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Analysis, model parameter extraction and optimization of planar inductors using MATLAB

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1. Introduction

The rising development of the microelectronic integrated circuits and technologies requires effective and flexible tools for modeling and simulation. Modeling of the planar inductors is a key problem in microelectronic design and requires precise implementation of the corresponding models for simulation and optimization. The application of a general-purpose matrix based software like MATLAB and the proper model implementation for such software is of great importance in the RF designer's everyday work.

The on-chip planar inductor is a very important constructive component of the contemporary CMOS microelectronics. In the CMOS SoC RFs the use of the planar inductors in designs like VCOs, mixers, RF amplifiers, impedance-matching circuits is widespread. Many papers, devoted on the on-chip spiral inductors analysis, model parameter extraction and optimization were published in the recent years.

The MATLAB environment can be successfully used in the circuit analysis. The implemented symbolic representation of the equations is of a significant importance for the description and simulation of multiparameter models in microelectronics.

Genetic Algorithm (GA) optimization tools are already implemented in leading general-purpose system analysing software like MATLAB. This gives the users another opportunity for solving various design optimization problems. The GA is a stochastic global search method, which is based on a mechanism resemble the natural biological evolution mechanism. GA operates on a population of solutions and the fitness of the solutions is determined from the objective function of some specific problem. Only the best fitted solutions remain in the population after a number of predefined cycles. In microelectronic technologies and in the microelectronic design the problems are often presented as mathematical functions of multiple variables. The optimization of the values of these variables is in some cases a complex problem, especially when the amount of the variables is huge. The big advantage of GA is that these algorithms do not require derivative information or other knowledge. Only the objective function and the corresponding fitness levels influence the direction of search. This makes the GA an useful tool for parameter optimization in microelectronics and especially for geometric optimization of microelectronic components, when the parameters of the technology are known.

A method for optimal design and synthesis of CMOS inductors for use in RF circuits is proposed in (Hershenson et al., 1999). The method is based on formulating the design problem as an optimization problem using geometric programming. The physical dimensions of the inductor are defined as design parameters. A variety of specifications are introduced including the required inductance value, as well as the minimum allowed values of the self-resonant frequency and the quality factor. Geometric constraints that can be handled include the maximum and the minimum values for each of the design parameters and a limit on total area.

The wide-band spiral inductor model, proposed in (Gil & Shin, 2003) is simple and has an excellent accuracy in comparison to the measured results. The main advantage of this model compared to the models with geometry dependent parameters developed in (Ashby et al., 1994; Yue et al. 1996; Mohan et al., 1999) consists in the frequency independence of the model parameters. Moreover, the models in (Ashby et al., 1994; Yue et al. 1996; Mohan et al., 1999) can not predict the drop-down characteristics in the series resistance at higher frequencies. The wide-band model was widely accepted and several extraction procedures were published to aid the verification and the easy implementation in the microelectronic designs (Kang et al., 2005; Chen et al., 2008). Several modifications of the model are proposed in (Sun et al., 2004 ; Wen & Sun, 2006). The application of the simple parameter extraction method (Kang et al., 2005) and the systematic model parameter extraction approach (Chen et al., 2008) lead to very accurate results. However, the application of these procedures takes time and the need of having automated extraction procedures using an industrial standard environment like MATLAB is an important problem to solve.

A bandwidth extension technique of gigahertz broadband circuitry is applied in (Mohan et al., 2000) by using optimized on-chip spiral inductors. A global optimization method, based on geometric programming, is discussed. As a result, the optimized on-chip inductors consume only 15% of the total area. A fast Sequential Quadratic Programming (SQP) approach to optimize the on-chip spiral inductors is proposed in (Zhan & Sapatnekar, 2004). A robust automated synthesis methodology to efficiently generate spiral inductor designs using multi-objective optimization techniques and surrogate functions to approximate Pareto surfaces in the design space is developed in (Nieuwoudt & Massoud, 2005). The obtained results indicate that the synthesis methodology efficiently optimizes inductor designs based on the defined requirements with an improvement of up to 51% in key inductor design constraints. The need to develop analysis and optimizations in one and the same environment leads to the usage of MATLAB and the implemented optimization toolboxes and GA toolbox (Chipperfield et al., 1994).

2. Wide-band planar inductor model analysis in MATLAB

The enhanced model of spiral inductor (Gil & Shin, 2003) can be treated as a parallel combination of an upper and a lower subcircuits (Fig. 1a). Because of the DC blocking property of the oxide capacitors C_{ox1} and C_{ox2}, the spiral inductor model can be separated into two parts: upper subcircuit and lower subcircuit. The inductor can be approximately characterized by the upper subcircuit for lower frequencies and by the lower subcircuit for high frequencies.

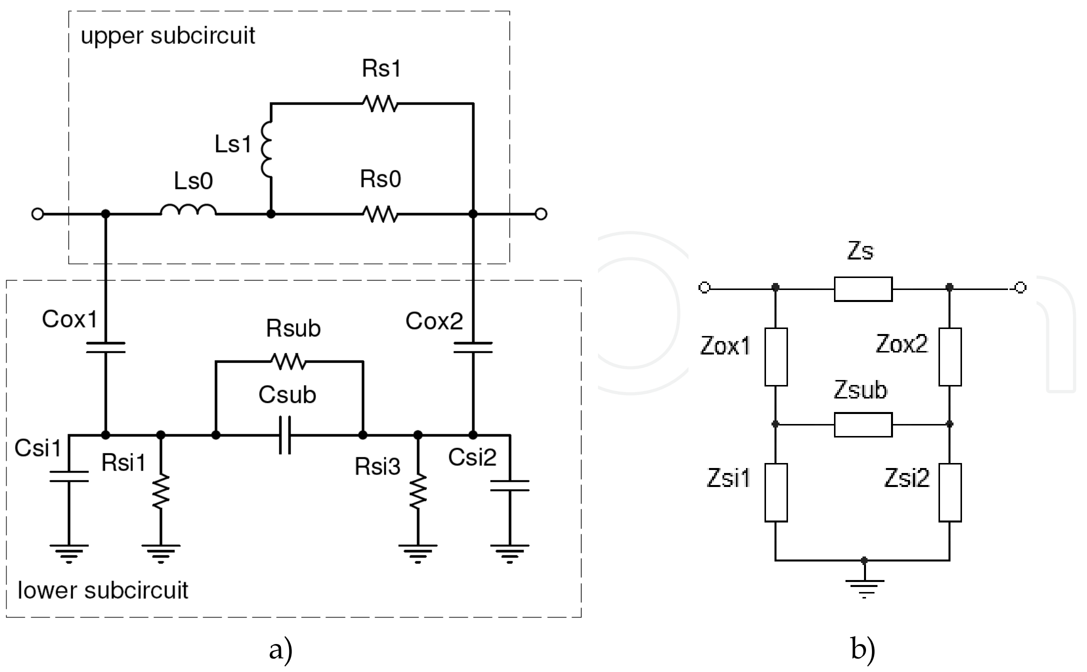

Fig. 1. The enhanced model of spiral inductor (a) and its schematic representation (b)

From the analysis point of view, the model can be represented using the equivalent schematic, shown in Fig. 1b (Durev et al., 2009). The following expressions can be written:

$$Z_{Ls0} = sL_{s0}\,; \qquad Z_{s1} = R_{s1} + sL_{s1}\,; \qquad Z_{Rs0} = R_{s0}\;; \qquad Z_s = Z_{Ls0} + \frac{1}{1/Z_{s1} + 1/Z_{Rs0}}\,, \tag{1}$$

$$Z_{ox1,2} = 1/sC_{ox1,2}\;; \qquad Z_{Rsi1,2} = R_{si1,2}\,; \qquad Z_{Csi1,2} = 1/sC_{si1,2}\,, \tag{2}$$

$$Z_{si1,2} = \frac{1}{1/Z_{Rsi1,2} + 1/Z_{Csi1,2}}\;; \quad Z_{Rsub} = R_{sub}\,; \quad Z_{Csub} = 1/sC_{sub}\,; \quad Z_{sub} = \frac{1}{1/Z_{Rsub} + 1/Z_{Csub}}\,. \tag{3}$$

The Y-parameters can be found using equations (1), (2) and (3). The corresponding schematics are shown in Fig. 2 for both cases – $\dot{V}_2 = 0$ (Fig. 2a) and $\dot{V}_1 = 0$ (Fig. 2b). According to the two-port definition for the Y-parameters, Y_{11} and Y_{21} are defined for the case in Fig. 2a, and Y_{22} and Y_{12} – for the case, shown in Fig. 2b.

