A Regressor-free Adaptive Control for Flexible-joint Robots based on Function Approximation Technique

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Abstract
An adaptive controller is presented in this paper to control an n-link flexible-joint manipulator with time-varying uncertainties. The function approximation technique (FAT) is utilized to represent time-varying uncertainties in some finite combinations of orthogonal basis. The tedious computation of the regressor matrix needed in traditional adaptive control is avoided in the new design, and the controller does not require the variation bounds of time-varying uncertainties needed in traditional robust control. In addition, the joint acceleration is not needed in the controller realization. Via the Lyapunov-like stability theory, adaptive update laws are derived to give convergence of the output tracking error. Moreover, the upper bounds of tracking errors in the transient state are also derived. A 2 DOF planar manipulator with flexible joints is used in the computer simulation to verify the effectiveness of the proposed controller.

Keywords: Adaptive control; Flexible-joint robot; FAT

1. INTRODUCTION

In practical applications, most controllers for robot manipulators equipped with harmonic devices are based on rigid-body dynamics formulation. To achieve high precision tracking performance, the joint flexibility should be carefully considered.[1] However, the modeling of flexible-joint robots is far more complex than that of rigid-joint robots. Besides, the mathematical model of the robot inevitably contains model inaccuracies such as parametric

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uncertainties, and unmodeled dynamics. Since these inaccuracies may degrade the performance of the closed-loop system, any practical design should consider their effects. Under the problems of joint flexibility and model inaccuracies, several strategies based on adaptive control or robust control for flexible-joint robots had been proposed.

Spong[2,3] proposed an adaptive controller for flexible-joint robots by using the singular perturbation formulation. Chen and Fu[4] presented a two-stage adaptive control scheme for a single-link robot based on a simplified dynamic model. Khorasani[5] designed an adaptive controller using the concept of integral manifolds for n-link flexible-joint robots. Without using the velocity measurements, Lim et. al.[6] proposed an adaptive integrator backstepping scheme for rigid-link flexible-joint robots. Dixon et. al.[7] designed an adaptive partial state feedback controller by using a nonlinear link velocity filter. Yim[8] suggested an output feedback adaptive controller based on the backstepping design. Kozlowski and Sauer[9,10] suggested an adaptive controller under the assumption of bounded disturbances to have semiglobal convergence. Tian and Goldenberg[11] proposed a robust adaptive controller with joint torque feedback. Jain and Khorrami[12] suggested a robust adaptive control for a class of flexible-joint robots that are transformable to a special strict feedback form. However, like most adaptive control strategies, the uncertainties should be linearly parameterizable into regressor form[13]. Availability of the regressor matrix is crucial to the derivation of adaptive controllers for robot manipulators. This is because traditional adaptive control strategies have a common assumption that the uncertain parameters should be constant or slowly time varying. Therefore, the robot dynamics is linearly parameterized into known regressor matrix and an unknown vector with constant parameters. In general, derivation of the regressor matrix for a given robot is tedious. Once it is obtained, we may find that, for most robots, elements in the unknown vector are simple combinations of system parameters such as link mass, link length and moment of inertia, and these are sometimes relatively easy to measure.[13]

Huang and Chen[14] proposed an adaptive backstepping-like controller based on FAT[15-28] for single-link flexible-joint robots with mismatched uncertainties. Similar to most backstepping designs, the derivation is too complex to robots with more joints. In this paper, we would like to propose a FAT based adaptive controller for n-link flexible-joint robots. The tedious computation of the regressor matrix is avoided in the new design. Moreover, the novel controller does not require the variation bounds of time-varying uncertainties needed in traditional robust control. In addition, the control strategy does not need to feedback joint acceleration. Convergence of the output error and the boundedness of all signals are proved using Lyapunov-like direct method with consideration of the effect of the approximation error.

This paper is organized as follows: in section 2, we derive the proposed adaptive controller in detail; section 3 presents simulation results of a 2-D flexible-joint robot using the proposed controller; finally, some conclusions are given in section 4.

2. MAIN RESULTS

The dynamics of an n-rigid link flexible-joint robot can be described by[29]

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q}) \tag{1}$$

$$\mathbf{J}\ddot{\boldsymbol{\theta}} + \mathbf{B}\dot{\boldsymbol{\theta}} + \mathbf{K}(\boldsymbol{\theta} - \mathbf{q}) = \mathbf{u} \tag{2}$$

where $\mathbf{q} \in \Re^n$ is the vector of link angles, $\boldsymbol{\theta} \in \Re^n$ is the vector of actuator angles, $\mathbf{u} \in \Re^n$ is the vector of actuator input torques, $\mathbf{D}(\mathbf{q})$ is the $n \times n$ inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ is an n-vector of centrifugal and Coriolis forces, and $\mathbf{g}(\mathbf{q})$ is the gravity vector. $\mathbf{J}$, $\mathbf{B}$ and $\mathbf{K}$ are $n \times n$ constant diagonal matrices of actuator inertias, damping and joint stiffness, respectively. Here, we would like to consider the case when the precise forms of $\mathbf{D}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ and $\mathbf{g}(\mathbf{q})$ are not available and their variation bounds are not given. This implies that traditional adaptive control and robust control cannot be applicable. In the following, we would like to use the function approximation technique to design an adaptive controller for the flexible-joint robot. Moreover, it is well-known that derivation of the regressor matrix for the adaptive control of high DOF rigid robot is generally tedious. For the flexible-joint robot in (1) and (2), its dynamics is much more complex than that of its rigid-joint counterpart. Therefore, the computation of the regressor matrix becomes extremely difficult. Different form the conventional adaptive control schemes for robot manipulators, the proposed FAT-based adaptive controller does not need the computation of the regressor matrix. This largely simplifies the implementation of the control loop.
Define $\boldsymbol{\tau} = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$ to be the vector of transmission torques, so (1) and (2) becomes[11]

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} \tag{3}$$

$$\mathbf{J}_t\ddot{\boldsymbol{\tau}} + \mathbf{B}_t\dot{\boldsymbol{\tau}} + \boldsymbol{\tau} = \mathbf{u} - \overline{\mathbf{q}}(\dot{\mathbf{q}}, \ddot{\mathbf{q}}) \tag{4}$$

where $\mathbf{J}_t = \mathbf{J}\mathbf{K}^{-1}$, $\mathbf{B}_t = \mathbf{B}\mathbf{K}^{-1}$ and $\overline{\mathbf{q}}(\dot{\mathbf{q}}, \ddot{\mathbf{q}}) = \mathbf{J}\ddot{\mathbf{q}} + \mathbf{B}\dot{\mathbf{q}}$. Define signal vector $\mathbf{s} = \dot{\mathbf{e}} + \boldsymbol{\Lambda}\mathbf{e}$ and $\mathbf{v} = \dot{\mathbf{q}}_d - \boldsymbol{\Lambda}\mathbf{e}$, where $\mathbf{q}_d \in \Re^n$ is the vector of desired states, $\mathbf{e} = \mathbf{q} - \mathbf{q}_d$ is the state error, and $\boldsymbol{\Lambda} = diag(\lambda_1, \lambda_2, ..., \lambda_n)$ with $\lambda_i > 0$ for all i=1, … n. Rewrite (3) in the form

$$\mathbf{D}\dot{\mathbf{s}} + \mathbf{C}\mathbf{s} + \mathbf{g} + \mathbf{D}\dot{\mathbf{v}} + \mathbf{C}\mathbf{v} = \boldsymbol{\tau} \tag{5}$$

A. Controller Design for Known Robot

Suppose $\mathbf{D}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ and $\mathbf{g}(\mathbf{q})$ are known, and we may design a proper control law such that $\boldsymbol{\tau}$ follows the trajectory below

$$\boldsymbol{\tau} = \mathbf{g} + \mathbf{D}\dot{\mathbf{v}} + \mathbf{C}\mathbf{v} - \mathbf{K}_d\mathbf{s} \tag{6}$$

where $\mathbf{K}_d$ is a positive definite matrix. Substituting (6) into (5), the closed loop dynamics becomes $\mathbf{D}\dot{\mathbf{s}}+\mathbf{C}\mathbf{s}+\mathbf{K}_d\mathbf{s}=\mathbf{0}$. Define a Lyapunov function candidate as $V=\frac{1}{2}\mathbf{s}^T\mathbf{D}\mathbf{s}$. Its time derivative along the trajectory of the closed loop dynamics can be computed as $\dot{V}=-\mathbf{s}^T\mathbf{K}_d\mathbf{s}+\mathbf{s}^T(\dot{\mathbf{D}}-2\mathbf{C})\mathbf{s}$. Since $\dot{\mathbf{D}}-2\mathbf{C}$ can be proved to be skew-symmetric, the above equation becomes $\dot{V}=-\mathbf{s}^T\mathbf{K}_d\mathbf{s}\le 0$. It is easy to prove that **s** is uniformly bounded and square integrable, and $\dot{\mathbf{s}}$ is also uniformly bounded. Hence, $\mathbf{s}\to\mathbf{0}$ as $t\to\infty$, or we may say $\mathbf{e}\to\mathbf{0}$ as $t\to\infty$. To make the actual $\boldsymbol{\tau}$ converge to the perfect $\boldsymbol{\tau}$ in (6), let us consider the reference model

$$\mathbf{J}_r\ddot{\boldsymbol{\tau}}_r+\mathbf{B}_r\dot{\boldsymbol{\tau}}_r+\mathbf{K}_r\boldsymbol{\tau}_r=\mathbf{K}_r\boldsymbol{\tau}_d+\mathbf{B}_r\dot{\boldsymbol{\tau}}_d+\mathbf{J}_r\ddot{\boldsymbol{\tau}}_d \tag{7}$$

where $\boldsymbol{\tau}_r\in\Re^n$ is the state vector of the reference model and $\boldsymbol{\tau}_d\in\Re^n$ is the desired states. Matrices $\mathbf{J}_r\in\Re^{n\times n}$, $\mathbf{B}_r\in\Re^{n\times n}$ and $\mathbf{K}_r\in\Re^{n\times n}$ are selected such that $\boldsymbol{\tau}_r\to\boldsymbol{\tau}_d$ exponentially. Define $\bar{\boldsymbol{\tau}}_d(\dot{\boldsymbol{\tau}}_d,\ddot{\boldsymbol{\tau}}_d)=\mathbf{K}_r^{-1}(\mathbf{B}_r\dot{\boldsymbol{\tau}}_d+\mathbf{J}_r\ddot{\boldsymbol{\tau}}_d)$, we may rewrite (4) and (7) in the state space form as

$$\dot{\mathbf{x}}_p=\mathbf{A}_p\mathbf{x}_p+\mathbf{B}_p\mathbf{u}-\mathbf{B}_p\bar{\mathbf{q}} \tag{8}$$

$$\dot{\mathbf{x}}_m=\mathbf{A}_m\mathbf{x}_m+\mathbf{B}_m(\boldsymbol{\tau}_d+\bar{\boldsymbol{\tau}}_d) \tag{9}$$

