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Rarefied natural gas transport

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1. Introduction

Natural gas (or simply gas), made up of around 82.0-89.6 mol% methane, 0.9-9.8 mol% nitrogen, 3.4-9.4 mol% ethane, 0.6-4.7 mol% propane, 0.1-1.7 mol% n-butane, and other gases (GPSA, 1998; Ivings et al., 2003; Schley et al., 2004), is a gaseous fossil fuel that has methane as its primary compound and is moved primarily by pipeline. It is desirable to understand the physical properties in rarefied natural gas transport and the corresponding behavior in pipelines.

Gas rarefaction was observed in many areas, such as gas bearings (Johnston & McCloskey, 1940; Carr, 1954; Burgdorfer, 1959; Hsia & Domoto, 1983), space vehicles (Ivanov & Gimelshein, 1998; Tsuboi & Matsumoto, 2005), microfluidic devices (Pfahler et al., 1991; Pong et al., 1994), etc. The effect of gas rarefaction is quantified by the Knudsen number Kn, which is defined as the ratio of the molecular mean free path to the characteristic length. Schaaf & Chambre (1961) have proposed the following ranges to determine the degree of gas rarefaction in a flow, as shown in Table 1. For $Kn \leq 0.01$, the flow is in the continuum regime, where the continuum hypothesis holds. The continuum field equations in conjunction with no-slip and no-jump boundary conditions describe the flow and thermal behavior. The rarefaction effect becomes noticeable when the value of Kn becomes greater than 0.01. The range 0.01-0.1 is referred to as the slip regime, where the conventional continuum field equations are still valid, but the conventional no-slip and no-jump boundary conditions may break down. It has been proven that the Maxwell-Smoluchowski (MS) slip and jump boundary conditions are valid (Arkilic et al., 1997; Beskok & Karniadakis, 1999; Ewart et al., 2007; Weng & Chen, 2008a). The range of $0.1 < Kn \leq 10$ represents the transition regime, where the molecular mean free path is comparable to the characteristic length, and the conventional continuum field equations may break down. For $10 < Kn$, the flow is in the free-molecular regime, where the collisions among the molecules are negligible. Although conventional transport systems operate in the continuum regime, pipelines can operate in the slip regime due to the reduction of working pressure or the enhancement of working temperature (the molecular mean free path is related to the pressure and the temperature).

In this chapter, first we will provide the property formulas of gases in power-law form and present the physical properties of methane at the standard reference state, so as to simply predict the physical properties of natural gases. Then we will discuss in some detail the use of the mass, momentum, and energy conservation equations as well as the slip and jump

boundary conditions as it is applied to fluid motion that is rarefied. Finally, we will develop the mathematical models of rarefied natural gas transport in basic driving mechanisms. We will obtain the analytical solutions of flow fields and characteristics, so as to realize the importance of gas rarefaction in natural gas transport.

After completing this chapter, you should be able to:

- use the property formulas of gases and the physical properties of methane at the standard reference state.
- use the mass, momentum, and energy conservation equations subject to the slip and jump boundary conditions.
- solve basic rarefied natural gas transport problems.
- use the analytical procedure shown in basic transport problems.
- apply the analytical solutions of basic transport problems.

Regime	Range
Continuum flow	$Kn \leq 0.01$
Slip flow	$0.01 < Kn \leq 0.1$
Transition flow	$0.1 < Kn \leq 10$
Free-molecular flow	$10 < Kn$

Table 1. Flow regimes of rarefied gas dynamics (see Schaaf & Chambre, 1961).

2. Physical Properties

Any characteristic of a substance that can be observed or measured without changing the identity of the substance is called physical property. Some important physical properties in rarefied gas transport are shear viscosity μ, thermal conductivity k, constant-pressure specific heat c_p, density ρ, and molecular mean free path λ. In this part, we provide the formulas for these properties of gases in power-law form and present the physical properties of methane at the standard reference state. Then you can use the formulas and the properties to present further properties of other hydrocarbons, such as ethane, propane, butane, etc., to calculate further physical properties of natural gases in most common operating states, and to conduct further analyses of theoretical and experimental researches. Various tests have been conducted to determine μ, k, c_p, and ρ of methane at different working pressures or temperatures (Mann & Dickins 1931; Johnston & McCloskey 1940; Clarke & Smith, 1969; Jansoone et al., 1970; Vennix et al., 1970; Younglove, 1974; Gammon & Douslin, 1976; Kerley, 1980; Kleinrahm & Wagner, 1986; Kleinrahm et al. 1986; Kurumov et al., 1988; Friend et al., 1989; Jin et al., 1992; Pátek & Klomfar, 2002; Hurly et al., 2003; Schley et al., 2004; Viswanathan, 2007). Except for density, temperature was found to be a relatively important factor in the gaseous state. It becomes necessary to have formulas for temperature-dependent physical properties of methane. The property formulas adopted in this chapter was originally proposed by Weng & Chen (2008b). According to the nature of gases, the power-law expressions for the shear viscosity, the thermal conductivity, the specific heat, and the density can be written, respectively, as ,

$$\mu = \mu_r \left(\frac{T}{T_r} \right)^{\omega} \tag{1}$$

$$k = k_r \left(\frac{T}{T_r} \right)^{\varpi} \tag{2}$$

$$c_p = c_{pr} \left(2 + s \left(\frac{T}{T_r} - 1 \right) - \left(\frac{T}{T_r} \right)^{\varpi - \omega} \right) \tag{3}$$

$$\rho = \rho_r \left(\frac{T}{T_r} \right)^{-1} \tag{4}$$

where the subscript r denotes the property values at a reference state, T is the temperature, ω and ϖ are the viscosity and conductivity indexes, respectively, and s is the specific-heat coefficient. Here, ideal gas behavior is assumed, which is quite reasonable in most common operating states. If the shear viscosity, thermal conductivity, and specific heat are known at two temperatures, one can determine the values of ω, ϖ and s. In Table 2, we use 300 K and 400 K for the constant determination of methane. In Fig. 1, we then verify the present formulas (1)–(4). The calculated results compare very well with the data listed in Friend et al. (1989) and Schley et al. (2004). The percentage absolute errors for μ, k, c_p, and ρ over $260\,\text{K} \le T \le 400\,\text{K}$ are less than 0.674, 2.01, 1.68, and 0.110, respectively. It should be noted that the data in the literature chosen for the comparison are modified to take into account the pressure effect.

