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# DeLiA: A New Family of Redundant Robot Manipulators

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## 1. Introduction

A fully decoupled parallel manipulator is a mechanism in which one output kinematic joint, degree of freedom, is affected by only one active or input kinematic pair, the perfect mechanism from a kinematic point of view due to the possibility to generate linear input-output kinematic constraint equations. Parallel manipulators with fewer than six-degrees-of-freedom frequently referred as limited-dof or defective parallel manipulators were the first class of parallel manipulators to be considered in that trend. Kong & Gosselin (2002a) introduced a class of translational fully decoupled parallel manipulators called Tripteron family. Carricato & Parenti-Castelli (2004) invented a two-degrees-of-freedom parallel wrist in which two interconnected linkages independently actuate one of the two angles associated to the orientation of the moving platform. Recently, Briot & Bonev (2009) proposed a fully decoupled translational parallel manipulator, called Pantopteron, for simple pick-and-place operations. Certainly, there is a significative number of contributions dealing with the study of limited-dof fully decoupled parallel manipulators, see for instance Carricato & Parenti-Castelli (2002), Kong & Gosselin (2002b, 2002c), Gosselin et al., (2004), Gogu (2005), Li et al., (2005), Ruggiu (2009) and so on. On the other hand, a fully decoupled six-degrees-of-freedom parallel manipulator is maybe, still in our days, an unrealistic task. In fact, the dream that in a Gough-Stewart platform one degree of freedom shall be affected by only one active kinematic joint is a far away reality, if sensors are not considered. In order to diminish such drawback, the term fully can be removed from the original concept meaning that a decoupled parallel manipulator is a mechanism in which the position and orientation, pose, of the moving platform with respect to the fixed platform can be computed separately. The decoupled motion can be achieved by introducing geometric conditions, e.g. Wohlhart (1994) studied a Gough-Stewart platform in which three of the six limbs share a common spherical joint over the moving platform, other topologies with uncoupled rotations and translations were investigated by Innocenti & Parenti-Castelli (1991), Zabalza et al., (2002), Yang et al., (2004), Takeda (2005) and so on. Despite the indisputable recent valuable advances in this subject, the development of decoupled parallel manipulators with simplified architectures preserving the well-known benefits of parallel manipulators such as higher stiffness and payload/capacity is a rather complicated task. At this point, and mainly due to the lack of an efficient mathematical resource to approach the forward kinematics of a general Gough-Stewart platform capable to determine the actual configuration of the manipulator, without using sensors, one can take into account that if

there is not essential the fully decoupled motion, then the development of partially decoupled parallel manipulators is a viable option to apply the benefits of mechanisms with nearly parallel kinematic structures, see for instance Briot et al., (2009), Altuzarra et al., (2010). It is interesting to note that mechanisms with mixed motions can be included in the class known as partially decoupled parallel manipulators.

In this chapter a new family of partially decoupled parallel manipulators endowed with an extra active kinematic joint is introduced. One member of this new family of robot manipulators is selected with the purpose to illustrate the methodology of kinematic analysis chosen to characterize the angular and linear kinematic properties, up to the acceleration analysis, of it. The forward position analysis of the robot, a challenging task for most parallel manipulators, is carried-out in a semi-closed form solution applying recursively the Sylvester dialytic elimination method that allows to determine all the feasible locations that the output platform can reach with respect to the fixed platform given a set of generalized coordinates. On the other hand, the velocity and acceleration analyses of the robot are approached by means of the theory of screws. With this mathematical tool, simple and compact expressions for computing the velocity and reduced acceleration states of the output platform are obtained taking advantage of the properties of reciprocal screws, via the Klein form of the Lie algebra  $e(3)$ . Finally, the robot is simulated as a virtual five-degrees-of-freedom parallel kinematic machine using special commercially available software like ADAMS©.

## 2. Description of the DeLiA robot family

Before the transcendental contributions of Gough (1957), Gough & Whitehall (1962) and Stewart (1965), it seems that a five-degrees-of-freedom spray painting machine was the first promissory industrial application of a parallel manipulator (Pollard, 1940; Bonev, 2003). Furthermore, many practical applications do not require the six degrees of freedom of a general Gough-Stewart platform, particularly five-degrees-of-freedom parallel manipulators had been proposed, among simple pointing devices, as multi-axis machine tools (Bohez, 2002; Zheng et al., 2005; Gao et al., 2005, 2006), bio-mechanical devices (Zhu et al., 2008; Gallardo-Alvarado, 2010) or new architectures for medical applications (Vlachos & Papadopoulos, 2005; Piccina, 2009).

With these considerations in mind and with the motivation that not always is essential a pure parallel kinematic topology, this work is intended to be a viable option to the development of a new class of five-degrees-of-freedom robots with a nearly parallel kinematic architecture, preserving the advantages of serial-parallel manipulators but with the possibility to mount all the active limbs on the fixed platform.

The proposed general topology is depicted in Fig. 1, it consists of a fixed platform, a coupler platform and an end-effector-platform also called output-platform. Please note that while in a general Gough-Stewart platform the generalized coordinates or active joints are necessarily coupled, in the proposed topology these motors can be decoupled into two different groups which allows to simplify the forward kinematics of the mechanisms at hand. Furthermore, the end-effector-platform is connected at the fixed platform by means of an active 6-dof three-legged parallel manipulator (XYS-type limb with  $X=RR,U,C,S$ ;  $Y=P,R,C$ ) whereas the coupler platform is connected at the fixed platform by means of an active 3-dof parallel manipulator (XYS-type limb with  $X=R,P$ ;  $Y=R,P$ ) and at the end-effector platform through a passive 3-dof parallel manipulator (XYS-type limb with  $X=R,P$ ;  $Y=R,P$ ). Interesting benefits can be observed in this topology:

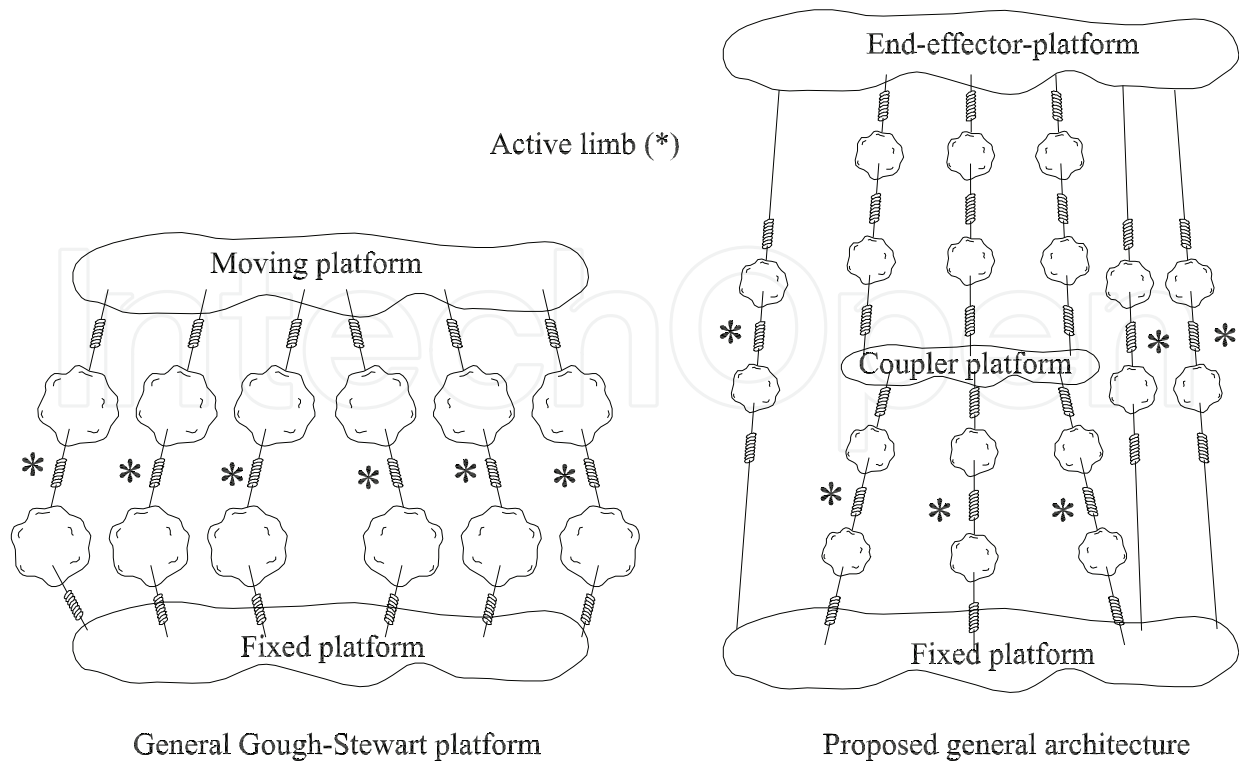


Fig. 1. General Gough-Stewart platform and the general proposed topology

- The motors can be mounted on the fixed platform
- The forward finite kinematics can be carried-out solving two decoupled systems of non-linear kinematic constraint equations
- The spherical joints attached at the end-effector platform allow to affirm that this topology is a non-overconstrained mechanism, e.g. does not require of additional tolerances of manufacture that ensure the intersection of screws

On the other hand, the 3-dof parallel manipulators chosen for this research, belong to the class known as zero-torsion parallel manipulators (Bonev, 2002).

