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# Workforce capacity planning using zero-one-integer programming

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## 1. Introduction

Planning for human resource needs is one of the greatest challenges facing managers and leaders. In order to meet this challenge, a uniform process that provides a disciplined approach for matching human resources with the anticipated needs of an organization is essential. Workforce planning is a fundamental planning tool, critical to quality performance that will contribute to the achievement of program objectives by providing a basis for justifying budget allocation and workload staffing levels. As an organization develops strategies to support the achievement of performance goals in the strategic plans, workforce planning should be included as a key management activity.

An important problem in workforce planning arises when management needs to employ the right number of people to perform daily tasks such as in a reception department. Each employee is weekly required to work a certain number of days (not necessarily consecutive) and the number of required employees varies from day to day according to demand. In particular, if every employee is required to work five days a week and takes any two days off, then scheduling employees to meet daily requirements becomes a formidable task for management. Many practical applications of workforce planning can be cited where such problems arise such as: planning of the number of employees in receptions in hospitals, service stations, call centers, and hotels; planning the number of doctors and nurses in hospitals; planning the number of workers in restaurants, etc. Assigning tasks to employees is a difficult task. Errors committed in such assignments can have far-reaching consequences, such as reduced efficiency due to absenteeism, lack of job satisfaction, formal grievances, and generally deteriorating labor relations.

The first part of the paper is this introduction, the second part is the literature review, the third part describes the statement of the problem considering an organization where each employee works six days per week and takes one day off, or he works five days per week, and takes two days off (consecutive, or not necessarily consecutive). The number of employees needed on each day of the week differs according to the different workload on each day. It is needed to minimize the total number of workers (workforce capacity), while maintaining the performance of keeping the minimum required number of employees on each day of the week. The fourth part introduces the proposed mathematical model formulation for each case. The model comprises the objective function and the problem constraints. The fifth part presents 15 real application examples, the examples are 12 hotels and 3 hospitals in Jeddah city, the data are taken for a period of 3 months. The six part presents the obtained results for the case studies, while the remaining parts are the conclusions and points for future research.

## 2. Literature Review

Workforce Planning (WFP) ensures that "the right people with the right skills are in the right place at the right time." This definition covers a methodical process that provides managers with a framework for making human resource decisions based on the organization's mission, strategic plan, budgetary resources, and a set of desired workforce competencies.

Various papers approach the problem from a spatial point of view. (Eiselt & Marianov, 2008) mapped the employees and the relevant tasks in a skill space, task assignments are determined, tasks are assigned to employees so as to minimize employee—task distances in order to avoid boredom, and minimize inequity between the individual employees' workloads, and minimize costs.

Constructing schedules is not also an easy task to accomplish in settings where work must be performed 24 hours per day and 7 days a week, such as in police and fire departments, or in emergency rooms of hospitals. (Knanth, 1996) studied the problem that one is faced when aiming to generate "good schedules" that satisfy many complicated rules, including ergonomic rules. Manpower scheduling in emergency rooms in hospitals is a famous and very critical problem in workforce capacity planning. (Carter & Lapierre, 2001) concluded that ergonomic constraints are very important in order to manage the circadian rhythm of the staff and it is critical to take them into account when building schedules. (Gendreau et al., 2007) discussed also this problem in five different hospitals of the Montreal, Canada area, the authors first propose generic forms for the constraints encountered in this context, then review several possible solution techniques that can be applied to physician scheduling problems, namely tabu search, column generation, mathematical programming and constraint programming, and examine their suitability for application depending on the specifics of the situation at hand.

(El-Quliti & Al-Darrab, 2009) address the problem of finding the optimal number of employees to be assigned each day of the week and determining the weekly schedule of each employee given that on each day at least a certain number of employees must be used to meet job or project requirements. The approach presented is to solve the problem in two stages. The first stage solves the problem with two consecutive off-days using a linear integer programming model. The second stage uses a zero-one integer programming model utilizing results of the first stage. Both mathematical formulation and solution to the problem are developed, and the LINDO computer package was used to solve an illustrative example. The optimum daily workforce size and schedule of every employee are thus obtained.

(El-Quliti & Al-Darrab, 2010) present some real world applications for the problem of finding the optimal number of employees to be assigned each day of the week, and determining the weekly schedule of each employee given that on each day at least a certain number of employees must be used to meet job requirements. 15 real case studies are presented in this research, 12 cases for receptions in hotels and 3 for emergency in hospitals in Jeddah city. LINGO computer package has been used to solve these case studies. The optimum daily workforce size and schedule of every employee have been obtained.

## 3. Statement of the Problem

Consider an organization where each employee works five (or six) days per week, and takes one (or two) day(s) off. Suppose that the number of employees needed on each day of the week differs according to the different workload on each day, and suppose that the required numbers are as follows:

| **Day:** | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|---|---|---|---|---|---|---|---|
| **Number required** | $n_1$ | $n_2$ | $n_3$ | $n_4$ | $n_5$ | $n_6$ | $n_7$ |
| **Number assigned** | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ |

It is needed to minimize the total number of workers (workforce capacity), while maintaining the performance of keeping the minimum required number of employees on each day of the week, the questions will be:

1. How many total employees should be assigned?
2. How many employees should work on each day?
3. What is the week schedule for each employee (working days)?

