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Active Vibration Control for a Nonlinear Mechanical System using On-line Algebraic Identification

Francisco Beltrán[1], Gerardo Silva[2], Andrés Blanco[3] and Esteban Chávez[4]

[1]Universidad Politécnica de la Zona Metropolitana de Guadalajara
[2]Centro de Investigación y de Estudios Avanzados del IPN
[3]Centro Nacional de Investigación y Desarrollo Tecnológico
[4]Universidad Politécnica de Guanajuato
México

1. Introduction

Many engineering systems undergo undesirable vibrations. Vibration control in mechanical systems is an important problem, by means of which vibrations are suppressed or at least attenuated. In this direction it has been common the use of passive and active dynamic vibration absorbers.

A dynamic vibration absorber is an inertia member coupled to a vibrating mechanical system by suitable linear and nonlinear coupling members (e.g., springs and dampers). For the passive case, the absorber only serves for a specific excitation frequency and stable operating conditions, but it is not recommended for variable frequencies and uncertain system parameters. An active dynamic vibration absorber achieves better dynamic performance by controlling actuator forces depending on feedback and feedforward information of the system obtained from sensors.

To cancel the exogenous harmonic vibrations on the primary system, the dynamic vibration absorber should apply an equivalent reaction force to the primary system equal and opposite to the exciting force causing the vibrations. This means that the vibration energy injected to the primary system is transferred to the absorber through the coupling elements. For more details about dynamic vibration absorber we refer to (Korenev & Reznikov, 1993) and references therein.

This chapter deals with the attenuation problem of harmonic mechanical vibrations in nonlinear mechanical systems by using active vibration absorbers and without employing vibration measurements. On-line algebraic identification is applied for the on-line estimation of the frequency and amplitude of exogenous vibrations affecting the nonlinear vibrating mechanical system. The proposed results are strongly based on the algebraic approach to parameter identification in linear systems reported by (Fliess & Sira-Ramírez, 2003), which employs differential algebra, module theory and operational calculus.

An important property of the algebraic identification is that the parameter and signal identification is not asymptotic but algebraic, that is, the parameters are computed as fast as the system dynamics is being excited by some external input or changes in its initial

conditions, in contrast to the well-known persisting excitation condition and complex algorithms required by most of the traditional identification methods (Ljung, 1987) and (Soderstrom, 1987).

The algebraic identification is combined with a certainty equivalence differential flatness based controller for asymptotic output tracking of an off-line and pre-specified output trajectory and cancellation of harmonic perturbations affecting directly the mechanical system. Numerical results show the dynamic and robust performance of the algebraic identification and the active vibration control scheme. Algebraic identification has been employed for parameter and signal estimation in linear vibrating mechanical systems by (Beltrán et al., 2010). Here numerical and experimental results show that the algebraic identification provides high robustness against parameter uncertainty, frequency variations, small measurement errors and noise.

2. Algebraic parameter identification

To illustrate the basic ideas of the algebraic identification methods proposed by (Fliess & Sira- Ramírez, 2003), it is considered the on-line parameter identification of a simple one degree-of freedom mass-spring-damper system as well as the parameters associated to an exogenous harmonic perturbation affecting directly its dynamics. The mathematical model of the mechanical system is described by the ordinary differential equation

$$m\ddot{x} + c\dot{x} + kx = u(t) + f(t) \tag{1}$$

where x denotes the displacement of the mass carriage, u is a control input (force) and $f(t) = F_0 \sin\omega t$ is a harmonic force (perturbation). The system parameters are the mass m, the stiffness constant of the linear spring k and the viscous damping c.

In spite of *a priori* knowledge of the mathematical model (1), it results evident that this is only an approximation for the physical system, where for large excursions of the mass carriage the mechanical spring has nonlinear stiffness function and close to the rest position there exist nonlinear damping effects (e.g., dry or Coulomb friction). Another inconvenient is that the information used during the identification process contains small measurement errors and noise. It is therefore realistic to assume that the identified parameters will represent approximations to equivalent values into the physical system. As a consequence the algorithms will have to be sufficiently robust against such perturbations. Some of these properties have been already analyzed by (Fliess & Sira-Ramírez, 2003).

2.1 On-line algebraic identification

Consider the unperturbed system (1), that is, when $f(t) \equiv 0$, where only measurements of the displacement x and the control input u are available to be used in the on-line parameter identification scheme. To do this, the differential equation (1) is described in notation of operational calculus (Fliess & Sira-Ramírez, 2003) as follows

$$m\left(s^2 x(s) - sx_0 - \dot{x}_0\right) + c\left(sx(s) - x_0\right) + kx(s) = u(s) \tag{2}$$

where $x_0 = x(t_0)$ and $\dot{x}_0 = \dot{x}(t_0)$ are unknown constants denoting the system initial conditions at $t_0 \geq 0$. In order to eliminate the dependence of the constant initial conditions, the equation (2) is differentiated twice with respect to the variable s, resulting in

$$m\left(2x + 4s\frac{dx}{ds} + s^2\frac{d^2x}{d^2s} \right) + c\left(2\frac{dx}{ds} + s\frac{d^2x}{ds^2} \right) + k\frac{d^2x}{d^2s} = \frac{d^2u}{ds^2} \tag{3}$$