where $\mathbf{x}_p=[\boldsymbol{\tau}\quad\dot{\boldsymbol{\tau}}]^T\in\Re^{2n}$ and $\mathbf{x}_m=[\boldsymbol{\tau}_r\quad\dot{\boldsymbol{\tau}}_r]^T\in\Re^{2n}$ are augmented state vectors. $\mathbf{A}_p=\begin{bmatrix}\mathbf{0} & \mathbf{I}_{n\times n}\\ -\mathbf{J}_t^{-1} & -\mathbf{J}_t^{-1}\mathbf{B}_t\end{bmatrix}\in\Re^{2n\times 2n}$ and $\mathbf{A}_m=\begin{bmatrix}\mathbf{0} & \mathbf{I}_{n\times n}\\ -\mathbf{J}_r^{-1}\mathbf{K}_r & -\mathbf{J}_r^{-1}\mathbf{B}_r\end{bmatrix}\in\Re^{2n\times 2n}$ are augmented system matrices. $\mathbf{B}_p=\begin{bmatrix}\mathbf{0}\\ \mathbf{J}_t^{-1}\end{bmatrix}\in\Re^{2n\times n}$ and $\mathbf{B}_m=\begin{bmatrix}\mathbf{0}\\ \mathbf{J}_r^{-1}\mathbf{K}_r\end{bmatrix}\in\Re^{2n\times n}$ are augmented input gain matrices, and the pair $(\mathbf{A}_m,\mathbf{B}_m)$ is controllable. Since all system parameters are assumed to be available at the present stage, we may select a controller in the form[30]

$$\mathbf{u}=\Theta\mathbf{x}_p+\Phi\boldsymbol{\tau}_d+\mathbf{h}(\bar{\boldsymbol{\tau}}_d,\bar{\mathbf{q}}) \tag{10}$$

where $\Theta\in\Re^{n\times 2n}$ and $\Phi\in\Re^{n\times n}$ satisfy $\mathbf{A}_p+\mathbf{B}_p\Theta=\mathbf{A}_m$ and $\mathbf{B}_p\Phi=\mathbf{B}_m$, respectively, and $\mathbf{h}(\bar{\boldsymbol{\tau}}_d, \bar{\mathbf{q}}) = \Phi\bar{\boldsymbol{\tau}}_d + \bar{\mathbf{q}}$. Substituting (10) into (8) and after some rearrangements, we may have the system dynamics

$$\dot{\mathbf{x}}_p = \mathbf{A}_m \mathbf{x}_p + \mathbf{B}_m(\boldsymbol{\tau}_d + \bar{\boldsymbol{\tau}}_d) \tag{11}$$

Define $\mathbf{e}_m = \mathbf{x}_p - \mathbf{x}_m$ and we may have the error dynamics directly from (9) and (11)

$$\dot{\mathbf{e}}_m = \mathbf{A}_m \mathbf{e}_m \tag{12}$$

Let $\mathbf{e}_\tau = \boldsymbol{\tau} - \boldsymbol{\tau}_r$ be the output vector of the error dynamics (12) as

$$\mathbf{e}_\tau = \mathbf{C}_m \mathbf{e}_m \tag{13}$$

where $\mathbf{C}_m \in \Re^{n \times 2n}$ is the augmented output matrix such that the pair $(\mathbf{A}_m, \mathbf{C}_m)$ is observable and the transfer function $\mathbf{C}_m (s\mathbf{I} - \mathbf{A}_m)^{-1} \mathbf{B}_m$ is strictly positive real. Since $\mathbf{A}_m$ is stable, (12) implies $\mathbf{e}_m \to 0$ as $t \to \infty$. This further gives $\boldsymbol{\tau} \to \boldsymbol{\tau}_d$ as $t \to \infty$.

B. Controller Design for Uncertain Robot

Suppose $\mathbf{D}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ and $\mathbf{g}(\mathbf{q})$ are not available, and $\ddot{\mathbf{q}}$ is not easy to measure, we would like to design a desired transmission torque $\boldsymbol{\tau}_d$ so that a proper controller $\mathbf{u}$ can be constructed to have $\boldsymbol{\tau} \to \boldsymbol{\tau}_d$.

Instead of (6), let us design a desired transmission torque $\boldsymbol{\tau}_d$ as

$$\boldsymbol{\tau}_d = \hat{\mathbf{g}} + \hat{\mathbf{D}}\dot{\mathbf{v}} + \hat{\mathbf{C}}\mathbf{v} - \mathbf{K}_d \mathbf{s} \tag{14}$$

where $\mathbf{K}_d > \frac{1}{4}\mathbf{I}_{n \times n}$, and $\hat{\mathbf{D}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{g}}$ are estimates of $\mathbf{D}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ and $\mathbf{g}(\mathbf{q})$, respectively. Substituting (14) into (5), we may have the closed loop dynamics

$$\mathbf{D}\dot{\mathbf{s}} + \mathbf{C}\mathbf{s} + \mathbf{K}_d \mathbf{s} = (\boldsymbol{\tau} - \boldsymbol{\tau}_\mathbf{d}) + (\hat{\mathbf{D}} - \mathbf{D})\dot{\mathbf{v}} + (\hat{\mathbf{C}} - \mathbf{C})\mathbf{v} + (\hat{\mathbf{g}} - \mathbf{g}) \tag{15}$$

If a proper controller $\mathbf{u}$ and update laws for $\hat{\mathbf{D}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{g}}$ can be designed, we may have $\boldsymbol{\tau} \to \boldsymbol{\tau}_d$, $\hat{\mathbf{D}} \to \mathbf{D}$, $\hat{\mathbf{C}} \to \mathbf{C}$ and $\hat{\mathbf{g}} \to \mathbf{g}$ so that (15) can give desired performance. Let us consider the control law

$$\mathbf{u} = \Theta\mathbf{x}_p + \Phi\boldsymbol{\tau}_d + \hat{\mathbf{h}} \tag{16}$$

where $\hat{\mathbf{h}}$ is an estimate of $\mathbf{h}$. Substituting (16) into (8), we may have the system dynamics

$$\dot{\mathbf{x}}_p = \mathbf{A}_m\mathbf{x}_p + \mathbf{B}_m(\boldsymbol{\tau}_d + \bar{\boldsymbol{\tau}}_d) + \mathbf{B}_p(\hat{\mathbf{h}} - \mathbf{h}) \tag{17}$$

Together with (9), we may have the error dynamics

$$\dot{\mathbf{e}}_m = \mathbf{A}_m\mathbf{e}_m + \mathbf{B}_p(\hat{\mathbf{h}} - \mathbf{h}) \tag{18}$$

$$\mathbf{e}_\tau = \mathbf{C}_m\mathbf{e}_m \tag{19}$$

If we may design an appropriate update law such that $\hat{\mathbf{h}} \to \mathbf{h}$, then (18) implies $\mathbf{e}_m \to 0$ as $t \to \infty$. This further implies $\boldsymbol{\tau} \to \boldsymbol{\tau}_d$ as $t \to \infty$. Since **D**, **C**, **g** and **h** are functions of time, traditional adaptive controllers are not directly applicable. To design the update laws, let us apply the function approximation representation[15-21]

$$\begin{aligned} \mathbf{D} &= \mathbf{W}_\mathbf{D}^T\mathbf{Z}_\mathbf{D} + \boldsymbol{\varepsilon}_\mathbf{D}, \quad & \mathbf{C} &= \mathbf{W}_\mathbf{C}^T\mathbf{Z}_\mathbf{C} + \boldsymbol{\varepsilon}_\mathbf{C}, \\ \mathbf{g} &= \mathbf{W}_\mathbf{g}^T\mathbf{Z}_\mathbf{g} + \boldsymbol{\varepsilon}_\mathbf{g}, \quad & \mathbf{h} &= \mathbf{W}_\mathbf{h}^T\mathbf{Z}_\mathbf{h} + \boldsymbol{\varepsilon}_\mathbf{h} \end{aligned} \tag{20a}$$

where $\mathbf{W}_\mathbf{D} \in \Re^{n^2\beta_D \times n}$, $\mathbf{W}_\mathbf{C} \in \Re^{n^2\beta_C \times n}$, $\mathbf{W}_\mathbf{g} \in \Re^{n\beta_g \times n}$, and $\mathbf{W}_\mathbf{h} \in \Re^{n\beta_h \times n}$ are weighting matrices, $\mathbf{Z}_\mathbf{D} \in \Re^{n^2\beta_D \times n}$, $\mathbf{Z}_\mathbf{C} \in \Re^{n^2\beta_C \times n}$, $\mathbf{Z}_\mathbf{g} \in \Re^{n\beta_g \times 1}$, and $\mathbf{Z}_\mathbf{h} \in \Re^{n\beta_h \times 1}$ are matrices of basis functions, and $\boldsymbol{\varepsilon}_{(\cdot)}$ are approximation error matrices. The number $\beta_{(\cdot)}$ represents the number of basis functions used. Using the same set of basis functions, the corresponding estimates can also be represented as

$$\begin{aligned} \hat{\mathbf{D}} &= \hat{\mathbf{W}}_\mathbf{D}^T\mathbf{Z}_\mathbf{D}, \quad & \hat{\mathbf{C}} &= \hat{\mathbf{W}}_\mathbf{C}^T\mathbf{Z}_\mathbf{C}, \\ \hat{\mathbf{g}} &= \hat{\mathbf{W}}_\mathbf{g}^T\mathbf{Z}_\mathbf{g}, \quad & \hat{\mathbf{h}} &= \hat{\mathbf{W}}_\mathbf{h}^T\mathbf{Z}_\mathbf{h} \end{aligned} \tag{20b}$$