## 4. Mathematical Model Formulation

### 4.1. Six-working days per week

The mathematical model is formulated by considering the number of employees that start working on Monday as $x_1$, and start on Tuesday as $x_2$, …, and start on Sunday as $x_7$, Fig. 1.
The objective function is clearly:

$$\text{Min. } z_1 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

The Problem Constraints:
For the number of employees working on Monday:

$$x^1 = x1 + x_3 + x_4 + x_5 + x_6 + x_7 \geq n_1$$

Similar constraints can be constructed for the other six days.
The complete Integer Program will have the following form:

$$\text{Min } z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \tag{1}$$

Subject to:
1) The Workload Constraints:

$$\begin{aligned}
x^1 &= x_1 + x_3 + x_4 + x_5 + x_6 + x_7 \geq n_1 \\
x^2 &= x_1 + x_2 + x_4 + x_5 + x_6 + x_7 \geq n_2 \\
x^3 &= x_1 + x_2 + x_3 + x_5 + x_6 + x_7 \geq n_3 \\
x^4 &= x_1 + x_2 + x_3 + x_4 + x_5 + x_7 \geq n_4 \\
x^5 &= x_1 + x_2 + x_3 + x_4 + x_5 + x_7 \geq n_5 \\
x^6 &= x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq n_6 \\
x^7 &= x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq n_7
\end{aligned} \tag{2}$$

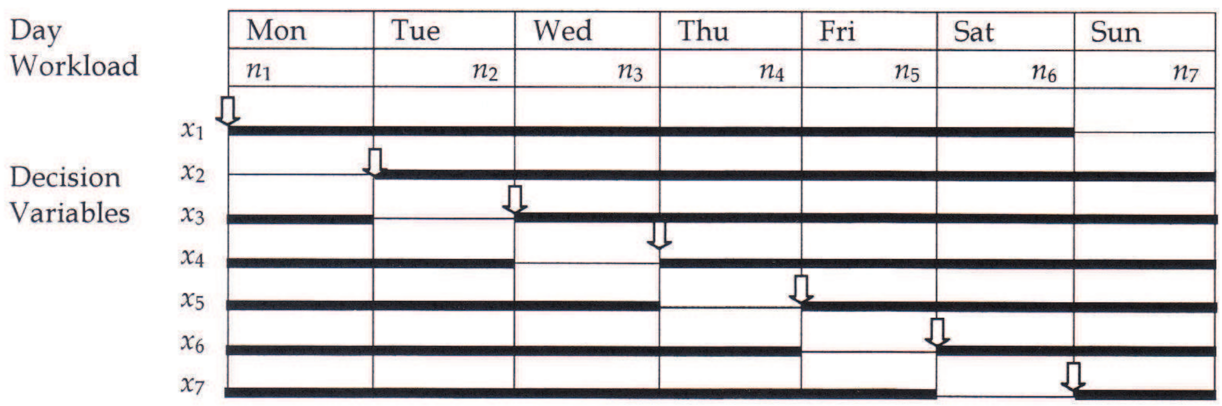



Fig. 1. Schematic diagram for 6-working days per week

2) The Minimum Workforce Capacity Constraints

Suppose that the minimum load including administrative and other related loads should not be less than a certain number $n_{min}$, in such a case, the workload constraints will have the following modified form:

$$x^i \geq \max\{n_i, n_{min}\},\ i = 1, 2, \ldots, 7 \tag{3}$$

3) Non-negativity and Integrality Constraints

$$x_1, x_2, x_3, x_4, x_5, x_6, \text{ and } x_7 \geq 0, \text{ integers} \tag{4}$$

The obtained optimum solution will state the optimum number of employees planned for each day of the week. The minimum required number of employees for the organization is then:

$$z^* = x_1^* + x_2^* + x_3^* + x_4^* + x_5^* + x_6^* + x_7^*$$



### 4.2. Five-consecutive working days per week

The mathematical model is formulated in a similar manner like in the case of 6-working days per week, see Fig. 2.

The objective function is clearly:

$$\text{Min. } z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

The Problem Constraints:

For the number of employees working on Monday:

$$x_1 + x_3 + x_4 + x_5 + x_6 + x_7 \geq n_1$$

Similar constraints can be constructed for the other six days.

The complete Integer Program will have the following form:

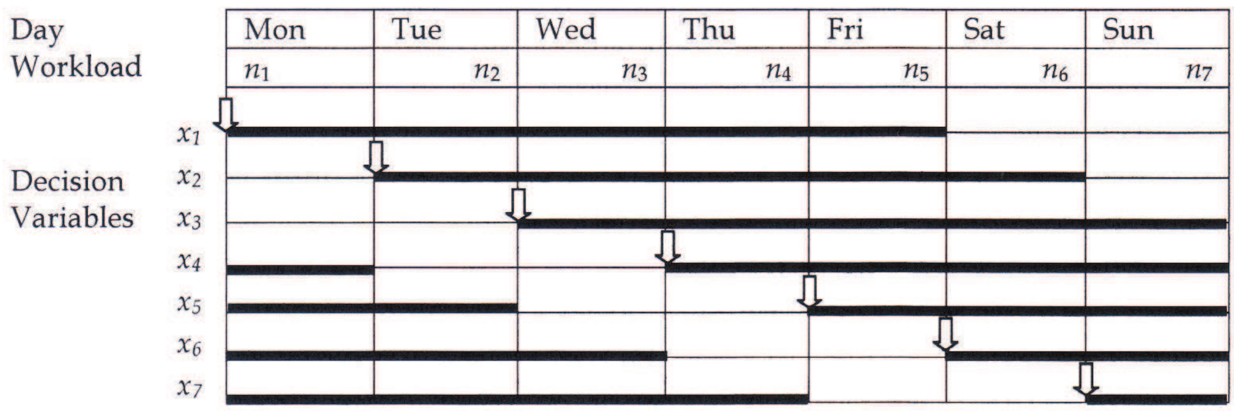



Fig. 2. Schematic diagram for 5-consecutive working days per week

$$\text{Min } z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \tag{5}$$

Subject to:

1) The Workload Constraints:

$$\begin{aligned} x^1 &= x_1 + x_4 + x_5 + x_6 + x_7 \geq n_1 \\ x^2 &= x_1 + x_2 + x_5 + x_6 + x_7 \geq n_2 \\ x^3 &= x_1 + x_2 + x_3 + x_6 + x_7 \geq n_3 \\ x^4 &= x_1 + x_2 + x_3 + x_4 + x_7 \geq n_4 \\ x^5 &= x_1 + x_2 + x_3 + x_4 + x_5 \geq n_5 \\ x^6 &= x_2 + x_3 + x_4 + x_5 + x_6 \geq n_6 \\ x^7 &= x_3 + x_4 + x_5 + x_6 + x_7 \geq n_7 \end{aligned} \tag{6}$$

2) The Minimum Workforce Capacity Constraints

Suppose that the minimum load including administrative and other related loads should not be less than a certain number $n_{min}$, in such a case, the workload constraints will have the following modified form:

$$x^i \geq \max\{n_i, n_{\min}\},\ i = 1, 2, \ldots, 7 \tag{7}$$

3) Non-negativity and Integrality Constraints

$$x_1, x_2, x_3, x_4, x_5, x_6, \text{ and } x_7 \geq 0, \text{ integers} \tag{8}$$

The obtained optimum solution will state the optimum number of employees planned for each day of the week. The minimum required number of employees for the organization is then:

$$z^* = x_1^* + x_2^* + x_3^* + x_4^* + x_5^* + x_6^* + x_7^*$$

### 4.3. Five-working days-not necessarily consecutive

The approach presented here is to solve the problem in two stages. The first stage solves the problem with two consecutive off-days using a linear integer programming model. As the obtained solution from the first stage constitutes a feasible solution to the original problem, the second stage uses a zero-one integer programming model utilizing results obtained from the first stage.

#### 4.3.1 Stage I of the Algorithm

Consider the situation where each employee works five consecutive days discussed before in section 4.2. The mathematical model is formulated by considering the number of employees that start working on Monday as $x_1$, and start on Tuesday as $x_2$, …, and start on Sunday as $x_7$, see Fig. 2.
The complete mathematical model was formulated in equations (5-8)
The obtained optimum solution will state the optimum number of employees planned for each day of the week. The minimum required number of employees for the organization is then:

$$z_1^* = x_1^* + x_2^* + x_3^* + x_4^* + x_5^* + x_6^* + x_7^*$$

To simplify notation in the following discussion, this minimum value will, henceforth, be denoted by $z_1$.

#### 4.3.2 Stage II of the Algorithm

In this stage we will consider the case where each employee works five days per week (not necessarily consecutive), and takes two days off (any two days in the week).
As the optimum solution obtained from stage I is a feasible solution to stage II, then the number of employees $z_1$ obtained from the first stage I can be considered as an upper bound for the total number of employees required for stage II. In this stage, we will begin with $z_1$ employees in the mathematical model, and then we will delete those employees who will be idle in the final solution.

Decision Variables:

Let $x_i^j$ denote binary decision variables, where:

$i$ = The day number such that:
1 = Monday, 2 = Tuesday, ..., and 7 = Sunday,

$j$ = The ID Number for an employee, $j$ = 1, 2, …, $z_1$, where $z_1$ is the minimum number of employees obtained from Stage I:

**Functions with N Possible Values**

(Hillier & Lieberman, 2005) considered the situation where a given function is required to take on any one of N given values. Denote this requirement by:

$$f(x_1, x_2, \dots, x_n) = d_1, d_2, \dots \text{ or } d_N.$$

The equivalent Binary Programming formulation of this requirement is the following:

$$f(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{N} d_i y_i ,$$

$$\sum_{i=1}^{N} y_i = 1, \text{ and}$$

$y_i$ is a binary variable, for $i = 1, 2, \ldots, N$.

This new set of constraints would replace the N possible values requirement in the statement of the overall problem. This set of constraints provides an equivalent formulation because exactly one $y_i$ must equal 1 and the others must equal 0, so exactly one $d_i$ is being chosen as the value of the function. In this case, there are N yes or no questions being asked, namely, should $d_i$ be the value chosen ($i = 1, 2, \ldots, N$)? Because the $y_i$'s respectively represent these yes-or-no decisions, the second constraint makes them mutually exclusive alternatives.

**Functions with Zero – Integer Values**

Consider the special case where N given functions are required to take on any one of only 2 given values, one of which is zero. Denote this requirement by:

$f_i(x_1, x_2, \ldots, x_n) = d_i$ or 0 for $i = 1, 2, \ldots, N$.

The equivalent Binary Programming formulation of this requirement is the following:

$f_i(x_1, x_2, \ldots, x_n) = d_i\, y_i$ for $i = 1, 2, \ldots, N$, and

$y_i$ is a binary variable for $i = 1, 2, \ldots, N$.

These constraints would replace the 2 possible values requirement in the statement of the problem. It provides an equivalent formulation because exactly the respective auxiliary binary variable $y_i$ must equal 0 or 1. In this case, there are 2N yes or no questions being asked, namely, for each one of the N functions: should $d_i$ be the value chosen? And should 0 be the value chosen? Because the variables $y_i$ represents these yes-or-no decisions, the binary constraints make them mutually exclusive alternatives so that each function $f_i(x_1, x_2, \ldots, x_n)$ will be equal to either $d_i$ or 0.

