L^T & 0 & \rho_2 A_{ni} L^T & 0 & \rho_2 D_i L^T & -\rho_2 L^T \\
\rho_3 A_i L^T & \rho_3 B_i G_j & 0 & \rho_3 A_{di} L^T & 0 & \rho_3 A_{ni} L^T & 0 & \rho_3 D_i L^T & -\rho_3 L^T \\
\rho_4 A_i L^T & \rho_4 B_i G_j & 0 & \rho_4 A_{di} L^T & 0 & \rho_4 A_{ni} L^T & 0 & \rho_4 D_i L^T & -\rho_4 L^T \\
\rho_5 A_i L^T & \rho_5 B_i G_j & 0 & \rho_5 A_{di} L^T & 0 & \rho_5 A_{ni} L^T & 0 & \rho_5 D_i L^T & -\rho_5 L^T \\
\rho_6 A_i L^T & \rho_6 B_i G_j & 0 & \rho_6 A_{di} L^T & 0 & \rho_6 A_{ni} L^T & 0 & \rho_6 D_i L^T & -\rho_6 L^T \\
\rho_7 A_i L^T & \rho_7 B_i G_j & 0 & \rho_7 A_{di} L^T & 0 & \rho_7 A_{ni} L^T & 0 & \rho_7 D_i L^T & -\rho_7 L^T \\
\rho_8 A_i L^T & \rho_8 B_i G_j & 0 & \rho_8 A_{di} L^T & 0 & \rho_8 A_{ni} L^T & 0 & \rho_8 D_i L^T & -\rho_8 L^T \\
\rho_9 A_i L^T & \rho_9 B_i G_j & 0 & \rho_9 A_{di} L^T & 0 & \rho_9 A_{ni} L^T & 0 & \rho_9 D_i L^T & -\rho_9 L^T
\end{bmatrix}, \\
\Theta_{12ij} &= \big[\ h_M \bar{N}_{ij} \quad h_M \bar{S}_{ij} \quad h_M \bar{M}_{ij} \quad \alpha_M \bar{W}_{ij} \quad \beta_M \bar{O}_{ij} \quad \gamma_M \bar{V}_{ij}\ \big], \\
\Theta_{22} &= \mathrm{diag}\big[\ -h_M \bar{Z}_1 \quad -h_M \bar{Z}_1 \quad -h_M \bar{Z}_2 \quad -\alpha_M \bar{Y}_1 \quad -\beta_M \bar{Y}_2 \quad -\gamma_M \bar{Y}_3\ \big].
\end{aligned}
$$

In this case, state feedback control gains in (2) are given by

$$K_i = G_i L^{-T},\ i = 1, \cdots, r. \tag{16}$$

Proof: We let $T_i = \rho_i \bar{L}$, $i = 1, \cdots, 9$ where each ρ_i is given and $\bar{L}$ is defined later, and substitute them into (5). If (5) holds, it follows that (9,9)-block of Φ_{11ij} must be negative definite. It follows that $T_9 + T_9^T = \rho_9(\bar{L} + \bar{L}^T) < 0$, which implies that $\bar{L}$ is nonsingular. Then, we define $\bar{L} = L^{-1}$ and calculate $\Theta_{ij} = \Sigma \Phi_{ij} \Sigma^T$ with $\Sigma = \mathrm{diag}[\,L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L\,]$. Defining $\bar{P}_i = LP_iL^T$, $\bar{R} = LRL^T$, $\bar{X} = LXL^T$, $\bar{Y}_i = LY_iL^T$, $i = 1,2,3$, $\bar{Q}_i = LQ_1L^T$, $\bar{Z}_i = LZ_iL^T$, $i = 1,2$, $\bar{N}_{kij} = LN_{kij}L^T$, $\bar{S}_{kij} = LS_{kij}L^T$, $\bar{M}_{kij} = LM_{kij}L^T$, $\bar{V}_{kij} = LV_{kij}L^T$, $\bar{W}_{kij} = LW_{kij}L^T$, $\bar{O}_{kij} = LO_{kij}L^T$, $i,j = 1, \cdots, r, k = 1, \cdots, 9$, we obtain Θ_{ij} in (15) where we let $G_j = K_jL^T$. If the conditions (15) hold, state feedback control gain matrix K_i is obviously given by (16).
We extend the result to the case of the uncertain system (1). The following lemma is necessary to prove Theorem 4.3.