Define $\tilde{\mathbf{W}}_{(\cdot)} = \mathbf{W}_{(\cdot)} - \hat{\mathbf{W}}_{(\cdot)}$, then equation (15) and (18) becomes

$$\mathbf{D}\dot{\mathbf{s}} + \mathbf{C}\mathbf{s} + \mathbf{K}_d\mathbf{s} = (\boldsymbol{\tau} - \boldsymbol{\tau}_d) - \tilde{\mathbf{W}}_\mathbf{D}^T\mathbf{Z}_\mathbf{D}\dot{\mathbf{v}} - \tilde{\mathbf{W}}_\mathbf{C}^T\mathbf{Z}_\mathbf{C}\mathbf{v} - \tilde{\mathbf{W}}_\mathbf{g}^T\mathbf{Z}_\mathbf{g} + \boldsymbol{\varepsilon}_1 \tag{21}$$

$$\dot{\mathbf{e}}_m = \mathbf{A}_m \mathbf{e}_m - \mathbf{B}_p \tilde{\mathbf{W}}_{\mathbf{h}}^T \mathbf{Z}_{\mathbf{h}} + \mathbf{B}_p \boldsymbol{\varepsilon}_2 \tag{22}$$

where $\boldsymbol{\varepsilon}_1 = \boldsymbol{\varepsilon}_1(\boldsymbol{\varepsilon}_{\mathbf{D}}, \boldsymbol{\varepsilon}_{\mathbf{C}}, \boldsymbol{\varepsilon}_{\mathbf{g}}, \mathbf{s}, \ddot{\mathbf{q}}_d)$ and $\boldsymbol{\varepsilon}_2 = \boldsymbol{\varepsilon}_2(\boldsymbol{\varepsilon}_h, \mathbf{e}_m)$ are lumped approximation errors. Since $\mathbf{W}_{(\cdot)}$ are constant vectors, their update laws can be easily found by proper selection of the Lyapunov-like function. Let us consider a candidate

$$\begin{aligned} V(\mathbf{s}, \mathbf{e}_m, \tilde{\mathbf{W}}_{\mathbf{D}}, \tilde{\mathbf{W}}_{\mathbf{C}}, \tilde{\mathbf{W}}_{\mathbf{g}}, \tilde{\mathbf{W}}_{\mathbf{h}}) &= \frac{1}{2}\mathbf{s}^T \mathbf{D}\mathbf{s} + \mathbf{e}_m^T \mathbf{P}_t \mathbf{e}_m \\ &+ \frac{1}{2} Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T \mathbf{Q}_{\mathbf{D}} \tilde{\mathbf{W}}_{\mathbf{D}} + \tilde{\mathbf{W}}_{\mathbf{C}}^T \mathbf{Q}_{\mathbf{C}} \tilde{\mathbf{W}}_{\mathbf{C}} + \tilde{\mathbf{W}}_{\mathbf{g}}^T \mathbf{Q}_{\mathbf{g}} \tilde{\mathbf{W}}_{\mathbf{g}} + \tilde{\mathbf{W}}_{\mathbf{h}}^T \mathbf{Q}_{\mathbf{h}} \tilde{\mathbf{W}}_{\mathbf{h}}) \end{aligned} \tag{23}$$

where $\mathbf{P}_t = \mathbf{P}_t^T \in \Re^{2n \times 2n}$ is a positive definite matrix satisfying the Lyapunov equation $\mathbf{A}_m^T \mathbf{P}_t + \mathbf{P}_t \mathbf{A}_m = -\mathbf{C}_m^T \mathbf{C}_m$. The matrices $\mathbf{Q}_{\mathbf{D}} \in \Re^{n^2\beta_{\mathbf{D}} \times n^2\beta_{\mathbf{D}}}$, $\mathbf{Q}_{\mathbf{C}} \in \Re^{n^2\beta_C \times n^2\beta_C}$, $\mathbf{Q}_{\mathbf{g}} \in \Re^{n\beta_g \times n\beta_g}$ and $\mathbf{Q}_{\mathbf{h}} \in \Re^{n\beta_h \times n\beta_h}$ are positive definite. The notation *Tr*(.) denotes the trace operation of matrices. The time derivative of *V* along the trajectory of (21) and (22) can be computed as

$$\begin{aligned} \dot{V} &= \mathbf{s}^T \mathbf{D}\dot{\mathbf{s}} + \frac{1}{2}\mathbf{s}^T \dot{\mathbf{D}}\mathbf{s} + \dot{\mathbf{e}}_m^T \mathbf{P}_t \mathbf{e}_m + \mathbf{e}_m^T \mathbf{P}_t \dot{\mathbf{e}}_m \\ &\quad - Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T \mathbf{Q}_{\mathbf{D}} \dot{\hat{\mathbf{W}}}_{\mathbf{D}} + \tilde{\mathbf{W}}_{\mathbf{C}}^T \mathbf{Q}_{\mathbf{C}} \dot{\hat{\mathbf{W}}}_{\mathbf{C}} + \tilde{\mathbf{W}}_{\mathbf{g}}^T \mathbf{Q}_{\mathbf{g}} \dot{\hat{\mathbf{W}}}_{\mathbf{g}} + \tilde{\mathbf{W}}_{\mathbf{h}}^T \mathbf{Q}_{\mathbf{h}} \dot{\hat{\mathbf{W}}}_{\mathbf{h}}) \\ &= -\mathbf{s}^T \mathbf{K}_d \mathbf{s} + \mathbf{s}^T \mathbf{e}_\tau - \mathbf{e}_\tau^T \mathbf{e}_\tau + \mathbf{s}^T \boldsymbol{\varepsilon}_1 + \mathbf{e}_m^T \mathbf{P}_t \mathbf{B}_p \boldsymbol{\varepsilon}_2 \\ &\quad - Tr[\tilde{\mathbf{W}}_{\mathbf{D}}^T (\mathbf{Z}_{\mathbf{D}} \dot{\mathbf{v}}\mathbf{s}^T + \mathbf{Q}_{\mathbf{D}} \dot{\hat{\mathbf{W}}}_{\mathbf{D}}) + \tilde{\mathbf{W}}_{\mathbf{C}}^T (\mathbf{Z}_{\mathbf{C}} \mathbf{v}\mathbf{s}^T + \mathbf{Q}_{\mathbf{C}} \dot{\hat{\mathbf{W}}}_{\mathbf{C}})] \\ &\quad - Tr[\tilde{\mathbf{W}}_{\mathbf{g}}^T (\mathbf{Z}_{\mathbf{g}} \mathbf{s}^T + \mathbf{Q}_{\mathbf{g}} \dot{\hat{\mathbf{W}}}_{\mathbf{g}}) + \tilde{\mathbf{W}}_{\mathbf{h}}^T (\mathbf{Z}_{\mathbf{h}} \mathbf{e}_m^T \mathbf{P}_t \mathbf{B}_p + \mathbf{Q}_{\mathbf{h}} \dot{\hat{\mathbf{W}}}_{\mathbf{h}})] \end{aligned} \tag{24}$$

According to the Kalman-Yakubovic Lemma, we have $\mathbf{e}_m^T \mathbf{P}_t \mathbf{B}_p = \mathbf{e}_\tau^T$ by picking $\mathbf{B}_m = \mathbf{B}_p$ [31]. According to (24), the update laws can be selected as

$$\begin{aligned} \dot{\hat{\mathbf{W}}}_{\mathbf{D}} &= -\mathbf{Q}_{\mathbf{D}}^{-1} \mathbf{Z}_{\mathbf{D}} \dot{\mathbf{v}}\mathbf{s}^T - \sigma_{\mathbf{D}} \hat{\mathbf{W}}_{\mathbf{D}}, \quad & \dot{\hat{\mathbf{W}}}_{\mathbf{C}} &= -\mathbf{Q}_{\mathbf{C}}^{-1} \mathbf{Z}_{\mathbf{C}} \mathbf{v}\mathbf{s}^T - \sigma_{\mathbf{C}} \hat{\mathbf{W}}_{\mathbf{C}}, \\ \dot{\hat{\mathbf{W}}}_{\mathbf{g}} &= -\mathbf{Q}_{\mathbf{g}}^{-1} \mathbf{Z}_{\mathbf{g}} \mathbf{s}^T - \sigma_{\mathbf{g}} \hat{\mathbf{W}}_{\mathbf{g}}, & \dot{\hat{\mathbf{W}}}_{\mathbf{h}} &= -\mathbf{Q}_{\mathbf{h}}^{-1} \mathbf{Z}_{\mathbf{h}} \mathbf{e}_\tau^T - \sigma_{\mathbf{h}} \hat{\mathbf{W}}_{\mathbf{h}} \end{aligned} \tag{25}$$

where $\sigma_{(\cdot)}$ are positive numbers. Then (24) becomes

$$\dot{V} = -[\mathbf{s}^T \quad \mathbf{e}_\tau^T]\mathbf{Q}\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\end{bmatrix} + [\mathbf{s}^T \quad \mathbf{e}_\tau^T]\begin{bmatrix}\boldsymbol{\varepsilon}_1\\ \boldsymbol{\varepsilon}_2\end{bmatrix} + \sigma_{\mathbf{D}} Tr(\widetilde{\mathbf{W}}_{\mathbf{D}}^T \hat{\mathbf{W}}_{\mathbf{D}}) + \sigma_{\mathbf{C}} Tr(\widetilde{\mathbf{W}}_{\mathbf{C}}^T \hat{\mathbf{W}}_{\mathbf{C}}) + \sigma_{\mathbf{g}} Tr(\widetilde{\mathbf{W}}_{\mathbf{g}}^T \hat{\mathbf{W}}_{\mathbf{g}}) + \sigma_{\mathbf{h}} Tr(\widetilde{\mathbf{W}}_{\mathbf{h}}^T \hat{\mathbf{W}}_{\mathbf{h}}) \tag{26}$$

where $\mathbf{Q} = \begin{bmatrix}\mathbf{K}_d & -\frac{1}{2}\mathbf{I}_{n\times n}\\ -\frac{1}{2}\mathbf{I}_{n\times n} & \mathbf{I}_{n\times n}\end{bmatrix}$ is positive definite due to proper selections of $\mathbf{K}_d$ and $\mathbf{K}_c$. Owing to the existence of $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ the definiteness of $\dot{V}$ cannot be determined. According to **Appendix** *Lemma A.1*、*Lemma A.4* and *Lemma A.7*, the right hand side of (26) can be divided into two parts to derive following inequalities

$$-[\mathbf{s}^T \quad \mathbf{e}_\tau^T]\mathbf{Q}\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\end{bmatrix} + [\mathbf{s}^T \quad \mathbf{e}_\tau^T]\begin{bmatrix}\boldsymbol{\varepsilon}_1\\ \boldsymbol{\varepsilon}_2\end{bmatrix} \le -\frac{1}{2}\left(\lambda_{\min}(\mathbf{Q})\left\|\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\end{bmatrix}\right\|^2 - \frac{1}{\lambda_{\min}(\mathbf{Q})}\left\|\begin{bmatrix}\boldsymbol{\varepsilon}_1\\ \boldsymbol{\varepsilon}_2\end{bmatrix}\right\|^2\right) \tag{27a}$$

$$Tr(\widetilde{\mathbf{W}}_{\mathbf{D}}^T \hat{\mathbf{W}}_{\mathbf{D}}) \le \frac{1}{2}Tr(\mathbf{W}_{\mathbf{D}}^T \mathbf{W}_{\mathbf{D}}) - \frac{1}{2}Tr(\widetilde{\mathbf{W}}_{\mathbf{D}}^T \widetilde{\mathbf{W}}_{\mathbf{D}}) \tag{27b}$$

$$Tr(\widetilde{\mathbf{W}}_{\mathbf{C}}^T \hat{\mathbf{W}}_{\mathbf{C}}) \le \frac{1}{2}Tr(\mathbf{W}_{\mathbf{C}}^T \mathbf{W}_{\mathbf{C}}) - \frac{1}{2}Tr(\widetilde{\mathbf{W}}_{\mathbf{C}}^T \widetilde{\mathbf{W}}_{\mathbf{C}}) \tag{27c}$$

$$Tr(\widetilde{\mathbf{W}}_{\mathbf{g}}^T \hat{\mathbf{W}}_{\mathbf{g}}) \le \frac{1}{2}Tr(\mathbf{W}_{\mathbf{g}}^T \mathbf{W}_{\mathbf{g}}) - \frac{1}{2}Tr(\widetilde{\mathbf{W}}_{\mathbf{g}}^T \widetilde{\mathbf{W}}_{\mathbf{g}}) \tag{27d}$$

$$Tr(\widetilde{\mathbf{W}}_{\mathbf{h}}^T \hat{\mathbf{W}}_{\mathbf{h}}) \le \frac{1}{2}Tr(\mathbf{W}_{\mathbf{h}}^T \mathbf{W}_{\mathbf{h}}) - \frac{1}{2}Tr(\widetilde{\mathbf{W}}_{\mathbf{h}}^T \widetilde{\mathbf{W}}_{\mathbf{h}}) \tag{27e}$$