Now, multiplying (3) by s^{-2} one obtains that

$$m\left(2s^{-2}x + 4s^{-1}\frac{dx}{ds} + \frac{d^2x}{d^2s} \right) + c\left(2s^{-2}\frac{dx}{ds} + s^{-1}\frac{d^2x}{ds^2} \right) + ks^{-2}\frac{d^2x}{d^2s} = s^{-2}\frac{d^2u}{ds^2} \tag{4}$$

and transforming back to the time domain leads to the integral equation

$$m\left[2\left(\int_{t_0}^{(2)}x \right) - 4\left(\int_{t_0}(\Delta t)x \right) + (\Delta t)^2 x \right] + c\left[-2\left(\int_{t_0}^{(2)}(\Delta t)x \right) + \left(\int_{t_0}(\Delta t)^2 x \right) \right] +$$
$$k\left(\int_{t_0}^{(2)}(\Delta t)^2 x \right) = \left(\int_{t_0}^{(2)}(\Delta t)^2 u \right) \tag{5}$$

where $\Delta t = t - t_0$ and $\left(\int_{t_0}^{(n)}\varphi(t) \right)$ are iterated integrals of the form $\int_{t_0}^{t}\int_{t_0}^{\sigma_1}\cdots\int_{t_0}^{\sigma_{n-1}}\varphi(\sigma_n)d\sigma_n\cdots d\sigma_1$,

with $\left(\int_{t_0}\varphi(t) \right) = \int_{t_0}^{t}\varphi(\sigma)d\sigma$ and n a positive integer.

The above integral-type equation (5), after some more integrations, leads to the following *linear* system of equations

$$A(t)\theta = b(t) \tag{6}$$

where $\theta = [m, c, k]^T$ denotes the parameter vector to be identified and $A(t)$, $b(t)$ are 3×3 and 3×1 matrices, respectively, which are described by

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & a_{13}(t) \\ a_{21}(t) & a_{22}(t) & a_{23}(t) \\ a_{31}(t) & a_{32}(t) & a_{33}(t) \end{bmatrix}, \quad b(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ b_3(t) \end{bmatrix}$$

whose components are time functions specified as

$$a_{11} = 2\int_{t_0}^{(2)}x - 4\int_{t_0}(\Delta t)x + (\Delta t)^2 x \qquad a_{31} = 2\int_{t_0}^{(4)}x - 4\int_{t_0}^{(3)}(\Delta t)x + \int_{t_0}^{(2)}(\Delta t)^2 x$$

$$a_{12} = -2\int_{t_0}^{(2)}(\Delta t)x + \int_{t_0}(\Delta t)^2 x \qquad a_{32} = -2\int_{t_0}^{(4)}(\Delta t)x + \int_{t_0}^{(3)}(\Delta t)^2 x$$

$$a_{13} = \int_{t_0}^{(2)}(\Delta t)^2 x \qquad a_{33} = \int_{t_0}^{(4)}(\Delta t)^2 x$$

$$a_{21} = 2\int_{t_0}^{(3)}x - 4\int_{t_0}^{(2)}(\Delta t)x + \int_{t_0}(\Delta t)^2 x \qquad b_1 = \int_{t_0}^{(2)}(\Delta t)^2 u$$

$$a_{22} = -2\int_{t_0}^{(3)}(\Delta t)x + \int_{t_0}^{(2)}(\Delta t)^2 x \qquad b_2 = \int_{t_0}^{(3)}(\Delta t)^2 u$$

$$a_{23} = \int_{t_0}^{(3)}(\Delta t)^2 x \qquad b_3 = \int_{t_0}^{(4)}(\Delta t)^2 u$$

From the equation (6) can be concluded that the parameter vector θ is *algebraically identifiable* if, and only if, the trajectory of the dynamical system is *persistent* in the sense established by

(Fliess & Sira-Ramírez, 2003), that is, the trajectories or dynamic behavior of the system (1) satisfy the condition

$$\det A(t) \neq 0$$

In general, this condition holds at least in a small time interval $(t_0, t_0 + \delta]$, where δ is a positive and sufficiently small value.

By solving the equations (6) it is obtained the following algebraic identifier for the unknown system parameters

$$\left. \begin{aligned} \hat{m} &= \frac{\Delta_1}{\det A(t)} \\ \hat{c} &= \frac{\Delta_2}{\det A(t)} \\ \hat{k} &= \frac{\Delta_3}{\det A(t)} \end{aligned} \right\} \forall t \in (t_0, t_0 + \delta] \tag{7}$$

where

$$\Delta_1 = b_1 a_{22} a_{33} - b_1 a_{23} a_{32} - b_2 a_{12} a_{33} + b_2 a_{13} a_{32} + b_3 a_{12} a_{23} - b_3 a_{13} a_{22}$$
$$\Delta_2 = -b_1 a_{21} a_{33} + b_1 a_{31} a_{23} + b_2 a_{11} a_{33} - b_2 a_{13} a_{31} - b_3 a_{11} a_{23} + b_3 a_{21} a_{13}$$
$$\Delta_3 = b_1 a_{21} a_{32} - b_1 a_{22} a_{31} - b_2 a_{11} a_{32} + b_2 a_{12} a_{21} + b_3 a_{11} a_{22} - b_3 a_{12} a_{21}$$
$$\det A(t) = a_{11} a_{22} a_{33} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} + a_{12} a_{31} a_{23} + a_{21} a_{13} a_{32} - a_{13} a_{22} a_{31}$$

2.2 Simulation and experimental results

The performance of the on-line algebraic identifier of the system parameters (7) is now evaluated by means of numerical simulations and experiments on a electromechanical platform (ECP^{TM} rectilinear plant) with a single degree-of-freedom mass-spring-damper system. The physical parameters were previously estimated through several experiments with different excitation inputs (natural and forced vibrations, step and sine sweep inputs, etc.) resulting in the following set of parameters:

$$m = 2.2685[kg], \ c = 4.1241[Ns/m], \ k = 356.56[N/m]$$

Nevertheless, it is convenient to remark that the real system clearly exhibits nonlinear effects like nonlinear stiffness and damping functions (hard springs and Coulomb friction on the slides) that were not considered during the synthesis of the algebraic identifier.