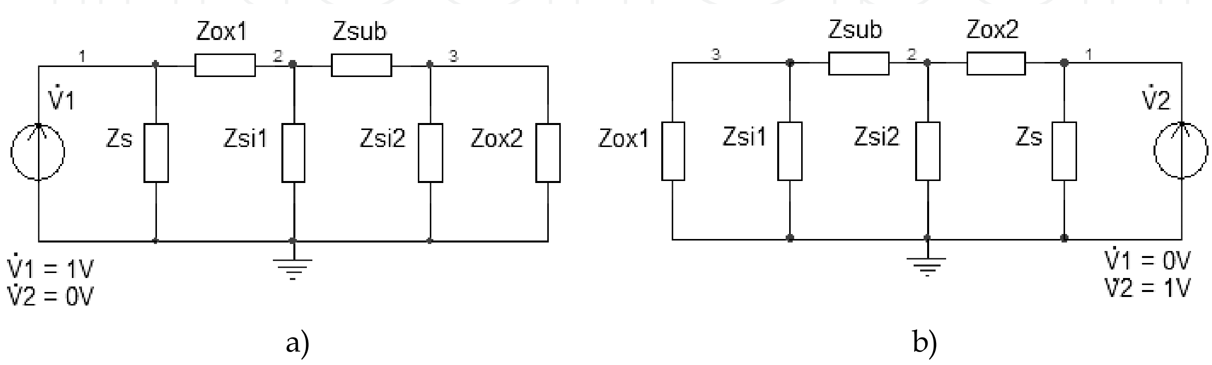

Fig. 2. Representation of the Y-parameter analysis: (a) $\dot{V}_2 = 0$; (b) $\dot{V}_1 = 0$

The equivalent impedance Z_{eqY11} for the schematic, shown in Fig. 2a can be found, using the following equations:

$$Z_{ox2si2} = \frac{1}{1/Z_{ox2} + 1/Z_{si2}}\ ;\ Z_{subox2si2} = Z_{ox2si2} + Z_{sub}\ ;\ Z_{subox2si2si1} = \frac{1}{1/Z_{si1} + 1/Z_{subox2si2}}, \quad (4)$$

$$Z_{subox2si2si1ox1} = Z_{ox1} + Z_{subox2si2si1}\ ; \qquad Z_{eqY11} = \frac{1}{1/Z_s + 1/Z_{subox2si2si1ox1}}. \quad (5)$$

The following expression for Y_{11} can be written:

$$Y_{11} = \dot{V}_1 / Z_{eqY11}\ . \quad (6)$$

The equivalent impedance Z_{eqY22} and Y_{22} are obtained similarly from the analysis of the schematic in Fig. 2b, when $\dot{V}_1 = 0$. The following expressions are valid:

$$Z_{eqY22} = \frac{1}{1/Z_s + 1/Z_{subox1si1si2ox2}}\ ; \quad Y_{22} = \dot{V}_2 / Z_{eqY22}\ . \quad (7)$$

Y_{21} can be easily expressed if the voltage $\dot{V}_{Zox2}$ across Z_{ox2} in Fig. 2a is known. It can be symbolically expressed using the symbolic analysis possibilities in MATLAB. If the input current, node voltages and admittances in Fig. 2 are declared as symbols using *syms*, the Modified Nodal Analysis (MNA) circuit matrix equation can be solved and the following expression can be written for $\dot{V}_{Zox2}$:

$$\dot{V}_{Zox2} = \frac{Y_{ox1}.Y_{sub}\dot{V}_1}{(Y_{ox1} + Y_{si1}).(Y_{sub} + Y_{ox2} + Y_{si2}) + Y_{sub}.(Y_{ox2} + Y_{si2})}\ . \quad (8)$$

In order to obtain Y_{21}, the currents $\dot{I}_{Zox2}$ and $\dot{I}_s$ are expressed, using the following equations:

$$\dot{I}_{Zox2} = \dot{V}_{Zox2} / Z_{ox2}\ ;\ \dot{I}_s = \dot{V}_1 / Z_s\ . \quad (9)$$

Using the expressions (9) and the Y_{21} two-port definition, Y_{21} is expressed in the form:

$$Y_{21} = -\left(\dot{I}_s + \dot{I}_{Zox2}\right)/\dot{V}_1\ . \quad (10)$$

Because of the symmetry of the model from Fig. 1b, the following expression is obtained for Y_{12} from the analysis of the circuit, shown in Fig. 2b when $\dot{V}_1 = 0$:

$$Y_{12} = -\left(\dot{I}_s + \dot{I}_{Zox1}\right)/\dot{V}_2 \quad (11)$$

3. Minimization the Parameter Extraction Errors Using GA

3.1. Purpose function and general structure

Once the basic circuit analysis and Y-parameter expressions are present, a GA approach can be applied to minimize the errors, coming from the parameter extraction procedure. The idea is to compare the Y-parameters, obtained for the model parameter values from the parameter extraction and the Y-parameters, obtained when the model parameter values are varied in a certain range. The actual comparison is done in the purpose function, which compares the absolute value for every frequency point of every two-port Y-parameter in one expression, using the sum of the least squares values:

$$G_{fun} = \sum_{i=1}^{n}\left(Y_{jk}(f_i) - Y_{jk}^{(m)}(f_i)\right)^2 , \tag{12}$$

where j, k = 1,2;

$Y_{jk}^{(m)}$ – measured Y-parameters;

Y_{jk} – simulated Y-parameters of the model;

n – number of the measured/simulated frequency points.

The GA optimization is done using 1000 iterations, generation gap of 0.7 for population of 200 individuals. (Durev, 2009). The function body contains two *for* cycles. The first cycle runs the calculations for every frequency point until the end (DATA_ROWS), and the second cycle runs the calculations for every generated individual (variable) for a given frequency point until the end (Nind).

3.2. Optimization procedure realized in MATLAB

The representation of equations (1 - 11) and the construction of the optimization procedure in MATLAB are shown below:

```
for i = 1:DATA_ROWS %Calculate the frequency response of the circuit
    s = j*2*pi*frequency(i);
for ix = 1:Nind
    ZLs0 = s.*Ls0(ix);  Zs1 = Rs1(ix) + s.*Ls1(ix);  ZRs0 = Rs0(ix);
    Zs = ZLs0 + ((Zs1.*ZRs0)./(Zs1 + ZRs0));  Zox1 = 1./(s.*Cox1(ix));
    Zox2 = 1./(s.*Cox2(ix));  ZRsi1 = Rsi1(ix);  ZCsi1 = 1./(s.*Csi1(ix));
    Zsi1 = (ZRsi1.*ZCsi1)./(ZRsi1 + ZCsi1);  ZRsi2 = Rsi2(ix);  ZCsi2 = 1./(s.*Csi2(ix));
    Zsi2 = (ZRsi2.*ZCsi2)./(ZRsi2 + ZCsi2);  ZRsub = Rsub(ix);  ZCsub = 1./(s.*Csub(ix));
    Zsub = (ZRsub.*ZCsub)./(ZRsub + ZCsub);
    %Calculate Y11, V1 = 1, V2 = 0
    Zox2si2 = (Zox2.*Zsi2)./(Zox2 + Zsi2);  Zsubox2si2 = Zox2si2 + Zsub;
    Zsubox2si2si1 = (Zsi1.*Zsubox2si2)./(Zsi1 + Zsubox2si2);
    Zsubox2si2si1ox1 = Zox1 + Zsubox2si2si1;
    ZeqY11 = (Zs.*Zsubox2si2si1ox1)./(Zs + Zsubox2si2si1ox1);
    Y11(ix, 1) = V1./ZeqY11;
    %Calculate Y22, V1 = 0, V2 = 1
    Zox1si1 = (Zox1.*Zsi1)./(Zox1 + Zsi1);  Zsubox1si1 = Zox1si1 + Zsub;
    Zsubox1si1si2 = (Zsi2.*Zsubox1si1)./(Zsi2 + Zsubox1si1);
    Zsubox1si1si2ox2 = Zox2 + Zsubox1si1si2;
    ZeqY22 = (Zs.*Zsubox1si1si2ox2)./(Zs + Zsubox1si1si2ox2);
```

```
    Y22(ix, 1) = U2./ZeqY22;
    Ysub = 1./Zsub;  Yox1 = 1./Zox1;  Yox2 = 1./Zox2;  Ysi1 = 1./Zsi1; Ysi2 = 1./Zsi2;
    %Calculate Y12, V1 = 0, V2 = 1
    %Expressions taken from symmetry considerations with Y21
    VZox1 = (Yox2.*Ysub)./((Yox2 + Ysi2).*(Ysub + Yox1 + Ysi1) + Ysub.*(Yox1 + Ysi1));
    IZox1 = VZox1./Zox1;  Is = V2./Zs; I1 = Is + IZox1;  Y12(ix, 1) = -I1./U2;
    %Calculate Y21, V1 = 1, V2 = 0
    %Expressions taken from the symbolic extraction of Y21
    VZox2 = (Yox1.*Ysub)./((Yox1 + Ysi1).*(Ysub + Yox2 + Ysi2) + Ysub.*(Yox2 + Ysi2));
    IZox2 = VZox2./Zox2;  Is = V1./Zs;  I2 = Is + IZox2;
    Y21(ix, 1) = -I2./U1;
  end
  %Least squares sum of the difference between the modules of the required and the current Y-parameters
  g_fun = g_fun + ((abs(Y11) - abs(Y11_req(i)))).^2 + ((abs(Y12) - abs(Y12_req(i)))).^2 + ((abs(Y21) - abs(Y21_req(i)))).^2 + ((abs(Y22) - abs(Y22_req(i)))).^2;
  end
```