Several combinations can be generated with the considerations above-mentioned and one of them, here after called D1 robot, is presented in Fig. 2.

D1 is a robot formed with an active 3-UPS parallel manipulator and two 3-RPS parallel manipulators, one active and the other passive. The nominal coordinates of the universal, prismatic, spherical and revolute joints of the chosen architecture are denoted respectively by  $U$ ,  $P$ ,  $S$  and  $R$  and are located by vectors  $\mathbf{U}$ ,  $\mathbf{P}$ ,  $\mathbf{S}$  and  $\mathbf{R}$ . In the rest of this work, the analysis is focused on the D1 robot.

### 3. Mobility analysis of the D1 robot

An exhaustive review of formulae addressing the mobility analysis of closed kinematic chains can be found in Gogu (2005) and the following is a variant of the well-known Kutzbach-Grübler formula for computing the degrees-of-freedom of spatial parallel manipulators

$$F = 6(n - j - 1) + \sum_{i=1}^j f_i \quad (1)$$

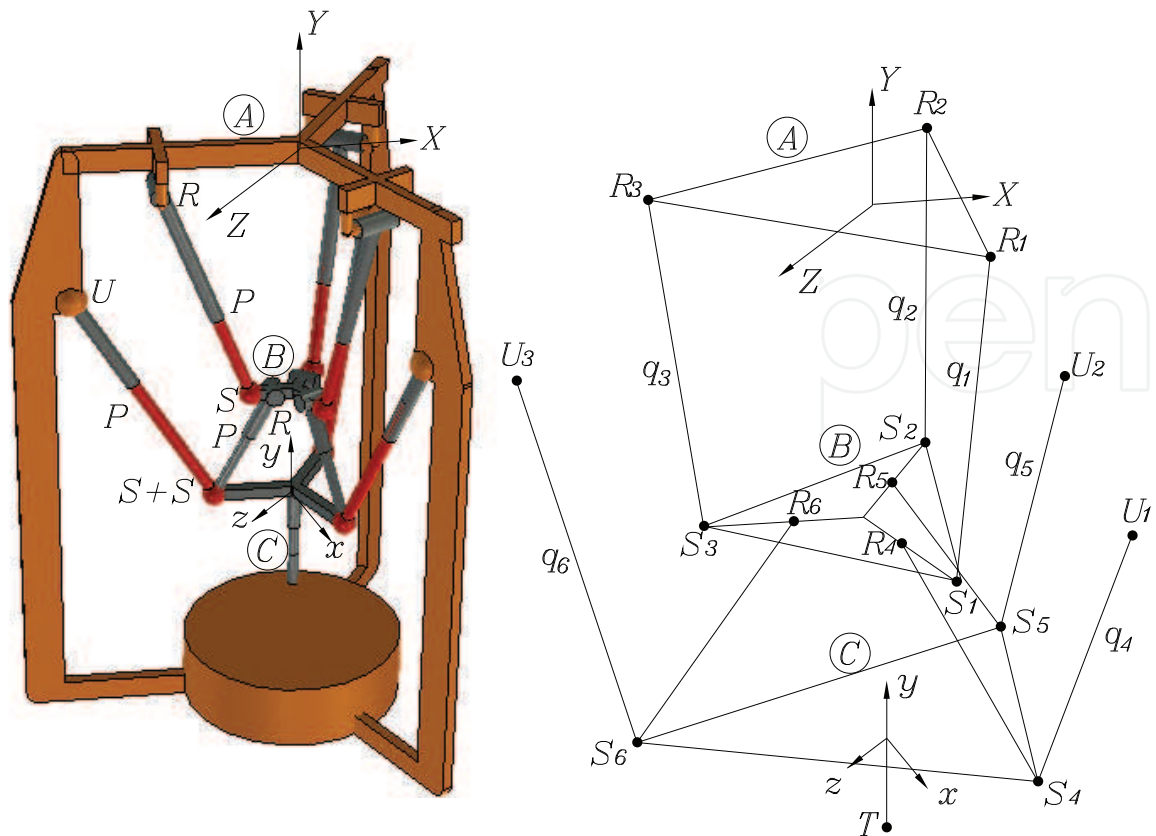


Fig. 2. D1, a member of the DeLiA robot family and its geometric scheme

where  $n$  is the number of links,  $j$  is the number of kinematic pairs, and  $f_i$  is the number of freedoms of the  $i$ -th kinematic pair. For the robot D1  $n = 21$ ,  $j = 6R + 9P + 3U + 9S = 27$ ,  $\sum_{i=1}^j f_i = 48$  and therefore  $F = 6$ , which is a wrong result. In fact, Dai et al. (2006) proved that in a 3-RPS parallel manipulator a basis representing the motions of the moving platform, with respect to the fixed platform, consists of three elements, two non-parallel coplanar rotations and one translation along an axis perpendicular to the plane formed by the spherical joints. According to this basis, the coupler platform of the robot D1 cannot rotate with respect to the fixed platform along an axis perpendicular to the plane formed by the spherical joints attached at the coupler platform. It is straightforward to demonstrate that such argument is valid too for the end-effector-platform and the coupler platform, in other words, the end-effector-platform has a rotation restricted with respect to the coupler platform due to the passive 3-RPS parallel manipulator connecting both platforms and the coupler platform has a rotation restricted with respect to the fixed platform due to the 3-RPS active parallel manipulator. With these considerations in mind, although the computed degrees of freedom of the robot at hand is six, the end-effector-platform does not accept arbitrary orientations with respect to the fixed platform, and therefore D1 is in reality a five-degrees-of-freedom redundant robot.

#### 4. Finite kinematics

In this section the position analysis of the proposed robot is presented.

#### 4.1 Forward position analysis

The forward position analysis (FPA) consists of finding the pose of the end-effector-platform with respect to the fixed platform given a set of six generalized coordinates  $q_i (i = 1, 2, \dots, 6)$ . Due to the decoupled architecture, the pose of the coupler platform, body B, with respect to the fixed platform, body A, is controlled by means of the internal generalized coordinates  $q_i (i = 1, 2, 3)$ . Furthermore, the pose of the coupler platform is easily determined through the computation of the coordinates of the centers of the spherical joints attached at the coupler platform, points  $S_i (i = 1, 2, 3)$ .

Let  $XYZ$  and  $xyz$  be two reference frames attached, respectively, at the fixed platform and at the end-effector-platform. With reference to Fig. 2, since the revolute joints attached at the fixed platform have a tangential arrangement, then it is possible to write three kinematic constraint equations as

$$\left. \begin{aligned} (S_1 - R_1) \bullet (R_2 - R_3) &= 0 \\ (S_2 - R_2) \bullet (R_3 - R_1) &= 0 \\ (S_3 - R_3) \bullet (R_2 - R_1) &= 0 \end{aligned} \right\} \quad (2)$$

where the dot ( $\bullet$ ) denotes the usual inner product of the three-dimensional vectorial algebra. Furthermore, three closure equations can be written as

$$(S_i - R_i) \bullet (S_i - R_i) = q_i^2 \quad i = 1, 2, 3 \quad (3)$$

Finally, according to the triangle  $\Delta S_1 S_2 S_3$  three compatibility equations are given by

$$(S_i - S_j) \bullet (S_i - S_j) = \overline{S_i S_j}^2 \quad i, j = 1, 2, 3 \quad \text{mod}(3) \quad (4)$$

