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Stochastic wind profiles determination for radioactive substances released from nuclear power plants

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Abstract
In this review we discuss a stochastic turbulent wind profile based on the three-dimensional stochastic Langevin equation for Gram-Chalier probability density function and a known mean wind velocity. Its solution permits to simulate radioactive substances dispersion in a turbulent regime, which is of interest for nuclear reactor accident scenarios and their related emergency actions. We discuss the stochastic Langevin equation together with an analytical method for solving the three-dimensional and time dependent equation which is then applied to radioactive substance dispersion for a stochastic turbulence model. The solution is obtained using the Adomian Decomposition Method, which provides a direct scheme for solving the problem without the need for linearisation and any transformation. The results of the model are compared to case studies with measured data and further compared to procedures and predictions from other approaches.

1. Introduction

Increasing energy demand and the related climate problem has beside other options reawaken nuclear energy as one possible pathway out of the as problematic predicted future perspectives, such as electricity shortages, fossil fuel price increases, global warming and heavy metal emissions from fossil fuel use among others. Estimates indicate that within the next two decades electrical energy consumption will double, which implies an increase in nuclear power plants. Experience gathered along the nuclear history has sharpened the rules and regulations that lead to the commissioning of latest generation nuclear technology. One of the issues is the choice of the site considering meteorological aspects as well as possible accident scenarios and their related emergency actions. In this line the following contribution focuses on the question of radioactive material dispersion after discharge from a nuclear power plant. The atmosphere is considered the principal vehicle by which radioactive materials that are either released from a nuclear power plant in experimental or eventually in accidental events could be dispersed in the environment and result in radiation exposure of plants, animals and last not least humans. Thus, the evaluation of airborne radioactive material transport in the atmosphere is one of the requirements for design and licensing of a nuclear power plant. In order to analyse the (possible) consequences of radioactive discharge atmospheric dispersion models are of need, which have to be tuned using specific meteorological parameters and conditions in the considered region. Moreover, they shall be subject to the local orography and supply with realistic information on radiological consequences of routine discharges and potential accidental releases of radioactive substances. Furthermore, case studies by model simulations may be used to establish limits for escape of radioactive material from the power plant into the atmosphere.

To this end in the present study, the wind profile with its turbulent properties for the different stability regimen in the planetary boundary layer are determined using the non-linear stochastic Langevin equation for a known average wind velocity field and probability density functions depending on the regimen in consideration. We show how the model is solved analytically using the decomposition method which then may provide short, intermediate and long term (normalized) concentrations and permit to assess the probability of occurrence of high contamination level case studies of accidental scenarios and additionally serve as a supplement for designing emergency response plans. The stochastic character of the process is implemented using the appropriate Gaussian, bi-Gaussian and Gram-Chalier probability distributions for the different stabilities. Exactness of the solution is manifest in stable convergence, which we control by a Lyapunov theory inspired criterion. The most adequate probability distribution is indicated by a novel statistical validation index, which from comparison to experimental data selects the most significant model. Comparison to the Copenhagen and to other deterministic approaches shows the advantage of the present analytical approach even for considerably rugged land relieves.

The stochastic character of turbulence is implemented using the Gram-Chalier probability distribution. Exactness of the solution is manifest in stable convergence, which we control by a Lyapunov theory inspired criterion. The most adequate probability distribution for the wind scenario of interest is indicated by a novel statistical validation index, which from comparison to experimental data selects the most significant approach. Comparison to the Copenhagen and to other deterministic approaches shows the advantage of the present analytical approach even for considerably rugged land relieves.

Our chapter is organised as follows. In section 2 we report on the state of the art of stochastic wind profile modelling, and show how a closed form solution may be obtained by Adomian's decomposition method. In 3, we present the numerical results for three probability density functions and in section 4 we come to our conclusions.

2. Stochastic Wind Profile Modelling

Dispersion of radiactive material in the planetary boundary layer is a stochastic process and thus obeys a stochastic law which may be expressed as a set of stochastic differential equations. For a time dependent regime considered in the present work, we assume that the associated Langevin equation adequately describes such a dispersion process, which we test by comparison to other methods in order to pin down computational errors and finally analyse for model adequacy. We are aware of the fact that up to date there do exist a variety of models and approaches to the problem, either based on numerical schemes, stochastic simulations or (semi-)analytical approaches and indicate in the further a selection of models. Numerical approaches may be found in the works of Tangerman Tangerman (1978), Brebbia Brebbia (1981), Chock et al. Chock et al. (1996), Sharan et al. Sharan et al. (1997) and Huebner et al. Huebner et al. (2001). There are various models that have been used effectively in the past to describe tracer dispersion Zannetti (1990), Seinfeld and Pandis (1998), Arya (2003), Arya (1999), and many of them make use of analytical approaches Lin and Hildemann (1997); Seinfeld and Pandis (1998); Sharan et al. (2003). One also finds semi-analytical methods, where we mention the works of Parlange Parlange (1971), Dike Dike (1975), Henry et al. Henry et al. (1991), Grisogono and Oerlemans Grisogono and Oerlemans (2001), Metha and Yadav Metha and Yadav (2003), Carvalho et al. Carvalho et al. (2005a) and Carvalho and Vilhena Carvalho and Vilhena (2005).

Upon developing a model one typically faces various problems. First one has to identify a differential equation that shall represent a model or a physical law. Once the law/model is accepted as the fundamental equation one challenges the task of solving the equation in many cases approximately and analyse the error of approximation and numerical errors in order to validate its prediction against experimental data. Experimental data of a stochastic process typically spread around average values, i.e. are distributed according to probability distributions. Hence, the model shall within certain limits reproduce the experimental findings. Since the fundamental equation is already a simplification deviations may occur which in general have their origin in a model error superimposed by numerical or approximation based errors. In case of a genuine convergence criterion one may pin down the error analysis essentially to a model validation. Since in general convergence is handled by heuristic convergence criteria, a model validation is not that obvious. Thus we show with the present discussion, that our semi-analytical approach does not only yield an acceptable solution to the Langevin equation but predicts tracer concentrations closer to observed values which is also manifest in the statistical analysis.

Simulation of substance dispersion in the Planetary Boundary Layer (PBL) through a Lagrangian particle model by the use of the Langevin equation and its diffusion equation limit (random displacement equation) usually has been solved by the method of Itō calculus Gardiner (1985); Rodean (1996). More recently one of the authors developed the Iterative Langevin Solution (ILS) method, which solves the Langevin equation in a semi-analytical manner by the method of successive approximations, known as Picard's iterative method. The method is principally characterised by the following steps: Application of Picard's procedure on the Langevin equation, to be more specific, integration, linearisation of the stochastic non-linear term and iterative solution of the resultant equation, which permits to evaluate an analytical expression for the velocity. Details of this approach may be found elsewhere Carvalho and Vilhena (2005); Carvalho et al. (2005a;b; 2007a;b); Szinvelski et al. (2006).

An alternative analytical method for solving linear and non-linear differential equations was developed by Adomian Adomian (1988), known as the decomposition method. The decomposition procedure permits to cast the solution into a convergent series by using the necessary number of iterations for both linear and non-linear deterministic and stochastic equations. The advantage of this method is that it provides a direct scheme for solving the problem without the need for linearisation or transformations. There exists a vast literature about applications of this method to a broad class of physical problems and we cite the works we considered relevant for the further discussion Adomian (1988; 1994; 1996); Dehghan (2004); El-Wakil et al. (2006); Eugene (1993); Inc (2004); Laffer and Abbaoui (1996). Thus the present work extends the list of methods that solve the Langevin equation assuming a Gram-Chalier turbulence condition by Adomian's approach. The variety of methods, numerical solution of the Langevin equation (integrated according to the Itō calculus), analytical solution of the Langevin equation (derivation of Uhlenbeck and Ornstein Uhlenbeck and Ornstein (1930)), Iterative Langevin Solution (ILS) and solution by decomposition (ADM) is validated considering the measured data of ground-level concentration from a tracer experiment Gryning and Lyck (1984).

