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ROBUST CONTROL DESIGN FOR TWO-LINK NONLINEAR ROBOTIC SYSTEM

Shield B. Lin[+] and Sheng-Guo Wang*

[+] *Prairie View A&M University, Prairie View, Texas*
USA
University of North Carolina at Charlotte, Charlotte, North Carolina
USA

1. Introduction

A good performance of robot control requires the consideration of efficient dynamic models and sophisticated control approaches. Traditionally, control law is designed based on a good understanding of system model and parameters. Thus, a detailed and correct model of a robot manipulator is needed for this approach [1, 2].

A two-link planar nonlinear robotic system is a well-used robotic system, e.g., for welding in manufacturing and so on. Generally, a dynamic model can be derived from the general Lagrange equation method. The modeling of a two-link planar nonlinear robotic system with assumption of only masses in the two joints can be found in the literature, e.g., [3, 4]. Here, the authors revise centrifugal and Coriolis force matrix in the literature [3, 4] as pointed out in the next section. Furthermore, in practice, the robot arms have their mass distributed along their arms, not only masses in the joints as assumed. Thus, it is desired to develop a detailed model for two-link planar robotic systems with the mass distributed along the arms. Distributed mass along robot arms was discussed by inertia in SCARA robot [5]. Here, we present a new detailed consideration of any mass distributions along robot arms in addition to the joint mass.

Moreover, it is also necessary to consider numerous uncertainties in parameters and modeling. Thus, robust control, robust adaptive control and learning control become important when knowledge of the system is limited. We need robust stabilization of uncertain robotic systems and furthermore robust performance of these uncertain robotic systems. Robust stabilization problem of uncertain robotic control systems has been discussed in [1-3, 5-6] and many others. Also, adaptive control methods have been discussed in [1, 7] and many others. Because the closed-loop control system pole locations determine internal stability and dominate system performance, such as time responses for initial conditions, papers [6, 8] consider a robust pole clustering in vertical strip on the left half s-plane to consider robust stability degree and degree of coupling effects of a slow subsystem (dominant model) and the other fast subsystem (non-dominant model) in a two-time-scale system. A control design method to place the system poles robustly within a vertical strip has been discussed in [6, 8-10], especially [6] for robotic systems. However, as mentioned

above, there is a need of a detailed and practical two-link planar robotic system modeling with the practically distributed robotic arm mass for control.

Therefore, this chapter develops a practical and detailed two-link planar robotic systems modeling and a robust control design for this kind of nonlinear robotic systems with uncertainties via the authors' developing robust control approach with both H∞ disturbance rejection and robust pole clustering in a vertical strip. The design approach is based on the new developing two-link planar robotic system models, nonlinear control compensation, a linear quadratic regulator theory and Lyapunov stability theory.

2. Modeling of Two-Link Robotic Systems

The dynamics of a rigid revolute robot manipulator can be described as the following nonlinear differential equation [1, 2, 6, 10]:

$$F_c = M(q)\ddot{q} + V(q,\dot{q})\dot{q} + N(q,\dot{q}) \tag{1.a}$$

$$N(q,\dot{q}) = G(q) + F_d\dot{q} + F_s(\dot{q}) \tag{1.b}$$

where $M(q)$ is an $n \times n$ inertial matrix, $V(q,\dot{q})$ an $n \times n$ matrix containing centrifugal and coriolis terms, $G(q)$ an $n \times 1$ vector containing gravity terms, $q(t)$ an $n \times 1$ joint variable vector, F_c an $n \times 1$ vector of control input functions (torques, generalized forces), F_d an $n \times n$ diagonal matrix of dynamic friction coefficients, and $F_s(\dot{q})$ an $n \times 1$ Nixon static friction vector.

However, the dynamics of the robotic system (1) in detail is needed for designing the control force, i.e., especially, what matrices $M(q)$, $V(q,\dot{q})$ and $G(q)$ are.

Consider a general two-link planar robotic system in Fig. 1, where the system has its joint mass m_1 and m_2 of joints 1 and 2, respectively, robot arms mass m_{1r} and m_{2r} distributed along arms 1 and 2 with their lengths l_1 and l_2, generalized coordinates q_1 and q_2, i.e., their rotation angles, $q = [q_1 \ q_2]'$, control torques (generalized forces) f_1 and f_2, $F_c = [f_1 \ f_2]'$.
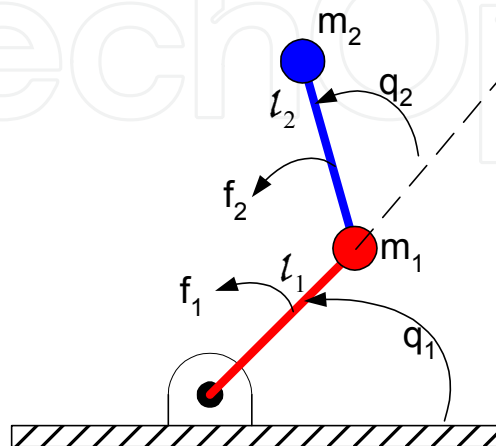