**Working Days for the Employees**

Some of the considered $z_1$ employees may not be needed in the final optimal solution, while the others will be needed to satisfy the required working load. The needed employees will work 5 days per week, and the extra ones (if any) will not work at all. To model this situation, we will introduce $z_1$ auxiliary binary variables $y_j$, each of which corresponds to an employee $j$, and we will consider these constraints:

$$\sum_{i=1}^{7} x_i^j = 5 y_j \quad , j = 1, 2, \ldots\ldots, z_1 \text{ ; and } y_j \text{ is a binary variable} \quad \text{for } j = 1, 2, \ldots, z_1.$$

For any $y_j$ = 1, $j$ = 1, 2, …, $z_1$; the corresponding employee $j$ will work 5 days, and for any $y_j$=0, $j$ = 1, 2, …, $z_1$; the corresponding employee $j$ will not work and he/she is not needed. So, the total number of needed employees, $z_2$, will be equal to:

$$z_2 = \sum_{j=1}^{z_1} y_j$$

**The objective Function**

As it is required to minimize the total number of needed employees, then the objective function will take the form:

$$Min \ \ z_2 = \sum_{j=1}^{z_1} y_j \tag{9}$$

**Problem Constraints**

**1) The Workload Constraints**

For the number of employees working on Monday:

$$\sum_{j=1}^{z_1} x_1^j \geq n_1$$

Similar constraints can be formulated for other days, so we will have:

$$\sum_{j=1}^{z_1} x_i^j \geq n_i \ , i = 1, 2, \ldots, 7 \tag{10}$$

**2) The Minimum Workforce Capacity Constraints**

Suppose that the minimum load including administrative and other related loads should not be less than a certain number $n_{\min}$, in such a case, the workload constraints will have the following modified form:

$$\sum_{j=1}^{z_1} x_i^j \geq \max\{n_i, n_{\min}\}, \ i = 1, 2, \ldots, 7 \tag{11}$$

**3) Working Days Constraints**

Each employee will work 5 days or he will not work at all:

$$\sum_{i=1}^{7} x_i^j = 5y_j \ , j = 1, 2, \ldots\ldots, z_1 \tag{12}$$

**4) Binary Constraints**

All the decision variables and the auxiliary variables are binary ones, so we have:

$$x_i^j, i = 1, 2, \ldots, 7, \text{ and } j = 1, 2, \ldots, z_1, \text{ and}$$

$$y_j, \; j = 1, 2, \ldots, z_1 \text{ are binary variables} \tag{13}$$

**The Working Schedule for Employees**

The working schedule for each employee $j$ will be known from the optimum solution of the model. When $x_i^j = 1$, then employee $j$ will work for day $i$, ($i = 1, 2, \ldots,$ or 7), and for $x_i^j = 0$, employee $j$ will not work for day $i$, ($i = 1, 2, \ldots,$ or 7).

### 4.4. Five-working days-not necessarily consecutive (direct approach)

The mathematical model is formulated in a similar way as in the case of 5-consecutive working days per week, but for all possible combinations of the two days off, see Fig. 3.

For the number of employees working on Monday:

$$x^1 = x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} \geq n_1$$

Similar constraints can be constructed for the other six days.

The complete Integer Program will have the following form:

$$\text{Min } z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \tag{14}$$

Subject to:

1) The Workload Constraints:

$$\begin{aligned}
x^1 &= x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} \geq n_1 \\
x^2 &= x_2 + x_3 + x_4 + x_5 + x_6 + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} \geq n_2 \\
x^3 &= x_1 + x_3 + x_4 + x_5 + x_6 + x_8 + x_9 + x_{10} + x_{11} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} \geq n_3 \\
x^4 &= x_1 + x_2 + x_4 + x_5 + x_6 + x_7 + x_9 + x_{10} + x_{11} + x_{13} + x_{14} + x_{15} + x_{19} + x_{20} + x_{21} \geq n_4 \\
x^5 &= x_1 + x_2 + x_3 + x_5 + x_6 + x_7 + x_8 + x_{10} + x_{11} + x_{12} + x_{14} + x_{15} + x_{17} + x_{18} + x_{21} \geq n_5 \\
x^6 &= x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + x_8 + x_9 + x_{11} + x_{12} + x_{13} + x_{15} + x_{16} + x_{18} + x_{20} \geq n_6 \\
x^7 &= x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_9 + x_{10} + x_{12} + x_{13} + x_{14} + x_{16} + x_{17} + x_{19} \geq n_7
\end{aligned} \tag{15}$$

$$x^i \geq \max\{n_i, n_{\min}\}, \; i = 1, 2, \ldots, 7 \tag{16}$$

$$x_1, x_2, \ldots\ldots\ldots\ldots, \text{ and } x_{21} \geq 0, \text{ integers} \tag{17}$$