Fig. 1 shows the simulation results using the algebraic identifier for a step input $u = 4$ [N]. Here it is clear how the parameter identification is quickly performed (before $t = 1.02$ s) and it is almost exact with respect to the real parameters. It is also evident the presence of singularities in the algebraic identifier, i.e., when the determinant $den = \det A(t)$ is zero. The first singularity, however, occurs about $t = 1.02$ s, that is too much time (more than 5 times) after the identification has been finished.

Fig. 2 presents the corresponding experimental results using the on-line algebraic identification scheme (7). In this case the actual system response is quite similar to the numerical simulation, resulting in the following (equivalent) parameters:
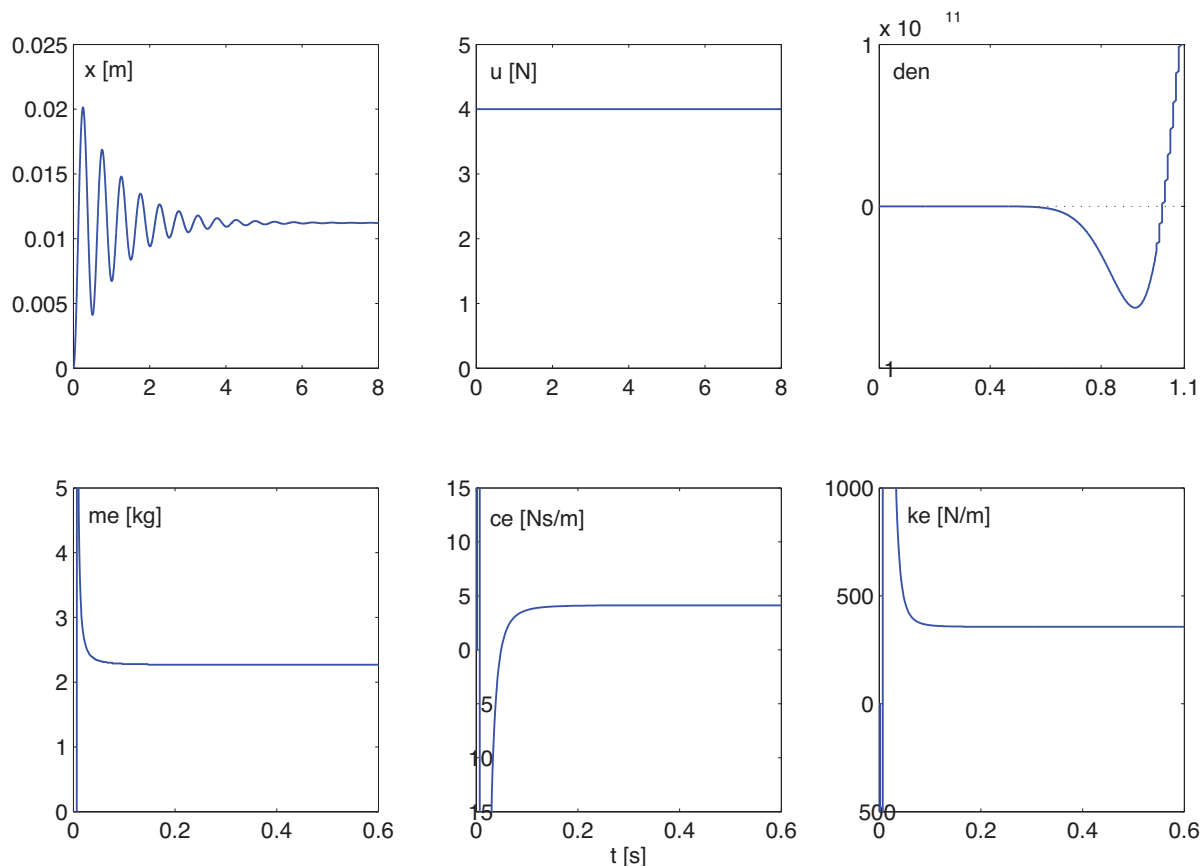
Fig. 1. Simulation results of the algebraic identifier. The subscript "*e*" denotes estimated values and *den* = det$A(t)$

$$m = 2.25[kg], \ c = 4.87[Ns \, / \, m], \ k = 362[Ns \, / \, m]$$

These values represent good approximations for the real parameters. Nevertheless, the identification process starts with some irregular behavior and the estimation takes more time (about $t = 0.4$ s), which we have attributed to several factors like neglected nonlinear effects (stiffness and friction), presence of noise on the output measurements and especially the computational algorithms based upon a sampled-time system with fast sampling time $t_s = 0.000884$ s and numerical integrations based on trapezoidal rules. Some of these problems in the parameter estimation have been already analyzed by (Sagara & Zhao, 1990). Many numerical and experimental results validate the good response of the on-line algebraic identification methods of unknown parameters. In addition, it can be proved the good robustness properties of the algebraic identifiers against stochastic perturbations, noisy measurements, small parameter variations and nonlinearities, which are not included here for space limitations. Moreover, because the algebraic identification process is quickly achieved with a high-speed DSP board, then any possible singularity does not affect significantly the identification results. Otherwise, close to any singularity or variations on the system dynamics, the algebraic identifier can be restarted.

