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Modelling of Transient Ground Surface Displacements Due to a Point Heat Source

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1. Introduction

Using Laplace-Hankel integral transformations, transient closed-form solutions of the thermally induced ground surface displacements, excess pore water pressure and temperature increment due to an instantaneous point heat source buried in an isothermal permeable half space are presented and discussed. The basic formulations of the governing equations are on the basis of Biot's three-dimensional consolidation theory of porous media. Numerical results show that the maximum ground surface horizontal displacement is around 38.5% of the maximum ground surface vertical displacement. The study concludes that the thermally induced horizontal displacement is significant. The solutions can be used to test numerical models and numerical simulations of the thermoelastic processes near the heat sources.

Heat source buried in the stratum leads to thermo-mechanical responses of fluid saturated porous medium. The heat source such as a canister of radioactive waste can cause temperature rise in the soil. The solid skeleton and pore fluid expand due to the heat source, and the volume increase of pore fluid is greater than that of the voids of solid matrix. This leads to an increase in pore fluid pressure and a reduction in effective stress. Therefore, thermal failure of soil will occur as a result of losing shear resistance due to the decrease in effective stress.

Attention is focused on the analytical solutions of the transient thermoelastic responses of an isotropic stratum due to an instantaneous point heat source. The responses of the stratum were satisfactorily modeled by assuming it as a thermoelastic porous continuum (Booker & Savvidou, 1985). It suggested that linear theory was adequate for a repository design based on technical conservatism. For example, Hueckel and Peano (1987) indicated that European guidelines require that temperature increments in the soil close to the heat source should not exceed 80°C while the temperature increments at the ground surface are limited to less than 1°C. Given these modest temperature increments, Hollister *et al.* (1981) observed that any significant non-linear behavior and/or plastic deformation of the soil would be confined to a relatively small volume of soil around the waste canister itself. In this case, a linear model can provide a reasonable approximation to the assessment of a proposed design (Smith & Booker, 1996). Hudson *et al.* (2005) given advices on how to incorporate thermo-hydro-mechanical coupled processes into performance and safety assessments and design studies for radioactive waste disposal in geological formations.

Governing equations of a fluid-saturated poroelastic solid in an isothermal quasi-static state were developed by Biot (1941, 1955). Lu and Lin (2006) displayed transient ground surface displacement produced by a point heat source/sink through analog quantities between poroelasticity and thermoelasticity. Booker and Savvidou (1984, 1985), Savvidou and Booker (1989) derived an extended Biot theory including the thermal effects and presented solutions of thermo-consolidation around the spherical and point heat sources. In their solutions, the isotropic or transversely isotropic flow properties are considered, whereas the isotropic elastic and thermal properties of the soils are introduced.

Based on Biot's three-dimensional consolidation theory of porous media, analytical solutions of the transient thermo-consolidation deformation due to a point heat source of constant strength buried in a saturated isotropic poroelastic half space were presented by Lu and Lin (2007). In this paper, instantaneous point heat source induced transient ground surface displacements are derived by using Laplace-Hankel integral transforms. The soil mass is modeled as a homogeneous isotropic saturated elastic half space of porous medium. Case of isothermal permeable half space boundary is investigated. Results are illustrated and compared to provide better understanding of the time dependent thermoelastic responses due to an instantaneous point heat source. The solutions can be used to test numerical models and the detailed numerical simulations of the thermoelastic processes near the buried heat sources.

2. Mathematical Model

2.1 Basic Equations

Figure 1 shows an instantaneous point heat source buried in a saturated porous stratum at depth h . The porous soil mass is considered as a homogeneous isotropic thermoelastic half space. The constitutive behaviours of the elastic soil skeleton are presented as:

$$\sigma_{ij} = 2G\varepsilon_{ij} + \frac{2G\nu}{1-2\nu}\varepsilon\delta_{ij} - \frac{2G(1+\nu)\alpha_s}{1-2\nu}\vartheta\delta_{ij} - p\delta_{ij} \; . \tag{1}$$

Here, σ_{ij} , ε_{ij} and ϑ are the total stress components, strain components and temperature increment measured from the reference state of the porous medium, respectively; ε is the volume strain of the porous medium; δ_{ij} is the Kronecker delta. The excess pore water pressure p is positive for compression. The constants ν , G and α_s are Poisson's ratio, shear modulus, and linear thermal expansion coefficient of the skeletal materials, respectively.

The strains ε_{ij} and displacement components u_i are given by the linear law:

$$\varepsilon_{ij} = \tfrac{1}{2}\left(u_{i,j} + u_{j,i}\right) \; . \tag{2}$$

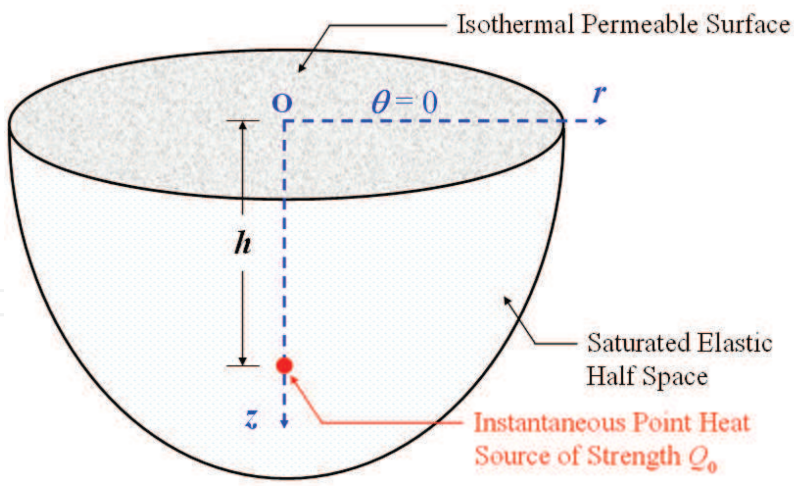

Fig. 1. Instantaneous point heat source buried in a homogeneous poroelastic half space