The presented procedure for optimization of the model parameters of the wide-band on-chip spiral inductor model is verified according to the published data in (Gil & Shin, 2003; Chen et al., 2008). The relative percentage error for the modules of the extracted and the measured Y-parameters is used to estimate the accuracy of the optimization procedure for various geometry RF spiral inductors. The maximal relative percentage error is calculated for the modules of the Y-parameters in the form:

$$\mathrm{RelErr}_{Y\max} = 100.\max_{i}\left|1 - \frac{Y_{jk}(f_i)}{Y_{jk}^{(m)}(f_i)}\right| , \tag{13}$$

where j, k = 1,2;

$Y_{jk}^{(m)}$ - measured Y-parameters;

Y_{jk} - simulated Y-parameters of the model.

The obtained results for the relative errors from the extraction procedure and after the optimization procedure are compared and the error improvement for each of the Y-parameters is shown in Table 1. For example, the extracted values for the case of 4.5 x 30 x 14.5 x 2 geometry (Table 1) are: R_{s0} = 6.680 Ω, R_{s1} = 7.588 Ω, L_{s0} = 3.02 nH, L_{s1} = 1.183 nH, C_{ox1} = 119.57 fF, C_{ox2} = 112.745 fF, R_{si1} = 291.376 Ω, R_{si2} = 286.831 Ω, C_{si1} = 34.133 fF, C_{si2} = 32.993 fF, R_{sub} = 946.544 Ω and C_{sub} = 72.409 fF.

Dimension (N x R x W x S)*	Relative error improvement, %			
	Y_{11}	Y_{12}	Y_{21}	Y_{22}
2.5 x 60 x 14.5 x 2	15	19	19	16
4.5 x 60 x 14.5 x 2	32	33	33	43
6.5 x 60 x 14.5 x 2	14	9	9	7
4.5 x 30 x 14.5 x 2	61	34	34	36
3.5 x 60 x 9 x 7.5	29	37	37	14

* N: number of turns, R: inner radius (μm), W: metal width (μm), S: spacing (μm)

Table 1. Error improvement after the application of GA optimization procedure

The corresponding relative errors are: $RelErr_{Y11max}$ = 0.254 %, $RelErr_{Y12max}$ = 0.062 %, $RelErr_{Y21max}$ = 0.062 %, $RelErr_{Y22max}$ = 0.239 %. After the GA optimization, the following model parameter values are obtained: R_{s0} = 6.697 Ω, R_{s1} = 7.558 Ω, L_{s0} = 3.02 nH, L_{s1} = 1.184 nH, C_{ox1} = 119.776 fF, C_{ox2} = 112.479 fF, R_{si1} = 292.638 Ω, R_{si2} = 285.733 Ω, C_{si1} = 33.997 fF, C_{si2} = 32.958 fF, R_{sub} = 944.4 Ω and C_{sub} = 72.516 fF.
The corresponding relative errors after the GA optimization are: $RelErr_{Y11max}$ = 0.099 %, $RelErr_{Y12max}$ = 0.041 %, $RelErr_{Y21max}$ = 0.041 %, $RelErr_{Y22max}$ = 0.154 %. Thus the improvement of the errors is 61%, 34%, 34% and 36% respectively. The improvement is achieved with 0.5% variation of the model parameter values. The proposed algorithm shows excellent agreement with the measured data over the whole frequency range.

4. Parameter extraction of the wide-band planar inductor model using MATLAB

Several extraction procedures for wide-band on-chip spiral inductor model are proposed in (Kang et al., 2005; Chen et al., 2008). An automated parameter extraction procedure using MATLAB is developed in (Gadjeva et al., 2009).
The input data is supplied to the MATLAB script as an Excel file .xls. The data is structured in columns, starting from the frequency column, followed by the real and imaginary parts of the measured two-port S-parameters: S_{11re}, S_{11im}, S_{12re}, S_{12im}, S_{21re}, S_{21im}, S_{22re} and S_{22im}.
The program has the option for the two-port Y-parameters to be the input data. In the most cases the input data are the measured S-parameters, which are easier to measure and the network analyzers provide this data. Once accepted from the .xls file, the S-parameters are converted to Y-parameters, as the parameter extraction procedure works with Y-parameters. The conversion is done using the MATLAB *s2y* function. Once converted, the input data is structured into five vectors: $[freq]_{n\times 1}$, $[Y11]_{n\times 1}$, $[Y12]_{n\times 1}$, $[Y21]_{n\times 1}$, $[Y22]_{n\times 1}$, where n is the number of points, measured as input data. The input data can be represented as a matrix $[INPUT\_DATA]_{DATA\_ROWS\ x\ 9}$ for nine input data vectors – frequency vector matrix and the real and the imaginary parts vectors for the two-port S- or Y-parameters.
The enhanced model of spiral inductor (Gil & Shin, 2003) shown in Fig. 1a can be approximately characterized by the upper subcircuit for lower frequencies. Such an approximation has been widely applied to calculate the series resistance and inductance (Kang et al., 2005; Huang et al., 2006). To minimize the error, caused by the approximation, the upper frequency limit must be calculated and fixed in the parameter extraction program. For this reason, the spiral inductor model shown in Fig. 1a can be represented by the equivalent schematics in Fig. 3 (π-network).

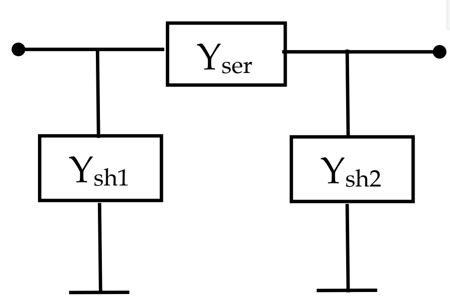

Fig. 3. Relation between the shunt and series admittances of the π-network for the spiral inductor model

In this network, the following expressions are valid (Chen et al., 2008):

$$Y_{ser} = -Y_{12}\ ; \quad Y_{sh1} = Y_{11} + Y_{12}\ ; \quad Y_{sh2} = Y_{22} + Y_{12}\ , \tag{14}$$

$$RAT_{1u} = 100\left|\frac{Y_{sh1}}{Y_{ser}}\right|; \quad RAT_{2u} = 100\left|\frac{Y_{sh2}}{Y_{ser}}\right|. \tag{15}$$

It is shown in (Chen et al., 2008) that the normalized magnitudes of the upper subcircuit RAT_{1u} and RAT_{2u} should be smaller than 1% in order to achieve accurate low frequency approximation. The frequency values in the $[freq]_{n\times1}$ data vector are in the range $[F_{min}; F_{max}]$. F_{min} is the start frequency and F_{max} is the end frequency at which the input two-port Y- or S-parameters are measured. The values of these two frequencies can be easily defined in MATLAB using *min* and *max* functions over the vector $[freq]_{n\times1}$:

```
Fmin = min(freq);
Fmax = max(freq);
```

The expressions (14) and (15), written as a MATLAB code are in the form:

```
Ysh1 = Y11 + Y12;    Ysh2 = Y22 + Y12;
rat1u = 100*abs(Ysh1./Y12);    rat2u = 100*abs(Ysh2./Y12);
```

The maximal frequencies, at which the normalized magnitudes of the vector components are less than 1%, can be found using the following code over the matrices $[freq]_{n\times1}$, $[RAT_{1u}]_{n\times1}$ and $[RAT_{2u}]_{n\times1}$:

```
for i = 1:DATA_ROWS
   if(rat1u(i) <= 1.0)
        freq_low_rat1u = freq(i);
   elseif(rat1u_min > 1.0)
        freq_low_rat1u = freq_low_rat1u_min;
        ErrorMsg
   end
   if(rat2u(i) <= 1.0)
        freq_low_rat2u = freq(i);
   elseif(rat2u_min > 1.0)
        freq_low_rat2u = freq_low_rat2u_min;
        ErrorMsg
   end
end
F1 = min(freq_low_rat1u, freq_low_rat2u);
```

Here n = DATA_ROWS and an error message ErrorMsg is written in case when there are no component values in $[RAT_{1u}]_{n\times1}$ and $[RAT_{2u}]_{n\times1}$, smaller than 1%. This occurs when the input data have not enough number of points or they are not precisely measured. The minimum values found in vectors $[RAT_{1u}]_{n\times1}$ and $[RAT_{2u}]_{n\times1}$ (freq_low_rat1u_min and freq_low_rat2u_min) are taken into account in this case and this could cause deviations in the calculations.