Physical Property	Symbol	Unit	Value
Shear Viscosity	μ	$\text{N} \cdot \text{s/m}^2$	1.114×10^{-5}
Thermal conductivity	k	$\text{W/m} \cdot \text{K}$	0.0348
Constant-pressure specific heat	c_p	$\text{J/K} \cdot \text{kg}$	2223
Density	ρ	kg/m^3	0.648
Mean free path	λ	m	0.542×10^{-7}
Ratio of specific heats	γ	–	1.304
Prnadtl number	Pr	–	0.712
Viscosity index	ω	–	0.835
Conductivity index	ϖ	–	1.225
Specific-heat coefficient	s	–	0.758

Table 2. Physical properties of methane at the standard reference state: 288.15 K and 101.325 kPa (see Johnston & McCloskey, 1940; Clarke & Smith, 1969; Haberman & John, 1980; Sonntag et al., 1998; GPSA, 1998; Ivings et al., 2003).

In addition, the average distance between molecular collisions, called the molecular mean free path, plays an important role in rarefied gas transport behavior, related to the temperature and pressure by

$$\lambda = \frac{\sqrt{\pi \hat{R} T / 2} \, \mu}{p} \tag{5}$$

where $\hat{R}$ is the specific gas constant. According to the power-law behavior considered here, the closed form is

$$\lambda = \lambda_r \left(\frac{T}{T_r} \right)^{\omega + 1/2} \tag{6}$$

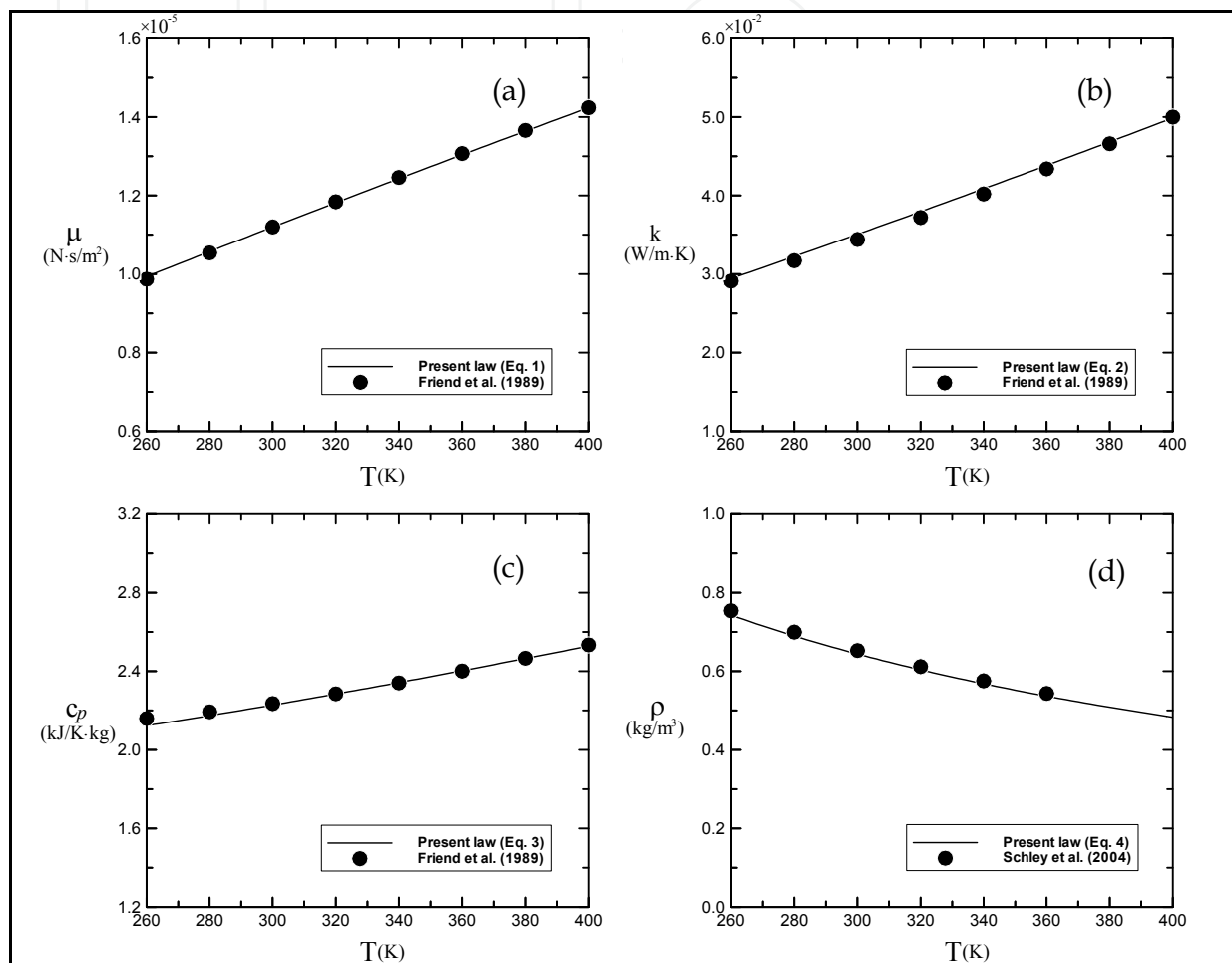Fig. 1. (a)–(d) Possible approximations of the physical property data listed in the literature. $T_r = 298.15$ K

3. Fundamental Principles

In this part, we introduce the fundamental principles of rarefied natural gas transport. Then you can use the mass, momentum, and energy conservation equations subject to the slip and jump boundary conditions to solve further problems involving mass or volume flow rate, to solve further problems involving force related to momentum change, and to solve further problems involving losses due to friction and energy input by compressors or extraction by turbine.

3.1 Mass balance principle

Conservation of mass is the key to tracking flowing fluid. The conservation statement of mass for a control volume is

$$\frac{\partial M_{cv}}{\partial t} = \sum \dot{m}_{in} - \sum \dot{m}_{out} \qquad (7)$$

where $\partial / \partial t$ is the time rate of change in a laboratory frame of reference, M is the instantaneous mass, $\dot{m}$ is the mass flow rate, the subscript cv denotes the control volume, and the subscripts in and out indicate the values for flow into and out of the control volume, respectively. The conservation statement requires that

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{u} = 0 \qquad (8)$$

where d / dt is the material derivative and $\mathbf{u}$ is the velocity vector.