The total stress components satisfy the following equilibrium equations:

$$\sigma_{ij,j} + b_i = 0 , \tag{3}$$

where b_i denote the body forces. From Eqs. (1) and (2), the equilibrium equations (3) for axially symmetric problem without body forces b_i can be expressed in terms of displacements u_i , excess pore water pressure p and temperature change ϑ of the thermoelastic half space in cylindrical coordinates (r, θ, z) as below:

$$G\nabla^2 u_r + \frac{G}{1-2\nu}\frac{\partial \varepsilon}{\partial r} - G\frac{u_r}{r^2} - \frac{\partial p}{\partial r} - \frac{2G(1+\nu)\alpha_s}{1-2\nu}\frac{\partial \vartheta}{\partial r} = 0 , \tag{4a}$$

$$G\nabla^2 u_z + \frac{G}{1-2\nu}\frac{\partial \varepsilon}{\partial z} - \frac{\partial p}{\partial z} - \frac{2G(1+\nu)\alpha_s}{1-2\nu}\frac{\partial \vartheta}{\partial z} = 0 , \tag{4b}$$

where the volume strain of the porous medium ε can be denoted as $\varepsilon = \partial u_r/\partial r + u_r/r + \partial u_z/\partial z$, while the Laplacian operator $\nabla^2 = \partial^2/\partial r^2 + 1/r\,\partial/\partial r + \partial^2/\partial z^2$. According to Darcy's law, the governing equation of the conservation of mass can be expressed as

$$-\frac{k}{\gamma_w}\nabla^2 p + \frac{\partial \varepsilon}{\partial t} + n\beta\frac{\partial p}{\partial t} + 3\alpha_u\frac{\partial \vartheta}{\partial t} = 0 , \tag{5}$$

where k and n are the permeability and porosity of the porous medium, respectively; β and γ_w are the compressibility and unit weight of pore water, respectively; $\alpha_u = (1-n)\alpha_s + n\alpha_w$, in which α_w is the coefficient of linear thermal expansion of the pore water.

For an instantaneous point heat source of strength Q_0 buried at point $(0,h)$, the uncoupled governing equation in axially symmetry is obtained from the conservation of energy and heat conduction law as following:

$$-\lambda_t \nabla^2 \vartheta + m\frac{\partial \vartheta}{\partial t} - \frac{Q_0}{2\pi r}\delta(r)\delta(z-h)\,\delta(t) = 0\,, \tag{6}$$

in which λ_t is the heat conduction coefficient of the porous stratum; the symbol $m = (1-n)\rho_s c_s + n\rho_w c_w$, c_s and c_w are the specific heats of the skeletal materials and pore water, while ρ_s, ρ_w are their densities, respectively; $\delta(r)$ or $\delta(t)$ is Dirac delta function. Eqs. (4a), (4b), (5) and (6) constitute the basic governing equations of the time-dependent axially symmetric thermoelastic responses of a saturated porous medium.

2.2 Boundary Conditions and Initial Conditions

The half space surface, $z=0$, is treated as a traction-free, isothermal and permeable boundary for all time $t \geq 0$. Its mathematical statements of the boundary conditions are:

$$\sigma_{rz}(r,0,t) = 0,\ \sigma_{zz}(r,0,t) = 0,\ p(r,0,t) = 0, \text{ and } \vartheta(r,0,t) = 0\,. \tag{7a}$$

It's reasonable to assume that the instantaneous point heat source has no effect on the far boundary at $z \to \infty$ for all time. Hence

$$\lim_{z\to\infty}\{u_r(r,z,t), u_z(r,z,t), p(r,z,t), \vartheta(r,z,t)\} = \{0,0,0,0\}\,. \tag{7b}$$

Assuming no initial change in displacements, temperature increment and seepage for the poroelastic medium, then the initial conditions at time $t=0$ of the mathematical model due to an instantaneous point heat source can be treated as:

$$u_r(r,z,0) = 0,\ u_z(r,z,0) = 0,\ p(r,z,0) = 0, \text{ and } \vartheta(r,z,0) = 0\,. \tag{8}$$

The transient ground surface displacements can be derived from the differential equations (4a), (4b), (5) and (6) corresponding with the boundary conditions at $z=0$, $z \to \infty$, and initial conditions at time $t=0$.

3. Analytic Solutions

3.1 Laplace-Hankel Transformations

Applying initial conditions of Eq. (8), the governing partial differential equations (4a), (4b), (5) and (6) are reduced to ordinary differential equations by performing appropriate Laplace-Hankel transforms (Sneddon, 1951) with respect to the time variable t and the radial coordinate r:

$$\left(\frac{d^2}{dz^2}-2\eta\xi^2\right)\tilde{u}_r-(2\eta-1)\xi\frac{d\tilde{u}_z}{dz}+\frac{1}{G}\xi\tilde{p}+\frac{2(1+\nu)\alpha_s}{1-2\nu}\xi\tilde{\vartheta}=0, \tag{9a}$$

$$(2\eta-1)\xi\frac{d\tilde{u}_r}{dz}+\left(2\eta\frac{d^2}{dz^2}-\xi^2\right)\tilde{u}_z-\frac{1}{G}\frac{d\tilde{p}}{dz}-\frac{2(1+\nu)\alpha_s}{1-2\nu}\frac{d\tilde{\vartheta}}{dz}=0, \tag{9b}$$

$$-\frac{k}{\gamma_w}\left(\frac{d^2}{dz^2}-\xi^2\right)\tilde{p}+s\left(\xi\tilde{u}_r+\frac{d\tilde{u}_z}{dz}\right)+n\beta s\tilde{p}+3\alpha_u s\tilde{\vartheta}=0, \tag{9c}$$