In this chapter the algebraic identification methodology is applied to estimate the parameters associated to exogenous perturbations affecting an nonlinear mechanical vibrating system.
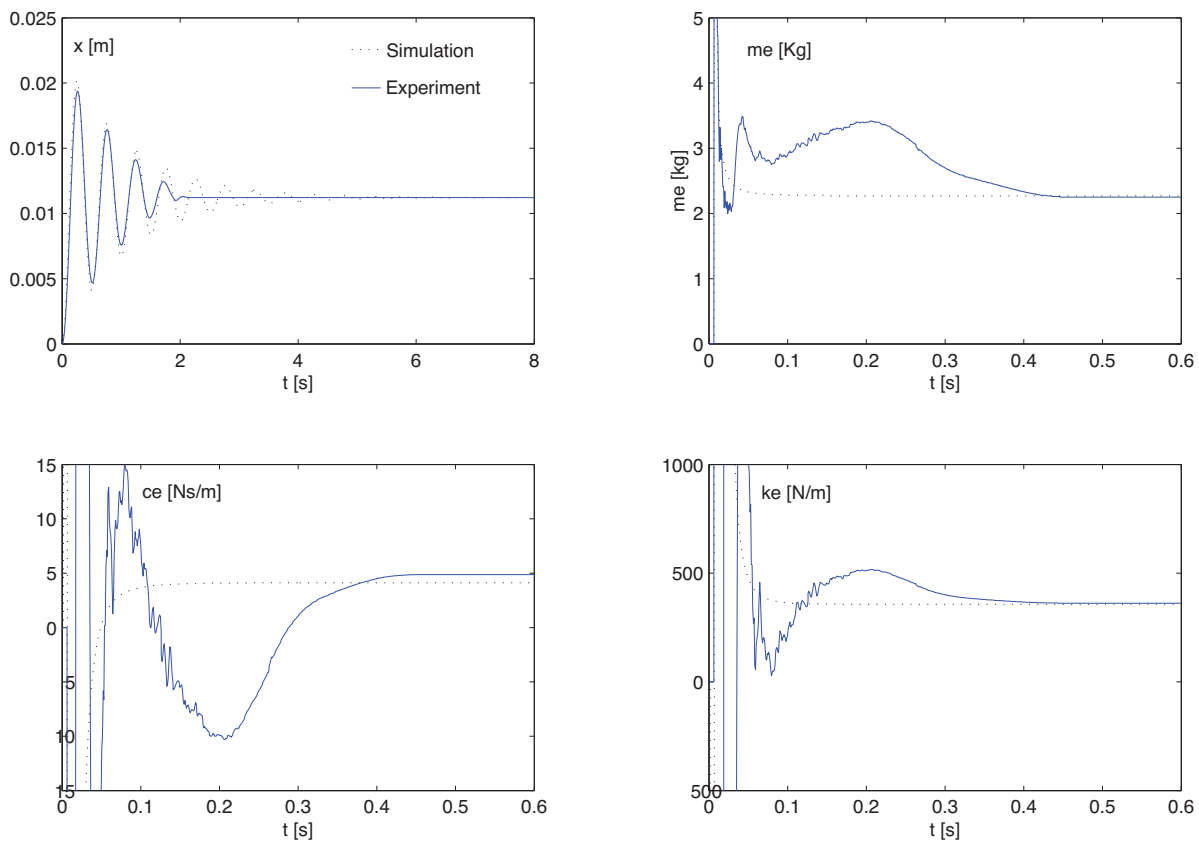
Fig. 2. Experimental results obtained on a ECP^{TM} Rectilinear Plant using algebraic identification methods.

3. An active vibration control scheme

Consider the nonlinear vibrating mechanical system shown in Fig. 3, which consists of an active nonlinear vibration absorber (secondary system) coupled to the perturbed mechanical system (primary system). The generalized coordinates are the displacements of both masses, x_1 and x_2, respectively. In addition, u represents the (force) control input and f an exogenous harmonic perturbation. Here m_1 and c_1 denote mass and linear viscous damping on the primary system; similarly, m_2 and c_2 denote mass and viscous damping of the active vibration absorber.

The two mechanical springs have the following nonlinear stiffness function

$$\mathcal{F}(x) = kx + k_p x^3$$

where x is the spring deformation, and k and k_p denote the linear and cubic stiffness, respectively.