2.1 The decomposition method for a stochastic process

A time dependent stochastic process μ is typically characterized by its time evolution, which depends on stochastic contributions, such as expectation values (E_n) of mean field character (E_0) and higher moments (here E_2), respectively. In our case we consider the Langevin equation to describe turbulence.

$$\mu(t+\tau) - \mu(t) = \int_t^{t+\tau} E_0(\mu(t'), t')dt' + \int_t^{t+\tau} \left(E_2(\mu(t'), t')\right)^{\frac{1}{2}} d\Sigma(t') \tag{1}$$

Here $d\Sigma$ is a stochastic measure for random motion and E_0 represents a drift like term, whereas E_2 is a measure for diffusion intensity, which satisfy the usual Lipschitz continuity condition in order to ensure the existence of a unique strong solution. In case of a Wiener process $\mu(t)$ is Markovian, but in our case we presume that the process is an Itō process, i.e. it depends on the present and previous values, hence the integral form of mean field and fluctuation contributions. Note, that the integral form will be used further down in order to set-up the solution following Adomian's prescription, which we resume in the following.
One may rewrite the stochastic equation from above (1) as a differential equation, upon using the limit $\tau \to 0$ and separating all terms depending on the process μ including the differential operator (LHS of equation (2)) from the noise generating term $G(t)$ (the stochastic contribution, last term in eq. (1)).

$$\mathcal{L}[\mu(t)] = \mathcal{L}_L[\mu(t)] + \mathcal{L}_N[\mu(t)] = G(t) \tag{2}$$

According to Adomian, one splits the linear operator, that includes the derivatives $\mathcal{L}_L$ with known inversion from the non-linear terms $\mathcal{L}_N$. Further we write $\mu(t)$ as a sum of a convergent sequence $\mu_i(t)$, still to be specified, and the non-linear term is cast into a sum of so called Adomian functional polynomials Adomian (1988).

$$\mu(t) = \sum_{i=0}^{\infty} \mu_i(t) \qquad\qquad \mathcal{L}_N[\mu] = \sum_{i=0}^{\infty} A_i \tag{3}$$

For the non linear part we use a normal convergent operator expansion

$$\frac{\partial^m}{\partial \mu^m}\left(\mathcal{L}_N[\mu]\right) = \frac{\partial^m F}{\partial \mu^m} = F^{(m)} \tag{4}$$

and rewrite the non-linear term as

$$\begin{aligned}
\mathcal{L}_N[\mu] &= \sum_{n=0}^{\infty} \frac{1}{n!} \underbrace{\left.\frac{\partial^n F}{\partial \mu^n}\right|_{\mu=\mu_0}}_{F_0^{(n)}} \left(\sum_{m=1}^{\infty} \mu_m \right)^n \\
&= \lim_{r\to\infty} \sum_{n=0}^{\infty} \frac{1}{n!} F_0^{(n)} \sum_{\substack{k_1,\cdots,k_r \\ \Sigma k_i = n}} \left(\binom{n}{\{k_i\}_1^r} \prod_{m=1}^{r} \mu_m^{k_m} \right) \\
&= F_0^{(0)} + \sum_{n=1}^{\infty} \left(F_0^{(1)} \mu_n + \sum_{j=2}^{n} \frac{1}{j!} F_0^{(j)} \sum_{\substack{k_1,\cdots,k_{n-1} \\ \Sigma k_i = j}} \left(\binom{j}{\{k_i\}_1^{n-1}} \prod_{m=1}^{n-1} \mu_m^{k_m} \right) \right)
\end{aligned} \tag{5}$$

where we introduced the shorthand notations for the derivative terms $F_0^{(n)}$ and the polynomial coefficients $\binom{n}{\{k_i\}_1^r} = \binom{n}{k_1,\ldots,k_r}$. Introducing these terms into the original differential equation permits to identify corresponding terms, that give rise to the iterative scheme in the spirit of Adomian as shown next.

$$\begin{aligned}
&\sum_{i=0}^{\infty} \mathcal{L}_L[\mu_i(t)] = G(t) - \\
&\quad -F_0^{(0)} - \sum_{n=1}^{\infty} \left(F_0^{(1)} \mu_n + \sum_{j=2}^{n} \frac{1}{j!} F_0^{(j)} \sum_{\substack{k_1,\cdots,k_{n-1} \\ \Sigma k_i = j}} \left(\binom{j}{\{k_i\}_1^{n-1}} \prod_{m=1}^{n-1} \mu_m^{k_m} \right) \right)
\end{aligned} \tag{6}$$

There are many possibilities to set up an iterative scheme which upon truncation to n terms in A_n and $n+1$ terms in μ_n yields an approximate solution in analytical form. Instead of solving the original Langevin equation we cast the problem into a set of simpler equations which may be solved because the integral operator $\mathcal{L}_L^{-1}$ is known.

$$\begin{aligned}
\mathcal{L}_L[\mu_0] &= G \\
\mathcal{L}_L[\mu_1] &= -A_0 = -F_0^{(0)} \\
\mathcal{L}_L[\mu_2] &= -A_1 = -F_0^{(1)} \mu_1 \\
\mathcal{L}_L[\mu_3] &= -A_2 = -F_0^{(1)} \mu_2 - \frac{1}{2} F_0^{(2)} \mu_1^2 \\
&\vdots \\
\mathcal{L}_L[\mu_{n+1}] &= -A_n = -F_0^{(1)} \mu_n - \sum_{j=2}^{n} \frac{1}{j!} F_0^{(j)} \sum_{\substack{k_1,\cdots,k_{n-1} \\ \Sigma k_i = j}} \left(\binom{j}{\{k_i\}_1^{n-1}} \prod_{m=1}^{n-1} \mu_m^{k_m} \right) \\
&\vdots
\end{aligned} \tag{7}$$

The way we have set up the iterative scheme defines the seed $\mu_0(t)$ of the functions iteration by the stochastic contribution as source term, whereas the remaining iterators are simply given by the Adomian functional polynomials as source terms of the equations to be solved. Note that in order to evaluate the i-th iteration step μ_i the μ_j with $j < i$ are known from the previous iteration steps. Moreover, the functional expansion of the non-linear term around the function μ_0 shows how the stochastic term effectively enters in the remaining terms μ_i with $i > 0$ from the non-linearity.

2.2 A convergent closed form solution

The iteration defines a convergent series towards μ for all t in a certain domain, thus the solution $\mu = \lim_{n\to\infty} \sum_{i=0}^{n} \mu_i$ is manifest exact. Since this scheme defines an explicit analytical expressions for the μ_i and A_i, respectively, one arrives at a procedure which permits to solve the differential equation without linearisation in closed form. The procedure has been applied to a variety of non-linear problems but an analytical procedure for testing convergence to the best of our knowledge has not been presented in literature, only numerical schemes may be found, see for instance refs. Inc (2004) and Aminataei and Hosseini (2007).

In general convergence is not guaranteed by the decomposition method, so that the solution shall be tested by a convenient criterion. Since standard convergence criteria do not apply for the present case due to the non-linearity and stochastic character, we present a method which is based on the reasoning of LyapunovBoichenko et al. (2005). While Lyapunov introduced this conception in order to test the influence of variations of the initial condition on the solution, we use a similar procedure to test the stability of convergence while starting from an approximate (initial) solution μ_0 (the seed of the iteration scheme).

Let us denote $|\delta Z_n| = \| \sum_{i=n+1}^{\infty} \mu_i \|$ the maximum deviation of the correct from the approximate solution $\Gamma_n = \sum_{i=0}^{n} \mu_i$, where $\| \cdot \|$ signifies the maximum norm. Then convergence occurs if there exists an n_0 such that the sign of λ is negative for all $n \geq n_0$.

$$\lambda = \frac{1}{\|\Gamma_n\|} \log \left(\frac{|\delta Z_n|}{|\delta Z_0|} \right) \tag{8}$$

In the further we apply the decomposition method as presented in general form above to the problem of tracer dispersion for three different turbulence probability density functions, i.e. Gaussian, bi-Gaussian and Gram-Chalier, respectively. The analysis of convergence is applied to all cases that shows that for $n_0 = 4$ the approach is convergent with an error less than 1%.

3. The Langevin Equation for Stochastic Turbulence

The stochastic equation (1) may be interpreted in terms of the Langevin equation, where μ represents the turbulent velocity vector with components u_i. In the Langevin equation Rodean (1996) the time evolution of the turbulent velocity is driven by a dissipative term and a second term which may be understood as the gradient of a potential that depends on the fluctuations of the turbulent velocity and represents a mean field interaction of the pollutant with the environment it is immersed. The last term represents the stochastic contribution due to a continuous series of particle collisions.

$$\frac{du_i}{dt} + \alpha_i u_i = \beta_i + \gamma_i u_i^2 + (C_0 \varepsilon)^{\frac{1}{2}} \xi_i(t) \, . \tag{9}$$

Here u_i with $i = 1, 2, 3$ is a Cartesian component of the turbulent velocity, which is related to the infinitesimal displacement and the wind velocity U_i by $dx_i = (U_i + u_i)dt$. The coefficients

$\alpha_i, \beta_i, \gamma_i$ of eq. (9) depend on the employed probability density function. Here C_0 is the Kolmogorov constant, ε is the rate of turbulence kinetic energy dissipation, and ξ_i is a random increment according to a probability density function.
Upon application of the described decomposition method from above (see 2.1) on equation (9), the turbulent velocity is decomposed into a series and the non-linear contribution is taken care of by Adomian's procedure.

$$\frac{d}{dt}\left(\sum_{n=0}^{\infty} u_{i,n}\right) + \alpha_i \left(\sum_{n=0}^{\infty} u_{i,n}\right) = \beta_i + (C_0\varepsilon)^{\frac{1}{2}}\xi_i(t) + \gamma_i \left(\sum_{n=0}^{\infty} A_{i,n}\right) , \tag{10}$$

where the non-linear term is $\sum_{n=0}^{\infty} A_{i,n} = u_i^2$.
In the iterative scheme the stochastic component is absorbed in the first term u_0 of the expansion and thus propagates through all subsequent terms, whereas the non-linear (mean field) term enters as a correction from the second term on. For any given truncation m the solution for the considered problem (9) is given in closed analytical form summing up the terms $\sum_{n=0}^{m} u_{i,n}$.
So far we have not defined the probability density function, that characterizes the type of turbulence which is correlated to the stability of the planetary boundary layer (PBL). In the studies of turbulent dispersion the stochastic behaviour may be classified according to stationarity or non-stationarity, according to spatial properties as homogeneity or non-homogeneity and according to the profile of the wind distribution, as Gaussian or non-Gaussian. When employing Lagrangian models one usually considers stationary and homogeneous turbulence in horizontal sheets and non-homogeneous and either Gaussian or non-Gaussian in the vertical direction depending on the stability condition. In stable or neutral conditions the velocity distribution may be considered Gaussian, whereas during convective conditions the velocity distribution is non-Gaussian because of the skewness of the turbulent velocity distribution, which has its origin in up- and down-drafts with different intensity. In the following we present the solutions for the Gram-Chalier probability density functions together with their model validation against the data from the Copenhagen experiment Gryning and Lyck (1984).