Lemma 4.2 *((10)) Given matrices $Q = Q^T$, H, E and $R = R^T > 0$ of appropriate dimensions*

$$Q + HF(t)E + E^T F^T(t)H^T < 0$$

for all $F(t)$ satisfying $F^T(t)F(t) \le R$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$Q + \frac{1}{\varepsilon}HH^T + \varepsilon E^T RE < 0.$$

Theorem 4.3 *Given scalars ρ_i, $i = 1, \cdots, 9$, the sampled-data controller (2) robustly stabilizes the uncertain system (1) if there exist matrices $\bar{P}_i > 0$, $\bar{Q}_i \ge 0$, $\bar{Z}_i > 0$, $i = 1,2$, $\bar{R} \ge 0$, $\bar{U} > 0$, $\bar{X} > 0$, $\bar{Y}_i > 0$, $i = 1,2,3$, L, G_j, $j = 1, \cdots, r$, and*

$$\begin{aligned}
\bar{N}_{ij} &= \begin{bmatrix} \bar{N}_{1ij}^T & \bar{N}_{2ij}^T & \bar{N}_{3ij}^T & \bar{N}_{4ij}^T & \bar{N}_{5ij}^T & \bar{N}_{6ij}^T & \bar{N}_{7ij}^T & \bar{N}_{8ij}^T & \bar{N}_{9ij}^T \end{bmatrix}^T, \\
\bar{S}_{ij} &= \begin{bmatrix} \bar{S}_{1ij}^T & \bar{S}_{2ij}^T & \bar{S}_{3ij}^T & \bar{S}_{4ij}^T & \bar{S}_{5ij}^T & \bar{S}_{6ij}^T & \bar{S}_{7ij}^T & \bar{S}_{8ij}^T & \bar{S}_{9ij}^T \end{bmatrix}^T, \\
\bar{M}_{ij} &= \begin{bmatrix} \bar{M}_{1ij}^T & \bar{M}_{2ij}^T & \bar{M}_{3ij}^T & \bar{M}_{4ij}^T & \bar{M}_{5ij}^T & \bar{M}_{6ij}^T & \bar{M}_{7ij}^T & \bar{M}_{8ij}^T & \bar{M}_{9ij}^T \end{bmatrix}^T, \\
\bar{V}_{ij} &= \begin{bmatrix} \bar{V}_{1ij}^T & \bar{V}_{2ij}^T & \bar{V}_{3ij}^T & \bar{V}_{4ij}^T & \bar{V}_{5ij}^T & \bar{V}_{6ij}^T & \bar{V}_{7ij}^T & \bar{V}_{8ij}^T & \bar{V}_{9ij}^T \end{bmatrix}^T, \\
\bar{W}_{ij} &= \begin{bmatrix} \bar{W}_{1ij}^T & \bar{W}_{2ij}^T & \bar{W}_{3ij}^T & \bar{W}_{4ij}^T & \bar{W}_{5ij}^T & \bar{W}_{6ij}^T & \bar{W}_{7ij}^T & \bar{W}_{8ij}^T & \bar{W}_{9ij}^T \end{bmatrix}^T, \\
\bar{O}_{ij} &= \begin{bmatrix} \bar{O}_{1ij}^T & \bar{O}_{2ij}^T & \bar{O}_{3ij}^T & \bar{O}_{4ij}^T & \bar{O}_{5ij}^T & \bar{O}_{6ij}^T & \bar{O}_{7ij}^T & \bar{O}_{8ij}^T & \bar{O}_{9ij}^T \end{bmatrix}^T, \ i,j = 1, \cdots, r
\end{aligned}$$

and scalars $\varepsilon_{ij} > 0$, $i,j = 1, \cdots, r$ such that

$$\begin{bmatrix} \Theta_{ij} + \varepsilon_i \bar{H}_i^T \bar{H}_i & \bar{E}_{ij}^T \\ \bar{E}_{ij} & -\varepsilon_{ij} I \end{bmatrix} < 0, \ i,j = 1, \cdots, r \tag{17}$$

where Θ_{ij} is given in Theorem 4.1, and

$$\begin{aligned}
\bar{H}_i &= -\begin{bmatrix} \rho_1 H_i^T & \rho_2 H_i^T & \rho_3 H_i^T & \rho_4 H_i^T & \rho_5 H_i^T & \rho_6 H_i^T & \rho_7 H_i^T & \rho_8 H_i^T \\ & & & & & \rho_9 H_i^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
\bar{E}_{ij} &= \begin{bmatrix} E_{1i}L^T & E_{bi}G_j & 0 & E_{2i}L^T & 0 & E_{3i}L^T & 0 & E_{di}L^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
\end{aligned}$$

In this case, state feedback control gains in (2) are given by (16).