Fig. 1. A two-link manipulator

Theorem 1. A general two-link planar robotic system has its dynamic model as in (1) with

$$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{2}$$

$$
\begin{aligned}
M_{11} &= (m_1 + \xi_1 m_{r1} + m_2 + m_{r2})l_1^2 \\
&\quad + (m_2 + \xi_2 m_{r2})l_2^2 + 2(m_2 + \zeta_2 m_{r2})l_1 l_2 \cos q_2
\end{aligned} \tag{3.a}
$$

$$
\begin{aligned}
M_{12} = M_{21} &= (m_2 + \xi_2 m_{r2})l_2^2 \\
&\quad + (m_2 + \zeta_2 m_{r2})l_1 l_2 \cos q_2
\end{aligned} \tag{3.b}
$$

$$M_{22} = (m_2 + \xi_2 m_{r2})l_2^2 \tag{3.c}$$

$$V(q, \dot{q}) = -(m_2 + \zeta_2 m_{r2})l_1 l_2 \dot{q}_2 \sin q_2 \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \tag{4}$$

$$G(q) = g \begin{bmatrix} (m_1 + \zeta_1 m_{r1} + m_2 + m_{r2})l_1 \cos q_1 + (m_2 + \zeta_2 m_{r2})l_2 \cos(q_1 + q_2) \\ (m_2 + \zeta_2 m_{r2})l_2 \cos(q_1 + q_2) \end{bmatrix} \tag{5}$$

where g is the gravity acceleration, m_1 and m_2 are joints 1 and 2 mass, respectively, m_{r1} and m_{r2} are total mass of arms 1 and 2, which are distributed along their arm lengths of l_1 and l_2, the scaling coefficients ξ_1, ξ_2, ζ_1 and ζ_2 are defined as follows:

$$
\begin{aligned}
\xi_i &= \int_0^{l_i} \rho_i(l) S_i(l) l^2 dl / m_{ri} l_i^2 , \quad \zeta_i = \int_0^{l_i} \rho_i(s) S_i(l) l dl / m_{ri} l_i , \\
m_{ri} &= \int_0^{l_i} \rho_i(l) S_i(l) dl , \quad i = 1, 2
\end{aligned} \tag{6}
$$

where $\rho_1(l)$ and $\rho_2(l)$ are the arm mass density functions along their length l, $S_1(l)$ and $S_2(l)$ are the arm cross-sectional area functions along the length l.

Proof: The proof is via Lagrange method and dynamic motion equations. The mass distribution can be various by introducing the above new scaling coefficients. Due to the page limit, detail of the proof is omitted.

Remark 1. From (2)–(4) in Theorem 1, $\dot{M}(q) = V(q, \dot{q})$. Theorem 1 is also different from the result in [3-6]. Especially, there are no corresponding items of ζ_i in [3-6].

Corollary 1. A two-link planar robotic system with consideration of only joint mass has its dynamic model as in (1) and Theorem 1, but with

$$m_{ri} = 0 , \quad \xi_i = 0 , \quad \zeta_i = 0 , \quad i = 1, 2 \tag{7}$$

It means that its inertia matrix $M(q)$ in (2), and

$$
\begin{aligned}
M_{11} &= (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2 m_2 l_1 l_2 \cos q_2 , \\
M_{12} = M_{21} &= m_2 (l_2^2 + l_1 l_2 \cos q_2) , \quad M_{22} = m_2 l_2^2
\end{aligned} \tag{8}
$$

$$V(q,\dot{q}) = -m_2 l_1 l_2 \dot{q}_2 \sin q_2 \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \qquad (9)$$

$$G(q) = g \begin{bmatrix} (m_1 + m_2) l_1 \cos q_1 + m_2 l_2 \cos(q_1 + q_2) \\ m_2 l_2 \cos(q_1 + q_2) \end{bmatrix} \qquad (10)$$

Remark 2. It is noticed that centrifugal and Coriolis matrix $V(q,\dot{q})$ in (26) is equivalent to:

$$V(q,\dot{q}) = m_2 l_1 l_2 \sin q_2 \begin{bmatrix} -\dot{q}_2 & -\dot{q}_1 - \dot{q}_2 \\ -\dot{q}_2 & 0 \end{bmatrix} \qquad (11)$$

in (1). Note that the Coriolis matrix is different from some earlier literatures in [3, 4].

Theorem 2. Consider a two-link planar robotic system having its robot arms with uniform mass distribution along the arm length. Thus, its dynamic model is as (1) – (6) of Theorem 1 with its scaling factors as follows:

$$\xi_1 = \xi_2 = 1/3 \text{ , and } \zeta_1 = \zeta_2 = 1/2 \qquad (12)$$

Proof: It can be proved by Theorem 1 and the uniform mass distribution in (6).

