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The Analytic Hierarchy and the Network Process in Multicriteria Decision Making: Performance Evaluation and Selecting Key Performance Indicators Based on ANP Model

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1. Introduction

AHP is a method for ranking decision alternatives and selecting the best one when the decision maker has multiple criteria (Taylor, 2004). In evaluation n competing alternatives A_1, A_2,…,A_n under a given criterion, it is natural to use the framework of pair-wise comparison by $n \times n$ square matrix from which a set of preference values for the alternatives is derived. Many methods for estimating the preference values from the pair-wise comparison matrix have been proposed and the effectiveness comparatively evaluated. Most of the estimating methods proposed and studied are with the paradigm of the analytic hierarchy process that presumes ratio-scaled preference values. AHP is one of the ways for deciding among the complex criteria structure in different levels. Fuzzy AHP is a synthetic extension of classical AHP method when the fuzziness of the decision maker is considered.
ANP is a new theory that extends the Analytic Hierarchy Process (AHP) to case of dependence and feedbacks introduced by Saaty (1980), with book in 1996 revised and extended in 2001. The ANP makes it possible to deal systematically with all kinds of dependence and feedback in decision system (Fiala, 2001; Chen, 2001). ANP allows for complex interrelationships among decision levels and attributes. The ANP feedback approach replaces hierarchies with networks in which the relationship between levels are not easily represented as higher or lower, dominated or being dominated, directly or indirectly (Meade & Sarkis, 1999). For instance, not only does the importance of the criteria determine the importance of the alternatives, as in hierarchy, but the importance of the alternatives may also have an impact on importance of the criteria (Saaty, 1996). Therefore, a hierarchical representation with a linear top-to-bottom structure is not suitable for complex system (Chung et al., 2005).
In literature, there exists numerous studies conduct with the aim of performing indicators within the boundaries of objective criteria. Sardana (2009) presents a business performance measurement framework, for organizational design, process management, quality management and recipient satisfaction, and defines an appropriate set of performance

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measures for small or medium enterprise. Hwang (2007) use Data Envelopment Analysis to measure the managerial performance of electronics industry in Taiwan. In multi-criteria decision making (MCDM) model for selecting the collecting centre location in the reverse logistics supply chain model (PLSCM) using the analytical hierarchy process and fuzzy analytical hierarchy process (FAHP) (Anand, et al., 2008). Faisal and Banwet (2009), the ANP, which utilizes the concept of dependence and feedback is proposed as a suitable technique for analyzing IT outsourcing decision. The synergistic integration of two techniques, the analytical network process and data envelopment analysis is application in a multi-phased supplier selection approach (Hasan et al., 2008). Lee (2007) construct an approach based on the analytical hierarchy process and balanced score card. It has four criteria of this study: financial perspective, customer perspective, internal business process perspective, and learning and growth perspective. In model of information system, Ballou et al. (1998) consider four criteria of information products: timeliness, data quality, cost and value. Niemir and Saaty (2004) argues performance indicators have: linked to strategy, quantitative, built on accessible data, easily understood, counterbalanced, relevant, and commonly defined. According the insights of literature a number of criteria have been defined: relevance, reliability, comparability and consistency, understandability and representational quality. As can be seen, the information manufacturing systems criteria (factor) are not independent of each other. Since the criteria (factor) weights are traditionally computed by assuming that the factors are independent, it is possible that the weights computed by including the dependent relations could be different. Therefore, it is necessary to employ analyses which measure and take the possible dependencies among factors into account in the information manufacturing system analysis.

2. Analytical Hierarchy Process (AHP)

2.1 AHP process

The analytic hierarchy process (AHP), developed at the Wharton School of Business by Thomas Saaty (1980), allows decision makers to model a complex problem in a hierarchical structure showing the relationships of the goal, objectives (criteria), sub-objectives, and alternatives (See Figure 1). Uncertainties and other influencing factors can also be included. Figure 1 – Decision Hierarchy AHP allows for the application of data, experience, insight, and intuition in a logical and thorough way. AHP enables decision-makers to derive ratio scale priorities or weights as opposed to arbitrarily assigning them. In so doing, AHP not only supports decision-makers by enabling them to structure complexity and exercise judgment, but allows them to incorporate both objective and subjective considerations in the decision process. AHP is a compensatory decision methodology because alternatives that are deficient with respect to one or more objectives can compensate by their performance with respect to other objectives. AHP is composed of several previously existing but unassociated concepts and techniques such as hierarchical structuring of complexity, pair-wise comparisons, redundant judgments, an eigenvector method for deriving weights, and consistency considerations. The AHP procedure involves six essential steps (Lee et al., 2008).

1. Define the unstructured problem
2. Developing the AHP hierarchy
3. Pair-wise comparison
4. Estimate the relative weights

5. Check the consistency
6. Obtain the overall rating

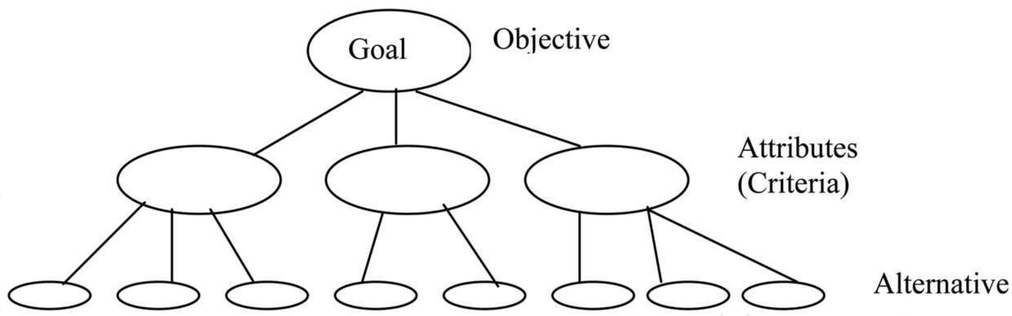


Fig. 1. Hierarchy structure of decision problem

Step 1: Define the unstructured problem

In this step the unstructured problem and their characters should be recognized and the objectives and outcomes stated clearly.

Step 2: Developing the AHP hierarchy

The first step in the AHP procedure is to decompose the decision problem into a hierarchy that consists of the most important elements of the decision problem (Boroushaki and Malczewski, 2008). In this step the complex problem is decomposed into a hierarchical structure with decision elements

Fig.2 represents this structure.

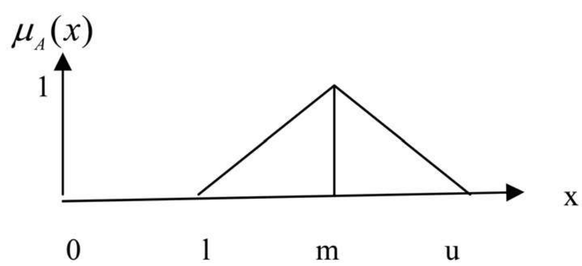


Fig. 2. Triangular membership function

Step 3: Pair-wise comparison

For each element of the hierarchy structure all the associated elements in low hierarchy are compared in pair-wise comparison matrices as follows:

$$A = \begin{bmatrix} 1 & \frac{w_1}{w_2} & \ldots \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & 1 & \ldots \frac{w_2}{w_n} \\ . & & \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \ldots 1 \end{bmatrix} \quad (1)$$

Where A = comparison pair-wise matrix,
w_1 = weight of element 1,
w_2 = weight of element 2,
w_n = weight of element n.

In order to determine the relative preferences for two elements of the hierarchy in matrix A, an underlying semantically scale is employs with values from 1 to 9 to rate (Table 1).

Preferences expressed in numeric variables	Preferences expressed in linguistic variables
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Intermediate values between adjacent scale values

Table 1. Scales for pair-wise comparison (Saaty, 1980)

Step 4: Estimate the relative weights

Some methods like eigenvalue method are used to calculate the relative weights of elements in each pair-wise comparison matrix. The relative weights (*W*) of matrix *A* is obtained from following equation:

$$A \times W = \lambda_{\max} \times W \tag{2}$$

Where λ_{max} = the biggest eigenvalue of matrix A, *I* = unit matrix.

Step 5: Check the consistency

In this step the consistency property of matrices is checked to ensure that the judgments of decision makers are consistent. For this end some pre-parameter is needed. Consistency Index (*CI*) is calculated as:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{3}$$

The consistency index of a randomly generated reciprocal matrix shall be called to the random index (*RI*), with reciprocals forced. An average *RI* for the matrices of order 1–15 was generated by using a sample size of 100 (Nobre et al., 1999). The table of random indexes of the matrices of order 1–15 can be seen in Saaty (1980). The last ratio that has to be calculated is *CR* (Consistency Ratio). Generally, if *CR* is less than 0.1, the judgments are consistent, so the derived weights can be used. The formulation of *CR* is:

$$CR = \frac{CI}{RI} \tag{4}$$

Step 6: Obtain the overall rating

In last step the relative weights of decision elements are aggregated to obtain an overall rating for the alternatives as follows:

$$w_i^s = \sum_{j=1}^{m} w_{ij}^s w_j \qquad , i = 1, \ldots, n \tag{5}$$

Where w_i^s = total weight of site i,

w_{ij}^{s} = weight of alternative (site) i associated to attribute (map layer) j,

w_j = weight of attribute j,

m = number of attribute,
n= number of site.