According to (23), we hav

$$\begin{aligned} V &= \frac{1}{2}[\mathbf{s}^T\mathbf{D}\mathbf{s} + \mathbf{e}_m^T\mathbf{P}_t\mathbf{e}_m + Tr(\widetilde{\mathbf{W}}_{\mathbf{D}}^T\mathbf{Q}_{\mathbf{D}}\widetilde{\mathbf{W}}_{\mathbf{D}} + \widetilde{\mathbf{W}}_{\mathbf{C}}^T\mathbf{Q}_{\mathbf{C}}\widetilde{\mathbf{W}}_{\mathbf{C}} + \widetilde{\mathbf{W}}_{\mathbf{g}}^T\mathbf{Q}_{\mathbf{g}}\widetilde{\mathbf{W}}_{\mathbf{g}} + \widetilde{\mathbf{W}}_{\mathbf{h}}^T\mathbf{Q}_{\mathbf{h}}\widetilde{\mathbf{W}}_{\mathbf{h}})] \\ &\le \frac{1}{2}\left[\lambda_{\max}(\mathbf{A})\left\|\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\end{bmatrix}\right\|^2 + \lambda_{\max}(\mathbf{Q}_{\mathbf{D}})Tr(\widetilde{\mathbf{W}}_{\mathbf{D}}^T\widetilde{\mathbf{W}}_{\mathbf{D}}) + \lambda_{\max}(\mathbf{Q}_{\mathbf{C}})Tr(\widetilde{\mathbf{W}}_{\mathbf{C}}^T\widetilde{\mathbf{W}}_{\mathbf{C}})\right. \\ &\quad \left. + \lambda_{\max}(\mathbf{Q}_{\mathbf{g}})Tr(\widetilde{\mathbf{W}}_{\mathbf{g}}^T\widetilde{\mathbf{W}}_{\mathbf{g}}) + \lambda_{\max}(\mathbf{Q}_{\mathbf{h}})Tr(\widetilde{\mathbf{W}}_{\mathbf{h}}^T\widetilde{\mathbf{W}}_{\mathbf{h}})\right] \end{aligned} \tag{28}$$

where $\mathbf{A} = \begin{bmatrix}\mathbf{D} & \mathbf{0}\\ \mathbf{0} & 2\mathbf{C}_m^T\mathbf{P}_t\mathbf{C}_m\end{bmatrix}$. With (27) and (28), (26) can be further written as

$$\dot{V} \le -\alpha V + \frac{1}{2}\left\{ [\alpha\lambda_{\max}(\mathbf{A}) - \lambda_{\min}(\mathbf{Q})] \left\| \begin{bmatrix} \mathbf{s} \\ \mathbf{e}_\tau \end{bmatrix} \right\|^2 + [\alpha\lambda_{\max}(\mathbf{Q_D}) - \sigma_\mathbf{D}] Tr(\tilde{\mathbf{W}}_\mathbf{D}^T \tilde{\mathbf{W}}_\mathbf{D}) \right.$$
$$+ [\alpha\lambda_{\max}(\mathbf{Q_C}) - \sigma_\mathbf{C}] Tr(\tilde{\mathbf{W}}_\mathbf{C}^T \tilde{\mathbf{W}}_\mathbf{C}) + [\alpha\lambda_{\max}(\mathbf{Q_g}) - \sigma_\mathbf{g}] Tr(\tilde{\mathbf{W}}_\mathbf{g}^T \tilde{\mathbf{W}}_\mathbf{g}) \tag{29}$$
$$+ [\alpha\lambda_{\max}(\mathbf{Q_h}) - \sigma_\mathbf{h}] Tr(\tilde{\mathbf{W}}_\mathbf{h}^T \tilde{\mathbf{W}}_\mathbf{h}) + \frac{1}{\lambda_{\min}(\mathbf{Q})} \left\| \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \end{bmatrix} \right\|^2$$
$$\left. + \sigma_\mathbf{D} Tr(\mathbf{W}_\mathbf{D}^T \mathbf{W}_\mathbf{D}) + \sigma_\mathbf{C} Tr(\mathbf{W}_\mathbf{C}^T \mathbf{W}_\mathbf{C}) + \sigma_\mathbf{g} Tr(\mathbf{W}_\mathbf{g}^T \mathbf{W}_\mathbf{g}) + \sigma_\mathbf{h} Tr(\mathbf{W}_\mathbf{h}^T \mathbf{W}_\mathbf{h}) \right\}$$

Although $\mathbf{D}$ and $\mathbf{L}$ are unknown, we know that $\exists \overline{D}$ and $\underline{D}$ s.t. $\underline{D} \le \|\mathbf{D}\| \le \overline{D}$ 、 $\exists \overline{L}$ and $\underline{L}$ s.t. $\underline{L} \le \|\mathbf{L}\| \le \overline{L}$, $\exists \overline{\eta}_A, \underline{\eta}_A > 0$ s.t. $\lambda_{\max}(\mathbf{A}) \le \overline{\eta}_A$ and $\lambda_{\min}(\mathbf{A}) \ge \underline{\eta}_A$ no 32.

Picking $\alpha \le \min\left\{ \frac{\lambda_{\min}(\mathbf{Q})}{\overline{\eta}_A}, \frac{\sigma_\mathbf{D}}{\lambda_{\max}(\mathbf{Q_D})}, \frac{\sigma_\mathbf{C}}{\lambda_{\max}(\mathbf{Q_C})}, \frac{\sigma_\mathbf{g}}{\lambda_{\max}(\mathbf{Q_g})}, \frac{\sigma_\mathbf{h}}{\lambda_{\max}(\mathbf{Q_h})} \right\}$, then we have

$$\dot{V} \le -\alpha V + \frac{1}{2\lambda_{\min}(\mathbf{Q})} \left\| \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \end{bmatrix} \right\|^2 + \frac{1}{2}[\sigma_\mathbf{D} Tr(\mathbf{W}_\mathbf{D}^T \mathbf{W}_\mathbf{D})$$
$$+ \sigma_\mathbf{C} Tr(\mathbf{W}_\mathbf{C}^T \mathbf{W}_\mathbf{C}) + \sigma_\mathbf{g} Tr(\mathbf{W}_\mathbf{g}^T \mathbf{W}_\mathbf{g}) + \sigma_\mathbf{h} Tr(\mathbf{W}_\mathbf{h}^T \mathbf{W}_\mathbf{h})] \tag{30}$$

Hence, $\dot{V} < 0$ whenever

$$(\mathbf{s}, \mathbf{e}_\tau, \tilde{\mathbf{W}}_\mathbf{D}, \tilde{\mathbf{W}}_\mathbf{C}, \tilde{\mathbf{W}}_\mathbf{g}, \tilde{\mathbf{W}}_\mathbf{h}) \in \{(\mathbf{s}, \mathbf{e}_\tau, \tilde{\mathbf{W}}_\mathbf{D}, \tilde{\mathbf{W}}_\mathbf{C}, \tilde{\mathbf{W}}_\mathbf{g}, \tilde{\mathbf{W}}_\mathbf{h}) | V >$$
$$\frac{1}{2\alpha}\left[\frac{1}{\lambda_{\min}(\mathbf{Q})} \sup_{\tau \ge t_0} \left\| \begin{bmatrix} \boldsymbol{\varepsilon}_1(\tau) \\ \boldsymbol{\varepsilon}_2(\tau) \end{bmatrix} \right\|^2 + \sigma_\mathbf{D} Tr(\mathbf{W}_\mathbf{D}^T \mathbf{W}_\mathbf{D}) \right.$$
$$\left. + \sigma_\mathbf{C} Tr(\mathbf{W}_\mathbf{C}^T \mathbf{W}_\mathbf{C}) + \sigma_\mathbf{g} Tr(\mathbf{W}_\mathbf{g}^T \mathbf{W}_\mathbf{g}) + \sigma_\mathbf{h} Tr(\mathbf{W}_\mathbf{h}^T \mathbf{W}_\mathbf{h}) \right]\}$$

This further concludes that $\mathbf{s}$, $\mathbf{e}_\tau$, $\mathbf{e}_i$, $\tilde{\mathbf{W}}_\mathbf{D}$, $\tilde{\mathbf{W}}_\mathbf{C}$, $\tilde{\mathbf{W}}_\mathbf{g}$, and $\tilde{\mathbf{W}}_\mathbf{h}$ are uniformly ultimately bounded(*u.u.b.*). The implementation of the desired transmission torque (14), control input (16) and update law (25) does not need to calculate the regressor matrix which is required in most adaptive designs for robot manipulators. The convergence of the parameters, however, can be proved to depend on the persistent excitation condition of the input.

