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Dynamic Trajectory-Tracking Control of an Omnidirectional Mobile Robot Based on a Passive Approach

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1. Introduction

The traditional control problems of trajectory-tracking and regulation have been extensively studied in the field of mobile robotics. In particular, the differential and the omnidirectional mobile robots, also known, respectively, as the (2,0) and the (3,0) robots (see (Bétourné & Campion, 1996), (Campion et al., 1996), have attracted the interest of many control researchers.

It is a common practice in mobile robotics to address control problems taking into account only a kinematic representation. In (Canudas et al., 1996) and (Campion et al., 1996) the kinematic models for diverse types of mobile robots are presented.

From a kinematic perspective, the trajectory-tracking control problem of (2,0)-type robot has been addressed and solved for example in (D'Andrea-Novel et al., 1992) following a dynamic feedback linearization approach. In (Oriolo et al., 2002) a real time implementation of a dynamic feedback linearization tracking-controller is presented. For the same class of robot, a discrete time approach is considered in (Niño-Suárez et al., 2006) where a path-tracking controller based on a sliding mode control technique is presented.

The regulation and trajectory-tracking problems for the omnidirectional mobile robot (3,0), have also received sustained attention. Considering, only its kinematic model, several control strategies have been proposed. In (Liu et al., 2003), it is designed a nonlinear controller based on a Trajectory Linearization strategy and in (Velasco-Villa et al., 2007), the remote control of the (3,0) mobile robot is developed based on a discrete-time strategy assuming a time-lag model of the robot. In (Velasco-Villa et al., 2007b) the trajectory-tracking problem is solved by means of an estimation strategy that predicts the future values of the system based on the exact nonlinear discrete-time model of the robot.

A more reduced number of contributions have been focused on the dynamic representation of the omnidirectional mobile robot. For example, in (Carter et al., 2001), it is described the mechanical design of a (3,0) robot and based on its dynamic model it is proposed a PID control for each robot wheel. Authors in (Bétourné & Campion, 1996) consider an Euler-

Lagrange model formulation and present an output feedback controller that solves the trajectory-tracking problem. In the same spirit, in (Williams et al., 2002) the dynamic model of the mobile robot is considered in order to study the slipping effects between the wheels of the vehicle and the working surface. In (Chung et al., 2003), the mobile robot is analyzed in the case of a vehicle supporting castor wheels. In (Vázquez & Velasco-Villa, 2008) the trajectory tracking problem is addressed and solved by considering a modification of the well known Computed-Torque strategy. Finally, in (Kalmár-nagy et al. 2004) the time-optimization problem of a desired trajectory is considered for a mobile robot subject to admissible input limits in order to obtain feedback laws that are based on the kinematic and dynamic models.

The analysis of Euler-Lagrange systems has produced several feedback strategies that have been developed mainly in the area of robot manipulators and, lately, in the area of power electronics. In particular, passivity-based control approaches that consider the energy managing structure of the system, for instance: the interconnection and damping assignment technique developed in (Ortega et al., 2001), and passivity-based approaches that exploit the passivity properties of the exact tracking error dynamics and its natural passive output are taken in (Ortega et al., 1998), (Sira-Ramírez & Rodríguez-Cortés, 2008), (Sira-Ramrez, 2005) and (Sira-Ramrez & Silva-Ortigoza, 2006).

We address and solve the trajectory-tracking problem of an omnidirectional mobile robot taking into account its dynamic model. Contrary to the differential case, the considered mobile robot it is not affected by non-holonomics constraints. Following ideas developed in the field of power electronics, in this work, we consider a passivity based control technique that exploit the passivity properties of the exact tracking error addressed as: Exact Tracking Error Dynamics Passive Output Feedback (ETEDPOF), (Sira-Ramrez, 2005), (Sira-Ramrez & Silva-Ortigoza, 2006). The performance of the proposed control strategy is contrasted through numerical simulations with the well-known Computed-Torque Control (Vázquez & Velasco-Villa, 2008) for a desired circular trajectory.

This paper is organized as follows: Section 2 presents the dynamic model of the omnidirectional mobile robot and some structural properties are stated. A brief recall of the Computed-Torque solution is also provided. Immediately, in Section 3 the trajectory-tracking problem is solved by the Exact Tracking Error Dynamics Passive Output Feedback. Closed loop stability is formally proven. In Section 4, the performance of the developed strategy is evaluated by means of numerical simulations and compared with the solution obtained by the Computed-Torque control technique. Section 5 presents some conclusions.

2. Omnidirectional Mobile Robot

A top view of the configuration of a (3,0) mobile robot is depicted in Figure 1. The mobile reference frame $X_m - Y_m$ is located at the center of mass of the vehicle with the X_m axis aligned with respect to the wheel 3. Wheels 1 and 2 are placed symmetrically with an angle $\delta = 30°$ with respect to the Y_m axis. The fixed reference frame $X - Y$ provides the absolute localization of the vehicle on the workspace. The mobile robot is of the type (Canudas et al., 1996) $(\delta_m, \delta_s) = (3,0)$, this is, it has three degrees of mobility and zero degrees of steerability allowing the displacements of the vehicle in all directions instantaneously.