2.2 Fuzzy process

2.2.1 A brief introduction to fuzzy set theory

Fuzzy set theory is a mathematical theory designed to model the vagueness or imprecision of human cognitive processes that pioneered. This theory is basically a theory of classes with unship boundaries. What is important to recognize is that any crisp theory can be fuzzified by generalizing the concept of a set within that theory to the concept of a fuzzy set. The stimulus for the transition from a crisp theory to a fuzzy one derives from the fact that both the generality of a theory and its applicability to real world problems are enhanced by replacing the concept of a crisp set with a fuzzy set (Zadeh, 1994).
Generally, the fuzzy sets are defined by the membership functions. The fuzzy sets represent the grade of any element x of X that have the partial membership to A. The degree to which an element belongs to a set is defined by the value between 0 and 1. If an element x really belongs to A if $\mu_A(x) = 1$ and clearly not if $\mu_A(x) = 0$. Higher is the membership value, $\mu_A(x)$, greater is the belongingness of an element x to a set A. The Fuzzy AHP presented in this paper applied the triangular fuzzy number through symmetric triangular membership function. A triangular fuzzy number is the special class of fuzzy number whose membership defined by three real numbers, expressed as (*l, m, u*).

$$\mu_A(x) = \begin{cases} (x-l)/(m-l), & l \le x \le m \\ (u-x)/(u-m), & m \le x \le u \\ 0 & otherwise \end{cases} \tag{6}$$

Since fuzziness and vagueness are common characteristics in many decision-making problems, a fuzzy AHP (FAHP) method should be able to tolerate vagueness or ambiguity (Mikhailov & Tsvetinov, 2004). In other word the conventional AHP approach may not fully reflect a style of human thinking because the decision makers usually feel more confident to give interval judgments rather than expressing their judgments in the form of single numeric values and so FAHP is capable of capturing a human's appraisal of ambiguity when complex multi-attribute decision making problems are considered (Erensal et al., 2006). This ability comes to exist when the crisp judgments transformed into fuzzy judgments. Zadeh (1965) published his work Fuzzy Sets, which described the mathematics of fuzzy set theory. This theory, which was a generalization of classic set theory, allowed the membership functions to operate over the range of real numbers [0, 1]. The main characteristic of fuzziness is the grouping of individuals into classes that do not have sharply defined boundaries. The uncertain comparison judgment can be represented by the fuzzy number.

2.2.2 Fuzzy AHP process

Step 1: Fuzzy pair-wise comparison matrix

Given a crisp pair-wise comparison matrix (CPM) A, having the values raging from 1/9 to 9, the ceisp PCM is fuzzified using the triangular fuzzy number (l, m u), which fuzzy the original PCM using the conversion number as indicated in the table below (Table 2). In order to construct pair-wise comparison of alternatives under each criterion or about criteria, like that was said for traditional AHP, a triangular fuzzy comparison matrix is defined as follows:

Crisp PCM value	Fuzzy PCM value	Crisp PCM value	Fuzzy PCM value
1	(1 1 1) if diagonal; (1 1 3) otherwise	1/1	(1 1 1) if diagonal; (1 1 3) otherwise
2	(1 2 4)	1/2	(1/4 1/2 1/1)
3	(1 3 5)	1/3	(1/5 1/3 1/1)
4	(2 4 6)	1/4	(1/6 1/4 1/2)
5	(3 5 7)	1/5	(1/7 1/5 1/3)
6	(4 6 8)	1/6	(1/8 1/6 1/4)
7	(5 7 9)	1/7	(1/9 1/7 1/5)
8	(6 8 10)	1/8	(1/10 1/8 1/6)
9	(7 9 11)	1/9	(1/11 1/9 1/7)

Table 2. Conversion of crisp to fuzzy PCM

$$\tilde{A} = (\tilde{a}_{ij})_{n\times n} = \begin{bmatrix} (111) & (l_{12}\ m_{12}\ u_{12})\ldots & (l_{1n}\ m_{1n}\ u_{1n}) \\ (l_{21}\ m_{21}\ u_{21}) & (111) & \ldots\ (l_{2n}\ m_{2n}\ u_{2n}) \\ & & \\ (l_{n1}\ m_{n1}\ u_{n1}) & (l_{n2}\ m_{n2}\ u_{n2})\ldots & (111) \end{bmatrix} \quad (7)$$

Where $\tilde{a}_{ij} = (l_{ij}\ m_{ij}\ u_{ij}),\ \tilde{a}_{ij}^{-1} = (1/u_{ji}\ 1/m_{ji}\ 1/l_{ji})$

For $i,j = 1,\ldots, n$ and $i \neq j$

Total weighs and preferences of alternatives can be acquired from different method. Two approaches will be posed in resumption.

Step 2: Fuzzy Extent Analysis

Chang's extent analysis: (Chang, 1996)

Different methods have been proposed in the literatures that one of most known of them is Fuzzy Extent Analysis proposed by Chang (1996). The steps of chang's extent analysis can be summarized as follows:

First step: computing the normalized value of row sums (i.e. fuzzy synthetic extent) by fuzzy arithmetic operations:

$$\tilde{s}_i = \sum_{j=1}^{n} \tilde{a}_{ij} \otimes [\sum_{k=1}^{n} \sum_{j=1}^{n} \tilde{a}_{ij}]^{-1} \quad (8)$$

Where ⊗ denotes the extended multiplication of two fuzzy numbers.

Second step: computing the degree of possibility of by following equation:

$$v(\tilde{s}_i \geq \tilde{s}_j) = \sup_{y \geq x}[\min(\tilde{s}_j(x), \tilde{s}_i(y))] \quad (9)$$

which can be equivalently expressed as,

$$v(\tilde{s}_i \geq \tilde{s}_j) = \begin{cases} 1 & m_i \geq m_j \\ \dfrac{u_i - l_j}{(u_i - m_i) + (m_j - l_j)} & l_j \leq u_i, i, j = 1, \ldots, n, j \neq i \\ 0 & otherwise \end{cases} \quad (10)$$

Where $\tilde{s}_i = (l_i \;\; m_i \;\; u_i)$ and $\tilde{s}_j = (l_j \;\; m_j \;\; u_j)$

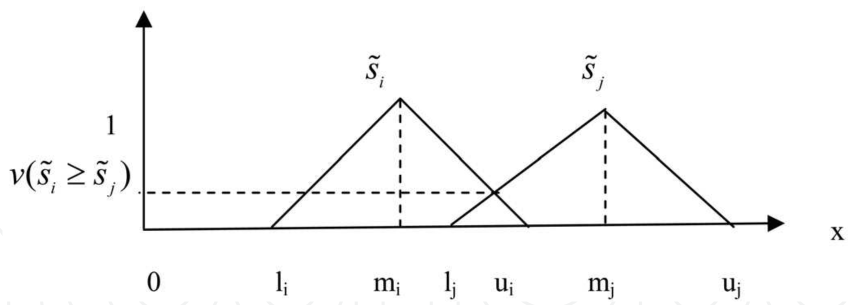


Fig. 3. The degree of possibility of $\tilde{s}_i \geq \tilde{s}_j$

Third step: calculating the degree of possibility of $\tilde{s}_i$ to be greater than all the other (n-1) convex fuzzy number $\tilde{s}_j$ by:

$$v(\tilde{s}_i \geq \tilde{s}_j \;\mid\; j=1,....,n, j \neq i) = \min_{j \in (1,....,n)\, j \neq i} v(\tilde{s}_i \geq \tilde{s}_j),\; i=1,....,n \tag{11}$$

Fourth step: defining the priority vector $W = (w_1, ..., w_n)^T$ of the fuzzy comparison matrix $\tilde{A}$ as:

$$w_i = \frac{v(\tilde{s}_i \geq \tilde{s}_j,\; j=1,...,n; j \neq i)}{\sum_{k=1}^{n} v(\tilde{s}_i \geq \tilde{s}_j,\; j=1,...,n; j \neq k)} \quad i=1,...,n \tag{12}$$

Jie, Meng and Cheong's extent analysis: (Jie, Meng and Cheong, 2006)

The fuzzy extent analysis is applied on the above fuzzy PCM to obtain the fuzzy performance matrix. The purpose of fuzzy extent analysis is to obtain the criteria importance and alternative performance by solving these fuzzified reciprocal PCMs.

$$\tilde{w} = (w_1 \; w_2 \qquad w_n)$$

$$\tilde{w}_i = (w_{il} \;\; w_{im} \;\; w_{iu}) \;\; i=1,....,n \tag{13}$$

$$w_{il} = \sum_{j=1}^{n} a_{ijl} \Big/ \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ijl} \;\; i=1,....,n$$

$$\tilde{x}_i = (x_{il} \;\; x_{im} \;\; x_{iu}) \;\; i=1,....,n$$

Step 3: α-cut based method

In this method fuzzy extent analysis is applies to get the fuzzy weights or performance matrix for both alternatives under each criteria context and criteria. After that, a fuzzy weighted sum performance matrix (p) for alternatives can thus be obtained by multiplying the fuzzy weight vector related to criteria with the decision matrix for alternatives under each criteria and summing up obtained vectors $\tilde{p} = \tilde{x}_i * \tilde{w}^T$.

$$\tilde{p} = \begin{bmatrix} (l_1 & m_1 & u_1 \\ (l_2 & m_2 & u_2 \\ & & \\ (l_n & m_n & u_n \end{bmatrix} \tag{14}$$

Where n is the number of alternative.