The frequency range, where the upper subcircuit from Fig. 1a represents the approximate behaviour of the spiral inductor model, is then fixed to [F_{min}; F_1], where F_1 is the minimal value of the frequencies freq_low_rat1u and freq_low_rat2u.

Once F_1 and F_{max} were calculated, the low and high frequency vectors: [freq_low]freq_low_rows x freq_low_columns and [freq_high]freq_high_rows x freq_high_columns can be found, which will be needed for the further indexing of the matrices in the calculations:

```
for i = 1:DATA_ROWS
  if(freq(i) <= F1)
      freq_low(i, 1) = freq(i);
  elseif(freq(i) <= Fmax)
      freq_high(i, 1) = freq(i);
  end
end
[freq_low_rows, freq_low_columns] = size(freq_low);
[freq_high_rows, freq_high_columns] = size(freq_high);
```

4.1. Extraction of L_{s0}, R_{s0}, L_{s1}, and R_{s1}

The equivalent resistance and inductance of the upper subcircuit are obtained for the frequency range [F_{min} ; F_1]:

$$R_{uf} = \Re[Z_u] \quad ; \quad L_{uf} = \frac{\Im[Z_u]}{\omega} \quad ; \quad Z_u = -1/Y_{12} \; . \tag{16}$$

The dc resistance and inductance R_{dc} and L_{dc} are calculated for $\omega = 0$. In the real case these values are calculated for $\omega = 2\pi F_{min}$ according to the following expressions:

$$R_{dc\max} = \max_i \left(\frac{R_{uf_i} F_{min}}{f_i} \right) \quad ; \quad L_{dc\max} = \max_i \left(L_{uf_i} \right). \tag{17}$$

The relation between the differences ΔR_{uf} and ΔL_{uf} gives the coefficient T from (Chen et al., 2008) defined in the form:

$$\Delta R_{uf} = R_{uf} - R_{dc\max} \; ; \quad \Delta L_{uf} = L_{dc\max} - L_{uf} \; , \tag{18}$$

$$T = \frac{\Delta R_{uf}}{\Delta L_{uf}} \quad ; \quad T_{max} = \max_i \left(\frac{f_i}{F_1} \cdot \frac{\Delta R_{uf_i}}{\Delta L_{uf_i}} \right). \tag{19}$$

The coefficient M and its maximal value M_{max} are obtained in the form (Chen et al., 2008):

$$M = \Delta R_{uf} \left(1 + T_\omega^2 \right) ; \quad M_{max} = \max_i \left(\Delta R_{uf_i} \left(1 + T_{\omega_i}^2 \right) \right) \quad ; \quad T_\omega = T/\omega \, . \tag{20}$$

The values of R_{s0}, R_{s1}, L_{s0} and L_{s1} are calculated directly from the obtained scalar values for R_{dcmax}, L_{dcmax}, T_{max} and M_{max} (Chen et al. , 2008):

$$R_{s0} = M_{max} + R_{dc\max} \; ; \quad R_{s1} = \frac{R_{s0} R_{dc\max}}{M_{max}} \, , \tag{21}$$

$$L_{s0} = L_{dc\max} - M_{\max}/T_{\max} \;;\quad L_{s1} = \frac{R_{s0} + R_{s1}}{T_{\max}}. \tag{22}$$

Using the equations (16) – (21), the upper subcircuit parameter extraction can be done with the following MATLAB source code:

```
Zu = -1./Y12;  Ruf = real(Zu([1 : freq_low_rows], 1)); Rdc = (Ruf*Fmin)./freq_low;
Luf = imag(Zu([1 : freq_low_rows], 1))./(w([1 : freq_low_rows], 1));
Rdc_max = max(Rdc) + 1.0e-15; Ldc_max = max(Luf) + 1.0e-15;
DRuf = Ruf - Rdc_max; DLuf = Ldc_max - Luf;
T = ((freq_low./F1).*DRuf)./DLuf; %Luf(x,x) = Ldc_max => Warning: Divide by zero.
T_max = max(T); Tw = T_max./w([1 : freq_low_rows], 1);
M = DRuf.*(1 + (Tw.*Tw)); M_max = max(M); Rs0 = M_max + Rdc_max;
Rs1 = (Rs0*Rdc_max)/M_max; Rt = Rs0 + Rs1; Ls0 = Ldc_max - (M_max/T_max);
Ls1 = (Rs0+Rs1)/T_max;
```

A small number of 1×10^{-15} is added to calculate R_{dcmax} and L_{dcmax} to avoid division by zero.

4.2. Extraction of C_{ox1} and C_{ox2}

Once the model parameters R_{s0}, R_{s1}, L_{s0} and L_{s1} are calculated, based on measured data in the frequency range [F_{min}, F_1], the equivalent series impedance Z_s and admittance Y_s of the upper subcircuit can be found using the following expressions:

$$Z_s = j\omega L_{s0} + \frac{1}{\frac{1}{R_{s0}} + \frac{1}{R_{s1} + j\omega L_{s1}}} \quad ; \qquad Y_s = \frac{1}{Z_s}. \tag{23}$$

For frequencies greater than F_1 the lower subcircuit has to be taken into account. The Y-matrix of the lower subcircuit $[Y_l]$ is obtained in the form:

$$[Y_l] = [Y] - [Y_s], \tag{24}$$

where $[Y_s]$ is the admittance matrix of the upper subcircuit;
[Y] - admittance matrix of the entire model.

$$Y_{11l} = Y_{11} - Y_s \;\;;\;\; Y_{12l} = Y_{12} + Y_s \;\;;\;\; Y_{22l} = Y_{22} - Y_s\,. \tag{25}$$

The lower subcircuit from Fig. 1a can be represented again as a π-network. The following expressions are valid for this subcircuit (Chen et al., 2008):

$$Y_{sh11} = Y_{11l} - Y_{12l}\,; \qquad Y_{sh21} = Y_{22l} - Y_{12l}\,; \qquad Y_{serl} = -Y_{12l}\,, \tag{26}$$

$$RAT_{1l} = 100\left|\frac{Y_{serl}}{Y_{sh11}}\right|; \qquad RAT_{2l} = 100\left|\frac{Y_{serl}}{Y_{sh21}}\right|. \tag{27}$$

It is shown in (Chen et al., 2008) that the normalized magnitudes of the lower subcircuit RAT_{1l} and RAT_{2l} should be smaller than 5% in order to achieve accurate high frequency approximation.

The expressions (23) – (27), written as a MATLAB code, are in the form:

```
Zs = j*w*Ls0 + (1./((1/Rs0) + (1./(Rs1 + j*w*Ls1))));   Ys = 1./Zs;    Y11l = Y11 - Ys;
Y12l = Y12 + Ys;    Y22l = Y22 - Ys;    Ysh1l = Y11l + Y12l;  Ysh2l = Y22l + Y12l;  Yserl = -Y12l;
 rat1l = 100.*abs(Yserl./Ysh1l);   rat2l = 100.*abs(Yserl./Ysh2l);
```

The maximal frequencies, at which the components of the vector of normalized magnitudes have values less than 5%, can be found using the following code over the matrices $[\mathrm{frequency}]_{n\times 1}$, $[\mathrm{RAT}_{1l}]_{n\times 1}$ and $[\mathrm{RAT}_{2l}]_{n\times 1}$:

```
for i = 1:DATA_ROWS
   if(freq(i) >= F1)
      if(rat1l(i) <= 5.0)
          freq_low_rat1l = freq(i);
      elseif(rat1l_min > 5.0)
          freq_low_rat1l = freq_low_rat1l_min;
          ErrorMsg
      end
      if(rat2l(i) <= 5.0)
          freq_low_rat2l = freq(i);
      elseif(rat2l_min > 5.0)
          freq_low_rat2l = freq_low_rat2l_min;
          ErrorMsg
      end
   end
end
F2 = min(freq_low_rat1l, freq_low_rat2l);
```

ErrorMsg is written in case there are no component values in $[\mathrm{RAT}_{1l}]_{n\times 1}$ and $[\mathrm{RAT}_{2l}]_{n\times 1}$, smaller than 5%. The minimum values found in vectors $[\mathrm{RAT}_{1l}]_{n\times 1}$ and $[\mathrm{RAT}_{2l}]_{n\times 1}$ namely freq_low_rat1l_min and freq_low_rat2l_min are taken into account in this case and this could cause deviations in the calculations.

The frequency range, where the lower subcircuit from Fig. 1a represents the approximate behaviour of the spiral inductor model is then fixed to [F_1; F_2], where F_2 is the minimal frequency between freq_low_rat1l and freq_low_rat2l.