2.1 Dynamic Model

Mobile robot velocity expressed in $X-Y$ coordinates is defined as (Campion et al., 1996),

$$\dot{q} = R^T(\phi)u \tag{1}$$

with,

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad R(\phi) = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
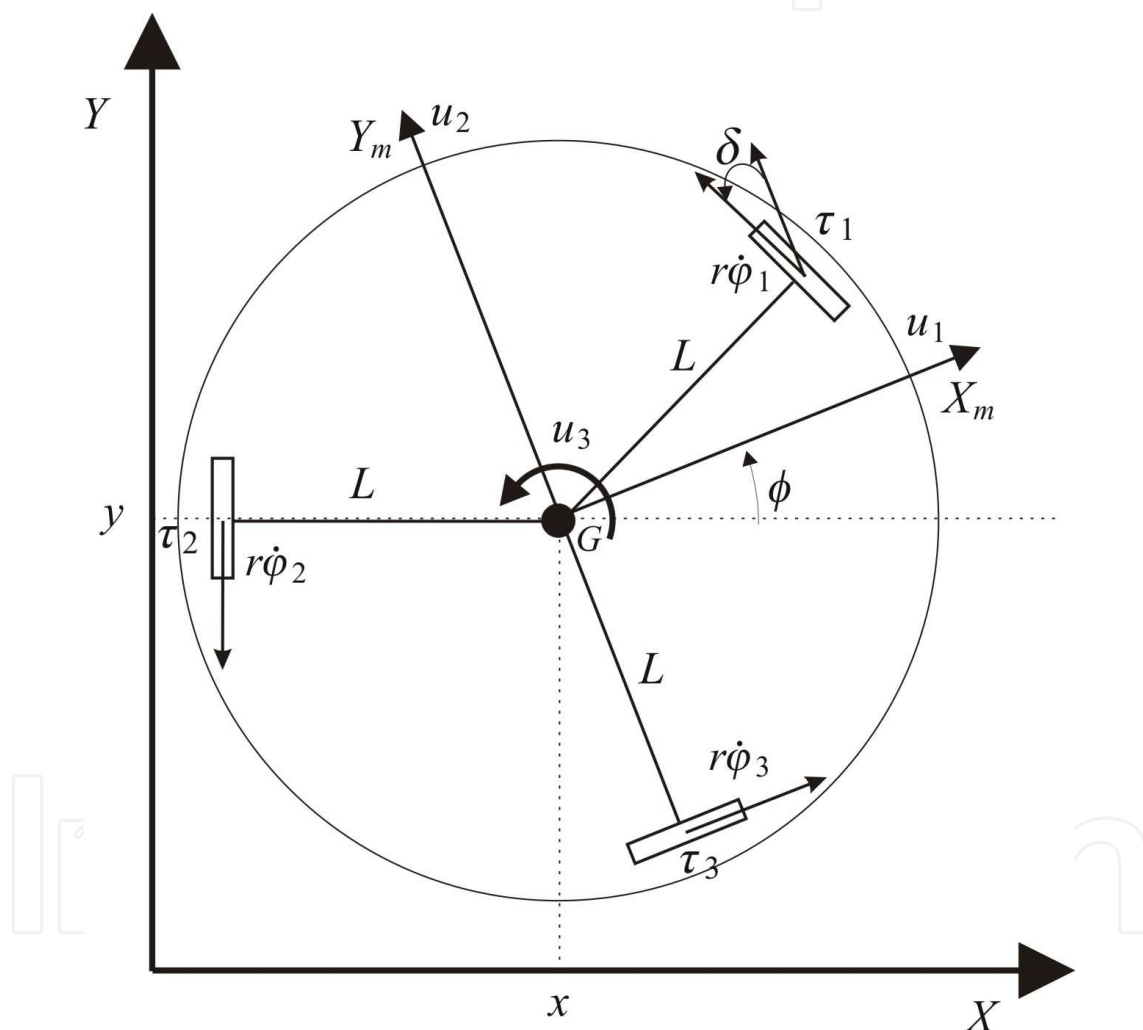

Fig. 1. Omnidirectional Mobile Robot

The point (x, y) is the position of the center of mass of the robot on the plane $X-Y$ and ϕ is the robot orientation with respect to the X-axis. u_1, u_2 are the mobile robot linear velocity expressed in the mobile reference system and u_3 is the rotational velocity measured in the mobile reference system.

A simple analysis of the velocity constrains on Figure 1 produces,

$$J_1 R(\phi)\dot{q} - J_2\dot{\varphi} = 0 \tag{2}$$

with,

$$J_1 = \begin{bmatrix} -\sin\delta & \cos\delta & L \\ -\sin\delta & -\cos\delta & L \\ 1 & 0 & L \end{bmatrix}, \quad J_2 = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix},$$

$\varphi = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 \end{bmatrix}^{\mathrm{T}}$, where φ_1, φ_2, φ_3 represent the angular displacements of wheels one, two and three, respectively; δ is the orientation of the i-wheel with respect to its longitudinal axis; L is the distance between the center of each wheel and the center of the vehicle and r is the radius of each wheel.

Following (Campion et al., 1996)-(Canudas et al., 1996), the kinetic energy of the robot is given by the wheel rotational energy plus the translational and rotational energy of the robot. Therefore, the Lagrangian of the system is obtained as,

$$\mathsf{L} = \frac{1}{2}\dot{q}^T R^T(\phi)MR(\phi)\dot{q} + \frac{1}{2}\sum_{i=1}^{3}\dot{\varphi}_i^{T}I_r\dot{\varphi}_i, \tag{3}$$

with $M = diag(M_p, M_p, I_p)$ and $I_r = diag(I_\varphi, I_\varphi, I_\varphi)$. M_p is the vehicle mass I_p the moment of inertia about the Z axis of the vehicle and I_φ is the moment of inertia of each wheel about its rotation axis.