Theorem 3. Consider a two-link planar robotic system having its robot arms with linear tapered-shapes respectively along the arm lengths as:

$$r_i(l) = r_{0i} - k_i l \text{ , } 0 \le l \le l_i \text{ , } S_i(l) = \mu_i r_i^2(l) \text{ , } i = 1,2 \qquad (13)$$

where $r_i(l)$ in length is a general measure of the arm cross-section at the arm length l, e.g., as a radius for a disk, a side length for a square, $\sqrt{ab}$ for a rectangular with sides a and b, etc., $S_i(l)$ is the cross-sectional area of arm i at its length position l, μ_i is a constant, e.g., as π for a circle and 1 for a square. Assume that arm 1 and arm 2 respectively have their two end cross-sectional areas as:

$$S_{01} = S_1(0) \text{ , } S_{t1} = S_1(l_1) \text{ , } S_{02} = S_2(0) \text{ , } S_{t2} = S_2(l_2) \qquad (14)$$

where $S_{0i} > S_{ti}$, $i = 1,2$. Their density functions are constants as $\rho_i(l) = \rho_i$, $i = 1,2$. Then, its dynamic model is as in (1) – (6) of Theorem 1 with its scaling factors:

$$\xi_i = \frac{S_{0i} + 6S_{ti} + 3\sqrt{S_{0i}S_{ti}}}{10(S_{0i} + S_{ti} + \sqrt{S_{0i}S_{ti}})} \text{ , } \zeta_i = \frac{S_{0i} + 3S_{ti} + 2\sqrt{S_{0i}S_{ti}}}{4(S_{0i} + S_{ti} + \sqrt{S_{0i}S_{ti}})} \qquad (15)$$

$i = 1,2$, and its arm mass:

$$m_{ri} = \rho_i l_i (S_{0i} + S_{ti} + \sqrt{S_{0i}S_{ti}})/3 \text{ , } i = 1,2 \qquad (16)$$

Proof: It is proved by substituting (13) and (14) into (6) in Theorem 1 and further derivations.

Remark 3. The scaling factors (15) and the arm mass (16) in Theorem 3 may have other equivalent formulas, not listed here due to the page limit. Here, we choose (15) and (16) because the two-end cross-sectional areas of each arm are easily found from the design parameters or measured in practice. The arm cross-sectional shapes can be general in (13) in Theorem 3.

3. Robust Control

In view of possible uncertainties, the terms in (1) can be decomposed without loss of any generality into two parts, i.e., one is known parts and another is unknown perturbed parts as follows [2, 6]:

$$M = M_0 + \Delta M , \quad N = N_0 + \Delta N , \quad V = V_0 + \Delta V \tag{17}$$

where M_0, N_0, V_0 are known parts, ΔM, ΔN, ΔV are

unknown parts. Then, the models in Section 2 can be used not only for the total uncertain robotic systems with uncertain parameters, but also for a known part with their nominal parameters of the systems.

Following our [6], we develop the torque control law as two parts as follows:

$$F_c = M_0(q)\ddot{q}_d + V_0(q,\dot{q})\dot{q} + N_0(q,\dot{q}) - M_0(q)u \tag{18}$$

where the first part consists of the first three terms in the right side of (18), the second part is the term of u that is to be designed for the desired disturbance rejection and pole clustering, q_d is the desired trajectory of q, however, the coefficient matrices are as (2) – (6) in Theorem 1 with all nominal parameters of the system. Define an error between the desired q_d and the actual q as:

$$e = q_d - q . \tag{19}$$

From (1) and (17)–(19), it yields:

$$\ddot{e} = M^{-1}(q)[\Delta M(q)\ddot{q}_d + \Delta V(q,\dot{q})\dot{q} + \Delta N(q,\dot{q}) + M_0(q)u]$$
$$= w + E\dot{e} + Fu + u \tag{20}$$

$$E = -M^{-1}(q)\Delta V(q,\dot{q}), \quad F = -M^{-1}(q)\Delta M(q)$$

$$w = -F\ddot{q}_d - E\dot{q}_d + M^{-1}\Delta N \tag{21}$$

From [6], we can have the fact that their norms are bounded:

$$\|w\| < \delta_w , \quad \|E\| < \delta_e , \quad \|F\| < \delta_f \tag{22}$$

Then, it leads to the state space equation as:

$$\dot{x} = Ax + Bu + B[0 \ E]x + BFu + Bw \tag{23}$$

$$x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} = [e_1 \ e_2 \ \dot{e}_1 \ \dot{e}_2]', \ A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{24}$$