Equation (8) is the mass conservation equation for compressible methane flow. In fact, the flow of a compressible fluid could be recognized as an incompressible flow while the flow speed is much less than the sound speed, i.e. Mach number<<1. For incompressible methane flow, the conservation equation is reduced to

$$\nabla \cdot \mathbf{u} = 0 \qquad (9)$$

3.2 Momentum balance principle

Newton's second law of motion is the conservation statement of momentum. It leads to the conclusion that forces can result from or cause changes in a flowing fluid's velocity magnitude and/or direction. The conservation statement of momentum for a control volume is

$$\frac{\partial (Mu_n)_{cv}}{\partial t} = \sum \dot{m}u_n)_+ - \sum \dot{m}u_n)_- + \sum F_n)_+ - \sum F_n)_- \qquad (10)$$

where u is the fluid velocity, Mu is the instantaneous momentum, $\dot{m}u$ is the momentum change rate associated with mass flow, F is the force acting on the control volume, the subscript n denotes the direction chosen for analysis, and the subscripts $+$ and $-$ indicate the values in the n or $-n$ direction on the control volume, respectively. The conservation statement requires that

$$\rho \frac{d\mathbf{u}}{dt} = \nabla \cdot \mathbf{T} + \rho \mathbf{f} \qquad (11)$$

where $\mathbf{T}$ is the stress tensor and $\mathbf{f}$ is the body force vector per unit mass.

The stress tensor for compressible fluids can be related to the flow fields by the following constitutive relation:

$$\mathbf{T} = \left(-p + \eta \nabla \cdot \mathbf{u} \right)\mathbf{I} + 2\mu \mathbf{D} \qquad (12)$$

where p is the pressure, η is the bulk viscosity, related to the viscosity μ by $\eta = -2\mu/3$, $\mathbf{I}$ is the Kronecker delta tensor, and $\mathbf{D}$ is the deformation rate tensor, $\mathbf{D} = \left((\nabla \mathbf{u})^T + \nabla \mathbf{u}\right)/2$. Substituting Eq. (12) into Eq. (11) gives

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \nabla(\eta \nabla \cdot \mathbf{u}) + 2\nabla \cdot (\mu \mathbf{D}) + \rho \mathbf{f} \qquad (13)$$

For incompressible methane flow with constant viscosity, the conservation equation is reduced to

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f} \qquad (14)$$

3.3 Energy balance principle

The first law of thermodynamics is the statement of conservation of energy. It leads to the conclusion that energies can result from or cause changes in a flowing fluid's velocity magnitude and/or temperature. The conservation statement of energy for a control volume is

$$\frac{\partial (Me)_{cv}}{\partial t} = \sum \dot{m}e)_{in} - \sum \dot{m}e)_{out} + \sum \dot{Q}_{in} - \sum \dot{Q}_{out} + \sum \dot{W} + \dot{O}, \qquad (15)$$

where e is the specific internal energy, Me is the instantaneous energy, $\dot{m}e$ is the energy change rate associated with mass flow, $\dot{Q}$ is the energy change rate associated with heat conduction, and $\dot{W}$ is the energy change rate associated with force, and $\dot{O}$ is the internal heat generation rate. The conservation statement requires that

$$\rho \frac{de}{dt} = -\nabla \cdot \mathbf{q} + \mathbf{T}^S : \mathbf{D} + \rho o \qquad (16)$$

where $\mathbf{q}$ is the heat flux vector, o is the internal heat generation rate per unit mass, and the superscript S indicates the symmetric tensor.

The heat flux vector can be related to the temperature field by the following constitutive relation (Fourier's law of conduction):

$$\mathbf{q} = -k\nabla T \qquad (17)$$

Substituting Eqs. (12) & (17) into Eq. (16) and recognizing $\rho de/dt = \rho c_p dT/dt - \beta T \dot{p} + (p/\rho) d\rho/dt$ (Bejan, 2004) gives

$$\rho c_p \frac{dT}{dt} = \nabla \cdot (k\nabla T) + \beta T \frac{dp}{dt} + \mu \Phi + \rho o \qquad (18)$$

where β is the thermal expansion coefficient and Φ is a function charactering the viscous dissipation, defined in index notation as

$$\Phi = \frac{\eta}{\mu}(u_{i,i})^2 + (u_{i,j}u_{i,j} + u_{i,j}u_{j,i}) \qquad (19)$$

For incompressible methane flow with constant thermal conductivity, the conservation equation is reduced to

$$\rho c_p \frac{dT}{dt} = k\nabla^2 T + \mu\Phi + \rho o \qquad (20)$$

where

$$\Phi = u_{i,j}u_{i,j} + u_{i,j}u_{j,i} \qquad (21)$$

3.4 Entropy balance principle

Any discussion of the fundamental principles must include the second law of thermodynamics because it is the basis for formulating and solving engineering problems. The second law of thermodynamics asserts that heat transfer processes are irreversible. To treat the second law quantitatively for processes, we use the conservation statement of entropy for a control volume:

$$\frac{\partial (Ms)_{cv}}{\partial t} = \sum \dot{m}s)_{in} - \sum \dot{m}s)_{out} + \sum \frac{\dot{Q}}{T}\bigg)_{in} - \sum \frac{\dot{Q}}{T}\bigg)_{out} + \dot{S}_{gen}, \qquad (22)$$

were S is the specific entropy, Ms is the instantaneous entropy, $\dot{m}s$ is the entropy rate associated with mass flow, $\dot{Q}/T$ is the entropy rate associated with heat conduction, and $\dot{S}_{gen}$ is the entropy generation rate. The irreversibility of the process is measured by the value of $\dot{S}_{gen}$. The conservation statement requires that

$$\rho \frac{ds}{dt} = -\nabla \cdot (\mathbf{q}/T) + \rho s_{gen} \qquad (23)$$

where s_{gen} is the entropy generation rate per unit mass. Substituting Eq. (17) into Eq. (23) gives

$$\rho \frac{ds}{dt} = \nabla \cdot (k\nabla T/T) + \rho s_{gen} \qquad (24)$$

3.5 Maxwell's and Smoluchowski's principles of boundary conditions

The effect of gas rarefaction is quantified by the Knudsen number Kn, which is defined as the ratio of the molecular mean free path λ to the characteristic length l_c. When the value of Kn is greater than 0.01, the conventional no-slip and no-jump boundary conditions may break down. The slip boundary conditions are based on Maxwell's expression (Maxwell, 1879):

$$u_{sl} = u_{wall} \pm \frac{2-\sigma_m}{\sigma_m}\frac{1}{\rho(2\hat{R}T_{gw}/\pi)^{1/2}}\tau_{rt} + \frac{3}{4}\frac{\gamma-1}{\gamma}\frac{\text{Pr}}{\rho\hat{R}T_{gw}}(-q_t) \qquad (25)$$

The jump boundary conditions are based on Smoluchowski's expression (Kennard, 1938):

$$T_{ju} = T_{wall} \pm \frac{2-\sigma_e}{\sigma_e} \frac{2(\gamma-1)}{\gamma+1} \frac{1}{\hat{R}\rho\left(2\hat{R}T_{gw}/\pi\right)^{1/2}} \left(-q_r\right) \tag{26}$$

Here, u_{sl} and T_{ju} are the velocity slip and temperature jump, respectively, u_{wall} and T_{wall} are the wall velocity and wall temperature, respectively, τ_{ij} is the stress tensor expressed in index notation, q_i is the heat flux vector expressed in index notation, σ_m and σ_e are the tangential momentum and thermal accommodation coefficients, respectively, the subscript gw denotes the gas values near the wall surface, and the subscripts r and t denote the gas values near the wall surface in the normal and tangential directions, respectively. Note that σ_m and σ_e are the parameters that describe the gas-wall interaction and related to the gas composition, temperature, and velocity over the surface as well as the solid surface temperature, roughness, and chemical state. The two accommodation coefficients range from near 0 to 1, where the two values, respectively, represent specular accommodation and complete (or diffuse) accommodation.