Proof: Replacing A_i, A_{di}, A_{ni}, B_i, D_i by $A_i + H_i F_i(t) E_{1i}$, $A_{di} + H_i F_i(t) E_{2i}$, $A_{ni} + H_i F_i(t) E_{3i}$, $B_i + H_i F_i(t) E_{bi}$, $D_i + H_i F_i(t) E_{di}$, we have

$$\Theta_{ij} + \bar{H}_i F_i(t) \bar{E}_{ij} + (\bar{H}_i F_i(t) \bar{E}_{ij})^T < 0, \ i,j = 1, \cdots, r.$$

It follows from Lemma 4.2 that the above LMIs hold if and only if there exist $\varepsilon_{ij} > 0$ such that

$$\Theta_{ij} + \varepsilon_{ij} \bar{H}_i \bar{H}_i + \frac{1}{\varepsilon_{ij}} \bar{E}_{ij} \bar{E}_{ij} < 0, \ i,j = 1, \cdots, r.$$

Applying the Schur complement formula, we have (17).

5. Numerical examples for controller design

Let us design robust sampled-data controllers for the system (1) with the following matrices.

$$
\begin{aligned}
& A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1.5 \end{bmatrix},\ A_{d1} = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix},\ A_{d2} = \begin{bmatrix} -1 & -1 \\ 0 & -1.4 \end{bmatrix}, \\
& A_{n1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},\ A_{n2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.1 \\ 1.2 \end{bmatrix}, \\
& D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix},\ H_1 = H_2 = 0.2I,\ E_{11} = E_{12} = 0.2I, \\
& E_{21} = E_{22} = 0.2I,\ E_{31} = E_{32} = 0.1I,\ E_{b1} = E_{b2} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},\ E_{d1} = E_{d2} = 0.1I.
\end{aligned}
$$

The grades are given by $\lambda_1(x_1) = \sin^2(x_1)$ and $\lambda_2(x_1) = 1 - \lambda_1(x_1)$. The maximum upper bound of the sampling time $h_M = 0.1$ and $d = 0.5$ are assumed. First, we let $\beta_M = \gamma_M = 0.1$. Theorem 4.3 with $\rho_1 = 5.46$, $\rho_2 = -0.01$, $\rho_3 = -2.19$, $\rho_4 = 0.60$, $\rho_5 = -0.01$, $\rho_6 = -0.01$, $\rho_7 = 0.50$, $\rho_8 = 0.10$, $\rho_9 = 1.96$ guarantees the existence of the sampled-data controller for the maximum upper bound of the time-delay $\alpha_M = 0.42$. In this case, control gains in (2) are given by

$$K_1 = \begin{bmatrix} -0.1800 & -0.9934 \end{bmatrix},\ K_2 = \begin{bmatrix} -0.1808 & -0.9942 \end{bmatrix}.$$

Next, we let $\alpha_M = \gamma_M = 0.1$. Theorem 4.3 with $\rho_1 = 5.74$, $\rho_2 = 0.50$, $\rho_3 = -2.19$, $\rho_4 = -0.60$, $\rho_5 = -0.01$, $\rho_6 = -0.42$, $\rho_7 = -0.50$, $\rho_8 = 0.16$, $\rho_9 = 1.96$ gives a robust sampled-data controller for the maximum upper bound $\beta_M = 3.43$. In this case, control gains in (2) are given by

$$K_1 = \begin{bmatrix} 0.1794 & -2.6198 \end{bmatrix},\ K_2 = \begin{bmatrix} 0.1795 & -2.6194 \end{bmatrix}.$$

Finally, we let $\alpha_M = \beta_M = 0.1$. Theorem 4.3 with $\rho_1 = 4.74$, $\rho_2 = -0.01$, $\rho_3 = -2.19$, $\rho_4 = -0.60$, $\rho_5 = -0.01$, $\rho_6 = 0.01$, $\rho_7 = -0.50$, $\rho_8 = 0.07$, $\rho_9 = 1.96$ gives a robust sampled-data controller for the maximum upper bound $\gamma_M = 2.90$. In this case, control gains in (2) are given by

$$K_1 = \begin{bmatrix} -0.0265 & -0.7535 \end{bmatrix},\ K_2 = \begin{bmatrix} -0.0260 & -0.7515 \end{bmatrix}.$$