The last three terms denote the total uncertainties in the system. The desired trajectory q_d for manipulators to follow is to be bounded functions of time. Its corresponding velocity $\dot{q}_d$ and acceleration $\ddot{q}_d$, as well as itself q_d, are assumed to be within the physical and kinematic limits of manipulators. They may be conveniently generated by a model of the type:

$$\ddot{q}_d(t) + K_v \dot{q}_d(t) + K_p q_d(t) = r(t) \tag{25}$$

where $r(t)$ is a 2-dimensional driving signal and the matrices K_v and K_p are stable.
The design objective is to develop a state feedback control law for control u in (18) as

$$u(t) = -Kx(t) \tag{26}$$

such that the closed-loop system:

$$\dot{x} = (A - BK + B \begin{bmatrix} 0 & E \end{bmatrix} - BFK)x + Bw \tag{27}$$

has its poles robustly lie within a vertical strip Ω:

$$\lambda(A_c) \in \Omega = \{s = x + jy | -\alpha_2 < x < -\alpha_1 \leq 0\} \tag{28}$$

and a δ-degree disturbance rejection from the disturbance w to the state x, i.e.,

$$\left\| T_{xw}(s) \right\|_\infty = \left\| (sI - A_c)^{-1} B \right\|_\infty \leq \delta \tag{29}$$

$$A_c = A - BK + B \begin{bmatrix} 0 & E \end{bmatrix} - BFK \tag{30}$$

From [6], we derive the following robust control law to achieve this objective.
Theorem 4. Consider a given robotic manipulator uncertain system (27) with (1)–(6), (17)-(22), (24), where the unstructured perturbations in (21) with the norm bounds in (22), the disturbance rejection index $\delta > 0$ in (29), the vertical strip Ω in (28) and a matrix Q>0. With the selection of the adjustable scalars ε_1 and ε_2, i.e.,

$$(1 - \delta_f)/\delta_e > \varepsilon_1 > 0, \ (1 - \delta_f - \varepsilon_1 \delta_e)\delta > \varepsilon_2 > 0 \tag{31}$$

there always exists a matrix $P > 0$ satisfying the following Riccati equation:

$$A'_{\alpha 1}P + PA_{\alpha 1} - (1 - \delta_f - \varepsilon_1 \delta_e - \varepsilon_2 / \delta)PBB'P$$
$$+ (\delta_e / \varepsilon_1)I + (1 / \varepsilon_2 \delta)I + Q = 0 \tag{32}$$

where

$$A_{\alpha 1} = A + \alpha_1 I = \begin{bmatrix} \alpha_1 I_2 & I_2 \\ 0 & \alpha_1 I_2 \end{bmatrix} \tag{33}$$

Then, a robust pole-clustering and disturbance rejection control law in (18) and (26) to satisfy (29) and (30) for all admissible perturbations E and F in (22) is as:

$$u = -Kx = -rB'Px \tag{34}$$

if the gain parameter r satisfies the following two conditions:

$$\text{(i)} \quad r \geq 0.5 \text{ and} \tag{35}$$

$$\text{(ii)} \quad \begin{matrix} 2\alpha_2 P + A'P + PA - (\delta_e / \varepsilon_1)I \\ -[2r(1 + \delta_f) + \varepsilon_1 \delta_e]PBB'P > 0 \end{matrix} \tag{36}$$

Proof: Please refer to the approach developed in [6, 8] and utilizing the new model in Section 2.
It is also noticed that:

$$BB' = \begin{bmatrix} 0 & 0 \\ 0 & I_2 \end{bmatrix} \tag{37}$$

It is evident that condition (i) is for the α_1-degree stability and δ-degree disturbance rejection, and condition (ii) is for the α_2-degree decay, i.e., the left vertical bound of the robust pole-clustering.

Remark 4. There is always a solution for relative stability and disturbance rejection in the sense of above discussion. It is because the Riccati equation (32) guarantees a positive definite solution matrix P, and thus there exists a Lyapunov function to guarantee the robust stability of the closed loop uncertain robotic systems. The nonlinear compensation part in (18) has a similar function to a feedback linearization. The feature differences of the proposed method from other methods are the new nominal model, and the robust pole-clustering and disturbance attenuation for the whole uncertain system family. It is further noticed that the robustly controlled system may have a good Bode plot for the whole frequency range in view of Theorem 4, inequality (29) and its H-infinity norm upper bound.

Remark 5. The tighter robust pole-clustering vertical strip $-\alpha_1 > \text{Re} \, \lambda(A_c) \geq -\underline{\alpha}_2$ has

$$\underline{\alpha}_2 = \alpha_1 + 0.5\overline{\lambda}\{P^{-1/2}[-A'P - PA_{\alpha 1} + (\delta_e / \varepsilon_1)I$$
$$+ (2r(1 + \delta_f) + \varepsilon_1 \delta_e)PBB'P]P^{-1/2}\} \tag{38}$$

4. Examples

Example 1. Consider a two-link planar manipulator example (Fig. 1). First, only joint link masses are considered for simplicity, as the one in [3, 6]. However, we take the correct model in Corollary 1 and Remark 2 into account. The system parameters are: link mass $m_1 = 2kg$, $m_2 = 10kg$, lengths $l_1 = 1m$, $l_2 = 1m$, angular positions q_1, q_2 (rad), applied torques f_1, f_2 (Nm). Thus, the nominal values of coefficient matrices for the dynamic equation (1) in Corollary 1 are:

$$M_0(q) = \begin{bmatrix} 22 + 20C_2 & 10(1 + C_2) \\ 10(1 + C_2) & 10 \end{bmatrix}, V_0(q, \dot{q}) = -10S_2\dot{q}_2\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

$$N_0(q, \dot{q}) = g\begin{bmatrix} 12C_1 + 10C_{12} \\ 10C_{12} \end{bmatrix} \tag{39}$$

where $C_i = \cos q_i$, $i = 1, 2$, $C_{12} = \cos(q_1 + q_2)$, $S_2 = \sin q_2$, and g is the gravity acceleration.