Considering that the kinematics restrictions (2) are satisfied for all t, it is possible to neglect the friction and slipping effects between the wheels and the working surface. Then, the Euler-Lagrange formulation produces the system representation,

$$[M + R^T(\phi)E^T I_r ER(\phi)]\ddot{q} + R^T(\phi)E^T I_r E\dot{R}(\phi)\dot{q} = R^T(\phi)E^T\tau$$

where $\tau = \begin{bmatrix} \tau_1 & \tau_2 & \tau_3 \end{bmatrix}^{\mathrm{T}}$, with τ_i the input torque of each wheel and $E = J_2^{-1}J_1$. Equivalently

$$D\ddot{q} + C(\dot{q})q = B\tau, \tag{4}$$

where,

$$D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_1 & 0 \\ 0 & 0 & d_3 \end{bmatrix}, \quad C(\dot{q}) = a\begin{bmatrix} 0 & \dot{\phi} & 0 \\ -\dot{\phi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B = \frac{1}{r}\begin{bmatrix} -\sin(\delta+\phi) & -\sin(\delta-\phi) & \cos\phi \\ \cos(\delta+\phi) & -\cos(\delta-\phi) & \sin\phi \\ L & L & L \end{bmatrix},$$

with $d_1 = M_p + \dfrac{3I_r}{2r^2}$, $d_3 = I_p + \dfrac{3I_rL^2}{r^2}$ and $a = \dfrac{3I_r}{2r^2}$.

2.2 Structural properties

In what follows, some structural properties of the dynamic model (4) are stated. These properties will be necessary to synthesize the proposed control strategy.

Property 1 *The vector* $C(\dot{q})\dot{q}$ *does not possess a unique representation, in particular, for the development of the feedback law, the following alternative parametrization will be considered,*

$$C(\dot{q})\dot{q} = a\begin{bmatrix} 0 & \dot{\phi} & 0 \\ -\dot{\phi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\dot{q}$$

$$= \frac{a}{2}\begin{bmatrix} 0 & \dot{\phi} & \dot{y} \\ -\dot{\phi} & 0 & -\dot{x} \\ -\dot{y} & \dot{x} & 0 \end{bmatrix}\dot{q} = C_a(\dot{q})\dot{q}.$$

Property 2 *The structure of matrix* $C_a(\dot{q})$ *is such that,*

$$\| C_a(\dot{q}) \| \leq k_c \| \dot{q} \|, \tag{5}$$

where it is easy to show that $k_c = \dfrac{a}{2}$.

2.3 Computed-Torque Control Solution

Before presenting the main result of the paper we briefly recalled the solution obtained by considering a modified version of the well know Computed-Torque control strategy. Following (Vázquez & Velasco-Villa, 2008), it is possible to consider for system (4) a feedback law of the form,

$$B\tau = D\ddot{q}_d + C(\dot{q}_d)\dot{q}_d - K_p\tilde{q} - K_d\dot{\tilde{q}} + C_r(\dot{q}_d)\dot{q} \tag{6}$$

where q_d is the desired trajectory and $\tilde{q} = q - q_d$ is the tracking error. K_p and K_d are diagonal and positive definite matrices of proportional and derivative gains and matrix $C_r(\dot{q}_d)$ is defined as,

$$C_r(\dot{q}_d) = \frac{a}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2\dot{y}_d & -2\dot{x}_d & 0 \end{bmatrix}.$$

Feedback (6) in closed loop loop with system (4) produces,

$$D\ddot{\tilde{q}} + C(\dot{q})\dot{q} - C(\dot{q}_d)\dot{q}_d + K_p\tilde{q} + K_d\dot{\tilde{q}} - C_r(\dot{q}_d)\dot{q} = 0,$$

for which, it is possible to formally proof asymptotic closed loop stability. In what follows, it will be presented an alternative feedback control law that solves the same problem.

3. Control design.

Consider the following general model of physical systems (Sira-Ramirez et al., 2006; Sira-Ramírez et al., 2008).

$$\mathsf{A}\dot{x} = \mathsf{J}(x,u)x - \mathsf{R}(x,u)x + \mathsf{B}(x)u + \mathsf{E}(t)$$
$$y = \mathsf{B}^\mathsf{T}x, \tag{7}$$

where x is an n dimensional vector, A is a constant symmetric, positive definite matrix, $\mathsf{J}(x,u)$ is a skew symmetric matrix, $\mathsf{R}(x,u)$ is a symmetric positive definite matrix and $\mathsf{E}(t)$ is a n-dimensional smooth vector function of t or sometimes, a constant vector. The input matrix $\mathsf{B}(x)$ is a $n \times m$ matrix and the output vector y is an m dimensional vector. Moreover, we assume that,

$$\mathsf{J}(x,u) = \mathsf{J}_0 + \sum_{j=1}^{m}\mathsf{J}_j^u u_j + \sum_{k=1}^{n}\mathsf{J}_k^x x_k$$

$$\mathsf{R}(x,u) = \mathsf{R}_0 + \sum_{j=1}^{m}\mathsf{R}_j^u u_j + \sum_{k=1}^{n}\mathsf{R}_k^x x_k \tag{8}$$

$$\mathsf{B}(x) = \mathsf{B}_0 + \sum_{k=1}^{n}\mathsf{B}_k x_k.$$

Define $e_u = u - u^*(t)$ and the following ,

$$\mathsf{M}^*(t) = \left[\left(\mathsf{J}_1^x - \mathsf{R}_1^x\right)x^* \quad \cdots \quad \left(\mathsf{J}_n^x - \mathsf{R}_n^x\right)x^*\right]$$
$$\mathsf{L}^*(t) = \left[B_1 x^* \quad \cdots \quad B_n x^*\right] \tag{9}$$
$$\mathsf{Q}^*(t) = \left[\left(\mathsf{J}_1^u - \mathsf{R}_1^u\right)x^* \quad \cdots \quad \left(\mathsf{J}_m^u - \mathsf{R}_m^u\right)x^*\right],$$