6. Application to observer design

In this section, using the results in the previous sections we consider an observer design for the system (1), which estimates the state variables of the system using sampled-data measurement outputs. Here, we assume that the system does not contain any uncertain parameters so that it is given by

$$
\begin{aligned}
\dot{x}(t) - \sum_{i=1}^{r} \lambda_i(\xi(t)) A_{ni} \dot{x}(t-\gamma) &= \sum_{i=1}^{r} \lambda_i(\xi(t)) \{ A_i x(t) \\
&\quad + A_{di} x(t - \alpha(t)) + D_i \int_{t-\beta}^{t} x(s) ds + B_i u(t) \}, \quad (18) \\
y(t) &= \sum_{i=1}^{r} \lambda_i(\xi(t)) C_i x(t) \quad (19)
\end{aligned}
$$

where all the time delays are assumed to be measurable.
The sampled-data measurement output may be represented as delayed measurement as follows:

$$y(t) = y_d(t_k) = y_d(t-(t-t_k)) = y_d(t-h(t)),\ t_k \le t \le t_{k+1}$$

where y_d is a zero-order measurement signal and the time-varying delay $0 \le h(t) = t - t_k$ is piecewise linear with the derivative $\dot{h}(t) = 1$ for $t \neq t_k$ as before. We consider the following rules for a system to estimate the state variables:

IF $\quad \xi_1(t_k)$ is M_{i1} and $\cdots$ and $\xi_p(t_k)$ is M_{ip},

THEN $\quad \dot{\hat{x}}(t) - A_{ni}\dot{\hat{x}}(t-\gamma) = A_i\hat{x}(t) + A_{di}\hat{x}(t-\alpha(t)) + D_i\int_{t-\beta}^{t}\hat{x}(s)ds$
$\quad +B_iu(t) + \bar{K}(y(t_k) - C_i\hat{x}(t_k)),\ i = 1,\cdots,r$

where $\hat{x}$ is the estimated state and $\bar{K} = \sum_{j=1}^{r}\lambda_j(\xi(t_k))\bar{K}_j$ is an observer gain to be determined. Then, the overall system is given by

$$\begin{aligned}\dot{\hat{x}}(t) - \sum_{i=1}^{r}\lambda_i(\xi(t))A_{ni}\dot{\hat{x}}(t-\gamma) \quad &= \quad \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\{A_i\hat{x}(t) + A_{di}\hat{x}(t-\alpha(t)) \\ &\quad +D_i\int_{t-\beta}^{t}\hat{x}(s)ds + B_iu(t) + \bar{K}_j(y(t_k) - C_i\hat{x}(t_k))\}.\end{aligned} \tag{20}$$

where we see the measurement output as

$$y(t) = \sum_{i=1}^{r}\lambda_i(\xi(t_k))C_ix(t-h(t)).$$

It follows from (18), (19) and (20) that the error $e(t) = x(t) - \hat{x}(t)$ satisfies

$$\begin{aligned}\dot{e}(t) - \sum_{i=1}^{r}\lambda_i(\xi(t))A_{ni}\dot{e}(t-\gamma) \quad &= \quad \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\{A_ie(t) + A_{di}e(t-\alpha(t)) \\ &\quad +D_i\int_{t-\beta}^{t}e(s)ds - \bar{K}_jC_ie(t_k)\}.\end{aligned} \tag{21}$$

We shall find conditions for (21) to be asymptotically stable. In this case, (20) becomes an observer for the system (18) and (19).
The following theorem gives conditions for the error system (21) to be asymptotically stable.

Theorem 6.1 *Given control gain matrices $\bar{K}_i$, $i = 1, \cdots, r$, the error system (21) is asymptotically stable if there exist matrices $P_i > 0$, $R \geq 0$, $X > 0$, $Y_i > 0$, $i = 1,2,3$, $Q_i \geq 0$, $Z_i > 0$, $i = 1,2$, and*

$$\begin{aligned}
N_{ij} &= \begin{bmatrix} N_{1ij}^T & N_{2ij}^T & N_{3ij}^T & N_{4ij}^T & N_{5ij}^T & N_{6ij}^T & N_{7ij}^T & N_{8ij}^T & N_{9ij}^T \end{bmatrix}^T, \\
S_{ij} &= \begin{bmatrix} S_{1ij}^T & S_{2ij}^T & S_{3ij}^T & S_{4ij}^T & S_{5ij}^T & S_{6ij}^T & S_{7ij}^T & S_{8ij}^T & S_{9ij}^T \end{bmatrix}^T, \\
M_{ij} &= \begin{bmatrix} M_{1ij}^T & M_{2ij}^T & M_{3ij}^T & M_{4ij}^T & M_{5ij}^T & M_{6ij}^T & M_{7ij}^T & M_{8ij}^T & M_{9ij}^T \end{bmatrix}^T, \\
V_{ij} &= \begin{bmatrix} V_{1ij}^T & V_{2ij}^T & V_{3ij}^T & V_{4ij}^T & V_{5ij}^T & V_{6ij}^T & V_{7ij}^T & V_{8ij}^T & V_{9ij}^T \end{bmatrix}^T, \\
W_{ij} &= \begin{bmatrix} W_{1ij}^T & W_{2ij}^T & W_{3ij}^T & W_{4ij}^T & W_{5ij}^T & W_{6ij}^T & W_{7ij}^T & W_{8ij}^T & W_{9ij}^T \end{bmatrix}^T, \\
O_{ij} &= \begin{bmatrix} O_{1ij}^T & O_{2ij}^T & O_{3ij}^T & O_{4ij}^T & O_{5ij}^T & O_{6ij}^T & O_{7ij}^T & O_{8ij}^T & O_{9ij}^T \end{bmatrix}^T, \; i,j = 1, \cdots, r,
\end{aligned}$$