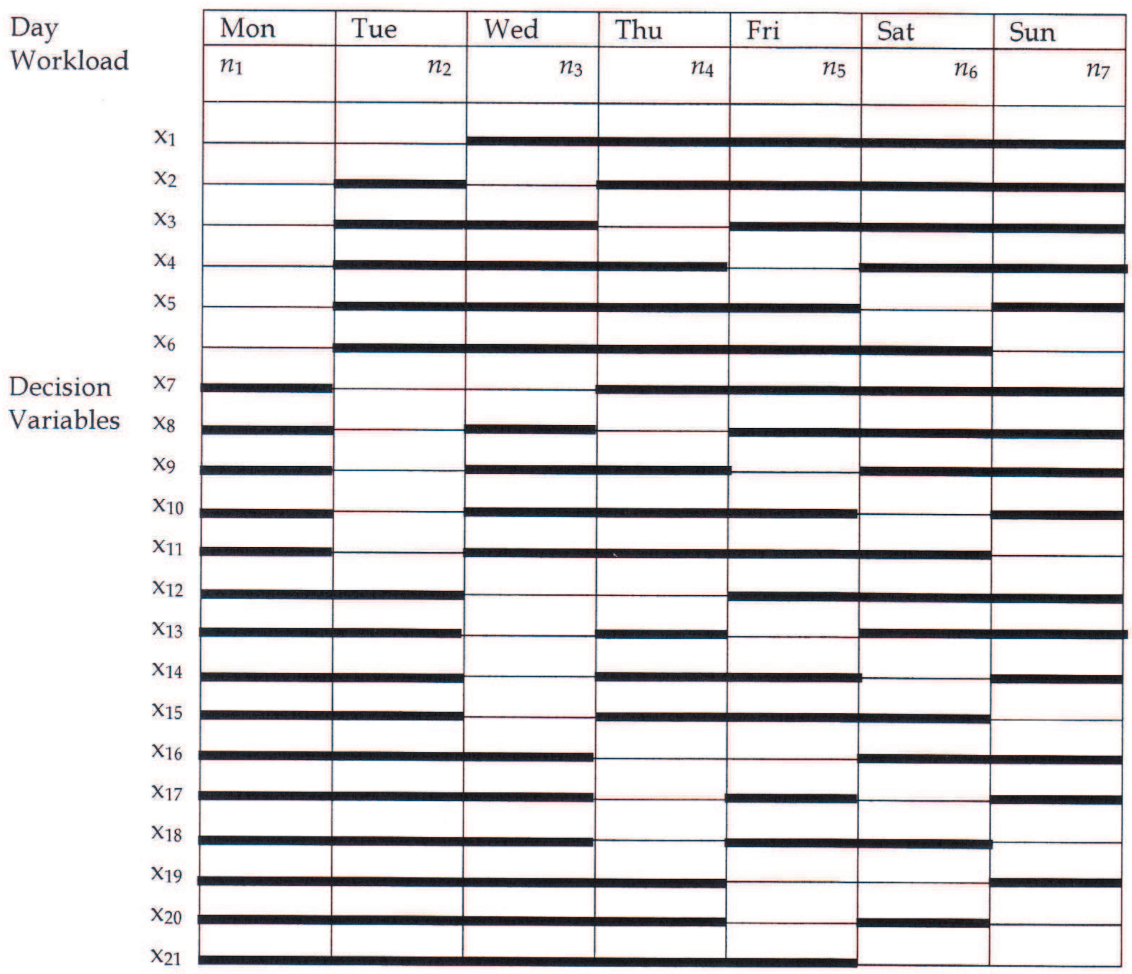


Fig. 3 Schematic diagram for 5-working days-not necessarily consecutive

## 5. Real Application Examples

| Saturday | Sunday | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|---|
| 194 | 159 | 176 | 156 | 193 | 196 | 213 |
| 178 | 140 | 189 | 174 | 194 | 173 | 231 |
| 183 | 166 | 178 | 142 | 199 | 184 | 183 |
| 196 | 162 | 183 | 168 | 200 | 194 | 215 |
| 197 | 144 | 203 | 179 | 211 | 202 | 254 |
| 212 | 182 | 243 | 187 | 220 | 228 | 242 |
| 217 | 169 | 209 | 194 | 222 | 223 | 237 |
| 224 | 189 | 207 | 198 | 229 | 204 | 245 |
| 215 | 173 | 225 | 192 | 223 | 178 | 182 |
| 161 | 138 | 158 | 153 | 162 | 181 | 184 |
| 175 | 144 | 173 | 142 | 166 | 172 | 211 |
| 171 | 146 | 174 | 152 | 180 | 178 | 196 |
| 162 | 147 | 162 | 145 | 173 | 180 | 183 |
| 183 | | | | | | |

Table 1. Number of guest in Jeddah Radisson Blu Hotel

15 real case studies are presented in this research, 12 cases for receptions in hotels and 3 for emergency in hospitals in Jeddah city, Kingdom of Saudi Arabia. Collecting data is the first and important step, the data here is the number of guests in the reception of a hotel, or the number of patients in the emergency section of a hospital. Data are collected for the hotels and hospitals during a period of 3 months. The number of guests or patients (working load) is different for different days of the week. For example, the number of guests for Jeddah Radisson Blu Hotel in the prescribed period is shown in Table 1.
The average number of guests in Radisson Blu Hotel Jeddah is shown in Figure 4.

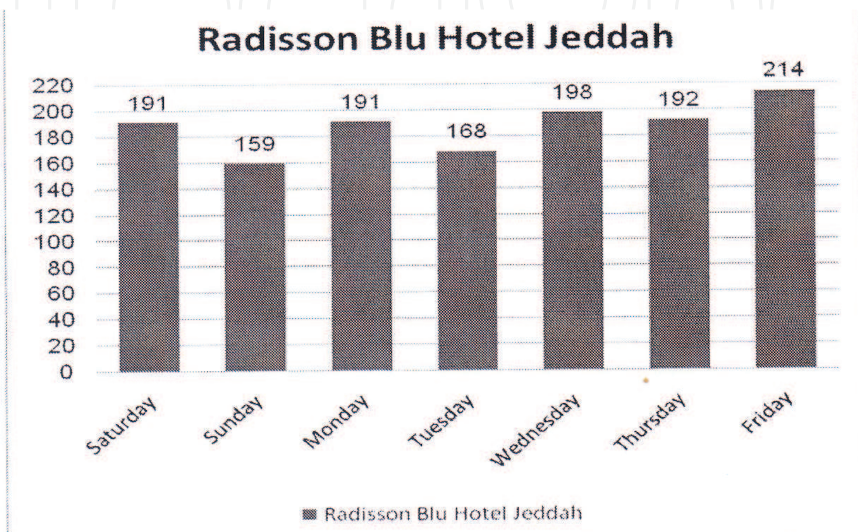



Fig. 4. Average number of guests in Radisson Blu Hotel, Jeddah

As an example of the data for hospitals, the number of patients in King Abdulaziz University Hospital in the prescribed period is shown in Table 2.
All the average numbers of guests in hotels or patients in hospitals are shown in Table 3 and in Figure 5.