where $x^* = x^*(t)$. Straightforward computations show that the error dynamics reads as,

$$\mathsf{A}\dot{e} = \left[\mathsf{J}^*(x,u,t) - \mathsf{R}^*(x,u,t)\right]e + \mathsf{B}^*(x,t)e_u$$
$$y_e = \mathsf{B}^*(x,t)^\mathsf{T}e \tag{10}$$

where,

$$
\begin{aligned}
\mathsf{J}^*(x,u,t) &= \mathsf{J}(x,u) + \frac{1}{2}\left[\mathsf{P}^*(t) - \mathsf{P}^*(t)^{\mathrm{T}}\right] \\
\mathsf{R}^*(x,u,t) &= \mathsf{R}(x,u) - \frac{1}{2}\left[\mathsf{P}^*(t) + \mathsf{P}^*(t)^{\mathrm{T}}\right] \\
\mathsf{B}^*(x,t) &= \mathsf{B}(x) + \mathsf{Q}^*(t),
\end{aligned}
\tag{11}
$$

with

$$
\mathsf{P}^*(t) = \mathsf{M}^*(t) + \mathsf{L}^*(t).
$$

We refer to (10) as the exact open loop error dynamics.
Following Sira-Ramirez et al., 2006; Sira-Ramírez et al., 2008 we have,

Assumption 3 *Given a feasible smooth bounded reference trajectory $x^*(t) \in R^n$, there exists a smooth open loop control input $u^*(t) \in R^m$, such that for all trajectories starting at $x(t_0) = x^*(t_0)$, the tracking error $e(t) = x(t) - x^*(t)$ is identically zero for all $t \geq t_0$.*

Assumption 4 *For any constant positive definite symmetric matrix K the following relation is uniformly satisfied*

$$
\mathsf{R}^*(x,u,t) + \mathsf{B}^*(x,t) K \mathsf{B}^*(x,t)^{\mathrm{T}} > 0.
$$

Theorem 5 *Consider the system (7)-(8) in closed loop with the controller,*

$$
u = u^*(t) - K \mathsf{B}^*(x,t) e.
\tag{12}
$$

Then, under Assumptions 3 and 4, the tracking error $e(t)$ is globally asymptotically stabilized to zero.

Proof. Take now the following Lyapunov function candidate,

$$
V = \frac{1}{2} e^{\mathrm{T}} \mathsf{A} e
\tag{13}
$$

whose time derivative is given by,

$$
\dot{V} = -e^{\mathrm{T}} \mathsf{R}^*(x,u,t) e + e^{\mathrm{T}} \mathsf{B}^*(x,t) e_u.
$$

Introducing (11) into the above equation, we have

$$
\dot{V} = -e^{\mathrm{T}}[\mathsf{R}^*(x,u,t) + \mathsf{B}^*(x,t) K \mathsf{B}^*(x,t)^{\mathrm{T}}] e,
$$

By Assumption 4, the proof is completed.

3.1 Omnidirectional mobile robot case

In the following we apply a slightly modified version of the result given in Theorem 5 to solve the trajectory-tracking control problem of the omnidirectional mobile robot. For this

purpose, we express the dynamic model of the mobile robot (4) in terms of (7). Defining initially the feedback,

$$\tau = B^{-1}u, \tag{14}$$

it is obtained,

$$D\ddot{q} + C_a(\dot{q})\dot{q} = u. \tag{15}$$

Consider now the state variables,

$$q = x_1 = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \quad \text{and} \quad \dot{q} = x_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \end{bmatrix}. \tag{16}$$

Therefore, system (15) can be rewritten as,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ D\dot{x}_2 &= -C_a(x_2)x_2 + u. \end{aligned} \tag{17}$$

Consider now a second feedback law of the form,

$$u = -x_1 - Rx_2 + v \tag{18}$$

with $R = diag\{r_1, r_2, r_3\}$. Replacing (18) into (17) we obtain the following closed-loop system,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ D\dot{x}_2 &= -C_a(x_2)x_2 - x_1 - Rx_2 + v, \end{aligned} \tag{19}$$

that can be rewritten in the form of (7) with,

$$\mathsf{A} = \begin{bmatrix} I & 0 \\ 0 & D \end{bmatrix}, \qquad \mathsf{J}(x,u) = \begin{bmatrix} 0 & I \\ -I & -C_a(x_2) \end{bmatrix},$$

$$\mathsf{R}(x,u) = \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix}, \qquad \mathsf{B}(x) = \begin{bmatrix} 0 \\ I \end{bmatrix},$$

and I a 3×3 unity matrix. Now, we obtain the dynamics of the tracking error. Straight forward computations show that,

$$\mathsf{L}^*(t) = \mathsf{Q}^*(t) = 0,$$

and

$$\mathsf{M}^*(t) = \begin{bmatrix} 0 & 0 \\ 0 & \mathsf{C}(t) \end{bmatrix}, \tag{20}$$

with

$$\mathsf{C}(t) = a \begin{bmatrix} 0 & -x_{23}^* & -x_{22}^* \\ x_{23}^* & 0 & x_{21}^* \\ -x_{22}^* & x_{21}^* & 0 \end{bmatrix}.$$