Let r and t denote the two coordinates near the wall surface and let u_r and u_t denote the components of velocity field. According to Eqs. (12) & (17), the shear stress τ_{rt}, the tangential heat flux q_t and the normal heat flux q_r are

$$\tau_{rt} = \mu\left(\frac{\partial u_r}{\partial t} + \frac{\partial u_t}{\partial r}\right)_{gw}, \quad q_t = -k\frac{\partial T}{\partial t}\bigg|_{gw}, \quad q_r = -k\frac{\partial T}{\partial r}\bigg|_{gw} \tag{27}$$

Substituting Eq. (27) into Eqs. (25) & (26) gives

$$u_{sl} = u_{wall} \pm \frac{2-\sigma_m}{\sigma_m}\lambda\frac{\partial u_t}{\partial r}\bigg|_{gw} + \frac{3}{2\pi}\frac{\gamma-1}{\gamma}\frac{c_p\rho}{\mu}\lambda^2\frac{\partial T}{\partial t}\bigg|_{gw} \tag{28}$$

$$T_{ju} = T_{wall} \pm \frac{2-\sigma_e}{\sigma_e}\frac{2\gamma}{\gamma+1}\frac{1}{Pr}\lambda\frac{\partial T}{\partial r}\bigg|_{gw} \tag{29}$$

4. Basic Transport Problems

Weng & Chen (2008c) have indicated that, on the basis of pressure or temperature difference, the three basic driving mechanisms are (1) pressure (2) buoyancy and (3) thermocreep, as shown in Fig. 2. Pressure and buoyancy are two conventional mechanisms. Thermal creep phenomenon (or simply transpiration) is a tangential flow along the wall surface from a cooler region to a hotter region. Previous work focuses mainly on creep effect on pressure-driven flow or buoyancy-driven flow. The effect during an externally applied pressure gradient or internally induced density variation is to hasten the flow, resulting in a streaming potential, whereas, in the absence of an external pressure gradient or internal density variation, the creep induces fluid flow when wall surface temperature gradient exists. In this part, we develop the mathematical models of rarefied natural gas transport in

the three basic driving mechanisms and obtain the analytical solutions of flow fields and characteristics. Then you can apply the analytical solutions of basic transport problems to determine further flow (or/and thermal) characteristics, predict and analyze further transport behavior of rarefied natural gas in pipelines, and understand why gas rarefaction in natural gas transport is so important; moreover, using the analytical procedure, you can conduct further analyses of theoretical researches.
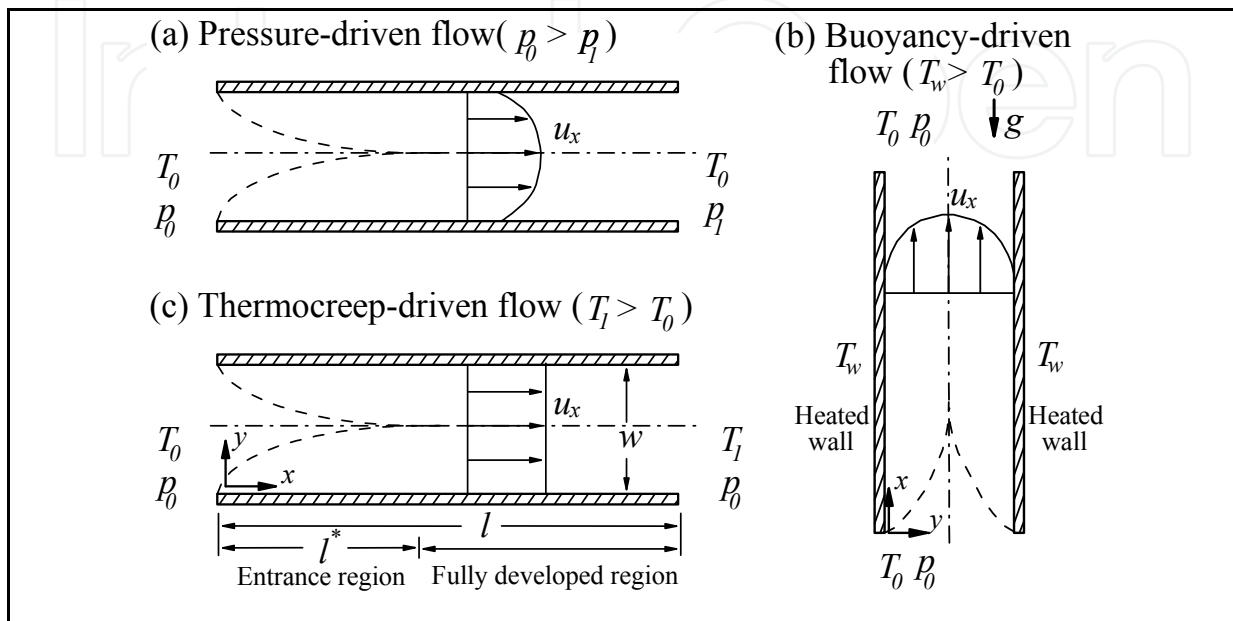

Fig. 2. Three basic driving mechanisms of rarefied gas dynamics (see Weng & Chen, 2008c).

Consider a sufficiently long open-ended parallel-plate channel of length l and width w. The flow section in the fully developed region is situated far from the entrance, and the flow is then fully developed. Assume that the flow originates from a reservoir in a reference state, enters the channel with a uniform velocity, and terminates in a discharge area. Let x and y denote the usual rectangular coordinates, let u_x and u_y denote the components of the velocity field, let the subscripts 0 and 1 denote the reservoir and discharge-area values, respectively, and let the subscript i denotes the inlet values.