Expressions (2), (3) and (4) are solved by applying the Sylvester dialytic elimination method (Tsai, 1999; Gallardo et al., 2007). Once the coordinates of the points  $S_i (i = 1, 2, 3)$  are calculated, the center of the triangle  $\Delta S_1 S_2 S_3$ , vector  ${}^A \rho^B$ , results in

$${}^A \rho^B = (S_1 + S_2 + S_3) / 3 \quad (5)$$

Finally, the pose of the coupler platform with respect to the fixed platform is summarized in the  $4 \times 4$  homogeneous transformation matrix  ${}^A T^B$ :

$${}^A T^B = \begin{bmatrix} {}^A R^B & {}^A \rho^B \\ \mathbf{O}_{1 \times 3} & 1 \end{bmatrix} \quad (6)$$

where  ${}^A R^B$  is the rotation matrix which is computed by means of the coordinates of the points  $S_i (i = 1, 2, 3)$ , for details see Gallardo-Alvarado et al. (2008). Following a similar procedure, the homogeneous transformation matrix of the end-effector-platform with respect to the fixed platform,  ${}^A T^C$ , is computed. To this end, consider the following closure equations

$$\left. \begin{aligned} (S_4 - R_4) \bullet (R_5 - R_6) &= 0 \\ (S_5 - R_5) \bullet (R_6 - R_4) &= 0 \\ (S_6 - R_6) \bullet (R_5 - R_4) &= 0 \\ (S_{i+3} - U_i) \bullet (S_{i+3} - U_i) &= q_{i+3}^2 \quad i = 1, 2, 3 \\ (S_i - S_j) \bullet (S_i - S_j) &= \overline{S_i S_j}^2 \quad i, j = 4, 5, 6 \quad \text{mod}(3) \end{aligned} \right\} \quad (7)$$

where the vectors  $R_i (i = 4, 5, 6)$  are computed by using the matrix  ${}^A T^B$ . Furthermore, the homogeneous transformation matrix between the end-effector- platform and the fixed platforms,  ${}^B T^C$ , can be calculated from

$${}^A T^C = {}^A T^B {}^B T^C \quad (8)$$

#### 4.2 Inverse position analysis

The inverse position analysis consists of finding the active limb lengths  $q^i (i = 1, 2, \dots, 6)$  of the robot given the pose of the end-effector- platform with respect to the fixed platform, matrix  ${}^A T^C$ . To this end, it is necessary to compute, as an intermediate step, the vectors  $S^i (i = 1, 2, \dots, 6)$ .

Immediately emerges that the coordinates of the points  $S_i (i = 4, 5, 6)$ , expressed in the fixed reference frame  $XYZ$ , attached at the end-effector- platform can be obtained from

$$\begin{bmatrix} S_i \\ 1 \end{bmatrix} = {}^A T^C \begin{bmatrix} s_i \\ 1 \end{bmatrix} \quad (9)$$

where  $s_i (i = 4, 5, 6)$  is the  $i$ -th point but expressed according to the moving reference frame  $xyz$ . Furthermore, the unknown vectors  $R_i (i = 4, 5, 6)$  and  $S_i (i = 1, 2, 3)$  can be computed according to the following closure equations

$$\left. \begin{aligned} (S_6 - R_6) \cdot (R_5 - R_4) &= 0 \\ (S_5 - R_5) \cdot (R_6 - R_4) &= 0 \\ (S_4 - R_4) \cdot (R_6 - R_5) &= 0 \\ (S_1 - R_1) \cdot (R_2 - R_3) &= 0 \\ (S_2 - R_2) \cdot (R_3 - R_1) &= 0 \\ (S_3 - R_3) \cdot (R_2 - R_1) &= 0 \\ S_1 + S_2 + S_3 &= R_4 + R_5 + R_6 \\ (S_i - S_j) \cdot (S_i - S_j) &= \overline{S_i S_j}^2 \quad i, j = 1, 2, 3 \quad \text{mod}(3) \\ (R_i - R_j) \cdot (R_i - R_j) &= \overline{R_i R_j}^2 \quad i, j = 4, 5, 6 \quad \text{mod}(3) \\ (S_i - R_{i+3}) \cdot (S_i - R_{i+3}) &= \overline{S_i R_{i+3}}^2 \quad i = 1, 2, 3 \end{aligned} \right\} \quad (10)$$

Finally, the limb lengths  $q_i (i = 1, 2, \dots, 6)$  result in

$$\left. \begin{aligned} q_i^2 &= (S_i - R_i) \cdot (S_i - R_i) \\ q_{i+3}^2 &= (S_{i+3} - U_i) \cdot (S_{i+3} - U_i) \quad i = 1, 2, 3 \end{aligned} \right\} \quad (11)$$

#### 5. Infinitesimal kinematics

In this section the velocity and acceleration analyses of the proposed robot are approached by means of the theory of screws. For detailed information of the kinematic analysis of closed chains and parallel manipulators, using such mathematical resource, the reader is referred to (Rico & Duffy, 2000; Gallardo et al., 2003). In particular screw theory is an

efficient mathematical resource to analyze five-degrees-of-freedom parallel manipulators (Li & Huang, 2002; Zhu et al., 2008; Gallardo-Alvarado et al., 2009). Furthermore, as a consideration for readers unfamiliar with the theory of screws an explanation of basic concepts dealing with it is also included in this section.

### 5.1 Preliminary concepts. Basic concepts of the screw theory

A screw  $\$ = (\hat{s}, s_O)$  is a six-dimensional vector composed of a vector  $\hat{s}$ , namely the primal part, denoting the direction of the screw axis and a vector  $s_O$ , namely the dual part, which is the moment produced by  $\hat{s}$  about a point  $O$  fixed to the reference frame. The moment  $s_O$  is calculated as follows

$$s_O = \hat{s}h + \hat{s} \times r_{O/P} \quad (12)$$

where  $h$  is the pitch of the screw and  $r_{O/P}$  is a vector directed from a point  $P$ , fixed to the screw axis, to point  $O$ . Note that if the pitch of the screw goes to infinity, then the screw represents a prismatic joint and it is represented by  $\$ = (0, \hat{s})$ . Any lower kinematic pair can be represented either by a screw or a group of screws. A cylindrical joint can be simulated by the combination of one revolute joint and one prismatic joint, whereas a spherical joint results of the action of three revolute joints whose axes, usually mutually orthogonal, intersect a common point.

Screw theory, which is isomorphic to the Lie algebra  $e(3)$  also referred as motor algebra, is the set of elements of the form  $\$ = (\hat{s}, s_O)$  with the following operations.

Let  $\$1 = (\hat{s}_1, s_{O1})$ ,  $\$2 = (\hat{s}_2, s_{O2})$ , and  $\$3 = (\hat{s}_3, s_{O3})$  be elements of the Lie algebra  $e(3)$  with  $\lambda_1, \lambda_2, \lambda_3 \in \mathfrak{R}$ . Then

1. Addition,  $\$1 + \$2 = (\hat{s}_1 + \hat{s}_2, s_{O1} + s_{O2})$
2. Multiplication by a scalar,  $\lambda \$1 = (\lambda \hat{s}_1, \lambda s_{O1})$
3. Lie product or dual motor product,  $[\$1 \$2] = (\hat{s}_1 \times \hat{s}_2, \hat{s}_1 \times s_{O2} - \hat{s}_2 \times s_{O1})$ .

The Lie product exhibits interesting properties like

- a. Nilpotent,  $[\$1 \$1] = (\mathbf{0}, \mathbf{0})$
- b. Non-commutative,  $[\$1 \$2] = -[\$2 \$1]$
- c. Distributive

$$[\$1 \lambda_2 \$2 + \lambda_3 \$3] = \lambda_2 [\$1 \$2] + \lambda_3 [\$1 \$3]$$

$$[\lambda \$1 + \lambda_2 \$2 \$3] = \lambda_1 [\$1 \$3] + \lambda_2 [\$2 \$3]$$

4. Jacobi identity,  $[\$1 [\$2 \$3]] + [\$3 [\$1 \$2]] + [\$2 [\$3 \$1]] = (\mathbf{0}, \mathbf{0})$

Furthermore, the Lie algebra  $e(3)$  is endowed with two symmetric bilinear forms

1. The Killing form,  $\{\$1; \$2\} = \hat{s}_1 \cdot \hat{s}_2$
2. The Klein form,  $\{\$1; \$2\} = \hat{s}_1 \cdot s_{O2} + \hat{s}_2 \cdot s_{O1}$

It is said that the screws  $\$1$  and  $\$2$  are reciprocal if  $\{\$1; \$2\} = 0$ , an interesting property that allows to simplify the forward infinitesimal kinematics of parallel manipulators.

Screw theory is a powerful mathematical tool modeling the kinematics of rigid bodies.