Finally, from equation (11) we have,

$$\mathsf{J}^*(x,t) = \begin{bmatrix} 0 & I \\ -I & J_{22}^* \end{bmatrix}, \quad \mathsf{R}^*(t) = \begin{bmatrix} 0 & 0 \\ 0 & R_{22}^* \end{bmatrix},$$
$$\mathsf{B}^* = \begin{bmatrix} 0 & I \end{bmatrix}^\mathrm{T}, \tag{21}$$

with

$$J_{22}^* = \begin{bmatrix} 0 & -ax_{23} - ax_{23}^* & -ax_{22} \\ ax_{23} + ax_{23}^* & 0 & ax_{21} \\ ax_{22} & -ax_{21} & 0 \end{bmatrix}$$

and

$$R_{22}^* = \begin{bmatrix} r_1 & 0 & ax_{22}^* \\ 0 & r_2 & -ax_{21}^* \\ ax_{22}^* & -ax_{21}^* & r_3 \end{bmatrix}.$$

Hence, the trajectory-tracking error dynamics is described by,

$$\begin{bmatrix} I & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \left\{ \begin{bmatrix} 0 & I \\ -I & J_{22}^* \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & R_{22}^* \end{bmatrix} \right\} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} e_v.$$

3.2 Trajectory-tracking solution: Initial proposal

From the previous developments, it is possible now to state a preliminary solution of the trajectory-tracking problem associated with the considered mobile robot.

Proposition 6 *Consider the dynamic model of the omnidirectional mobile robot (4) in closed loop with the controller,*

$$\tau = B^{-1} \left[-x_1 - Rx_2 + v(t)^* - K\mathsf{B}^* e_2 \right] \tag{22}$$

where $K = diag\{0, K_2\}$ *and* K_2 *a symmetric positive definite matrix. Assume that* $x_1(t)^*$, $x_2(t)^*$ *and* $v(t)^*$ *satisfy Assumption 3 and*

$$\frac{\lambda_m(R) + \lambda_m(K_2)}{a} > \left\| x_2^* \right\|. \tag{23}$$

Then, the closed-loop system (4)-(22) renders the equilibrium point $e_1 = x_1 - x_1^* = 0$, $e_2 = x_2 - x_2^* = 0$ asymptotically stable.

Proof. The proof of this result can be seen as a particular case of the solution given in the next subsection and it is left to the interested reader.

Remark 7 *It is possible to see that by defining controller (22) as a function only of the velocity error the convergence of e_1 is slow. This affects the overall performance of the controller.*

3.3 Main trajectory-tracking solution
In order to improve the controller convergence produced by the feedback law (22), consider now,

$$e_v = -K_1 e_1 - K_2 e_2 \tag{24}$$

with $K_1 = diag\{k_{11}, k_{12}, k_{13}\}$ and $K_2 = diag\{k_{21}, k_{22}, k_{23}\}$, obtaining the closed loop system,

$$\dot{e}_1 = e_2$$
$$D\dot{e}_2 = -e_1 + (J_{22}^* - R_{22}^*)e_2 - K_1 e_1 - K_2 e_2. \tag{25}$$

To show the convergence of the tracking error notice first that $(e_1, e_2) = (0,0)$ is an equilibrium point of system (25). Consider, now a candidate Lyapunov function of the form,

$$V(e_1, e_2) = \frac{1}{2}e_2^T D e_2 + \frac{1}{2}e_1^T e_1 + \varepsilon e_1^T D e_2 + \frac{1}{2}\varepsilon e_1^T K_2 e_1. \tag{26}$$

It is not difficult to see that this function is positive definite for sufficiently small ε. Taking the time derivative of equation (26) along (25) it is obtained,

$$\dot{V} = -\varepsilon e_1^T [I + K_1] e_1 - e_2^T [R_{22}^* + K_2 - \varepsilon D] e_2$$
$$+ \varepsilon e_1^T [(J_{22}^* - R_{22}^*) - K_1] e_2. \tag{27}$$

Note now that R_{22}^* can be written as $R_{22}^* = R + aF(x_{2d})$ with,

$$F(x_{2d}) = \begin{bmatrix} 0 & 0 & x_{22d} \\ 0 & 0 & -x_{21d} \\ x_{22d} & -x_{21d} & 0 \end{bmatrix},$$

and also, $J_{22}^* - R_{22}^*$ can be rewritten as $J_{22}^* - R_{22}^* = -R + C_a(e_2) + aG(x_{2d})$ with

$$G(x_{2d}) = \begin{bmatrix} 0 & -x_{23d} & -x_{22d} \\ x_{23d} & 0 & x_{21d} \\ 0 & 0 & 0 \end{bmatrix}.$$

From the fact that,

$$\|F(x_{2d})\| \le \|x_{2d}\| \quad \text{and} \quad \|G(x_{2d})\| \le \|x_{2d}\|,$$

lengthy but simple computations show that,

$$\dot{V} \le -\{[\lambda_m(K_1) + \varepsilon]\|e_1\|^2 + [\lambda_m(K_2) + \lambda_m(R_{22})$$
$$- \varepsilon\lambda_M(D) - a\|x_{2d}\| - \varepsilon a\|e_1\|]\|e_2\|^2$$
$$- [\varepsilon\lambda_M(R_{22}) + \varepsilon a\|x_{2d}\| + \lambda_M(k_1)]\|e_1\|\|e_2\|\}.$$