4.1 Pressure-driven flow
If the driving mechanism is pressure, as shown in Fig. 2(a), then the reservoir pressure p_0 is greater than the discharge-area pressure p_1 and the discharge-area temperature T_1 is equal to the reservoir temperature T_0. Modeling the flow as a two-dimensional isothermal steady compressible flow, the field equations are given by (Eqs., 8 & 13)

$$\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} = 0 \tag{30}$$

$$\rho\left(u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu_r\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{1}{3}\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_y}{\partial x\partial y}\right)\right) \tag{31}$$

$$\rho\left(u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_r\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{1}{3}\left(\frac{\partial^2 u_x}{\partial x\partial y}+\frac{\partial^2 u_y}{\partial y^2}\right)\right) \tag{32}$$

The corresponding slip boundary conditions for a stationary unheated channel (Eq., 28) are

$$u_{sl}=\pm\frac{2-\sigma_m}{\sigma_m}\lambda\frac{\partial u_x}{\partial y}\bigg|_{gw} \tag{33}$$

Here, we have neglected the body force for simplicity. By using a perturbation analysis (Weng & Chen, 2008a), the momentum conservation equations (31) & (32) for the flow through a sufficiently long channel can be reduced to the form:

$$\frac{dp}{dx}=\mu_r\frac{\partial^2 u_x}{\partial y^2} \tag{34}$$

Proceeding with the analysis, we introduce the following dimensionless parameters:

$$\left.\begin{array}{l}X=\dfrac{x}{l_c},\ Y=\dfrac{y}{l_c},\ U=\dfrac{u_x}{u_c},\ V=\dfrac{u_y}{u_c},\ P=\dfrac{p}{p_c},\\[3mm]\mathrm{Kn}=\dfrac{\lambda}{l_c},\ \ \mathrm{We}=\dfrac{\rho_r\left(p_i-p_o\right)l_c^2}{\mu_r^2},\ \Lambda=\dfrac{\rho_r\sqrt{\pi\hat{R}T/2}l_c}{\mu_r}\end{array}\right\} \tag{35}$$

where We is the pressure drop from the entrance to the exit, Λ is a material constant, and the subscript c denotes the characteristic values. Here, the characteristic length l_c, velocity u_c, and pressure p_c are, respectively, defined as

$$l_c=w,\ u_c=\frac{\left(p_i-p_o\right)l_c}{\mu_r},\ p_c=\frac{\mu_r^2}{\rho_r l_c^2} \tag{36}$$

Substituting Eq. (35) into Eqs. (30), (33), (34) gives

$$\frac{\partial(PU)}{\partial X}+\frac{\partial(PV)}{\partial Y}=0 \tag{37}$$

$$\frac{dP}{dX}=\mathrm{We}\frac{\partial^2 U}{\partial Y^2} \tag{38}$$

$$\frac{u_{sl}}{u_c}=\pm\frac{2-\sigma_m}{\sigma_m}\mathrm{Kn}\frac{\partial U}{\partial Y}\bigg|_{gw} \tag{39}$$

Solving momentum conservation equation (38) subject to the slip boundary conditions (39) gives

$$U(X,Y) = \frac{1}{2\text{We}} \frac{dP(X)}{dX}\left(Y^2 - Y - \frac{2-\sigma_m}{\sigma_m}\text{Kn}(X)\right) \qquad (40)$$

Substituting Eq. (40) into mass conservation equation (37) and integrating once in Y, we can derive an equation for the cross-flow velocity:

$$V(X,Y) = -\frac{1}{24\text{We}}\left(\frac{1}{P}\frac{d^2P^2(X)}{dX^2}\left(2Y^3 - 3Y^2\right) - 12\left(\frac{2-\sigma_m}{\sigma_m}\text{Kn}(X)\frac{d^2P(X)}{dX^2}\right)Y\right) \qquad (41)$$

Evaluating this at $Y=1$, where V must vanish, we can derive an equation for the pressure:

$$\frac{1}{12}P^2 + \frac{2-\sigma_m}{\sigma_m}\Lambda P = C_1 X + C_0 \qquad (42)$$

where

$$\left.\begin{aligned} C_0 &= \frac{1}{12}\left(\text{We} + P_o\right)^2 + \frac{2-\sigma_m}{\sigma_m}\Lambda\left(\text{We} + P_o\right), \\ C_1 &= -\frac{1}{12L}\left(\text{We}^2 + 2P_o\text{We}\right) - \frac{1}{L}\frac{2-\sigma_m}{\sigma_m}\Lambda\text{We} \end{aligned}\right\} \qquad (43)$$

The corresponding mass flow rate is

$$\dot{M} = \frac{p\int_0^w u_x dy}{p_c u_c l_c} = P\int_0^1 U dY = \frac{1}{24L}\left(\text{We} + 2\left(P_o + 6\frac{2-\sigma_m}{\sigma_m}\Lambda\right)\right) \qquad (44)$$

or

$$\dot{M} = \frac{\overline{P}}{12L}\left(1 + 6\frac{2-\sigma_m}{\sigma_m}\overline{\text{Kn}}\right) \qquad (45)$$

where L is the dimensionless channel length, $\overline{P}$ is the dimensionless average pressure, $\overline{P} = (P_i + P_o)/2$, and $\overline{\text{Kn}}$ is the average Knudsen number calculated at $\overline{P}$.

4.2 Buoyancy-driven flow

If the driving mechanism is buoyancy, as shown in Fig. 2(b), then the wall temperature T_w is greater than the ambient temperature (that is, $T_1 = T_0$ and $T_w > T_0$) and the discharge-area pressure p_1 is equal to the reservoir pressure p_0. Modeling the flow as a two-dimensional steady incompressible flow, the field equations under the Boussinesq approximation (Boussinesq, 1903) are given by (Eqs., 9, 14, 20)

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \tag{46}$$

$$\rho_r \left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} \right) = -\frac{\partial \hat{p}}{\partial x} + \rho_r g \beta_r (T - T_r) + \mu_r \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} \right) \tag{47}$$

$$\rho_r \left(u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} \right) = -\frac{\partial \hat{p}}{\partial y} + \mu_r \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} \right) \tag{48}$$

$$\rho_r c_{pr} \left(u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} \right) = k_r \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{49}$$

where $\hat{p}$ is the pressure defect, related to p by $\hat{p} = p - p_h$, where p_h is the hydrostatic pressure. The corresponding slip and jump boundary conditions for a stationary isothermally heated channel (Eqs., 28 & 29) are

$$u_{sl} = \pm \frac{2 - \sigma_m}{\sigma_m} \lambda_r \frac{\partial u_x}{\partial y}\bigg|_{gw} + \frac{3}{2\pi} \frac{\gamma_r - 1}{\gamma_r} \frac{c_{pr}\rho_r}{\mu_r} \lambda_r^2 \frac{\partial T}{\partial x}\bigg|_{gw} \tag{50}$$

$$T_{ju} = T_w \pm \frac{2 - \sigma_e}{\sigma_e} \frac{2\gamma_r}{\gamma_r + 1} \frac{1}{\mathrm{Pr}_r} \lambda_r \frac{\partial T}{\partial y}\bigg|_{gw} \tag{51}$$

Here, we have neglected the internal heat generation for simplicity. It should be noted that assuming a small temperature difference between the wall and the ambient gas supports the constant-property assumption (Weng & Chen 2008b) and that considering the low-speed flow of a low-Prandtl-number fluid supports the neglect of viscous dissipation in the energy equation (Chen & Weng 2005; Weng & Chen 2008b).