| Saturday | Sunday | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|---|
| 108 | 80 | 86 | 75 | 67 | 38 | 34 |
| 99 | 61 | 59 | 73 | 76 | 46 | 33 |
| 84 | 68 | 63 | 75 | 69 | 48 | 34 |
| 57 | 79 | 103 | 70 | 64 | 37 | 37 |
| 89 | 51 | 92 | 74 | 69 | 62 | 41 |
| 99 | 75 | 83 | 72 | 71 | 58 | 38 |
| 88 | 79 | 96 | 84 | 62 | 53 | 59 |
| 90 | 64 | 84 | 86 | 74 | 30 | 42 |
| 67 | 54 | 40 | 57 | 35 | 12 | 19 |
| 63 | 56 | 26 | 36 | 51 | 12 | 2 |
| 37 | 43 | 44 | 58 | 33 | 8 | 6 |
| 55 | 42 | 49 | 39 | 40 | 17 | 4 |
| 42 | 35 | 50 | 43 | | 11 | 7 |
| 42 | | | | | | 5 |

Table 2. Number of patients in KAU Hospital

| Name | Sat | Sun | Mon | Tue | Wed | Thu | Fri |
|---|---|---|---|---|---|---|---|
| Radison Blu Hotel | 191 | 159 | 191 | 168 | 198 | 192 | 214 |
| Le Meridien Hotel | 197 | 256 | 251 | 287 | 233 | 229 | 250 |
| Sunset Hotel Hotel | 81 | 114 | 140 | 83 | 113 | 83 | 111 |
| Al-Hamra Hotel | 110 | 85 | 112 | 119 | 129 | 113 | 100 |
| Golden Tulip Hotel | 50 | 51 | 47 | 48 | 55 | 39 | 44 |
| Moevenpick Hotel | 180 | 161 | 197 | 146 | 194 | 183 | 202 |
| Trident Hotel Hotel | 135 | 119 | 124 | 107 | 112 | 123 | 115 |
| Hilton Hotel Hotel | 293 | 232 | 351 | 327 | 342 | 352 | 321 |
| Crowne Plaza Hotel | 225 | 213 | 232 | 235 | 248 | 243 | 236 |
| Ramada Hotel | 131 | 114 | 105 | 114 | 95 | 92 | 105 |
| Marriott Hotel | 199 | 189 | 166 | 170 | 196 | 181 | 190 |
| Al-Salam Hospital | 215 | 219 | 191 | 198 | 191 | 206 | 198 |
| King Abdulaziz Univ. Hospital | 73 | 59 | 62 | 60 | 51 | 31 | 26 |
| Al-Rafeea Hospital | 41 | 31 | 37 | 34 | 28 | 18 | 11 |
| Al-Salam Hospital | 33 | 29 | 31 | 32 | 32 | 16 | 19 |

Table 3. Average Number of Guests and Patients

In all the case studies, each reception employee in a hotel or a nurse in a hospital is working 6 days per week. Each hotel employee can serve about 40 guests in his shift, and each nurse in a hospital can serve 5 patients. The mathematical model is formulated for each case according to equations 1-4 stated before.