Notice now that the above relation can be rewritten as:

$$\dot{V} \le -\begin{bmatrix} \|e_1\| & \|e_2\| \end{bmatrix} P \begin{bmatrix} \|e_1\| \\ \|e_2\| \end{bmatrix}$$

with

$$P = \begin{bmatrix} [\lambda_m(K_1) + \varepsilon] & p_{12} \\ p_{12} & p_{22} \end{bmatrix},$$

where

$$p_{12} = -\frac{1}{2}[\varepsilon\lambda_M(R_{22}) + \varepsilon a\|x_{2d}\| + \lambda_M(K_1)]$$

$$p_{22} = \lambda_m(K_2) + \lambda_m(R_{22}) - \varepsilon\lambda_M(D) - a\|x_{2d}\| - \varepsilon a\|e_1\|.$$

Therefore, the closed-loop system will be stable if the conditions,

$$(i) \quad \lambda_m(K_1) + \varepsilon > 0$$
$$(ii) \quad \det\{P\} > 0$$

are satisfied. Condition (i) is trivially satisfied while condition (ii) can be written as a
second order function of ε, that is,

$$-\gamma_1\varepsilon^2 + \gamma_2\varepsilon + \gamma_3 > 0 \tag{28}$$

where,

$$\gamma_1 = \lambda_M(D) + a\|e_1\| + \frac{1}{4}[\lambda_M(R_{22}) + a\|x_{2d}\|]^2$$

$$\gamma_2 = \lambda_m(K_2) + \lambda_m(R_{22}) - a\|x_{2d}\| - \lambda_m(K_1)[\lambda_M(D) + a\|e_1\|]$$
$$- \lambda_M(K_1)[\lambda_M(R_{22}) + a\|x_{2d}\|]$$

$$\gamma_3 = \lambda_m(K_1)[\lambda_m(K_2) + \lambda_m(R_{22}) - a\|x_{2d}\|] - \frac{1}{4}\lambda_M^2(K_1).$$

It is clear now, from equation (28), that the system will be asymptotically stable for a sufficiently small ε. Notice that when $\varepsilon \to 0$ the required stability condition is reduced to $\gamma_3 > 0$ that can be easily obtained by an adequate selection of the control gains together with a bounded desired velocity. Hence, we have shown:

Proposition 8 *Consider the dynamic model of the omnidirectional mobile robot (4) in closed loop with the controller*

$$\tau = B_a^{-1}[-x_1 - Rx_2 + v(t)^* - K_1 e_1 - K_2 e_2] \qquad (29)$$

where K_1 and K_2 are symmetric positive definite matrices. Assume that $x_1(t)^*$, $x_2(t)^*$ and $v(t)^*$ satisfy Assumption 3 and $\gamma_3 > 0$. Then, the closed-loop system (4)-(22) renders the equilibrium point $e_1 = 0$, $e_2 = 0$ asymptotically stable.

4. Numerical simulations

We carried out numerical simulations to assess the performance of the controller given in Proposition 8. The values of the parameters correspond to a laboratory prototype built in our institution and they are $M_p = 9.58$ Kg, $I_r = 0.52$ Kgm^2, $L = 0.1877$ m, $r = 0.03812$ m and $\delta = 30^o$. The initial conditions of the mobile robot are $x_1(0) = [0, \quad 0, \quad 1.5]^T$ and $x_2(0) = [0, \quad 0, \quad 0]^T$. Finally, the controller parameters are summarized in Table 1.

Parameter	Value	Parameter	Value
k_{11}	200	k_{21}	200
k_{12}	200	k_{22}	200
k_{13}	200	k_{23}	100
r_1, r_2	200	r_3	30

Table 1. Feedback control law parameters

It is desired to follow a circular trajectory or radius 0.5 m centered at the origin with initial conditions $x_{1d}(0) = [0.5, \quad 0, \quad \pi/2]^T$.