$$-\lambda_t\left(\frac{d^2}{dz^2}-\xi^2\right)\tilde{\vartheta}+ms\tilde{\vartheta}-\frac{Q_0}{2\pi}\delta(z-h)=0, \tag{9d}$$

where ξ and s are Hankel and Laplace transform parameters, respectively; $\eta=(1-\nu)/(1-2\nu)$; and the symbols $\tilde{u}_r$, $\tilde{u}_z$, $\tilde{p}$, $\tilde{\vartheta}$ are defined as:

$$\tilde{u}_r(z;\xi,s)=\int_0^\infty\int_0^\infty ru_r(r,z,t)exp(-st)J_1(\xi r)dtdr, \tag{10a}$$

$$\tilde{u}_z(z;\xi,s)=\int_0^\infty\int_0^\infty ru_z(r,z,t)exp(-st)J_0(\xi r)dtdr, \tag{10b}$$

$$\tilde{p}(z;\xi,s)=\int_0^\infty\int_0^\infty rp(r,z,t)exp(-st)J_0(\xi r)dtdr, \tag{10c}$$

$$\tilde{\vartheta}(z;\xi,s)=\int_0^\infty\int_0^\infty r\vartheta(r,z,t)exp(-st)J_0(\xi r)dtdr, \tag{10d}$$

in which $J_\alpha(x)$ represents the first kind of Bessel function of order α.
The general solutions of equations (9a)-(9d) are obtained as below:

$$\begin{aligned}\tilde{u}_r(z;\xi,s)=&(A_1+A_2z)e^{\xi z}+(A_3+A_4z)e^{-\xi z}+A_5e^{\sqrt{\xi^2+\frac{s}{c_1}}z}+A_6e^{-\sqrt{\xi^2+\frac{s}{c_1}}z}+A_7e^{\sqrt{\xi^2+\frac{s}{c_2}}z}+A_8e^{-\sqrt{\xi^2+\frac{s}{c_2}}z}\\&+\frac{c_2Q_0}{8\pi\eta G\lambda_t}\left(-\frac{c_a}{s}e^{-\xi|z-h|}-\frac{c_b}{s}\xi\sqrt{\xi^2+\frac{s}{c_1}}^{-1}e^{-\sqrt{\xi^2+\frac{s}{c_1}}|z-h|}+\frac{c_c}{s}\xi\sqrt{\xi^2+\frac{s}{c_2}}^{-1}e^{-\sqrt{\xi^2+\frac{s}{c_2}}|z-h|}\right),\end{aligned} \tag{11a}$$

$$\begin{aligned}\tilde{u}_z(z;\xi,s)=&\left[-A_1+\frac{1+(2\eta+1)n\beta G}{1+(2\eta-1)n\beta G}\frac{1}{\xi}A_2-A_2z\right]e^{\xi z}+\left[A_3+\frac{1+(2\eta+1)n\beta G}{1+(2\eta-1)n\beta G}\frac{1}{\xi}A_4+A_4z\right]e^{-\xi z}\\&-\frac{1}{\xi}\sqrt{\xi^2+\frac{s}{c_1}}A_5e^{\sqrt{\xi^2+\frac{s}{c_1}}z}+\frac{1}{\xi}\sqrt{\xi^2+\frac{s}{c_1}}A_6e^{-\sqrt{\xi^2+\frac{s}{c_1}}z}-\frac{1}{\xi}\sqrt{\xi^2+\frac{s}{c_2}}A_7e^{\sqrt{\xi^2+\frac{s}{c_2}}z}+\frac{1}{\xi}\sqrt{\xi^2+\frac{s}{c_2}}A_8e^{-\sqrt{\xi^2+\frac{s}{c_2}}z}\\&\pm\frac{c_2Q_0}{8\pi\eta G\lambda_t}\left(-\frac{c_a}{s}e^{-\xi|z-h|}-\frac{c_b}{s}e^{-\sqrt{\xi^2+\frac{s}{c_1}}|z-h|}+\frac{c_c}{s}e^{-\sqrt{\xi^2+\frac{s}{c_2}}|z-h|}\right),\end{aligned} \tag{11b}$$

$$\tilde{p}(z;\xi,s)=\frac{2G}{1+(2\eta-1)n\beta G}\left(-A_2e^{\xi z}+A_4e^{-\xi z}\right)$$

$$-2\eta G\frac{1}{\xi}\frac{s}{c_1}\left(A_5 e^{\sqrt{\xi^2+\frac{s}{c_1}}z}+A_6 e^{-\sqrt{\xi^2+\frac{s}{c_1}}z}\right)-2\eta G\frac{1}{\xi}\frac{c_b}{c_c}\frac{s}{c_1}\left(A_7 e^{\sqrt{\xi^2+\frac{s}{c_2}}z}+A_8 e^{-\sqrt{\xi^2+\frac{s}{c_2}}z}\right)$$

$$+\frac{Q_0}{4\pi\lambda_t}\frac{c_b c_2}{c_1}\left(\sqrt{\xi^2+\frac{s}{c_1}}^{-1}e^{-\sqrt{\xi^2+\frac{s}{c_1}}|z-h|}-\sqrt{\xi^2+\frac{s}{c_2}}^{-1}e^{-\sqrt{\xi^2+\frac{s}{c_2}}|z-h|}\right), \tag{11c}$$

$$\tilde{\vartheta}(z;\xi,s)=2\eta G\frac{1}{\xi}\frac{1}{c_c}\frac{s}{c_2}\left(A_7 e^{\sqrt{\xi^2+\frac{s}{c_2}}z}+A_8 e^{-\sqrt{\xi^2+\frac{s}{c_2}}z}\right)+\frac{Q_0}{4\pi\lambda_t}\sqrt{\xi^2+\frac{s}{c_2}}^{-1}e^{-\sqrt{\xi^2+\frac{s}{c_2}}|z-h|}, \tag{11d}$$