The mathematical model of the two degree-of-freedom system is described by two coupled nonlinear differential equations

$$m_1 \ddot{x}_1 + k_1 x_1 + k_{1p} x_1^3 + c_1 \dot{x}_1 - k_2 (x_2 - x_1) - k_{2p} (x_2 - x_1)^3 = f(t)$$
$$m_2 \ddot{x}_2 + k_2 (x_2 - x_1) + k_{2p} (x_2 - x_1)^3 = u(t)$$
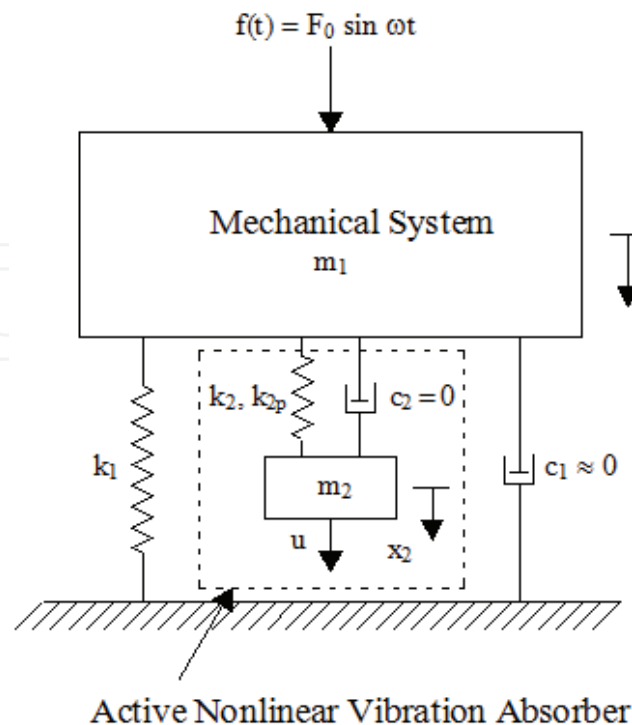
(8)

Fig. 3. Schematic diagram of the vibrating mechanical system.

where $f(t) = F_0 \sin \omega t$. In order to simplify the analysis we have assumed that $c_1 \approx 0$ and $c_2 \equiv 0$. Defining the state variables as $z_1 = x_1$, $z_2 = \dot{x}_1$, $z_3 = x_2$ and $z_4 = \dot{x}_2$, one obtains the following state space description

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = -\frac{k_1}{m_1}z_1 - \frac{k_{1p}}{m_1}z_1^3 + \frac{k_2}{m_1}(z_3 - z_1) + \frac{k_{2p}}{m_1}(z_3 - z_1)^3 + \frac{1}{m_1}f(t)$$
$$\dot{z}_3 = z_4 \tag{9}$$
$$\dot{z}_4 = -\frac{k_2}{m_2}(z_3 - z_1) - \frac{k_{2p}}{m_2}(z_3 - z_1)^3 + \frac{1}{m_2}u$$

In what follows we will apply the algebraic identification method to estimate the harmonic force $f(t)$ and design an active vibration controller based on state feedback and feedforward information obtained from $f(t)$.

3.1 Differential flatness-based control

The system (9) is differentially flat, with flat output given by $y = z_1$ and further denoted as L. Then, all the state variables and the control input can be parameterized in terms of the flat output $L = z_1$ and a finite number of its time derivatives (Fliess et al., 1993).

Indeed, under the assumption of perfect knowledge of L, the second equation in (9) actually represents a reduced cubic algebraic equation from where the mass position of the vibration absorber z_3 can be obtained. The only real root of such a cubic equation is readily obtained as

$$z_3 = L + \frac{1}{6k_{2p}}\left[k_{2p}^2\left(108d + 12\sqrt{3\frac{4k_2^3 + 27k_{2p}d^2}{k_{2p}}} \right) \right]^{1/3}$$

$$-2k_2\left[k_{2p}^2\left(108d + 12\sqrt{3\frac{4k_2^3 + 27k_{2p}d^2}{k_{2p}}} \right) \right]^{-1/3} \tag{10}$$

with $d = m_1\ddot{L} + k_1 L + k_{1p}L^3 - f(t)$. Note that the differentially parameterized expression for z_3, in (10), implies that its second time derivative, $\dot{z}_4$, can be expressed as a function denoted by $\phi(L, \dot{L}, \ddot{L}, L^{(3)}, L^{(4)}, f, \dot{f}, \ddot{f})$. Then, from the fourth equation in (9), the control input, u, can be parameterized in terms of differential functions of L as

$$u = m_2\dot{z}_4 + k_2\left(z_3\left(L,\ddot{L},f\right) - L \right) + k_{2p}\left(\phi\left(L,\dot{L},\ddot{L},L^{(3)},L^{(4)},f,\dot{f},\ddot{f}\right) - L \right)^3 \tag{11}$$

Therefore, all system variables are expressible as differential functions of the flat output.
From (11) one obtains the following differential flatness-based controller to asymptotically track a desired reference trajectory $L^*(t)$:

$$u = m_2\dot{z}_4 + k_2\left(z_3\left(L,\ddot{L},f\right) - L \right) + k_{2p}\left(\phi\left(L,\dot{L},\ddot{L},L^{(3)},v,f,\dot{f},\ddot{f}\right) - L \right)^3$$
$$v = \left(L^*\right)^{(4)}(t) - \beta_3\left[L^{(3)} - \left(L^*\right)^{(3)}(t) \right]$$
$$-\beta_2\left[\ddot{L} - \ddot{L}^*(t) \right] - \beta_1\left[\dot{L} - \dot{L}^*(t) \right] - \beta_0\left[L - L^*(t) \right] \tag{12}$$

The use of this controller yields the following closed-loop dynamics for the trajectory tracking error $e = L - L^*(t)$ as follows

$$e^{(4)} + \beta_3 e^{(3)} + \beta_2\ddot{e} + \beta_1\dot{e} + \beta_0 e = 0 \tag{13}$$

Therefore, selecting the design parameters β_i, $i = 0, ...3$, such that the associated characteristic polynomial for (13) be *Hurwitz*, one guarantees that the error dynamics be globally asymptotically stable.