Once F_2 is calculated, the middle frequency vector $[\mathrm{freq\_mid}]_{\mathrm{freq\_mid\_rows\ x\ freq\_mid\_columns}}$ can be found, which will be needed for the further indexing of the matrices in the calculations:

```
for i = 1:DATA_ROWS
   if(freq(i) <= F2)
        freq_mid(i, 1) = freq(i);
   end
end
[freq_mid_rows, freq_mid_columns] = size(freq_mid);
```

Then C_{ox1} and C_{ox2} can be extracted as maximal values in the range [F_1; F_2] using the expressions from (Chen et al., 2008):

$$C_{ox1} = \frac{-1}{\omega\Im\left[1/\left(Y_{11l} + Y_{12l}\right)\right]}; \qquad C_{ox2} = \frac{-1}{\omega\Im\left[1/\left(Y_{22l} + Y_{12l}\right)\right]}. \tag{28}$$

The following MATLAB source code is used for the calculation of C_{ox1} and C_{ox2}:

```
Cox1 = max(-1./(imag(1./(Y11l([freq_low_rows : freq_mid_rows], 1) + Y12l([freq_low_rows : freq_mid_rows], 1))).*w([freq_low_rows : freq_mid_rows], 1)));
Cox2 = max(-1./(imag(1./(Y22l([freq_low_rows : freq_mid_rows], 1) + Y12l([freq_low_rows : freq_mid_rows], 1))).*w([freq_low_rows : freq_mid_rows], 1)));
```

4.3. Extraction of R_{si1}, R_{si2}, C_{si1}, C_{si2}, R_{sub} and C_{sub}

The lower subcircuit represents the behavior of the model in the range [F_2; F_{max}]. It is analyzed for the extraction of R_{si1}, R_{si2}, C_{si1}, C_{si2}, R_{sub} and C_{sub} based on the relations between the lower subcircuit Y-parameters and the input and output voltages $\dot{V}_1$ and $\dot{V}_2$ using the previously extracted values for the model parameters C_{ox1} and C_{ox2} .

$$\dot{V}_{1a} = \left(1 - \frac{Y_{11l}}{j\omega C_{ox1}}\right)\dot{V}_1 \ ; \qquad \dot{V}_{2a} = \left(-\frac{Y_{12l}}{j\omega C_{ox2}}\right)\dot{V}_1 , \tag{29}$$

$$\dot{V}_{2b} = \left(1 - \frac{Y_{22l}}{j\omega C_{ox2}}\right)\dot{V}_2 \ ; \qquad \dot{V}_{1b} = \left(-\frac{Y_{12l}}{j\omega C_{ox1}}\right)\dot{V}_2 , \tag{30}$$

$$\Delta_V = \dot{V}_{1a}\dot{V}_{2a} - \dot{V}_{1b}\dot{V}_{2a} \ ; \qquad \Delta_{V1} = (Y_{11} + Y_{12})\dot{V}_{2b} - (Y_{22} + Y_{12})\dot{V}_{2a} , \tag{31}$$

$$\Delta_{V2} = (Y_{22} + Y_{12})\dot{V}_{1a} - (Y_{11} + Y_{12})\dot{V}_{1b} . \tag{32}$$

As a result, the model parameters R_{si1}, R_{si2}, C_{si1}, C_{si2} can be extracted directly as follows:

$$R_{si1} = \max_i\left(\frac{f_i/F_{max}}{\Re(\Delta_{V1}/\Delta_V)}\right) \ ; \ R_{si2} = \max_i\left(\frac{f_i/F_{max}}{\Re(\Delta_{V2}/\Delta_V)}\right), \tag{33}$$

$$C_{si1} = \max_i\left(\frac{(f_i/F_{max})\Im(\Delta_{V1}/\Delta_V)}{\omega_i}\right); \qquad C_{si2} = \max_i\left(\frac{(f_i/F_{max})\Im(\Delta_{V2}/\Delta_V)}{\omega_i}\right). \tag{34}$$

Expressions (29) – (34) are represented in MATLAB using the following code over matrices operations:

```
Zcox1 = 1./(j*w*Cox1);  Zcox2 = 1./(j*w*Cox2);  V1a = 1 - (Y11l.*Zcox1);
V2a = -(Y12l.*Zcox2); V2b = 1 - (Y22l.*Zcox2); V1b = -(Y12l.*Zcox1);
DV = (V1a.*V2b) - (V1b.*V2a);
DV1 = ((Y11 + Y12).*V2b) - ((Y22 + Y12).*V2a);
DV2 = (V1a.*(Y22 + Y12)) - (V1b.*(Y11 + Y12));
Rsi1 = max((freq([freq_mid_rows : freq_high_rows], 1)/Fmax)./real(DV1([freq_mid_rows : freq_high_rows], 1)./DV([freq_mid_rows : freq_high_rows], 1)));
Rsi2 = max((freq([freq_mid_rows : freq_high_rows], 1)/Fmax)./real(DV2([freq_mid_rows : freq_high_rows], 1)./DV([freq_mid_rows : freq_high_rows], 1)));
Csi1 = max((freq([freq_mid_rows : freq_high_rows], 1)/Fmax).*imag(DV1([freq_mid_rows : freq_high_rows], 1)./DV([freq_mid_rows : freq_high_rows], 1))./w([freq_mid_rows : freq_high_rows], 1));
```

Csi2 = max((freq([freq_mid_rows : freq_high_rows], 1)/Fmax).*imag(DV2([freq_mid_rows : freq_high_rows], 1)./DV([freq_mid_rows : freq_high_rows], 1))./w([freq_mid_rows : freq_high_rows], 1));

The following expression for the admittance Y_{sub} can be used to extract R_{sub} and C_{sub}:

$$Y_{sub} = \frac{\left(\Delta_{V2}/\Delta_V + j\omega C_{ox2}\right)}{\left(\dot{V}_{1a} / \dot{V}_{2a} - 1\right)}; \quad (35)$$

$$R_{sub} = \left\{\max_i\left[\left(f_i/F_{max}\right)\Re\left(Y_{sub}\right)\right]\right\}^{-1}; \qquad C_{sub} = \max_i\left(\frac{\Im\left(Y_{sub}\right)}{\omega_i}\right). \quad (36)$$

Expressions (35) and (36) are represented in MATLAB using the following code over matrix operations:

```
Ysub = ((DV2./DV) + j*w*Cox2)./((V1a./V2a) - 1);
Rsub = 1/max((freq([freq_mid_rows : freq_high_rows], 1)/Fmax).*
real(Ysub([freq_mid_rows : freq_high_rows], 1)));
Csub = max(imag(Ysub([freq_mid_rows : freq_high_rows], 1))./w([freq_mid_rows :
freq_high_rows], 1));
```

4.4. Results of the extraction procedure of the wide-band inductor model

The presented procedure for parameter extraction of wide-band on-chip spiral inductor model in MATLAB was verified according to the published data in (Gil & Shin, 2003). The relative error over the frequency range for the real and the imaginary part of the measured and the extracted Y-parameters is used to estimate the accuracy of the extraction procedure for various geometry RF spiral inductors. The maximal relative error is calculated over the modules of the Y-parameters, using formula (13). The obtained results are presented in Table 2. They are in agreement with the measured results from (Gil & Shin, 2003; Chen et al., 2008). The maximal relative error is less than 0.5% which makes the parameter on-chip spiral inductor parameter extraction procedure very accurate. For example the extracted values for the case of 4.5 x 30 x 14.5 x 2 geometry (Table 2) are: R_{s0} = 6.681 Ω, R_{s1} = 7.589 Ω, L_{s0} = 3.02 nH, L_{s1} = 1.183 nH, C_{ox1} = 119.571 fF, C_{ox2} = 112.745 fF, R_{si1} = 291.377 Ω, R_{si2} = 286.832 Ω, C_{si1} = 34.134 fF, C_{si2} = 32.994 fF, R_{sub} = 946.544 Ω and C_{sub} = 72.41 fF.

Dimension (N x R x W x S)	$RelErr_Y$, %			
	Y_{11}	Y_{12}	Y_{21}	Y_{22}
2.5 x 60 x 14.5 x 2	0.173	0.054	0.054	0.164
4.5 x 60 x 14.5 x 2	0.451	0.101	0.101	0.391
6.5 x 60 x 14.5 x 2	0.479	0.151	0.151	0.436
4.5 x 30 x 14.5 x 2	0.254	0.062	0.062	0.239
3.5 x 60 x 9 x 7.5	0.235	0.065	0.065	0.216

* N: number of turns, R: inner radius (µm), W: metal width (µm), S: spacing (µm)

Table 2. Error estimation of the extraction procedure

5. Parameter Extraction of Physical Geometry Dependent RF Planar Inductor Model

The physical model of planar spiral inductor on silicon (Yue et al., 1996) is a very popular model used in microelectronic design. Its model parameter values can be expressed directly using the geometry of the spiral inductor. The skin effect at high frequencies is modeled using a frequency dependent series resistance. Several extraction procedures are developed for the physical spiral inductor model – direct procedures (Shih et al., 1992), optimization based procedures (Post, 2000). A number of approaches to geometry optimization of spiral inductors are proposed based on geometric programming optimization (Wenhuan & Bandler, 2006), parametric analysis (Hristov et al., 2003), etc. An approach is developed in (Durev et al., 2010) to direct parameter extraction based on the measured S-parameters. The approach gives excellent results for frequencies around the working frequency. GA based approach is used to refine the simulated S-parameters and to minimize the post extraction errors for the full investigated frequency range.