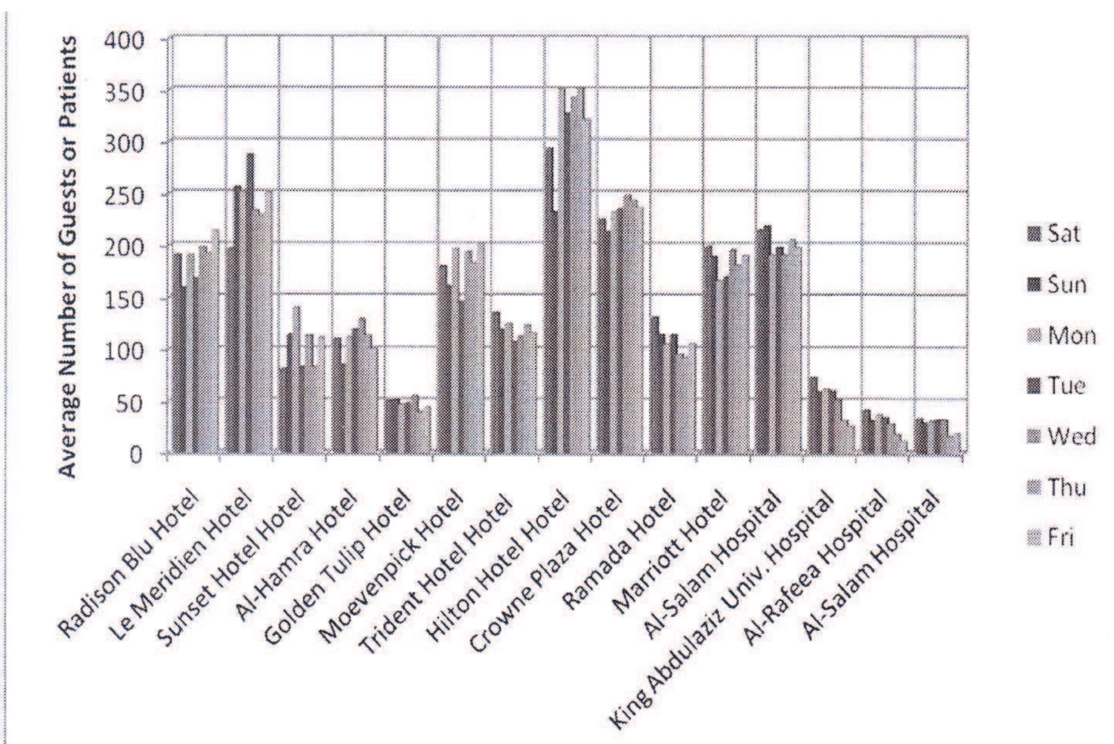



Fig. 5. Average Number of Guests and Patients

For example, for Radisson Blu Hotel Jeddah, Table 4 shows the required number of employees in each day of the week calculated according to the working load (average number of guests) and the number of guests that one employee can serve per one shift (40).

| Day | Sat | Sun | Mon | Tue | Wed | Thu | Fri |
|---|---|---|---|---|---|---|---|
| **Required Number of Employees** | 5 | 4 | 5 | 5 | 5 | 5 | 6 |

Table 4. Required Number of Employees for Radisson Blu Hotel Jeddah

The mathematical model for this case is as follows:

$$\text{Min } Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

Subject to:

$$x_1 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 5,$$
$$x_1 + x_2 + x_4 + x_5 + x_6 + x_7 \geq 4,$$
$$x_1 + x_2 + x_3 + x_5 + x_6 + x_7 \geq 5,$$
$$x_1 + x_2 + x_3 + x_4 + x_6 + x_7 \geq 5,$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_7 \geq 5,$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 5,$$
$$x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 6,$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \text{ integers.}$$

Similar mathematical models are formulated for the other case studies in the same manner according to the data for each case.

## 6. Results for the Application Examples

LINGO software is used to solve the obtained mathematical models for the case studies. An example of the obtained results are diagrammatically shown in Figure 6 for Radisson Blu Hotel Jeddah.

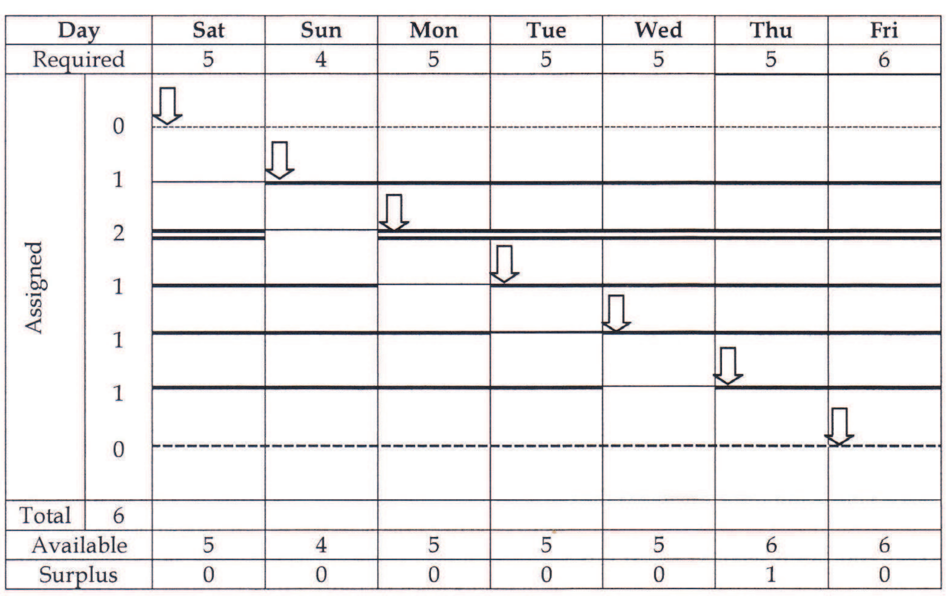



Fig. 6. Optimum solution for Radisson Blu Hotel Jeddah

The solution for all hotels and hospitals cases (needed number of employees) and the surplus (difference between assigned and the required number of employees) are shown in Table 5.

| No. | Place Name | Needed Employees | Surplus (Days) | Load/employee |
|---|---|---|---|---|
| 1 | Radison Blu Hotel | 6 | 0(6), 1(1) | 40 guests |
| 2 | Le Meridien Hotel | 8 | 0(6), 1(1) | |
| 3 | Sunset Hotel | 4 | 0(2), 1(5) | |
| 4 | Sofitel Jeddah Al Hamra | 4 | 0(6), 1(1) | |
| 5 | Golden Tulip Hotel | 3 | 0(3), 1(4) | |
| 6 | Moevenpick Hotel | 6 | 0(7) | |
| 7 | Trident Hotel | 4 | 0(7) | |
| 8 | Hilton Hotel | 10 | 0(6), 1(1) | |
| 9 | Crowne Plaza Hotel | 8 | 0(7) | |
| 10 | Ramada Continental Hotel | 4 | 0(7) | |
| 11 | Marriott Hotel | 6 | 0(6), 1(1) | |
| 12 | Holiday Inn Jeddah Al Salam | 7 | 0(7) | |
| 13 | KAU Hospital | 15 | 0(4), 2(1), 4(1), 8(1), | 5 patients |
| 14 | Al-Rafeea Hospital | 9 | 0(4), 2(1), 3(1), 5(1) | |
| 15 | Al-Salam Hospital | 7 | 0-7 | |