It is evident, however, that the controller (12) requires the perfect knowledge of the exogenous signal $f(t)$ and its time derivatives up to second order, revealing several disadvantages with respect to other control schemes. Nevertheless, one can take advantage of the algebraic identification methods: *i)* to estimate the force $f(t)$ and reconstruct an estimated signal $\hat{f}(t)$, or *ii)* when the structure of the signal is well-known (e.g., harmonic force $f(t) = F_0\sin\omega t$) to estimate its associated parameters (F_0,ω) and then reconstruct it. As a consequence, the combination of the feedback and feedforward control (12) with algebraic identification methods will improve the robustness properties against variations on the amplitude and/or excitation frequency.

4. Algebraic identification of harmonic vibrations

Consider the nonlinear mechanical system (9) with perfect knowledge of its system parameters and, that the whole set of state vector components and the control input u are

available for the identification process of the harmonic signal $f(t) = F_0 \sin \omega t$. In this case we proceed to synthesize algebraic identifiers for the excitation frequency ω and amplitude F_0. For simplicity, we also suppose that $c_1 = c_2 \equiv 0$.

4.1 Identification of the excitation frequency ω

Consider the second equation in (9)

$$m_1 \frac{d}{dt} z_2 + \varphi = F_0 \sin \omega t \tag{14}$$

where

$$\varphi = k_1 z_1 + k_{1p} z_1^3 - k_2 (z_3 - z_1) - k_{2p}^3 (z_3 - z_1)^3$$

In order to eliminate the presence of the amplitude F_0, we differentiate the equation (14) twice with respect to time t, resulting

$$\frac{d^2}{dt^2} \left(m_1 \frac{d}{dt} z_2 + \varphi \right) = -F_0 \omega^2 \sin \omega t \tag{15}$$

Multiplication of (14) by ω_2 and adding it to (15), leads to

$$\omega^2 \left(m_1 \frac{d}{dt} z_2 + \varphi \right) + \frac{d^2}{dt^2} \left(m_1 \frac{d}{dt} z_2 + \varphi \right) = 0 \tag{16}$$

Multiplying (16) by the quantity t^3 and integrating the result three times with respect to time t, one gets

$$\omega^2 \left(\int_{t_0}^{(3)} t^3 \left(m_1 \frac{d}{dt} z_2 + \varphi \right) \right) + \left(\int_{t_0}^{(3)} t^3 \frac{d^2}{dt^2} \left(m_1 \frac{d}{dt} z_2 + \varphi \right) \right) = 0 \tag{17}$$

where $\left(\int_{t_0}^{(n)} \eta(t) \right)$ are iterated integrals of the form $\int_{t_0}^{t} \int_{t_0}^{\sigma_1} \cdots \int_{t_0}^{\sigma_{n-1}} \varphi(\sigma_n) d\sigma_n \cdots d\sigma_1$, with $\left(\int_{t_0} \eta(t) \right) = \int_{t_0}^{t} \eta(\sigma) d\sigma$ and n a positive integer.

Using integration by parts, one gets

$$\omega^2 \left(m_1 \left(\int_{t_0}^{(3)} t^3 \frac{d}{dt} z_2 \right) + \left(\int_{t_0}^{(3)} t^3 \varphi \right) \right) + m_1 \left(\int_{t_0}^{(3)} t^3 \frac{d^3}{dt^3} z_2 \right)$$
$$+ \left(\int_{t_0}^{(2)} t^3 \frac{d}{dt} \varphi \right) - 3 \left(\int_{t_0}^{(3)} t^2 \frac{d}{dt} \varphi \right) = 0 \tag{18}$$

where

$$\left(\int_{t_0}^{(3)} t^3 \frac{d}{dt} z_2 \right) = \left(\int_{t_0}^{(2)} t^3 z_2 \right) - 3 \left(\int_{t_0}^{(3)} t^2 z_2 \right)$$

$$\left(\int_{t_0}^{(3)} t^3 \frac{d^3}{dt^3} z_2 \right) = t^3 z_2 - 9 \left(\int_{t_0} t^2 z_2 \right) + 18 \left(\int_{t_0}^{(2)} t z_2 \right) - 6 \left(\int_{t_0}^{(3)} z_2 \right)$$

$$\frac{d}{dt} \varphi = k_1 z_2 + 3 k_{1p} z_1^2 z_2 - \left[k_2 + 3 k_{2p} (z_3 - z_1)^2 \right] (z_4 - z_2)$$

Finally, solving for the excitation frequency ω in (18) leads to the following on-line algebraic identifier for the excitation frequency:

$$\omega_e^2 = -\frac{N_1(t)}{D_1(t)}, \ \forall t \in (t_0, t_0 + \delta_0] \tag{19}$$

where

$$N_1(t) = m_1\left(\int_{t_0}^{(3)} t^3 \frac{d^3}{dt^3} z_2\right) + \left(\int_{t_0}^{(2)} t^3 \frac{d}{dt}\varphi\right) - 3\left(\int_{t_0}^{(3)} t^2 \frac{d}{dt}\varphi\right)$$

$$D_1(t) = m_1\left(\int_{t_0}^{(3)} t^3 \frac{d}{dt} z_2\right) + \left(\int_{t_0}^{(3)} t^3 \varphi\right)$$

Therefore, when the condition $D_1(t) \neq 0$ be satisfied at least for a small time interval $(t_0, t_0 + \delta_0]$ with $\delta_0 > 0$, we can find from (19) a closed-form expression for the estimated excitation frequency.