The velocity state  ${}^n V_O^m$  of a rigid body  $m$  as it is observed from another body  $n$  or reference frame is a twist about screw (Ball, 1900), indeed  ${}^n V_O^m = \omega^n \$^m$ , given by



$${}^n V_O^m = \begin{bmatrix} {}^n \omega^m \\ {}^n v_O^m \end{bmatrix} \quad (13)$$

where  ${}^n \omega^m$  and  ${}^n V_O^m$  are, respectively, the angular and linear velocities of the body under study and  $O$  is a point of the body  $m$  that is instantaneously coincident with the origin of the reference frame  $n$ , point  $O$  is also known as the reference pole. Furthermore, in an open kinematic chain, e.g. a serial manipulator, the velocity state of the end-effector, labeled body  $m$ , with respect to the base link, labeled body  $n$ , can be written as a linear combination of the involved infinitesimal screws associated to the kinematic pairs as follows

$${}^n \omega_{n+1} {}^n \$^{n+1} + {}_{n+1} \omega_{n+2} {}^{n+1} \$^{n+2} + \dots + {}_{m-1} \omega_m {}^{m-1} \$^m = {}^n V_O^m \quad (14)$$

On the other hand, the reduced acceleration state  ${}^n A_O^m$  of body  $m$  with respect to body  $n$ , also known as accelerator, is a six-dimensional vector given by

$${}^n A_O^m = {}_n \dot{\omega}_m {}^n \$^m = \begin{bmatrix} {}^n \dot{\omega}^m \\ {}^n a_O^m - {}^n \omega^m \times {}^n v_O^m \end{bmatrix} \quad (15)$$

where  $\dot{\omega}$  and  ${}^n a_O^m$  are the angular and linear accelerations of body  $m$  with respect to body  $n$  taking  $O$  as the reference pole. Furthermore, in a serial manipulator the reduced acceleration state of the end-effector with respect to the base link is given by

$${}^n \dot{\omega}_{n+1} {}^n \$^{n+1} + {}_{n+1} \dot{\omega}_{n+2} {}^{n+1} \$^{n+2} + \dots + {}_{m-1} \dot{\omega}_m {}^{m-1} \$^m + {}^n \mathcal{L}^m = {}^n A_O^m \quad (16)$$

where

$$\begin{aligned} {}^n \mathcal{L}^m = & [ {}_n \omega_{n+1} {}^n \$^{n+1} \quad {}_{n+1} \omega_{n+2} {}^{n+1} \$^{n+2} + \dots + {}_{m-1} \omega_m {}^{m-1} \$^m ] \\ & + [ {}_{n+1} \omega_{n+2} {}^{n+1} \$^{n+2} + {}_{n+2} \omega_{n+3} {}^{n+2} \$^{n+3} + \dots + {}_{m-1} \omega_m {}^{m-1} \$^m ] \\ & + \dots + [ {}_{m-2} \omega_{m-1} {}^{m-2} \$^{m-1} \quad {}_{m-1} \omega_m {}^{m-1} \$^m ] \end{aligned} \quad (17)$$

is the Lie screw or complementary six-dimensional vector of the reduced acceleration state.

It is worth mentioning that even though its compactness, Eq. (16) contains all the terms involved in the acceleration analysis of a rigid body. In fact, e.g. Eq. (16) contains the terms of the acceleration of Coriolis and one does not need to make a distinction of it. Furthermore, Eq. (16) can be easily translated into computer codes approaching the kinematic analysis of robot manipulators. This expression was introduced for the first time by Rico-Martínez & Duffy (1996) and its correctness was validated by the author of this work with the publication of several papers in well known journals. Before the pioneering contribution of Rico-Martínez & Duffy (1996) the screw theory was confined to the so-called first order analysis (velocity analysis). Its introduction almost fifteen years ago opened the possibility to extend the screw theory to the so-called higher order kinematic analyses.

## 5.2 Velocity analysis

The modeling of the screws of three representative limbs of the robot is depicted in Fig. 3. It must be noted that due to the existence of compound joints in the output platform, e.g.

Spherical + Spherical, these spherical joints require each one more of the usual three infinitesimal screws indicating concurrent revolute joints. Furthermore, before do any further, in order to solve the inverse velocity and acceleration analyses, it is necessary the introduction of auxiliary screws with the purpose to satisfy an algebraic requirement, with this consideration in mind the revolute joints are modeled as cylindrical joints, in which the corresponding translational velocities are equal to zero. In other words,  ${}^0\omega_1 = {}^6\omega_7 = 0 (i = 1, 2, 3)$ .

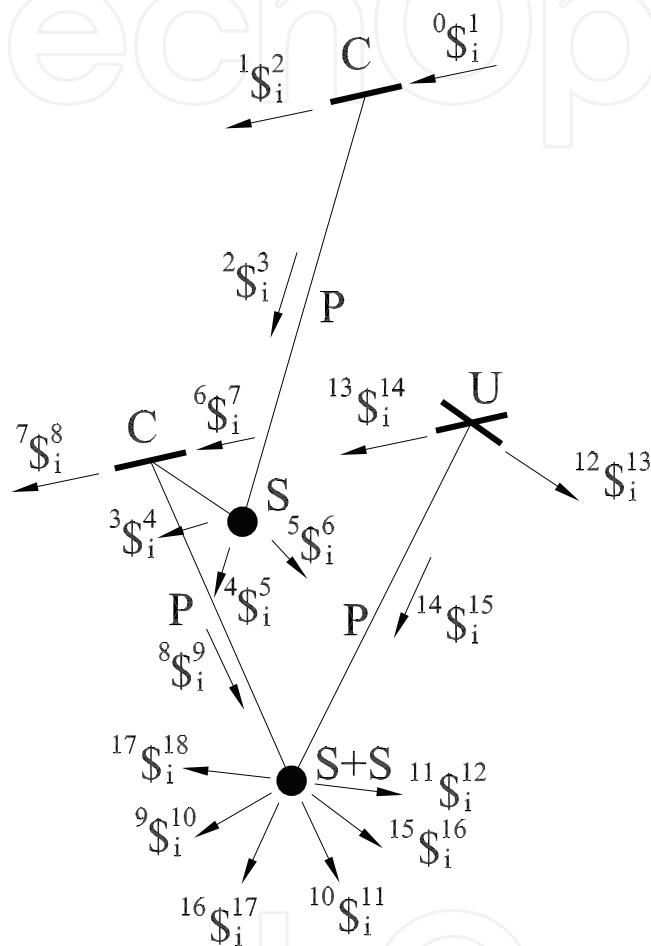


Fig. 3. Infinitesimal screws of three representative limbs of the robot D1

Let  ${}^A\omega^C$  and  ${}^A v_O^C$  be, respectively, the angular and linear velocities of a point  $O$  attached at the end-effector-platform. The velocity state, or twist about a screw, of the end-effector-platform with respect to the fixed platform, six-dimensional vector  ${}^A V_O^C = [{}^A\omega^C, {}^A v_O^C]^T$ , can be obtained through the coupler and fixed platforms as

$${}^A V_O^C = {}^A V_O^B + {}^B V_O^C \tag{18}$$

where  ${}^A V_O^B$  is the velocity state of the coupler platform with respect to the fixed platform, and  ${}^B V_O^C$  is the velocity state of the end-effector-platform with respect to the coupler platform. Furthermore, these kinematic states can be written in screw form as

$$V = J_i \Omega, V \in \{{}^A V_O^B, {}^B V_O^C, {}^A V_O^C\} \quad i = 1, 2, 3 \tag{19}$$

where the Jacobians  $\mathbf{J}_i \in \{ {}^A\mathbf{J}_i^B, {}^B\mathbf{J}_i^C, {}^A\mathbf{J}_i^C \}$  are given by

$$\left. \begin{aligned} {}^A\mathbf{J}_i^C &= [{}_{12}^{13}\bar{\$}_i, {}_{13}^{14}\bar{\$}_i, {}_{14}^{15}\bar{\$}_i, {}_{15}^{16}\bar{\$}_i, {}_{16}^{17}\bar{\$}_i, {}_{17}^{18}\bar{\$}_i] \\ {}^B\mathbf{J}_i^C &= [{}_{6}^7\bar{\$}_i, {}_{7}^8\bar{\$}_i, {}_{8}^9\bar{\$}_i, {}_{9}^{10}\bar{\$}_i, {}_{10}^{11}\bar{\$}_i, {}_{11}^{12}\bar{\$}_i] \\ {}^A\mathbf{J}_i^B &= [{}_{0}^1\bar{\$}_i, {}_{1}^2\bar{\$}_i, {}_{2}^3\bar{\$}_i, {}_{3}^4\bar{\$}_i, {}_{4}^5\bar{\$}_i, {}_{5}^6\bar{\$}_i] \end{aligned} \right\} \quad (20)$$

whereas  $\boldsymbol{\Omega}_i \in \{ {}^A\boldsymbol{\Omega}_i^B, {}^B\boldsymbol{\Omega}_i^C, {}^A\boldsymbol{\Omega}_i^C \}$  are matrices containing the joint rate velocities. In fact:

$$\left. \begin{aligned} {}^A\boldsymbol{\Omega}_i^C &= [{}_{12}^{-i}\bar{\omega}_{13}, {}_{13}^i\omega_{14}, {}_{14}^i\omega_{15}, {}_{15}^i\omega_{16}, {}_{16}^i\omega_{17}, {}_{17}^i\omega_{18}]^T \\ {}^B\boldsymbol{\Omega}_i^C &= [{}_{6}^{-i}\bar{\omega}_7, {}_{7}^i\omega_8, {}_{8}^i\omega_9, {}_{9}^i\omega_{10}, {}_{10}^i\omega_{11}, {}_{11}^i\omega_{12}]^T \\ {}^A\boldsymbol{\Omega}_i^B &= [{}_{0}^{-i}\bar{\omega}_1, {}_{1}^i\omega_2, {}_{2}^i\omega_3, {}_{3}^i\omega_4, {}_{4}^i\omega_5, {}_{5}^i\omega_6]^T \end{aligned} \right\} \quad (21)$$