5.1. Analysis of the spiral inductor model

The physical model of spiral inductor (Yue et al., 1996) is shown in Fig. 4. The model parameters are R_s, R_{si}, C_s, C_{ox}, C_{si} and L_s. The series resistance takes into account the skin depth of the conductor. L_s is the inductance of the spiral, C_{ox} represents the capacitance between the spiral and the substrate. R_{si} and C_{si} model the resistance and the capacitance of the substrate, and C_s models the parallel-plate capacitance between the spiral and the center-tap underpass. The presented extraction procedure is based on the measured two-port *S*-parameters.

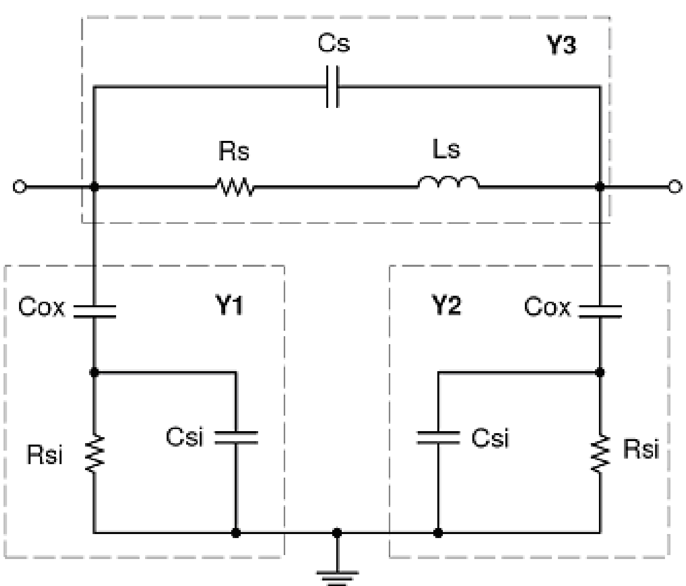

Fig. 4. Physical model of spiral inductor

As the model parameters can be easily expressed by the two-port *Y*-parameters, the measured S-parameters S_{ijm} are converted to Y-parameters Y_{ijm}, i, j = 1, 2. The parameter extraction procedure is based on determination of the admittances Y_1, Y_2 and Y_3 (Fig. 4) as a function of the two-port Y-parameters. The next step is to express the admittances Y_1 and Y_3 as well as the corresponding impedances Z_1 and Z_3 by the model parameters.

The parameters R_s , L_s , R_{si} and C_{ox} are obtained for lower frequencies ($f = f_l$). The parameter C_{si} is determined for high frequencies ($f = f_h$):

$$R_s = \Re(Z_{31}) \;; \qquad L_s = \Im(Z_{31})/\omega_l \;; \qquad R_{si} \approx \Re(Z_{11}), \tag{37}$$

$$C_{ox} \approx -\frac{1}{\omega_l \Im(Z_{11})}; \qquad C_{si} \approx -\frac{1}{\omega \Im(Z_{1h}) + \frac{1}{C_{ox}}}. \tag{38}$$

The series resistance R_{sw} is obtained for the working frequency f_w. C_s is obtained for the resonant frequency:

$$R_{sw} \approx (\omega_w L_s)^2 \Re(Y_{3w}) \;; \qquad C_s \approx \frac{L_s}{(\omega_0 L_s)^2 + R_{sw}^2}. \tag{39}$$

The series resistance R_s in Fig. 4 is frequency dependent. If the geometry of the extracted spiral inductor is known, R_s can be calculated using the formula (Yue et al., 1996):

$$R_s = \frac{l}{\sigma w \delta \left(1 - e^{-t/\delta}\right)}, \tag{40}$$

where w is the width of the metal strips, δ is the skin-effect depth into the metal layers, σ is the conductivity of metal layers, l is the length of the spiral, t is the thickness of the metal layer of the spiral (Yue et al., 1996). In case when the geometry of the spiral inductor is not known, R_s can be calculated using the formula (Ashby et al., 1994):

$$R_s = R_0 \left(1 + K_1 f^{K_2}\right), \tag{41}$$

where the coefficients K_1 and K_2 determine the frequency dependence of R_s.
The model parameters results after the application of the described direct extraction procedure are given in Table 3.

Model Param.	Extraction Results		
	(N x R x W x S) 6.5 x 60 x 14.5 x 2 f_w = 1.09GHz	(N x R x W x S) 4.5 x 60 x 14.5 x 2 f_w = 1.81GHz	(N x R x W x S) 3.5 x 60 x 9 x 7.5 f_w = 2.91GHz
R_{s0}(Ω)	7.6	5.06	5.68
R_{sw}(Ω)	9.08	6.351	5.7
L_s(nH)	11.9	5.67	3.56
C_{ox}(fF)	232.92	157.02	86.78
R_{si}(Ω)	138.62	225.48	340.28
C_{si}(fF)	123.59	79.98	67.58
C_s(fF)	45.76	96.04	152.99

* N: number of turns, R: internal radius (μm), W: metal width (μm), S: spacing (μm)

Table 3. Model parameter values after the application of the direct extraction procedure

5.2. Error Estimation

The error estimation is given in Table 4. The relative RMS error is used over the investigated frequency range between the measured and the obtained S-parameters for three different geometries spiral inductors, published in (Gil & Shin, 2003):

$$RMSErr_S = 100 \times \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{S_{jk}}{S_{jk}^{(m)}}\right)^2}, \quad (42)$$

where j, k = 1, 2; $S_{jk}^{(m)}$ – measured S-parameters (Gil & Shin, 2003);

S_{jk} – obtained S-parameters;

n – number of frequency points.

Geometry (N x R x W x S) Freq. range	RMSErrS, %	
	$\lvert S_{11}\rvert$	$\lvert S_{12}\rvert$
6.5 x 60 x 14.5 x 2 50MHz ÷ 1.3GHz	2.94	1.21
4.5 x 60 x 14.5 x 2 50MHz ÷ 2.1GHz	7.02	1.85
3.5 x 60 x 9 x 7.5 50MHz ÷ 3.2GHz	10.19	4.07

* N: number of turns, R: internal radius (μm), W: metal width (μm), S: spacing (μm)

Table 4. Error estimation of the direct extraction procedure

Because of the determination of C_s for the working frequency f_w the frequency ranges in Table 3 are limited. To enlarge the frequency ranges a GA approach is applied to optimize the model parameter values.

5.3. Optimization of the Model Parameters Based on GA

The model parameter values are varied in a certain range by the GA according to the value of its purpose function. This range is determined to be ±20% around the values from Table 3. As the capacitance C_s is determined at the working frequency, it is expected to decrease at high frequencies. This determines its broader range of variation. The purpose function minimizes the difference between the measured and the obtained Y-parameters:

$$G_{fun} = \sum_{i=1}^{n}\left|\Re\left[Y_k(f_i)\right] - \Re\left[Y_k^{(req)}(f_i)\right]\right| + \sum_{i=1}^{n}\left|\Im\left[Y_k(f_i)\right] - \Im\left[Y_k^{(req)}(f_i)\right]\right|, \quad (43)$$

where

k = 1, 3;

$Y_k(f_i)$ – current admittances;

$Y_k^{(req)}(f_i)$ – admittances obtained by S- to Y-transformation of the measured S-parameters;

n – number of frequency points.

The optimization procedure is realized using the GA Toolbox (Chipperfield et al., 1994) in MATLAB. The GA procedure has the following parameters: NIND = 300, MAXGEN = 200, NVAR = 5, PRECI = 200, GGAP = 0.7, where NIND is the number of individuals, MAXGEN is the maximal number of iterations, NVAR is the number of the optimized model parameters, PRECI is the precision factor, and GGAP is the generation gap (Chipperfield et al., 1994).
The obtained results are given in Table 5. As the model in Fig. 4 is verified up to 10GHz (Yue et al., 1996), the values of the optimized model parameters for geometries 6.5 x 60 x 14.5 x 2 and 4.5 x 60 x 14.5 x 2 preserve their dependence on the geometry of the inductor. The model parameters for geometry 3.5 x 60 x 9 x 7.5 are optimized in the frequency range 50MHz ÷ 14.4GHz and they do not preserve the dependence on the geometry of the inductor.
The comparison between the measured (Gil & Shin, 2003) and GA optimized results of S_{11} and S_{12} of 6.5 x 60 x 14.5 x 2 inductor is shown in Fig. 5. As a result of the GA optimization the obtained relative RMS error is less than 5%.