4.2 Identification of the amplitude F_0

To synthesize an algebraic identifier for the amplitude F_0 of the harmonic vibrations acting on the mechanical system (9), consider again the equation (14).
Multiplying (14) by the quantity t and integrating the result once with respect to time t, we have that

$$m_1 \int_{t_0}^t \left(t \frac{d}{dt} z_2\right) dt + \int_{t_0}^t (t\varphi) dt = F_0 \int_{t_0}^t (t \sin \omega t) dt \tag{20}$$

By integrating by parts, the equation (20) is equivalent to

$$m_1\left((t - t_0) z_2 - \int_{t_0}^t z_2 dt\right) + \int_{t_0}^t (t\varphi) dt = F_0 \int_{t_0}^t (t \sin \omega t) dt \tag{21}$$

At this point we assume that the excitation frequency has been previously estimated, during a small time interval $(t_0, t_0 + \delta_0]$, using (19). The estimated result is therefore $\omega_e(t_0 + \delta_0)$. After the time $t = t_0 + \delta_0$ it is started the on-line identifier for the amplitude, obtained from (21) as follows

$$F_{0e} = \frac{N_2(t)}{D_2(t)} \tag{22}$$

where

$$N_2(t) = m_1 z_2 \Delta t + \int_{t_0 + \delta_0}^t (t\varphi - m_1 z_2) dt$$

$$D_2(t) = \left(\int_{t_0 + \delta_0}^t t \sin[\omega_e(t_0 + \delta_0)t] dt\right)$$

Such an estimation is valid if the condition $D_2(t) \neq 0$ holds for a sufficiently small time interval $[t_0 + \delta_0, t_0 + \delta_1]$ with $\delta_1 > \delta_0 > 0$.

5. An adaptive-like controller with algebraic identification

The differential flatness based active vibration control (12) can be combined with the online identification of harmonic vibrations (19),(22), resulting the following *certainty equivalence* feedback control law

$$u = m_2 \dot{z}_4 + k_2 \left(z_3 \left(L, \ddot{L}, f_e \right) - L \right) + k_{2p} \left(\phi \left(L, \dot{L}, \ddot{L}, L^{(3)}, v, f_e, \dot{f}_e, \ddot{f}_e \right) - L \right)^3$$
$$v = \left(L^* \right)^{(4)} (t) - \beta_3 \left[L^{(3)} - \left(L^* \right)^{(3)} (t) \right]$$
$$- \beta_2 \left[\ddot{L} - \ddot{L}^* (t) \right] - \beta_1 \left[\dot{L} - \dot{L}^* (t) \right] - \beta_0 \left[L - L^* (t) \right]$$

(23)

where $f_e(t) = F_{0e} \sin \omega_e t$. Note that, in accordance with the algebraic identification approach, providing fast identification for the parameters associated to the harmonic vibration (ω, F_0) and, as a consequence, fast estimation of this perturbation signal, the proposed controller (23) resembles an adaptive control scheme. From a theoretical point of view, the algebraic identification is instantaneous (Fliess & Sira-Ramírez, 2003). In practice, however, there are modeling and computational errors as well as other factors that can inhibit the precise algebraic computation. Fortunately, the identification algorithms and closed-loop system are robust against such difficulties (Beltrán et al., 2010).

6. Simultion results

Some simulations were performed to show the on-line identification of harmonic vibrations and its use in an adaptive-like vibration control (23). The parameters for the ECP^{TM} rectilinear control system are given in Table 1.

$m_1 = 10kg$	$m_2 = 2kg$
$k_1 = 1000N/m$	$k_2 = 200N/m$
$k_{1p} = 100N/m$	$k_{2p} = 50N/m^3$

Table 1. System parameters.

The controller (23) was specified such that one could observe how the active vibration absorber cancels the vibrations on the primary system and the asymptotic output tracking of an off-line and prespecified reference trajectory, towards the desired equilibrium.
The planned trajectory for the flat output $y = z_1$ is given by

$$L^* (t) = \begin{cases} 0 & \text{for } 0 \le t < T_1 \\ \psi(t, T_1, T_2) \overline{L} & \text{for } T_1 \le t \le T_2 \\ \overline{L} & \text{for } t > T_2 \end{cases}$$

(24)

where $\overline{L}$ = 0.01 [m], T_1 = 5 [s], T_2 = 10 [s] and $\psi(t, T_1, T_2)$ is a Bézier polynomial, with $\psi(T_1, T_1, T_2) = 0$ and $\psi(T_2, T_1, T_2) = 1$, described by

$$\psi(t) = \left(\frac{t - T_1}{T_2 - T_1}\right)^5 \left[r_1 - r_2\left(\frac{t - T_1}{T_2 - T_1}\right) + r_3\left(\frac{t - T_1}{T_2 - T_1}\right)^2 - \ldots - r_6\left(\frac{t - T_1}{T_2 - T_1}\right)^5\right]$$

with $r_1 = 252$, $r_2 = 1050$, $r_3 = 1800$, $r_4 = 1575$, $r_5 = 700$, $r_6 = 126$.