The inverse velocity analysis consists of finding the joint rate velocities of the robot given a prescribed velocity state of the end-effector-platform with respect to the fixed platform,  ${}^A\mathbf{V}_O^C$ . This analysis is solved directly by means of expressions (18) and (19), however the loss freedom of the end-effector-platform must be taken into proper account in order to obtain the desired velocity state. Furthermore, it is important to emphasize that the Jacobians  ${}^A\mathbf{J}_i^B$ ,  ${}^B\mathbf{J}_i^C$  and  ${}^A\mathbf{J}_i^C$  must be invertible, otherwise the robot is at singular configuration.

On the other hand, the forward velocity analysis consists of finding the velocity state  ${}^A\mathbf{V}_O^C$ , given the active joint rate velocities of the robot. It is interesting to note that due to the decoupled architecture, the velocity state  ${}^A\mathbf{V}_O^B$  depends only of the three active joints  ${}_{2}\omega_3^i (i=1,2,3)$ . Furthermore, since  ${}_{3}^4\bar{\$}_i$  and  ${}_{4}^5\bar{\$}_i$  are reciprocal to the remaining screws representing the revolute joints in the same limbs, the application of the Klein form,  $\{*; *\}$ , between the velocity state  ${}^A\mathbf{V}_O^B$  and these reciprocal screws, the reduction of terms leads to

$$\mathbf{J}_1^T \Delta {}^A\mathbf{V}_O^B = [0 \quad {}_{2}\omega_3^1 \quad 0 \quad {}_{2}\omega_3^2 \quad 0 \quad {}_{2}\omega_3^3]^T \quad (22)$$

where

$$\mathbf{J}_1 = [{}_{3}^4\bar{\$}_1, {}_{4}^5\bar{\$}_1, {}_{3}^4\bar{\$}_2, {}_{4}^5\bar{\$}_2, {}_{3}^4\bar{\$}_3, {}_{4}^5\bar{\$}_3] \quad (23)$$

is the active Jacobian matrix between the middle and fixed platforms and

$$\Delta = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ \mathbf{I} & \mathbf{O} \end{bmatrix} \quad (24)$$

is an operator of polarity defined by the  $3 \times 3$  identity matrix  $\mathbf{I}$  and the  $3 \times 3$  zero matrix  $\mathbf{O}$ . Hence, the velocity state  ${}^A\mathbf{V}_O^B$  is obtained directly from the input/output velocity equation (22). In order to compute the velocity state  ${}^A\mathbf{V}_O^C$  please note that the screws  ${}_{16}^{17}\bar{\$}_i (i=1,2,3)$  are reciprocal to the remaining screws, in the same limb, representing the revolute joints of the UPS-type limbs. Thus, after applying the Klein form between these screws and the velocity state  ${}^A\mathbf{V}_O^C$ , the reduction of terms yields

$$\{^{16}\$^i; {}^A V_O^C\} = {}_{14}\underline{\omega}_{15}^i \quad i = 1, 2, 3 \quad (25)$$

Similarly, the application of the Klein form of the screws  ${}^9\$^i$  ( $i = 1, 2, 3$ ) to both sides of Eq. (18) allows to write

$$\{^9\$^i; {}^A V_O^C\} = \{^9\$^i; {}^A V_O^B\} \quad i = 1, 2, 3 \quad (26)$$

Finally, casting in a matrix-vector form Eqs. (25) and (26) one obtains

$$\underline{J}_2^T \Delta {}^A V_O^C = \begin{bmatrix} {}_{14}\underline{\omega}_{15}^1 \\ {}_{14}\underline{\omega}_{15}^2 \\ {}_{14}\underline{\omega}_{15}^3 \\ \{^9\$^1; {}^A V_O^B\} \\ \{^9\$^2; {}^A V_O^B\} \\ \{^9\$^3; {}^A V_O^B\} \end{bmatrix} \quad (27)$$

where

$$\underline{J}_2 = [{}^{16}\$^1, {}^{16}\$^2, {}^{16}\$^3, {}^9\$^1, {}^9\$^2, {}^9\$^3] \quad (28)$$

is the active Jacobian matrix between the output and fixed platforms.

Therefore the velocity state  ${}^A V_O^C$  can be computed directly from the input/ output velocity equation (27). Please note that the forward velocity analysis requires that the active Jacobian matrices  $\underline{J}_1$  and  $\underline{J}_2$  must be invertible, otherwise the manipulator is at a singular configuration.

### 5.3 Redundancy analysis of the robot D1

In what follows the redundancy of the robot under study is briefly explained. Firstly, consider that according to section 3  ${}^A \omega^B \cdot {}^A \tau^B = {}^B \omega^C \cdot {}^B \tau^C = 0$  where  ${}^A \tau^B$  and  ${}^B \tau^C$  are, respectively, normal vectors to the planes  $S_1S_2S_3$  and  $S_4S_5S_6$ . Furthermore, taking into account that  ${}^A \omega^C = {}^A \omega^B + {}^B \omega^C$ , then the loss rotation of the robot leads to

$${}^A \omega^C \cdot ({}^A \tau^B + {}^B \tau^C) - {}^A \omega^B \cdot {}^B \tau^C - {}^B \omega^C \cdot {}^A \tau^B = 0 \quad (29)$$

Equation (29) is called a *zero-torsion condition* and indicates that one element of the angular velocity  ${}^A \omega^C$  can be written as a linear combination of its remaining components. With this consideration in mind the velocity state  ${}^A V_O^C$  can be considered, by using a proper reference frame, as a five-dimensional vector which implies that it is possible to write, according to Eqs. (22) and (27),  ${}^A V_O^C$  in terms of first order coefficients (Gallardo-Alvarado & Rico-Martínez, 2001) as

$${}^A V_O^C = \mathbf{G}\mathbf{Q} + \mathbf{Q}_* \quad (30)$$

where  $\mathbf{Q}$  is a  $5 \times 1$  matrix containing five of the six generalized or active joint rate velocities which is affected by the  $5 \times 5$  matrix  $\mathbf{G}$  whose elements are the corresponding first order coefficients of the chosen active joints while  $\mathbf{Q}_*$  is a  $5 \times 1$  matrix formed with the remaining active joint multiplied by its corresponding first order coefficients. Given a prescribed

velocity state  ${}^A V_O^C$ , expression (30) indicates that the user can select five of the six active joints and the remaining one can be used in order to avoid/escape from possible singularities, if any. Furthermore, the extra active joint can be used with the purpose to optimize trajectories. This feature is one of the main benefits of the robot D1.