in which

$$c_a=\frac{c_1}{c_3}-\frac{2G(1+\nu)\alpha_s}{1-2\nu}, \tag{12a}$$

$$c_b=\frac{c_1^2}{c_3(c_2-c_1)}, \tag{12b}$$

$$c_c=\frac{c_1c_2}{c_3(c_2-c_1)}-\frac{2G(1+\nu)\alpha_s}{1-2\nu}, \tag{12c}$$

where $c_a+c_b=c_c$ and

$$c_1=\frac{k}{\gamma_w}\frac{2\eta G}{2\eta Gn\beta+1}, \tag{13a}$$

$$c_2=\frac{\lambda_t}{m}, \tag{13b}$$

$$c_3=\frac{k}{\gamma_w}\frac{1-\nu}{3(1-\nu)\alpha_u+(1+\nu)\alpha_s}. \tag{13c}$$

The constants $A_i\,(i=1,2,\cdots,8)$ in Eqs. (11a)-(11d) are functions of the transformed variables ξ and s which must be determined from the transformed mechanical, flow and thermal boundary conditions. The upper and lower signs in equation (11b) are for the conditions of $(z-h)\geq 0$ and $(z-h)<0$, respectively.

3.2 Transformed Boundary Conditions

Taking Laplace-Hankel transforms for the boundary conditions at $z=0$, the Eq. (7a), yields the transformed boundary conditions as following:

$$\frac{d\tilde{u}_r(0;\xi,s)}{dz}-\xi\tilde{u}_z(0;\xi,s)=0,\ \eta\frac{d\tilde{u}_z(0;\xi,s)}{dz}+(\eta-1)\xi\tilde{u}_r(0;\xi,s)=0,$$

$$\tilde{p}(0;\xi,s)=0,\ \text{and}\ \tilde{\vartheta}(0;\xi,s)=0. \tag{14a}$$

In this manipulation, the boundary conditions at $z\to\infty$ are used to perform the integral transformations as below:

$$\lim_{z\to\infty}\left\{\tilde{u}_r\left(z;\xi,s\right),\tilde{u}_z\left(z;\xi,s\right),\tilde{p}\left(z;\xi,s\right),\tilde{\vartheta}\left(z;\xi,s\right)\right\}=\left\{0,0,0,0\right\}. \tag{14b}$$

Here, $\tilde{u}_r$, $\tilde{u}_z$, $\tilde{p}$ and $\tilde{\vartheta}$ follow the definitions of Eqs. (10a)-(10d).

The constants $A_i\left(i=1,2,\cdots,8\right)$ of the general solutions can be determined by the transformed half space boundary conditions at $z=0$ and the remote boundary conditions at $z\to\infty$. Finally, the desired quantities u_r, u_z, p and ϑ are obtained by applying appropriate inverse Laplace-Hankel transformations with the help of mathematical handbook (Erdelyi *et al.*, 1954).

3.3 Expressions for Ground Surface Displacements

The study is focused on horizontal and vertical displacements of the ground surface, $z=0$, due to an instantaneous point heat source. The transformed transient ground surface displacements $\tilde{u}_r\left(0;\xi,s\right)$ and $\tilde{u}_z\left(0;\xi,s\right)$ due to an instantaneous point heat source are derived from the transformed general solutions (11a)-(11b) and mechanical boundary conditions at $z=0$ and $z\to\infty$, the Eqs. (14a)-(14b), as below:

$$\tilde{u}_r\left(0;\xi,s\right)=\frac{Q_0}{2\left(2\eta-1\right)\pi Gm}\left[-\frac{c_a}{s}exp\left(-\xi h\right)-\frac{c_b}{s}exp\left(-\sqrt{\xi^2+\frac{s}{c_1}}h\right)+\frac{c_c}{s}exp\left(-\sqrt{\xi^2+\frac{s}{c_2}}h\right)\right], \tag{15a}$$

$$\tilde{u}_z\left(0;\xi,s\right)=\frac{Q_0}{2\left(2\eta-1\right)\pi Gm}\left[\frac{c_a}{s}exp\left(-\xi h\right)+\frac{c_b}{s}exp\left(-\sqrt{\xi^2+\frac{s}{c_1}}h\right)-\frac{c_c}{s}exp\left(-\sqrt{\xi^2+\frac{s}{c_2}}h\right)\right]. \tag{15b}$$

The Laplace-Hankel inversion formulae for displacements are defined as following:

$$u_r\left(r,z,t\right)=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}\int_0^{\infty}\xi\tilde{u}_r\left(z;\xi,s\right)J_1\left(\xi r\right)exp\left(st\right)d\xi ds, \tag{16a}$$

$$u_z\left(r,z,t\right)=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}\int_0^{\infty}\xi\tilde{u}_z\left(z;\xi,s\right)J_0\left(\xi r\right)exp\left(st\right)d\xi ds. \tag{16b}$$