$$T = \begin{bmatrix} T_1^T & T_2^T & T_3^T & T_4^T & T_5^T & T_6^T & T_7^T & T_8^T & T_9^T \end{bmatrix}^T$$

such that

$$\Phi_{ij} = \begin{bmatrix} \Phi_{11ij} & \Phi_{12ij} \\ \Phi_{12ij}^T & \Phi_{22} \end{bmatrix} < 0, \; i,j = 1, \cdots, r \tag{22}$$

where

$$\begin{aligned}
\Phi_{11ij} &= \Phi_1 + \Phi_{2ij} + \Phi_{2ij}^T + \Phi_{3ij} + \Phi_{3ij}^T, \\
\Phi_1 &= \begin{bmatrix}
\Phi_{111} & 0 & 0 & 0 & 0 & 0 & 0 & P_2 & P_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -R & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1-d)Q_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -P_2 & 0 \\
0 & 0 & 0 & 0 & 0 & -Q_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
P_2 & 0 & 0 & 0 & -P_2 & 0 & 0 & -X - \frac{1}{\beta_M}U & 0 \\
P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{199}
\end{bmatrix}, \\
\Phi_{111} &= Q_1 + R + \beta_M U + \beta_M^2 X, \\
\Phi_{199} &= Q_2 + \alpha_M Y_1 + \beta_M Y_2 + \gamma_M Y_3 + h_M(Z_1 + Z_2), \\
\Phi_{2ij} &= \begin{bmatrix} N_{ij} + M_{ij} + W_{ij} + O_{ij} + V_{ij} & -N_{ij} + S_{ij} & -M_{ij} - S_{ij} & -W_{ij} \end{bmatrix. \\
&\qquad\qquad \begin{matrix} -O_{ij} & 0 & -V_{ij} & 0 & 0 \end{matrix} \big], \\
\Phi_{3ij} &= \begin{bmatrix} -TA_i & T\bar{K}_j C_i & 0 & -TA_{di} & 0 & -TA_{ni} & 0 & -TD_i & T \end{bmatrix}, \\
\Phi_{12ij} &= \begin{bmatrix} h_M N_{ij} & h_M S_{ij} & h_M M_{ij} & \alpha_M W_{ij} & \beta_M O_{ij} & \gamma_M V_{ij} \end{bmatrix}, \\
\Phi_{22} &= \mathrm{diag}\begin{bmatrix} -h_M Z_1 & -h_M Z_1 & -h_M Z_2 & -\alpha_M Y_1 & -\beta_M Y_2 & -\gamma_M Y_3 \end{bmatrix}.
\end{aligned}$$

Proof: Proof is similar to that of Theorem 3.1. We first note that the following is true for any matrix T.

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\bar{\zeta}^T(t)T\big[\dot{e}(t)-A_ie(t)$$
$$-A_{di}e(t-\alpha(t))-A_{ni}\dot{e}(t-\gamma)-D_i\int_{t-\beta}^{t}e(s)ds-\bar{K}_jC_ie(t-h(t))\big]=0 \tag{23}$$

where

$$\bar{\zeta}(t)=\begin{bmatrix} e^T(t) & e^T(t-h(t)) & e^T(t-h_M) & e^T(t-\alpha(t)) & e^T(t-\beta) \\ & & \dot{e}(t-\gamma) & e(t-\gamma) & \int_{t-\beta}^{t}e^T(s)ds & \dot{e}^T(t)\end{bmatrix}^T.$$