The desired trajectory is $q_d(t)$ in (25) with $K_v = 0$, and

$K_p = I$, the signal $r(t) = \begin{bmatrix} 0.5 & 1 \end{bmatrix}'$, the initial values of the desired states $q_d(0) = \begin{bmatrix} 2 & 2 \end{bmatrix}'$, $\dot{q}_d(0) = 0$, i.e.,

$$q_{d1}(t) = 1.5\cos t + 0.5, \quad \text{and} \quad q_{d2}(t) = \cos t + 1 \tag{40}$$

The initial states are set as $q_1(0) = q_2(0) = -2$, and $\dot{q}_1(0) = \dot{q}_2(0) = 0$, i.e., $e_1(0) = 4$, $e_2(0) = 4$, $\dot{e}_1(0) = 0$, and $\dot{e}_2(0) = 0$. The state variable is $x = [e' \ \dot{e}']'$ where $e = q_d - q$. The parametric uncertainties are assumed to satisfy (22) with $\delta_f = 0.5$, $\delta_e = 40$, $\delta_N = 10$. Select the adjustable parameters $\varepsilon_1 = 0.012$, $\varepsilon_2 = 0.0015$ from (31), disturbance rejection index $\delta = 0.1$, the relative stability index $\alpha_1 = 0.1$, and the left bound of vertical strip $\alpha_2 = 2000$ since we want a fast response. By Theorem 4, we solve the Riccati equation (32) to get the solution matrix P and the gain matrix as:

$$P = \begin{bmatrix} 12693I_2 & 1584I_2 \\ 1584I_2 & 1643I_2 \end{bmatrix}, K = rB'P = [950.1823I_2 \ 985.7863I_2]$$

with $r = 0.6$. The eigenvalues of the closed-loop main system matrix $A - BK$ are {-0.9648, -0.9648, -984.8215, -984.8215}. Remark 5 gives the result $-\underline{\alpha}_2 = -1873$. The uncertain closed-loop system has its $-\alpha_2 < \text{Re}[\lambda(A_c)] < -\alpha_1$ robustly.

The total control input (law) is

$$F_c = F_T = M_0\ddot{q}_d + V_0(q, \dot{q})\dot{q} + N_0 - M_0 u$$

$$= \begin{bmatrix} 22 + 20C_2 & 10(1 + C_2) \\ 10(1 + C_2) & 10 \end{bmatrix} \begin{bmatrix} -1.5\cos t \\ -\cos t \end{bmatrix}$$

$$+ 10S_2\dot{q}_2 \begin{bmatrix} -2 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + g \begin{bmatrix} 12C_1 + 10C_{12} \\ 10C_{12} \end{bmatrix} +$$

$$\begin{bmatrix} 22 + 20C_2 & 10(1 + C_2) \\ 10(1 + C_2) & 10 \end{bmatrix} \begin{bmatrix} 950.1823I_2 & 985.7863I_2 \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \qquad (41)$$