Using integral transform handbook (Erdelyi *et al.*, 1954) and integral inversions listed in Eqs. (16a)-(16b), the transient horizontal displacement $u_r\left(r,0,t\right)$ and vertical displacement $u_z\left(r,0,t\right)$ of the ground surface due to an instantaneous point heat source of strength Q_0 are obtained as follows:

$$\begin{aligned}u_r\left(r,0,t\right)=\frac{Q_0}{2\left(2\eta-1\right)\pi Gm}&\left\{-\frac{c_a r}{\left(h^2+r^2\right)^{3/2}}-\frac{c_b}{c_1}\int_0^{c_1 t}\frac{c_1 hr}{16\tau^3}e^{-\frac{2h^2+r^2}{8\tau}}\left[I_0\left(\frac{r^2}{8\tau}\right)-I_1\left(\frac{r^2}{8\tau}\right)\right]d\tau\right.\\&\left.+\frac{c_c}{c_2}\int_0^{c_2 t}\frac{c_2 hr}{16\tau^3}e^{-\frac{2h^2+r^2}{8\tau}}\left[I_0\left(\frac{r^2}{8\tau}\right)-I_1\left(\frac{r^2}{8\tau}\right)\right]d\tau\right\},\end{aligned} \tag{17a}$$

$$u_z(r,0,t)=\frac{Q_0}{2(2\eta-1)\pi Gm}\left\{\frac{c_a h}{\left(h^2+r^2\right)^{2/3}}+\frac{c_b}{c_1}\left[\frac{c_1 h}{h^2+r^2}\frac{1}{\sqrt{\pi c_1 t}}e^{-\frac{h^2+r^2}{4c_1 t}}+\frac{c_1 h}{\left(h^2+r^2\right)^{3/2}}erfc\left(\frac{\sqrt{h^2+r^2}}{2\sqrt{c_1 t}}\right)\right]\right.$$
$$\left.-\frac{c_c}{c_2}\left[\frac{c_2 h}{h^2+r^2}\frac{1}{\sqrt{\pi c_2 t}}e^{-\frac{h^2+r^2}{4c_2 t}}+\frac{c_2 h}{\left(h^2+r^2\right)^{3/2}}erfc\left(\frac{\sqrt{h^2+r^2}}{2\sqrt{c_2 t}}\right)\right]\right\}, \tag{17b}$$

where $erfc(x)$ denotes the complementary error function; $I_\alpha(x)$ is known as the modified Bessel function of the first kind of order α. The transient ground surface horizontal and vertical displacements shown in Eqs. (17a)-(17b) vanished when $t\to\infty$ in this linear elastic model.
The maximum ground surface horizontal displacement $u_{r\,max}$ of the half space due to an instantaneous point heat source is derived from Eq. (17a) by letting $r=h/\sqrt{2}\approx 0.707h$. After doing so, we have

$$u_{r\,max}=u_r\left(h/\sqrt{2},0,0^+\right)=-\frac{\sqrt{3}c_a Q_0}{9(2\eta-1)\pi Gmh^2}, \tag{18}$$

in which the value $r=h/\sqrt{2}$ is derived when $du_r\left(r,0,0^+\right)/dr$ is equal to zero.
The maximum ground surface vertical displacement $u_{z\,max}$ of the isothermal permeable half space due to an instantaneous point heat source is derived from Eq. (17b) by letting $r=0$. Hence

$$u_{z\,max}=u_z\left(0,0,0^+\right)=\frac{c_a Q_0}{2(2\eta-1)\pi Gmh^2}. \tag{19}$$

The absolute value of the ratio of $u_{r\,max}/u_{z\,max}$ can be derived from Eqs. (18) and (19) as below:

$$\left|\frac{u_{r\,max}}{u_{z\,max}}\right|\times 100\%=\frac{2\sqrt{3}}{9}\times 100\%\cong 38.5\%. \tag{20}$$

The above result shows the maximum ground surface horizontal displacement is around 38.5% of the maximum vertical displacement for the isothermal permeable ground surface due to an instantaneous point heat source.

3.4 Expressions for Excess Pore Water Pressure and Temperature Increment of the Stratum

The study also addressed the excess pore water pressure and temperature increment of the poroelastic half space due to an instantaneous point heat source. The transformed excess pore water pressure and temperature increment are obtained from Eqs. (11c)-(11d) with the help of transformed hydraulic the thermal boundary conditions in equations (14a)-(14b) and can be expressed as following:

$$\tilde{p}(z;\xi,s)=\frac{Q_0}{4\pi\lambda_t}\frac{c_b c_2}{c_1}\left\{\sqrt{\xi^2+\frac{s}{c_1}}^{-1}\left[exp\left(-\sqrt{\xi^2+\frac{s}{c_1}}|z-h|\right)-exp\left(-\sqrt{\xi^2+\frac{s}{c_1}}(z+h)\right)\right]\right.$$
$$\left.-\sqrt{\xi^2+\frac{s}{c_2}}^{-1}\left[exp\left(-\sqrt{\xi^2+\frac{s}{c_2}}|z-h|\right)-exp\left(-\sqrt{\xi^2+\frac{s}{c_2}}(z+h)\right)\right]\right\}, \tag{21a}$$

$$\tilde{\vartheta}(z;\xi,s)=\frac{Q_0}{4\pi\lambda_t}\sqrt{\xi^2+\frac{s}{c_2}}^{-1}\left[exp\left(-\sqrt{\xi^2+\frac{s}{c_2}}|z-h|\right)-exp\left(-\sqrt{\xi^2+\frac{s}{c_2}}(z+h)\right)\right]. \tag{21b}$$

The Laplace-Hankel inversion formulae for $\tilde{p}(z;\xi,s)$ and $\tilde{\vartheta}(z;\xi,s)$ are defined as below:

$$p(r,z,t)=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}\int_0^{\infty}\xi\tilde{p}(z;\xi,s)J_0(\xi r)exp(st)d\xi ds, \tag{22a}$$

$$\vartheta(r,z,t)=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}\int_0^{\infty}\xi\tilde{\vartheta}(z;\xi,s)J_0(\xi r)exp(st)d\xi ds. \tag{22b}$$