Figure 2 shows the evolution on the plane of the mobile robot when it is considered the control strategy proposed on this paper (P) and the one obtained when the Computed-Torque control (CT) (6) is considered. The torque input signals are shown on Figure 3 for the passive approach and on Figure 4 for the Computed-Torque control. It is clear that by selecting the control gains under the restriction of equivalent torque magnitude, the control strategy proposed in this work has a better performance than the one obtained by the Computed-Torque scheme. Finally, the evolution of the position and velocity errors for the passivity control strategy are shown on Figures 5 and 6 respectively.
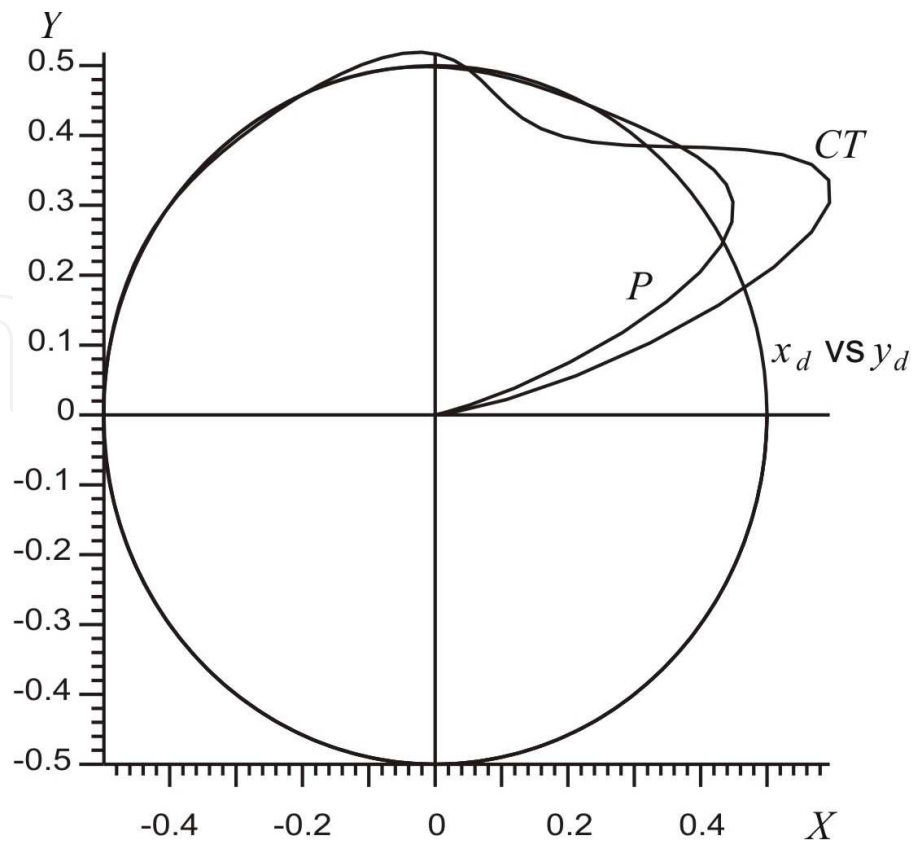
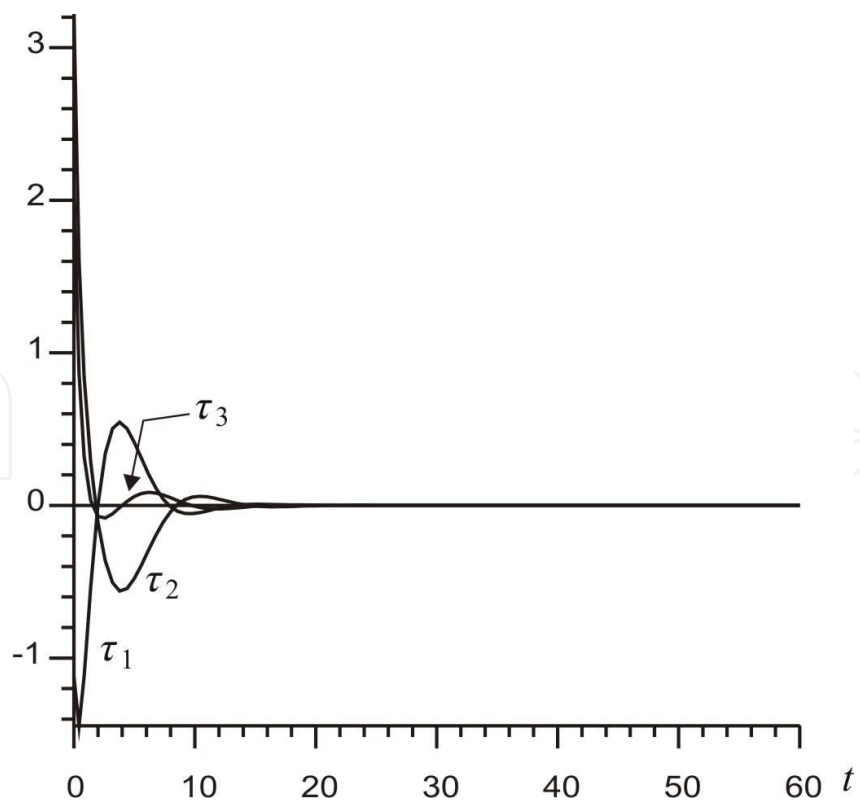Fig. 2. Evolution on the plane of the mobile robot.

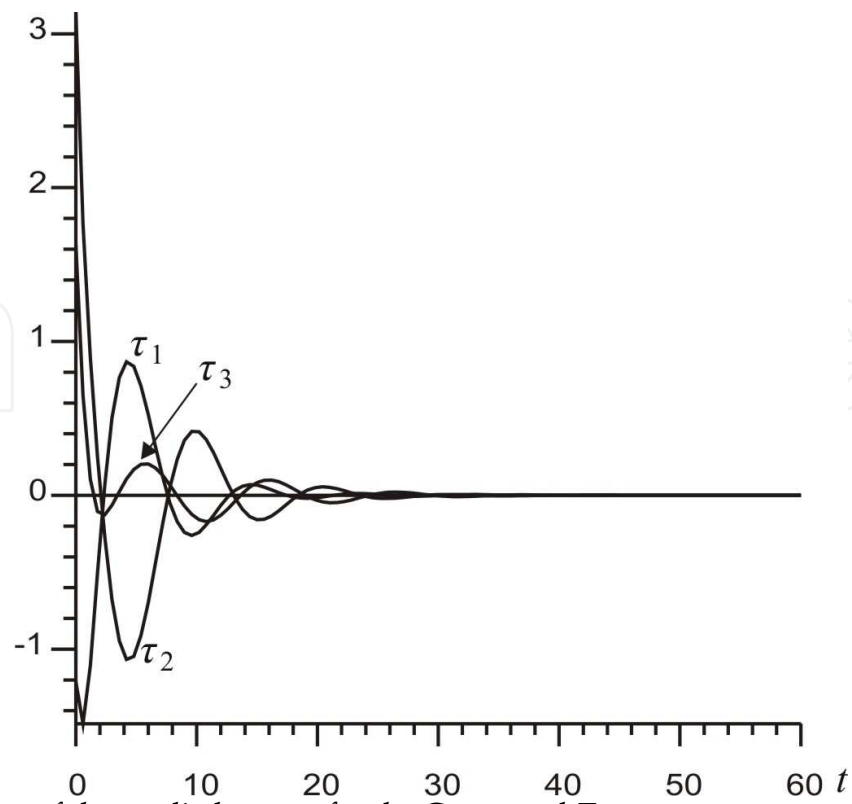Fig. 3. Evolution of the applied torque for the passive strategy.