Model Param.	Extraction Results from MATLAB		
	(N x R x W x S) 6.5 x 60 x 14.5 x 2 f_w = 1.09GHz	(N x R x W x S) 4.5 x 60 x 14.5 x 2 f_w = 1.81GHz	(N x R x W x S) 3.5 x 60 x 9 x 7.5 f_w = 2.91GH
$R_{s0}(\Omega)$	Calculated using expression (40)		
L_s(nH)	11.7	5.48	3.5
C_{ox}(fF)	220	140	40
$R_{si}(\Omega)$	150	240	360
C_{si}(fF)	150	76.7	20
C_s(fF)	6.22	20	15.7

* N: number of turns, R: internal radius (μm), W: metal width (μm), S: spacing (μm)

Table 5. Model parameter values after the GA optimization in MATLAB

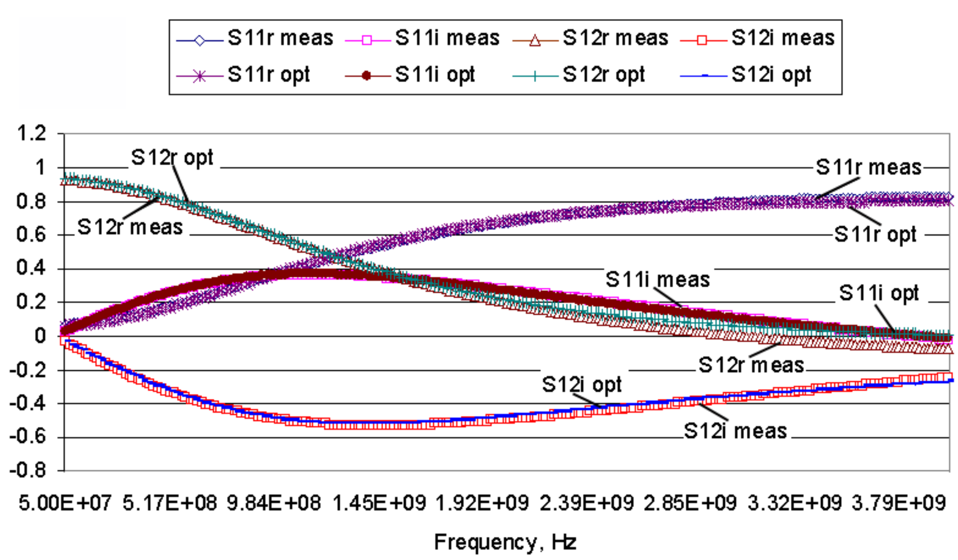

Fig. 5. Comparison between the measured (Gil & Shin, 2003) and GA optimized S-parameters of 6.5 x 60 x 14.5 x 2 inductor

6. Optimization of geometric parameters of spiral inductors using genetic algorithms in MATLAB

Based on the physical model for planar spiral inductors (Yue et al., 1996; Sieiro et al., 2002; Nieuwoudt et al., 2005) and the simple accurate expressions for the inductance (Mohan et al., 1999), a geometry optimization procedure is proposed in (Post, 2000). It allows the obtaining optimal trace width that maximizes the quality factor of the spiral inductor with a required inductance at a given frequency. Optimal design of the parameters of spiral inductors is proposed in (Gadjeva & Hristov, 2004), based on PSpice model and parametric analysis. Computer models are developed in (Gadjeva et al., 2006) using the possibilities of MATLAB and GA toolbox (Chipperfield et al., 1994). GA program designed using GA toolbox in MATLAB is used for optimization the geometric parameters of spiral inductors.

6.1. Optimal design of spiral inductors using GA

The optimal design of the spiral inductors is based on a precise mathematical model, which consists of multiple parameters – dependent and independent. The independent geometry parameters characterizing the spiral inductor are the number of turns n, the trace width w, the spacing sp, and the outer diameter D_{out} (Fig. 6).

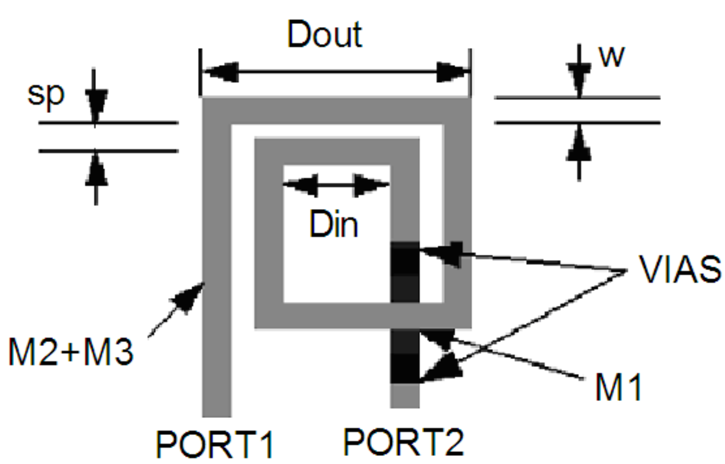

Fig. 6. The geometry parameters of a square spiral inductor

Each of the independent parameters has its own value range, which depends on the used microelectronic technology. The independent geometry parameters - number of turns n, the trace width w, the spacing sp, and the outer diameter D_{out} are fixed in the range:

n = 7 ± 50%;
w = 13e-6 ± 50%;
sp = 7e-6 ± 50%;
Dout = 300e-6 ± 50%;

The variables PERL and PERU fix the variation range to ± 50%.
In MATLAB this is done in the following way (Chipperfield et al., 1994):
%Variation percent

```
PERL = 0.5; % lower limit
PERU = 1.5; % upper limit
%       %Dout              w                 sp               n
FieldD = [PRECI            PRECI             PRECI            PRECI;
        300.0e-6*PERL     13.0e-6*PERL      7.0e-6*PERL      7.0*PERL;
        300.0e-6*PERU     13.0e-6*PERU      7.0e-6*PERU      7.0*PERU;
        1                 1                 1                1;
        0                 0                 0                0;
        1                 1                 1                1;
        1                 1                 1                1];
```

Based on these parameters, the dependent inductor parameters are calculated: the inner diameter D_{in}, the average diameter $D_{avg} = 0.5(D_{out} + D_{in})$, the trace length $l = 4nD_{avg}$, the area (Mohan et al., 1999; Post, 2000). The computer-aided design model is based on the two-port equivalent circuit of the spiral inductor shown in Fig. 4 (Yue et al., 1996). The parameters R_s, R_{si}, C_s, C_{ox} and C_{si} are in the form (Yue et al., 1996):

$$R_s = \frac{l}{\sigma w \delta\left(1 - e^{-t/\delta}\right)}; \qquad \delta = \sqrt{\frac{2}{\omega \sigma \mu_0}}, \tag{44}$$

$$C_s = \frac{\varepsilon_{ox} n w^2}{t_{oxM1,M2}}; \qquad C_{ox} = \frac{\varepsilon_{ox} l w}{2 t_{ox}}, \tag{45}$$

$$R_{si} = \frac{2}{G_{sub} l w}; \qquad C_{si} = \frac{C_{sub} l w}{2}, \tag{46}$$

where σ is the metal conductivity at dc, t is the metal thickness, δ is the metal skin depth, t_{ox} is the oxide thickness between spiral and substrate, $t_{ox\ M1\ M2}$ is the oxide thickness between spiral and centertap, l is the overall length of spiral, w is the line width, C_{sub} is the ubstrate capacitance per unit area, and G_{sub} is the substrate conductance per unit area.
The parallel equivalent circuit of the spiral inductor shown in Fig. 7 is used for calculating the quality factor Q of the inductor (Mohan et al., 1999):

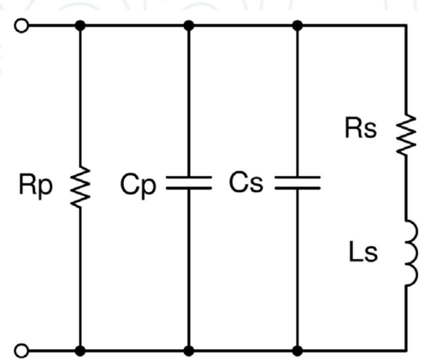

Fig. 7. The parallel equivalent circuit of the spiral inductor

$$Q = \frac{\omega L_s}{R_s} \cdot \frac{R_p}{R_p + \left[(\omega L_s / R_s)^2 + 1\right] R_s} \cdot \left[1 - \frac{R_s^2 (C_s + C_p)}{L_s} - \omega^2 L_s (C_s + C_p)\right], \quad (47)$$

where

$$R_p = \frac{1}{\omega^2 C_{ox}^2 R_{si}} + R_{si}\left(1 + \frac{C_{si}}{C_{ox}}\right)^2, \quad (48)$$

$$C_p = C_{ox} \frac{1 + \omega^2 (C_{ox} + C_{si}) C_{si} R_{si}^2}{1 + \omega^2 (C_{ox} + C_{si})^2 R_{si}^2} \quad . \quad (49)$$