We can think of the fully developed region as the flow section situated far from the entrance such that u_y is negligible. Based on this characterization, the mass conservation equation (46) requires that $\partial u_x / \partial x = 0$. In most treatments, $u_y = 0$ and $\partial u_x / \partial x = 0$ are taken as a starting point in the analysis of fully developed flow. The momentum conservation equations (47) & (48) then become

$$0 = \rho_r g \beta_r (T - T_r) + \mu_r \frac{d^2 u_x}{dy^2} \tag{52}$$

A solution of Eq. (52) in the form $u_x(Y)$ is only possible if T is a function of y position only, i.e., $\partial T / \partial X = 0$. It implies that the assumption of a hydrodynamically fully developed flow necessarily means that the flow is also thermally fully developed. The energy conservation equations (49) and the slip boundary conditions (50) then be reduced to

$$0 = \frac{d^2 T}{dy^2} \tag{53}$$

$$u_{sl} = \pm \frac{2 - \sigma_m}{\sigma_m} \lambda_r \frac{\partial u_x}{\partial y}\bigg|_{gw} \tag{54}$$

Proceeding with the analysis, we introduce the following dimensionless parameters:

$$X = \frac{x}{l_c}, \quad Y = \frac{y}{l_c}, \quad U = \frac{u_x}{u_c}, \quad \Theta = \frac{T - T_r}{T_w - T_r} \tag{55}$$

Here, the characteristic length l_c and velocity u_c are, respectively, defined as

$$l_c = w, \quad u_c = \frac{\rho_r g \beta_r (T_w - T_r) l_c^2}{\mu_r} \tag{56}$$

Substituting Eq. (55) into Eqs. (51)–(54) gives

$$\frac{d^2 U}{dY^2} = -\Theta \tag{57}$$

$$\frac{d^2 \Theta}{dY^2} = 0 \tag{58}$$

$$\frac{u_{sl}}{u_c} = \pm \frac{2 - \sigma_m}{\sigma_m} \mathrm{Kn}_r \frac{dU}{dY}\bigg|_{gw} \tag{59}$$

$$\frac{T_{ju} - T_r}{T_w - T_r} = 1 \pm \frac{2 - \sigma_e}{\sigma_e} \frac{2\gamma_r}{\gamma_r + 1} \frac{1}{\mathrm{Pr}_r} \mathrm{Kn}_r \frac{d\Theta}{dY}\bigg|_{gw} \tag{60}$$

Equations (57) & (58) subject to (59) & (60) have the following velocity and temperature analytical solutions:

$$U(Y) = -\frac{1}{2}\left(Y^2 - Y - \frac{2 - \sigma_m}{\sigma_m} \mathrm{Kn}_r\right) \tag{61}$$

$$\Theta(Y) = 1 \tag{62}$$

The corresponding mass flow rate is

$$\dot{M} = \frac{\int_0^w u_x dy}{u_c l_c} = \int_0^1 U dY = \frac{1}{2}\left(\frac{2 - \sigma_m}{\sigma_m} \mathrm{Kn} + \frac{1}{6}\right), \tag{63}$$

4.3 Thermocreep-driven flow

If the driving mechanism is thermocreep, as shown in Fig. 2(c), then the reservoir temperature T_0 is less than the discharge-area pressure T_1 and the discharge-area pressure p_1 is equal to the reservoir pressure p_0. Modeling the flow as a two-dimensional steady incompressible constant-property flow, the momentum and energy equations under the fully developed flow limit ($u_y = 0$ and $\partial u_x / \partial x = 0$) are given by (Eqs., 9, 14, 20)

$$0 = -\frac{dp}{dx} + \mu_r \frac{d^2 u_x}{dy^2} \tag{64}$$

$$\rho_r c_{pr} u_x \frac{\partial T}{\partial x} = k_r \frac{\partial^2 T}{\partial y^2} + \mu_r \left(\frac{du_x}{dy}\right)^2 \tag{65}$$

Here, we have used the Prandtl boundary layer theory (Prandtl, 1904) to omit nonessential terms shown in field equations. It should be noted that assuming a small temperature difference between the reservoir and the discharge area supports the constant-property assumption.

Proceeding with the analysis, we introduce the following dimensionless parameters:

$$\left.\begin{array}{l} X = \dfrac{x}{l_c \mathrm{Gr}}, \ Y = \dfrac{y}{l_c}, \ U = \dfrac{u_x}{u_c}, \ \Theta = \dfrac{T - T_r}{T_1 - T_0}, \ P = \dfrac{p}{p_c}, \\[3mm] \mathrm{Gr} = \dfrac{\rho_r^2 g \beta_r (T_1 - T_0) l_c^3}{\mu_r^2}, \ \mathrm{Ec} = \dfrac{u_c^2}{c_{pr}(T_1 - T_0)} \end{array}\right\} \tag{66}$$

Here, the characteristic length l_c, velocity u_c, and pressure p_c are, respectively, defined as

$$l_c = w, \ u_c = \frac{\rho_r g \beta_r (T_1 - T_0) l_c^2}{\mu_r}, \ p_c = \rho_r u_c^2 \tag{67}$$

Substituting Eq. (66) into Eqs. (64), (65), and (50) gives

$$\frac{dP}{dX} - \frac{d^2 U}{dY^2} = 0 \tag{68}$$

$$\mathrm{Pr}_r U \frac{\partial \Theta}{\partial X} = \frac{\partial^2 \Theta}{\partial Y^2} + \mathrm{Pr}_r \, \mathrm{Ec} \left(\frac{dU}{dY}\right)^2 \tag{69}$$

$$\frac{u_{sl}}{u_c} = \pm \frac{2 - \sigma_m}{\sigma_m} \mathrm{Kn}_r \left.\frac{dU}{dY}\right|_{gw} + \frac{3}{2\pi} \frac{\gamma_r - 1}{\gamma_r} \frac{1}{\mathrm{Ec}} \mathrm{Kn}_r^2 \left.\frac{\partial \Theta}{\partial X}\right|_{gw} \tag{70}$$

From Eqs. (68) and (70), a solution of Eq. (68) in the form $U(Y)$ is only possible if dP/dX and $\partial \Theta / \partial X$ are constants (let C_0 and C_1, respectively). The constant value in temperature

gradient implies that the flow under the assumption of hydrodynamically fully developed flow is also thermally fully developed.