3.1 The Copenhagen experiment

The Copenhagen tracer experiment Gryning and Lyck (1984) was carried out in the northern part of Copenhagen. A tracer (SF_6) was released without buoyancy from a tower at a height of $115m$ and collected at the ground-level positions in up to three crosswind arcs of tracer sampling units. The sampling units were positioned $2km - 6km$ from the point of release. A total of nine tracer experiment runs were performed in stability conditions as shown in table 1. The site was mainly residential with a roughness length of $0.6m$. Wind speeds at 10 and 115 meters were used to calculate the coefficient for the vertical exponential wind profile, which is used to model the wind speed.

$$\begin{aligned} \chi &= \left[\frac{\log\left(\frac{U(115)}{U(10)}\right)}{\log\left(\frac{115}{10}\right)}\right] \\ U(z) &= U(0)\left[\frac{z}{10}\right]^{\chi} \end{aligned} \tag{11}$$

where $U(10)$ is the wind speed in $10m$ and $U(115)$ is the wind speed in $115m$, respectively.

Run	L (m)	z_i (m/s)	u_* (m/s)	w_* (m/s)	$U(10)$ (m/s)	$U(115)$ m	h
1	-37	1980	0.36	1.8	2.1	3.4	1980
2	-292	1920	0.73	1.8	4.9	10.6	1920
3	-71	1120	0.38	1.3	2.4	5.0	1120
4	-133	390	0.38	0.7	2.5	4.6	390
5	-444	820	0.45	0.7	3.1	6.7	820
6	-432	1300	1.05	2.0	7.2	13.2	1300
7	-104	1850	0.64	2.2	4.1	7.6	1850
8	-56	810	0.69	2.2	4.2	9.4	810
9	-289	2090	0.75	1.9	5.1	10.5	2090

Table 1. Meteorological parameters measured during the Copenhagen experiment. L is the Monin-Obukohv length, z_i the convective boundary layer height, u_* is the local friction velocity, w_* is the convective velocity scale, $U(10)$ is the wind speed in $10m$ and $U(115)$ is the wind speed in $115m$ and h is the PBL height.

For the simulations, the turbulent flow is assumed inhomogeneous only in the vertical direction and the transport is realized by the longitudinal component of the mean wind velocity. The horizontal domain was determined according to sampler distances and the vertical domain was set equal to the observed PBL height. The time step was maintained constant and was obtained according to the value of the Lagrangian decorrelation time scale ($\Delta t = \tau_L / c$), where τ_L must be the smaller value among τ_{L_u}, τ_{L_v}, τ_{L_w} and C is an empirical coefficient set equal to 10. In Equation (10), the product $C_0\varepsilon$ is calculated in terms of the turbulent velocity variance σ_i^2 and the Lagrangian decorrelation time scale τ_{L_i} Hinze (1986); Tennekes (1982), which are parametrised according to a scheme developed by Degrazia et al. (Degrazia et al. (2000)). These parametrisations are based on Taylor's statistical diffusion theory and the observed spectral properties. The concentration field is determined by counting the particles in a cell or imaginary volume in the position x, y, z. The integration eq. (10) was computed by the Romberg method.

3.2 Solution for Gaussian turbulence

In the case where a Gaussian probability density function describes best the stochastic turbulence the coefficients of the Langevin equation (9) and (10) are

$$\alpha_i = \frac{C_0\varepsilon}{2\sigma_i^2}\,, \qquad \beta_i = \frac{1}{2}\frac{\partial\sigma_i^2}{\partial x_j}\,, \qquad \gamma_i = \frac{1}{2\sigma_i^2}\left(\frac{\partial\sigma_i^2}{\partial x_j}\right)\,. \tag{12}$$

In Table (2) we compare the experimental findings with the model predictions by the proposed procedure (ADM - Adomian Decomposition Method), by the Itō method Rodean (1996), by the ILS method Carvalho and Vilhena (2005) and the early analytical derivation (ANA) by Uhlenbeck and Ornstein Uhlenbeck and Ornstein (1930). From the comparison one observes a reasonable agreement among the models and also with the experimental data. In the following table the numerical convergence of the ADM approach for a Gaussian probability density function (pdf) is indicated. The convergence analysis shows that already a few terms represent an analytical solution with spurious error only. The figures 1 show the Lyapunov exponent of

Exp.	Distance (m)	Observed $(\mu g m^{-2})$	Predictions C_y $(\mu g m^{-2})$ ADM	ILS	Itō	ANA
1	1900	2074	2092	2770	1486	2320
1	3700	739	1281	725	1001	2026
2	2100	1722	1496	1699	1344	1290
2	4200	944	850	1489	1117	1059
3	1900	2624	2601	2710	2415	2366
3	3700	1990	1605	2136	1649	2066
3	5400	1376	1273	1328	1073	2062
4	4000	2682	2379	2726	1947	1565
5	2100	2150	2586	2138	2042	2090
5	4200	1869	1818	2484	1967	1701
5	6100	1590	1568	2206	1690	1819
6	2000	1228	951	915	872	853
6	4200	688	619	775	718	651
6	5900	567	488	673	612	622
7	2000	1608	1172	1606	1015	1320
7	4100	780	680	1290	660	1145
7	5300	535	554	933	548	1170
8	1900	1248	1228	1252	1099	726
8	3600	606	723	522	887	667
8	5300	456	489	416	737	682
9	2100	1511	1433	1660	1330	1334
9	4200	1026	884	1135	1162	1068
9	6000	855	630	894	962	1115

Table 2. Concentrations of nine runs with various positions of the Copenhagen experiment and model prediction by the approaches ADM, ILS, Itō and ANA, using a Gaussian probability density function.

the Adomian approach depending on the number of terms for the 9 experimental runs. Note, that the more negative the exponent λ the more stable is convergence. Figure (2) shows the dispersion of the Copenhagen experimental data in comparison with their model predictions by ADM, Itō, ILS, ANA. Note, that the closer the data are grouped to the bisector the better is the agreement between experiment and prediction.
In figure 3 we show the linear regression of each model, where the closer their intersect is to the origin and the closer the slope is to unity the better is the approach. By comparison one observes that the present approach yields the best description of the data. Details of the regression may be found in table 4. In order to perform a model validation we introduce an index κ which if identical zero there is a perfect match between the model and the experimental findings.

$$\kappa = \sqrt{(a-1)^2 + \left(\frac{b}{\bar{C}_o}\right)^2} \qquad \text{with} \qquad \bar{C}_o = \frac{1}{n}\sum_{i=1}^{n} C_{oi} \tag{13}$$

Here a is the slope, b the intersection, C_{oi} the experimental data and $\bar{C}_o$ the arithmetic mean. Since both the experiment and the model are of stochastic character, fluctuations are present,

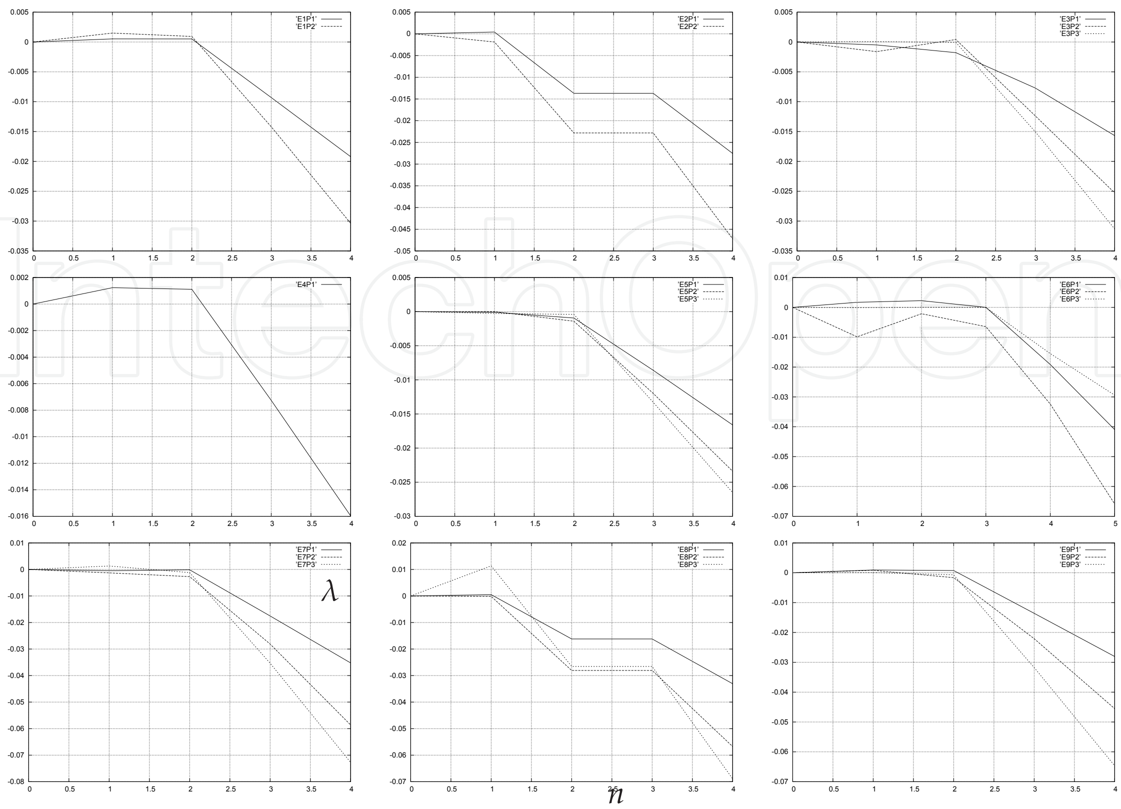

Fig. 1. Lyapunov exponent λ of the Adomian approach depending on the number n of terms for the 9 experimental runs.