According to Wang (1997), in order to checking and comparing fuzzy number, α-cut based method is need for checking and comparing fuzzy number. The α-cut based method 1 stated that if let A and B be fuzzy numbers with α-cut, $A_\alpha = [a_\alpha^-, a_\alpha^+]$ *and* $[b_\alpha^-, b_\alpha^+]$. It say A is smaller than B depend by $A \le B$, if $a_\alpha^- < b_\alpha^-$ and $a_\alpha^+ < b_\alpha^+$. for all $\alpha \in (0,1]$. The advantage of this method is conclusion is less controversial. The α cut analysis is applied to transform the total weighted performance matrices into interval performance matrices which is showed with α Left and α Right for each alternatives as follows:

$$\tilde{p}_\alpha = \begin{bmatrix} (\alpha Left_1 \;\; \alpha Right_1) \\ (\alpha Left_2 \;\; \alpha Right_2) \\ \\ (\alpha Left_n \;\; \alpha Right_n) \end{bmatrix} \tag{15}$$

$$\alpha Left = [\alpha * (m - l)] + l$$

$$\alpha Right = u - [\alpha * (u - m)]$$

Step 4: λ Function and Crisp values Normalization

It is done by applying the Lambda function which represents the attribute of the decision maker that is maybe optimistic, moderate or pessimistic. Decision maker with optimistic attribute will take the medium lambda and the pessimistic person will take the minimum lambda in the range of [0, 1] as follows:

$$c_\lambda = \begin{bmatrix} c_{\lambda 1} \\ c_{\lambda 2} \\ \\ c_{\lambda n} \end{bmatrix} \tag{16}$$

$$c_\lambda = \lambda * \alpha Right + [(1 - \lambda) * \alpha Left]$$

Where c_λ is crisp value

Finally, the crisp values need to be normalized, because the elements of different scales.

$$c_{\lambda i} = \frac{c_{\lambda i}}{\Sigma c_{\lambda i}} \tag{17}$$

3. From AHP to ANP

The AHP is comprehensive framework that is designed to cope with the intuitive, the rational, and the irrational when we make multi-objective, multi-criterion, and multi-actor decisions with and without certainty of any number of alternatives. The basic assumption of AHP is the condition of functional independence of the upper part, or cluster (see Figure 4), of the hierarchy, from all its lower parts, and form the criteria or items in each level (Lee & Kim, 2000). In Figure 4, a network can be organized to include source clusters, intermediate clusters and sink clusters. Relationship in network are represented by arcs, where the

directions of arcs signify directional dependence (Chang et al., 2006 and Sarkis, 2002). Inner dependencies among the elements of a cluster are represented by looped arcs (Sarkis, 2002).
In ANP the hierarchical relation between criteria and alternatives are generalized to networks. Many decision problems cannot be structured hierarchically, because they involve the interaction and dependence of high-level elements on lower-level elements. Not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives themselves determines the importance of the criteria. Thus, in ANP the decision alternatives can depend on criteria and each other as well as criteria can depend on alternatives and other criteria (Saaty, 2001). Technically, in ANP, the system structure is presented graphically and by matrix notations. The graphic presentation describes the network of influences among the elements and clusters by nodes and arcs. The results of pair wise comparisons (weights in priority vectors) are stored to matrices and further to a supper matrix consisting of the lower level matrices. In ANP interdependence can occur in several ways: (1) uncorrelated elements are connected, (2) uncorrelated levels are connected and (3) dependence of two levels is two-way i.e. bi-directional). By incorporating interdependence, Meade and Sarkis (1999) suggest to develop "super-matrix". The super-matrix adjusts the relative importance weights in individual matrices to form a new overall matrix with the eigenvectors of the adjusted relative importance weights.

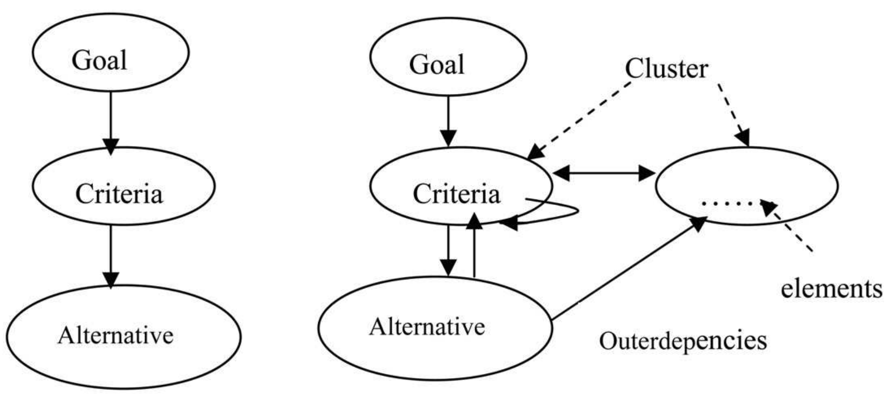


Fig. 4. Hierarchy and network (a) Hierarchy (b) network

3.1 Proposed ANP algorithm

Step 1. model construction and problem structuring: The problem should be stated and be decomposed into a rational system, like a network. The network structure can be obtained by decision-makers through brainstorming or other appropriate methods. An example of the format of network is shown in Figure 4.

Step 2. Pair-wise comparison matrices and priority vectors: In ANP, like AHP, decision elements at each component are compared pair-wise which respect to their importance towards their control criteria. The components (clusters) themselves are also compared pair-wise with respect to their contribution to the goal. Decision makers are asked to respond to a series of pair-wise comparisons where two elements or two components at a time will be compared in terms of how they contribute to their particular level criterion (Meade & Sarkis, 1999). In addition, interdependencies among elements of cluster must also be examined pair-wise; the influence of each element on other elements can be represented by an eigenvector. The relative importance values are determined with Saaty's 1-9 scale (Table 3), where a score of 1 represents equal importance between the two elements and a score 9 indicates the extreme importance of one element (row component in the matrix) compared to the other on (column component in the matrix) (Meade and Sarkis, 1999). A reciprocal value is assigned to the inverse comparison, that is $a_{ij} = 1/a_{ij}$, where $a_{ij}(a_{ji})$ denotes the importance of the *i*th (*j*th) element. Like with AHP, pairwise comparison in ANP is performed in the framework of a matrix, and a local priority vector can be derived as an estimate of the relative importance associated with the elements (or clusters) being compared by solving the following equation:

$$A \times W = \lambda_{\max} \times W \tag{18}$$

Where the matrix of pair-wise comparison is A, w is the eigenvector, and $\lambda_{\max}$ is the large eigenvalue of A. Saaty (1980) proposes several algorithms for approximating W. The numerical pair-wise comparison matrices are calculated as per the following equations as, described by Saaty (1980)

$$\tilde{w}_i = \sqrt[n]{\prod_{j=1}^{n} a_{ij}} \tag{19}$$

Where, $\tilde{w}_i$ is the eigenvector of the pair-wise comparison matrix, a_{ij} is the element of the pair-wise comparison matrix.

$$w_i = \tilde{w}_i \Big/ \sum_{i=1}^{n} \tilde{w}_i \tag{20}$$

Equation (20) is to normalize $\tilde{w}_i$

$$\lambda_{\max} = \sum_{i=1}^{n} \frac{(Aw)_i}{nw_i} \tag{21}$$

Where, $\lambda_{\max}$ is the eigenvalue.

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{22}$$

$$CR = \frac{CI}{RI} \tag{23}$$

Where, CR denotes the consistency ratio, CI denotes the consistency index, RI denotes the average random consistency index. The value of RI is denoted by the order n of the matrix referring to Table 4.