The momentum equation (68) can be integrated twice to obtain the streamwise velocity:

$$U(Y) = \frac{1}{2}C_0 Y^2 + C_2 Y + C_3 \tag{71}$$

Substituting Eq. (71) into the energy equation (69) and integrating the resultant with respect to Y twice and the thermally fully developed condition $\partial\Theta/\partial X = C_1$ with respect to X once, we obtain the temperature:

$$\begin{aligned}
\Theta(X, Y) &= \Pr C_1 \left(\frac{1}{24}C_0 Y^4 + \frac{1}{6}C_2 Y^3 + \frac{1}{2}C_3 Y^2 \right) \\
&\quad - \Pr \text{Ec} \left(\frac{1}{12}C_0^2 Y^4 + \frac{1}{3}C_0 C_2 Y^3 + \frac{1}{2}C_2^2 Y^2 \right) + C_4 Y + C_5 + C_1 X.
\end{aligned} \tag{72}$$

Integrating the pressure gradient $dP/dX = C_0$ with respect to X once, we obtain the pressure:

$$P(X) = P_0 + C_6 + C_0 X \tag{73}$$

By applying the boundary conditions given in Eq. (70), the inlet condition $P(0) = P_0 - \dot{M}^2/2$ (Chen & Weng, 2006), the outlet conditions $P(L) = P_0$, the midline condition $\partial\Theta(X, 1/2)/\partial Y = 0$, and the edge conditions $\Theta(L, 0) = 1$ and $\Theta(0, 0) = 0$, the six unknown constants C_0, C_1, C_2, C_3, C_4, C_5, and C_6 can be obtained as

$$\left. \begin{aligned}
&C_0 = \frac{1}{2}\frac{\dot{M}^2}{L}, \ C_1 = \frac{1}{L}, \\
&C_2 = -\frac{1}{2}C_0, \ C_3 = -\frac{1}{2}\beta_m C_0 + \beta_{mc} C_1, \\
&C_4 = \frac{\Pr_r}{24}\left((1 + 6\beta_m)C_0 C_1 - 12\beta_{mc} C_1^2 + \text{Ec}C_0^2 \right), \\
&C_5 = 0, \ C_6 = -\frac{1}{2}\dot{M}^2,
\end{aligned} \right\} \tag{74}$$

where

$$\beta_m = \frac{2 - \sigma_m}{\sigma_m}\text{Kn}, \ \beta_{mc} = \frac{3}{2\pi}\frac{\gamma - 1}{\gamma}\frac{1}{\text{Ec}}\text{Kn}^2, \ L = \frac{l}{l_c \text{Gr}}, \ \dot{M} = \frac{u_i}{u_c} = \int_0^1 U dY \tag{75}$$

By using the flow-rate expression $\dot{M} = \int_0^1 U dY$, the channel length L can be obtained as

$$L = -\frac{1}{24}(1 + 6\beta_m)\dot{M} + \frac{\beta_{mc}}{\dot{M}} \tag{76}$$

5. Summary

In this chapter, the property formulas of natural gases are provided in power-law form. To simply predict the physical properties of natural gases, the physical properties of methane at the standard reference state are presented. The basic flows are analyzed by using important principles including conservation of mass, Newton's second law of motion, and the first and second laws of thermodynamics.

The following checklist provides a study guide for this chapter. When your study of the entire chapter has been completed, you should be able to

- use the property formulas of gases to present further properties of other hydrocarbons, such as ethane, propane, butane, etc.
- use the physical properties of methane, in conjunction with the properties of other gases as necessary, to calculate further physical properties of natural gases in most common operating states.
- use the property formulas of gases and the physical properties of methane at the standard reference state to conduct further analyses of theoretical and experimental researches.
- use the mass conservation equation to solve further problems involving mass or volume flow rate.
- use the momentum conservation equation subject to the slip boundary conditions to solve further problems involving force related to momentum change.
- use the energy conservation equation subject to the jump boundary conditions to solve further problems involving losses due to friction and energy input by compressors or extraction by turbine.
- use the analytical procedure shown in basic transport problems to conduct further analyses of theoretical researches.
- apply the analytical solutions of basic transport problems to determine further flow (or/and thermal) characteristics, predict and analyze further transport behavior of rarefied natural gas in pipelines, and understand why gas rarefaction in natural gas transport is so important.

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7. References

Arkilic, E. B.; Schmidt, M. A. & Breuer, K. S. (1997). Gaseous slip flow in long microchannels. *J. Microelectromech. Systems*, 6, 167–178.

Bejan, A., (2004). *Convection heat transfer*, John Wiley & Sons, 0471271500, 3rd edition.

Beskok, A. & Karniadakis, G. E. (1999). A model for flows in channels, pipes, and ducts at micro and nano scales. *Microscale Thermophys. Eng.*, 3, 43–77.

Boussinesq, J. (1903). *Theorie analytique de la chaleur heat dissipation of parallel plates by free convection*, Gauthier-Villars, Paris.

Burgdorfer, A. (1959). The influence of molecular mean free path on the performance of hydrodynamic gas lubricated bearings. *ASME J. Basic Eng.,* 81, 94–100.

Carr, N. L. (1954). Viscosity of hydrocarbon gases under pressure. *J. Pet. Technol.,* 6, 47–55.

Chen, C.-K. & Weng, H. C. (2005). Natural convection in a vertical microchannel. *ASME J. Heat Transfer*, 127, 1053–1056.

Chen, C.-K. & Weng, H. C. (2006). Developing natural convection with thermal creep in a vertical microchannel. *J. Phys. D,* 39, 3107–3118.

Clarke, A. G. & Smith, E. B. (1969). Low-temperature viscosities and intermolecular forces of simple gases. *J. Chem. Phys.,* 51, 4156–4161.

Ewart, T.; Perrier, P.; Graur, I. & Méolans, J. G. (2007). Mass flow rate measurements in a microchannel, from hydrodynamic to near free molecular regimes. *J. Fluid Mech.,* 584, 337–356.

Friend, D. G.; Ely, J. F. & Ingham, H. (1989). Thermophysical properties of methane. *J. Phys. Chem. Ref. Data,* 18, 583–638.

Gammon, B. E. & Douslin, D. R. (1976). The velocity of sound and heat capacity in methane from near-critical to subcritical conditions and equation-of-state implications. J. Chem. Phys., 64, 203–218.