Now, we take the derivative of $V(e_t)$, which is defined as $V(x_t)$ with replacing x_t by e_t, with respect to t along the solution of the error system (21) and add the left-hand-sides of (6)-(11) with replacing x by e and (23):

$$\begin{aligned}
\dot{V}(e_t) \quad \le \quad & \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\Big\{\bar{\zeta}^T(t)\Xi_{ij}\bar{\zeta}(t) \\
& -\int_{t-h(t)}^{t}\Big[\bar{\zeta}^T(t)N_{ij}+\dot{e}^T(s)Z_1\Big]Z_1^{-1}\Big[N_{ij}^T\bar{\zeta}(t)+Z_1\dot{e}(s)\Big]ds \\
& -\int_{t-h_M}^{t-h(t)}\Big[\bar{\zeta}^T(t)S_{ij}+\dot{e}^T(s)Z_1\Big]Z_1^{-1}\Big[S_{ij}^T\bar{\zeta}(t)+Z_1\dot{x}(s)\Big]ds \\
& -\int_{t-h_M}^{t}\Big[\bar{\zeta}^T(t)M_{ij}+\dot{x}^T(s)Z_2\Big]Z_2^{-1}\Big[M_{ij}^T\bar{\zeta}(t)+Z_2\dot{e}(s)\Big]ds \\
& -\int_{t-\alpha(t)}^{t}\Big[\bar{\zeta}^T(t)W_{ij}+\dot{e}^T(s)Y_1\Big]Y_1^{-1}\Big[W_{ij}^T\bar{\zeta}(t)+Y_1\dot{e}(s)\Big]ds \\
& -\int_{t-\beta}^{t}\Big[\bar{\zeta}^T(t)O_{ij}+\dot{e}^T(s)Y_2\Big]Y_2^{-1}\Big[O_{ij}^T\bar{\zeta}(t)+Y_2\dot{e}(s)\Big]ds \\
& -\int_{t-\gamma}^{t}\Big[\bar{\zeta}^T(t)V_{ij}+\dot{e}^T(s)Y_3\Big]Y_3^{-1}\Big[V_{ij}^T\bar{\zeta}(t)+Y_3\dot{e}(s)\Big]ds\Big\}
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
\Xi_{ij} \quad = \quad & \Theta_{11ij}+h_M(N_{ij}Z_1^{-1}N_{ij}^T+S_{ij}Z_1^{-1}S_{ij}^T+M_{ij}Z_2^{-1}M_{ij}^T)+\alpha_M W_{ij}Y_1^{-1}W_{ij}^T \\
& +\beta_M O_{ij}Y_2^{-1}O_{ij}^T+\gamma_M V_{ij}Y_3^{-1}V_{ij}^T.
\end{aligned}$$

Now, if (22) is satisfied, then by Schur complement formula we have

$$\Theta_{ij}<0,\ i,j=1,\cdots,r. \tag{25}$$

If (25) holds, we have $\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\bar{\zeta}^T(t)\Psi_{ij}\bar{\zeta}(t)<0$, which implies that $\dot{V}(x_t)$ <0 because $Y_i>0$, $Z_i>0$, $i=1,2$ and the last five terms in (24) are all less than 0. This proves that conditions (22) suffice to show the asymptotic stability of the system (21).
Theorem 6.1 still does not propose an observer design method. Hence, we give the following result.

Theorem 6.2 *Given scalars ρ_i, $i = 1, \cdots, 9$, (20) becomes an observer for the nominal system (18) and (19) if there exist matrices $P_i > 0$, $R \geq 0$, $U > 0$, $X > 0$, $Y_i > 0$, $i = 1,2,3$, $Q_i \geq 0$, $Z_i > 0$, $i = 1,2$, L, G_j, $j = 1, \cdots, r$,*

$$\begin{aligned}
N_{ij} &= \begin{bmatrix} N_{1ij}^T & N_{2ij}^T & N_{3ij}^T & N_{4ij}^T & N_{5ij}^T & N_{6ij}^T & N_{7ij}^T & N_{8ij}^T & N_{9ij}^T \end{bmatrix}^T, \\
S_{ij} &= \begin{bmatrix} S_{1ij}^T & S_{2ij}^T & S_{3ij}^T & S_{4ij}^T & S_{5ij}^T & S_{6ij}^T & S_{7ij}^T & S_{8ij}^T & S_{9ij}^T \end{bmatrix}^T, \\
M_{ij} &= \begin{bmatrix} M_{1ij}^T & M_{2ij}^T & M_{3ij}^T & M_{4ij}^T & M_{5ij}^T & M_{6ij}^T & M_{7ij}^T & M_{8ij}^T & M_{9ij}^T \end{bmatrix}^T, \\
V_{ij} &= \begin{bmatrix} V_{1ij}^T & V_{2ij}^T & V_{3ij}^T & V_{4ij}^T & V_{5ij}^T & V_{6ij}^T & V_{7ij}^T & V_{8ij}^T & V_{9ij}^T \end{bmatrix}^T, \\
W_{ij} &= \begin{bmatrix} W_{1ij}^T & W_{2ij}^T & W_{3ij}^T & W_{4ij}^T & W_{5ij}^T & W_{6ij}^T & W_{7ij}^T & W_{8ij}^T & W_{9ij}^T \end{bmatrix}^T, \\
O_{ij} &= \begin{bmatrix} O_{1ij}^T & O_{2ij}^T & O_{3ij}^T & O_{4ij}^T & O_{5ij}^T & O_{6ij}^T & O_{7ij}^T & O_{8ij}^T & O_{9ij}^T \end{bmatrix}^T, \quad i,j = 1, \cdots, r
\end{aligned}$$