CR is used to test the consistency of the pair-wise comparison. If the value of CR is less than 0.1, this indicates the pair-wise comparison matrix achieves satisfactory consistency. In this paper, Expert Choice Software (2000) is used to compute the eigenvectors from the pair-wise comparison matrices and to determine the consistency ratios. Another method is discussed by (Chang et. al., 2006). The following three-step procedure is used to synthesize priorities (Chang et al., 2006).

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the
3	Moderate	objective
5	importance	Experience and judgment slightly favor one
7	Strong importance	over another
	Very strong	Experience and judgment strongly favor one
9	importance	over another
		Activity is strongly favored and its dominance
2,4,6,8	Absolute	is demonstrated in practice
	importance	Importance of one over another affirmed on
		the highest possible order
	Intermediate values	Used to represent compromise between the
		priorities list above
Reciprocal of above non-zero number	If activity i has one of the above non-zero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i	

Table 3. Saaty's 1-9 scale for AHP performance

n	1	2	3	4	5	6	7	8	9	10	11
RI	0	0	0.58	0. 90	1.12	1.24	1.32	1.41	1.45	1.48	1.49

Table 4. Average random consistency index (Saaty, 1980)

1. Sum the value in each column of the pair-wise matrix.
2. Divide each element in a column by the sum of its respective column. The resultant matrix is referred to as the normalized pair-wise comparison matrix.
3. Sum the elements in each row of the normalized pair-wise comparison matrix, and divide the sum by the n elements in the row. These final numbers provide an estimate of the relative priorities of the elements being compared with respect to its upper level criterion.

Step 3. Super-matrix formation: The super-matrix concept is similar to the Markov chain process (Saaty, 1996). To obtain global priorities in a system with interdependent influence, the local priority vectors are entered in the appropriate columns of matrix. As a result, a super-matrix is actually a partitioned matrix, where each matrix segment represents a relationship between two clusters in a system. Let the clusters of a decision system be $c_k, k = 1, 2, ..., n$, and each cluster k has m_k elements, denoted by $e_{k1}, e_{k2}, ...e_{kmk}$. The local priority vectors obtained in step 2 are grouped and placed in the appropriate positions in a super matrix on the flow of influence from one cluster to another, or from a cluster to itself, as in the loop.

Step 4. Selection of the best alternatives: If the super-matrix formed in step 3 covers the whole network, the priority weights of the alternatives can be found in the column of alternatives in the normalized super-matrix. On the other hand, if a super-matrix only comprises of components that are interrelated, additional calculation must be made to obtain the overall priorities of the alternatives. The alternative with the large overall priority should be the one selection.

The outcome of step 3 is the un-weighted supper-matrix. In order to rank the alternative factors, the limit priority of the alternative factors should be derived through the following process. The un-weighted supper-matrix must first be transformed to a matrix where each of columns is a stochastic column (Saaty, 2006). This is known as the weighted supper-matrix. Then, the weighted supper-matrix must be transformed to a limit matrix which contents the limit priorities of the alternative factors. The alternative factors can then be ranked according to their limit priorities.

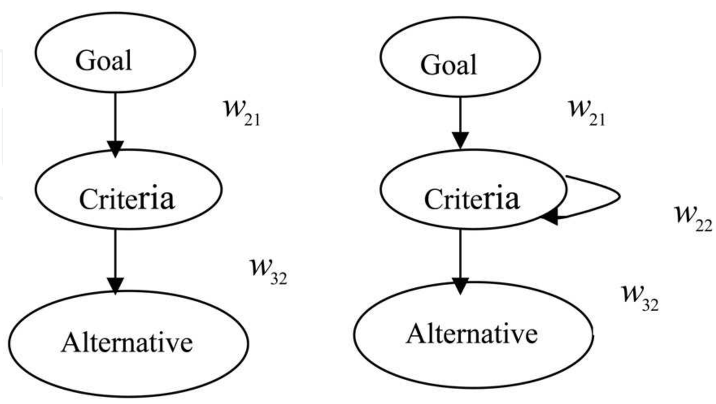


Fig. 5. Hierarchy and network (a) Hierarchy (b) network

$$w = \begin{array}{cc} & \\ c_1 & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1m1} \end{array} \\ c_k & \begin{array}{c} e_{k1} \\ e_{k2} \\ \vdots \\ e_{kmk} \end{array} \\ c_n & \begin{array}{c} e_{n1} \\ e_{n2} \\ \vdots \\ e_{nmn} \end{array} \end{array} \overset{e_{11}\ e_{12}\ \ldots e_{m1} \ldots e_{k1}\ e_{k2} \ldots e_{kmk} \ldots e_{n1}\ e_{n2} \ldots e_{nmn}}{\begin{bmatrix} w_{11} & w_{1k} & w_{1n} \\ w_{k1} & w_{kk} & w_{kn} \\ w_{n1} & w_{nk} & w_{nn} \end{bmatrix}} \tag{24}$$

As an example, the super-matrix representation of a hierarchy with three levels as show in Figure 5(a), is as follows:

$$w_n = \begin{bmatrix} 0 & 0 & 0 \\ w_{21} & 0 & 0 \\ 0 & w_{32} & I \end{bmatrix} \tag{25}$$

In this matrix, w_{21} is a vector which represents the impact of the goal on the criteria, w_{32} is a matrix that represents the impact of the criteria on each of the alternatives, I is the identity matrix, and zero entries correspond to those elements having no influence. For example give above, if the criteria are interrelated, the hierarchy is replaced with the network shown in Figure 5(b). The interdependency is exhibited by the presence of the matrix w_{22} of the supper-matrix w_n (Saaty, 1996).

$$w_n = \begin{bmatrix} 0 & 0 & 0 \\ w_{21} & w_{22} & 0 \\ 0 & w_{32} & I \end{bmatrix} \quad (26)$$

3.2 Proposed fuzzy ANP algorithm

The process of Fuzzy ANP (FANP) comprises four major steps as follows:

Step 1: Establish model and problem

The problem should be stated clearly and decomposed into a rational system like a network. The structure can be obtained by the opinion of decision makers through brainstorming or other appropriate methods.

Step 2: Establish the triangular fuzzy number

A fuzzy set is a class of objectives with a continuum of grades of membership. Such a set is characterized by membership function, which assigns to each object a grade of membership ranging between zero and one. A triangular fuzz number (TNN) is denoted simply as (l, m, u). The parameters l, m and u, respectively, denote the smallest possible value, the most promising value and the large possible value describe a fuzzy event. Let $[A_{ij}^k]_{n\times n}$ be a represents a judgment of expert k for the relative importance of two criteria C_i and C_j

$$[A_{ij}^k]_{n\times n} = \begin{bmatrix} \tilde{a}_{11}^k & \tilde{a}_{12}^k & \dots & \tilde{a}_{1n}^k \\ \tilde{a}_{21}^k & \tilde{a}_{22}^k & \dots & \tilde{a}_{2n} \\ & & & \\ \tilde{a}_{n1}^k & \tilde{a}_{n2}^k & \dots & \tilde{a}_{nn}^k \end{bmatrix}, \text{ k=1, 2, …, m} \quad (27)$$

The triangular fuzzy numbers $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$ and $l_{ij}, m_{ij}, u_{ij} \in [1/9, 9]$ are established as follows:

$$l_{ij} = \min_k(\tilde{a}_{ij}^k),\ m_{ij} = \sqrt[m]{\prod_{k=1}^{m} \tilde{a}_{ij}^k},\ u_{ij} = \max_k(\tilde{a}_{ij}^k) \quad (28)$$

Step 3: Establish the fuzzy Pair-wise Comparison Matrix

From Equation ( ), we have

$$\tilde{A} = (\tilde{a}_{ij})_{n\times n} = \begin{bmatrix} (1 1 1) & (l_{12}\ m_{12}\ u_{12}) \dots & (l_{1n}\ m_{1n}\ u_{1n}) \\ (l_{21}\ m_{21}\ u_{21}) & (1 1 1) \quad \dots & (l_{2n}\ m_{2n}\ u_{2n}) \\ & & \\ (l_{n1}\ m_{n1}\ u_{n1}) & (l_{n2}\ m_{n2}\ u_{n2}) \dots & (1 1 1) \end{bmatrix} \quad (29)$$

Step 4: α -cut based method and

According to Lious and Wang (1992) and Wang (1997) in order to checking and comparing fuzzy number, α-cut based method is need for checking and comparing fuzzy number. The α can be viewed as a stable or fluctuating condition. The range of uncertainty is the greatest when α = 0. The decision making environment stabilizes when increasing α while, simultaneously, the variance for decision making decreases. Additionally, α can be any number between 0 and 1, an analysis is normally set as the following ten numbers, 0.1, 0.2,…, 1 for uncertainty emulation.