The above derivation only demonstrates the boundedness of the closed loop system, but in practical applications the transient performance is also of great importance. For further development, we may apply the comparison lemma[32] to (30) to have the upper bound for V as

$$V(t) \le e^{-\alpha(t-t_0)}V(t_0) + \frac{1}{2\alpha}\left[\frac{1}{\lambda_{\min}(\mathbf{Q})}\sup_{t_0<\tau<t}\left\|\begin{bmatrix}\boldsymbol{\varepsilon}_1(\tau)\\ \boldsymbol{\varepsilon}_2(\tau)\end{bmatrix}\right\|^2 + \sigma_{\mathbf{D}}Tr(\mathbf{W}_{\mathbf{D}}^T\mathbf{W}_{\mathbf{D}}) + \sigma_{\mathbf{C}}Tr(\mathbf{W}_{\mathbf{C}}^T\mathbf{W}_{\mathbf{C}}) + \sigma_{\mathbf{g}}Tr(\mathbf{W}_{\mathbf{g}}^T\mathbf{W}_{\mathbf{g}}) + \sigma_{\mathbf{h}}Tr(\mathbf{W}_{\mathbf{h}}^T\mathbf{W}_{\mathbf{h}})\right] \tag{31}$$

From (23), we obtain

$$V \ge \frac{1}{2}\left[\lambda_{\min}(\mathbf{A})\left\|\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\end{bmatrix}\right\|^2 + \lambda_{\min}(\mathbf{Q_D})Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\tilde{\mathbf{W}}_{\mathbf{D}}) + \lambda_{\min}(\mathbf{Q_C})Tr(\tilde{\mathbf{W}}_{\mathbf{C}}^T\tilde{\mathbf{W}}_{\mathbf{C}}) + \lambda_{\min}(\mathbf{Q_g})Tr(\tilde{\mathbf{W}}_{\mathbf{g}}^T\tilde{\mathbf{W}}_{\mathbf{g}}) + \lambda_{\min}(\mathbf{Q_h})Tr(\tilde{\mathbf{W}}_{\mathbf{h}}^T\tilde{\mathbf{W}}_{\mathbf{h}})\right] \tag{32}$$

Thus, the bound of $\left\|\begin{bmatrix}\mathbf{s}^T & \mathbf{e}_\tau^T\end{bmatrix}^T\right\|^2$ for $t \ge t_0$ can be derived from (31) and (32) as

$$\begin{aligned}
\left\|\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\end{bmatrix}\right\|^2 &\le \frac{1}{\underline{\eta}_A}[V - \lambda_{\min}(\mathbf{Q_D})Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\tilde{\mathbf{W}}_{\mathbf{D}}) - \lambda_{\min}(\mathbf{Q_C})Tr(\tilde{\mathbf{W}}_{\mathbf{C}}^T\tilde{\mathbf{W}}_{\mathbf{C}}) \\
&\quad - \lambda_{\min}(\mathbf{Q_g})Tr(\tilde{\mathbf{W}}_{\mathbf{g}}^T\tilde{\mathbf{W}}_{\mathbf{g}}) - \lambda_{\min}(\mathbf{Q_h})Tr(\tilde{\mathbf{W}}_{\mathbf{h}}^T\tilde{\mathbf{W}}_{\mathbf{h}}) \\
&\le \frac{1}{\underline{\eta}_A}\Bigg\{2e^{-\alpha(t-t_0)}V(t_0) + \frac{1}{\alpha}\left[\frac{1}{\lambda_{\min}(\mathbf{Q})}\sup_{t_0<\tau<t}\left\|\begin{bmatrix}\boldsymbol{\varepsilon}_1(\tau)\\ \boldsymbol{\varepsilon}_2(\tau)\end{bmatrix}\right\|^2 + \sigma_{\mathbf{D}}Tr(\mathbf{W}_{\mathbf{D}}^T\mathbf{W}_{\mathbf{D}})\right. \\
&\quad \left. + \sigma_{\mathbf{C}}Tr(\mathbf{W}_{\mathbf{C}}^T\mathbf{W}_{\mathbf{C}}) + \sigma_{\mathbf{g}}Tr(\mathbf{W}_{\mathbf{g}}^T\mathbf{W}_{\mathbf{g}}) + \sigma_{\mathbf{h}}Tr(\mathbf{W}_{\mathbf{h}}^T\mathbf{W}_{\mathbf{h}})\right] \\
&\quad - \lambda_{\min}(\mathbf{Q_D})Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\tilde{\mathbf{W}}_{\mathbf{D}}) - \lambda_{\min}(\mathbf{Q_C})Tr(\tilde{\mathbf{W}}_{\mathbf{C}}^T\tilde{\mathbf{W}}_{\mathbf{C}}) \\
&\quad - \lambda_{\min}(\mathbf{Q_g})Tr(\tilde{\mathbf{W}}_{\mathbf{g}}^T\tilde{\mathbf{W}}_{\mathbf{g}}) - \lambda_{\min}(\mathbf{Q_h})Tr(\tilde{\mathbf{W}}_{\mathbf{h}}^T\tilde{\mathbf{W}}_{\mathbf{h}})\Bigg\}
\end{aligned} \tag{33}$$

From the derivations above, we can conclude that the proposed design is able to give bounded tracking with guaranteed transient performance. The following theorem is a summary of the above results.

Theorem 1: Consider the n-rigid link flexible-joint robot (1) and (2) with unknown parameters **D**, **C**, and **g** then desired transmission torque (14), control input (16) and update law (25) ensure that

(i) error signals **s**, $\mathbf{e}_\tau$, $\tilde{\mathbf{W}}_\mathbf{D}$, $\tilde{\mathbf{W}}_\mathbf{C}$, $\tilde{\mathbf{W}}_\mathbf{g}$, and $\tilde{\mathbf{W}}_\mathbf{h}$ are *u.u.b*.

(ii) the bound of the tracking error vectors for $t \geq t_0$ can be derived as the form of (33), if the Lyapunov-like function candidates are chosen as (23).

Remark 1: The term with $\sigma_{(\cdot)}$ in (25) is to modify the update law to robust the closed-loop system for the effect of the approximation error[17]. Suppose a sufficient number of basis functions $\beta_{(\cdot)}$ is selected so that the approximation error can be neglected then we may have $\sigma_{(\cdot)} = 0$, and (26) becomes

$$\dot{V} = -[\mathbf{s}^T \quad \mathbf{e}_\tau^T]\mathbf{Q}\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\end{bmatrix} \leq 0 \tag{34}$$

It is easy to prove that **s** and $\mathbf{e}_\tau$ are also square integrable. From (21) and (22), $\dot{\mathbf{s}}$ and $\dot{\mathbf{e}}_\tau$ are bounded; as a result, asymptotic convergence of **s** and $\mathbf{e}_\tau$ can easily be shown by Barbalat's lemma. This further implies that $\boldsymbol{\tau} \to \boldsymbol{\tau}_d$ and $\mathbf{q} \to \mathbf{q}_d$ even though **D**, **C**, and **g** are all unknown.

Remark 2: Suppose $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ cannot be ignored but their variation bounds are available[16,17] i.e. there exists positive constants δ_1 and δ_2 such that $\|\boldsymbol{\varepsilon}_1\| \leq \delta_1$, and $\|\boldsymbol{\varepsilon}_2\| \leq \delta_2$. To cover the effect of these bounded approximation errors, the desired transmission torque (14) and the control input (16) are modified to be

$$\boldsymbol{\tau}_d = \hat{\mathbf{D}}\dot{\mathbf{v}} + \hat{\mathbf{C}}\mathbf{v} + \hat{\mathbf{g}} - \mathbf{K}_d\mathbf{s} + \boldsymbol{\tau}_{robust1} \tag{35}$$

$$\mathbf{u} = \boldsymbol{\Theta}\mathbf{x}_p + \boldsymbol{\Phi}\boldsymbol{\tau}_{td} + \hat{\mathbf{h}} + \boldsymbol{\tau}_{robust2} \tag{36}$$

where $\boldsymbol{\tau}_{robust1}$ and $\boldsymbol{\tau}_{robust2}$ are robust terms to be designed. Let us consider the Lyapunov-like function candidate (23) and the update law (25) again. The time derivative of V can be computed as

$$\dot{V} = -[\mathbf{s}^T \quad \mathbf{e}_\tau^T]\mathbf{Q}\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\end{bmatrix} + \delta_1\|\mathbf{s}\| + \delta_2\|\mathbf{e}_\tau\| + \mathbf{s}^T\boldsymbol{\tau}_{robust1} + \mathbf{e}_\tau^T\boldsymbol{\tau}_{robust2} \tag{37}$$

By picking $\boldsymbol{\tau}_{robust1} = -\delta_1[\mathrm{sgn}(s_1) \quad \cdots \quad \mathrm{sgn}(s_n)]^T$, where s_k, k=1,…,n is the k-th element of **s**, and $\boldsymbol{\tau}_{robust2} = -\delta_2[\mathrm{sgn}(e_{\tau_1}) \quad \cdots \quad \mathrm{sgn}(e_{\tau_n})]^T$ where e_{τ_k}, k=1,…,2n is the k-th element of $\mathbf{e}_\tau$, we may have $\dot{V} \leq 0$, and asymptotic convergence of the state error can be concluded by Barbalat's lemma.

3. SIMULATION STUDY

Consider a planar robot (Fig.1) with two rigid links and two flexible joints represented by the differential equation (1), and (2). The quantities m_i, l_i, l_{ci} and I_i are mass, length, gravity center distance and inertia of link i, respectively. Actual values of link parameters in the simulation are[18] m_1=0.5kg, m_2=0.5kg, l_1=l_2=0.75m, l_{c1}=l_{c2}=0.375m, I_1=0.09375kg-m^2, and I_2=0.046975kg-m^2. The actuator inertias, damping, and joint stiffness are $\mathbf{J} = diag(0.02,0.01)(kg \cdot m^2)$, $\mathbf{B} = diag(5,4)(Nm \cdot \sec/rad)$ and $\mathbf{K} = diag(100,100)(Nm/rad)$ respectively. We would like the end-point to track a 0.2m-radius circle centered at (0.8 m, 1.0 m) in 10 seconds. To have more challenge, we pick the initial condition of the link angles and the motor angles as $\mathbf{q} = [-0.184 \quad 1.94 \quad 0 \quad 0]^T$ and $\boldsymbol{\theta} = [-0.184 \quad 1.94 \quad 0 \quad 0]^T$ that are significantly away from the desired trajectory. The initial value of the reference model state vector is $\boldsymbol{\tau}_r = [0.39 \quad -0.72 \quad 0 \quad 0]^T$ which is the same as the initial value of the desired reference input $\boldsymbol{\tau}_d$. The controller gains are selected as $\mathbf{K}_d = diag(0.1, 0.1)$ and $\boldsymbol{\Lambda} = diag(5,5)$. Each element of **D**, **C**, **g** and **h** is approximated by the first 41 terms of the Fourier series. The simulation results are shown in Fig. 2 to 8. Fig. 2 shows the tracking performance of the end-point and the desired trajectory in the Cartesian space. It is observed that the end-point trajectory converges nicely to the desired trajectory, although the initial position error is quite large. Fig. 3 is the joint space tracking performance. It shows that the transient response vanishes very quickly. Fig. 4 is the actuator inputs in N-m. Fig. 5 to 8 are the performance of function approximation for **D**, **C**, **g** and **h** respectively. Since the reference input does not satisfy the persistent excitation condition, some estimates do not converge to their actual values but remain bounded as desired. It is worth to note that in designing the controller we do not need much knowledge for the system. All we have to do is to pick some controller parameters and some initial weighting matrices.

4. CONCULSIONS

In this paper, we have proposed a FAT-based adaptive controller for a flexible joint robot containing time-varying uncertainties. The new design is free from regressor calculation and knowledge of bounds of uncertainties.

Feedback of the joint acceleration is also avoided. The function approximation technique is used to deal with time-varying uncertainties. Using the Lyapunov like analysis, rigorous proof of the closed loop stability has been investigated with consideration of the approximation error. Computer simulation results justify its feasibility of giving satisfactory tracking performance on a 2-D flexible-joint robot although we do not know much knowledge about the system model.