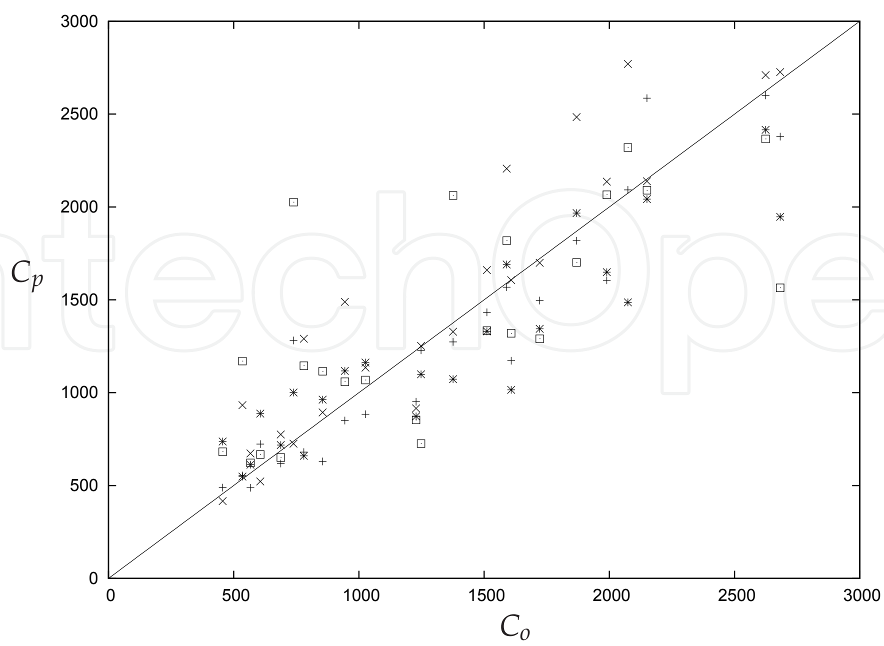

Fig. 2. Dispersion diagram of predicted (C_p) against measured values (C_p) by ADM ($+$), ILS ($\times$), Itō ($\ast$), ANA ($\boxdot$).

Run	Terms	C_y $(\mu g m^{-2})$		
	u_0	2063.595	1289.481	
	u_0+u_1	2010.773	1340.828	
1	$u_0+u_1+u_2$	2011.426	1308.431	
	$u_0+u_1+u_2+u_3$	2092.073	1281.515	
	$u_0+u_1+u_2+u_3+u_4$	2092.073	1281.515	
	u_0	1417.238	823.5428	
	u_0+u_1	1356.679	855.5662	
2	$u_0+u_1+u_2$	1495.957	850.2274	
	$u_0+u_1+u_2+u_3$	1495.957	850.2274	
	$u_0+u_1+u_2+u_3+u_4$	1495.957	850.2274	
	u_0	2549.781	1563.213	1292.831
	u_0+u_1	2615.559	1607.727	1250.595
3	$u_0+u_1+u_2$	2600.684	1526.362	1253.927
	$u_0+u_1+u_2+u_3$	2601.178	1604.603	1272.520
	$u_0+u_1+u_2+u_3+u_4$	2601.178	1604.603	1272.520
	u_0	2376.284		
	u_0+u_1	2444.419		
4	$u_0+u_1+u_2$	2427.065		
	$u_0+u_1+u_2+u_3$	2379.459		
	$u_0+u_1+u_2+u_3+u_4$	2379.459		
	u_0	2134.523	1525.608	1454.858
	u_0+u_1	2215.876	1523.247	1491.563
5	$u_0+u_1+u_2$	2544.441	1794.193	1626.543
	$u_0+u_1+u_2+u_3$	2586.452	1817.632	1567.856
	$u_0+u_1+u_2+u_3+u_4$	2586.452	1817.632	1567.856
	u_0	959.1522	567.4748	471.1268
	u_0+u_1	912.4229	619.4894	518.6852
6	$u_0+u_1+u_2$	890.4201	605.0680	454.2511
	$u_0+u_1+u_2+u_3$	942.7131	620.3289	483.5103
	$u_0+u_1+u_2+u_3+u_4$	951.0098	619.3738	1488.264
	$u_0+u_1+u_2+u_3+u_4+u_5$	951.0098	619.3738	1488.264
	u_0	1087.322	699.6638	585.6924
	u_0+u_1	1122.203	687.8445	624.9547
7	$u_0+u_1+u_2$	1098.108	682.8063	537.3536
	$u_0+u_1+u_2+u_3$	1171.588	679.6330	554.0372
	$u_0+u_1+u_2+u_3+u_4$	1171.588	679.6330	554.0372
	u_0	1184.016	787.5058	489.2957
	u_0+u_1	1150.614	780.2734	502.6539
8	$u_0+u_1+u_2$	1228.163	722.6319	489.3400
	$u_0+u_1+u_2+u_3$	1228.163	722.6319	489.3400
	$u_0+u_1+u_2+u_3+u_4$	1228.163	722.6319	489.3400
	u_0	1404.454	853.9096	679.9700
	u_0+u_1	1332.897	825.2356	681.6997
9	$u_0+u_1+u_2$	1523.163	876.8478	661.7837
	$u_0+u_1+u_2+u_3$	1433.129	884.0126	630.3093
	$u_0+u_1+u_2+u_3+u_4$	1433.129	884.0126	630.3093

Table 3. Numerical Convergence of ADM using a Gaussian pdf.

but in the average model and experiment shall coincide, thus the introduced index represents a genuine model validation.

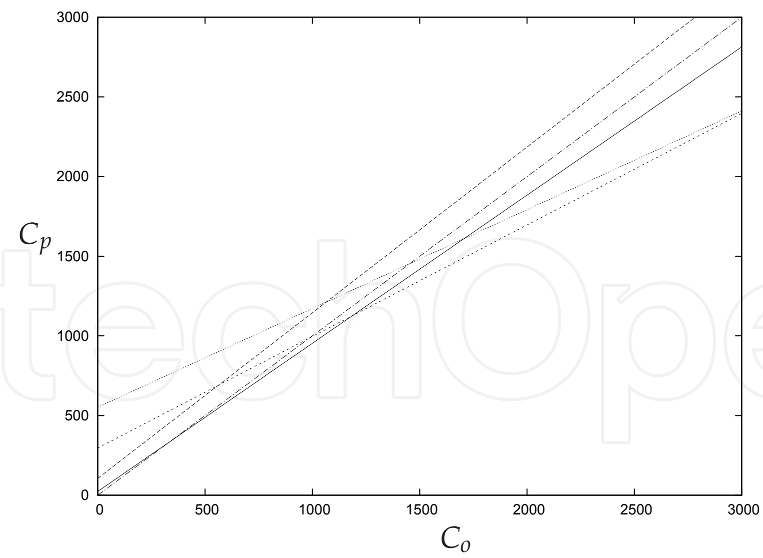

Fig. 3. Linear regression for the ADM (——), ILS (– – –), Itō (- - - -) and ANA (· · · · · ·) with a Gaussian pdf. The bisector (– · – ·) was added as an eye guide.

Modelo	Regression	R^2	κ
ADM	$y = 0,93x + 23,50$	$0,89$	$0,07$
ILS	$y = 1,04x + 105,51$	$0,87$	$0,09$
Itō	$y = 0,70x + 296,13$	$0,83$	$0,37$
ANA	$y = 0,62x + 552,32$	$0,33$	$0,56$

Table 4. Comparison of the linear regressions of ADM, ILS, Itō and ANA for a Gaussian pdf.

3.3 Solution for bi-Gaussian turbulence

In the convective boundary layer, the heating of the air layer close to the ground produces turbulent flux which gives origin to the so-called up- and down-drafts. This phenomenon is not symmetric but has a more intensive contribution from the up-drafts. Because of mass conservation the down-drafts occupy a larger area. As a consequence the stochastic term shall be asymmetric which excludes the Gaussian probability density as a convenient function. There is no indication for a unique probability density function so far, nevertheless the following characteristics shall be present.

- The probability density shall have an enhanced tail towards higher velocities, that indicate the more energetic up-drafts, but with a smaller integral proportion than down-drafts.
- The probability density shall have a pronounced maximum at negative velocities, i.e. the down-drafts.

One finds typically two types of asymmetric probability density functions in the literature, the bi-Gaussian and the Gram-Chalier distribution, where the latter is represented by a truncated series of Hermite polynomials.