Besides, when α = 0 represents the upper-bound u_{ij} and lower-bound l_{ij} of triangular fuzzy numbers, and while, α =1 represents the geometric meanv m_{ij} .

$$\text{The } \alpha\text{-cut of } \tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij}) \text{ is } [L_\alpha(l_{ij}), R_\alpha(u_{ij})] \quad (30)$$

Where $L_\alpha(l_{ij}) = \alpha(m_{ij} - l_{ij}) + l_{ij}$, $R_\alpha(u_{ij}) = u_{ij} - \alpha(u_{ij} - m_{ij})$

$L_\alpha(l_{ij})$ represents the left-end value of α -cut for $\tilde{a}_{ij}$, $R_\alpha(u_{ij})$ represents the right-end value of α -cut for $\tilde{a}_{ij}$

Step 5: λ Function and Crisp Pair-wise Comparison Matrix

Various defuzzication methods are available, and the method adopted herein was derived form Liou and Wang (1992), the method can be clearly express fuzzy perception.

$$g_{\lambda,\alpha}(\tilde{a}_{ij}) = \lambda \times L_\alpha(l_{ij}) + (1-\lambda) R_\alpha(u_{ij}), \quad 0 \le \alpha \le 1, 0 \le \lambda \le 1$$

$$g_{\lambda,\alpha}(\tilde{a}_{ij}) = 1/\ g_{\lambda,\alpha}(\tilde{a}_{ji}),\ 0 \le \alpha \le 1, 0 \le \lambda \le 1 \quad (31)$$

λ can be viewed as the degree of decision maker's pessimism. When λ =0, the decision maker is more optimistic and, thus, the expert consensus is upper-bound u_{ij} of the triangular fuzzy number. When λ =1, the decision maker is pessimistic, and the number ranges froe 0 to 1. However, five numbers 01., 0.3, 0.5, 0.7, and 0.9, are used to emulate the state of mind of decision makers.

The pair-wise comparison matrix is expressed in Equation (32).

$$g_{\lambda,\beta}(\tilde{A}) = g_{\lambda,\alpha}([\tilde{a}_{ij}])_{n \times n} = \begin{bmatrix} 1 & g_{\lambda,\alpha}(\tilde{a}_{12})\ldots & & g_{\lambda,\alpha}(\tilde{a}_{1n}) \\ g_{\lambda,\alpha}(\tilde{a}_{21}) & 1 & \ldots & g_{\lambda,\alpha}(\tilde{a}_{2n}) \\ & & & \\ g_{\lambda,\alpha}(\tilde{a}_{n1}) & g_{\lambda,\alpha}(\tilde{a}_{n2})\ldots & & 1 \end{bmatrix} \quad (32)$$

Step 6: Determine Eigenvector and Suppermarix Formation

Let $\lambda_{\max}$ be the eigenvalue of the pair-wise comparison matrix $g_{\lambda,\beta}(\tilde{A})$.

$$g_{\lambda,\beta}(\tilde{A}) \bullet \mathrm{W} = \lambda_{\max} \bullet \mathrm{W} \quad (33)$$

Wheer W denotes the eigenvector of $g_{\lambda,\beta}(\tilde{A})$, $0 \le \alpha \le 1, 0 \le \lambda \le 1$.

4. An illustrative example

4.1 Selecting key performance indicators based on ANP mode

4.1.1 Proposed ANP for Information manufacturing system

The network model developed in order to find out weights of the factors that are to be used in Information manufacturing system performance indicator is shown in Figure 6.

The following criteria have been identified to select relevant performance indicators useful for decision making.

C1. Relevance: A relevant performance indicator provides information to make a difference in decision by helping user to either form prediction about the outcomes of past, present, and future events or to confirm or correct prior expectations. In accounting standard board

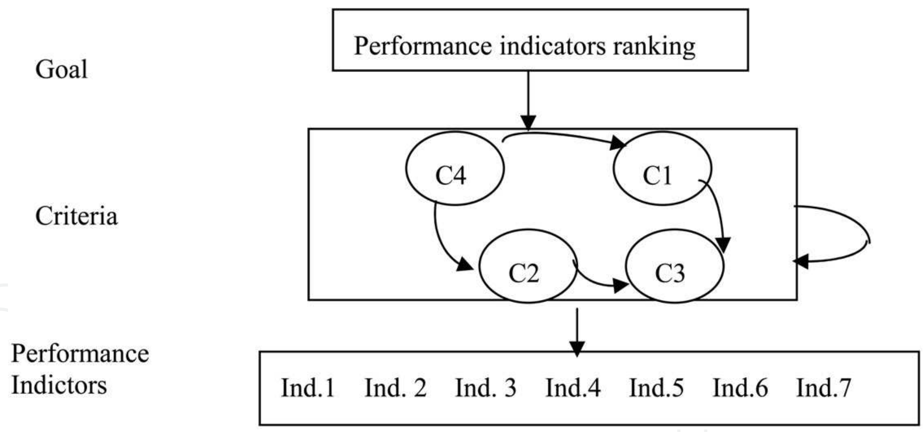


Fig. 6. ANP model for information manufacturing system

(1980), a criteria feature of the relevance has the timeliness, predictive value, and feedback value.
C2: Reliability: Reliability is the ability of a system or component to perform its required functions under stated conditions for a specified period of time. It refers to quality of a performance indicator that assures that it is reasonable free from error and bias and faithfully represents what it purports to represent. In accounting standard board (1980), the reliability of information has verifiability, representational faithfulness, and neutrality.
C3: Comparability and Consistency: Comparability refers to the quality of information related to a performance indicator that enables users to identify similarities and difference between two sets of economic phenomena, while the consistency is the conformity of an indicator from period to period with unchanging policies and procedures. In accounting standard board (1980), Information about a particular enterprise gains greatly in usefulness if it can be compared with similar information about other enterprise and with similar information about the same enterprise for some other period or some other point in time. Comparability between enterprise and consistency in the application of methods over time increase the information value of comparisons of relative economic opportunities or performance.
C4: Understandability and Representational quality: These criteria deals with aspects related to the meaning and format of data collected to build a performance indicator. The performance indicators have to be interpretable as well as easy to understand for user.
The group of performance indicators to be evaluated has been indicated by the top managers of the company.

Ind.1: Actual leather consumptions- Estimated leather consumptions (daily)
Ind.2: Employees' expenses / turnover (monthly)
Ind.3: Number of claims occurred during the process (daily)
Ind.4: Number of supplies' claims (daily)
Ind.5: Number of shifts of the delivery dates of orders / planned orders (daily)
Ind.6: Working minutes for employee / estimated minutes (daily)
Ind.7: Working minutes for department / estimated minutes (daily)

The general sub-matrix notation for Information manufacturing system model used in this study is as follows:

$$w = \begin{matrix} \text{Goal} \\ \text{Criteria} \\ \text{Indictor} \end{matrix} \begin{bmatrix} 0 & 0 & 0 \\ w_1 & w_2 & 0 \\ 0 & w_3 & I \end{bmatrix} \tag{34}$$

Where w_1 is a vector that resents the impact of the goal. w_2 is the matrix that represents the inner dependence of the Information manufacturing system criteria, and w_3 is the matrix that denotes the impact of the criteria on each of the indicators. To apply the ANP to matrix operations in order to determine the overall priorities of the indicator with Information manufacturing system analysis, the proposed algorithm is as follows:

Step 1. Identify Information manufacturing system indicators according to criteria.

Step 2. Assume that there is no dependence among the Information manufacturing system criteria; determine the importance degree of the criteria with 1-9 scale (i.e. calculate w_1).

Step 3. Determine, with 1-9 scale, the inner dependence matrix of each Information manufacturing system criteria with respect to the other criteria (i.e. calculate w_2).

Step 4. Determine the interdependence priorities of the Information manufacturing system criteria (i.e. calculate $w_{criteria} = w_2 \times w_1$).

Step 5. Determine the importance degree of the indicator with respect to each Information manufacturing system criteria with a 1-9 scales (i.e. calculate w_3).

Step 6. Determine the overall priorities of the indicator, reflecting the interrelationships within the manufacturing system criteria (i.e. calculate $w_{indictor} = w_3 \times w_{criteria}$).

4.1.2 Application of the proposed ANP model

Step 1. The problem is converted into a hierarchy structure in order to transform criteria and the indicator into a state in which they can be measured by the ANP technique. The schematic structure established is shown in Figure 6.

Step 2. Assume that there is no dependence among the information manufacturing system criteria; determine the importance degree of the criteria with 1-9 scale is made with respect to the goal. The comparison results are showed in Table 5. All pairwise comparisons in the application are performed by the expert team mentioned in the beginning of this study. In addition, the consistency ration (CR) is provided in the last row of the matrix.