#### 5.4 Acceleration analysis

Let  ${}^A \dot{\omega}^C$  and  ${}^A a_O^C$  be, respectively, the angular and linear accelerations of a point  $O$  of the end-effector-platform. The reduced acceleration state, or accelerator, of the end-effector-platform with respect to the fixed platform,  ${}^A A_O^C = [{}^A \dot{\omega}^C, {}^A a_O^C - {}^A \omega^C \times {}^A v_O^C]^T$ , can be obtained through the coupler and fixed platforms as

$${}^A A_O^C = {}^A A_O^B + {}^B A_O^C + [{}^A V_O^B \quad {}^B V_O^C] \quad (31)$$

where  ${}^B A_O^C$  is the accelerator of the coupler platform with respect to the fixed platform,  ${}^B A_O^C$  is the accelerator of the end-effector-platform with respect to the coupler platform. Furthermore, these accelerators can be written in screw form as follows

$$A = J_i \dot{\Omega}_i + \mathcal{L}_i \quad A \in \{{}^A A_O^B, {}^B A_O^C, {}^A A_O^C\} \quad i = 1, 2, 3 \quad (32)$$

where  $\dot{\Omega}_i \in \{{}^A \dot{\Omega}_i^B, {}^B \dot{\Omega}_i^C, {}^A \dot{\Omega}_i^C\}$  are matrices containing the joint rate accelerations of the corresponding limbs, whereas  $\mathcal{L}_i \in \{{}^A \mathcal{L}_i^B, {}^B \mathcal{L}_i^C, {}^A \mathcal{L}_i^C\}$  are composed Lie products given by

$$\left. \begin{aligned} {}^A \mathcal{L}_i^C &= \sum_{j=12}^{16} \left[ {}^j \omega_{j+1}^i \quad {}^j \mathcal{L}_i^{j+1} \quad \sum_{k=j+1}^{17} {}^k \omega_{k+1}^i \quad {}^k \mathcal{L}_i^{k+1} \right] \\ {}^B \mathcal{L}_i^C &= \sum_{j=6}^{10} \left[ {}^j \omega_{j+1}^i \quad {}^j \mathcal{L}_i^{j+1} \quad \sum_{k=j+1}^{11} {}^k \omega_{k+1}^i \quad {}^k \mathcal{L}_i^{k+1} \right] \\ {}^A \mathcal{L}_i^B &= \sum_{j=0}^4 \left[ {}^j \omega_{j+1}^i \quad {}^j \mathcal{L}_i^{j+1} \quad \sum_{k=j+1}^5 {}^k \omega_{k+1}^i \quad {}^k \mathcal{L}_i^{k+1} \right] \end{aligned} \right\} \quad (33)$$

The inverse acceleration analysis consists of finding the joint rate accelerations of the robot given a prescribed accelerator  ${}^A A_O^C$ . This analysis is carried-out by means of expressions (31) and (32).

On the other hand the forward acceleration analysis consists of finding the accelerator of the end-effector-platform with respect to the fixed platform,  ${}^A A_O^C$ , given the active joint rate accelerations of the robot. This analysis is very close to the presented to solve the forward velocity analysis, therefore only the obtained expressions are included here.

The accelerator  ${}^A A_O^B$  can be computed upon the input/output acceleration expression

$$J_{-1}^T \Delta {}^A A_O^B = \begin{bmatrix} \{ {}^3 \mathcal{L}_1^4, {}^A \mathcal{L}_1^B \} \\ {}_2 \dot{\omega}_3^1 + \{ {}^4 \mathcal{L}_1^5, {}^A \mathcal{L}_1^B \} \\ \{ {}^3 \mathcal{L}_2^4, {}^A \mathcal{L}_2^B \} \\ {}_2 \dot{\omega}_3^2 + \{ {}^4 \mathcal{L}_2^5, {}^A \mathcal{L}_2^B \} \\ \{ {}^3 \mathcal{L}_3^4, {}^A \mathcal{L}_3^B \} \\ {}_2 \dot{\omega}_3^3 + \{ {}^4 \mathcal{L}_3^5, {}^A \mathcal{L}_3^B \} \end{bmatrix} \quad (34)$$

whereas the reduced acceleration state  ${}^A A_O^C$  can be obtained from the input/ output acceleration relationship

$$\mathbf{J}_2^T \Delta {}^A A_O^C = \begin{bmatrix} {}_{14}\dot{\omega}_{15}^1 + \{{}^{16}\$1^7; {}^A \mathcal{L}_1^C\} \\ {}_{14}\dot{\omega}_{15}^2 + \{{}^{16}\$1^7; {}^A \mathcal{L}_2^C\} \\ {}_{14}\dot{\omega}_{15}^3 + \{{}^{16}\$1^7; {}^A \mathcal{L}_3^C\} \\ \{{}^9\$1^{10}; {}^A A_O^B + [{}^A V_O^B {}^B V_O^C] + {}^B \mathcal{L}_1^C\} \\ \{{}^9\$2^{10}; {}^A A_O^B + [{}^A V_O^B {}^B V_O^C] + {}^B \mathcal{L}_2^C\} \\ \{{}^9\$3^{10}; {}^A A_O^B + [{}^A V_O^B {}^B V_O^C] + {}^B \mathcal{L}_3^C\} \end{bmatrix} \quad (35)$$

Finally, please note that the computation of the accelerators  ${}^A A_O^B$  and  ${}^A A_O^C$ , by means respectively of expression (34) and (35), does not require the values of the passive joint rate accelerations of the robot. Furthermore, once these reduced acceleration states are calculated, the accelerator  ${}^B A_O^C$  is obtained using expression (31).

## 6. Computer aided kinematic simulations

With the purpose to exemplify the performance of the D1 robot, in this section the kinematic behavior of a virtual prototype, left image provided in Fig. 2, is simulated by means of the commercially available software ADAMS<sup>®</sup>.

The parameters of the robot, using hereafter SI units, are given by

$$\left. \begin{array}{l} R_1 = (0.3, 0, 0) \\ R_2 = (-0.125, 0, -0.216) \\ R_3 = (-0.125, 0, 0.216) \\ U_1 = (0.5, -0.25, 0) \\ U_2 = (-0.215, -0.25, -0.337) \\ U_3 = (-0.215, -0.25, 0.337) \\ \overline{S_1 S_2} = \overline{S_1 S_3} = \overline{S_3 S_2} = 0.173 \\ \overline{S_4 S_5} = \overline{S_4 S_6} = \overline{S_5 S_6} = 0.311 \\ \overline{S_1 R_4} = \overline{S_2 R_5} = \overline{S_3 R_6} = 0.125 \end{array} \right\}$$

whereas in the home position the coordinates of the spherical joints  $S_i (i = 1, 2, \dots, 6)$  and of the tool tip  $T$  are given by

$$\left. \begin{array}{l} S_1 = (0.1, -0.5, 0) \\ S_2 = (-0.05, -0.05, 0.086) \\ S_3 = (-0.05, -0.05, -0.086) \\ S_4 = (0.18, -0.75, 0) \\ S_5 = (-0.09, -0.75, -0.155) \\ S_6 = (-0.09, -0.75, 0.155) \\ T = (0, -1.0, 0) \end{array} \right\}$$

The inverse kinematics of the robot is proved simulating the D1 robot as a parallel kinematic machine tool. To this end, two tasks are assigned to the tool tip:

- The robot will drill three holes
- The robot will mill a hexagon

In order to achieve these tasks, only five of the six available generalized coordinates are required, therefore one of them, e.g.  $q_3$ , should be locked. After, the required instantaneous variations of the generalized coordinates satisfying such operations are provided in Fig. 4.

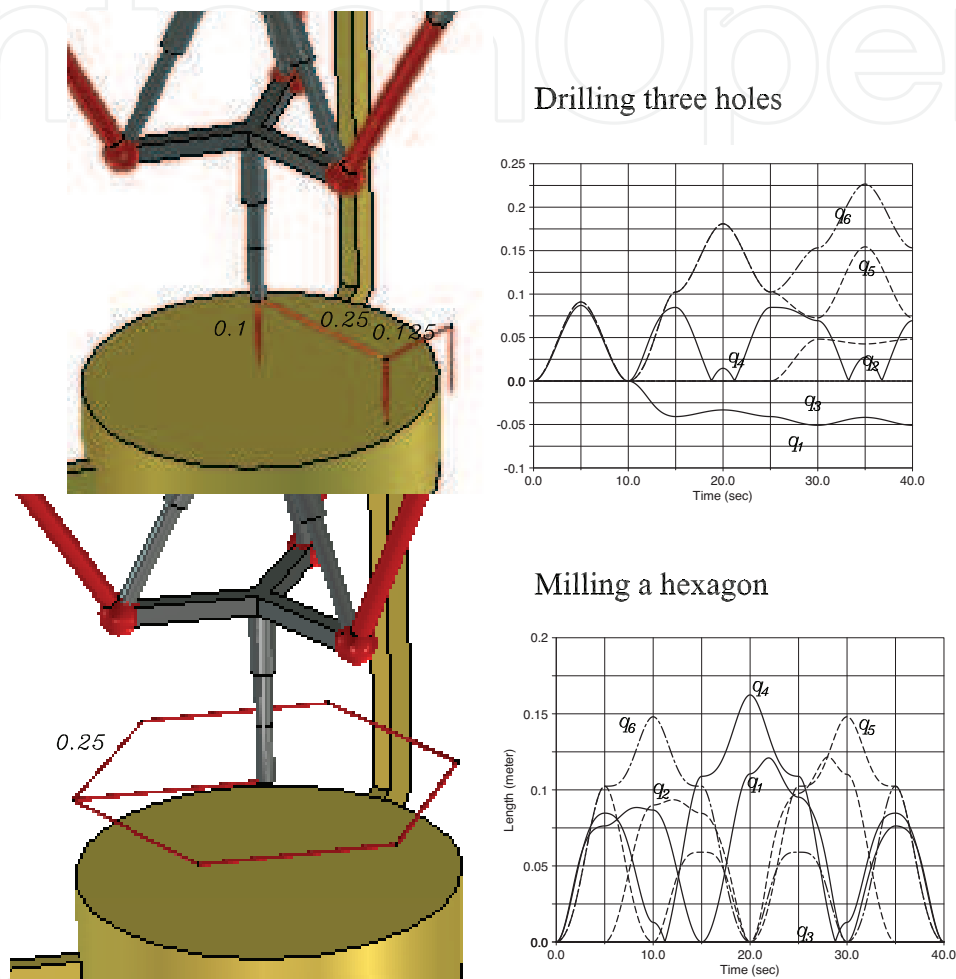


Fig. 4. The D1 robot working as a non-redundant five-degrees-of-freedom parallel kinematic machine