A simulation for this example is taken with $\Delta M(q) = 0.4M_0(q)$, i.e., 40% disturbance, $\left\| M^{-1}(q)\Delta M(q) \right\| = 0.2857 < \delta_f = 0.5$, $\Delta V_m(q,\dot{q}) = 0.2V_{m0}(q,\dot{q})$ with 20% disturbance, and $\Delta N(q) = 0.2N_0(q)$ with 20% disturbance. The simulation results by MATLAB and Simulink are shown in Figs. 2-3.
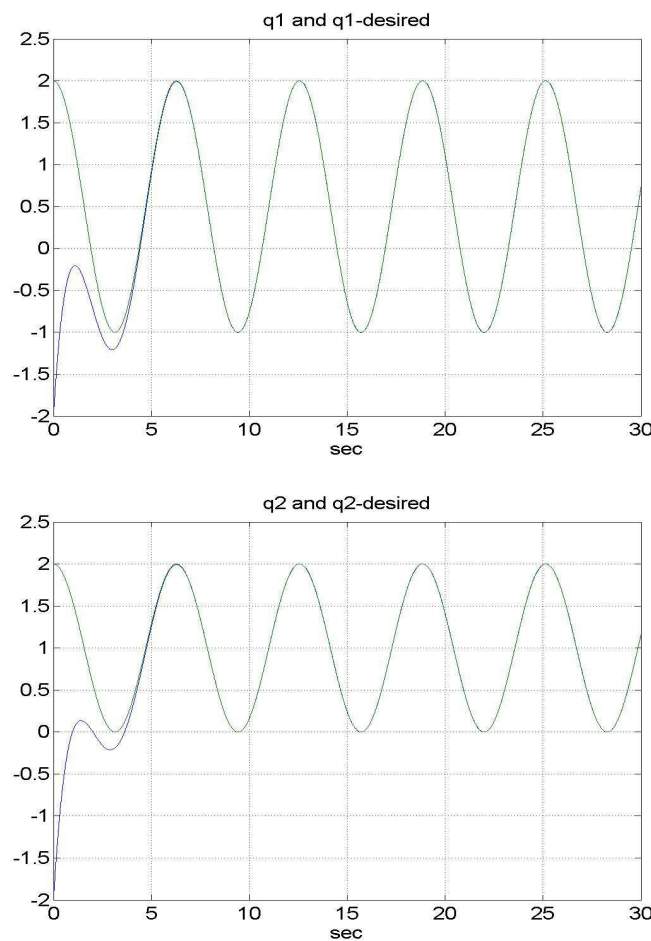
Fig. 2. States and their desired states: (a) $q_1(t)$ & $q_{1d}(t)$, (b) $q_2(t)$ & $q_{2d}(t)$ in Example 1
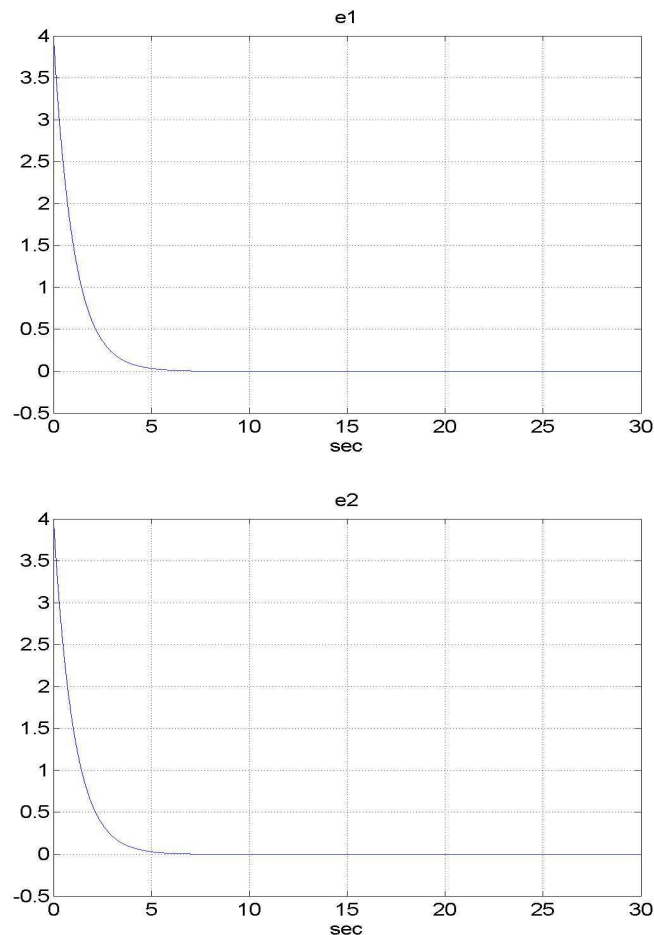
Fig. 3. Error signals: (a) $e_1(t)$, (b) $e_2(t)$ in Example 1

Example 2. Consider a two-link planar robotic system example with the joint mass and the arm mass along the arm length. The mass of joint 1 is $m_1 = 1\,\text{kg}$, and the mass of joint 2 is $m_2 = 0.5\,\text{kg}$. The dimensions of two robot arms are linearly reduced round rods. The two terminal radii of the arm rod 1 are $r_{01} = 3\,\text{cm}$ and $r_{t1} = 2\,\text{cm}$. The two terminal radii of the arm rod 2 are $r_{02} = 2\,\text{cm}$ and $r_{t1} = 1\,\text{cm}$. Their end cross-sectional areas are $S_{01} = 9\pi\,\text{cm}^2$, $S_{t1} = 4\pi\,\text{cm}^2$, $S_{02} = 4\pi\,\text{cm}^2$, and $S_{t2} = 1\pi\,\text{cm}^2$. The arm length and mass are $l_1 = 1\,\text{m}$, $l_2 = 1\,\text{m}$, $m_{r1} = 5.2\,\text{kg}$, and $m_{r2} = 1.9\,\text{kg}$. By Theorem 3, it leads to:

$$\xi_1 = 0.2684, \ \xi_2 = 0.2286, \ \zeta_1 = 0.4342, \ \zeta_2 = 0.3929 \tag{42}$$

By Theorem 3, the nominal model parameters are:

$$M_0(q) = \begin{bmatrix} 5.7301 + 2.4929C_2 & 0.9343 + 1.2464C_2 \\ 0.9343 + 1.2464C_2 & 0.9343 \end{bmatrix} \ V_0(q, \dot{q}) = -1.2464S_2\dot{q}_2 \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

$$N_0(q, \dot{q}) = g \begin{bmatrix} 5.6579C_1 + 1.2464C_{12} \\ 1.2464C_{12} \end{bmatrix} \tag{43}$$

The desired trajectory $q_d(t)$ is the same as in Example 1. The initial states are set as $q_1(0) = -2$, $q_2(0) = 0$, $\dot{q}_1(0) = \dot{q}_2(0) = 0$, i.e., $e_1(0) = 4$, $e_2(0) = 2$, $\dot{e}_1(0) = 0$, $\dot{e}_2(0) = 0$.