such that

$$\Theta_{ij} = \begin{bmatrix} \Theta_{11ij} & \Theta_{12ij} \\ \Theta_{12ij}^T & \Theta_{22} \end{bmatrix} < 0, \quad i,j = 1, \cdots, r \tag{26}$$

where

$$\begin{aligned}
\Theta_{11ij} &= \Theta_1 + \Theta_{2ij} + \Theta_{2ij}^T + \Theta_{3ij} + \Theta_{3ij}^T, \\
\Theta_1 &= \begin{bmatrix}
\Theta_{111} & 0 & 0 & 0 & 0 & 0 & 0 & P_2 & P_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -R & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1-d)Q_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -P_2 & 0 \\
0 & 0 & 0 & 0 & 0 & -Q_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
P_2 & 0 & 0 & 0 & -P_2 & 0 & 0 & -X - \frac{1}{\beta_M}U & 0 \\
P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Theta_{199}
\end{bmatrix}, \\
\Theta_{111} &= Q_1 + R + \beta_M U + \beta_M^2 X, \\
\Theta_{199} &= Q_2 + \alpha_M Y_1 + \beta_M Y_2 + \gamma_M Y_3 + h_M(Z_1 + Z_2), \\
\Theta_{2ij} &= \begin{bmatrix} N_{ij} + M_{ij} + W_{ij} + O_{ij} + V_{ij} & -N_{ij} + S_{ij} & -M_{ij} - S_{ij} & -W_{ij} \end{bmatrix. \\
&\qquad\qquad \begin{matrix} -O_{ij} & 0 & -V_{ij} & 0 & 0 \end{matrix} \Big],
\end{aligned}$$

$$\Theta_{3ij} = -\begin{bmatrix} \rho_1 LA_i & -\rho_1 G_j C_i & 0 & \rho_1 LA_{di} & 0 & \rho_1 LA_{ni} & 0 & \rho_1 LD_i & -\rho_1 L \\ \rho_2 LA_i & -\rho_2 G_j C_i & 0 & \rho_2 LA_{di} & 0 & \rho_2 LA_{ni} & 0 & \rho_2 LD_i & -\rho_2 L \\ \rho_3 LA_i & -\rho_3 G_j C_i & 0 & \rho_3 LA_{di} & 0 & \rho_3 LA_{ni} & 0 & \rho_3 LD_i & -\rho_3 L \\ \rho_4 LA_i & -\rho_4 G_j C_i & 0 & \rho_4 LA_{di} & 0 & \rho_4 LA_{ni} & 0 & \rho_4 LD_i & -\rho_4 L \\ \rho_5 LA_i & -\rho_5 G_j C_i & 0 & \rho_5 LA_{di} & 0 & \rho_5 LA_{ni} & 0 & \rho_5 LD_i & -\rho_5 L \\ \rho_6 LA_i & -\rho_6 G_j C_i & 0 & \rho_6 LA_{di} & 0 & \rho_6 LA_{ni} & 0 & \rho_6 LD_i & -\rho_6 L \\ \rho_7 LA_i & -\rho_7 G_j C_i & 0 & \rho_7 LA_{di} & 0 & \rho_7 LA_{ni} & 0 & \rho_7 LD_i & -\rho_7 L \\ \rho_8 LA_i & -\rho_8 G_j C_i & 0 & \rho_8 LA_{di} & 0 & \rho_8 LA_{ni} & 0 & \rho_8 LD_i & -\rho_8 L \\ \rho_9 LA_i & -\rho_9 G_j C_i & 0 & \rho_9 LA_{di} & 0 & \rho_9 LA_{ni} & 0 & \rho_9 LD_i & -\rho_9 L \end{bmatrix},$$

$$\Theta_{12ij} = \begin{bmatrix} h_M N_{ij} & h_M S_{ij} & h_M M_{ij} & \alpha_M W_{ij} & \beta_M O_{ij} & \gamma_M V_{ij} \end{bmatrix},$$

$$\Theta_{22} = \mathrm{diag}\begin{bmatrix} -h_M Z_1 & -h_M Z_1 & -h_M Z_2 & -\alpha_M Y_1 & -\beta_M Y_2 & -\gamma_M Y_3 \end{bmatrix}.$$