In the further we discuss the bi-Gaussian probability density function, which contains a linear superposition of two Gaussian functions, one with maximum probability at a positive velocity, the other one at a negative value as for instance in ref. Baerentsen and Berkowicz

(1984). Baerentsen e Berkowicz (1984) used a pair of Langevin equations, one for up- and one for down-drafts, each with its specific Gaussian function. In this work we condense this phenomenon in one equation, introducing a sum of two Gaussian functions with different parameters and relative weight.

$$P(z,w) = A_1 P_1(z,w) + A_2 P_2(z,w) \tag{14}$$

where A_1 and A_2 define the relative proportions between up- (P_1) and down-drafts (P_2) for the vertical turbulent velocities (w).

$$P(z,w) = \frac{1}{\sqrt{2\pi}}\frac{A_1}{\sigma_1}\exp\left[-\frac{1}{2}\left(\frac{w-m_1}{\sigma_1}\right)^2\right] + \frac{1}{\sqrt{2\pi}}\frac{A_2}{\sigma_2}\exp\left[-\frac{1}{2}\left(\frac{w-m_2}{\sigma_2}\right)^2\right] \tag{15}$$

Here, m_1, m_2 are the average probabilities of P_1 and P_2, respectively, and σ_1 and σ_2 represent the standard deviations of each distribution. The mean up- and down-draft velocities are

$$m_1 = \langle w_1 \rangle \qquad \text{and} \qquad m_2 = \langle w_2 \rangle \ , \tag{16}$$

and the respective standard deviations are

$$\sigma_1 = \left(\langle w_1^2 \rangle\right)^{\frac{1}{2}} \qquad \text{and} \qquad \sigma_2 = \left(\langle w_2^2 \rangle\right)^{\frac{1}{2}} \ . \tag{17}$$

A general prescription on how to determine the parameters A_1, A_2, m_1, m_2, σ_1 and σ_2 consists in the usage of generating functional of moments.

$$\langle w_n \rangle = \int_{-\infty}^{\infty} w_n P(z,w)\, dw \tag{18}$$

From the normalisation and the first four statistical momenta one obtains an equation system which eliminates the unknowns.

$$A_1 + A_2 = 1 \tag{19}$$
$$A_1 m_1 + A_2 m_2 = 0 \tag{20}$$
$$A_1(m_1^2 + \sigma_1^2) + A_2(m_2^2 + \sigma_2^2) = \sigma_w^2 \tag{21}$$
$$A_1(m_1^3 + 3m_1\sigma_1^3) + A_2(m_2^3 + 3m_2\sigma_2^3) = \langle w^3 \rangle \tag{22}$$
$$A_1(m_1^4 + 6m_1^2\sigma_1^2 + 3\sigma_1^4) + A_2(m_2^4 + 6m_2^2\sigma_2^2 + 3\sigma_2^4) = \langle w^4 \rangle \tag{23}$$

Upon application of the bi-Gaussian probability density function the expression for the deterministic coefficient of the vertical dimension in the Langevin equation is then,

$$a_w = -w\frac{A_1P_1\sigma_1^2 + A_2P_2\sigma_2^2}{\sigma_1^2\sigma_2^2}\frac{C_0\varepsilon}{2P} + \left(\frac{A_1w_1P_1}{\sigma_1^2} + \frac{A_2w_2P_2}{\sigma_2^2}\right)\frac{C_0\varepsilon}{2P} + \frac{\phi}{P} \ . \tag{24}$$

Using the deterministic coefficient the Langevin equation reads

$$\frac{dw}{dt} + \frac{A_1P_1\sigma_1^2 + A_2P_2\sigma_2^2}{\sigma_1^2\sigma_2^2}\frac{C_0\varepsilon}{2P}w = \left(\frac{A_1w_1P_1}{\sigma_1^2} + \frac{A_2w_2P_2}{\sigma_2^2}\right)\frac{C_0\varepsilon}{2P} + \frac{\phi}{P} + (C_0\varepsilon)^{\frac{1}{2}}\,\xi_w(t) \ , \tag{25}$$

where ϕ is obtained upon application of the bi-Gaussian probability density function Luhar et al. (1996):

$$\phi = -\frac{1}{2}\left(A_1\frac{\partial w_1}{\partial z} + w_1\frac{\partial A_1}{\partial z}\right)\mathrm{erf}\left(\frac{w-w_1}{\sqrt{2}\sigma_1}\right)$$
$$+w_1P_1\left[A_1\frac{\partial w_1}{\partial z}\left(\frac{w^2}{\sigma_1^2}+1\right)+w_1\frac{\partial A_1}{\partial z}\right]+\frac{1}{2}\left(A_2\frac{\partial w_2}{\partial z}+w_2\frac{\partial A_2}{\partial z}\right)\mathrm{erf}\left(\frac{w-w_2}{\sqrt{2}\sigma_2}\right)$$
$$+w_2P_2\left[A_2\frac{\partial w_2}{\partial z}\left(\frac{w^2}{\sigma_2^2}+1\right)+w_2\frac{\partial A_2}{\partial z}\right]. \tag{26}$$

In a more compact form this yields for the Langevin equation with a bi-Gaussian probability density function (25)

$$\frac{dw}{dt} + \alpha_w w = \beta_w + \gamma_w + (C_0\varepsilon)^{\frac{1}{2}}\xi_w(t)\,, \tag{27}$$

where

$$a_w = \frac{A_1P_1\sigma_1^2 + A_2P_2\sigma_2^2}{\sigma_1^2\sigma_2^2}\frac{C_0\varepsilon}{2P}\,, \beta_w = \left(\frac{A_1w_1P_1}{\sigma_1^2}+\frac{A_2w_2P_2}{\sigma_2^2}\right)\frac{C_0\varepsilon}{2P}\gamma_w = \frac{\phi}{P}\,.$$

In Table (5) the concentrations of the measurements together with theoretical predictions of ADM, ILS and Itō are presented. Table (6) shows the numerical convergence of the ADM method. As already evident in the previous case also for the bi-Gaussian probability density function only a few terms are necessary in order to represent a solution.

Figure (5) shows the dispersion plot of the experimental values against the theoretical predicted values by ADM, ILS and the Itō calculus.

We also apply the model validation as introduced in the previous section to the model application with the bi-Gaussian probability density function. One observes that the all three approaches are more or less close to the bisector, however the comparison with the model validation from the previous case shows that the Gaussian probability density function seems more adequate for the stability condition of the experiment which is also manifest in the smallest κ for ADM.

From the comparison of the regressions in table 7 one recognizes that the three approaches behave similar with respect to R^2 but show larger values for κ in comparison to the case where the Gaussian probability density function defined the stochastic character of the turbulence.

3.4 Solution for Gram-Chalier turbulence

The use of the Gram-Chalier probability density function for stochastic Lagrangian models was proposed by Ferrero e Anfossi (1998) Ferrero et al. (2000), which makes use of an expansion in Hermite polynomials. In the present discussion we use the series until the fourth resulting in an asymmetric probability density function for the vertical turbulent velocities.

$$P(r,z) = \frac{e^{-\frac{r^2}{2}}}{\sqrt{2\pi}}(1 + c_3H_3 + c_4H_4)\,, \tag{28}$$

where

$$c_3 = \frac{1}{6}\mu_3\,, \qquad c_4 = \frac{1}{24}(\mu_4 - 6\mu_2 + 3)\,, \tag{29}$$

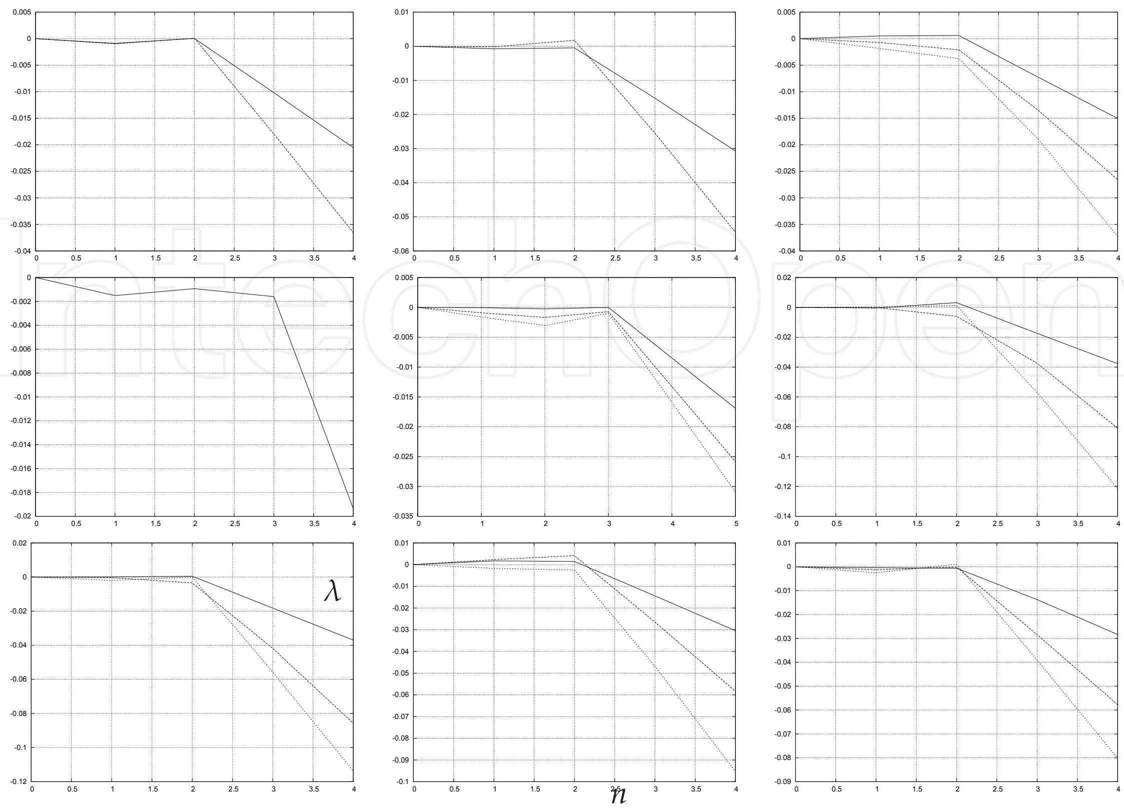

Fig. 4. Lyapunov exponent λ of the Adomian approach depending on the number n of terms for the 9 experimental runs using the bi-Gaussian pdf.