The following monomial expression (Post, 2000) is used to model the inductance L_s:

$$L_s = \beta D_{out}^{\alpha 1} w^{\alpha 2} D_{avg}^{\alpha 3} n^{\alpha 4} sp^{\alpha 5} . \quad (50)$$

The dimensions are in μm and the inductance is in nH. The coefficients β, α_i , i = 1,2,...,5 depend on the inductor shape – square, hexagonal and octagonal (Mohan et al., 1999). The data for square inductor are β =1.62e-3, α_1= –1.21, α_2 = – 0.147, α_3 =2.4, α_4 =1.78, α_5 = – 0.03.
The expression ensures good accuracy and agreement between the calculated inductor value and the measured one (Mohan et al., 1999; Post, 2000).
The required inductance L_{sreq}, the frequency f_s , the technological parameters and the coefficients β, α_i , i = 1,2,...,5, are introduced as input data in MATLAB m-file. The optimization was done for L_{sreq} = 7.28 nH and f_s = 2 GHz.
The object function finds the global minimum g_fun for its expression:

$$g\_fun = Q_{req} + W.|L_s - L_{sreq}| , \quad (51)$$

where Q_{req} = 1/Q and W is a weighting coefficient. The minimum value of g_fun guarantees the maximal value for the Q-factor for Lsreq = 7.28 nH. The implementation of the procedure is done in the following way:

1. Introducing the input data:

```
mju = 1.256e-6;  beta = 1.62e-3;
al1 = -1.21;  al2 = -0.147;   al3 = 2.4;
al4 = 1.78;  al5 = -0.03;  Eox = 3.45e-11;
toxM1M2 = 1.3e-6;
tox = 4.5e-6;  Csub = 1.6e-6;
Gsub = 4.0e4; sigma = 1/3e-8;
ro_spec = 1/sigma;  t = 1e-6;
frequency = 2e9; %2GHz
math_pi=3.1415965;  omg = 2*math_pi*frequency;
delta = sqrt(2.0/(omg*mju*sigma));
Lsreq = 7.28e-9;
```

2. Explicitly enter the independent geometric parameters in order to be recognized from the GA:

```
Dout = Chrom(:,1)
w = Chrom(:,2)
sp = Chrom(:,3)
n = Chrom(:,4)
```

3. Enter the sequence of calcuations in order to obtain the members which take part in the expression for the objective function:

```
Din = (Dout - 2.0.*(n.*(sp + w) - sp));
Davg = 0.5.*(Dout + Din);
L = 4.0.*n.*Davg;
Cs = (n.*(w.^2).*Eox)./toxM1M2;
delta = sqrt(2.0/(omg*mju*sigma));
Cox = (0.5.*L.*w.*Eox)./tox;
Csi = 0.5.*L.*w.*Csub;
Rsi = 2.0./(L.*w.*Gsub);
Rs = L./(w.*sigma.*delta.*(1.0 - exp(-t./delta)));
Cs = (n.*(w.^2).*Eox)./toxM1M2;
Rp = (1./(omg.^2.*Cox.^2.*Rsi)) + (Rsi.*(Cox + Csi).^2)./Cox.^2;
Cp = Cox.*((1 + omg.^2.*(Cox + Csi).*Csi.*Rsi.^2)./(1 + omg.^2.*(Cox + Csi).^2.*Rsi.^2));
Ls = beta*((Dout*1.0e6).^al1).*((w*1.0e6).^al2).*((Davg*1.0e6).^al3).*(n.^al4).*
((sp*1.0e6).^al5).*1.0e-9;
Q = (omg.*Ls./Rs).*(Rp./(Rp + ((omg.*Ls./Rs).^2 + 1).*Rs)).*(1 - Rs.^2.*(Cs + Cp)./Ls -
omg.^2.*Ls.*(Cs + Cp));
Qrec = abs(1.0./Q);
```

4. Calculation the objective function:

```
g_fun = Qrec + 10e8*abs(Ls - Lsreq);
```

Fig. 8 represents the optimization of the Q-factor using the genetic algorithm in MATLAB, described above. After some itterations the GA finds the global minimum for its objective function, which gives the optimized value for the Q-factor.
The optimized geometry parameters using MATLAB GA are: sp = 7.99 μm , w = 13.15 μm , n = 3.74, Dout = 365.64 μm .
The GA procedure has the following parameters: NIND = 100, MAXGEN = 100, NVAR = 4, PRECI = 100, GGAP = 0.7. The obtained results are in agreement with the simulated and test results obtained in (Mohan et al., 1999; Post, 2000; Gadjeva & Hristov, 2004).

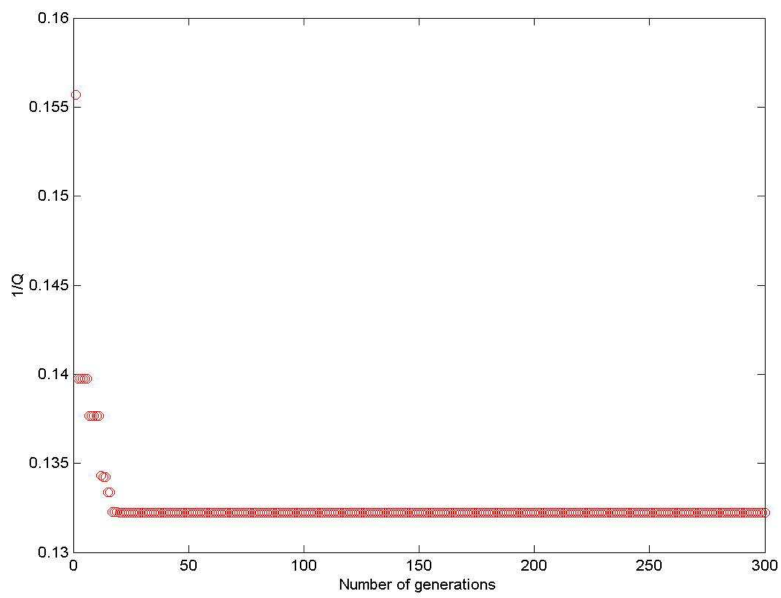

Fig. 8. Optimization of the Q-factor of the spiral inductor using GA in MATLAB

7. Conclusion

The extended possibilities of the general-purpose software MATLAB for modeling, simulation and optimization can be successfully used in RF microelectronic circuit design. Based on description of the device models, various optimization problems can be solved. Automated model parameter extraction procedure for on-chip wide-band spiral inductor model has been developed and realized in the MATLAB environment. The obtained results for the simulated two-port Y- and S-parameters of the spiral inductor model using extracted parameters are compared with the measurement data. The achieved maximal relative error is less than 0.5% which makes the developed parameter extraction procedure of the on-chip spiral inductor model very accurate.

Based on genetic algorithm and GA tool in MATLAB, optimization of geometric parameters of planar spiral inductors for RF applications is performed with respect to the quality factor Q. The optimization maximizes the Q-factor for a given value range of the input independent geometric parameters. The methodology is useful in microelectronics, as every mathematical technological model can be analysed in similar way, which gives an advantage in solving complex problems, based on the technology parameters optimization.

The automated approaches to model parameter extraction and optimizaton of on-chip spiral inductors in the MATLAB environment are universal and flexible and can be similarly applied to various passive and active microelectronic components such as planar transformers, MOSFETs, heterojunction transistors (HBT), etc.

The rich possibilities for analysis and optimization of MATLAB are of great importance in the design process of RF circuits at component and system level.

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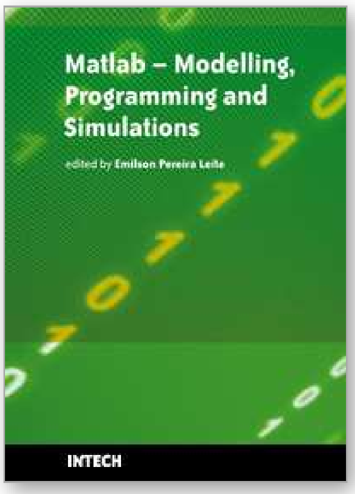

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This book is a collection of 19 excellent works presenting different applications of several MATLAB tools that can be used for educational, scientific and engineering purposes. Chapters include tips and tricks for programming and developing Graphical User Interfaces (GUIs), power system analysis, control systems design, system modelling and simulations, parallel processing, optimization, signal and image processing, finite different solutions, geosciences and portfolio insurance. Thus, readers from a range of professional fields will benefit from its content.