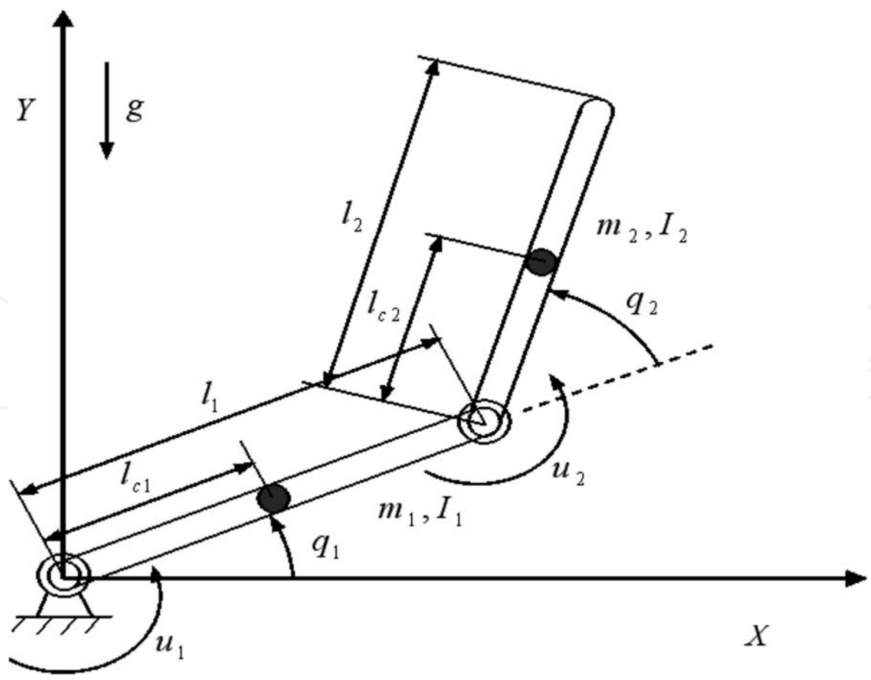

Fig. 1. 2-DOF planar robot

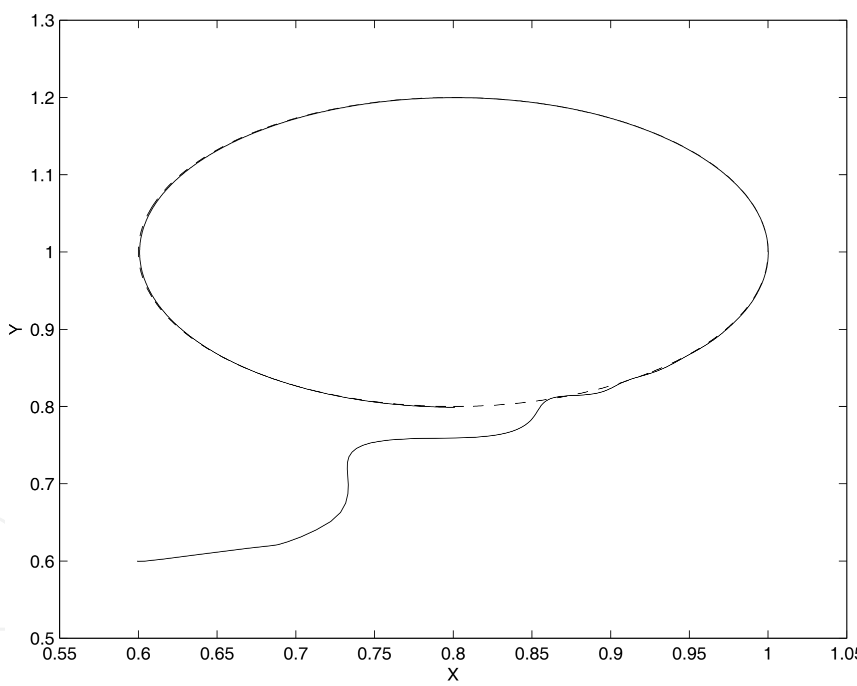

Fig. 2. Tracking performance of end-point in the Cartesian space (— actual; --- desired). Initial position of end-point is at the point (0.6*m*, 0.6*m*). After some transient, the tracking error is very small, although we do not know precise dynamics of the robot.

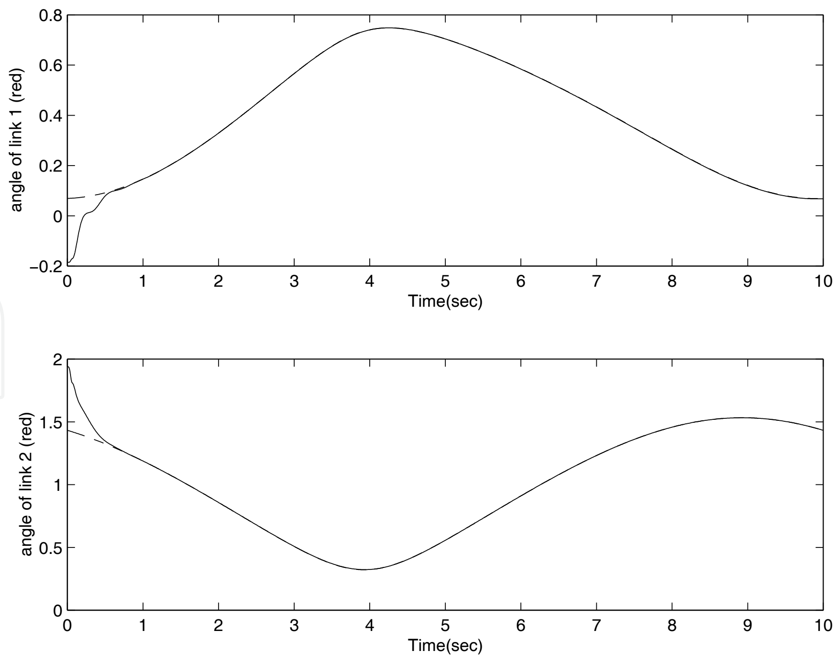

Fig. 3. The joint space tracking performance(— actual; --- desired). The real trajectory converges very quickly.

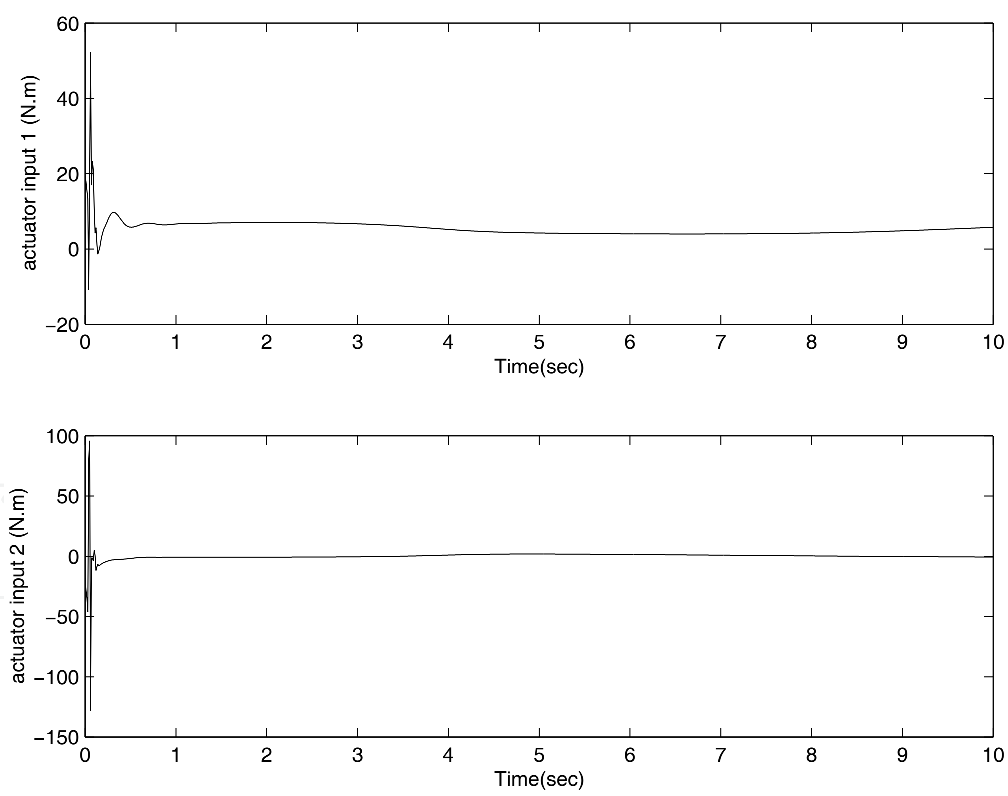

Fig. 4. Actuator input.

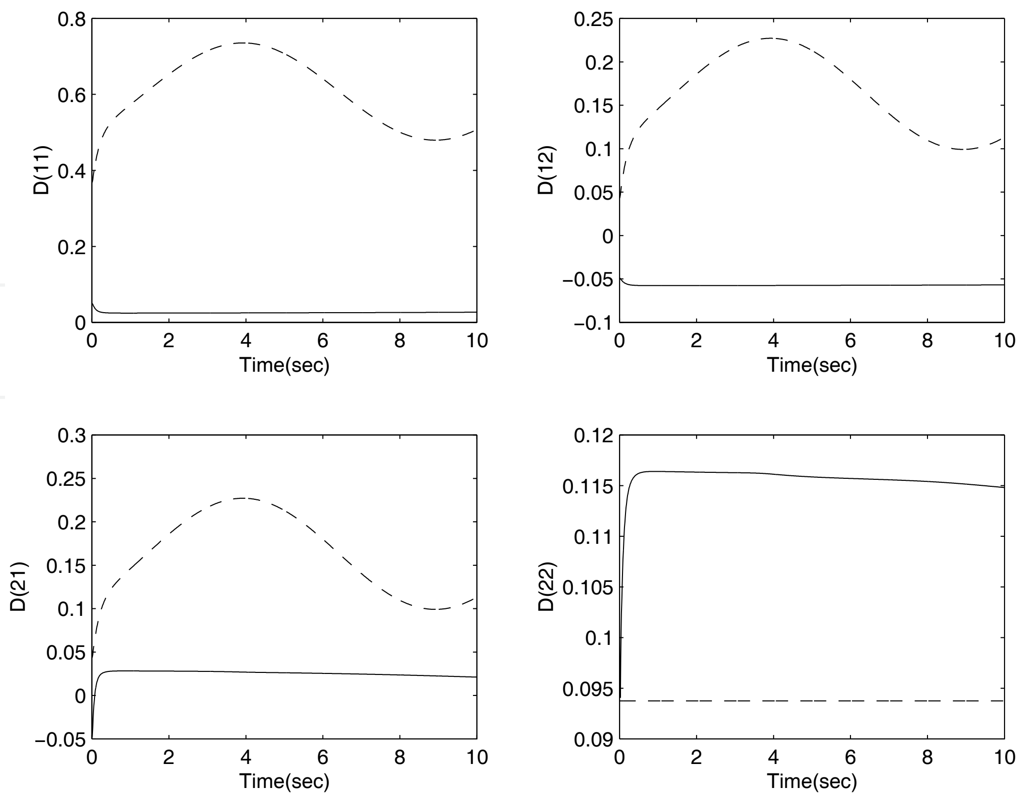

Fig. 5. Approximation of D matrix(— estimate; --- real). Although the estimated values do not converge to the true values, they are bounded and small.