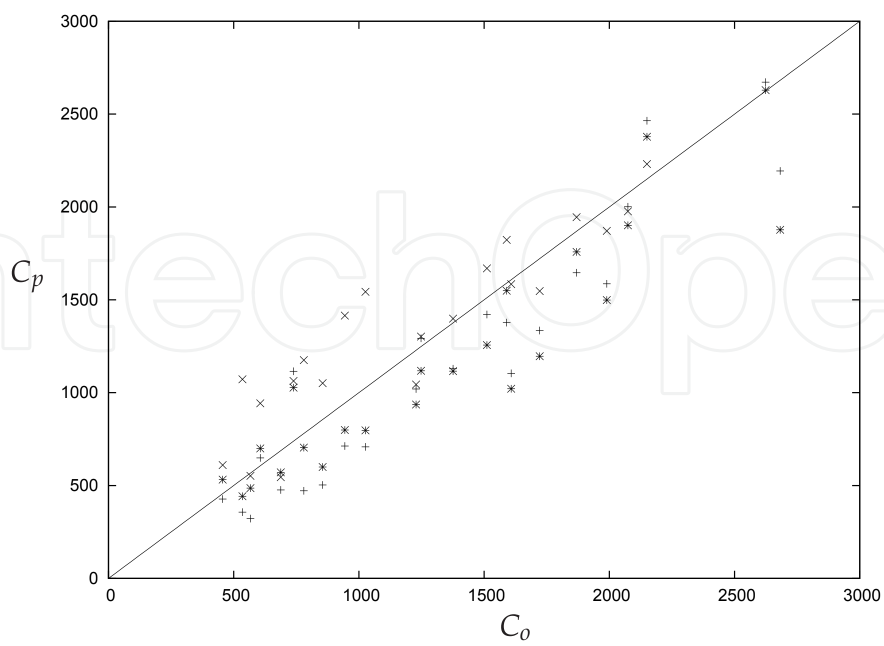

Fig. 5. Dispersion diagram of predicted (C_p) measured against measured (C_o) values by by ADM ($+$), ILS ($\times$) and Itō ($\ast$) for a bi-Gaussian pdf.

	Distance	Observation	Prediction C_y $(\mu g m^{-2})$		
Exp.	(m)	$(\mu g m^{-2})$	Bi-Gaussian	ILS	Itō
1	1900	2074	2001	1976	1901
1	3700	739	1115	1063	1027
2	2100	1722	1335	1547	1196
2	4200	944	713	1415	799
3	1900	2624	2672	3020	2629
3	3700	1990	1586	1871	1499
3	5400	1376	1129	1399	1116
4	4000	2682	2194	3001	1877
5	2100	2150	2464	2231	2378
5	4200	1869	1646	1945	1758
5	6100	1590	1377	1823	1549
6	2000	1228	1020	1044	936
6	4200	688	476	545	571
6	5900	567	322	552	486
7	2000	1608	1104	1584	1021
7	4100	780	472	1175	704
7	5300	535	357	1072	442
8	1900	1248	1293	1302	1118
8	3600	606	649	943	700
8	5300	456	427	610	532
9	2100	1511	1421	1669	1256
9	4200	1026	708	1543	797
9	6000	855	503	1051	600

Table 5. Concentration from the Copenhagen experiment and predictions from ADM, ILS and Itō using a bi-Gaussian pdf.

$$H_3 = r^3 - 3r \, , \qquad H_4 = r^4 - 6r^2 + 3 \, , \tag{30}$$

and $r = u_i/\sigma_i$. In the case of Gaussian turbulence equation (28) recovers the normal distribution with c_3 and c_4 equal zero. The Gram-Charlier probability density function of the third order is obtained by the choice $c_4 = 0$. Upon application of equation (28) in the equation of the deterministic coefficients yields,

$$a(x_i, u_i) = \frac{f_i}{h_i}\frac{\sigma_i}{\tau_{L_i}} + \sigma_i \frac{\sigma_i}{x_j}\frac{g_i}{h_i} \, , \tag{31}$$

where $j = 1, 2, 3$ and $j \neq i$, τ_{L_i} is the Lagrangian correlation time scale and f_i, g_i and h_i are expressions as shown below.

$$f_i = -3C_3 - r_i(15C_4 + 1) + 6C_3 r_i^2 + 10C_4 r_i^3 - C_3 r_i^4 - C_4 r_i^5 \tag{32}$$

$$g_i = 1 - C_4 - r_i^2(1 + C_4) - 2C_3 r_i^3 - 5C_4 r_i^4 + C_3 r_i^5 + C_4 r_i^6 \tag{33}$$

$$h_i = 1 - 3C_4 - 3C_3 r_i - 6C_4 r_i^2 + C_3 r_i^3 + C_4 r_i^4 \tag{34}$$

Run	Terms	C_y $(\mu g m^{-2})$		
	u_0	1930,605	1164,849	
	u_0+u_1	1988,625	1097,584	
1	$u_0+u_1+u_2$	1923,805	1169,695	
	$u_0+u_1+u_2+u_3$	2000,73	1114,916	
	$u_0+u_1+u_2+u_3+u_4$	2000,73	1114,916	
	u_0	1268,718	705,3707	
	u_0+u_1	1310,076	706,2147	
2	$u_0+u_1+u_2$	1299,548	687,1277	
	$u_0+u_1+u_2+u_3$	1335,249	713,1350	
	$u_0+u_1+u_2+u_3+u_4$	1335,249	713,1350	
	u_0	2645,291	1388,329	934,7424
	u_0+u_1	2569,380	1520,158	1152,943
3	$u_0+u_1+u_2$	2545,731	1592,848	1132,120
	$u_0+u_1+u_2+u_3$	2671,856	1585,870	1129,320
	$u_0+u_1+u_2+u_3+u_4$	2671,856	1585,870	1129,320
	u_0	1918,952		
	u_0+u_1	2183,475		
4	$u_0+u_1+u_2$	2156,671		
	$u_0+u_1+u_2+u_3$	2201,707		
	$u_0+u_1+u_2+u_3+u_4$	2193,560		
	u_0	2342,774	1341,859	1061,910
	u_0+u_1	2573,369	1572,562	1340,005
5	$u_0+u_1+u_2$	2527,639	1664,515	1371,924
	$u_0+u_1+u_2+u_3$	2585,538	1545,632	1290,100
	$u_0+u_1+u_2+u_3+u_4$	2464,081	1646,118	1376.838
	$u_0+u_1+u_2+u_3+u_4+u_5$	2464,081	1646,118	1376.838
	u_0	1024,609	470,6826	312,2561
	u_0+u_1	1016,280	481,9288	311,2449
6	$u_0+u_1+u_2$	925,6591	476,7062	308,4274
	$u_0+u_1+u_2+u_3$	1019,558	476,3795	321,7595
	$u_0+u_1+u_2+u_3+u_4$	1019,558	476,3795	321,7595
	u_0	1048,864	510,2945	401,3791
	u_0+u_1	1042,545	439,8970	378,0348
7	$u_0+u_1+u_2$	1012,276	479,4384	400,1880
	$u_0+u_1+u_2+u_3$	1104,080	472,1360	357,2161
	$u_0+u_1+u_2+u_3+u_4$	1104,080	472,1360	357,2161
	u_0	1280,617	646,3909	378,3521
	u_0+u_1	1193,205	662,8212	402,9780
8	$u_0+u_1+u_2$	1219,809	702,0541	443,7521
	$u_0+u_1+u_2+u_3$	1293,085	649,3011	427,3928
	$u_0+u_1+u_2+u_3+u_4$	1293,085	649,3011	427,3928
	u_0	1452,223	652,5918	469,1815
	u_0+u_1	1438,980	729,6335	493,3640
9	$u_0+u_1+u_2$	1433,919	656,2526	448,3562
	$u_0+u_1+u_2+u_3$	1420,842	707,7180	503,2054
	$u_0+u_1+u_2+u_3+u_4$	1420,842	707,7180	503,2054

Table 6. Numerical convergence of ADM for a bi-Guassian pdf.