The transient excess pore water pressure $p(r,z,t)$ and temperature increment $\vartheta(r,z,t)$ of the saturated isothermal permeable half space due to an instantaneous point heat source are obtained as following:

$$p(r,z,t)=\frac{Q_0}{8\pi\lambda_t}\frac{c_1c_2}{c_3(c_2-c_1)}\left\{\frac{1}{\sqrt{\pi c_1 t^3}}\left[exp\left(-\frac{r^2+(z-h)^2}{4c_1t}\right)-exp\left(-\frac{r^2+(z+h)^2}{4c_1t}\right)\right]\right.$$
$$\left.-\frac{1}{\sqrt{\pi c_2 t^3}}\left[exp\left(-\frac{r^2+(z-h)^2}{4c_2t}\right)-exp\left(-\frac{r^2+(z+h)^2}{4c_2t}\right)\right]\right\}, \tag{23a}$$

$$\vartheta(r,z,t)=\frac{Q_0}{8\pi\lambda_t}\frac{1}{\sqrt{\pi c_2 t^3}}\left[exp\left(-\frac{r^2+(z-h)^2}{4c_2t}\right)-exp\left(-\frac{r^2+(z+h)^2}{4c_2t}\right)\right]. \tag{23b}$$

4. Numerical Results

Following Ma and Hueckel (1992, 1993), Bai and Abousleiman (1997), the selected representative parameters are listed in Table 1 to verify the proposed solutions. The constants $c_i\,(i=1,2,3,a,b,c)$ are derived as shown in Table 2 by using the parameters listed in Table 1, Eqs. (12a)-(12c) and Eqs. (13a)-(13c).

Symbol	Value	Unit
Shear modulus of the skeletal material, G	100	*MPa*
Poisson's ratio of the skeletal material, ν	0.23	Dimensionless
Porosity of the porous medium, n	0.2	Dimensionless
Coefficient of heat conduction of the porous medium, λ_t	1.69	*J/sm°C*
Linear thermal expansion coefficient of the pore water, α_w	3.33×10^{-6}	$^{o}C^{-1}$
Linear thermal expansion coefficient of the skeletal material, α_s	3.33×10^{-7}	$^{o}C^{-1}$
Specific heat of the pore water, c_w	500	*J/kg°C*
Specific heat of skeletal material, c_s	200	*J/kg°C*
Density of the pore water, ρ_w	1,000	kg/m^3
Density of the skeletal material, ρ_s	2,000	kg/m^3
Permeability of porous medium, k	1×10^{-11}	*m/s*
Unit weight of the pore water, γ_w	9,810	N/m^3
Compressibility of the pore water, β	5×10^{-10}	Pa^{-1}

Table 1. Selected representative parameters (Ma and Hueckel, 1992, 1993; Bai and Abousleiman, 1997)

Symbol	Value	Unit
c_1	2.826×10^{-7}	m^2/s
c_2	4.024×10^{-6}	m^2/s
c_3	3.062×10^{-10}	$^{o}Cm^4/Ns$
c_a	7.714×10^{2}	$N/^{o}Cm^2$
c_b	6.974×10^{1}	$N/^{o}Cm^2$
c_c	8.411×10^{2}	$N/^{o}Cm^2$