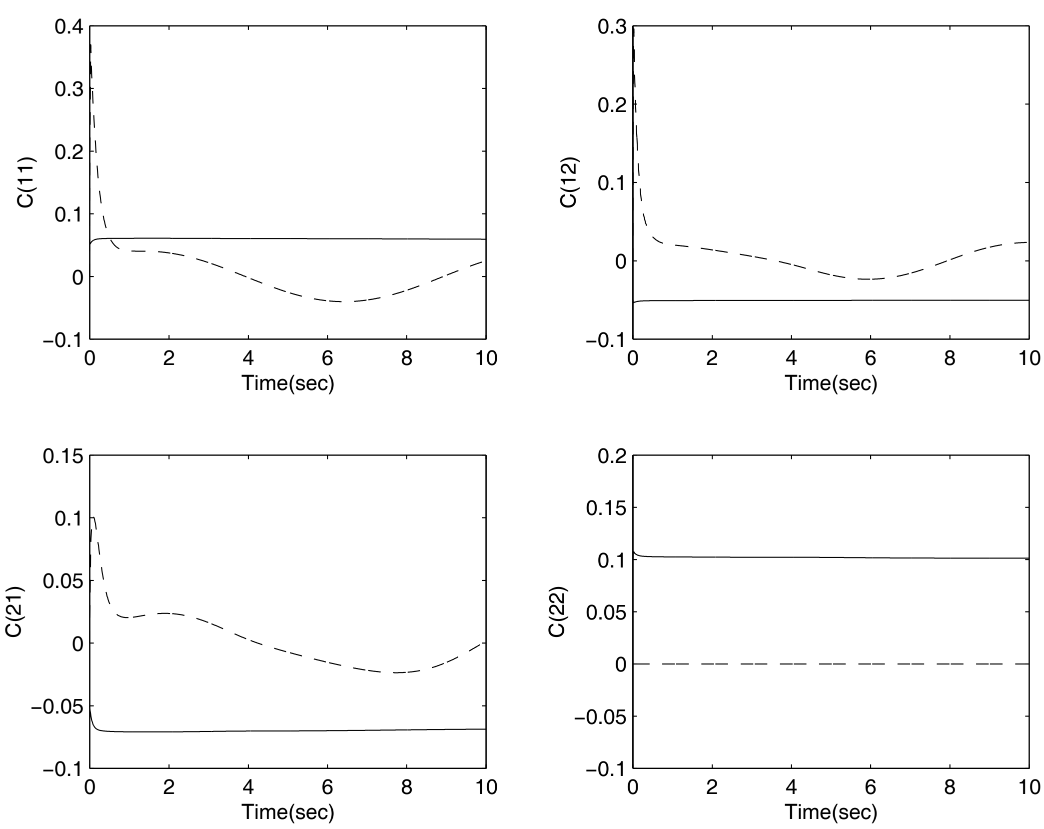

Fig. 6. Approximation of **C** matrix(— estimate; --- real).

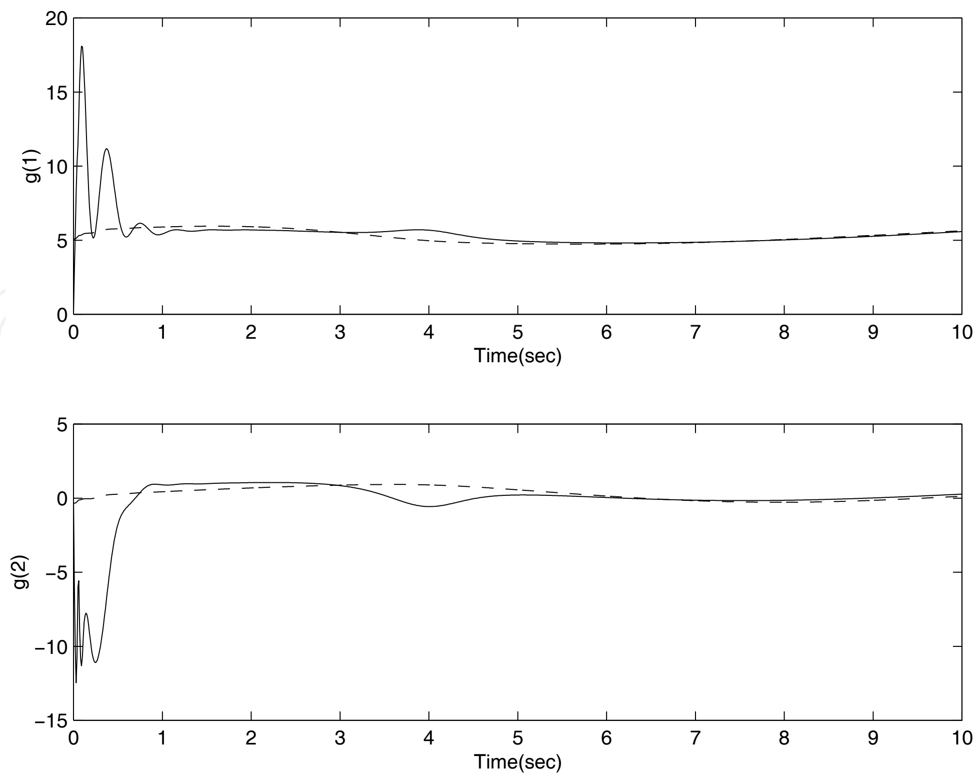

Fig. 7. Approximation of vector **g**(— estimate; --- real).

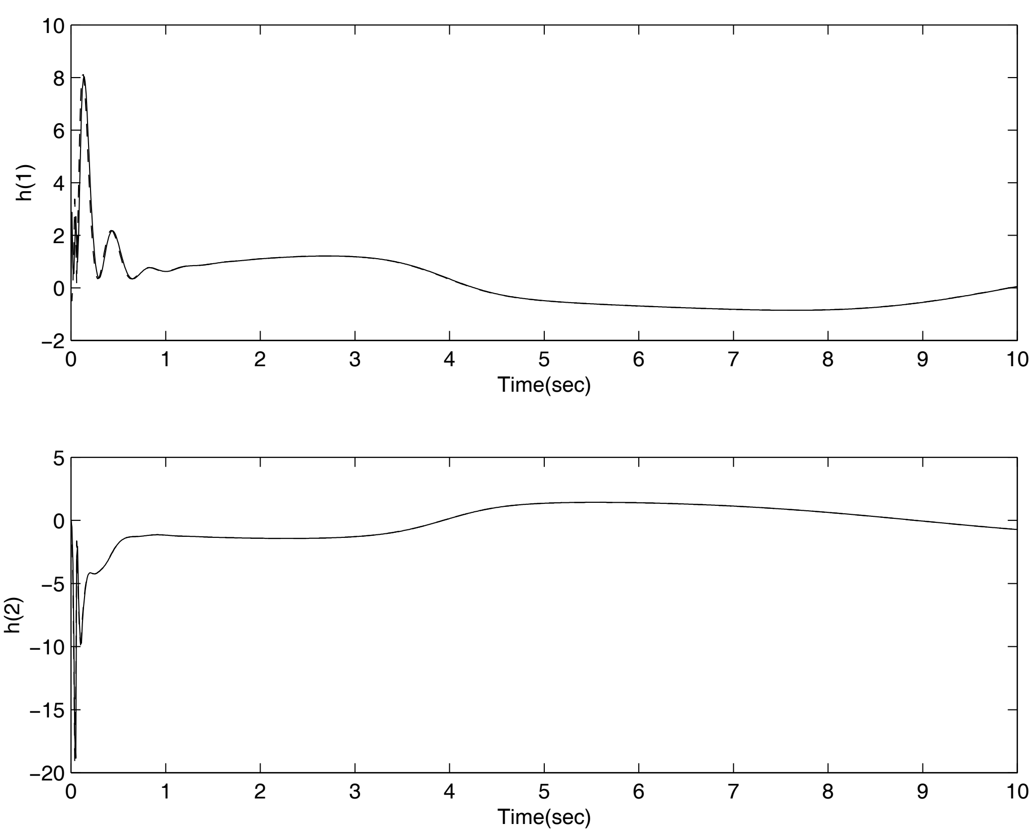

Fig. 8. Approximation of vector **h**(— estimate; --- real).

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APPENDIX

Lemma A.1:

Let $\mathbf{s} \in \Re^n$, $\boldsymbol{\varepsilon} \in \Re^n$ and $\mathbf{K}$ is the $n \times n$ positive definite matrix. Then,

$$-\mathbf{s}^T\mathbf{K}\mathbf{s} + \mathbf{s}^T\boldsymbol{\varepsilon} \le \frac{1}{2}[\lambda_{\min}(\mathbf{K})\|\mathbf{s}\|^2 - \frac{\|\boldsymbol{\varepsilon}\|^2}{\lambda_{\min}(\mathbf{K})}]. \tag{A.1}$$

Proof:

$$\begin{aligned}-\mathbf{s}^T\mathbf{K}\mathbf{s} + \mathbf{s}^T\boldsymbol{\varepsilon} &\le [-\lambda_{\min}(\mathbf{K})\|\mathbf{s}\| + \|\boldsymbol{\varepsilon}\|]\|\mathbf{s}\| \\ &= -\frac{1}{2}[\sqrt{\lambda_{\min}(\mathbf{K})}\|\mathbf{s}\| - \frac{\|\boldsymbol{\varepsilon}\|}{\sqrt{\lambda_{\min}(\mathbf{K})}}]^2 \\ &\quad -\frac{1}{2}[\lambda_{\min}(\mathbf{K})\|\mathbf{s}\|^2 - \frac{\|\boldsymbol{\varepsilon}\|^2}{\lambda_{\min}(\mathbf{K})}] \\ &\le -\frac{1}{2}[\lambda_{\min}(\mathbf{K})\|\mathbf{s}\|^2 - \frac{\|\boldsymbol{\varepsilon}\|^2}{\lambda_{\min}(\mathbf{K})}]\end{aligned}$$

Q.E.D.

Lemma A.2:

Let $\mathbf{w}_i^T = [w_{i1} \quad w_{i2} \quad \cdots \quad w_{in}] \in \Re^{1\times n}$, i=1,…,m and $\mathbf{W}$ is a block diagonal matrix defined as $\mathbf{W} = diag\{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_m\} \in \Re^{mn\times m}$. Then,

$$Tr(\mathbf{W}^T\mathbf{W}) = \sum_{i=1}^{m}\|\mathbf{w}_i\|^2 \tag{A.2}$$

The notation $Tr(.)$ denotes the trace operation.