Inserting the deterministic coefficient (31) into the Langevin equation renders the latter

$$\frac{du_i}{dt} = \frac{f_i}{h_i}\frac{\sigma_i}{\tau_{L_i}} + \sigma_i \frac{\partial \sigma_i}{\partial x_j}\frac{g_i}{h_i} + (C_0\varepsilon)^{\frac{1}{2}}\xi_i(t) \ . \tag{35}$$

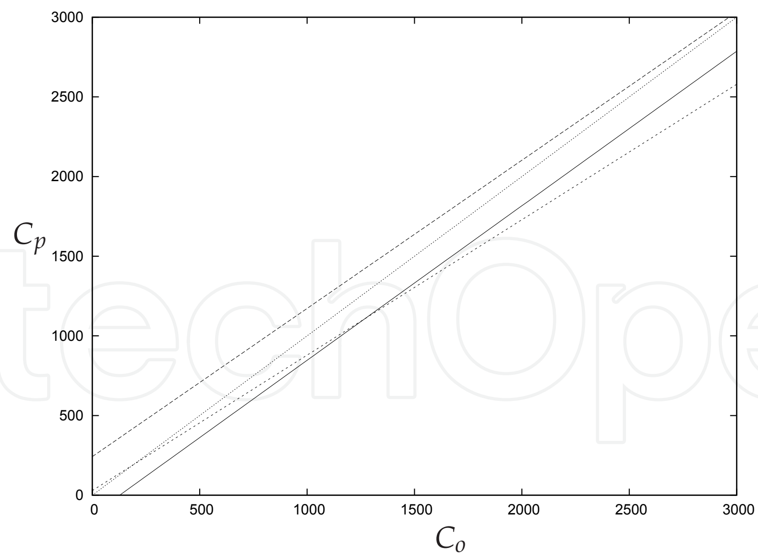

Fig. 6. Linear regression for the ADM (——), ILS (– – –) and Itō (- - - -) with a Bi-Gaussian pdf. The bisector (– · – ·) was added as an eye guide.

Model	Regression	R^2	κ
ADM	$y = 0,97x - 123,47$	$0,89$	$0,10$
ILS	$y = 0,93x + 242,34$	0.89	$0,19$
Itō	$y = 0,85x + 29,07$	0.86	$0,15$

Table 7. Comparison of the linear regressions using the bi-Gaussian probability density function.

In short hand notation this reads

$$\frac{du_i}{dt} = \alpha_i + \beta_i + (C_0\varepsilon)^{\frac{1}{2}}\xi_i(t), \tag{36}$$

where

$$\alpha_i = \frac{f_i}{h_i}\frac{\sigma_i}{\tau_{L_i}}\ , \tag{37}$$

$$\beta_i = \sigma_i\frac{\partial\sigma_i}{\partial x_j}\frac{g_i}{h_i}\ . \tag{38}$$

In table (8) we present the concentrations of the Copenhagen experiment together with the results from the ADM, ILS and Itō approaches.

Table 9 shows the numerical convergence of the ADM method. As in the two previous cases only a few terms reproduce with considerable fidelity the exact solution with a Gram-Chalier probability density function. Figure (8) shows the dispersion plot of observed against predicted data. In Figure (9) are shown the linear regression for the three approaches. All three methods, ADM, ILS and Itō reproduce reasonably well the expected bisector. Using the model validation index κ shows that for all three probability density functions the ADM approach yields results closest to the expected concentration profile.

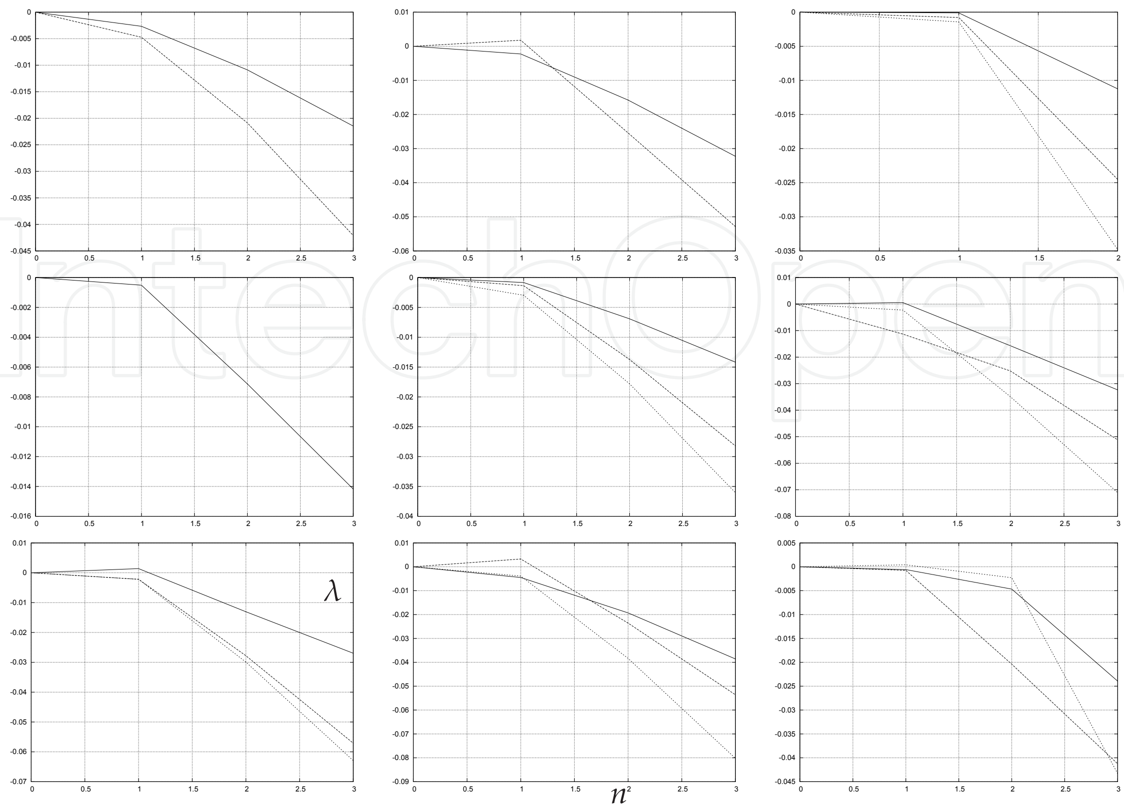

Fig. 7. Lyapunov exponent λ of the Adomian approach depending on the number of terms n for the 9 experimental runs using the Gram-Chalier pdf.

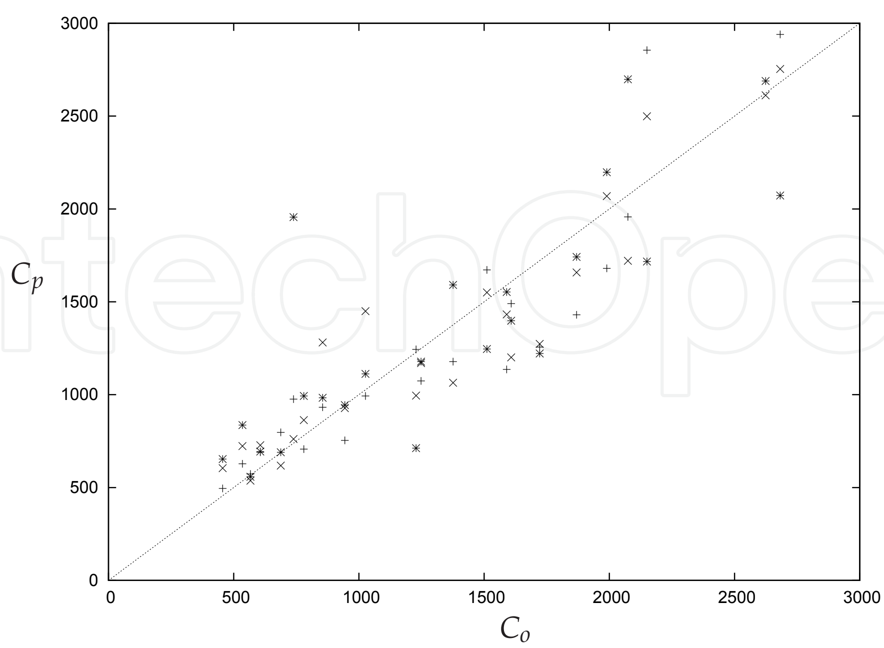

Fig. 8. Dispersion diagram of predicted (C_p) against observed values (C_o) with a Gram-Chalier probability density function.

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	Distance	Observation	Prediction C_y $(\mu g m^{-2})$		
Exp.	(m)	$(\mu g m^{-2})$	ADM	ILS	Ito
1	1900	2074	1957	1721	2698
1	3700	739	976	761	1956
2	2100	1722	1256	1273	1222
2	4200	944	754	928	944
3	1900	2624	3426	2612	2689
3	3700	1990	1680	2069	2198
3	5400	1376	1178	1064	1591
4	4000	2682	2940	2754	2072
5	2100	2150	2855	2499	1717
5	4200	1869	1430	1658	1742
5	6100	1590	1136	1432	1553
6	2000	1228	1244	995	712
6	4200	688	797	618	690
6	5900	567	573	537	558
7	2000	1608	1490	1201	1398
7	4100	780	707	863	993
7	5300	535	628	723	836
8	1900	1248	1074	1170	1178
8	3600	606	690	728	694
8	5300	456	495	604	653
9	2100	1511	1672	1550	1246
9	4200	1026	993	1450	1112
9	6000	855	932	1281	983

Table 8. Concentration of the Copenhagen experiment in comparison to the predictions by ADM, ILS and Itō using a Gram-Chalier pdf.