Table 2. Values of $c_i\left(i = 1, 2, 3, a, b, c\right)$

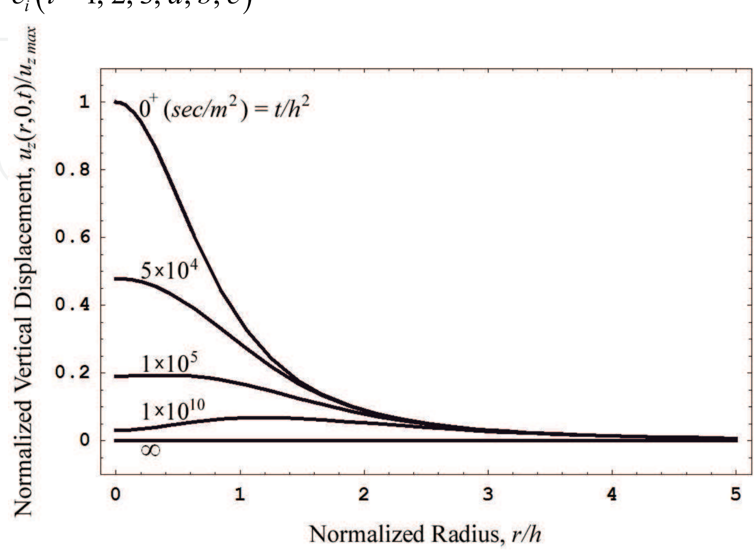

Fig. 2. Vertical displacement profile at the ground surface $z = 0$

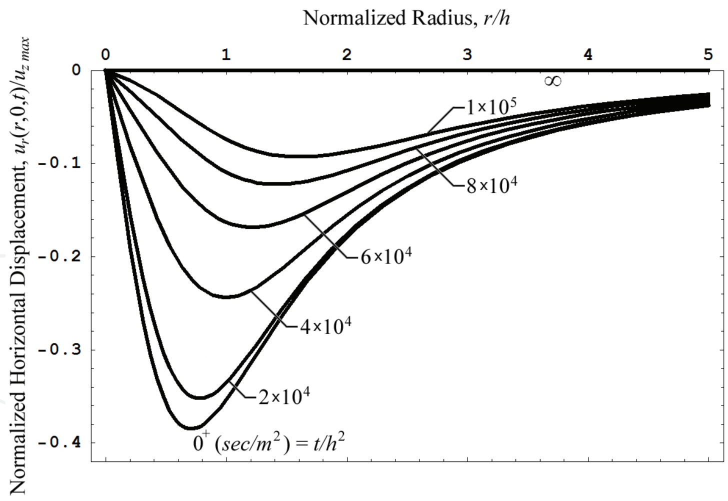

Fig. 3. Horizontal displacement profile at the ground surface $z = 0$

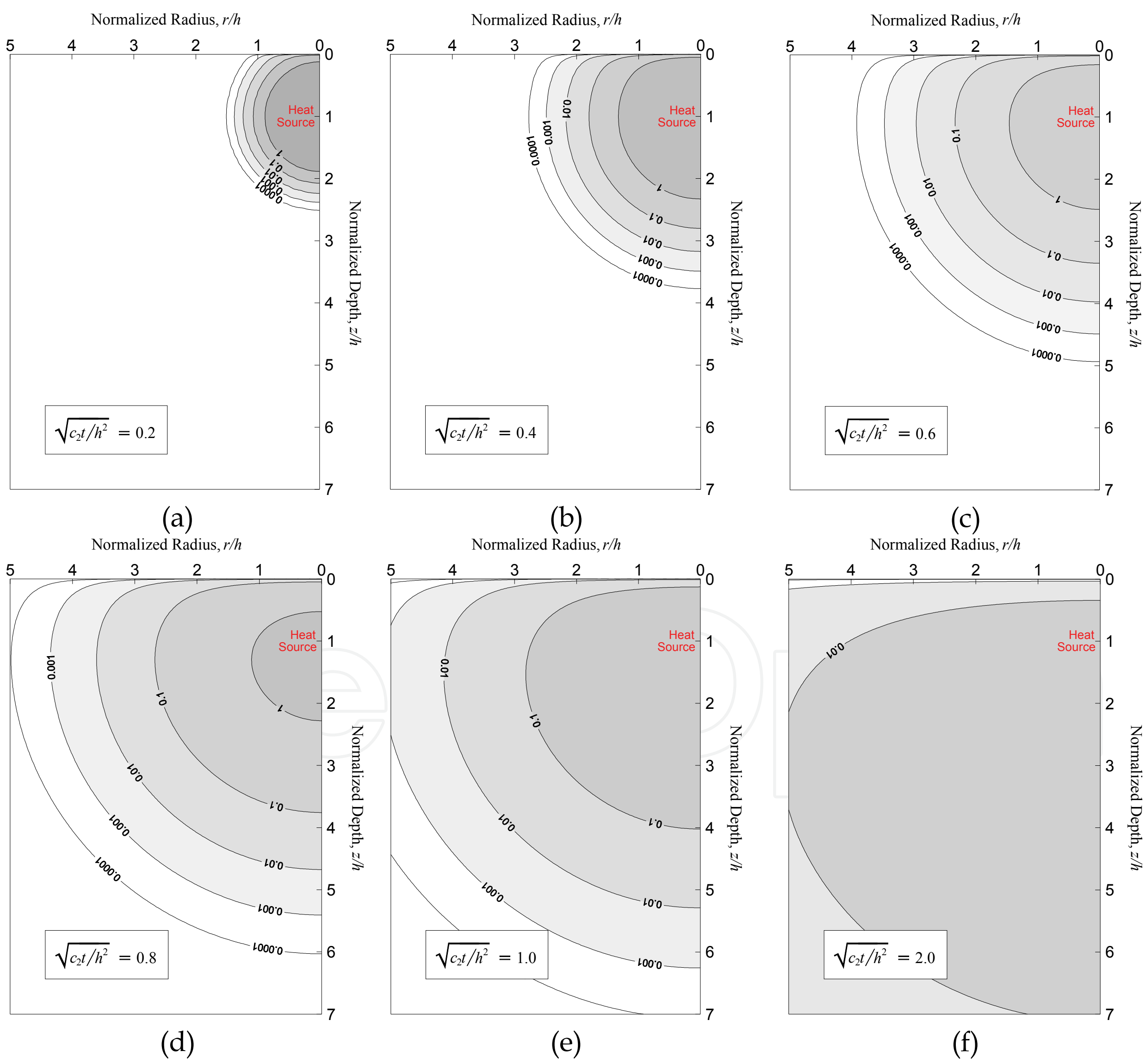

Fig. 4. Distribution of normalized temperature increments $\vartheta(r,z,t)/\left[c_2Q_0/8\pi^{1.5}\lambda_t h^3\right]$

The profiles of vertical and horizontal displacements at the ground surface $z=0$ are normalized by $u_{z\,max}$ as shown in Figures 2 and 3, respectively. The results shown in Figures 2 and 3 indicate that the ground surface displacements due to instantaneous point heat source can reach its extreme values initially, and then the displacements decreases gradually. Figure 3 shows that the maximum ground surface horizontal displacement is around 38.5% of the maximum ground surface vertical displacement. Figures 2 and 3 also concluded that the long-term thermoelastic ground surface deformations due to an instantaneous point heat source vanished in this linear elastic model.

From Eq. (23b), the profiles of normalized temperature increment $\vartheta(r,z,t)\Big/\left[c_2 Q_0/8\pi^{1.5}\lambda_t h^3\right]$ of isothermal permeable half space at six different dimensionless time factor $\sqrt{c_2 t/h^2}$ = 0.2, 0.4, 0.6, 0.8, 1.0 and 2.0 are illustrated in Figures 4(a)-(f), respectively. The changes in temperature increment have positive value of ϑ which is caused by the heating of instantaneous point heat source. It's observed that the positive temperature change increases to a wider region of the half space initially and then gradually decreased. The stratum temperature rise caused by instantaneous point heat finally disappeared, and the elastic deformations due to instantaneous point heat source fully recovered as the temperature increment vanished.

The presented closed-form solutions can be used to test numerical models for thermoelastic processes. It can also be used in more detailed numerical simulations of the processes near the buried heat sources.

5. Conclusions

Using Laplace-Hankel transformations, the transient closed-form solutions of the thermoelastic consolidation due to an instantaneous point heat source in an isothermal permeable half space are obtained. The results show:

1. The maximum ground surface horizontal displacement is around 38.5% of the maximum ground surface vertical displacement of the isothermal permeable half space at $r = h/\sqrt{2} \approx 0.707h$.
2. It's observed that the positive temperature change increases to a wider region of the half space initially and then gradually decreased. The stratum temperature rise caused by instantaneous point heat finally disappeared, and the elastic deformations due to instantaneous point heat source fully recovered as the temperature increment vanished.