Criteria	C1	C2	C3	C4	Importance degree of information manufacturing system criteria
C1	1	2	3	3	0.447
C2		1	2	3	0.282
C3			1	2	0.163
C4				1	0.105

CR = 0.03

Table 5. Pair-wise comparison of information manufacturing system criteria that there is no dependence along them

$$w_1 = \begin{bmatrix} c1 \\ c2 \\ c3 \\ c4 \end{bmatrix} = \begin{bmatrix} 0.447 \\ 0.282 \\ 0.163 \\ 0.105 \end{bmatrix} \tag{35}$$

Step 3. Inner dependence matrix of each information manufacturing system criteria with respect to the other criteria is determined by analyzing the impact of each criteria on every other criteria using pair-wise comparisons. The dependencies among the information manufacturing system criteria, which are presented schematically in Figure 3, are determined. Based on the inner dependencies presented in Figure 3, pair-wise comparison matrices are formed for the criteria (Table 6)

Criteria	C1	C2	Relative importance weights
C1	1	6	0.857
C2		1	0.142

CR = 0.00

Table 6. The inner dependence matrix of information manufacturing system criteria with respect to C3

Step 4. In this step, the interdependent priorities of the information manufacturing system criteria are calculated as follows:

$$w_{criteria} = w_2 \times w_1 = \begin{bmatrix} 1 & 0 & 0.857 & 0 \\ 0 & 1 & 0.142 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.447 \\ 0.282 \\ 0.163 \\ 0.105 \end{bmatrix} = \begin{bmatrix} 0.310 \\ 0.162 \\ 0.086 \\ 0.442 \end{bmatrix} \quad (36)$$

Step 5. In this step, we calculate the importance degrees of the indicators with respect to each criteria. Using Expert Choice software, the eigenvectors are computed by analyzing the matrices and the w_4 matrix.

$$w_3^T = \begin{bmatrix} 0.1220 & 0.1056 & 0.2401 & 0.2407 & 0.1276 & 0.1208 & 0.1047 \\ 0.2690 & 0.3722 & 0.2881 & 0.3089 & 0.3475 & 0.3474 & 0.3329 \\ 0.5070 & 0.3722 & 0.3885 & 0.3089 & 0.3828 & 0.3768 & 0.4082 \\ 0.1067 & 0.1501 & 0.0832 & 0.1416 & 0.1420 & 0.1549 & 0.1543 \end{bmatrix} \quad (37)$$

Step 6. Finally, the overall priorities of the indicator, reflecting the interrelationships within the criteria, are calculated as follows:

$$w_{indictor} = w_3 \times w_2 = \begin{bmatrix} 0.1721 \\ 0.1714 \\ 0.1914 \\ 0.2138 \\ 0.1915 \\ 0.1945 \\ 0.1896 \end{bmatrix} \quad (38)$$

The main results of the ANP application were the overall priorities of the indicators obtained by the synthesizing the priorities of the indicators from the entire network.

4.1.3 Comparing the AHP and ANP results

According the ANP analysis, indicators are ordered as Ind. 4 -Ind. 5 -Ind. 6- Ind. 7 – Ind. 1 – Ind. 2. The sample example is analyze with the hierarchical model given in Figure 5(a) by assuming is no dependence among the criteria.

The overall priorities computed for the alternative are presented below. The same pair-wise comparison matrices are used to compute the AHP priority values. (see Table 7)

$$w_{indictor(AHP)} = w_3 \times w_2 = \begin{bmatrix} 0.2242 \\ 0.2238 \\ 0.2608 \\ 0.2599 \\ 0.2323 \\ 0.2296 \\ 0.2234 \end{bmatrix} \tag{39}$$

In AHP analysis, indicators are ordered as Ind. 3-ind. 4 -Ind. 5 - Ind. 1 – Ind. 2 – Ind. 7. .

	Ind. 1	Ind. 2	Ind. 3	Ind. 4	Ind.5	Ind. 6	Ind. 7
Weights in AHP Ranking in AHP	0.2242 5	0.2238 6	0.2608 1	0.2599 2	0.2323 3	0.2296 4	0.2234 7
Weights in ANP Ranking in ANP	0.1721 6	0.1714 7	0.1914 4	0.2138 1	0.1953 2	0.1945 3	0.1896 5

Table 7. Weights and ranking of information manufacture systems with AHP and ANP

4.2 Performance evaluation based on ANP model and BSC

4.2.1 Balanced Score Card (BSC)

The BSC is a conceptual framework for translating an organization's vision into a set of performance indicators distributed among four perspectives: Financial, Customer, Internal Business Processes, and Learning and Growth. Indicators are maintained to measure an organization's progress toward achieving its vision; other indicators are maintained to measure the long term drivers of success. Through the BSC, an organization monitors both its current performance (finances, customer satisfaction, and business process results) and its efforts to improve processes, motivate and educate employees, and enhance information systems--its ability to learn and improve. The four perspectives and explained briefly as follows (Kaplan and Norton, 1996)

- Financial perspective: The financial addresses the question of how shareholders view the firm and which financial goals are desired from the shareholder's perspective. The measurement criteria are usually profit, cash flow, ROI, return on invested capital, and economic value added.
- Customer perspective: Customer is the source of business profits; hence, satisfying customer needs is the objective purposed by companies. This perspective provides data regarding the internal business results against measures that lead to financial success and satisfied customers. To meet the organizational objectives and customers expectations, organizations must identify the key business processes at which they must excel. Key processes are monitored to ensure that outcomes are satisfactory.

Internal business processes are the mechanisms through which performance expectations are achieved. Some examples of the core or genetic measures are customer satisfaction, customer retention, new customer acquisition, market position and market share in targeted segment.

- Internal Business process perspective: The objective of this perspective is to satisfy shareholders and customers by excelling at some business process. These are the processes in which the firm must concentrate its efforts to excel. In determining the objectives and measures, the first step should be corporate value-chain analysis. Some examples of the core or genetic measures are innovation, operation and after-sale services.
- Learning and Growth perspective: The objective of this perspective is to provide the infrastructure for achieving the objectives of the other three perspectives and for creating long-term growth and improvement through people, systems and organizational procedures. Some examples of the core or genetic measures are employee satisfaction, continuity, training and skills. The criteria include turnover rate of workers, expenditures on new technologies, expenses on training, and lead time for introducing innovation to a market.

4.2.2 A model of performance evaluation based on ANP and BSC

In order to deal with the performance evaluation problem of enterprise, it is required to employ multiple criteria decision-making methods (MCDM). According to Opricovic and Tzeng (2004), solving MCDM problems is essential to establish evaluation criteria and alternatives, and to apply a normative multi-criteria analysis method in to select a favorable alternative. Since the ANP can be used to select the metrics of the BSC and to help understand the relative importance of metrics. Therefore, the procedures of proposed method are mainly divided the following steps:

Step 1. Define the decision goals

Decision-making is the process of defining the decision goals, gathering relevant information, and selecting the optimal alternative.

Step 2. Establish evaluation clusters

After defining the decision goals, it is required to generate and establish evaluation clusters which is alike a chain of the criteria cluster (purposes), the sub-criteria cluster (evaluators), and the alternatives cluster. Using the theory and methodology of BSC, it creates an adaptive performance evaluation system. According Kanan and Norton (1998), four important factors for evaluating enterprise strategies can be obtained, including: financial perspective (S_1), customer perspective (S_2), Internal Business process perspective (S_3), and Learning and Growth perspective (S_4). In financial perspective, three important factors (sub-critical) are: net asset income ratio (C_{11}), sales net ratio (C_{12}), sales growth ratio (C_{13}). In customer perspective, four important factors (sub-critical) are: customer profitability (C_{21}), market share (C_{22}), customer retention ratio (C_{23}), and customer satisfaction (C_{24}). In Business process perspective, four important factors (sub-critical) are: product improvement (C_{31}), Product Place (C_{32}), product quality (C_{33}), Business process (C_{34}). In Learning and Growth perspective, our important factors (sub-critical) are: employee motivation (C_{41}), Employee Training (C_{42}), Employee satisfaction (C_{43}), Information feedback (C_{44}). As for the alternatives cluster, there are: A_1, A_2, and A_3.

Step 3. Establish network structure

According to step 2, it is assumed that the four selection criteria are independent. Figure 7 illustrates the ANP network component.