The parametric uncertainties in practice are assumed to satisfy (22) with $\delta_f = 0.25$, $\delta_e = 10$, $\delta_N = 10$. Select the adjustable parameters $\varepsilon_1 = 0.0375$ and $\varepsilon_2 = 0.0188$ from (31), the disturbance rejection index $\delta = 0.1$, the relative stability index $\alpha_1 = 0.1$, and the left bound of vertical strip $\alpha_2 = 100$. By Theorem 4, the solution matrix P to (32) and the gain matrix K are

$$P = \begin{bmatrix} 898.87I_2 & 13.55I_2 \\ 13.55I_2 & 12.47I_2 \end{bmatrix}, K = rB'P = [65.0589I_2 \ 59.8659I_2]$$

with $r = 4.8$. The eigenvalues of the closed-loop main system matrix $A - BK$ are $\{-1.1072, -58.7587, -1.1072, -58.7587\}$. The uncertain system has $-\alpha_2 < \text{Re}[\lambda(A_c)] < -\alpha_1$ robustly.

The total control input (law) is :

$$f = M_0\ddot{q}_d + V_0(q,\dot{q})\dot{q} + N_0 - M_0 u$$

$$= \begin{bmatrix} 5.7301 + 2.4929C_2 & 0.9343 + 1.2464C_2 \\ 0.9343 + 1.2464C_2 & 0.9343 \end{bmatrix} \cdot \begin{bmatrix} -1.5\cos t \\ -\cos t \end{bmatrix}$$

$$- 1.2464S_2\dot{q}_2\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + g\begin{bmatrix} 5.6579C_1 + 1.2464C_{12} \\ 1.2464C_{12} \end{bmatrix}$$

$$+ \begin{bmatrix} 5.7301 + 2.4929C_2 & 0.9343 + 1.2464C_2 \\ 0.9343 + 1.2464C_2 & 0.9343 \end{bmatrix}$$

$$\cdot [65.0589I_2 \quad 59.8659I_2] \cdot [e' \quad \dot{e}']' \qquad (44)$$

The simulation is taken with $\Delta M(q) = 0.25M_0(q)$, $\Delta V_m(q,\dot{q}) = 0.1V_{m0}(q,\dot{q})$, and $\Delta N(q) = 0.1N_0(q)$. The results are shown in Figs. 4-5. It is noticed that the error may be reduced when the gain parameter r is set large.
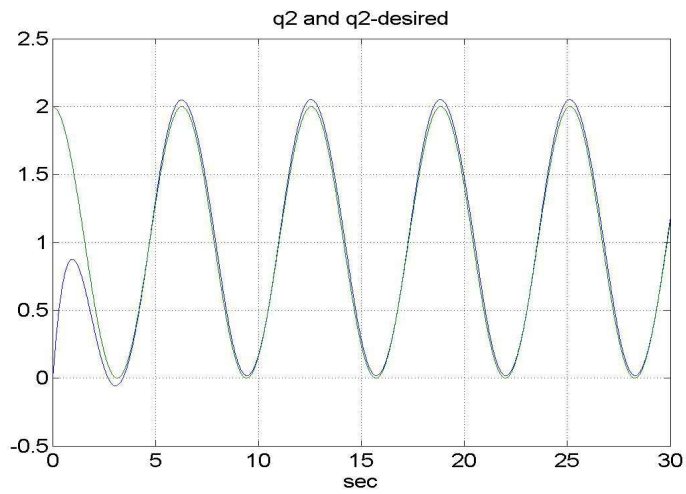