On the other hand, the forward kinematics of the robot is simulated by using the six generalized coordinates. In other words, the robot D1 is used as a redundant manipulator with six active joints to realize five degrees of freedom in the output platform. To this end, upon the home position of the robot D1, the active limbs are conditioned to the following periodical variations

$$\left. \begin{array}{l} \Delta q_1 = -0.01 \sin(t), \Delta q_2 = 0.0125 \sin(t) \\ \Delta q_3 = 0.015 \sin(t) \cos(t), \Delta q_4 = 0.1 \sin(t) \cos(t) \\ \Delta q_5 = 0.075 \sin(t), \Delta q_6 = 0.125 \sin^2(t) \\ 0.0 \leq t \leq 2\pi \end{array} \right\}$$

With these data the most representative results of the simulation are provided in Fig. 5.

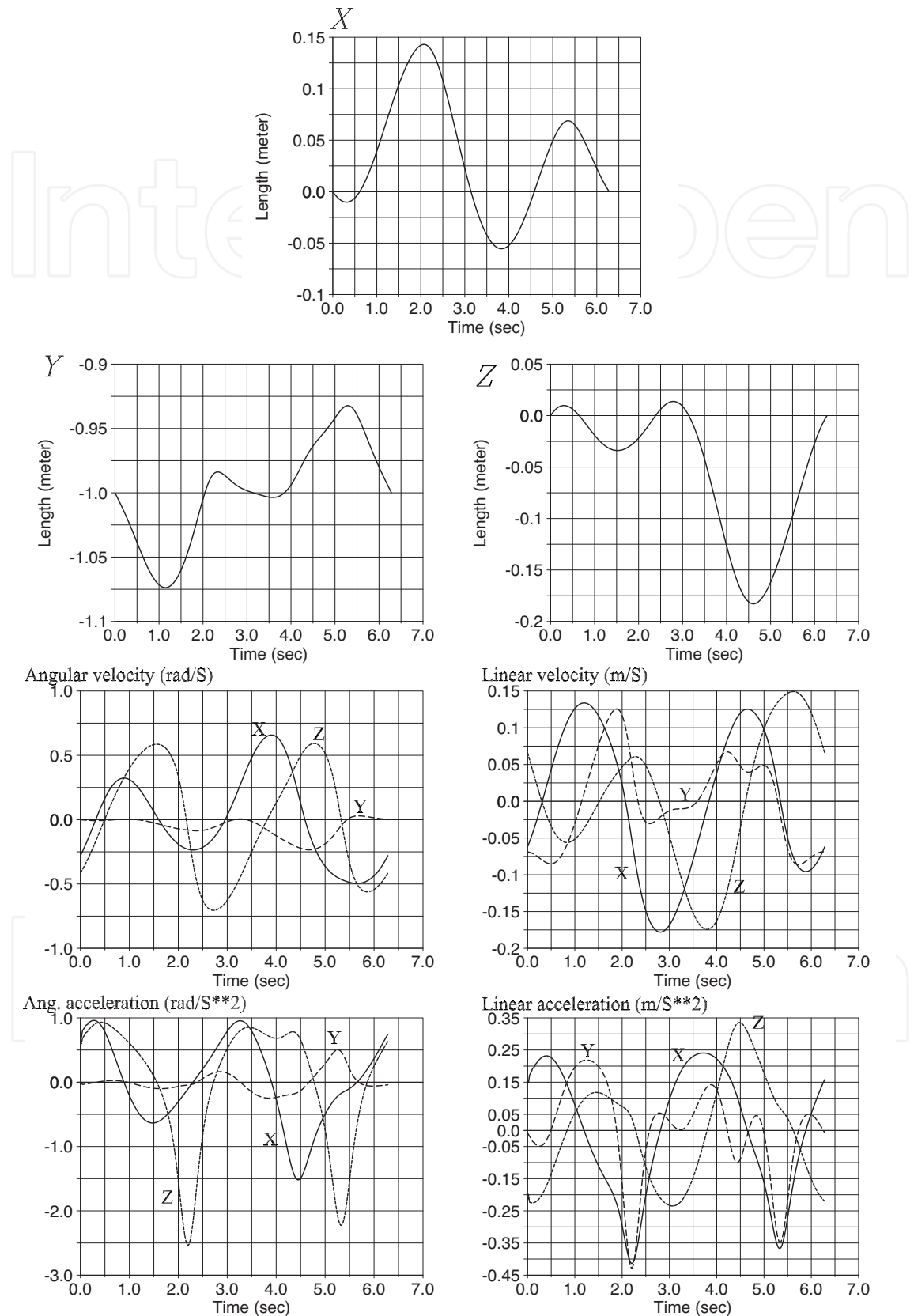


Fig. 5. Forward kinematics, six active joints to realize five degrees of freedom



Finally, of course a virtual prototype is not the final word about the correctness performance of a proposed manipulator, but it is an advisable option before the construction of a real prototype.

## 7. Conclusions

In this work a new class of redundant robot manipulators called DeLiA is introduced. The main features of the proposed robots are:

- Symmetry
- Decoupled architecture. Only three of the six active limbs connect the end-effector-platform to the fixed platform
- The forward position analysis, a challenging task of most parallel manipulators, is carried-out by solving two sets of non-linear equations
- The proposed robot does not require of special conditions of mechanical assembly like intersection of screws or similar
- The six active limbs are mounted on the fixed platform, simplifying the kinematics and control of the robot
- Redundancy, the robot is endowed with an extra degree of freedom which can be used with the purpose to avoid/escape from singular configurations, as well to optimize trajectories. Any of the six active joints can play this role

Finally, D1, a member of the DeLiA robot family, is simulated as a five-degrees-of-freedom parallel kinematic machine tool with the aid of commercially available software like ADAMS®.

## 8. Acknowledgement

This work was supported by National Council of Science and Technology of México, Conacyt.

## 9. References

- Altuzarra, O.; Loizaga, M.; Pinto, C. & Petuya, V. (2010). Synthesis of partially decoupled multi-level manipulators with lower mobility, *Mechanism and Machine Theory*, Vol. 45, No. 1, 106–118.
- Ball, R.S. (1999, Reprinted 1998). *A Treatise on the Theory of Screws*, Cambridge University Press, Cambridge.
- Bohez, E.L.J. (2002). Five-axis milling machine tool kinematic chain design and analysis, *International Journal of Machine Tools and Manufacture*, Vol. 42, No. 4, 505–520.
- Bonev, I.A. (2002). *Geometric Analysis of Parallel Mechanisms*, thèse de doctorat, Université Laval, Canada.
- Bonev, I. (2003). The true origins of parallel robots. Available at Parallel Mic Center, ParalleMIC online review, available at <http://www.parallemic.org/Reviews/Review007.html>
- Briot, S.; Arakelian, V. & Guégan, S. (2009). PAMINSA: a new family of partially decoupled parallel manipulators, *Mechanism and Machine Theory*, Vol. 44, No. 2, 425–444.
- Briot, S. & Bonev, C.M. (2009). Pantopteron: a new fully decoupled 3DOF translational parallel robot for pick-and-place applications, *ASME Journal of Mechanisms and Robotics*, Vol. 1, May, paper 021001, 1–9.