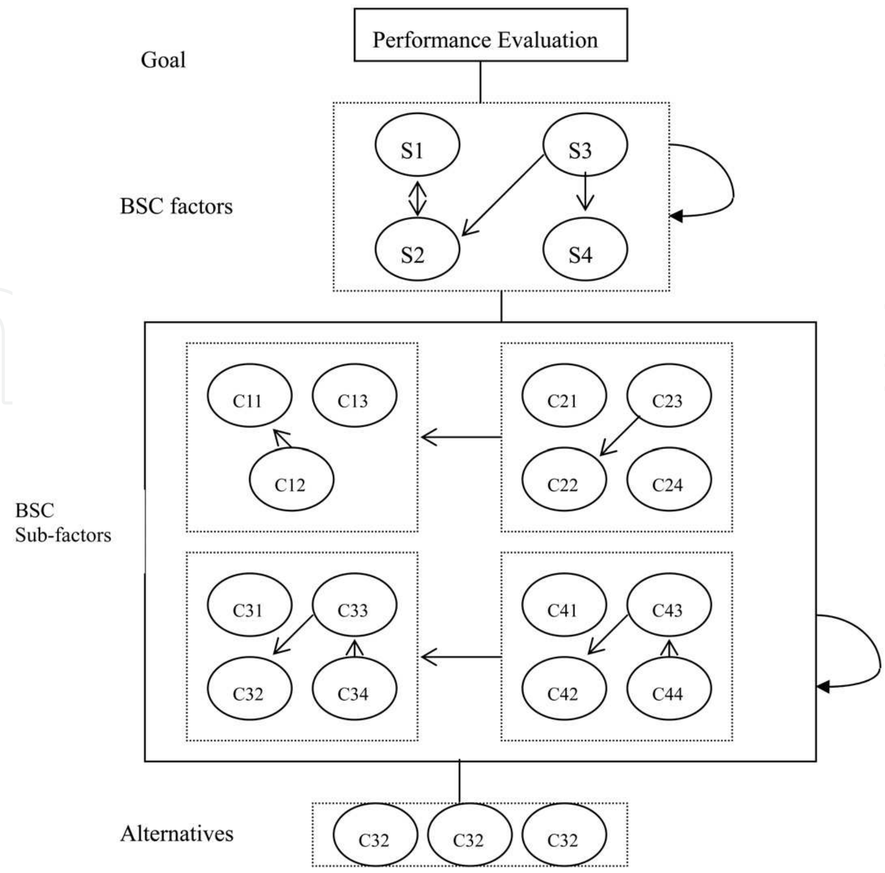


Fig. 7. ANP model for Performance evaluation

Step 4. Pair-wise comparisons matrices and priority vectors

Saaty (1980) proposed several algorithms to approximate W. In this study, Expert Choice (2000) is used to compute the eigenvectors from the pair-wise comparison matrices and to determine the consistency rations.

Step 5. Super-matrix formulation

The super-matrix will be an un-weighted one. In each column, it consists of several eigenvectors which of them sums to one and hence entire column of matrix may sums to an integer greater than one.

In this study, the super-matrix structure is shown in Equation (40). The network model according to the determined criteria is given in Figure 1 W_1 is the local importance degrees of the BSC factors; W_2 is the inner independence matrix of each BSC factor with respect to the other factors by using the schematic representation of the inner dependence among the BSC factors; W_3 is the local importance degrees of the BSC sub-factors; W_4 is the inner independence matrix of each BSC sub-factors with respect to the other sub-factors by using the schematic representation of the inner dependence among the BSC sub-factors; W_5 is the local importance degrees of the alternative strategies with respect to each BSC sub-factors. Also the clusters, which have no interaction, are shown in the supper-matrix with zero (0).

$$\mathbf{W}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ W_1 & W_2 & 0 & 0 \\ 0 & W_3 & W_4 & 0 \\ 0 & 0 & W_5 & I \end{bmatrix} \quad (40)$$

Step 5-1. Calculate W_1

Assuming that there is no dependence among the BSC factors, pair-wise comparison of BSC factor using as 1-9 scale is made with respect to the goal.

$$W_1 = [\ 0.447\ \ 0.282\ \ 0.1\ 0.105]^T \tag{41}$$

Step 5-2. Calculate W_2

The inner independence matrix of each BSC factor with respect to the other factors is the schematic representation of the inner dependence among the BSC factors. The inner dependence matrix of the BSC factors with respect to S_1, S_2, S_3, S_4.

The inner dependence matrix of the BSC factors (W_2) is found.

$$W_2=\begin{bmatrix} 1.000 & 0.625 & 0.900 & 0.857 \\ 0.068 & 1.000 & 0.000 & 0.142 \\ 0.681 & 0.238 & 1.000 & 0.000 \\ 0.249 & 0.126 & 0.100 & 1.000 \end{bmatrix} \tag{42}$$

Step 5-3. Calculate W_3

In this step, local priorities of the BSC sub-factors are calculated using the pair-wise comparison matrix. A priority vector obtained by analyzing the pair-wise comparison is shown below.

Step 5-4. Calculate W_4

The inner independence matrix of each BSC sub-factor with respect to the other sub-factors is the schematic representation of the inner dependence among the BSC sub-factors.

$$W_3 = \begin{matrix} C_{11} \\ C_{12} \\ C_{13} \\ C_{21} \\ C_{22} \\ C_{23} \\ C_{24} \\ C_{31} \\ C_{32} \\ C_{33} \\ C_{34} \\ C_{41} \\ C_{42} \\ C_{43} \\ C_{44} \end{matrix} \begin{bmatrix} 0.618 & 0.000 & 0.000 & 0.000 \\ 0.242 & 0.000 & 0.000 & 0.000 \\ 0.130 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.410 & 0.000 & 0.000 \\ 0.000 & 0.311 & 0.000 & 0.000 \\ 0.000 & 0.098 & 0.000 & 0.000 \\ 0.000 & 0.180 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.513 & 0.000 \\ 0.000 & 0.000 & 0.210 & 0.000 \\ 0.000 & 0.000 & 0.112 & 0.000 \\ 0.000 & 0.000 & 0.164 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.573 \\ 0.000 & 0.000 & 0.000 & 0.111 \\ 0.000 & 0.000 & 0.000 & 0.242 \\ 0.000 & 0.000 & 0.000 & 0.073 \end{bmatrix} \tag{43}$$

$$W_4=\begin{bmatrix} W_{41} & 0 & 0 & 0 \\ 0 & W_{42} & 0 & 0 \\ 0 & 0 & W_{43} & 0 \\ 0 & 0 & 0 & W_{44} \end{bmatrix} \tag{44}$$

Where

$$W_{41}=\begin{bmatrix}0.000 & 0.000 & 0.000\\ 0.333 & 0.000 & 0.000\\ 0.667 & 0.000 & 0.000\end{bmatrix}\quad W_{42}=\begin{bmatrix}1.00 & 0.335 & 0 & 0\\ 0.00 & 0.349 & 0 & 0\\ 0.00 & 0.085 & 0 & 0\\ 0.00 & 0.418 & 0 & 0\end{bmatrix}$$

$$W_{43}=\begin{bmatrix}0.228 & 0.331 & 0.000 & 0.206\\ 0.000 & 0.334 & 0.228 & 0.407\\ 0.354 & 0.101 & 0.354 & 0.096\\ 0.418 & 0.234 & 0.234 & 0.291\end{bmatrix}\quad W_{44}=\begin{bmatrix}0.564 & 0.083 & 0.256 & 0.413\\ 0.133 & 0.338 & 0.257 & 0.091\\ 0.242 & 0.535 & 0.117 & 0.259\\ 0.060 & 0.043 & 0.376 & 0.239\end{bmatrix}$$

Step 5-5. Calculate W_5

In this step, local priorities of the four alternatives with respect to each sub--factors are calculated using the pair-wise comparison matrix. A priority vector obtained by analyzing the pair-wise comparison is shown below.

$$W_5=[\,W_{51}\;\; W_{52}\;\; W_{53}\;\; W_{54}] \tag{45}$$

$$W_{51}=\begin{bmatrix}0.07 & 0.47 & 0.81\\ 0.65 & 0.08 & 0.07\\ 0.28 & 0.45 & 0.12\end{bmatrix}\quad W_{52}=\begin{bmatrix}0.18 & 0.20 & 0.75 & 0.18\\ 0.59 & 0.40 & 0.06 & 0.59\\ 0.23 & 0.40 & 0.19 & 0.23\end{bmatrix}$$

$$W_{53}=\begin{bmatrix}0.80 & 0.69 & 0.77 & 0.73\\ 0.12 & 0.22 & 0.07 & 0.08\\ 0.08 & 0.09 & 0.16 & 0.19\end{bmatrix}\quad W_{54}=\begin{bmatrix}0.40 & 0.12 & 0.75 & 0.69\\ 0.20 & 0.42 & 0.18 & 0.09\\ 0.40 & 0.46 & 0.07 & 0.22\end{bmatrix}$$

Step 6. Limit matrix

After entering the sub-matrices into the super-matrix and completing the column stochastic, the super-matrix is often raised to sufficient large power until convergence occur (Satty, 1996; Meade & Sarkis, 1998). The priority of alternatives, A_1 = 0.478, A_2 = 0.280, A_3 = 0.244.