GPSA (1998). *Engineering Data Book*, Section 23–Physical properties, Gas Processors Suppliers Association, Tulsa, Oklahoma, 11th edition.

Haberman, W. L. & John, J. E. A. (1980) *Engineering thermodynamics,* Allyn and Bacon, Inc., Boston.

Hurly, J. J.; Gillis, K. A.; Mehl, J. B. & Moldover, M. R. (2003). The viscosity of seven gases measured with a greenspan viscometer. *Int. J. Thermophys.,* 24, 1441–1474.

Hsia, Y. T. & Domoto, G. A. (1983). An experimental investigation of molecular rarefaction effects in gas lubricated bearings at ultra-low clearances. *ASME J. Lubrication Tech.,* 105, 120–130.

Ivanov, M. S. & Gimelshein, S. F. (1998). Computational hypersonic rarefied flows. *Ann. Rev. Fluid Mech.,* 30, 469–505.

Ivings, M. J.; Lea, C. J. & Ledin, H. S. (2003). Outstanding safety questions concerning the analysis of ventilation and gas dispersion in gas turbine enclosures: best practice guidelines for CFD, *Technical Report CM/03/12*, Health and Safety Laboratory.

Jansoone, V.; Gielen, H. & de Boelpaep, J. (1970). The pressure-temperature-volume relationship of methane near the critical point. *Physica.,* 46, 213–221.

Johnston, H. L. & McCloskey, K. E. (1940). Viscosities of Several Common Gases between 90°K. and Room Temperature. J. Phys. Chem., 44, 1038–1058.

Jin, G. X.; Tang, S. & Sengers, J. V. (1992). Thermodynamic properties of methane in the critical region. *Int. J. Thermophys.,* 13, 671–684.

Kennard, E. H. (1938) *Kinetic theory of gasses,* McGraw-Hill, New York.

Kerley, G. I. (1980). A theoretical equation of state for methane," *J. Appl. Phys.,* 51, 5368–5374.

Kleinrahm, R. & Wagner, W. (1986). Measurement and correlation of the equilibrium liquid and vapour densities and the vapour pressure along the coexistence curve of methane. *J. Chem. Thermodynam.,* 18, 739–760.

Kleinrahm, R.; Duschek, W. & Wagner, W. (1986) (Pressure, density, temperature) measurements in the critical region of methane. *J. Chem. Thermodynam.,* 18, 1103–1114.

Kurumov, D. S.; Olchowy, G. A. & Sengers, J. V. (1988). Thermodynamic properties of methane in the critical region. *Int. J. Thermophys.*, 9, 73–84.

Mann, W. B. & Dickins, B. G. (1931). The Thermal Conductivities of the Saturated Hydrocarbons in the Gaseous State. *Proc. R. Soc. Lond. A,* 134, 77–96.

Maxwell, J. C. (1879). On stress in rarefied gases from inequalities of temperature. *Philos. Trans. R. Soc. London,* 170, 231–256.

Pátek, J. & Klomfar, J. (2002). Measurement of the thermal conductivity of argon and methane: a test of a transient hot-wire apparatus. *Fluid Phase Equilib.,* 198, 147–163.

Pfahler, J.; Harley, J. C.; Bau, H. & Zemel, J. N. (1991). Gas and liquid flow in small channels, *Winter Annual Meeting of the American Society of Mechanical Engineers,* 32, 49–60, 0791808637, Atlanta, December 1991, ASME, New York.

Pong, K. C.; Ho, C. M.; Liu, J. & Tai, Y. C. (1994). Nonlinear pressure distribution in uniform microchannels, *Proceedings of the 1994 International Mechanical Engineering Congress and Exposition*, 197, pp. 51–56, Chicago, November 1994, ASME, New York.

Prandtl, L. (1904) On fluid motions with very small friction (in German), *Proceedings of the 3rd International Mathematical Congress*, pp. 484–491, Heidelberg, Germany.

Schaaf, S. A. & Chambre, P. L. (1961). *Flow of rarefied gases,* Princeton University Press, New Jersey.

Schley, P.; Jaeschke, M.; Kuchenmeister, C. & Vogel, E. (2004). Viscosity measurements and predictions for natural gas.

Sonntag, R. E.; Borgnakke, C. & Wylen, G. J., (1998). *Fundamentals of thermodynamics*, John Wiley & Sons, 047118361X, New York, 5th edition.

Tsuboi, N. & Matsumoto, Y. (2005). Experimental and numerical study of hypersonic rarefied gas flow over flat plates. *AIAA Journal,* 43, 1243–1255.

Vennix, A. J.; Leland Jr., T. W. & Kobayashi, R. (1970). Low-temperature volumetric properties of methane. *J. Chem. Eng. Data,* 15, 238–243.

Viswanathan, A. (2007). *Viscosities of natural gases at high pressures and high temperatures,* MS thesis, Texas A&M University, College. Station, Texas.

Weng, H. C. & Chen, C.-K. (2008a). A challenge in Navier–Stokes-based continuum modeling: Maxwell–Burnett slip law. *Phys. Fluids,* 20, 106101.

Weng, H. C. & Chen, C.-K. (2008b). Variable physical properties in natural convective gas microflow. *ASME J. Heat Transf.,* 130, 082401.

Weng, H. C. & Chen, C.-K. (2008c). Fully developed thermocreep-driven gas microflow. *Appl. Phys. Lett.,* 92, 094105.

Younglove, B. A. (1974). The specific heats cp and cv of compressed and liquified methane. *J. Res. Natl. Bur. Stand.,* 78A, 401–410.
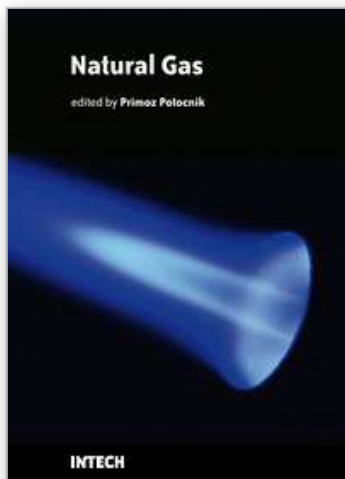
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The contributions in this book present an overview of cutting edge research on natural gas which is a vital component of world's supply of energy. Natural gas is a combustible mixture of hydrocarbon gases, primarily methane but also heavier gaseous hydrocarbons such as ethane, propane and butane. Unlike other fossil fuels, natural gas is clean burning and emits lower levels of potentially harmful by-products into the air. Therefore, it is considered as one of the cleanest, safest, and most useful of all energy sources applied in variety of residential, commercial and industrial fields. The book is organized in 25 chapters that cover various aspects of natural gas research: technology, applications, forecasting, numerical simulations, transport and risk assessment.

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