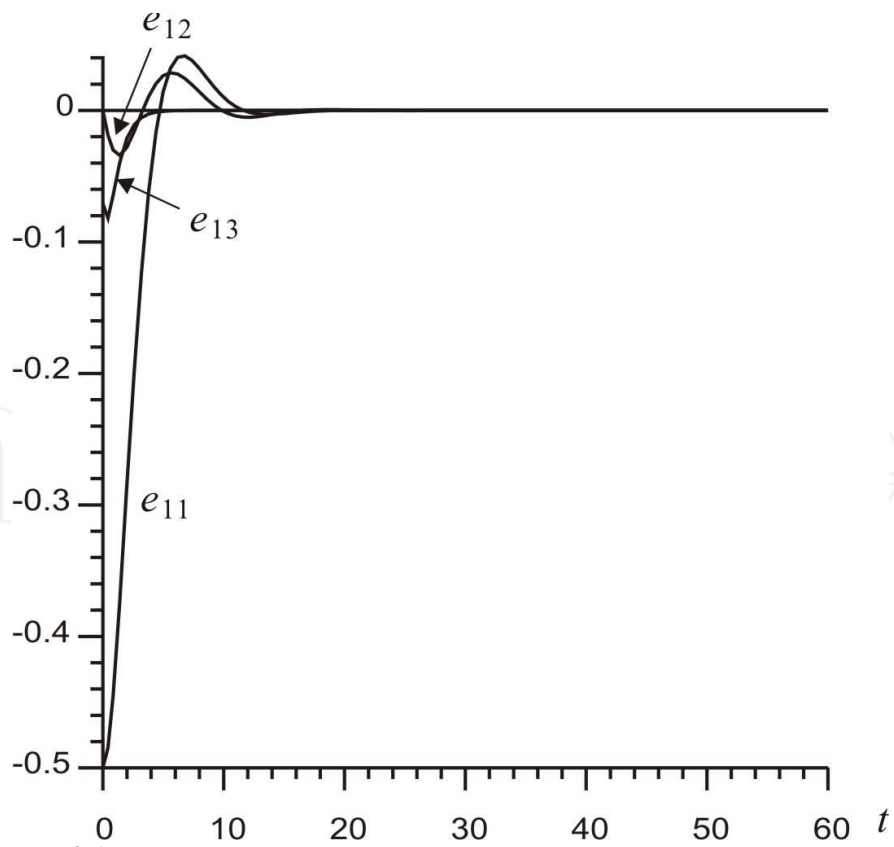Fig. 4. Evolution of the applied torque for the Computed-Torque strategy.

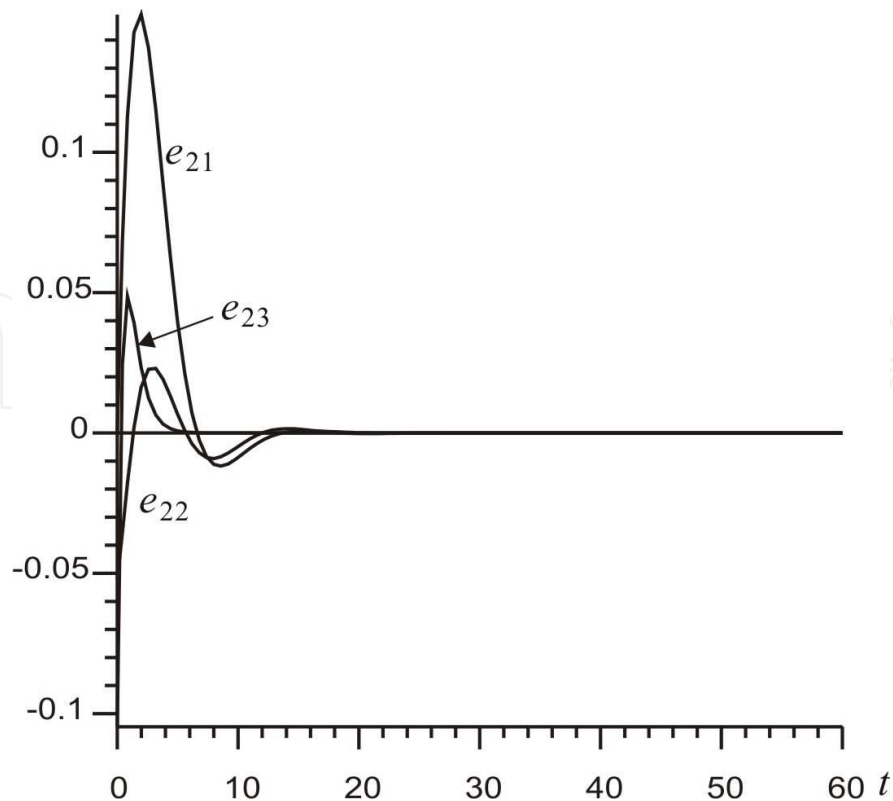Fig. 5. Evolution of the position errors.

Fig. 6. Velocity errors.

5. Conclusions

The trajectory-tracking problem for the omnidirectional mobile robot considering its dynamic model has been addressed and solved by means of a full state information time varying feedback based on a methodology that exploits the passivity properties of the exact tracking error dynamics. The asymptotic stability of the closed loop system is formally proved. Numerical simulations are proposed to illustrate the properties of the closed-loop system showing a better performance than the control obtained by the well known Computed-Torque approach.

6. Acknowledgment

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Edited by Ernest Hall

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