q1 and q1-desired
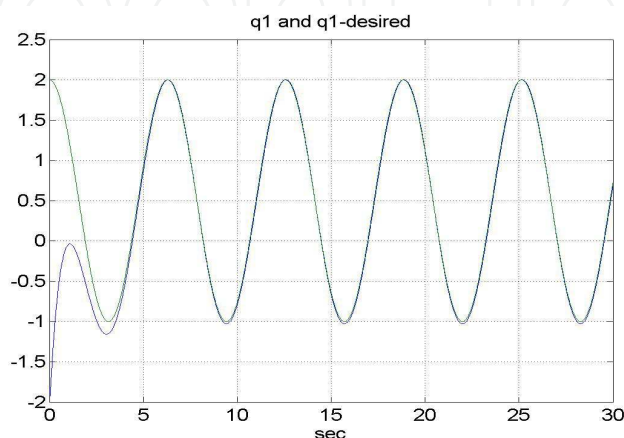
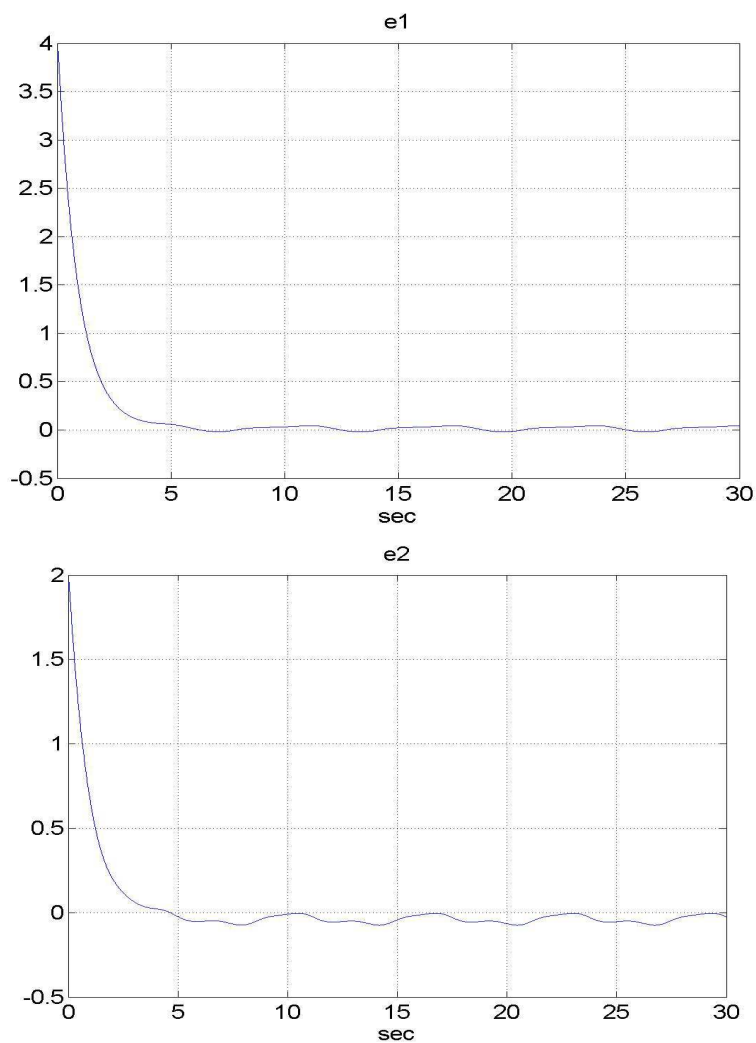Fig. 4. States and their desired states: (a) $q_1(t)$ & $q_{1d}(t)$, (b) $q_2(t)$ & $q_{2d}(t)$

Fig. 5. Error signals: (a) $e_1(t)$, & (b) $e_2(t)$

5. Conclusion

The chapter develops the practical models of two-link planar nonlinear robotic systems with their arm distributed mass in addition to the joint-end mass. The new scaling coefficients are introduced for solving this problem with the distributed mass along the arms. In addition, Theorems 2 and 3 respectively present two special cases: a uniform arm shape (i.e., uniform distributed mass) and a linear reduction of arm shape along the arm length.

Based on the presented new models, an approach to design a continuous nonlinear control law with a linear state-feedback control for the two-link planar robotic uncertain nonlinear systems is presented in Theorem 4. The designed closed-loop systems possess the properties of robust pole-clustering within a vertical strip on the left half s-plane and disturbance rejection with an H_∞-norm constraint. The suggested robust control for the uncertain nonlinear robotic systems can guarantee the required robust stability and performance in face of parameter errors, state-dependent perturbations, unknown parameters, frictions, load variation and disturbances for all allowed uncertainties in (22). The presented robust control does always exist as pointed out in Remark 4. The adjustable scalars ε_i, i=1, 2, provide some flexibility in finding a solution of the algebraic Riccati equation. The designed uncertain system has α_1-degree robust stabilization and δ-degree disturbance rejection. The controller gain parameter r is selected such that the designed uncertain linear system achieves robust pole-clustering within a vertical strip. The examples illustrate excellent results. This design procedure may be used for designing other control systems, modeling, and simulation.

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Advances in Robot Manipulators

Edited by Ernest Hall

The purpose of this volume is to encourage and inspire the continual invention of robot manipulators for science and the good of humanity. The concepts of artificial intelligence combined with the engineering and technology of feedback control, have great potential for new, useful and exciting machines. The concept of eclecticism for the design, development, simulation and implementation of a real time controller for an intelligent, vision guided robots is now being explored. The dream of an eclectic perceptual, creative controller that can select its own tasks and perform autonomous operations with reliability and dependability is starting to evolve. We have not yet reached this stage but a careful study of the contents will start one on the exciting journey that could lead to many inventions and successful solutions.

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