- Carricato, M. & Parenti-Castelli, V. (2002). Singularity-free fully-isotropic translational parallel mechanisms, *The International Journal of Robotics Research*, Vol. 21, No. 2, 161-174.
- Carricato, M. & Parenti-Castelli, V. (2004). A novel fully decoupled two-degrees-of-freedom parallel wrist, *The International Journal of Robotics Research*, Vol. 23, No. 6, 661-667.
- Dai, J.S.; Huang, Z. & Lipkin, H. (2006). Mobility of overconstrained parallel mechanisms, *ASME Journal of Mechanical Design*, Vol. 128, No. 1, 220-229.
- Gallardo, J.; Orozco, H.; Rodríguez, R. & Rico, J.M. (2007). Kinematics of a class of parallel manipulators which generates structures with three limbs, *Multibody System Dynamics*, Vol. 17, No. 1, 27-46.
- Gallardo, J.; Rico, J.M.; Frisoli, A.; Checcacci, D. & Bergamasco, M. (2003). Dynamics of parallel manipulators by means of screw theory, *Mechanism and Machine Theory*, Vol. 38, No. 11, 1113-1131.
- Gallardo-Alvarado, J., & Rico-Martínez, J.M. (2001) Jerk influence coefficients, via screw theory, of closed chains, *Meccanica*, Vol. 36, No. 2, 213-228.
- Gallardo-Alvarado, J.; Aguilar-Nájera, C.R.; Casique-Rosas, L.; Pérez-González, L. & Rico-Martínez, J.M. (2008). Solving the kinematics and dynamics of a modular spatial hyper-redundant manipulator by means of screw theory, *Multibody System Dynamics*, Vol. 20, No. 4, 307-325.
- Gallardo-Alvarado, J.; Rojas-Garduño, H. & Arroyo-Ramírez, B. (2009). Kinematics of a five-degrees-of-freedom parallel manipulator using screw theory, *The International Journal of Advance Manufacturing Technology*, Vol. 45, No. (7-8), 830-840.
- Gallardo-Alvarado, J. & Lasso-Arroyo, R. (2009). Jerk analysis of a module of an artificial spine by means of screw theory, *Journal of Advanced Research and Technology*, Vol. 7, No. 3, 249-258.
- Gao, F.; Peng, B.; Li, W. & Zhao, H. (2005). Design of a novel 5-DOF fully parallel kinematic machine tool based on workspace, *Robotica*, Vol. 23, No. 1, 35-43.
- Gao, F.; Peng, B.; Zhao, H. & Li, W. (2006). A novel 5-DOF fully parallel kinematic machine tool, *The International Journal of Advanced Manufacturing Technology*, Vol. 31, No. (1-2), 201-207.
- Gogu, G. (2005). Singularity-free fully-isotropic parallel manipulators with Schönflies motions, *Proceedings of the ICAR International Conference on Advanced Robotics*, pp. 194-201, Seattle, WA.
- Gogu, G. (2005). Mobility of mechanisms: a critical review, *Mechanism and Machine Theory*, Vol. 40, No. 9, 1068-1097.
- Gosselin, C.M.; Kong, X.; Foucault, S. & Bonev, I.A. (2004). A fully decoupled 3-DOF translational parallel mechanism, *Proceedings PKM International Conference*, pp. 595610, Chemnitz, Germany.
- Gough, V.E. (1957). Contribution to discussion to papers on research in automobile stability and control and in type performance, *Proceedings Automation Division Institution of Mechanical Engineers*, 392-395.
- Gough, V.E. & Whitehall, S.G. (1962). Universal tyre test machine, *Proceedings of the FISITA Ninth International Technical Congress*, pp. 117-137.
- Innocenti, C. & Parenti-Castelli, V. (1991). Direct kinematics of the 6-4 fully parallel manipulator with position and orientation uncoupled, *Proceedings European Robotics and Intelligent Systems Conference*, pp. 23-28, Corfou.
- Kong, X. & Gosselin, C.M. (2002a). A class of 3-DOF translational parallel manipulators with linear input-output equations, *Proceedings of the Workshop on Fundamental*

- Issues and Future Research Directions for Parallel Mechanisms and Manipulators*, pp. 2532, Quebec City, Quebec, Canada.
- Kong, X. & Gosselin, C.M. (2002b). Type synthesis of linear translational parallel manipulators, In: *Advances in Robot Kinematics Theory and Applications*, Lenarčič, J. & Thomas, F. (Eds.), Kluwer Academic Publishers, 411420.
- Kong, X. & Gosselin, C.M. (2002c). Kinematics and singularity analysis of 3- CRR 3-DOF translational parallel manipulators, *The International Journal of Robotics Research*, Vol. 21, No. 9, 791798.
- Li, Q. & Huang, Z. (2003). Type synthesis of 5-DOF parallel manipulators, *Proceedings of the 2003 IEEE International Conference on Robotics & Automation*, pp. 1203–1208, Taipei, Taiwan.
- Li, W.M.; Gao, F. & Zhang, J.J. (2005). R-CUBE, a decoupled parallel manipulator only with revolute joints, *Mechanism and Machine Theory*, Vol. 40, No. 4, 467–473.
- Piccina, O.; Bayle, B.; Maurin B. & de Mathelin, M. (2009). Kinematic modeling of a 5-DOF parallel mechanism for semi-spherical workspace, *Mechanism and Machine Theory*, Vol. 44, No. 8, 1485–1496.
- Pollard, W.L.G. (1940). *Spray painting machine*, US Patent No. 2,213,108, August 26.
- Rico-Martínez, J.M. & Duffy, J. (1996). An application of screw algebra to the acceleration analysis of serial chains, *Mechanism and Machine Theory*, Vol. 31, No. 4, 445–457.
- Rico, J.M. & Duffy, J. (2000). Forward and inverse acceleration analyses of in-parallel manipulators, *ASME Journal of Mechanical Design*, Vol. 122, No. 3, 299–303.
- Ruggiu, M. (2009). Kinematic analysis of a fully decoupled translational parallel manipulator, *Robotica*, Vol. 27, 961–969.
- Stewart, D. (1965). A platform with six degrees of freedom, *Proceedings Institution of Mechanical Engineers Part I*, Vol. 180, No. 15, 371–386.
- Takeda, Y.; Kamiyama, K.; Maki, Y.; Higuchi, M. & Sugimoto, K. (2005). Development of position-orientation decoupled spatial in-parallel actuated mechanisms with six degrees of freedom, *Journal of Robotics and Mechatronics*, Vol. 17, No. 1, 59–68.
- Tsai, L.W. (1999). *Robot Analysis*, John Wiley & Sons, New York.
- Vlachos, K. & Papadopoulos, E. (2005). Endpoint-Side optimization of a five degrees-of-freedom haptic mechanism, *Proceedings of the 13th Mediterranean Conference on Control and Automation Limassol*, pp. 27–29 Cyprus.
- Wohlhart, K. (1994). Displacement analysis of the general spherical Stewart platform, *Mechanism and Machine Theory*, Vol. 29, No. 4, 581–589.
- Yang, G.; Chen, I-M.; Chen, W. & Lin, W. (2004). Kinematic design of a six-dof parallel-kinematics machine with decoupled-motion architecture, *IEEE Transactions on Robotics*, Vol. 20, No. 5, 876–884.
- Zabalza, I.; Ros, J.; Gil, J.J.; Pintor, J.M. & Jiménez, J.M. (2002). Tri-Scott. A new kinematic structure for a 6-dof decoupled parallel manipulator, *Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators*, pp. 12–15, Québec, Canada.
- Zheng, K.-J.; Gao, J.-S. & Zhao, Y.S. (2005). Path control algorithms of a novel 5-DOF parallel machine tool, *Mechatronics and Automation, 2005 IEEE International Conference*, pp. 1381–1385, Niagara Falls, Ont., Canada.
- Zhu, S.J.; Huang, Z. & Zhao, M.Y. (2008). Feasible human-spine motion simulators based on parallel manipulators, In: *Parallel Manipulators, Towards New Applications*, Wu, H., ed., I-Tech Education and Publishing, Vienna, Austria.



## **Advanced Strategies for Robot Manipulators**

Edited by S. Ehsan Shafiei

ISBN 978-953-307-099-5

Hard cover, 428 pages

**Publisher** Sciyo

**Published online** 12, August, 2010

**Published in print edition** August, 2010

Amongst the robotic systems, robot manipulators have proven themselves to be of increasing importance and are widely adopted to substitute for human in repetitive and/or hazardous tasks. Modern manipulators are designed complicatedly and need to do more precise, crucial and critical tasks. So, the simple traditional control methods cannot be efficient, and advanced control strategies with considering special constraints are needed to establish. In spite of the fact that groundbreaking researches have been carried out in this realm until now, there are still many novel aspects which have to be explored.

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Jaime Gallardo-Alvarado (2010). DeLiA: a New Family of Redundant Robot Manipulators, *Advanced Strategies for Robot Manipulators*, S. Ehsan Shafiei (Ed.), ISBN: 978-953-307-099-5, InTech, Available from: <http://www.intechopen.com/books/advanced-strategies-for-robot-manipulators/delia-a-new-family-of-redundant-robot-manipulators>

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