5. Reference

[1] Anand, M. D., Kumanan, T.S.S, Johnny, M. A., (2008), Application of multi-criteria decision making for selection of robotic system using fuzzy analytic hierarchy process, International Journal of Management and Decision Making, Vol. 9, No. 1, pp. 75-98.

[2] Ballou, D., Wang, R., Pazer, H., and Tayi, G. K., (1998), Modeling information manufacturing systems to determine information product quality, Management Science, Vol. 44. No. 4, pp. 462-484.

[3] Boroushaki, S. and Malczewski, J., (2008), Implementing an extension of the analytical hierarchy process using ordered weighted averaging operators with fuzzy quantifiers in ArcGIS, Computer and Geosciences, Vol. 34, pp. 399-410.

[4] Brans, j. p., Vincke, Ph. And Mareschal, B., (1986), How to select and how to rank projects: the PROMETHEE method, European Journal of Operational Research, Vol. 24, pp. 228-238.

[5] Chang, D. Y., (1996), Applications of the extent analysis method on fuzzy AHP, European Journal of Operation Research, Vol. 95, pp. 649-655.

[6] Chang, S. L., Wang, R. C. and Wang, S. Y. (2006). Applying fuzzy linguistic quantifier to select supply chain partners at different phases of product life cycle, International Journal of Production Economics, 100(2), pp. 348-359.
[7] Chen, Y. W.,(2001), Formulation of a Learning Analytical Network Process, Proceeding of the Sixth International Symposium on The AHP, ISAHP 2001,(Bem-Switzerland), pp. 73-78.
[8] Cheng, W. L. and Li, H., (2004), Contractor selection using the analytic network process, Construction Management and Economics, December, 22, pp. 1021-1032.
[9] Chung, S. F., Lee, A. H. L., and Pearn, W. L.., (2005), Analytical network process (ANP) approach for mix planning in semiconductor fabricator, International Journal of Production Economics, 96, pp. 15-36.
[10] Erensal, Y. C., Oncan, T. and Demircan, M. L., (2006), Determining key capabilities in technology management using fuzzy analytic hierarchy process: a case of Turkey, Information Science, Vol. 176, pp. 2755-2770.
[11] Expert Choice, Expert Choice, Analytical Hierarchy Process (AHP) Software, Version 9.5, Pittsburg, 2000.
[12] Faisal, M. N., Banwet, D. K., (2009), Analysis alternatives for information technology outsourcing decision: an analytic network process approach, International Journal of Business of Information Systems, Vol. 4, No. 1, pp. 47-62.
[13] Fiala, P. and Jablonsky, J., (2001), Performance Analysis of Network Production System by ANP Approach, Proceeding of the Sixth International Symposium on the AHP, ISAHP 2001, (Bem-Switzerland), pp. 101-103.
[14] Hasan, M. A., Shankar, R., Sarkis, J., (2008), Supplier selection in an agile manufacturing environment using data envelopment analysis and analytical network process, International Journal of Logistics Systems and Management, Vol. 4, No. 5, pp. 523-550.
[15] Hwang, S. N., (2007), An application of data envelopment analysis to measure the managerial performance of electronics industry in Taiwan, International Journal of Technology Management, Vol. 40, No, 1/2/3, pp. 215-228.
[16] Jie, L. H., Meng, M. C., and Cheomg, C. W., (2006), Web based fuzzy multi-criteria decision making tool, International Journal of the computer, the Internet and management, Vol. 14, N0. 2, pp. 1-14.
[17] Kaplan, R. S. and Norton, D. P., (1992), The balanced scorecard to work, Harvard Business Review, Vo;. 70, No. 1, pp. 71-79.
[18] Kaplan, R. S. and Norton, D. P., (1996), Using balanced scorecard as a strategic management system,, Harvard Business Review, Vol., 74, No. 1, pp. 75-85.
[19] Lee A. H. I., Chen, W. C. and Chang, C. J., (2008), A fuzzy AHP and BSC approach for evaluating performance of IT department in manufacturing industry in Taiwan, Expert Systems `with Application, Vol. 34, pp. 96-107.
[20] Lee, J. W. and Kim, S. H., (2000), Using analytical network process and goal programming for interdependent information system project selection, Computer and Operations Research, 27, pp. 367-382.
[21] Lee, J. W. and Kim, S. H., (2001), An integrated approach for independent information system projection, International Journal of Project Management, 19, pp. 111-118.
[22] Lee, M. C., (2007), A method of performance Evaluation by Using the Analytic Network Process and Balanced Score Card, Proceedings of the 2007 International Conference

on Convergence Information Technology, 21-23 Nov. IEEE Computer Society pp.235 – 240.

[23] Lious, T. S. and Wang, M. J. J., Ranking Fuzzy Numbers with Integral Value, Fuzzy Sets and Systems, Vol. 50, pp.247-55, 1992.

[24] Meade, L. M. and Sariks, J., (1999), Analyzing organizational project alternatives for agile manufacturing processes: An analytical network approach, International Journal o Production Research, 37, pp. 241-261

[25] Mikhailov, L. and Tsvetinov, P., (2004), Evaluation of services using a fuzzy analytic hierarchy process, Applied Soft Computing, Vol. 5, pp. 23-33.

[26] Niemira, M. P. and Sadty, T. L., (2004), An analytical network process model for financial-crisis forecasting, International Journal of Forecasting, Vol. 20, pp. 578-587.

[27] Nobre, F. F., Trotta, L. T. F., Gomes, L. F. A. M., (1999), Multi-criteria decision making: an approach to setting priorities in health care, Symposium on statistical bases for public health decision making, Vol. 18, No. 23, pp.3345-3354.

[28] Opricovic, S. and Tzeng, G. H.,(2004), Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS, European Journal of Operational Research, 156, pp. 445-455.

[29] Saaty, T. J. and Vargas, L. G., (1980), Decision Making with the analytic Network process: economics, political, social and technological application with benefits, opportunities, costs and risks, Spring Science + Business, USA

[30] Saaty, T. J, (1996), Decision making in Complex Environments, The Analytical Hierarchy Process for decision Making with Dependence and Dependence and Feedback (USA: RWS Publications).

[31] Saaty, T. L. (1999), Fundamentals of analytic network process, Japan, Kobe: The International Symposium on the Analytic Hierarchy Process.

[32] Saaty, T. L., (2001), The Analytic Network Process: Decision Making With Dependence and Feedback, RWS Publications, Pittsburgh.

[33] Sarkis, J., (2002), Quantitative models for performance measurement system-alternate considerations, International Journal of Production Economics, 86, pp. 81-90.

[34] Saaty, R. W., (2003), The analytical hierarchy process (AHP) for decision making and the analytical network process (ANP) for decision making with dependence and feedback, Creative Decisions Foundation 2003.

[35] Saaty, T. L., (2006), Rank from comparisons and from ratings in the analytic hierarchy/network processes, European Journal of Operational Research, Vol. 168, 2006, pp. 557-570.

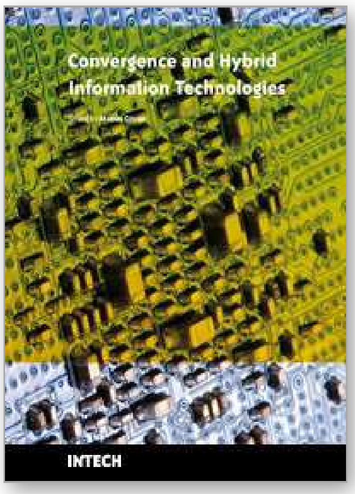
[36] Sardana, G. D., (2009), Evaluating the business performance of an SME: a conceptual framework, International Journal of Globalisation and Small Business, Vol. 3, No. 2, pp. 137-159.

[37] Taylor, B. W., (2004), Introduction to Management Science, Pearson Education Inc., New Jersey.

[38] Wang, L. X., (1997), A course in fuzzy systems and control, United States of America, Prentice-Hall.

[39] Zadeh, L. A. (1965), Fuzzy sets, Information and Control, Vol. 8, No. 3, pp. 338-353.

[40] Zadeh, L. A. (1994), Fuzzy logic, neural networks, and soft computing. Communications of the ACM vol. 37, No. 3, pp. 77-84



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Starting a journey on the new path of converging information technologies is the aim of the present book. Extended on 27 chapters, the book provides the reader with some leading-edge research results regarding algorithms and information models, software frameworks, multimedia, information security, communication networks, and applications. Information technologies are only at the dawn of a massive transformation and adaptation to the complex demands of the new upcoming information society. It is not possible to achieve a thorough view of the field in one book. Nonetheless, the editor hopes that the book can at least offer the first step into the convergence domain of information technologies, and the reader will find it instructive and stimulating.