6. Acknowledgements

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8. Notation of Symbols

$b_i(i=r,\theta,z)$ Body forces of the poroelastic half space (*N/m³*)

c_1 Parameter, $c_1 = 2\eta Gk/(2\eta Gn\beta+1)\gamma_w$ (m^2/s)

c_2 Parameter, $c_2 = \lambda_t/m$ (m^2/s)

c_3 Parameter, $c_3 = (1-\nu)k/\left[3(1-\nu)\alpha_u + (1+\nu)\alpha_s\right]\gamma_w$ ($^oCm^4/Ns$)

c_a Parameter, $c_a = \dfrac{c_1}{c_3} - \dfrac{2G(1+\nu)\alpha_s}{1-2\nu}$ ($N/^oCm^2$)

c_b Parameter, $c_b = \dfrac{c_1^2}{c_3(c_2-c_1)}$ ($N/^oCm^2$)

c_c Parameter, $c_c = \dfrac{c_1c_2}{c_3(c_2-c_1)} - \dfrac{2G(1+\nu)\alpha_s}{1-2\nu}$ ($N/^oCm^2$)

c_s Specific heat of the skeletal material (J/kg^oC)

c_w Specific heat of the pore water (J/kg^oC)

$erfc(x)$ Complementary error function (Dimensionless)

G Shear modulus of the isotropic poroelastic half space (*Pa*)

h Buried depth of instantaneous point heat source (*m*)

$I_\nu(x)$ Modified Bessel function of the first kind of order ν (Dimensionless)

$J_\nu(x)$ First kind of the Bessel function of order ν (Dimensionless)

k Permeability of the isotropic poroelastic half space (*m/s*)

m Thermal parameter, $m = (1-n)\rho_s c_s + n\rho_w c_w$ ($J/^oCm^3$)

n Porosity of the poroelastic half space (Dimensionless)

p Excess pore fluid pressure of the isotropic poroelastic half space (*Pa*)

$\tilde{p}$ Laplace-Hankel transforms of p, Eq. (10c)

Q_0 Strength of instantaneous point heat source (*J*)

(r,θ,z) Cylindrical coordinates system (*m*, *radian*, *m*)

s Laplace transform parameter (s^{-1})

t Time (*s*)

$u_i(i=r,z)$ Displacement components of the poroelastic half space (*m*)

$u_{r\,max}$, $u_{z\,max}$ Maximum ground surface horizontal/vertical displacement of the poroelastic half space (*m*)

$\tilde{u}_i(i=r,z)$ Laplace-Hankel transforms of u_i, Eqs. (10a)-(10b)

α_s Linear thermal expansion coefficient of skeletal of the stratum ($^oC^{-1}$)

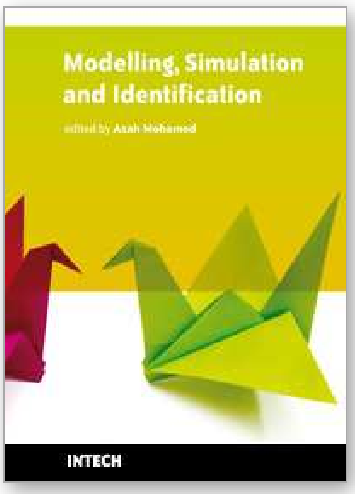
α_u Linear thermal expansion factor, $\alpha_u = (1-n)\alpha_s + n\alpha_w$ ($^oC^{-1}$)

α_w Linear thermal expansion coefficient of pore water ($^oC^{-1}$)

β Compressibility of pore water (Pa^{-1})

γ_w Unit weight of pore water (*N/m³*)

$\delta(t)$	Dirac delta function (s^{-1})
$\delta(x)$	Dirac delta function (m^{-1})
δ_{ij}	Kronecker delta (Dimensionless)
ε	Volume strain of the poroelastic half space (Dimensionless)
$\varepsilon_{ij}(i,j=r,\theta,z)$	Strain components of the poroelastic half space (Dimensionless)
η	Parameter, $\eta=(1-\nu)/(1-2\nu)$ (Dimensionless)
ϑ	Temperature change of the poroelastic half space (°C)
$\tilde{\vartheta}$	Laplace-Hankel transform of ϑ, Eq. (10d)
λ_t	Thermal conductivity of the poroelastic half space ($J/sm°C$)
ν	Poisson's ratio of the isotropic poroelastic half space (Dimensionless)
ξ	Hankel transform parameter (m^{-1})
ρ_s	Density of skeletal material (kg/m^3)
ρ_w	Density of pore water (kg/m^3)
$\sigma_{ij}(i,j=r,\theta,z)$	Total stress components of the poroelastic half space (Pa)
∇^2	Laplacian operator, $\nabla^2=\partial^2/\partial r^2+1/r\,\partial/\partial r+\partial^2/\partial z^2$ (m^{-2})



Modelling, Simulation and Identification

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Modeling, simulation and identification has been actively researched in solving practical engineering problems. This book presents the wide applications of modeling, simulation and identification in the fields of electrical engineering, mechanical engineering, civil engineering, computer science and information technology. The book consists of 17 chapters arranged in an order reflecting multidimensionality of applications related to power system, wireless communication, image and video processing, control systems, robotics, soil mechanics, road engineering, mechanical structures and workforce capacity planning. New techniques in signal processing, adaptive control, non-linear system identification, multi-agent simulation, eigenvalue analysis, risk assessment, modeling of dynamic systems, finite difference time domain modeling and visual feedback are also presented. We hope that readers will find the book useful and inspiring by examining the recent developments in the applications of modeling, simulation and identification.