Proof: The proof is straightforward as below:

$$\mathbf{W}^T\mathbf{W} = \begin{bmatrix} w_{11} & \cdots & w_{1n} & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & w_{21} & \cdots & w_{2n} & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & w_{m1} & \cdots & w_{mn} \end{bmatrix} \begin{bmatrix} w_{11} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ w_{1n} & 0 & \cdots & 0 \\ 0 & w_{21} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & w_{2n} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{m1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{w}_1^T & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_2^T & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_m^T \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_m \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{w}_1^T\mathbf{w}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_2^T\mathbf{w}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_m^T\mathbf{w}_m \end{bmatrix}$$

$$= \begin{bmatrix} \|\mathbf{w}_1\|^2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \|\mathbf{w}_2\|^2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \|\mathbf{w}_m\|^2 \end{bmatrix}$$

The last equality holds because by definition $\mathbf{w}_i^T\mathbf{w}_i = w_{i1}^2 + w_{i2}^2 + \ldots + w_{im}^2 = \|\mathbf{w}_i\|^2$.

Therefore, we have $Tr = (\mathbf{W}^T\mathbf{W}) = \sum_{i=1}^{m} \|\mathbf{w}_i\|^2$.

Q.E.D.

Lemma A.3:

Suppose $\mathbf{w}_i^T = [w_{i1} \quad w_{i2} \quad \cdots \quad w_{in}] \in \Re^{1\times n}$ and $\mathbf{v}_i^T = [v_{i1} \quad v_{i2} \quad \cdots \quad v_{in}] \in \Re^{1\times n}$, i=1,…,m. Let **W** and **V** be block diagonal matrices that are defined as $\mathbf{W} = diag\{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_m\} \in \Re^{mn\times m}$ and $\mathbf{V} = diag\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_m\} \in \Re^{mn\times m}$, respectively. Then,

$$Tr(\mathbf{V}^T\mathbf{W}) \le \sum_{i=1}^{m} \|\mathbf{v}_i\|\|\mathbf{w}_i\| \tag{A.3}$$

Proof: The proof is also straightforward:

$$\mathbf{V}^T\mathbf{W} = \begin{bmatrix} \mathbf{v}_1^T & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{v}_2^T & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{v}_m^T \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_m \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{v}_1^T\mathbf{w}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{v}_2^T\mathbf{w}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{v}_m^T\mathbf{w}_m \end{bmatrix}$$

Hence,

$$\begin{aligned} Tr(\mathbf{V}^T\mathbf{W}) &= \mathbf{v}_1^T\mathbf{w}_1 + \mathbf{v}_2^T\mathbf{w}_2 + \ldots + \mathbf{v}_m^T\mathbf{w}_m \\ &\le \|\mathbf{v}_1\|\|\mathbf{w}_1\| + \|\mathbf{v}_2\|\|\mathbf{w}_2\| + \ldots + \|\mathbf{v}_m\|\|\mathbf{w}_m\| \\ &= \sum_{i=1}^{m} \|\mathbf{v}_i\|\|\mathbf{w}_i\| \end{aligned}$$

Q.E.D.

Lemma A.4:

Let $\mathbf{W}$ be defined as in *Lemma A.2*, and $\tilde{\mathbf{W}}$ is a matrix defined as $\tilde{\mathbf{W}} = \mathbf{W} - \hat{\mathbf{W}}$, where $\hat{\mathbf{W}}$ is a matrix with proper dimension. Then

$$Tr(\tilde{\mathbf{W}}^T\hat{\mathbf{W}}) \le \frac{1}{2}Tr(\mathbf{W}^T\mathbf{W}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}). \tag{A.4}$$

Proof:

$$
\begin{aligned}
Tr(\tilde{\mathbf{W}}^T \hat{\mathbf{W}}) &= Tr(\tilde{\mathbf{W}}^T \mathbf{W}) - Tr(\tilde{\mathbf{W}}^T \tilde{\mathbf{W}}) \\
&\le \sum_{i=1}^{m} (\|\tilde{\mathbf{w}}_i\| \|\mathbf{w}_i\| - \|\tilde{\mathbf{w}}_i\|^2) \qquad \text{(by } \textit{Lemma A.2} \text{ and } \textit{A.3}\text{)} \\
&= \frac{1}{2} \sum_{i=1}^{m} [\|\mathbf{w}_i\|^2 - \|\tilde{\mathbf{w}}_i\|^2 - (\|\tilde{\mathbf{w}}_i\| - \|\mathbf{w}_i\|)^2] \\
&\le \frac{1}{2} \sum_{i=1}^{m} (\|\mathbf{w}_i\|^2 - \|\tilde{\mathbf{w}}_i\|^2) \\
&= \frac{1}{2} Tr(\mathbf{W}^T \mathbf{W}) - \frac{1}{2} Tr(\tilde{\mathbf{W}}^T \tilde{\mathbf{W}}) \quad \text{(by } \textit{Lemma A.2}\text{)}
\end{aligned}
$$

Q.E.D.

In the above lemmas, we consider properties of a block diagonal matrix. In the following, we would like to extend the analysis to a class of more general matrices.

Lemma A.5:

Let $\mathbf{W}$ be a matrix in the form $\mathbf{W}^T = [\mathbf{W}_1^T \quad \mathbf{W}_2^T \quad \cdots \quad \mathbf{W}_p^T] \in \Re^{pmn \times m}$ where $\mathbf{W}_i = diag\{\mathbf{w}_{i1}, \mathbf{w}_{i2}, \cdots, \mathbf{w}_{im}\} \in \Re^{mn \times m}$, i=1,…,p, are block diagonal matrices with the entries of vectors $\mathbf{w}_{ij}^T = [w_{ij1} \quad w_{ij2} \quad \cdots \quad w_{ijn}] \in \Re^{1 \times n}$, j=1,…,m. Then, we may have

$$
Tr(\mathbf{W}^T \mathbf{W}) = \sum_{i=1}^{p} \sum_{j=1}^{m} \|\mathbf{w}_{ij}\|^2 . \tag{A.5}
$$

Proof:

$$
\begin{aligned}
\mathbf{W}^T \mathbf{W} &= [\mathbf{W}_1^T \quad \cdots \quad \mathbf{W}_p^T] \begin{bmatrix} \mathbf{W}_1 \\ \vdots \\ \mathbf{W}_p \end{bmatrix} \\
&= \mathbf{W}_1^T \mathbf{W}_1 + \cdots + \mathbf{W}_p^T \mathbf{W}_p
\end{aligned}
$$

Hence, we may calculate the trace as

$$
\begin{aligned}
Tr(\mathbf{W}^T \mathbf{W}) &= Tr(\mathbf{W}_1^T \mathbf{W}_1) + \cdots + Tr(\mathbf{W}_p^T \mathbf{W}_p) \\
&= \sum_{j=1}^{m} \|\mathbf{w}_{1j}\|^2 + \cdots + \sum_{j=1}^{m} \|\mathbf{w}_{pj}\|^2 \qquad \text{(by } \textit{Lemma A.1}\text{)} \\
&= \sum_{i=1}^{p} \sum_{j=1}^{m} \|\mathbf{w}_{ij}\|^2
\end{aligned}
$$

Q.E.D.

Lemma A.6:

Let **V** and **W** be matrices defined in *Lemma A.5*, Then,

$$Tr(\mathbf{V}^T\mathbf{W}) \le \sum_{i=1}^{p}\sum_{j=1}^{m}\|\mathbf{v}_{ij}\|\|\mathbf{w}_{ij}\|. \tag{A.6}$$

Proof:

$$\begin{aligned} Tr(\mathbf{V}^T\mathbf{W}) &= Tr(\mathbf{V}_1^T\mathbf{W}_1) + \cdots + Tr(\mathbf{V}_p^T\mathbf{W}_p) \\ &\le \sum_{j=1}^{m}\|\mathbf{v}_{1j}\|\|\mathbf{w}_{1j}\| + \cdots + \sum_{j=1}^{m}\|\mathbf{v}_{pj}\|\|\mathbf{w}_{pj}\| \qquad \text{(by } \textit{Lemma A.3}\text{)} \\ &= \sum_{i=1}^{p}\sum_{j=1}^{m}\|\mathbf{v}_{ij}\|\|\mathbf{w}_{ij}\| \end{aligned}$$

Q.E.D.

Lemma A.7:

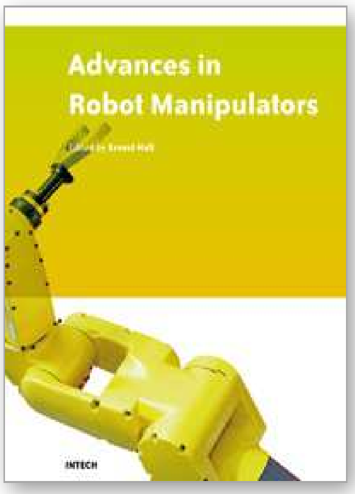
Let **W** be defined as in *Lemma A.5*, and $\tilde{\mathbf{W}}$ is a matrix defined as $\tilde{\mathbf{W}} = \mathbf{W} - \hat{\mathbf{W}}$, where $\hat{\mathbf{W}}$ is a matrix with proper dimension. Then

$$Tr(\tilde{\mathbf{W}}^T\hat{\mathbf{W}}) \le \frac{1}{2}Tr(\mathbf{W}^T\mathbf{W}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}). \tag{A.7}$$

Proof:

$$\begin{aligned} Tr(\tilde{\mathbf{W}}^T\hat{\mathbf{W}}) &= Tr(\tilde{\mathbf{W}}^T\mathbf{W}) - Tr(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}) \\ &\le \sum_{i=1}^{p}\sum_{j=1}^{m}(\|\tilde{\mathbf{w}}_{ij}\|\|\mathbf{w}_{ij}\| - \|\tilde{\mathbf{w}}_{ij}\|^2) \qquad \text{(by } \textit{Lemma A.5} \text{ and } \textit{A.6}\text{)} \\ &= \frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{m}[\|\mathbf{w}_{ij}\|^2 - \|\tilde{\mathbf{w}}_{ij}\|^2 - (\|\tilde{\mathbf{w}}_{ij}\| - \|\mathbf{w}_{ij}\|)^2] \\ &\le \frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{m}(\|\mathbf{w}_{ij}\|^2 - \|\tilde{\mathbf{w}}_{ij}\|^2) \\ &= \frac{1}{2}Tr(\mathbf{W}^T\mathbf{W}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}) \qquad \text{(by } \textit{Lemma A.5}\text{)} \end{aligned}$$

Q.E.D



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The purpose of this volume is to encourage and inspire the continual invention of robot manipulators for science and the good of humanity. The concepts of artificial intelligence combined with the engineering and technology of feedback control, have great potential for new, useful and exciting machines. The concept of eclecticism for the design, development, simulation and implementation of a real time controller for an intelligent, vision guided robots is now being explored. The dream of an eclectic perceptual, creative controller that can select its own tasks and perform autonomous operations with reliability and dependability is starting to evolve. We have not yet reached this stage but a careful study of the contents will start one on the exciting journey that could lead to many inventions and successful solutions.

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