Table 5. Solution for all hotels and hospitals

## 7. Conclusions

1. Many real world application examples in hotels, hospitals, call centers, and many other organizations address the problem of workforce planning. The problem facing the management is to find the minimum number of employees to be assigned each day of the week given that a certain number must be assigned to meet job requirements on each day, and each employee should work a certain number of days per week (5 or 6).
2. The problem constraints are the workload in different days of the week**,** the minimum workforce capacity that should exist each day, the number of working days per week (consecutive or not), and the integrality constraints. The objective is to minimize the total number of needed employees, and
3. A mathematical formulation for the problem is illustrated for three cases: 6 working days per week, 5 consecutive working days per week, and 5 working days but not necessarily consecutive. All pattern possibilities for working different days are investigated for each case, and the number of employees assigned accordingly is considered as the decision variables. Another approach is proposed to solve such a problem in two stages. The first stage solves the problem with two consecutive off-days using a linear integer programming model. The second stage uses a zero-one integer programming model utilizing results of the first stage as it represents a feasible solution for the second stage.
4. 15 real case studies are presented, 12 for reception in hotels, and 3 for emergency in hospitals. The employees in hotels and the nurses in hospitals are working 6 days per

week and they have one day off. Data are collected for each case for a time period of 3 months, the data represents the working load in each day of the week.

**5.** The obtained mathematical model is an Integer Linear Programming and the solution to the problem are developed, and the LINDO computer package is used to solve the case studies. The obtained solutions represent the required number of employees and the scheduling for each group.

## 8. Points for Future Research

1. To categorize the employees into two categories: senior and junior, where some of the tasks can be done only by seniors.
2. To enhance the mathematical model formulation by including supervisors' constraints whom should be distributed allover days of the week.
3. To perform sensitivity analysis for the problem.
4. To consider the case involving stochastic constraints.
5. To perform a complete decision support system in order to help decision makers finding the optimum solutions for such practical applications.

## 9. References

Carter, M. & Lapierre, S. (2001). Scheduling emergency room physicians. *Health Care, Management Science, Vol.* 4, (2001) 347–360.

Eiselt, H., & Marianov, V. (2008). Employee positioning and workload allocation. *Computers & Operations Research*, Vol. 35, (Issue 2, February 2008) 513-524.

El-Quliti, S. & Al-Darrab, I. (2009). A Zero-One Integer Programming model for the optimum workforce capacity planning with workload constraints. *Proceedings of The* 20th. *IASTED International Conference on Modelling and Simulation*,"*MS 2009*", Banff, Alberta, Canada, July 2009, IASTED.

El-Quliti, S. & Al-Darrab, I. (2010). Optimum Workforce Capacity Planning With Real World Applications Using Integer Programming. *Proceedings of The First International Symposium on Computing in Science & Engineering,* Kuşadası- İzmi, June 2010.

Gendreau, M., Ferland, J. , Gendron,B. , Hail ,N., Jaumard, B. , Lapierre, S., Pesant, G. & Soriano, P. (2007). *Physician Scheduling in Emergency Rooms,* Lecture Notes in Computer Science, Springer Berlin / Heidelberg.

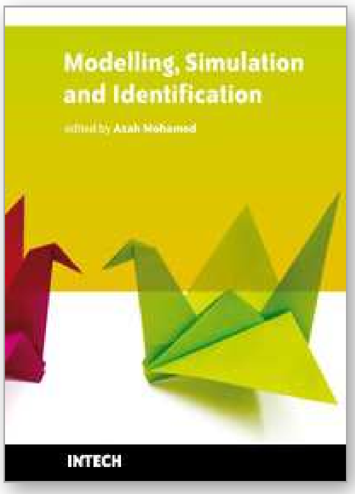
Hillier, F. & Lieberman, G. (2005).Introduction to Operations Research, 8th edition, McGraw-Hill International Edition, Industrial & Plant Engineering Series, ISBN 0-07-232169-5, Singapore.

Knaunth, P. (1996). Design better shift systems. *Applied Ergonomics*, Vol. 27 (1996) 39–44.

LINDO Systems Corporation (2010). LINGO 11.0, *Optimization Modeling Software for Linear, Nonlinear, and Integer Programming*,

http://www.lindo.com/index.php?option=com_content&view=article&id=2&Itemid=10, visited on 03 April (2010).

Taha, H. (2003). *Operations Research: An Introduction*, 8th. Edition, Pearson Education, Inc.,ISBN-10: 0-13-136014-0, New Jersey.




**Modelling, Simulation and Identification**

Edited by Azah Mohamed

ISBN 978-953-307-136-7
Hard cover, 354 pages
**Publisher** Sciyo
**Published online** 18, August, 2010
**Published in print edition** August, 2010

Modeling, simulation and identification has been actively researched in solving practical engineering problems. This book presents the wide applications of modeling, simulation and identification in the fields of electrical engineering, mechanical engineering, civil engineering, computer science and information technology. The book consists of 17 chapters arranged in an order reflecting multidimensionality of applications related to power system, wireless communication, image and video processing, control systems, robotics, soil mechanics, road engineering, mechanical structures and workforce capacity planning. New techniques in signal processing, adaptive control, non-linear system identification, multi-agent simulation, eigenvalue analysis, risk assessment, modeling of dynamic systems, finite difference time domain modeling and visual feedback are also presented. We hope that readers will find the book useful and inspiring by examining the recent developments in the applications of modeling, simulation and identification.