In Fig. 4 is depicted the identification process of the harmonic vibrations $f(t) = 2\sin(12t)$ [N] and the dynamic behavior of the adaptive-like control scheme (23). We can observe a good and fast estimation ($t \ll 0.1s$) and how the active vibration absorber dissipates all the vibrating energy H_1 and allows that the output follows the desired reference trajectory given by (24).

The controller parameters $\{\beta_0, \beta_1, \beta_2\}$ were chosen to be in correspondence with the fourth order closed-loop tracking error dynamics characteristic polynomial:

$$\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)^2 = s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s + \beta_0$$
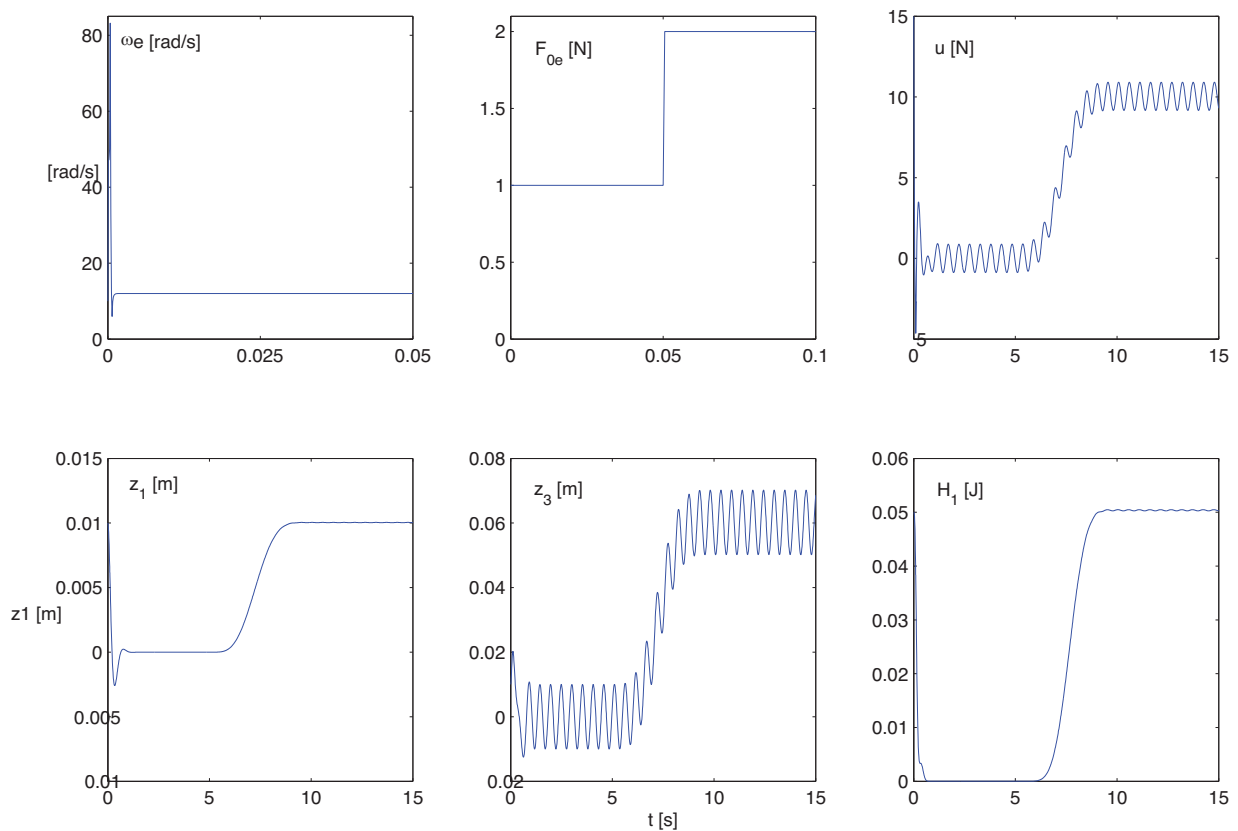
with $\zeta = 0.7071$ y $\omega_n = 10$.

Fig. 4. Controlled system responses and identification of frequency and amplitude of $f(t) = F_0 \sin\omega t$.

7. Conclusions

The design of active dynamic vibration absorbers is performed by using feedback and feedforward control. The differential flatness property of the mechanical system is employed to synthesize an active vibration controller, simplifying the trajectory tracking problem with the application of a static state feedback controller based on linear pole placement and perturbation feedforward. Since this active controller requires information of the exogenous harmonic vibrations, an algebraic identification approach is proposed for the on-line estimation of the frequency and amplitude of vibrations affecting the mechanical system. This approach is quite promising, in the sense that from a theoretical point of view, the algebraic identification is practically instantaneous and robust with respect to parameter uncertainty, frequency variations, small measurement errors and noise. Thus the algebraic identification is combined with the differential flatness based controller to get an adaptive-like controller, which results quite precise, fast and robust against parameter uncertainty and variations on the excitation frequency and amplitude of exogenous perturbations.

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Vibration Control

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Vibrations are a part of our environment and daily life. Many of them are useful and are needed for many purposes, one of the best example being the hearing system. Nevertheless, vibrations are often undesirable and have to be suppressed or reduced, as they may be harmful to structures by generating damages or compromise the comfort of users through noise generation of mechanical wave transmission to the body. the purpose of this book is to present basic and advanced methods for efficiently controlling the vibrations and limiting their effects. Open-access publishing is an extraordinary opportunity for a wide dissemination of high quality research. This book is not an exception to this, and I am proud to introduce the works performed by experts from all over the world.

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