As already mentioned before, the model validation indicates the Gaussian probability density function implemented together with the ADM approach as the most adequate description for the Copenhagen experiment by virtue of $\kappa = 0.07$ being significantly smaller than all other realizations. This was also to be expected from the stability conditions given in table 1, which characterize the turbulence regime as strong convective. It is worth mentioning that since convergence is genuinely controlled the present procedure permits to pin down model limitations which in other approaches are hidden in numerical imprecision or approximations.

4. Conclusions

In this paper we presented an analytical solution of the three-dimensional stochastic Langevin equation applied to radioactive substance dispersion for Gaussian, bi-Gaussian and Gram-Chalier turbulence, respectively. The solution was obtained using the Adomian Decomposition Method (ADM) whose principal advantage relies in the fact that the non-linearity can be taken care of without linearisation or simplifications. Further, the stochastic part is absorbed in the initial term of the iteration and thus propagates through all the subsequent iteration terms. For the Langevin equation the non-trivial questions of uniqueness and convergence

Run	Terms	C_y $(\mu g m^{-2})$		
	u_0	2134,374	905,1679	
	u_0+u_1	1958,222	975,6816	
1	$u_0+u_1+u_2$	1957,265	976,3862	
	$u_0+u_1+u_2+u_3$	1957,265	976,3862	
	u_0	1215,582	733,3223	
	u_0+u_1	1258,735	683,5825	
2	$u_0+u_1+u_2$	1256,363	754,4139	
	$u_0+u_1+u_2+u_3$	1256,363	754,4139	
	u_0	3431,128	1602,438	1114,993
	u_0+u_1	3422,063	1700,270	1165,802
3	$u_0+u_1+u_2$	3425,766	1679,948	1177,613
	$u_0+u_1+u_2+u_3$	3425,766	1679,948	1177,613
	u_0	3066,27		
	u_0+u_1	2911,51		
4	$u_0+u_1+u_2$	2939,85		
	$u_0+u_1+u_2+u_3$	2939,85		
	u_0	2817,202	1396,448	1075,217
	u_0+u_1	2858,730	1434,506	1134,203
5	$u_0+u_1+u_2$	2855,275	1429,748	1136,310
	$u_0+u_1+u_2+u_3$	2855,275	1429,748	1136,310
	u_0	1273,45	851,0902	525,1321
	u_0+u_1	1304,966	797,4598	559,2511
6	$u_0+u_1+u_2$	1243,54	797,4534	572,6406
	$u_0+u_1+u_2+u_3$	1243,54	797,4534	572,6406
	u_0	1461,67	672,4225	613,2297
	u_0+u_1	1868,749	699,8157	631,2928
7	$u_0+u_1+u_2$	1489,976	707,4423	627,6641
	$u_0+u_1+u_2+u_3$	1489,976	707,4423	627,6641
	u_0	973,0740	691,4625	512,5447
	u_0+u_1	1074,354	702,0862	497,4521
8	$u_0+u_1+u_2$	1073,538	690,2837	494,8708
	$u_0+u_1+u_2+u_3$	1073,538	690,2837	494,8708
	u_0	1647,435	1054,789	898,2956
	u_0+u_1	1662,513	963,0107	883,1010
9	$u_0+u_1+u_2$	1671,778	992,9380	936,4106
	$u_0+u_1+u_2+u_3$	1671,788	992,9380	932,4106
	$u_0+u_1+u_2+u_3+u_4$	1671,788	992,9380	932,4106

Table 9. Numerical convergence of ADM using a Gram-Chalier pdf.

Model	Regression	R^2	κ
ADM	$y = 1,09x + 113,83$	0,85	0,12
ILS	$y = 0,90x + 112,17$	0,87	0,13
ITO	$y = 0,78x + 324,52$	0,62	0,33

Table 10. Comparison of the linear regressions for the ADM, ILS and Itō approach using the Gram-Chalier probability density function.

for the Adomian approach in stochastic problems is given since the drift and dispersion terms

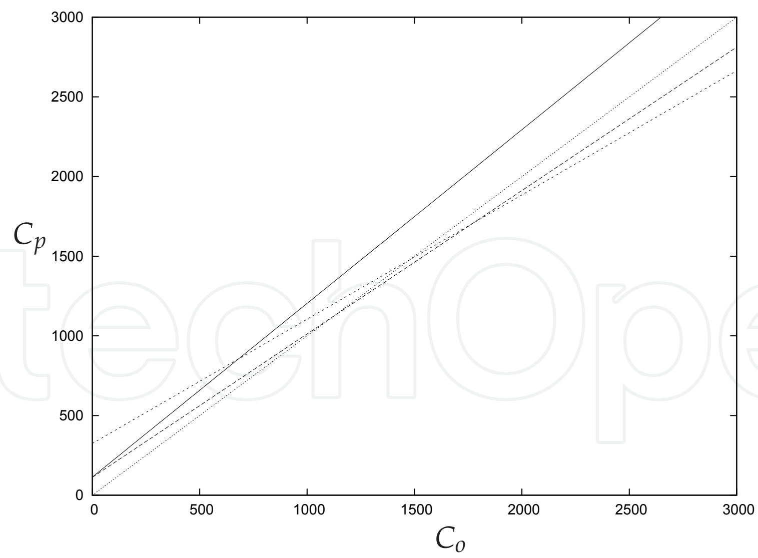

Fig. 9. Linear regression using the Gram-Chalier probability density function

satisfy a Lipschitz condition.

We showed in a general form how to construct a recursive scheme where convergence is understood. A genuine criterion was introduced based on Lyapunov's theory, that in our case tests stability of convergence. Application of that criterion showed that in all three cases only up to five terms are necessary so that the approximate solution differs from the real solution by less than one percent. On the one hand, the generality of the proposed solution with respect to the considered probability density functions on the other hand the controlled convergence permits to validate the model in question. In this line we introduced a novel index, that describes the deviation of the model prediction from the one represented by the experimental data, using the relation between predicted to observed tracer concentrations. Based on this index we verify that the model with the Gaussian density function yields within the phenomenon inherent fluctuations the best agreement between model and observation. Among the three probability distributions the Gaussian one is from the physics point of view considered the most adequate for the Copenhagen experiment.

Furthermore, the ADM solution was validated by comparison with experimental data against the prediction of other models, i.e. the numerical solution of the Langevin equation by integration according to the Itō calculus, the analytical solution of the Langevin equation (ANA) following the derivation of Uhlenbeck and Ornstein and the Iterative Langevin Solution (ILS). A statistical analysis showed good agreement between predicted and measured data and all values are within the range that are characteristic for other state-of-the-art approaches. The ranking from the analysis defines the sequence ADM, ILS, Itō, ANA, which is manifest in the fact that the ratio of predicted to observed concentrations in the ADM approach was reasonably close to the bisector. Thus, the present approach may be considered a valuable procedure to simulate tracer dispersion in the atmosphere until new improvements will alter the present picture.

We believe that we have done a step into a new direction with the present contribution, that may be useful to analyse meteorological aspects as well as simulate possible scenarios, for the purpose to analyse (possible) consequences of radioactive discharge and its relation to radiological consequences of routine discharges and potential accidental releases of radioactive

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substances from nuclear power plants. Furthermore, these case studies by model simulations may be used to establish limits for escape of radioactive material from the power plant into the atmosphere. Since measurements are typically performed in a limited set of positions a calibrated model is able to reconstruct the three dimensional wind velocity field considering especially the contributions by turbulence. To the best of our knowledge up-to-date the tracer technique is not used for site evaluation, but could supply valuable information on the wind properties for a given region of interest and its time-behaviour.

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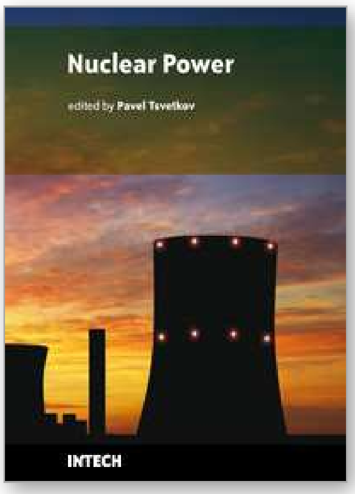

Nuclear Power
Edited by Pavel Tsvetkov

ISBN 978-953-307-110-7
Hard cover, 388 pages
Publisher Sciyo
Published online 17, August, 2010
Published in print edition August, 2010

The world of the twenty first century is an energy consuming society. Due to increasing population and living standards, each year the world requires more energy and new efficient systems for delivering it. Furthermore, the new systems must be inherently safe and environmentally benign. These realities of today's world are among the reasons that lead to serious interest in deploying nuclear power as a sustainable energy source. Today's nuclear reactors are safe and highly efficient energy systems that offer electricity and a multitude of co-generation energy products ranging from potable water to heat for industrial applications. The goal of the book is to show the current state-of-the-art in the covered technical areas as well as to demonstrate how general engineering principles and methods can be applied to nuclear power systems.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

Marco Tullio Vilhena, Bardo Bodmann and Kelen Berra De Mello (2010). Stochastic Wind Profiles Determination for Radioactive Substances Released from Nuclear Power Plants, Nuclear Power, Pavel Tsvetkov (Ed.), ISBN: 978-953-307-110-7, InTech, Available from: http://www.intechopen.com/books/nuclear-power/stochastic-wind-profiles-determination-for-radioactive-substances-released-from-nuclear-power-plants

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