In this case, observer gains in (20) are given by

$$\bar{K}_i = L^{-1} G_i,\ i = 1, \cdots, r. \tag{27}$$

Proof: We let $T_i = \rho_i \bar{L}$, $i = 1, \cdots, 9$ where each ρ_i is given and $\bar{L}$ is defined later, and substitute them into (26). If (26) holds, it follows that (9,9)-block of Θ_{11ij} must be negative definite. It follows that $T_9 + T_9^T = \rho_9(\bar{L} + \bar{L}^T) < 0$, which implies that $\bar{L}$ is nonsingular. Then, we calculate $\Theta_{ij} = \Sigma \Phi_{ij} \Sigma^T$ with $\Sigma = \mathrm{diag}[\, L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L \,]$, and obtain Θ_{ij} in (26) where we let $G_j = L\bar{K}_j$. If the conditions (26) hold, observer gain matrix $\bar{K}_i$ is obviously given by (27).

7. Numerical examples for observer design

Let us design sampled-data observers for the system (18) and (19) with the following matrices.

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1.5 \end{bmatrix},\ A_{d1} = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix},\ A_{d2} = \begin{bmatrix} -1 & -1 \\ 0 & -1.4 \end{bmatrix},$$
$$A_{n1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},\ A_{n2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.1 \\ 1.2 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.3 & 1.2 \end{bmatrix},\ D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}.$$

The grades are given by $\lambda_1(x_1) = \frac{1}{1+e^{-x_1}}$ and $\lambda_2(x_1) = 1 - \lambda_1(x_1)$. The maximum upper bound of the sampling time $h_M = 0.1$ and $d = 0.4$ are assumed. We let $\alpha_M = \beta_M = 0.2$. Theorem 6.2 with $\rho_1 = 4.76$, $\rho_2 = -0.03$, $\rho_3 = -2.19$, $\rho_4 = -0.60$, $\rho_5 = -0.01$, $\rho_6 = 0.01$, $\rho_7 = -0.50$, $\rho_8 = 0.09$, $\rho_9 = 1.96$ guarantees the existence of the sampled-data observer for the maximum upper bound of the time-delay $\gamma_M = 2.41$. In this case, observer gains in (2) are given by

$$\bar{K}_1 = \begin{bmatrix} -1.1239 \\ 0.7925 \end{bmatrix},\ \bar{K}_2 = \begin{bmatrix} -1.1254 \\ 0.7916 \end{bmatrix}.$$

8. Conclusion

In this paper, robust sampled-data control and observer design for uncertain fuzzy systems with discrete, neutral and distributed delays has been considered. Less conservative robust stability conditions were obtained as LMI conditions via time-varying delay system approach. Then, a controller design method was proposed via LMI conditions. As a dual result, an observer design method was also given. Finally, some examples were given to illustrate our design approach.

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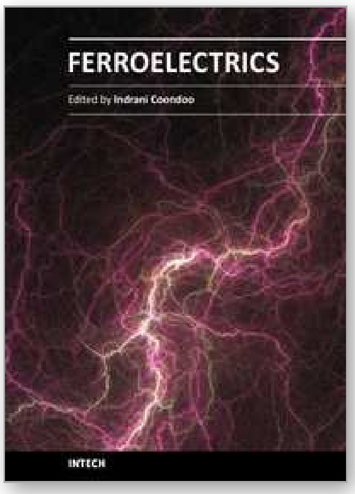

Ferroelectrics

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Ferroelectric materials exhibit a wide spectrum of functional properties, including switchable polarization, piezoelectricity, high non-linear optical activity, pyroelectricity, and non-linear dielectric behaviour. These properties are crucial for application in electronic devices such as sensors, microactuators, infrared detectors, microwave phase filters and, non-volatile memories. This unique combination of properties of ferroelectric materials has attracted researchers and engineers for a long time. This book reviews a wide range of diverse topics related to the phenomenon of ferroelectricity (in the bulk as well as thin film form) and provides a forum for scientists, engineers, and students working in this field. The present book containing 24 chapters is a result of contributions of experts from international scientific community working in different aspects of ferroelectricity related to experimental and theoretical work aimed at the understanding of ferroelectricity and their utilization in devices. It provides an up-to-date insightful coverage to the recent advances in the synthesis, characterization, functional properties and potential device applications in specialized areas.